THESIS

A STATISTICAL STUDY OF BED FORMS IN ALLUVIAL CHANNELS

By

James H. Algert

In partial fulfillment of the requirements

for the Degree of Master of Science

in Civil Engineering

Colorado State University

Fort Collins, Colorado

June 1965

CER65JHA26

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ABSTRACT OF THESIS

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The techniques of correlation and spectral density analysis of random processes were applied to the problem of describing and predicting dune profiles in a sand bed channel. The data analyzed were taken from three sources: a 0.4-ft flume for this study; a 8-ft flume for a previous study; a conveyance channel on the Rio Grande near Bernardo, New Mexico. For all of this data the sand sizes were between $d_{50} = .23$ mm and $d_{50} = .34$ mm. Covariance and spectral density functions were computed using the IBM 1620 and a program written for this study. Models for these functions were derived and computed. The first three values of the covariance functions were found to be sufficient for computing approximate models and two other parameters which describe the goodness of fit of the models or how well the process of bed elevation as a function of distance downstream is described by a second order autoregressive scheme. The first value of covariance, which is the variance of the process, relates well with the flow parameter of unit discharge. The next two values of covariance show a less definite relationship with the same flow parameter.

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June 1965

ACKNOWLEDGEMENTS

The author is indebted to his major professor, Dr. D. B. Simons, for guidance during the course of this research. Thanks are also due to members of the committee, Dr. V. M. Yevdjevich and Dr. M. M. Siddiqui for their review of the thesis.

Special acknowledgement is due to Dr. M. M. Siddiqui for a major part of the theoretical development used in this study.

Thanks are also due to Mr. H. P. Guy, Mr. C. F. Nordin, Mr. E. V. Richardson, and Mr. W. W. Sayre of the U.S. Geological Survey for their helpful comments and review during the data collection and writing of the thesis.

Financial support for this study was provided by the U.S. Geological Survey.

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PARTIAL LIST OF SYMBOLS

Symbol	Definition	Units
α	Autoregressive coefficient	none
β	Autoregressive coefficient	none
C_λ	Autocovariance function	ft^2
C_0	Variance or σ_x^2	ft^2
E(f)	Expected value of f	none
ϵ_ℓ	Random portion of the process, X_ℓ	ft
f	Frequency used for spectral density function	cycles/unit lag
g(f)	Spectral density function model	ft^2
G(f)	Spectral density function from data	ft^2
H	equals $4\ C_0^{0.5}$ or 4 standard deviations	ft
ℓ	Longitudinal distance in profile	ft
λ	Lag interval	ft or in.
L	Total distance interval considered	ft
M	Maximum lag used for a profile	none
N	Number of data points for a profile	none
R^2	Multiple correlation between X_ℓ and $(X_{\ell-1}, X_{\ell-2})$	none
r_k	Correlation coefficient = C_k/C_0	none
σ_ϵ^2	Variance of ϵ_ℓ	ft^2
X_ℓ	The process being studied--bed elevation X as a function of distance downstream ℓ	ft
$z_x(f)$	A random process with orthogonal increments	none

INTRODUCTION

The problem of measuring roughness in sand channels is as old as the study of river hydraulics. For estimating flow velocity, boundary shear, and sediment transport, some method of evaluating friction must be available. This has usually been done with empirical relationships and coefficients based largely on an engineer's judgment. Within the last fifteen years, more rational methods have been developed which are based partly on simplified models of form roughness, but if relationships using the mechanics of flow around submerged objects are to be developed, mathematical descriptions of the movable and flow-molded bed forms must first be found.

The purpose of this study is to apply statistical techniques to bed-profile data to try to find some significant statistical parameters which will be useful in constructing mathematical models of bed forms. The bulk of the data used in this study was limited to the dune range with one uniform sand size. The most severe limitation of the data, however, was the use of a very narrow channel in order to insure two-dimensional bed forms. Data from three-dimensional bed-forms in a large flume and a conveyance channel are included mainly for comparison.

REVIEW OF LITERATURE

The earliest descriptions of roughness were developed for fixed-boundary flow. Most common of these descriptions are the Manning and Chezy equations which use an empirical coefficient of roughness. These coefficients can be approximately related to the size of the roughness.

$$\frac{1.49\ R^{1/6}}{C} = n = \frac{\sqrt[6]{K_s}}{29.3} \tag{1}$$

A theoretically more acceptable relationship is

$$V/V_* = 5.75\ \log_{10}\ (12.2\ \frac{R}{K_s}\ x) \tag{2}$$

In the above equations,

C	=	the Chezy discharge coefficient
n	=	the Manning resistance coefficient
K_s	=	the height of a representative roughness element
R	=	the hydraulic radius
V	=	the mean velocity
V_*	=	the shear velocity, $\sqrt{gRS}$
x	=	a correction factor for the transition region for smooth to rough boundaries, usually given as a function of K_s/δ
δ	=	the theoretical thickness of the laminar sublayer, which is equal to $11.6\ \nu/V_*$
ν	=	the kinematic velocity

S = the slope of the energy gradient

g = the acceleration due to gravity.

These equations strictly apply only to uniform-size roughness on a fixed boundary, but they have been applied extensively to sand channels. One fairly recent approach (Einstein-Barbarossa, 1952) considers the various types of roughness separately and then sums them much as in pipe flow with loss in bends, etc. The two main types of roughness in this analysis were sand-grain roughness and form roughness. The first of these can be evaluated by means of Eq. 1, using grain diameter for K_s. The form roughness was evaluated by means of a simple hydraulic model: Assume that form roughness from ripples, dunes, or bars on the bed of a channel is equivalent to the form roughness arising from equally spaced blocks of uniform size. With N blocks in the area, LP, and with cross sectional area A per block, the following relationship was derived:

$$V/V_*'' = \frac{2LP}{NAC_D} \tag{3}$$

in which

L = a unit length of channel

P = the wetted perimeter

C_D = a drag coefficient

V/V_*'' = a dimensionless resistance coefficient.

The quantity V/V_*'' is equivalent to the dimensionless Chezy coefficient, $C/\sqrt{g} = V/V_*$, and is a measure of the flow resistance which is attributable to the form roughness arising from the ripples, dunes, and other irregularities on the bed. V/V_*'' was determined from field data to be a function of a dimensionless shear stress parameter. This functional relation developed appears to give good results for some natural streams (Einstein and Barbarossa, 1952). However, Simons and Richardson (1962, p. 1000) have shown that the Einstein-Barbarossa bar resistance curve is not generally satisfactory for the entire range of flow conditions and bed configurations.

The flow regimes for the various types of bed forms have been studied extensively (Simons and others, 1961, p. A36), to determine the effects of the flow parameters and bed-material size on the bed forms. In general, the flow was classified as (1) a lower flow regime, which includes plane bed with bed-material transport, ripples, dunes with ripples superposed, and dunes; and (2) an upper flow regime, which includes a plane bed with high sediment transport, standing waves, antidunes, and chute-and-pool type antidunes. The occurrence of these bed forms has been classified in several ways, and the three most important variables were found to be slope, depth, and the fall velocity of the bed material in the water-sediment mixture. Thus, even though the flow is turbulent, the viscosity is quite important and a change in viscosity can easily shift the bed roughness from one form to another. The variation of a friction factor as a function of the bed form was also presented in a qualitative manner for various flow conditions (Simons and Richardson, 1963, Fig. 3). A useful consequence of these investigations is the ability to usually predict the type of bed configuration from the approximate hydraulic conditions. This, in conjunction with some knowledge of the roughness for each type of bed form, allows an engineer to estimate the type of flow to be expected in a sand channel.

In a paper on the geometrical properties of sand waves (Yalin, 1964), expressions were derived for average amplitude and wave length from dimensional analysis and flume data. The dimensional analysis was based on the predominant importance of the critical tractive force from the Shields' curve. Although experimental data from many sources actually failed to produce any significant relationship, an equation was presented (Yalin, 1964, Eqs. 28 and 30) which shows wave length and amplitude for ripples and dunes being dependent only on depth, critical depth, and grain size.

An excellent summary of recent practices and current literature on roughnesses in open channel is given by the American Society of Civil Engineers Task Committee on Hydromechanics (Am. Soc. Civil Engineers, Committee on Hydromechanics, 1963).

Correlation functions and spectral density representations have just recently come into use for fluid mechanics and hydraulic problems. More extensive applications are found in such fields as radio wave propagation (Siddiqui, 1962) and communication theory. Medical research and the natural sciences are also finding these statistical techniques useful in conjunction with digital computers for analyzing vast amounts of data.

Several physical processes similar to sand waves have recently been studied with similar methods. In the design of military vehicles for off-road hard-ground conditions (Kozin and others, 1963), spectral representations of survey profiles along with the measured dynamical response of vehicles led to the design of optimum wheel-base lengths and suspension systems. In the studies by Pierson and Moskowitz (1964) on wind seas, curves were fitted empirically to the power spectrums with good results for the wind speeds studied. These relationships can be used for fairly accurate wave forecasting.

ANALYTICAL DEVELOPMENT

The basic approach used in this study follows the methods outlined by Blackman and Tukey (1958) for spectral density and covariance analysis and by Siddiqui (1962, and written communication, 1965) for determination of models for the process. The expression used for the spectral density, however, is due to Bartlett (1950).

Some of the definitions and corresponding assumptions which need to be satisfied within certain limits for the analysis to be valid are outlined here. The process frequently referred to in this paper is the bed elevation X as a function of the distance ℓ downstream in a profile, X_ℓ. This is a Gaussian process, if for every n, ℓ_1, ℓ_2, . . . , ℓ_n, the joint probability distribution function of X_{ℓ_1}, X_{ℓ_2}, . . . , X_{ℓ_n} is a n-dimensional Gaussian or normal distribution.

Each such distribution would then be completely determined by the ensemble averages:

$$X_{\ell_i} = \text{ave.} \left\{ X_{\ell_i} \right\} \tag{4}$$

and by the covariances:

$$C_{ij} = \text{ave} \left\{ \left[X_{\ell_i} - \bar{X}_{\ell_i} \right] \cdot \left[X_{\ell_j} - \bar{X}_{\ell_j} \right] \right\} \tag{5}$$

The assumption of a normal process, however, is not necessary for the following analysis. The process is assumed to be a stationary process:

$$C_{ij} = C\ (\ell_i - \ell_j) \tag{6}$$

That is, if the bed profile is a stationary process, the mean value and covariance function are dependent only on the length of the distance interval, $\ell_i - \ell_j$. For this study, the functions should be approximately the same for all profiles of a given run. This is the erogodic property that averages across an ensemble are equivalent to averages over distance along a single function of infinite extent. This property is implied whenever a process is stationary and has zero averages and a continuous spectral density function.

If the above conditions are not strictly met, the process is not completely determined by the functions, but the analysis may still be useful. Thus, one of the important parts of the investigation is to determine the type of process, or profiles, being analyzed.

The autocovariance function is given by Blackman and Tukey (1958):

$$C_\lambda = \lim_{L \to \infty} \frac{1}{L} \int_{-L/2}^{L/2} X(\ell) \cdot X(\ell+\lambda) d\ell \tag{7}$$

in which the mean value of the data, $X(\ell)$, has been removed, and λ is the lag distance or interval.

The autocovariance function can be estimated for digital data by:

$$C_\lambda \approx \frac{1}{N-\lambda} \sum_{\ell=1}^{N-\lambda} X(\ell) \cdot X(\ell+\lambda) \tag{8}$$

for $\lambda = 0, 1, \ldots, M$ (M is the maximum lag interval) where $M \to \infty$ as $N \to \infty$, but $M/N \to 0$. N is the number of data points for the profile.

In particular the variance of the process is:

$$C_0 = \frac{1}{N} \sum_{\ell=1}^{N} X(\ell)^2 \tag{9}$$

An estimate of the power spectrum is given by Bartlett (1950):

$$G(f) = C_0 + 2 \sum_{\ell=1}^{M-1} (1-\ell/M) \cdot C_0 \cos \frac{2\pi \ell j}{M} \tag{10}$$

for $j = 0, 1, \ldots, M/2$, and $f = 2j/M$.

