

FOLIO

TA 7

C6

no. 64-17

cop. 2

SCIENCES

QUALITY OF EFFLUENT FROM DRAINS
IN AN AQUIFER CONTAINING SALINE WATER

by

D. E. L. Maasland

and

M. W. Bittinger

A final report
covering work conducted
under contract with the
Bureau of Reclamation, U. S.
Department of Interior dated
July 1, 1963

Civil Engineering Section
Engineering Research Center
Foothills Campus
Colorado State University
Fort Collins, Colorado
May 1964

LIBRARIES
COLORADO STATE UNIVERSITY
FORT COLLINS, COLORADO 80521

CER64DELM-MWB17

QUALITY OF EFFLUENT FROM DRAINS
IN AN AQUIFER CONTAINING SALINE WATER

by

D. E. L. Maasland

and

M. W. Bittinger

A final report
covering work conducted
under contract with the
Bureau of Reclamation, U. S.
Department of Interior dated
July 1, 1963

Civil Engineering Section
Engineering Research Center
Foothills Campus
Colorado State University
Fort Collins, Colorado
May 1964

CER64DELM-MWB17

LIST OF FIGURES

<u>Figure</u>		<u>Page</u>
1	Position of the saline-fresh water interface at different times (uniform aquifer)	6
2	Saline-fresh water interface position at different times intervals (uniform aquifer)	8
3	Percentage of salt water remaining in the model at any time (uniform aquifer, $Q = 1.05 \text{ cm}^3/\text{sec}$) . .	10
4	Quality of effluent as a function of time (uniform aquifer, $Q = 1.05 \text{ cm}^3/\text{sec}$)	11
5	Relationship between model discharge and model time for different values of quality of effluent (uniform aquifer)	13
6	Quality of drain effluent as a function of time for some specific situations (uniform aquifer)	14
7	Relationship between model discharge and model time for different values of quality of effluent. Two part aquifer. $K_{\text{top}}/K_{\text{bottom}} = 1/11.5$	15
8	Quality of effluent as a function of time for some specific situations. Two part aquifer. $K_{\text{top}}/K_{\text{bottom}} = 1/11.5$	16
9	Relationship between model discharge and model time for different values of quality of effluent. Two part aquifer $K_{\text{top}}/K_{\text{bottom}} = 10.5/1$	18
10	Quality of drain effluent as a function of time for some specific situations. Two part aquifer. $K_{\text{top}}/K_{\text{bottom}} = 10.5/1$	19
11	Relationship between model discharge and model time for different percentages of aquifer cleaning (uniform aquifer)	21
12	Percentage of salt water removed from model as a function of the amount of water applied $Q = 1.00 \text{ cm}^3/\text{sec}$	22

QUALITY OF EFFLUENT FROM DRAINS
IN AN AQUIFER CONTAINING SALINE WATER.

INTRODUCTION

Many aquifers in arid and semi-arid regions contain saline water. When irrigation water is applied to the land overlying these aquifers the water table will begin to rise. To prevent waterlogging and salt accumulation near the surface a drainage system has to be installed.

When a drainage system consisting of nearly horizontal parallel drains is used, the quality of the effluent will be determined by the ratio of fresh and saline water flowing into the drains at any time. Pertinent variables influencing the ratio of fresh to salt water inflow into the drains are (1) the spacing between drains, (2) aquifer characteristics (permeability, layering, effective porosity), (3) recharge rate, and (4) the vertical distance between the drains and an impermeable layer forming the lower boundary of flow in the aquifer.

OBJECTIVE OF STUDY

Under contract with the Bureau of Reclamation, U.S. Department of Interior, dated July 1, 1963, it was the objective of this undertaking to evaluate the applicability of a viscous analogy model (Hele-Shaw) to studies involving changes in quality of drainage effluent under various drainage conditions.

THE MODEL

The Hele-Shaw model used in this experiment consists of two parallel plates of plexiglass. The plates are separated by spacers in order to obtain a uniform interspace. Details of construction, materials, and dimensions of the model are presented in Appendix B.

The theory of the model is based upon the similarity between the laws governing ground water flow through porous media and viscous flow between parallel plates. For ground water flow Darcy's law applies (for list of symbols see Appendix A):

$$g_s = - K_s \frac{\partial \phi}{\partial s} \quad (1)$$

Similarly for viscous flow of a liquid between two parallel plates Poiseuille's Law applies:

$$q_s = - \frac{g}{12} \frac{b^2}{\nu} \frac{\partial \phi}{\partial s} \quad (2)$$

Comparing equations (1) and (2), the permeability of the model may be derived:

$$K_m = \frac{g}{12} \frac{b^2}{\nu} \quad (3)$$

Equation (3) indicates any desired model permeability may be obtained by selecting an appropriate combination of interspace width and liquid kinematic viscosity.

Bear¹ has derived the scales for the Hele-Shaw model. Scales pertinent to the present study are reproduced below. The equations of motion of the free surface of ground water flow and model flow are, respectively:

$$K_{xp} \left(\frac{\partial \phi_p}{\partial x_p} \right)^2 + K_{zp} \left[\left(\frac{\partial \phi_p}{\partial z_p} \right)^2 - \frac{\partial \phi_p}{\partial z_p} \right] = n_p \frac{\partial \phi_p}{\partial t_p} \quad (4)$$

and

$$K_m \left(\frac{\partial \phi_m}{\partial x_m} \right)^2 + K_m \left[\left(\frac{\partial \phi_m}{\partial z_m} \right)^2 - \frac{\partial \phi_m}{\partial z_m} \right] = n_m \frac{\partial \phi_m}{\partial t_m} \quad (5)$$

Denoting the scaling factors for each of the variables in equations (4) and (5) by a subscript r, we have,

$$K_{xr} = \frac{K_m}{K_{xp}}, \quad \phi_r = \frac{\phi_m}{\phi_p}, \quad x_r = \frac{x_m}{x_p}, \quad K_{zr} = \frac{K_m}{K_{zp}},$$

$$z_r = \frac{z_m}{z_p}, \quad n_r = \frac{n_m}{n_p} \quad \text{and} \quad t_r = \frac{t_m}{t_p} \quad (6)$$

Substituting equations (6) into equation (4) gives:

$$\frac{x_r^2}{K_{xr} \phi_r^2} K_m \left(\frac{\partial \phi_m}{\partial x_m} \right)^2 + \frac{z_r^2}{K_{zr} \phi_r^2} K_m \left(\frac{\partial \phi_m}{\partial z_m} \right)^2 - \frac{z_r}{K_{zr} \phi_r} K_m \left(\frac{\partial \phi_m}{\partial z_m} \right) = \frac{t_r}{n_r \phi_r} n_m \frac{\partial \phi_m}{\partial t_m} \quad (7)$$

Comparing equations (5) and (7) it follows directly that in order to have similarity:

$$\frac{x_r^2}{K_{xr} \phi_r^2} = \frac{z_r^2}{K_{zr} \phi_r^2} = \frac{z_r}{K_{zr} \phi_r} = \frac{t_r}{n_r \phi_r} \quad (8)$$

¹Bear, Jacob, 1960. Scales of viscous analogy models for ground water studies: Am. Soc. Civil Engineers Proc. v. HY2, No. 2364, p. 11-23.

Rearranging the first equations shows the effect of distorting the length scale:

$$\frac{x_r}{z_r} = \frac{\sqrt{K_{xr}}}{\sqrt{K_{zr}}} \quad (9)$$

In other words, distortion of the model geometry compared to the prototype (x_r/z_r not equal to 1) will require unequal values of K_{xp} and K_{zp} . Since generally x_r will be taken smaller than z_r , the permeability of the prototype being modeled will be assumed larger in the horizontal direction than in the vertical direction. Fortunately, this is in accordance with usual field conditions.

The time scale may also be determined from equation (8) (keeping in mind that $\phi_r = z_r$):

$$t_r = \frac{n_r x_r^2}{K_{xr} z_r} \quad (10)$$

Substitution of equations (3) and (6) in equation (10) and taking $n_m = 1$, gives:

$$t_m = \frac{12}{g} \frac{K_{xp}}{n_p} \frac{x_r^2}{z_r} \frac{v}{b^2} t_p \quad (11)$$

The discharge scale may be obtained by comparison of Darcy's and Poiseuille's Law:

$$Q_{xp} = -K_{xp} b_p z_p \frac{\partial \phi_p}{\partial x_p} \quad (12)$$

and

$$Q_{xm} = -K_{xm} b_m z_m \frac{\partial \phi_m}{\partial x_m} \quad (13)$$

Thus,

$$Q_{xr} = \frac{K_{xr} b_r z_r^2}{x_r} = \frac{g}{12} \frac{b^3}{v} \frac{1}{K_{xp} b_p} \frac{z_r^2}{x_r} \quad (14)$$

In a part of this study it was desired to model a layered aquifer system with permeability in one-half the aquifer 10 times the other half. The discharge scale for both parts of the aquifer must be equal. Denoting the two layers of different permeability by subscripts 1 and 2 we obtain, using equation (14):

$$\left(\frac{g}{12} \frac{b^3}{v} \frac{1}{K_{xp} b_p} \frac{z_r^2}{x_r} \right)_1 = \left(\frac{g}{12} \frac{b^3}{v} \frac{1}{K_{xp} b_p} \frac{z_r^2}{x_r} \right)_2 \quad (15)$$

Since, ν , g and b_0 are constant and z_r and x_r are fixed throughout the model we may^p rewrite equation (15)^r as:

$$\left(\frac{b^3}{K_{xp}} \right)_1 = \left(\frac{b^3}{K_{xp}} \right)_2 \quad (16)$$

To obtain layer permeability ratios of 1 to 10, b_2 must be approximately $2.16 b_1$ [$(2.16)^3 = 10$]. To satisfy equation (16) and to fulfill the requirement of a constant time scale (equation 11) the ratio b/n_p must be constant. Therefore, changes in interspace width in the model necessitates assumed changes in effective porosity of the prototype.