It was determined from inspection of the covariance function that the process could be represented by the second order autoregressive scheme, or the Markov second order linear model:

$$X_\ell = \alpha X_{\ell-1} - \beta X_{\ell-2} + \epsilon_\ell \tag{11}$$

where ϵ_ℓ is a random variable, and independent of $X_{\ell-1}, X_{\ell-2} \ldots$. Expressions for α and β are outlined by Siddiqui (written communication, 1965):

$$\alpha = \frac{C_0 C_1 - C_1 C_2}{C_0^2 - C_1^2} \tag{12}$$

$$\beta = \frac{C_1^2 - C_0 C_2}{C_0^2 - C_1^2} \tag{13}$$

For this to be valid, $4\beta > \alpha^2$ must hold true. The total variance = $\sigma_x^2 = C_0$ and from Eq. 11, for σ_ϵ^2 = variance of ϵ_ℓ, it follows that:

$$C_0 = \frac{(1+\beta)\sigma_\epsilon^2}{(1-\beta)\left\{(1+\beta)^2 - \alpha^2\right\}} \tag{14}$$

or,
$$\sigma_\epsilon^2 = \frac{C_0 (1 - \beta)\left\{(1 + \beta)^2 - \alpha^2\right\}}{(1 + \beta)} \tag{15}$$

The term ϵ_ℓ is the random component of the process, $X(\ell)$. The amplitude parameter H which is termed the significant wave height (Moskowitz, 1964, p. 5177) was found from the total variance:

$$H = 4 \left(C_0\right)^{1/2} = 4 \sigma_X \tag{16}$$

A model for the covariance function was calculated by:

$$C_k = \alpha C_{k-1} - \beta C_{k-2} \tag{17}$$

A theoretical spectral density function was obtained from the following expressions:

$$X_\ell = \int_{-1/2}^{1/2} e^{2\pi i \ell} dz_X (f) \tag{18}$$

$$\epsilon_\ell = \int_{-1/2}^{1/2} e^{2\pi i \ell} dz_\epsilon (f) \tag{19}$$

The derivation of this model of the spectral density function is found in the appendix. The results were as follows:

$$g(f) = \frac{\sigma_\epsilon^2}{\left| 1 - \alpha e^{-2\pi i f} + \beta e^{-4\pi i f} \right|^2} \tag{20}$$

for $f = 0, .05, .1, \ldots, 1.0$; or twice the number of frequencies used for $G(f)$. The evaluation of this expression is also found in the appendix.

The performance of the models is indicated by the value of the coefficients R^2 , where R^2 is the multiple correlation between X_ℓ and $(X_{\ell-1}, X_{\ell-2})$, (Siddiqui, written communication, 1965):

$$R^2 = \frac{r_1^2 + 2 r_1^2 r_2 + r_2^2}{1 - r_1^2} \tag{21}$$

where $r_k = \frac{C_k}{C_o}$, and $1 - R^2$ = the proportion of the variance which is unaccounted for by C_o : $1 - R^2 = \sigma_\epsilon{}^2/C_o$.

It should be noted that the coefficients r_k , $k = 0, 1, \ldots, M$, form the autocorrelation function which represents the same properties of the process as the autocovariance functions. With the variance C_o removed, as in the autocorrelation function, the functions for all profiles would have the same initial ordinate value of 1.0.

EQUIPMENT AND EXPERIMENTAL TECHNIQUES

Most of the sand-bed profile data used for this study were collected in a recirculating flume 2-ft wide, 2-1/2-ft deep, and 60-ft long in the CSU hydraulics laboratory (see Fig. 1). The flume is described in detail by Guy and others (1965, Fig. 2). The flume was modified for this study by the installation of a plastic-coated plywood partition, Figure 2, to give essentially two-dimensional bed forms. This provided an effective flume section 0.4-ft wide and 2.5-ft deep with nearly the same wall roughness on each side. The flow conditions were nevertheless distorted with respect to wide-channel flow because of the large side-wall effect.

Because of the small scale of the flume, a feed and trap system for the sand was possible which eliminated the need for a recirculating system. Sand was fed into the flume at the upper end by an adjustable vibration feeder which if the sand level in the hopper remained fixed, gave a constant feed rate. The level in the hopper was maintained by a gravity-feed supply tank. The sand was trapped at the lower end of the flume.

The sediment transport was measured indirectly by the sonic-sounder data. The sonic records of the bed profiles were planimetered to give the total amount of sand in the bed within 5 percent. When the amount of sand in the bed became constant the bed was assumed to be in equilibrium and the transport rate equal to the feed rate. The other criterion used for equilibrium of the system was the water surface slope. When the water surface slope reached a stable value as near uniform flow as possible and the volume of sand in the bed became constant, the collection of data was begun.

A fairly uniform sand with median diameter, d_{50}, of 0.34 mm was used for this study (Fig. 3). A uniform sand was selected to eliminate the effects of sorting and gradation which could produce a systematic variation in the bed configuration along the flume.

Hydraulic conditions were chosen to span a range of discharge for a depth of about 0.5 ft in the various flumes. The only variable parameters in the study, as the water temperature was fairly stable, were the sand feed-rate, the water discharge, the depth, and the resulting bed and water-surface slopes.

A sonic-sounder (Richardson and others, 1961) and a Mosely recorder were used to measure the bed profiles. The probe was mounted on the carriage (Fig. 2) and a coaxial cable led to the sounder. The sounder was calibrated in place for a 1-ft range by placing metal blocks in flume. This was then checked on the sand bed with the carriage stationary and with the carriage in motion to evaluate distortion of the recorder output. Each profile was begun at a metal reference block at the upper end of the flume to check for possible drift of the sounder. The drift was almost negligible.

A synchronous-motor was installed on the carriage (Fig. 2) to provide an accurate carriage velocity of up to 1 fps. The velocity used in this study was 1/3 fps ± 2%. Any higher speed would have caused too much distortion of the sonic-sounder data due to the lag-time of the recorder. The worst distortion was in the trough depth (up to 10% at 1/3 fps carriage velocity). An event marker was installed on the carriage to provide a check on the carriage speed and to help define the horizontal scale on the sonic-sounder strip charts.

The V-notch weir box at the lower end of the flume (Fig. 1) served two purposes: as an accurate flow measuring device for low discharge, and later in the run as a sand-trap.

A typical run was conducted as follows. The discharge and approximate depth were established, then the sand feed was begun. This flow was maintained until the sand bed was approximately at its final level in the flume, and the flow was turned off - usually after about six hours. The following day's operation was about ten hours long. The flow rates were established and water surface and bed profiles were then taken every 1/2 hour to record any changes in the bed volume or slope and to check for a fairly uniform flow of about 0.5-ft depth. Usually within four hours the collection of bed profile data was begun. About 12 of these sonic records were taken at 15 minute intervals for each run. Water surface slopes were measured every half hour. Three of such runs were used in this study.

Data from two other sources were also used in this study. The two stream bed profiles are from the Rio Grand Conveyance Channel near Bernardo, New Mexico (J. K. Culbertson, 1965, written communication). The channel is about 70-ft wide. These profiles were taken along a 350-ft reach at 43 ft and 56 ft from one bank (Figs. 12, 15). The sounding records were collected using a lightweight sonic-sounder (Karaki and others, 1961) in a boat that was reeled downstream at approximately constant speed. An event marker was used to show actual longitudinal stationing. The dunes had 1 to 3 ft amplitudes and wavelengths of 20 to 40 ft, thus providing a much different magnitude of bed forms for comparison with flume data. Dunes were also studied intensively in the 8-ft flume at Colorado State University (Guy and others, 1965), and three profiles each from two of these runs are also included in this study (Figs. 13, 15 and Tables 1, 2).

The slope of the bed is regarded as an undesirable trend for the analysis, and it was removed from the profiles (Fig. 4) during the analog-digital conversion of the data.

EQUIPMENT USED IN THE ANALYSIS OF THE DATA

The analysis of data for this study was made possible through the use of an analog-digital converter and an IBM 1620 digital computer at Colorado State University.

The analog to digital converter is used by following the trace with a rheostat-controlled pen as the trace moves by at a constant speed of 2 inches per minute. The voltage fed into the converter is proportional to the height of the signal. This voltage is averaged over a certain period and recorded at set intervals (0.1 sec. and 3.0 sec. in this case). This gives a lag interval of 0.25 ft of bed profile, or is equivalent to point-gage data taken every three inches.

These data were then processed in the 1620 computer. The mean, autocovariance function, spectral density function, and an estimate of the variance of the spectral density were determined by the program which was written for this study. The program is available at the U.S. Geological Survey, Colorado State University, Fort Collins, Colorado.

DISCUSSION OF RESULTS

In reviewing the data presented it is necessary to keep in mind the degree of precision and the subsequent limitations on the results. It was possible to maintain constant water and sand discharge, but for all three runs in the narrow flume the other parameters, such as velocity, depth and slope, changed almost constantly but usually within 6 percent of the mean. It is difficult to define equilibrium conditions and obtain uniform flow for such a short sand-bed reach. Equilibrium flow is flow with constant discharge and constant energy gradient with the average energy gradient being parallel to the average slope of the bed. There appears to be oscillation about average values of slope and depth. As long as no definite trend toward a changing water or bed slope or depth was found, the data was assumed to be as good a representation of equilibrium conditions as possible. The hydraulic data is given in Table 1.

Due to entrance conditions the water surface was not parallel to the bed surface as far downstream as station 16. From that point on the water surface slope was approximately a straight line. Little draw-down or back-water effect was measurable at the lower end. In general, the profile from about station 18 to station 52 was used in the analysis to give as many dunes as possible, even though the flow may not always have been uniform throughout this reach. Longer profiles are needed to better demonstrate the assumed stationarity of the process. That is, more dunes per profile or a longer flume are needed to give consistent and reliable results.

In the discussion of the results, several important goals are considered. Mathematical justification for the use of the techniques of time series analysis for this stochastic process of bed elevation along a longitudinal profile with the assumptions of equation 6 is presented but in a limited form because of insufficient data. It is shown that some statistically significant parameters can be calculated (Eqs. 12, 13, 15, 16) from the first three values of the covariance function, C_0, C_1, and C_2. These parameters are used to derive models which resemble the covariance and spectral density functions. The coefficients α and β are major parts of the process model of Eq. 11. The total variance, C_0, relates to the dune amplitudes (Eq. 16) and is referred to as the significant wave height. The possibility of relating the three values of covariance to the flow conditions is discussed with the hope that a method of prediction of the theoretical models of covariance and spectral density functions based only on hydraulic and sand-size data can be found.

The statistical functions evaluated in this study are summarized partially in Table 2 and Figs. 5-13. The units and a brief description of the functions may be useful in visualizing their physical significance. Study of Eqs. 8 and 10 reveals that the units of the ordinates are ft^2. The absicissa for the autocovariance functions is in units of lag interval and is equal to 1/4 times the lag in ft. Covariance functions are plotted in two ways to show this relationship (Figs. 7 and 14). The abscissa for the spectral density can be represented in several ways: as frequency in cycles per unit horizontal distance, or as wave length itself. The damped oscillation of the autocovariance and the shape of the spectral density curve demonstrate that the process can be thought of as a band of modulated frequencies or wave lengths. The spectral density also gives the power or variance density contained in the various bands. The power can be thought of as the amount of oscillatory movement in the process attributed to the particular band of the wave length spectrum. Thus the process is a continuous spectrum of wave lengths of changing variance density. The existence of the spectrum is a consequence of some degree of statistical stationarity in the process which can be taken as an indication of the suitability of the process for this type of analysis. If a record of ripples superposed on dunes were analyzed, for example, the spectrum would show two modes or peaks. The low frequency peak measures the power in the long wave length range, and the high frequency peak indicates the lesser power contained in the ripple range of wave lengths.

Some of the individual covariance and spectral densities taken from the 0.4-ft flume data are given in Figs. 5-10. The importance of accurately removing the mean value of the profile is demonstrated by comparing the asymmetry of Fig. 5 with the shapes in Figs. 6 and 7.

The average values of the spectral density functions for all the data are shown in Figs. 11-13, along with the corresponding model functions. The two very flat average value functions indicate that this method of averaging does not give a good representation of the process. The models give a much better picture in these cases. Average value and model functions for the covariance of the 0.4-ft flume data are given in Fig. 14. There are discrepancies between some of the individual functions, but this is to be expected in consideration of the varying flow conditions encountered within a run. If longer profiles were available, they would have demonstrated a higher degree of stationarity. The unit lag interval for all runs in the 0.4-ft flume is 0.25 ft. The unit lag intervals for the 8-ft flume data and the conveyance channnel data respectively are 0.5 ft and 2.5 ft. The maximum lag for runs 2 and 3 is 30, and the maximum lag for all others is 20. This is apparent in the abscissas for the spectral density function (Figs. 11-14).

The spectral densitites give the clearest representation of the process, but there is a problem in the interpretation of these functions. The precision in locating the peak for the spectral densities is limited because of the wide spacing of points on the abscissa. The peak will always occur at a frequency which is a multiple of $2/M$, where M is the maximum number of lags. To help define this peak, a larger M with the same N should be used; but for statistical reasons M should be limited to about ten percent of N, the total number of data points.

This points out one advantage in using an analog computer for further studies. However, digital computing techniques are still more advanced and proficient. The frequency interval between points for the model is arbitrary as this can be programed for any set of values of f in Eq. 20 between zero and one. The program for this study was set up for frequency intervals which were half the size used for the actual spectral densities.. By using the model functions of Fig. 11, the short dune lengths of run 3 can be distinguished from the longer dune forms of runs 2 and 4. The wave lengths of greatest power are thus found to be 3.25, 2.70, and 2.50 feet for runs 2, 4, and 3 respectively. The values derived from the models correlate with the larger, middle and lower average dune lengths given in Table 1.

The difference in dune amplitude for the various runs is evident by the different scales on the ordinates of the spectral density and covariance functions. The value of C_0 or the covariance for zero lag interval is most significantly related to the flow conditions of depth and velocity. Relations for C_1 and C_2 are not as well defined but are still significant (Figs. 15 and 16).

The models for covariance agree quite well with the average values of C_i, $i = 0 \rightarrow M$ (see Fig. 14). Values of R^2 (Eq. 21) were computed for all the profiles, and $1-R^2$ agrees reasonably well ($\pm$ 5 percent) with σ_ϵ^2/C_0. The high values of R (Table 2) indicate good performance of the models. This is a good indication that the process can be represented fairly accurately by

Eq. 11. If C_0, C_1 and C_2 can be estimated, as shown in Figs. 15 and 16, the covariance and spectral density functions can be estimated. This plot of H, $4\,C_1^{0.5}$, and $4\,C_2^{0.5}$ vs V·D utilizes all the available data with average values used for each run from all the profiles. There are actually only five points on each line though, because the three runs in the 0.4-ft flume are so closely grouped. This makes further verification necessary with more of the same type of data. The 0.4-ft flume data plotted separately (Fig. 16) show good relationships but with different slopes than in Fig. 15. The reason for this is not apparent, but it could be due to the large side-wall effect in the narrow flume.

The models for the spectral density functions show some contrast in their agreement with the actual functions. The models for the field data agree quite well as does the model for run 4 (Figs. 11-13). The agreement for the 8-ft flume data is fairly close, but the models for runs 2 and 3 do not correspond with respect to peak values to the averages of the actual spectral densities in Figs. 8 and 9. This lack of correspondence between the model and the average values of the spectral density functions indicates either that the process is not stationary for these flow conditions or that the flow conditions themselves are changing. Because these runs are for flows on the upper and lower limits of the dune range of flow conditions, it might be expected that the conditions would not be as stable as for run 4 which is in the middle range for dunes. Another indication that the process itself is stationary is that for each individual value of α and β the necessary relationship for a stationary process holds:

$$4\beta > \alpha^2$$

That is, this condition for a stationary process is met for each individual profile. Therefore it is reasoned, that the flow conditions must be constantly changing, with no observable trend, to produce the two wide, flat spectrums of Fig. 11 which are not good representations of the process.

It has not been possible in the past to successfully relate average dune height and length to flow parameters. In this study significantly different descriptions of dune forms were used, and these yielded very promising relationships with the flow parameter of unit discharge. The significant wave height, H , is a more logical choice than average value of amplitude because the effect of one large dune on flow conditions is greater than several small ones in most cases. The value of H has been found to correlate quite well with the average of the upper one-third values of ocean wave heights (Moskowitz, 1964), and this same correlation can be expected to apply in this study. The values for C_1 and C_2 were also related to this same flow parameter and this result enables one to generate a spectral density function for a reach knowing only unit discharge for the dune range of flow conditions. Using this function a significant wave length is found from the frequency corresponding to the peak of the spectral density. This wave length description is quite different than average wave length, being about 30 percent more than the average as found by the length of profile divided by the number of dunes. The depth used for the relationships of Figs. 15 and 16 is also a more logical value for calculating unit discharge than the average depth over a cross-section. The depth was evaluated using the mean value of the bed-elevation profile which was then referenced to the distance below the water surface of the sounding probe.

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

In this study methods have been presented and used for analyzing sand-bed profiles as the realizations of random processes. The assumptions to be met for such an analysis to be valid were described in the analytical development. The main assumptions are included in the conditions for ergodicity. In this analysis it was assumed that averages, such as $\bar{C}_0$, $\bar{C}_1$ and $\bar{C}_2$, across an ensemble are equivalent to averages over distance along a single function of infinite extent. This is strongly implied whenever a process is stationary and has zero averages and a continuous spectral density. Another necessary condition for ergodicity is that $C(\lambda) \rightarrow 0$ as $\lambda \rightarrow \infty$. This appears to be true for the process studied here, as the function, $C(\lambda)$, does damp out and tend to zero in all cases. Also the individual spectral density functions are all continuous, and in most cases the processes have zero averages (the exceptions being some of the profiles of run 2, Fig. 5). The necessary condition for stationarity, $4\beta > \alpha^2$, holds for all cases, but this essential property has not been proven in this study.

The result of this study is a method of predicting and describing statistically significant parameters of wave amplitude and length and functions of covariance and spectral density. For the sand sizes used in this study, the only hydraulic information necessary is unit discharge. However, these techniques have only been applied to uniform, straight, sand-bed channels with well developed dune profiles. Meandering, bars, and various other phenomena often found in alluvial channels have not been considered in this study.

Future studies are necessary for a better general understanding of the process and the assumptions necessary for the analysis. In particular, if a second order autoregressive scheme is the best model for the process, the physical significance of this result should be investigated. This could lead to a better knowledge of the mechanics of the movement of dunes in a channel. Since the covariance values of C_1 and C_2 are obviously dependent on the length of the unit lag chosen for the analog-digital conversion of the sonic records, a consistent value of ten percent of the average dune length is suggested for the magnitude of the unit lag. This may improve the relations for C_1 and C_2 in Fig. 15. The major requirement for future data is merely that the profiles be as long as possible, containing 25 or more dunes if possible.

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APPENDIX A

A model for the spectral density is given by Siddiqui (1962). Expressions for the two parts of the process are:

$$X_\ell = \int_{-1/2}^{1/2} e^{2\pi i \ell f} \, d\, z_x(f); \quad \epsilon_\ell = \int_{-1/2}^{1/2} e^{2\pi i \ell f} \, d\, z_\epsilon(f)$$

where $z_x(f)$ and $z_\epsilon(f)$ are random processes with orthogonal increments and other properties described by Siddiqui (1963, p. 572), f is the frequency in cycles per unit lag and $i = \sqrt{-1}$. The above limits of integration were shown to be valid (Siddiqui, 1962, p. 574) for discrete observations at equal intervals of a continuous process.

Taking expected values:

$$E\left[X_{\ell+k} \cdot \overline{X}_\ell\right] = \int_{-1/2}^{1/2} e^{2\pi ikf} \, E\left[dz_x(f) \cdot dz_x(f)\right]$$

$$C_k = \int_{-1/2}^{1/2} e^{2\pi ikf} \, g(f)\, df$$

where $g(f)\,df$ is the spectral density function and $g_\epsilon(f) = \sigma_\epsilon^{\,2}$, $-1/2 \leq f \leq 1/2$.

$$\int_{-1/2}^{1/2} e^{2\pi i\ell f} \, dz_x(f) - \alpha \int_{-1/2}^{1/2} e^{2\pi i(\ell-1)f} \, dz_x(f)$$

$$+ \beta \int_{-1/2}^{1/2} e^{2\pi i(\ell-2)f} \, dz_x(f) = \int_{-1/2}^{1/2} e^{2\pi i\ell f} \, dz_\epsilon(f)$$

or $$\int_{-1/2}^{1/2} e^{2\pi i \ell f}\, dz_{\epsilon}(f) = \int_{1/2}^{1/2} e^{2\pi i \ell f}\, dz_{x}(f) \left[1 - \alpha e^{-2\pi if} + \beta e^{-4\pi if}\right]$$

$$E\left|dz_{\epsilon}(f)\right|^2 = E\left|dz_{x}(f)\left[1 - \alpha e^{-2\pi if} + \beta e^{-4\pi if}\right]\right|^2$$

$$E\left|dz_{x}(f)\right|^2 = g(f)$$

$$\sigma_{\epsilon}^2 = g(f) \quad \left|1 - \alpha e^{-2\pi if} + \beta e^{-4\pi if}\right|^2$$

or $$g(f) = \frac{\sigma_{\epsilon}^2}{\left|1 - \alpha e^{-2\pi if} + \beta e^{-4\pi if}\right|^2}$$

The denominator $= \left|1 - \alpha \cos 2\pi f + \beta \cos 4\pi f + i\, \alpha \sin 2\pi f\right.$

$\left. -i\, \beta \sin 4\pi f\right|^2$

This was simplified and the model for the spectral density function is as follows:

$$g(f) = \frac{\sigma_{\epsilon}^2}{\left[1 + \alpha^2 + \beta^2 + 2\beta \cos 4\pi f - 2\alpha \cos 2\pi f\ (1+\beta)\right]}$$

APPENDIX B
Tables and Figures

Table 1.--Summary - Hydraulic Data

Run	Slope x 10^2 Bed	Water	Discharge (cfs)	Velocity (fps)	Depth (ft)	Manning n	Number of Profiles	Temperature (°C)	Number dunes used	Wave height* (H) ft	Wave length ($\bar{\lambda}$)
2[a]	0.46	0.44	0.421	1.82	0.58		10	17.8	131	0.192	2.72
4[a]	.40	.38	.345	1.78	.485		13	15.6	235	.139	2.26
3[a]	.39	.37	.280	1.74	.400		11	18.4	213	.108	1.94
16[b]		.134	17.23	2.11	1.02	0.0224	3	17.8	16	.700	8.63
17[b]		.136	10.01	1.92	.65	.01935	3	16.7	30	.462	4.73
5[c]		.058	575	2.61	3.24	.0284	1	8.34	12	2.98	25.4
6[c]		.058	575	2.61	3.24	.0284	1	8.34	13	2.58	24.6

a) 0.4-ft flume (d_{50} = 0.34 mm)

b) 8-ft flume (d_{50} = 0.28 mm)

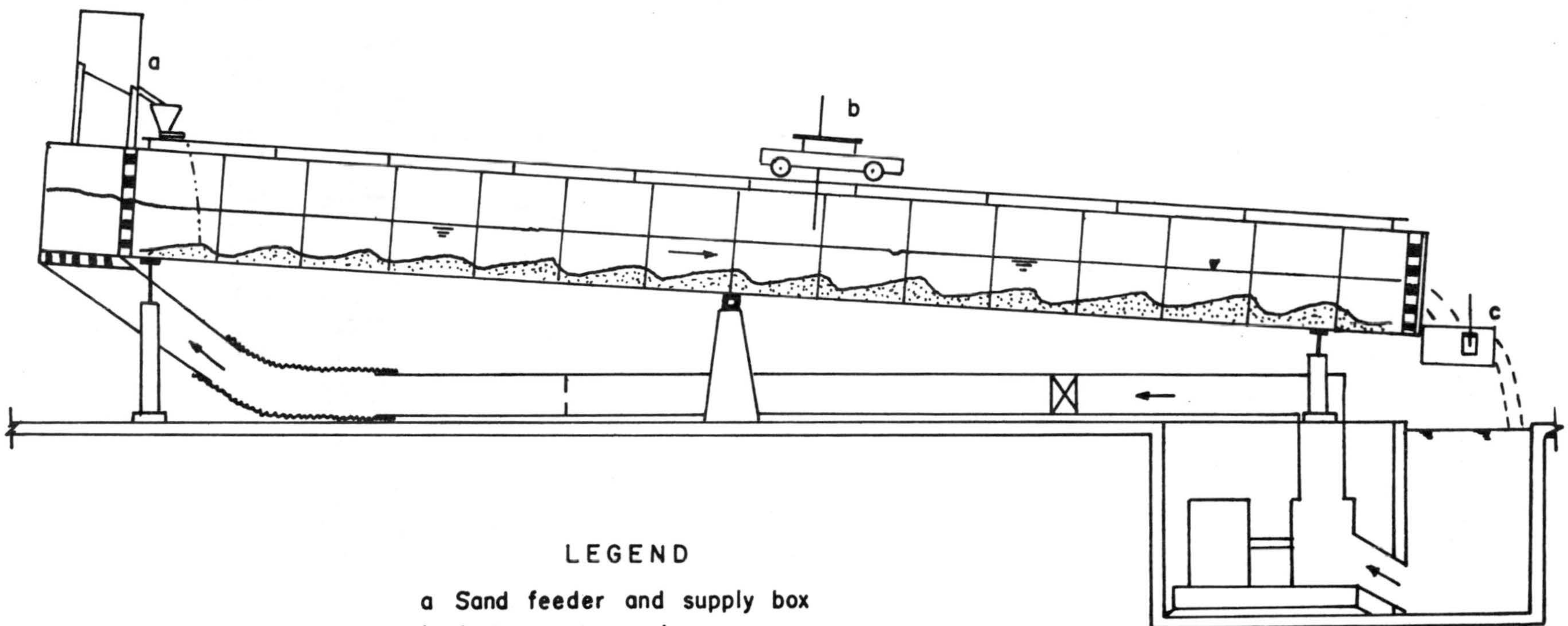
c) Conveyance channel near Bernardo, New Mexico (d_{50} = 0.23 mm)