The density difference between saline and fresh water is an important variable in this study. Bear has shown that the necessary conditions for similitude are:

$$\left(\frac{\nu_m}{\nu_p} \right)_f = \left(\frac{\nu_m}{\nu_p} \right)_s \quad (17)$$

and

$$\left(\frac{\gamma_f - \gamma_s}{\gamma_f} \right)_m = \left(\frac{\gamma_f - \gamma_s}{\gamma_f} \right)_p \quad (18)$$

Since fresh and saline water were used for the experiments reported here no difficulties were experienced in meeting these conditions.

SYSTEMS STUDIED

Three systems were examined in the present study:

1. Parallel drains in an aquifer of uniform permeability.
2. Parallel drains in a two-part aquifer. The permeability of the lower half of the aquifer was approximately ten times the permeability of the upper half.
3. Parallel drains in a two-part aquifer. The permeability of the upper half of the aquifer was approximately ten times the permeability of the lower half.

In all three systems it was assumed the aquifer initially was saturated with water containing 15,000 ppm of sodium chloride over its entire depth, i.e., from the drain down to the impermeable layer. Application of water to the ground surface above the water table was assumed to be continuous. This continuous rate of application was equal in volume to several intermittent applications of water during a year's period. The recharge water was assumed to be free of salt.

The ratio of drain spacing to the height of drain above the aquifer bottom was maintained nearly constant throughout the tests. Therefore, application of these data to different prototype geometry requires an assumption of low vertical permeability compared to horizontal permeability as computed by equation (9).

For systems (2) and (3) described above, the assumption of an effective porosity in one-half the prototype aquifer equal to over two times the other half was necessary in order to maintain a constant b/m_p ratio.

EXPERIMENTAL PROCEDURE

Since theoretically there is no flow across the vertical plane midway between drains, it was sufficient to model only half of the distance between drains. The width of the interspace was determined by adding known volume increments of water and noting the height to which it filled the model.

At the beginning of each run the model was filled to the height of the drain with water containing 15,000 ppm of sodium chloride and a trace of dye (potassium permanganate).

At time zero, application of fresh water was started through 45 calibrated capillary tubes. The capillary tubes were connected to a 96 inch-long lucite tube. Water entered the lucite tube at a constant rate. A few minutes after the start of each experiment a fresh water-salt water interface was established. This interface moved downward during the course of a run until an approximate steady state was obtained.

Photographs were taken of the model at appropriate time intervals in order to obtain a time record of the salt-fresh water interface. (Figure 1) Simultaneously, measurements were taken of the rate of drain discharge.

After the completion of a run the procedure was repeated at a different rate of flow. In total, 12 runs were conducted with the uniform aquifer, 13 runs with the less permeable part on top and 15 runs with the more permeable part on top.

Dimensions and characteristics of the model and liquid are given in Table 1.

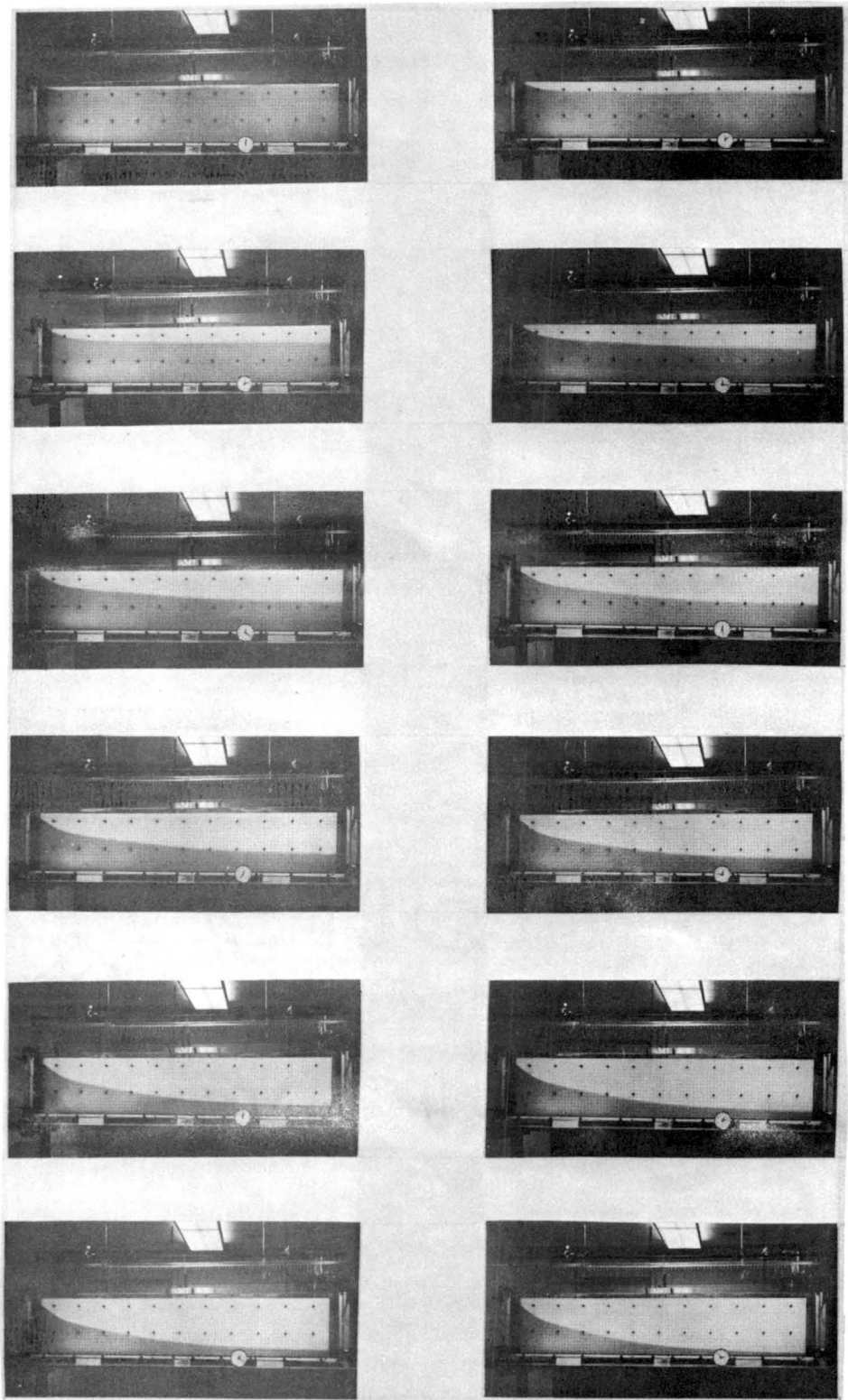


FIGURE 1. POSITION OF THE FRESH-SALINE WATER INTERFACE AT DIFFERENT TIMES (UNIFORM AQUIFER)

TABLE 1

DIMENSIONS AND CHARACTERISTICS OF MODEL AND LIQUID

K = Constant	$K_{\text{top}}/K_{\text{bottom}} = 1/10$	$K_{\text{top}}/K_{\text{bottom}} = 10/1$
	<u>TOP Half</u>	<u>TOP Half</u>
$g = 980 \text{ cm/sec}^2$	$g = 980 \text{ cm/sec}^2$	$g = 980 \text{ cm/sec}^2$
$x_m = 235 \text{ cms}$	$x_m = 236 \text{ cms}$	$x_m = 236 \text{ cms}$
$z_m = 44.5 \text{ cms}$	$z_m = 21.3 \text{ cms}$	$z_m = 20.9 \text{ cms}$
$v_{\text{water}} = 0.0090 \text{ cm}^2/\text{sec}$	$v_{\text{water}} = 0.0090 \text{ cm}^2/\text{sec}$	$v_{\text{water}} = 0.0090 \text{ cm}^2/\text{sec}$
$b_m = 0.086 \text{ cms}$	$b_m = 0.073 \text{ cms}$	$b_m = 0.186 \text{ cms}$
	<u>BOTTOM Half</u>	<u>BOTTOM Half</u>
	$g = 980 \text{ cm/sec}^2$	$g = 980 \text{ cm/sec}^2$
	$x_m = 236 \text{ cms}$	$x_m = 236 \text{ cms}$
	$z_m = 21.3 \text{ cms}$	$z_m = 20.9 \text{ cms}$
	$v_{\text{water}} = 0.0090 \text{ cm}^2/\text{sec}$	$v_{\text{water}} = 0.0090 \text{ cm}^2/\text{sec}$
	$b_m = 0.165 \text{ cms}$	$b_m = 0.085 \text{ cms}$

CHANGE OF QUALITY COMPUTATION

The change in quality of the drain effluent was determined from the change of the salt-fresh water interface with time. The procedure was as follows:

1. For each run a graph was prepared showing the position of the interface at different times. Eleven points were plotted for every interface position and the complete interface was obtained by linear interpolation between adjacent points. A typical chart of step 1 is shown in Figure 2.

2. The water table for each set of runs was assumed to be horizontal at a uniform height for every discharge. Although this assumption is not in accordance with the real situation, the volumetric error incurred is small compared to the volume of the entire aquifer.

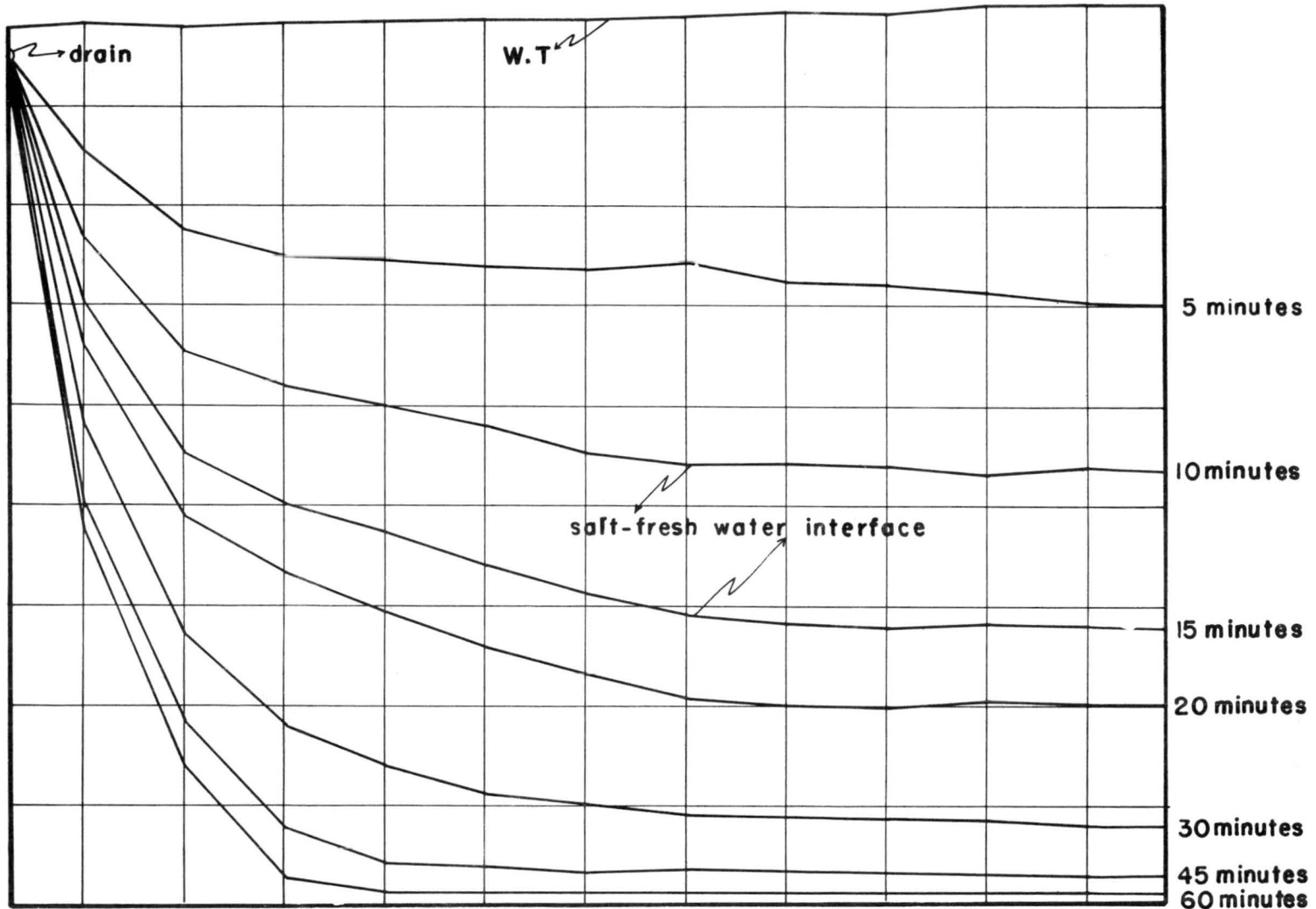


Figure 2. Saline-fresh water interface position at different time intervals (uniform aquifer) $Q = 1.05 \text{ cm}^3/\text{sec}$.

The area enclosed between the water table and each successive interface with time was determined by means of a planimeter. The ratios between each of these areas (or volumes) and the total aquifer area (or volume) were then determined, giving the percentage of the aquifer cleaned at different times. For each discharge the percentage of the aquifer cleaned was plotted against time. A typical graph of this kind is shown in Figure 3.

3. The average quality during any minute interval was determined from the following expression:

$$\text{quality (in ppm)} = \frac{[A] \left[\text{Volume model (cm}^3\text{)} \right] \left[15,000 \text{ ppm} \right]}{[100] \left[\text{Discharge (cm}^3\text{/sec)} \right]}$$

$$A = \Delta \left\{ (\% \text{ cleaned})_{\text{time}(t)\text{min}} - (\% \text{ cleaned})_{\text{time}(t-1)\text{min}} \right\}$$

The mean slope (or quality) for each one minute interval of each graph (Figure 3) was determined. These data were plotted against time. Figure 4 gives an example of this.

4. Finally, from the complete set of graphs as exemplified in Figure 4, the model times were determined for different discharges at which the quality was equal. These data were used to construct a graph of discharge in the model against model time. Thus a set of curves was obtained with quality as parameter. (Figures 5, 7, and 9).

The rate of recharge, $h \times b_p$, must equal the rate of drain discharge, Q_p . Substitution in equation (14) gives:

$$Q_m = \frac{g}{12} \frac{b_m^3}{v} \frac{h}{K_{xp}} \frac{z_m^2}{z_p^2} \frac{x_p^2}{x_m^2} \quad (19)$$

and according to equation (11) we are able to determine the quality of the effluent from drains in the prototype under different conditions of drain spacing, depth of aquifer, effective porosity, permeability, and rate of irrigation application.

RESULTS

(1) Parallel drains in an aquifer of constant permeability.

Figure 4-5 shows the results of the experiments with the uniform aquifer.

The constants $\frac{g}{12} \frac{b^3}{v} \frac{z_m^2}{x_m^2}$ and $\frac{12}{g} \frac{x_m^2}{z_m} \frac{v}{b^2}$ have magnitudes of 48.6 cm³/sec and 0.308 minutes (see equations (19) and (11) and Table 1) respectively.

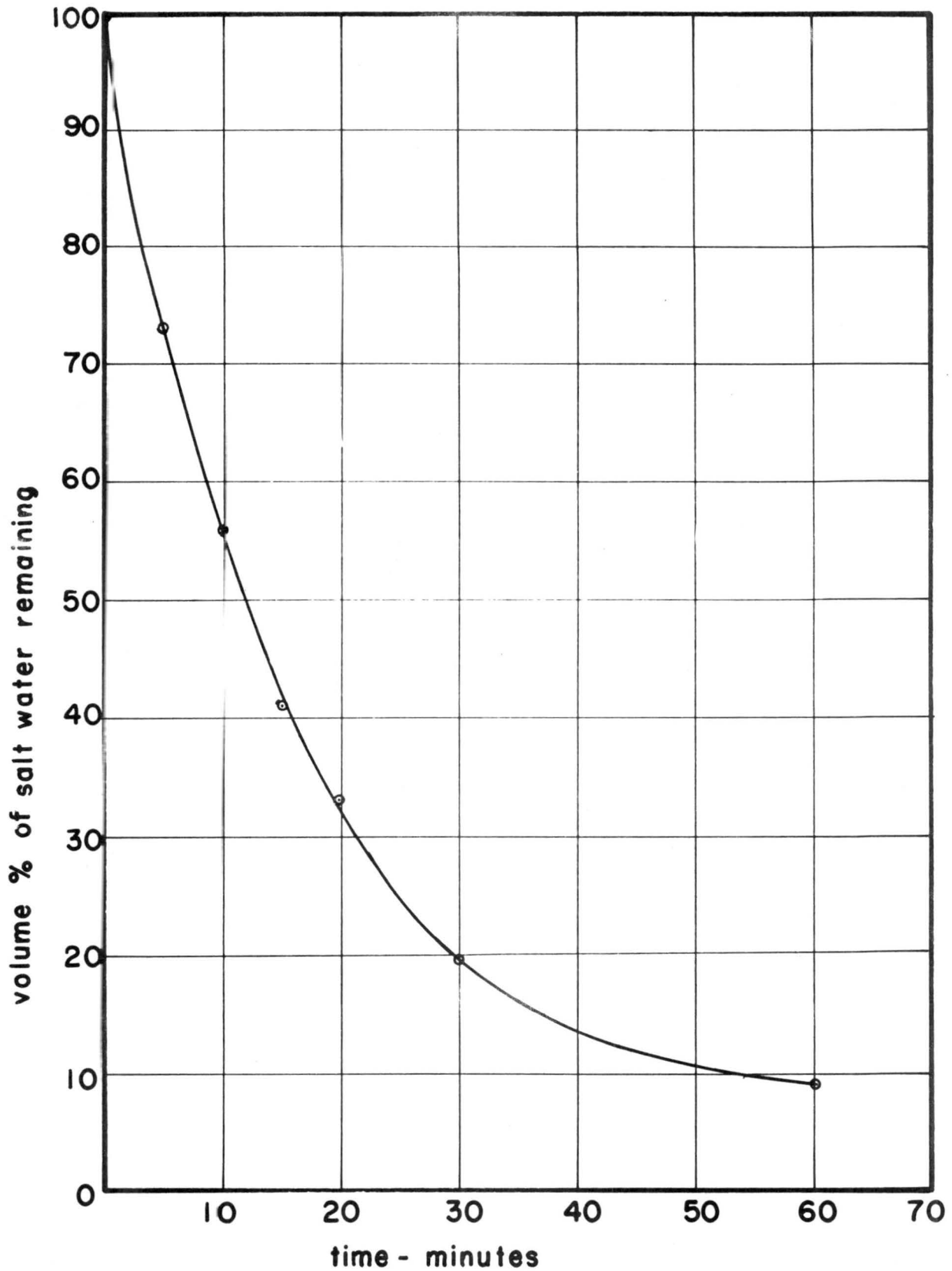


Figure 3. Percentage of salt water remaining in the model at any time (uniform aquifer, $Q = 1.05 \text{ cm}^3/\text{sec}$).