* Significant wave height (see text)

Table 2.--Summary - Statistical Parameters

Profile	Run 2 α	Run 2 β	Profile	Run 3 α	Run 3 β	Profile	Run 4 α	Run 4 β
1	1.22	0.662	1	1.14	0.372	1	1.225	0.614
2	1.32	.590	2	.961	.459	2	1.320	.660
3	1.32	.576	3	1.27	.560	3	1.280	.567
4	1.32	.583	4	1.061	.414	4	1.341	.584
5	1.34	.618	5	1.040	.340	5	1.220	.544
6	1.33	.596	6	1.032	.590	7	1.165	.638
7	1.35	.617	7	1.032	.564	8	1.255	.645
8	1.36	.600	8	1.040	.412	9	1.14	.683
9	1.28	.512	9	1.109	.517	10	1.114	.557
10	1.25	.574	10	1.072	.531	11	1.120	.660
			11	1.034	.498	12	1.180	.600
						13	1.160	.678
						14	1.142	.588
Mean	1.309	0.593		1.072	0.478		1.205	0.617
	$\overline{C_o} = 0.00228$			$\overline{C_o} = 0.000726$			$\overline{C_o} = 0.00119$	
	$\overline{\sigma^2_\varepsilon} = 0.000483$			$\overline{\sigma^2_\varepsilon} = 0.000264$			$\overline{\sigma^2_\varepsilon} = 0.000326$	
	$\overline{R} = 0.893$			$\overline{R} = 0.797$			$\overline{R} = 0.852$	

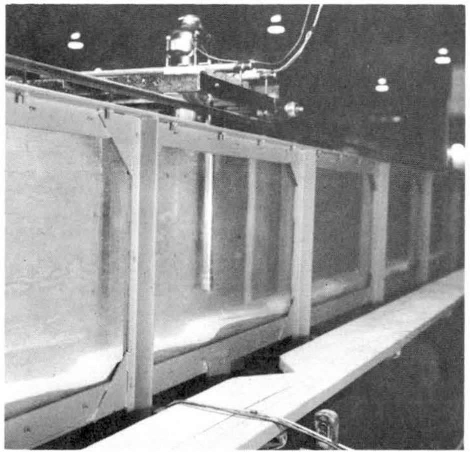
Profile	8' Flume Run 16 α	8' Flume Run 16 β	Profile	8' Flume Run 17 α	8' Flume Run 17 β	Profile	Conveyance Channel α	Conveyance Channel β
1	1.150	0.398	1	1.250	0.392	5	1.095	0.397
2	1.050	.371	2	1.720	.871		$C_o = 0.555$	
3	1.310	.622	3	1.470	.578		$\sigma^2_\varepsilon = 0.184$	
Mean	1.170	0.464	Mean	1.480	0.614		$\overline{R} = 0.822$	
	$\overline{C_o} = 0.03052$			$\overline{C_o} = 0.01333$		6	1.270	0.518
	$\overline{\sigma^2_\varepsilon} = 0.003782$			$\overline{\sigma^2_\varepsilon} = 0.003062$			$C_o = 0.419$	
	$\overline{R} = 0.936$			$\overline{R} = 0.856$			$\sigma^2_\varepsilon = 0.0918$	
							$\overline{R} = 0.887$	



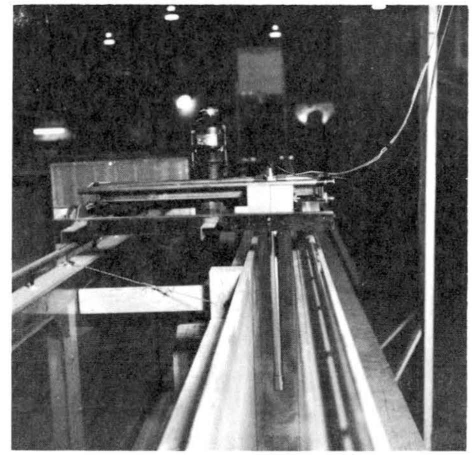
LEGEND

a Sand feeder and supply box
b Instrument carriage
c V-notch weir box

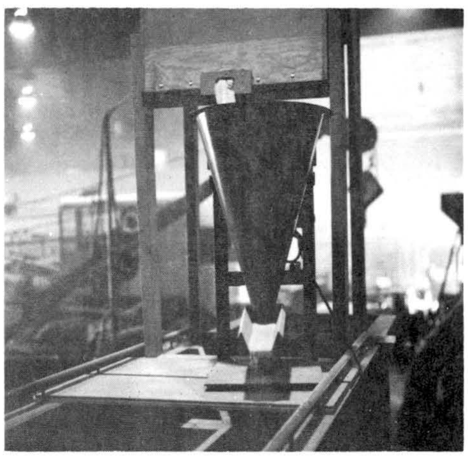
Figure 1.--Schematic diagram of the 2-ft flume



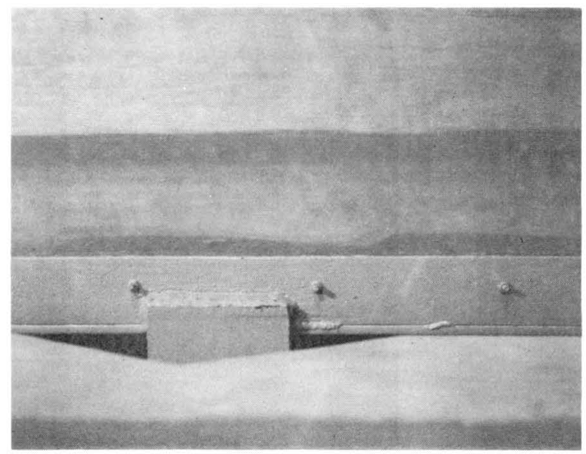
Instrument carriage and dry bed after run 4.



Partition for 0.4-ft. wide channel.



Constant rate feeder and supply box.



Typical view of run 3. Scale 1" = 1 ft.

Figure 2.--Special features of the 0.4-ft. channel.

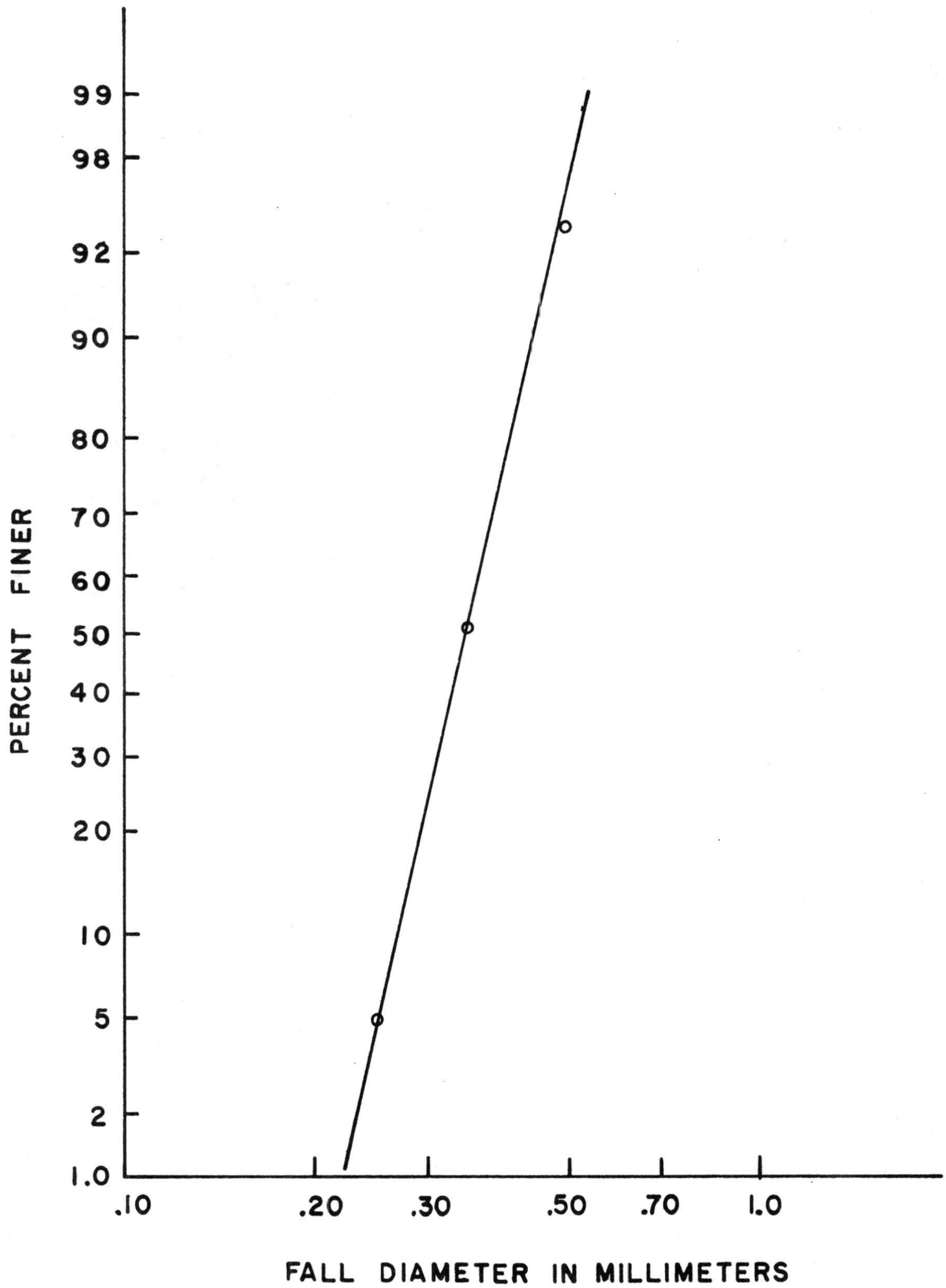


Figure 3.--Size distribution of sand used in the 0.4-ft flume.

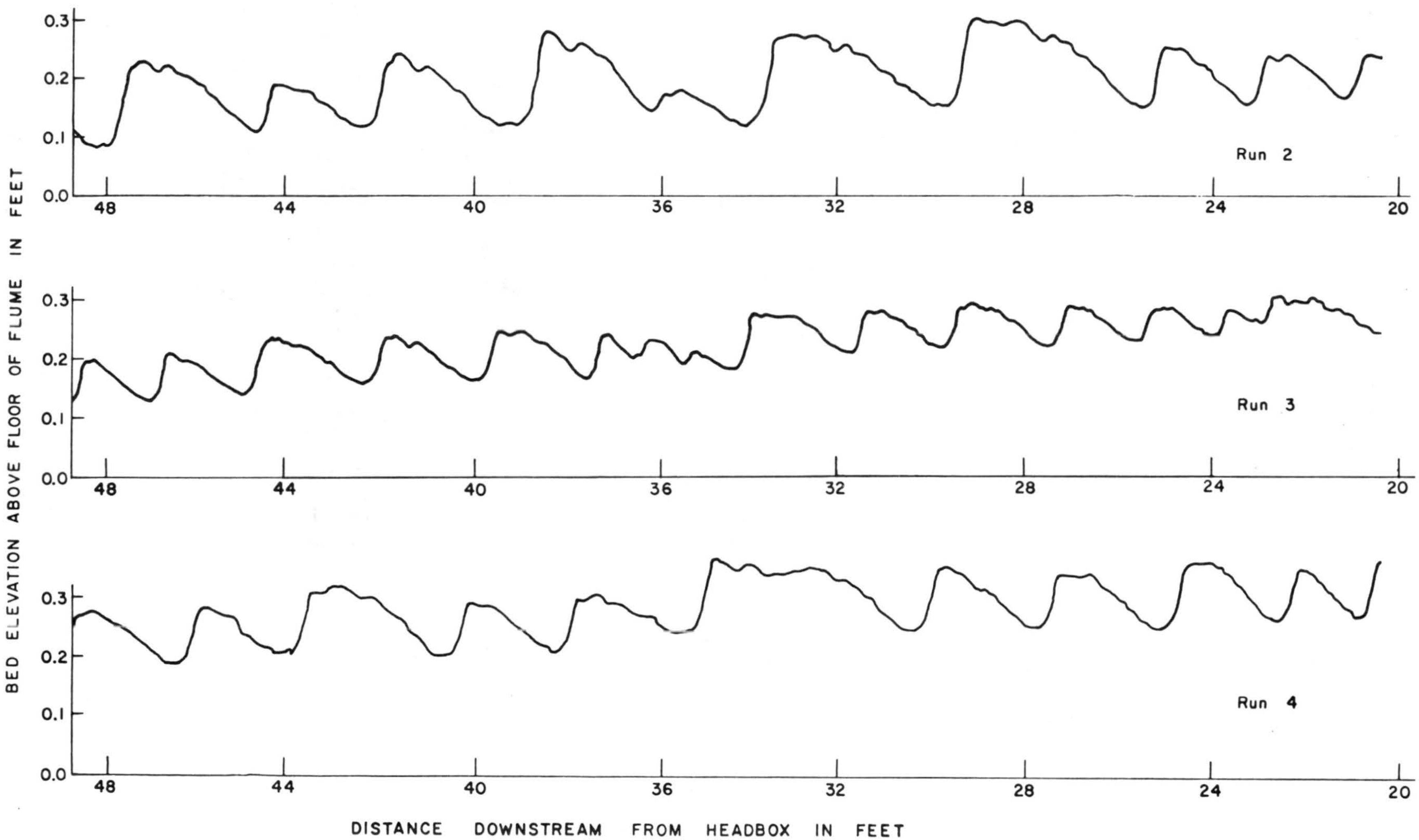


Figure 4.--Typical sonic-sounder profiles for runs 2, 3, & 4.

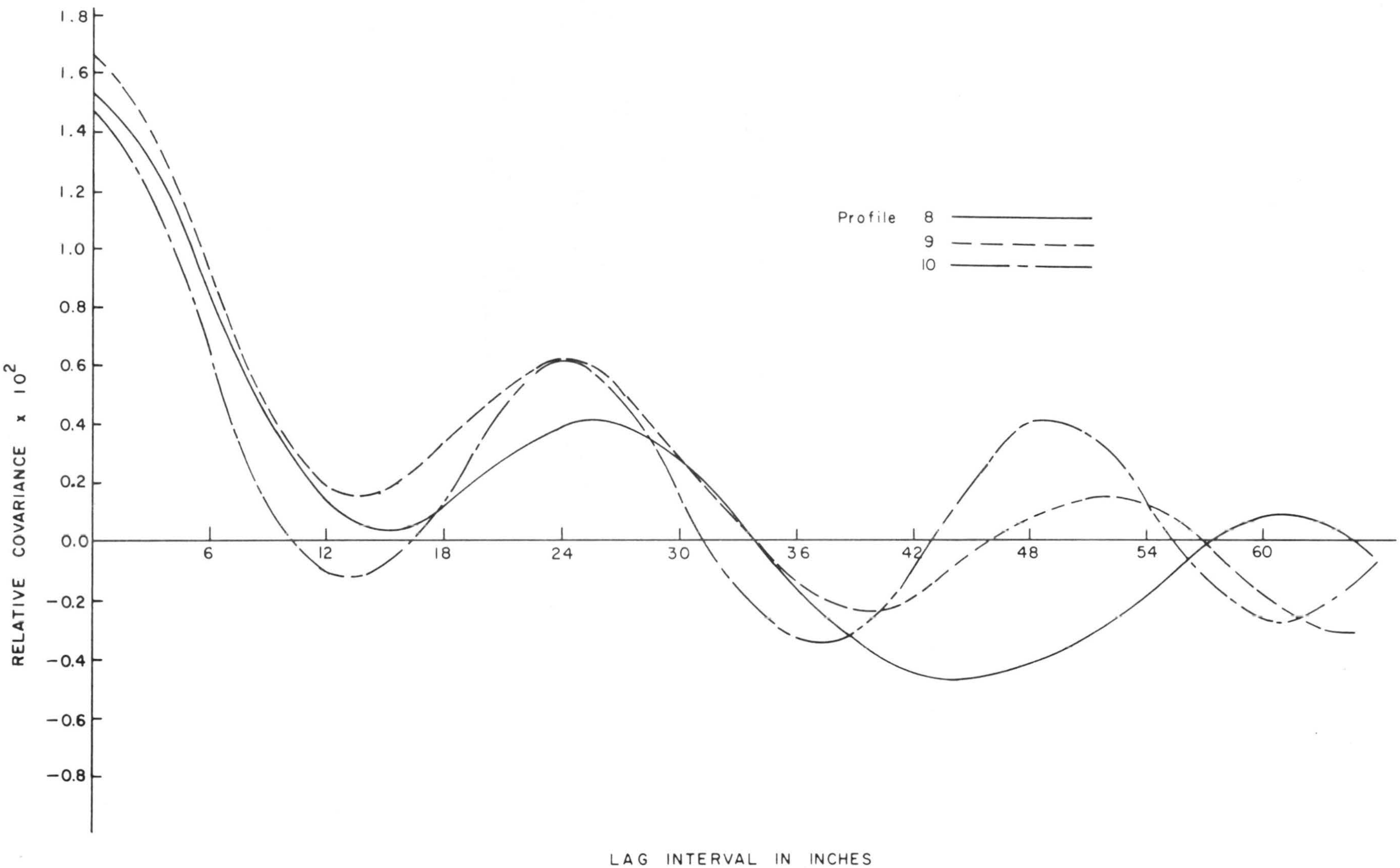


Figure 5.--Typical covariance functions for run 2.

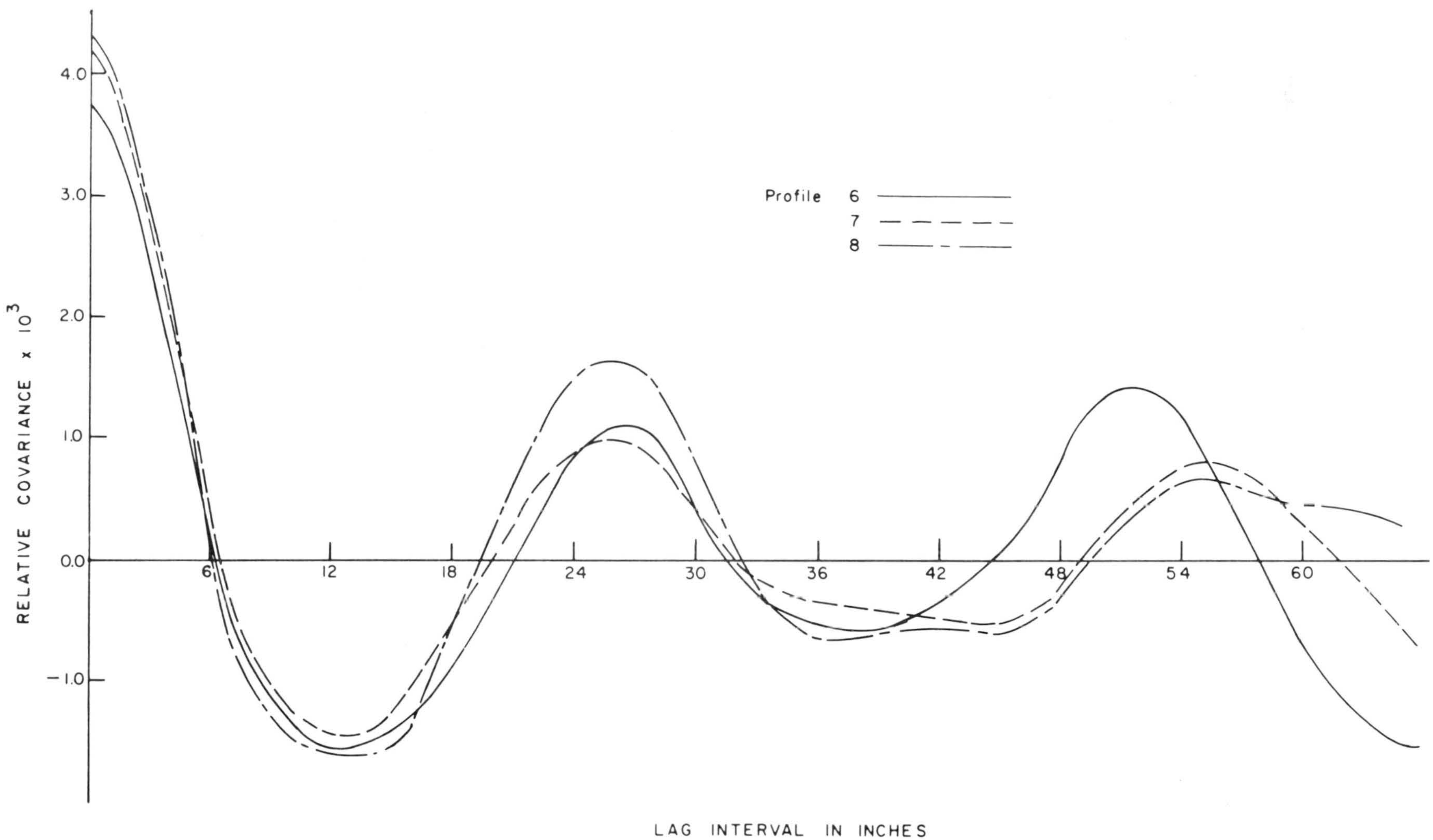


Figure 6.--Typical covariance functions for run 3.