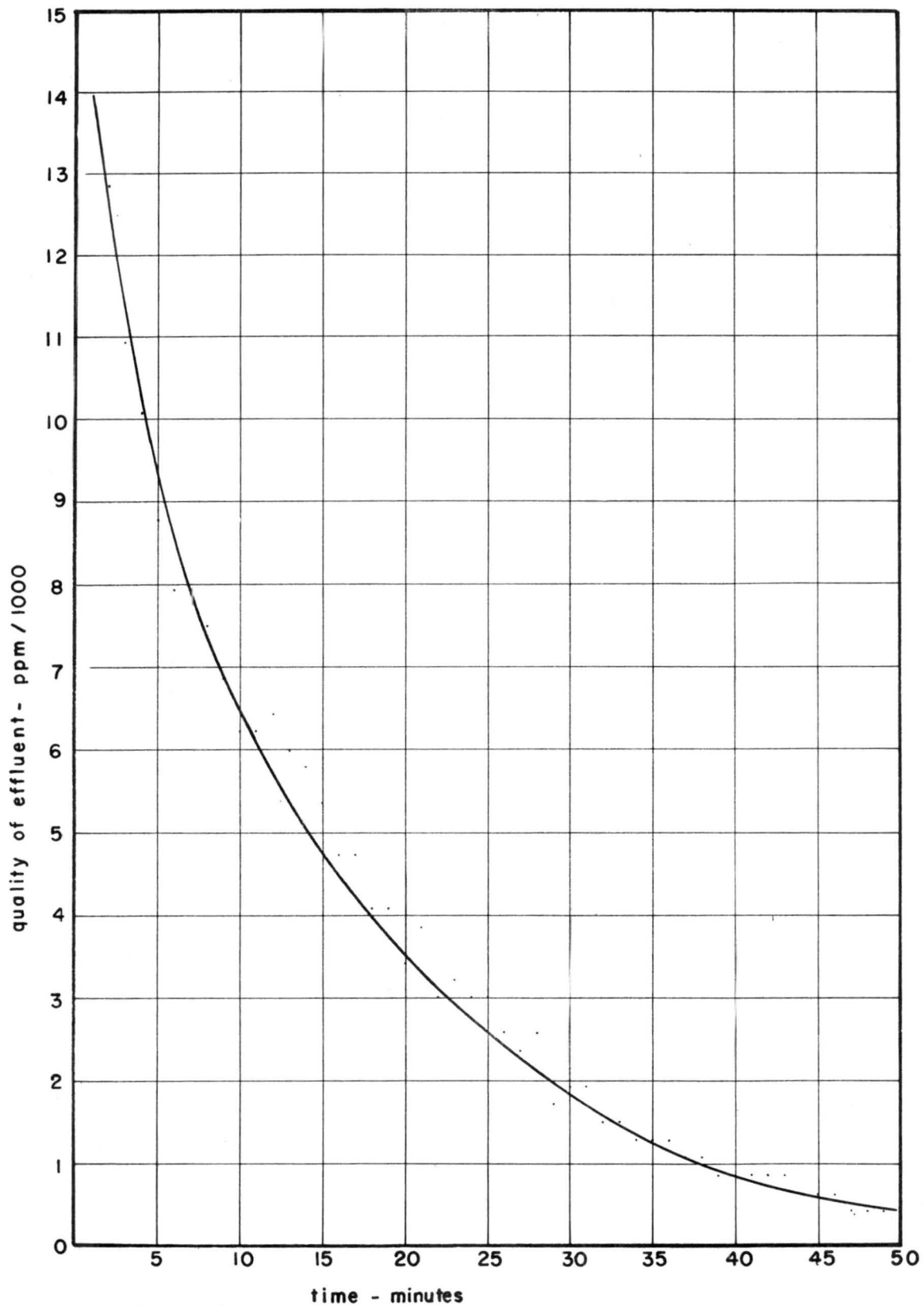


Figure 4. Quality of effluent as a function of time
(uniform aquifer, $Q = 1.05 \text{ cm}^3/\text{sec}$)

The numbers on the abscissa represent values of $0.308 \left(\frac{K_{xp}}{n_p} \frac{z_p}{x_p^2} t_p \right)$ and the numbers on the ordinate are values of $48.6 \left(\frac{h}{K_{xp}} \frac{x_p^2}{z_p^2} \right)$. The parameter on the curves represents the quality of the effluent in ppm. Since the constants 0.308 and 48.6 have dimensions of minutes and cm^3/sec , respectively (the same as t_m and Q_m), any consistent system of units may be used for the remaining variables.

Some specific examples derived from Figure 5 are shown in Figure 6. The curve A in Figure 6 shows the change of quality of effluent with time for the following conditions: distance between drains 1200 ft, depth of aquifer 60 feet, permeability 15,000 ft/year, recharge rate 1 ft/year and effective porosity 0.2. Curve B differs from curve A only in the distance between drains, which has been doubled (2400 ft). The effect of doubling the drain distance is to increase the concentration of effluent, especially during later years. Increasing the fresh water application rate (curve C) results in a marked decrease in salt concentration at any time.

(2) Parallel drains in a two-part aquifer - the permeability of the lower half of the aquifer approximately ten times the permeability of the upper half.

The spacing between plates of the upper half was 0.073 cms, of the lower half 0.165 cms. Using equation (16) the resulting permeability ratio was 1/11.5. Since b/n_p must be constant throughout the model the effective porosity of the lower half of the aquifer must be 2.26 times greater than the upper half.

The results of this two-part system are shown in Figure 7. The values of $\frac{g}{12} \frac{b^3}{v} \frac{z_m^2}{x_m}$ and $\frac{12}{g} \frac{x_m^2}{z_m} \frac{v}{b^2}$

for the upper half of the aquifer are respectively 6.79 cm^3/sec and 0.9 minutes. Although these values will be different for the lower part, the use of proper values of K_{xp} and n_p will permit them to be used for the lower part of the aquifer as well. Some specific examples are shown in Figure 8. Curve A shows the case for a drain spacing of 1200 ft, depth of the upper part (and hence of the lower part) 30 ft, permeability 9000 ft/year, recharge rate 1 ft/year, and effective porosity 0.15. The effect of reducing the drain spacing to 600 ft is again to reduce the salt content of the drainage effluent (curve B). The rate at which irrigation water is applied is an important factor in determining the quality of the effluent ($h = 0.5$ ft/year in curve C). Although the volume of effluent at a low rate of recharge will be small, the quality of the effluent will be considerably poorer at any time.