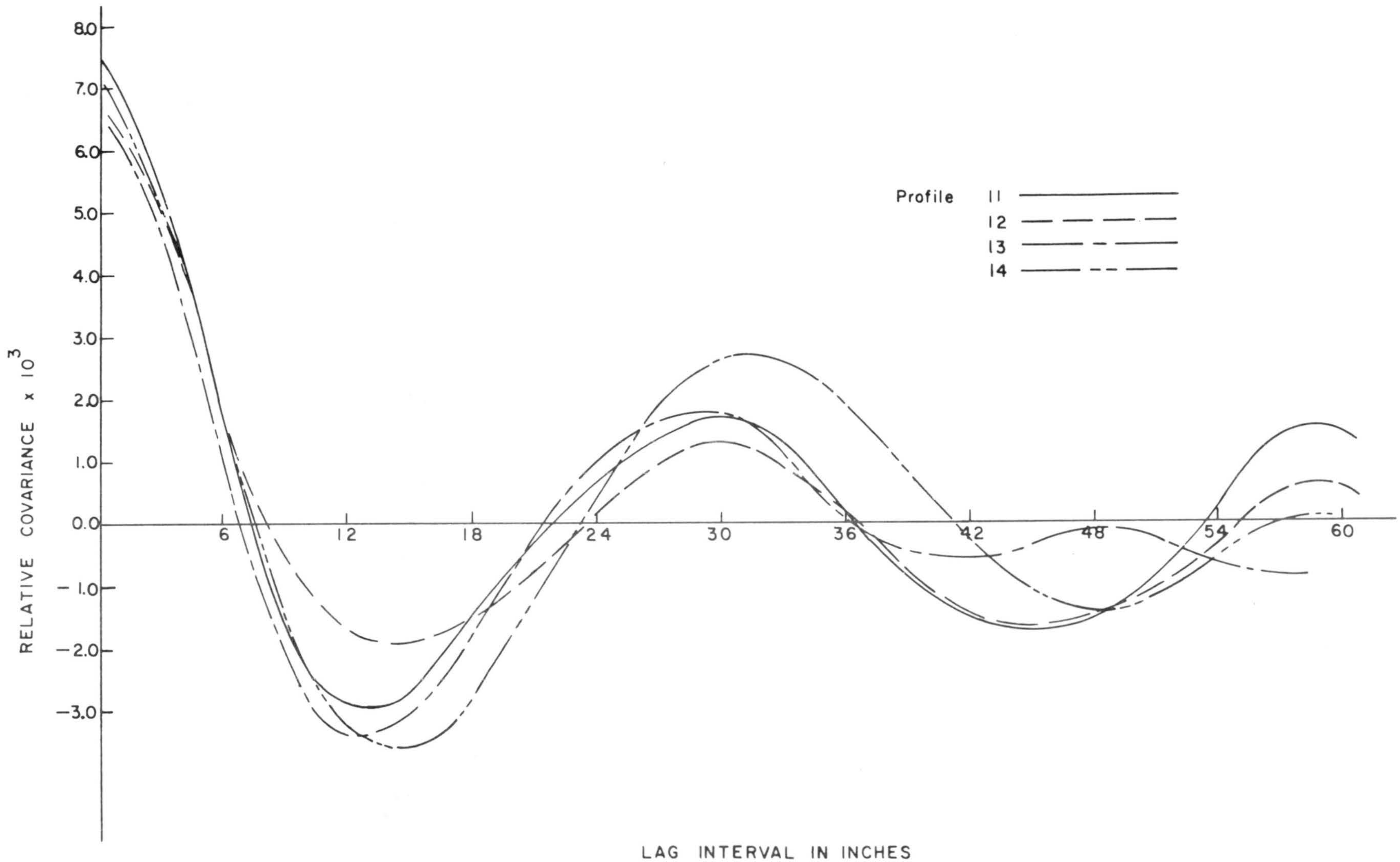


Figure 7.--Typical covariance functions for run 4.

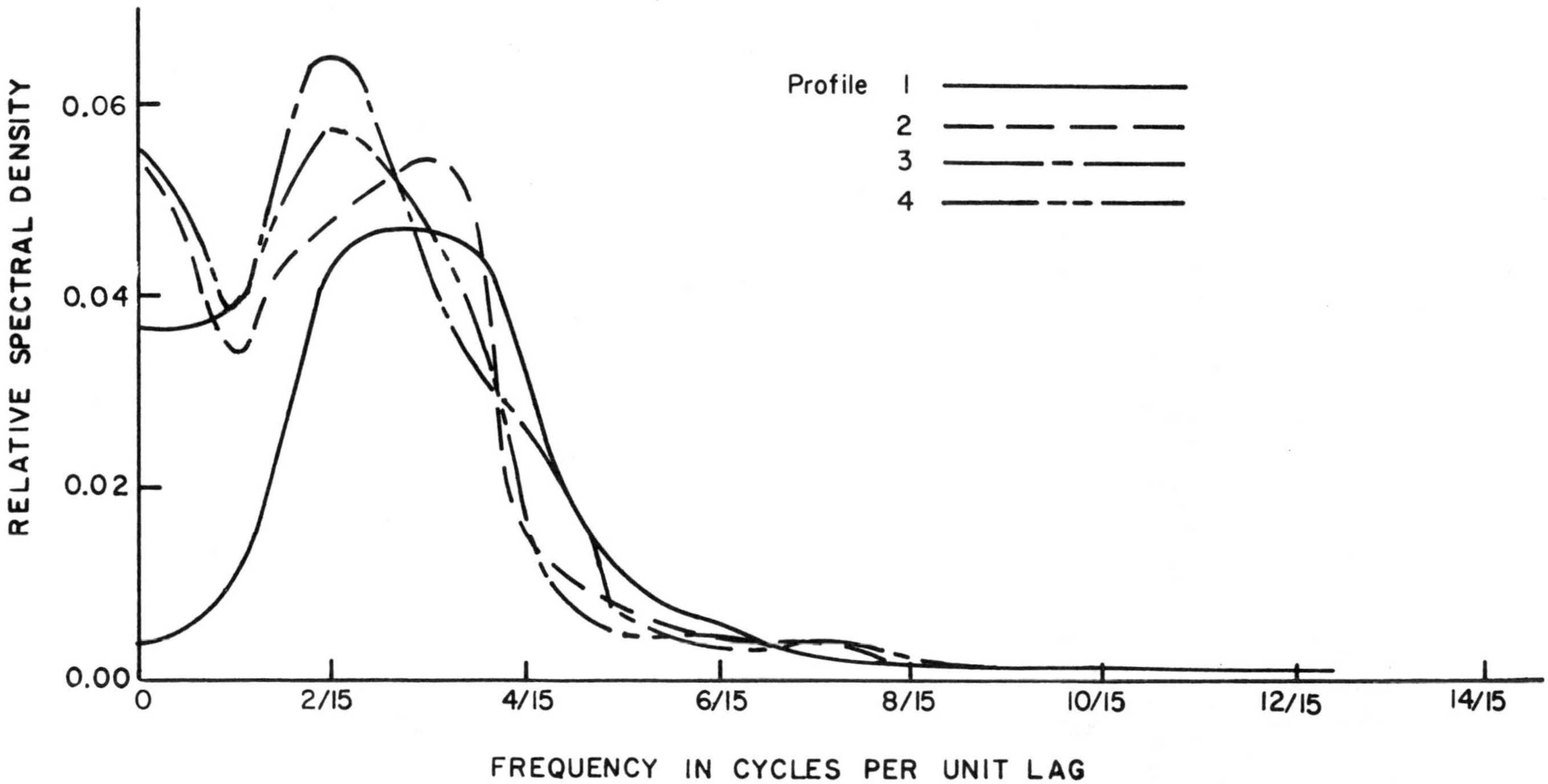


Figure 8.--Typical spectral density functions.

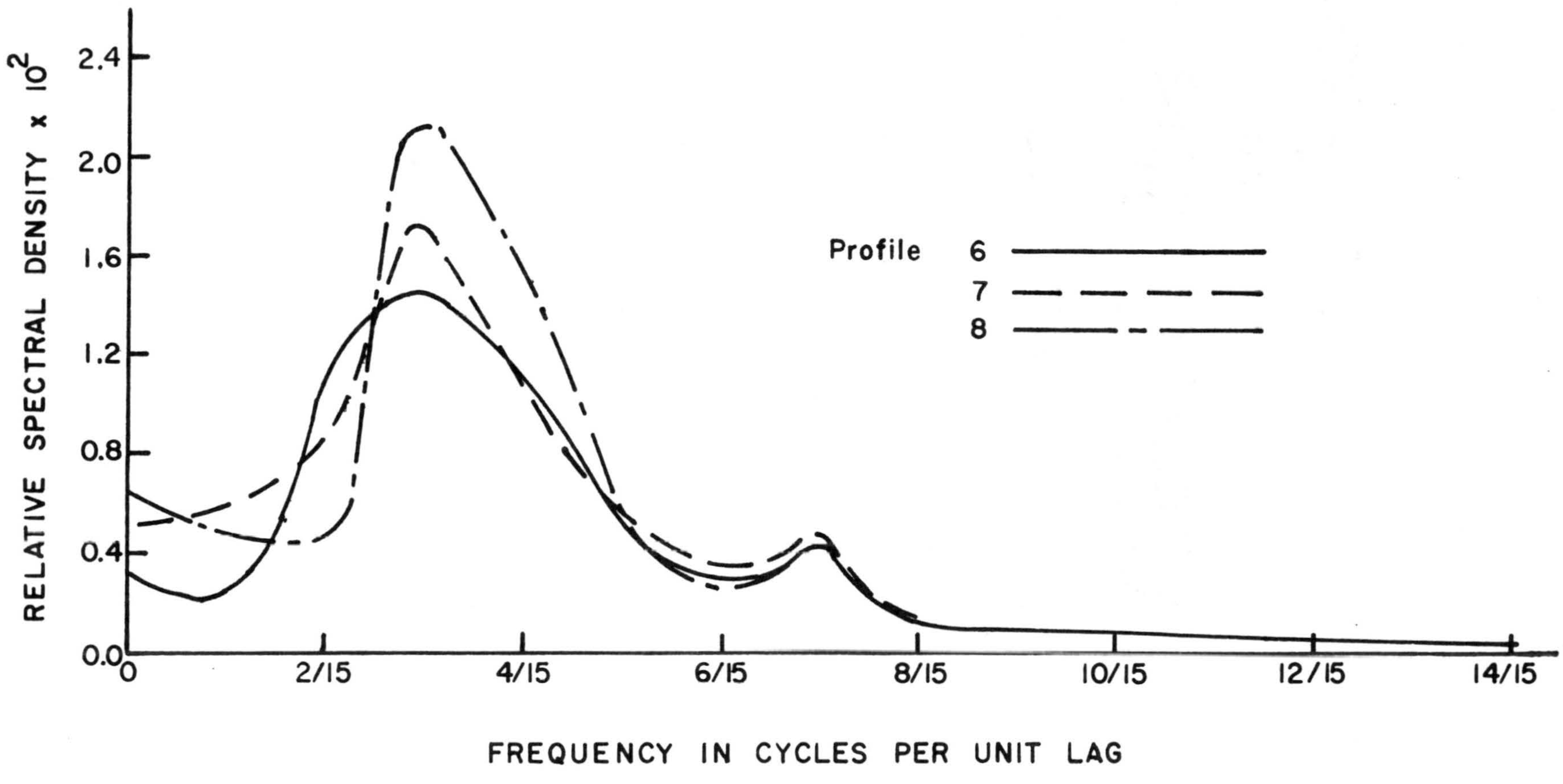


Figure 9. Typical spectral density functions.

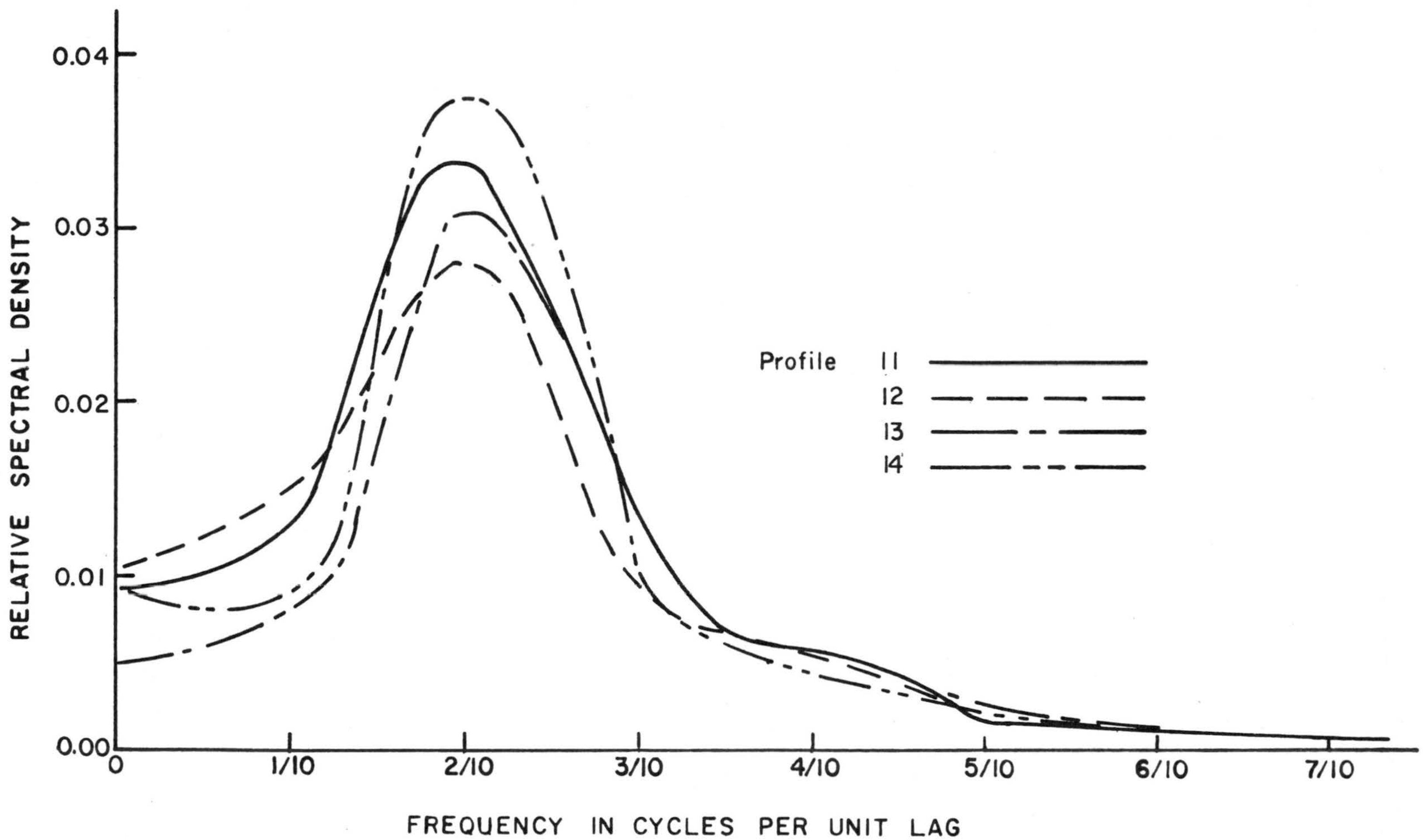


Figure 10.--Typical spectral density functions.