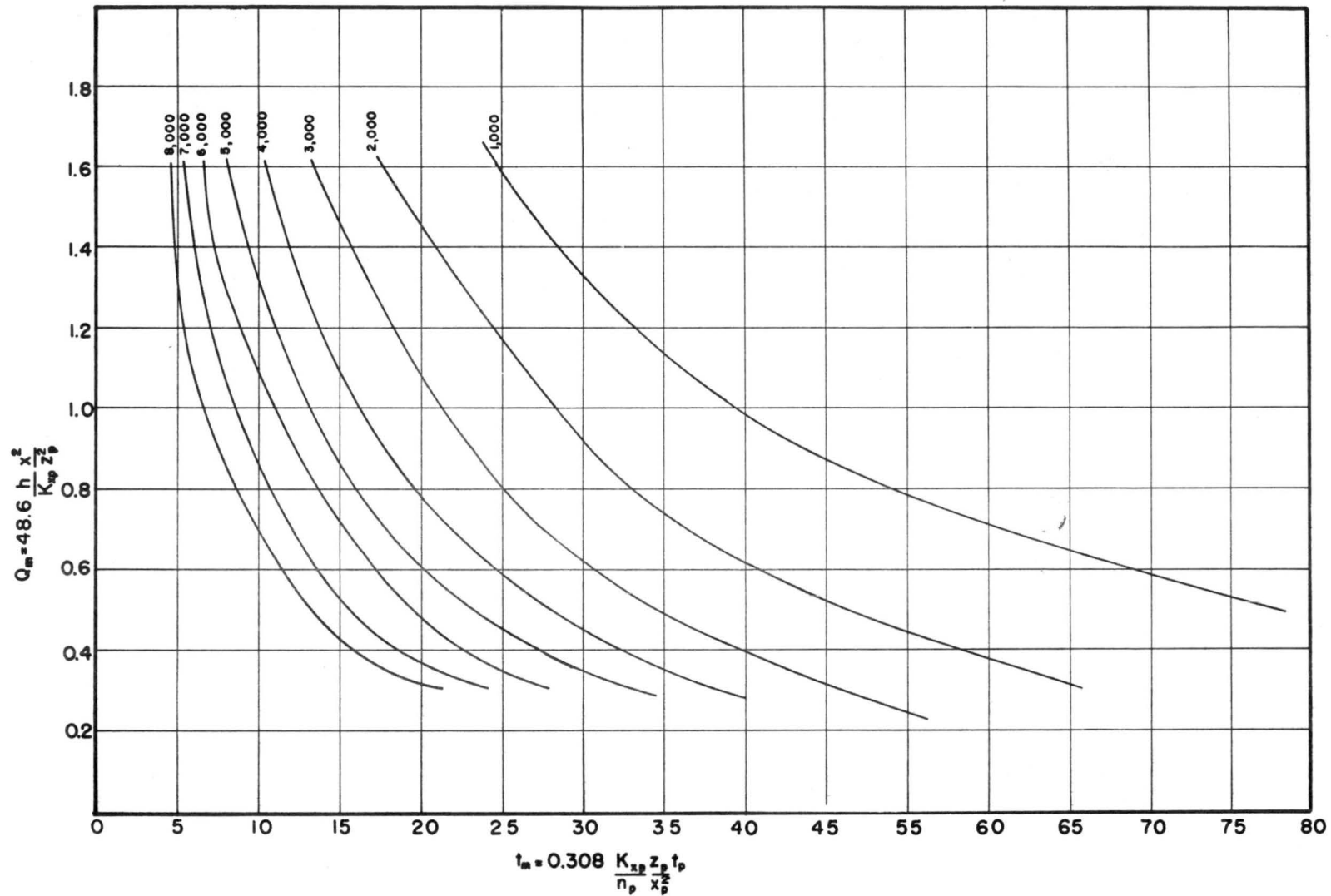


Figure 5. Relationship between model discharge and model time for different values of quality of effluent. Uniform aquifer.

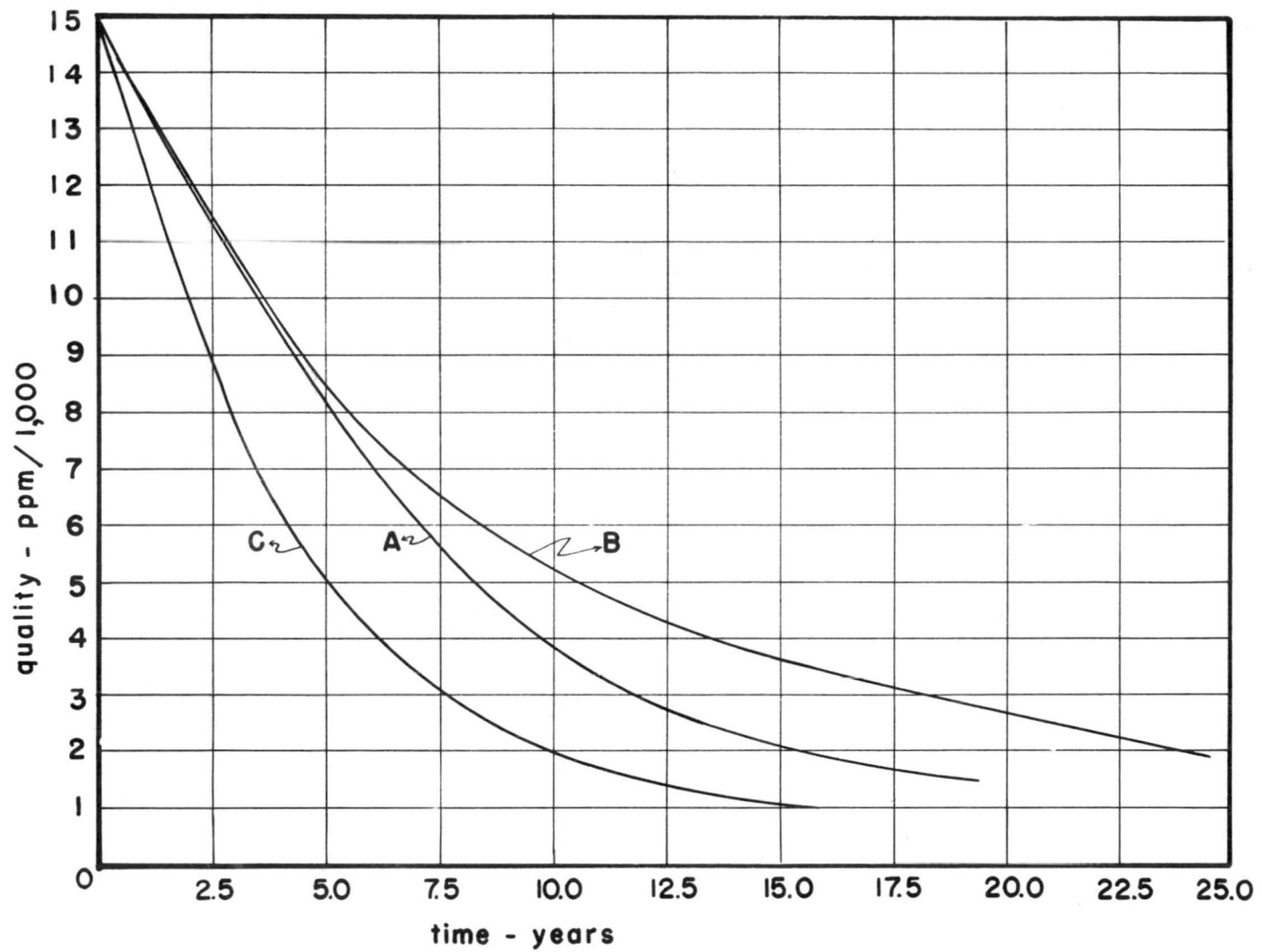


Figure 6. Quality of drain effluent as a function of time for some specific situations (uniform aquifer)

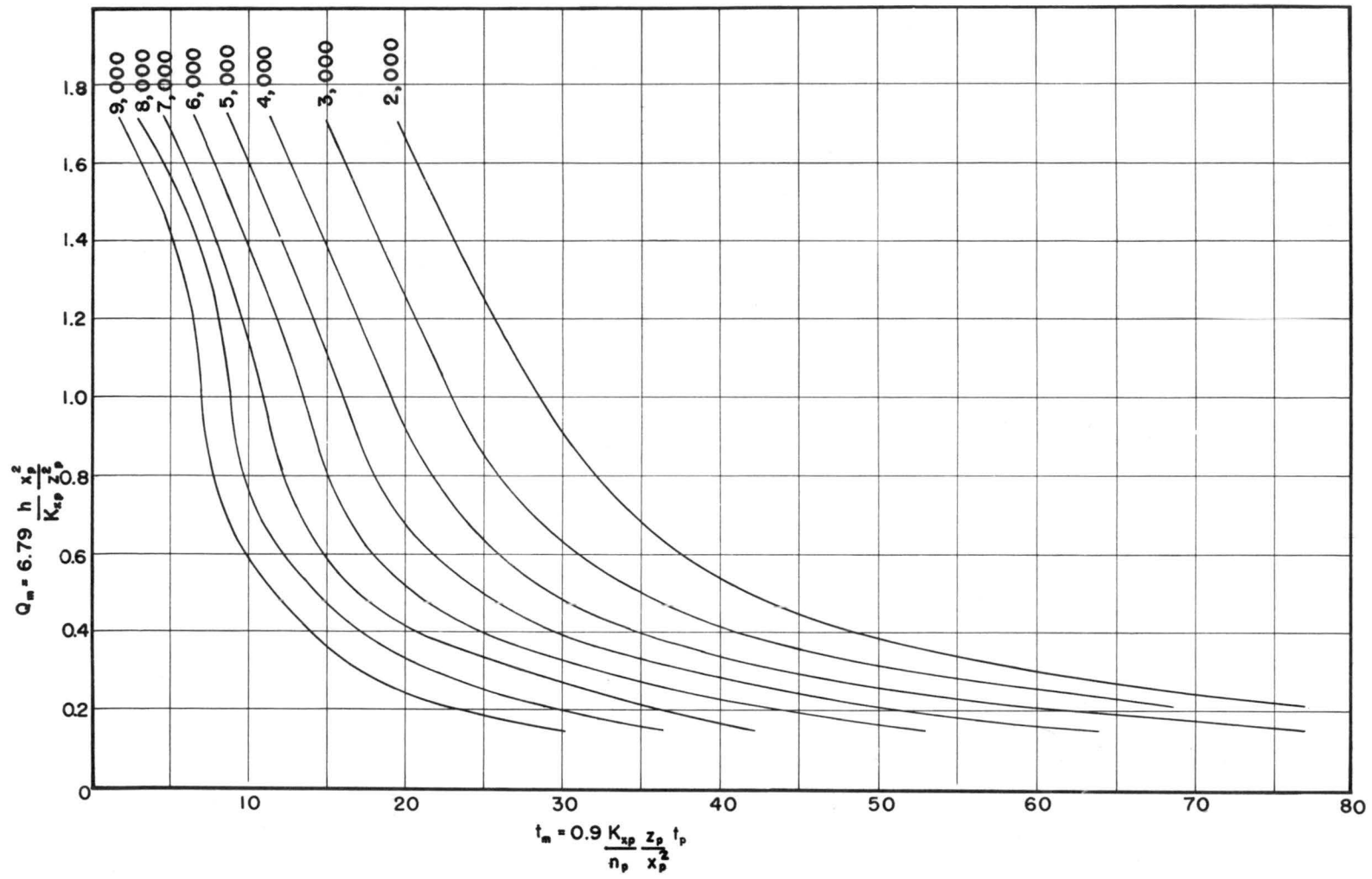


Figure 7. Relationship between model discharge and model time for different values of quality of effluent. Two part aquifer. $K_{top}/K_{bottom} = 1/11.5$. Use values of K_{xp} , n_p , and z_p for top half of aquifer (consistent units).

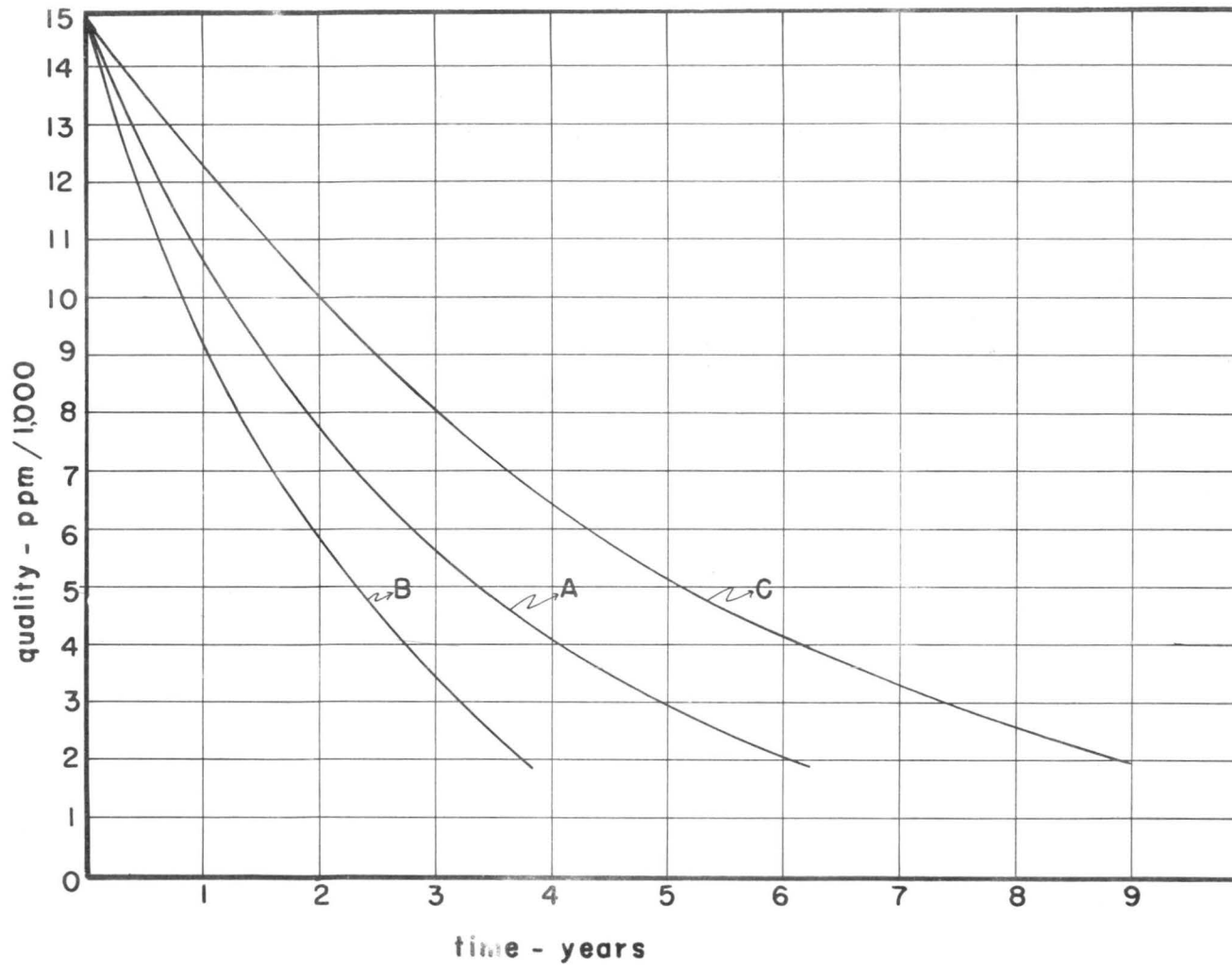


Figure 8. Quality of drain effluent as a function of time for some specific situations. Two part aquifer. $K_{top}/K_{bottom} = 1/11.5$. See text for meaning of A, B, and C.

(3) Parallel drains in a two-part aquifer - the permeability of the upper half of the aquifer approximately ten times the permeability of the lower half.

The spacing between plates of the upper half was 0.186 cms, and of the lower half 0.085 cms. The resulting permeability ratio in this situation was therefore 10.5/1.0 (see equation (16)) and the constant b/n_p ratio requirement fixes the effective porosity ratio at 2.19/1.0.

Figure 9 shows the results of this system. The values of $\frac{g}{12} \frac{b^3}{v} \frac{z_m^2}{x_m}$ and $\frac{12}{g} \frac{x_m^2}{z_m} \frac{v}{b^2}$ for the upper part of the aquifer are respectively 108 cm³/sec and 0.141 minutes. Some examples are presented in Figure 10. Curve A denotes a situation for which the drain spacing is 1200 ft, depth of the upper part (and hence lower part) 30 ft, permeability 90,000 ft/year, recharge rate 1 ft/year and effective porosity 0.3. The result of a two-fold increase in drain spacing is shown by curve B. A two-fold increase in recharge rate lowers the concentration at any time (Curve C).

DISCUSSION

The results of the experiment have been summarized in the charts of Figures 5, 7, and 9. Values of the concentration change with time can be obtained from these figures for different combinations of drain spacing, aquifer depth, permeability, effective porosity and recharge rate.

Although these results are rather general in their application, some serious questions must be raised in connection with their use. The scaling in a model of this type is accomplished by using a number of equations, say n , containing a number of variables (m), where m is greater than n . When we assign values to $(m-n)$ variables, the remaining variables (n) become fixed (for example see Santing¹, 1951). In this study we were interested in investigating the effect of change of drain spacing and thickness of aquifer upon the quality of drainage effluent. However, by changing either of these variables one changes also the ratio between horizontal and vertical permeability. Although not much is known of the effect of anisotropy, results obtained by Brown and Skibitzke² would indicate that the effect is considerable.

¹Santing, G., 1951, Modèle pour l'étude des problèmes de l'écoulement simultané des eaux souterraines douces et salées: A.I.H.S. Assemblée générale de Bruxelles, v. II, p. 184-193.

²Brown, Russel H., and Skibitzke, Herbert E., 1963, Ground water movement in fluvial deposits: unpublished report. U.S. Geological Survey, Water Resources Division, Phoenix, Arizona, 18 p.

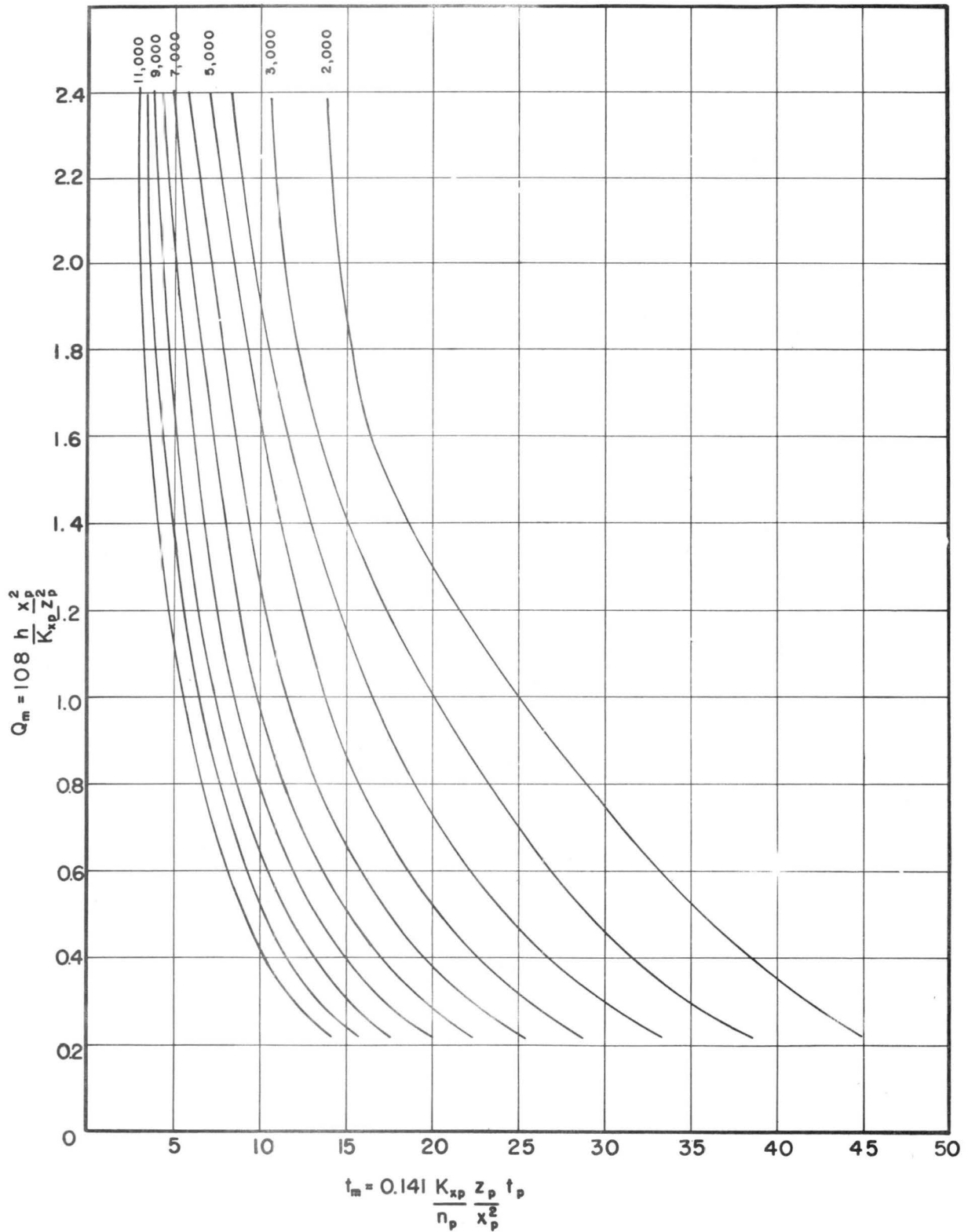


Figure 9. Relationship between model discharge and model time for different values of quality of effluent. Two part aquifer $K_{top}/K_{bottom} = 10.5/1$.