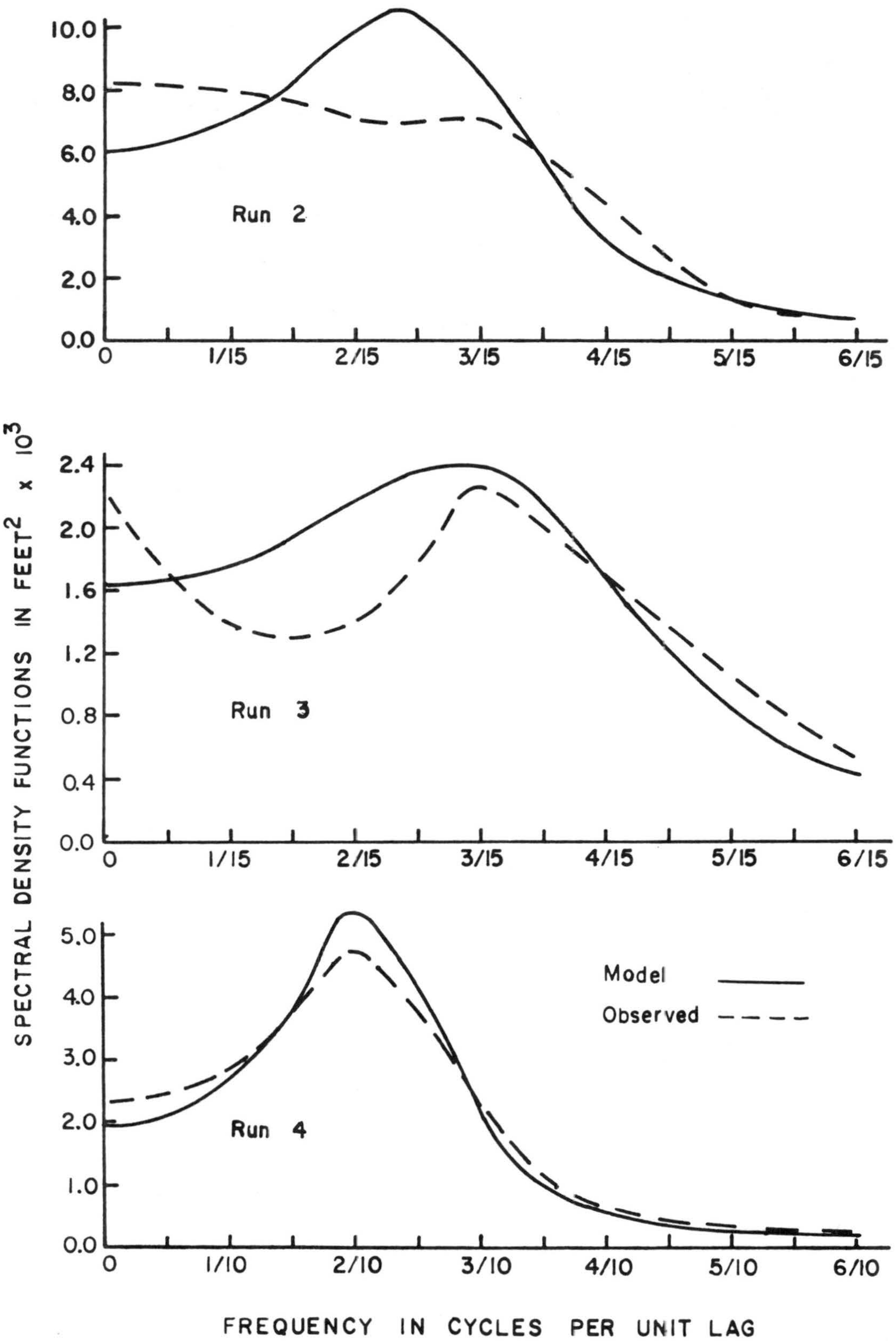


Figure 11.--Models and average values of spectral density functions for runs 2, 3, & 4.

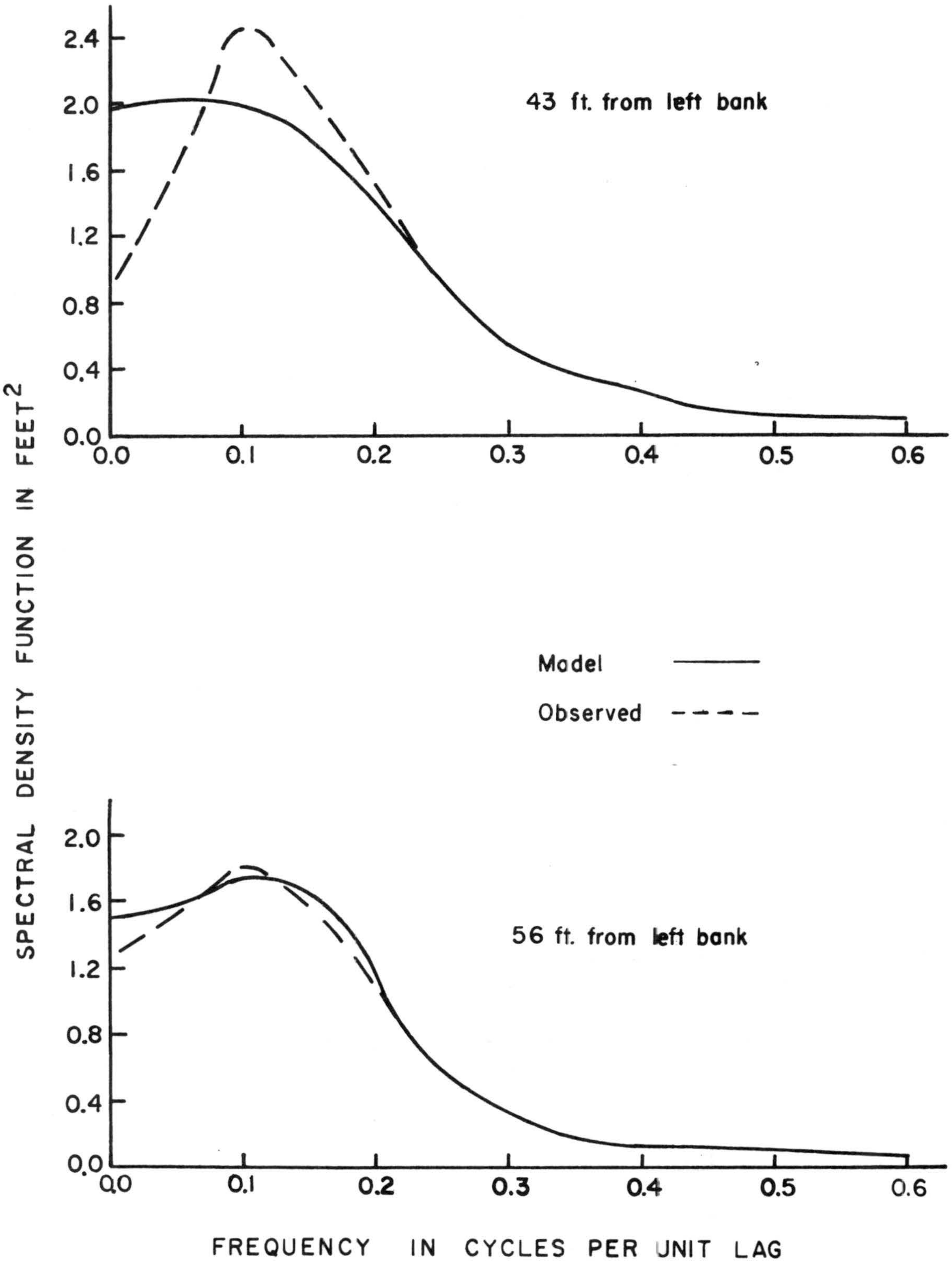


Figure 12.--Models and actual values of spectral density functions for two profiles from the Rio Grande Conveyance Channel near Bernardo, New Mexico.

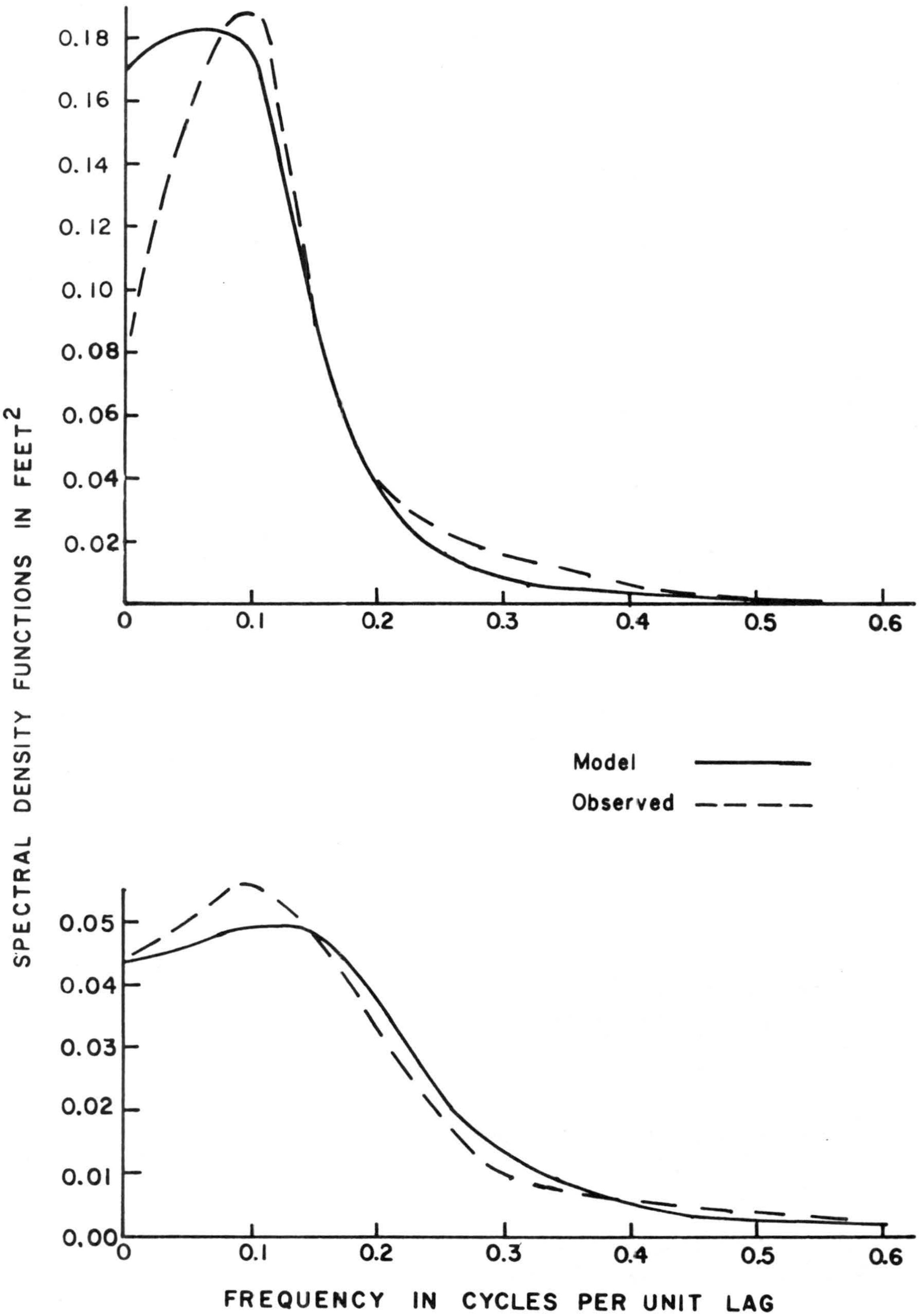


Figure 13.--Models and average values of spectral density functions for the 8-ft flume data.