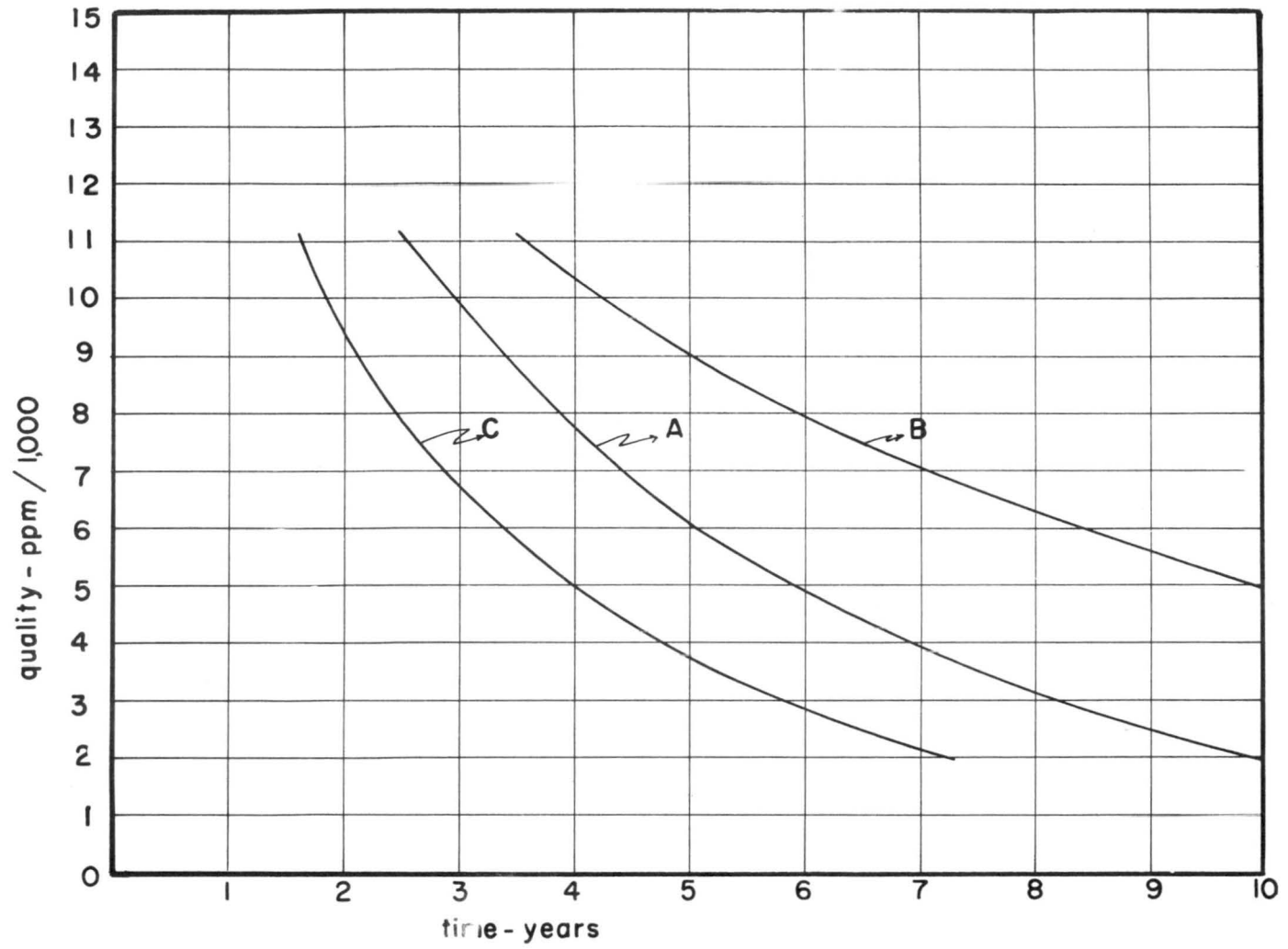


Figure 10. Quality of drain effluent as a function of time for some specific situations. Two part aquifer, $K_{top}/K_{bottom} = 10.5/1$.

Even more serious is the requirement of a constant b_m/n ratio in case of a two-part aquifer. In the present study, where b_m/n ratios of permeability were used, the resulting porosity ratios modeled were approximately 2 to 1. This, of course, is not necessarily true in the field. Since the magnitude of the effective porosity is directly proportional to the amount of water to be displaced, the results of this study should be judged in the light of this.

Figures 5, 7, and 9 cover a range of Q_m from approximately 0.2 to 2.0 cm^3/sec . Q_m involves the variables h , K_{xp} , x_p , and z_p . Obviously only those combinations of the four variables, that will result in a value of Q_m in the indicated range, may be used. Making an appropriate choice of h , K_{xp} , z_p , one will in general not be able to more than double x_p (half the drain spacing). This fact plus the additional difficulty of a different K_x/K_z ratio makes it difficult to directly compare the results. Not much would be gained by using higher model discharge rates, since the salt water - fresh water interface would be displaced so fast as to make it impossible to make accurate observations. A better way of increasing the range of Q_m and still be able to obtain accurate results is to use a model liquid having a higher kinematic viscosity. (Glycerine-water mixture, oil, etc.). The use of a more viscous liquid would allow a wider interspace which would enhance the accuracy because small deviations from the average spacing would cause a relatively smaller error in the model permeability.

The points from which the curves in Figures 5, 7, and 9 were drawn showed a considerable scatter. This scatter disappeared to a large extent when the means of both model discharge and time were taken for several points. The scatter encountered was probably due to the following: (1) the method employed to compute the quality, (2) non-uniformity of the spacing between plates, (3) inaccuracy of discharge measurements and (4) small changes in temperature of the water in the model during runs. It was also noted that the rate of discharge decreased slightly during each run. In a well-controlled setup it would be possible to correct for all of these factors.

The results of this study have been plotted in a different manner (for the homogeneous aquifer) in Figure 11. The abscissa and ordinate are the same as in Figure 5, but the parameter on the curves shows the percentage of the salt water that has been removed, Figure 12 has been derived from Figure 11 and it shows the cumulative volume of water in units of model volume versus the percentage of salt water removed for a flow rate of 1.0 cm^3/sec . Figure 12 contradicts the belief that aquifers are flushed substantially free of salt water when the quantity of applied fresh water is equal to the volume of salt water originally present. Because of the dimensionless character of the curves, Figure 12 would equally apply to a field situation with drain spacing of 6000 ft, depth of aquifer 60 ft, permeability of 4780 ft/year, effective porosity 0.314 and recharge rate of 1 ft/year or any other combination of variables

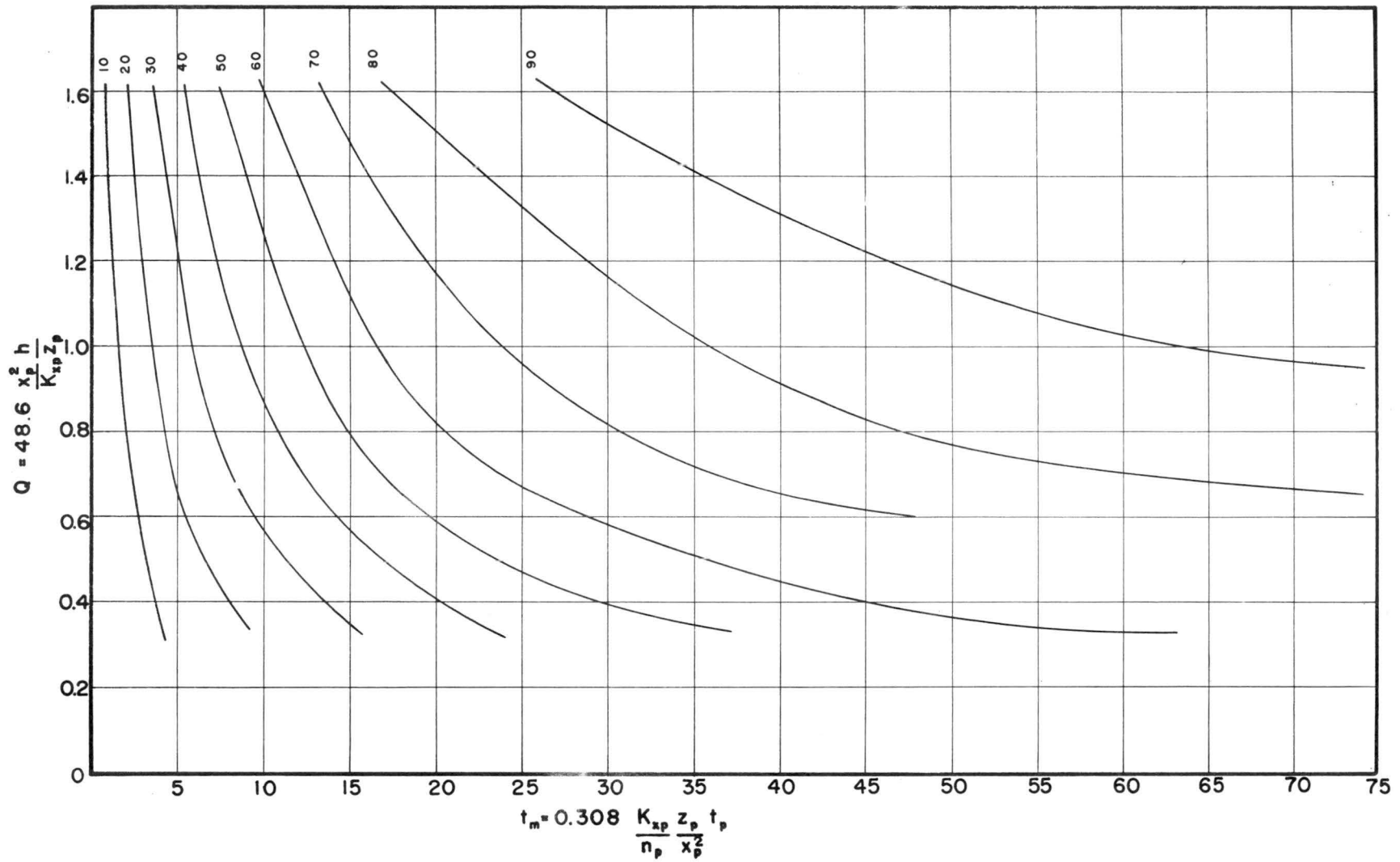


Figure 11. Relationship between model discharge and model time for different percentages of aquifer cleaning (uniform aquifer)

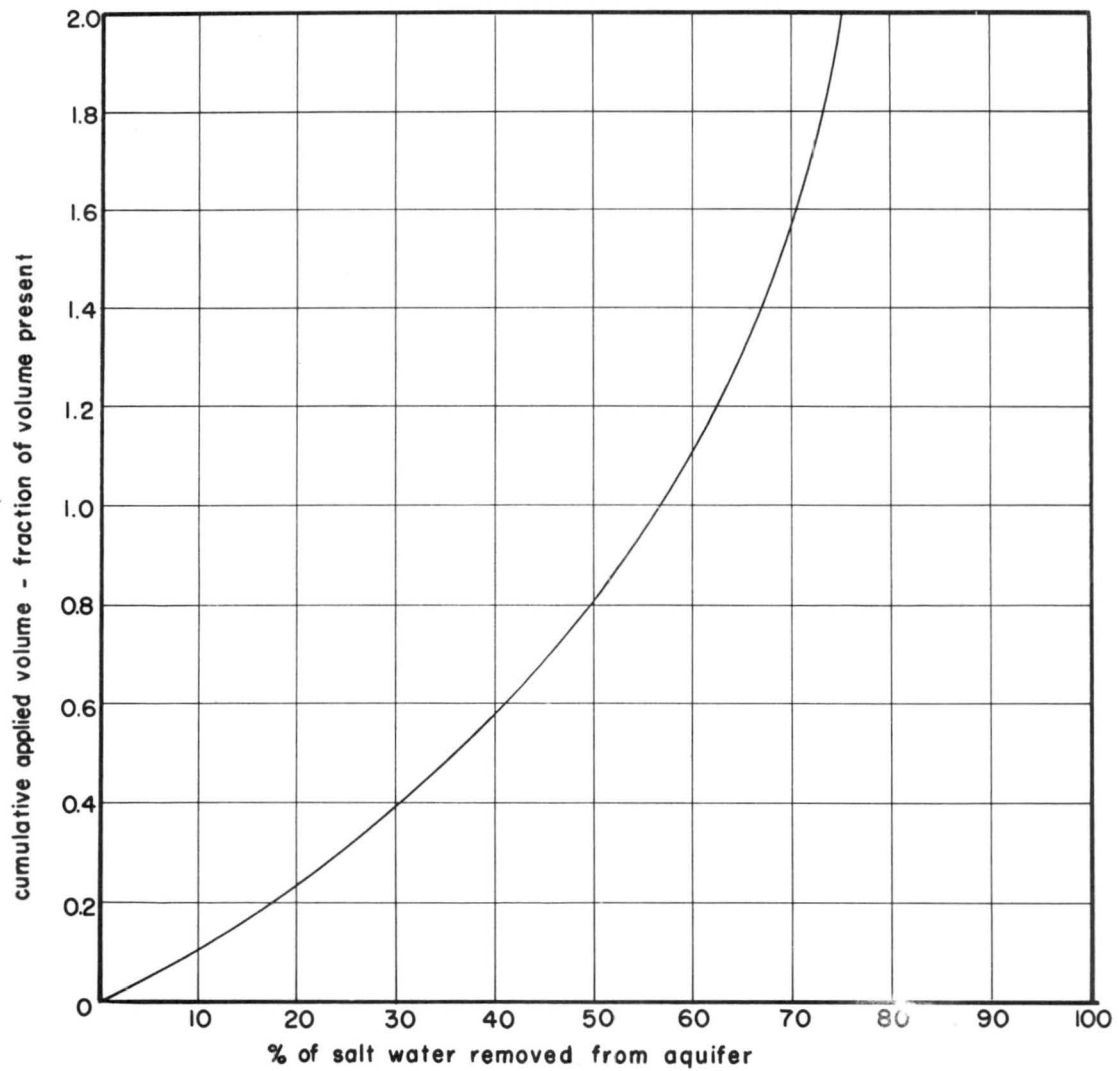


Figure 12. Percentage of salt water removed from model as a function of the amount of water applied. $Q = 1.00 \text{ cm}^3/\text{sec}$.

which fixes the value of Q_m at $1 \text{ cm}^3/\text{sec}$. In the numerical example above $x_r = 600/235 = 2.54$ and $z_r = 60/44.5 = 1.35$. According to equation (9), $K_{xr}/K_{zr} \approx 6.5$ or $K_{xp}/K_{zp} \approx 6.5$, a ratio of horizontal to vertical permeability within reason under many field conditions.

APPENDIX A

Notation

<u>Symbol</u>		<u>Units</u>
b	Width	L
g	Acceleration of gravity	LT^{-2}
h	Rate of recharge	LT^{-1}
K_x, K_z	Permeability in horizontal and vertical direction	LT^{-1}
m	Subscript used in conjunction with a variable denoting the magnitude of the variable in the model	
n	Effective porosity	Dimensionless
p	Subscript used in conjunction with a variable denoting the magnitude of the variable in the prototype	
q_s	Flux in directions	LT^{-1}
Q	Rate of flow	L^3T^{-1}
r	Subscript used in conjunction with a variable denoting the scaling factor of the variable	
s	Direction	L
t	Time	L
x	Horizontal distance	L
z	Vertical distance	L
γ_f	Specific weight fresh water	FL^{-3}
γ_s	Specific weight salt water	FL^{-3}
ϕ	Potential	L
v	Kinematic viscosity	L^2T^{-1}

APPENDIX B

Construction and Assembly of Model

The Hele-Shaw model consists of two 8 ft x 1.5 ft x $\frac{3}{8}$ inch plexiglass plates. To one of these plates two 2- $\frac{1}{2}$ inch diameter plexiglass discharge collecting tubes were glued, 2 inches from each end. A series of $\frac{1}{4}$ inch drain holes, $\frac{1}{2}$ inch on center, were drilled through the contact between the plate and the cylinder. Uniform spacing was obtained by locating three horizontal rows of $\frac{1}{4}$ inch spacer holes at 8 inch centers. The rows were located 8, 16, and 19 inches respectively from the bottom of the plate.

The model was assembled as follows:

(1) On each $\frac{1}{4}$ inch bolt a washer and a rubber gasket were placed, and then the bolts were inserted into the spacer holes of the front plate (the plate without the cylinders). The plate was then placed flat on a table.

(2) Brass spacer washers, $\frac{1}{32}$ inch thick were placed over each bolt. Spacer strips (hoop brass, 1 inch x $\frac{11}{32}$ inch) were placed along sides and bottom of the plate about $\frac{1}{2}$ inch from the edge. Plasticine beads ($\frac{1}{4}$ inch diameter) were laid on the edge along sides and bottom of the plate.

(3) On one end all drain holes except one near the top were with electricians tape. On the other end all holes were covered. The back plate was then placed in position on top of the front plate. The spacer holes were sealed by placing a rubber gasket, a washer, and a nut on each spacer bolt.

(4) After tightening nuts the model was placed into an aluminum frame in vertical position and secured to the frame with C-clamps on the bottom and sides.