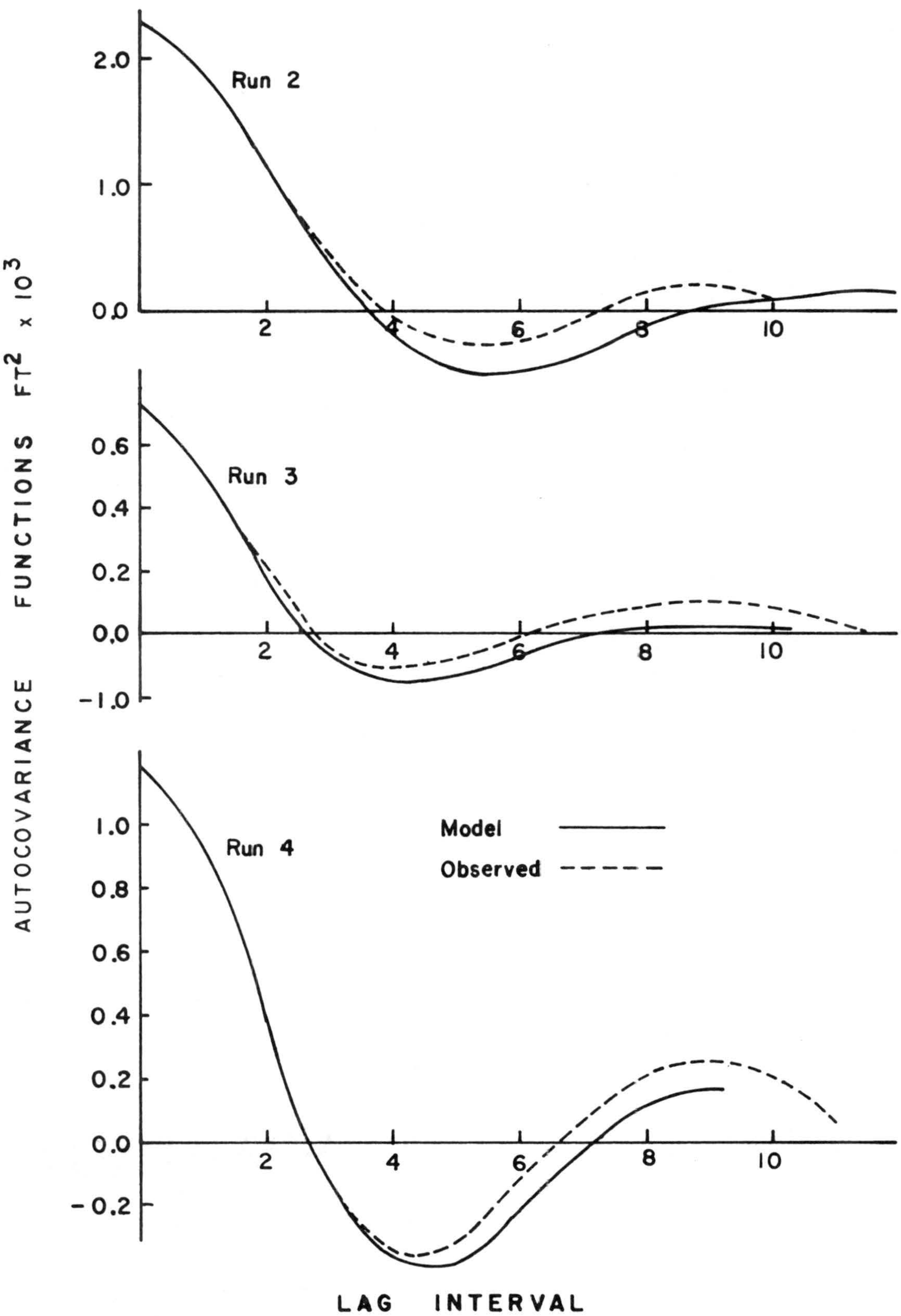


Figure 14.--Models and average values of spectral density functions for the 0.4-ft flume data.

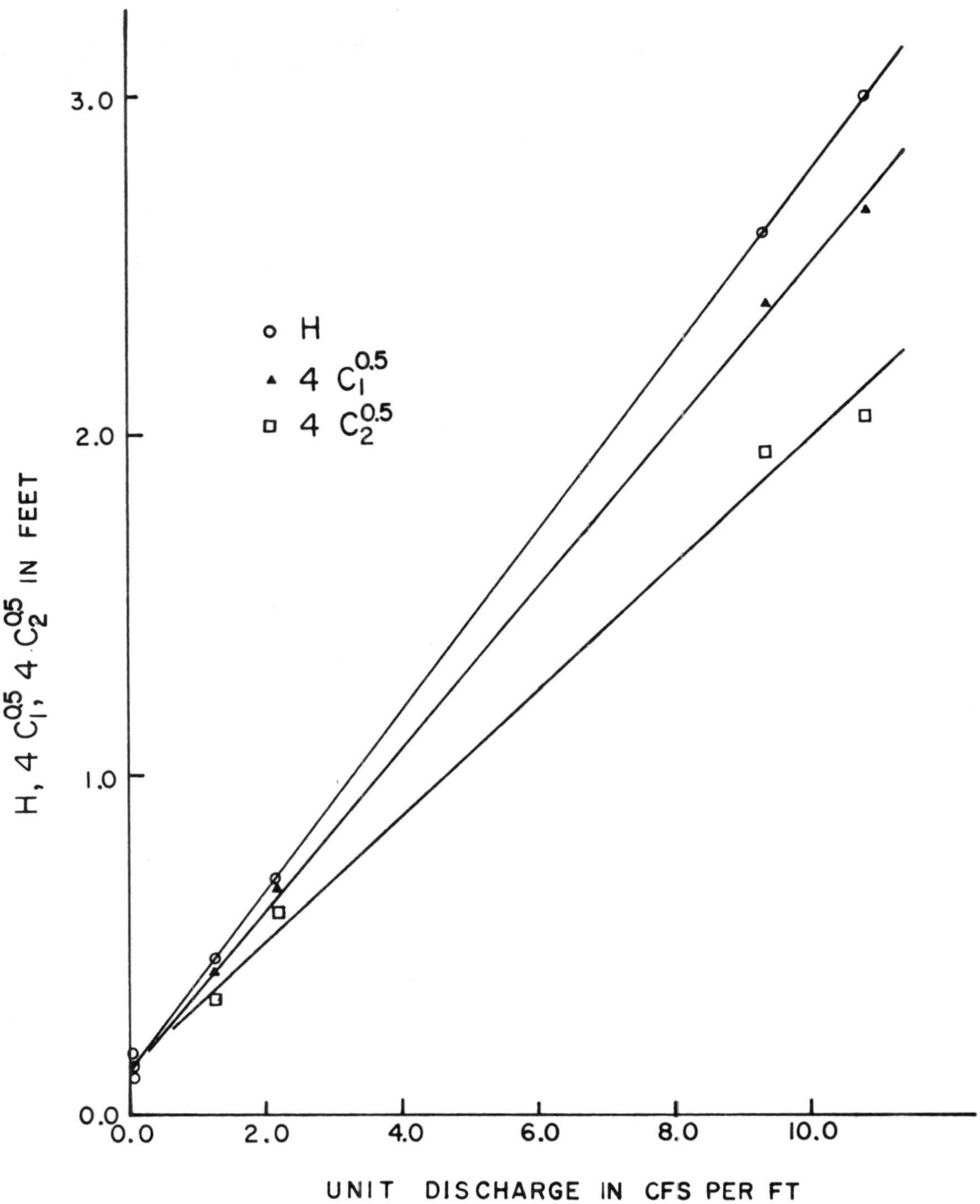


Figure 15.--Significant wave height H, $4C_1^{0.5}$, & $4C_2^{0.5}$ vs velocity times depth for the seven runs used in this analysis.

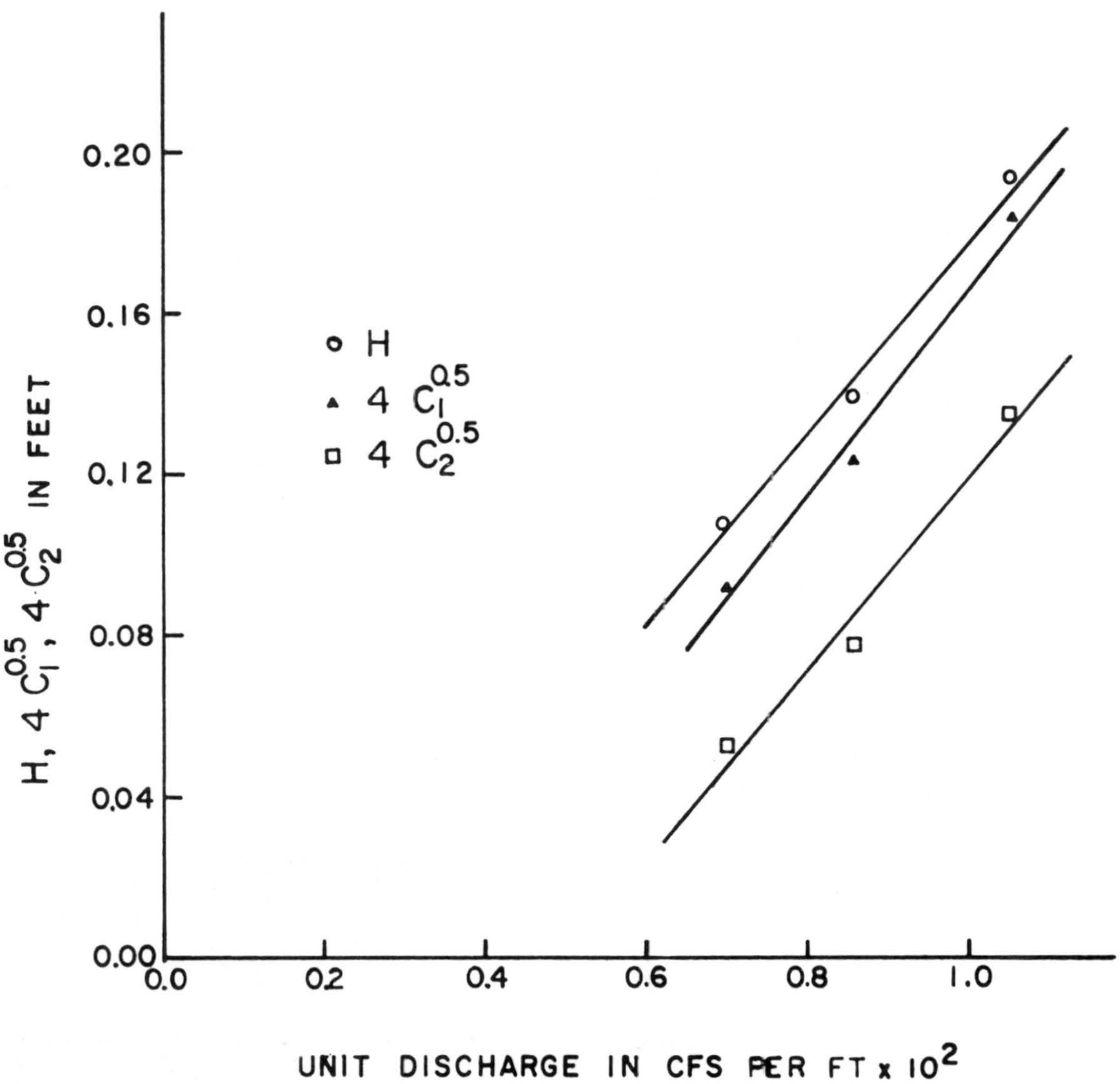


Figure 16.--Significant wave height H, $4C_1^{0.5}$, & $4C_2^{0.5}$ vs velocity times depth for the 0.4-ft flume data.