THESIS

EVENTUALITY-BASED INTERVAL SEMANTICS AND FREE LOGIC: WHAT IF THERE, LIKE, IS NO FUTURE, MAN?

Submitted by

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ABSTRACT

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Future contingent propositions have famously been a source of trouble for philosophers and logicians committed to any variety of indeterminism on which facts about the future are not yet fixed. One possible answer to the problem involves presupposition—namely, that propositions lack truth-value when other propositions that they presuppose are false. This paper explores the plausibility of such an answer, beginning with a brief discussion of the problem of future contingent propositions and presupposition. From there, an in-depth discussion of Free Logic lays the groundwork of logical tools for the project, exploring the motivation for Free Logic's development and examples of Free Logic semantics. Subsequently, this paper discusses the history and usefulness of events-based semantics in analyzing English sentences. Using the tools of events-based semantics and formal logic, this paper formally models this approach to sentences in English by defining a semantics which can capture both tense and aspect of such sentences and which allows for truth-valueless future contingent propositions while preserving logical truths like the law of excluded middle.

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CHAPTER 1

1.1 Introduction

In standard systems of logic, the principle of excluded middle (EM) states that, given any proposition *p*, either *p* is true or its formal negation ¬*p* is true–colloquially, all propositions are either true of false. Those philosophically committed to the view that the future is open and its facts are not yet fixed, then, face an interesting dilemma when confronted with propositions about the future. Take a (contingent) future proposition, "I will visit New York next week." If that proposition has truth-value now, then there seems to be no basis to think the future is open with respect to my visiting New York next week. If it is true, then I cannot fail to visit New York; if it is false, then I *must* fail to visit New York next week. This, then, would hold for all future contingent propositions. If all such propositions have truth-value now, then a complete description of every truth in the universe *now* must therefore entail a complete description of the universe at any point in the future. Hence, there is only one possible future, and determinism must be true. But suppose we hold the view that (contingent) future propositions are, as of now, neither true nor false–that it is genuinely, metaphysically open whether or not I will visit New York next week. If future contingents lack truth-value, then for any contingent future proposition *r, r* is neither true nor false. If *r* is neither true nor false, then, ¬*r* is also neither true nor false.[1] If neither '*r*' nor '¬*r*' is true, then the classically valid formula '*r* v ¬*r*' is

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1. One might object that if r is neither true nor false, then in particular it is not true, so ¬p would be true. To endorse this approach is to reject more features of classical logic—in this case, double negation. On classical models, if the formula '¬p' is true, the formula '¬¬p' is false, and if '¬¬p' is false, then by double negation, 'p' is false.

not true, thus EM is not valid. The following positions, then, appear to be in tension: (a) EM is logically valid, and (b) some propositions about the future are neither true nor false.

1.2 Presupposition

In his work *On Referring*, Peter Strawson argues against the Russellian analysis of the following sentence *s*:

s The present king of France is bald.

On Bertrand Russell's analysis, *s* is best analyzed as a disguised existential claim that there presently exists a king of France, there is exactly one king of France, and the king of France is bald. Translated into first order logic, the Russellian analysis of *s* reads as s_L:

s_L $\exists x(KFx \wedge \forall y(KFy \rightarrow y=x) \wedge Bx)$.

The Strawsonian analysis agrees with the Russellian analysis that *s* is meaningful and that anyone who utters *s* would be uttering a true statement if and only if there actually existed at the time of utterance exactly one king of France and that king of France was bald. But on the Russellian analysis, anyone who utters it when there exists no king of France would be making a false assertion.

Strawson disagrees, arguing that someone who utters it need not be saying something true or something false. If someone were to seriously and genuinely say to you, "The king of France is bald," Strawson points out that "That's untrue" would be an unnatural and inappropriate response; the most natural response would be to say, "You're confused. France is a democratic republic. It has no king." This isn't to contradict *s*, but to point out that some other proposition that *s* takes for granted is false. Presupposition, then, can be understood in the following way: some sentence (a) presupposes some sentence (b) when, if sentence (a) is either true or false, then sentence (b) is true. Sentence *s*, then, can be said to presuppose sentence *s2*:

s2 There presently exists a king of France.

While definite descriptions are perhaps the most well-known species of presupposition, others have been established in the literature on presupposition and pragmatics. Take, for example, the following sentences:

s3 Steve thought his sister was a good singer.

s4 Joshua believed that Kate knew where he parked his car.

Sentence *s3* presupposes that Steve has a sister (or, minimally, that Steve believes he has a sister). Likewise, *s4* presupposes that Joshua has a car (or, minimally, Joshua believes he has a car), and plausibly that Joshua (believes he) knows someone named Kate. *s3* is true only if Steve believes he has a sister; *s4* is true only if Joshua believes that he has a car, and that Joshua (believes that) knows someone named Kate. When a proposition—in this case, *s*, *s3*, or *s4*—presupposes the truth of some other proposition (or propositions) which happens to be false, then the former propositions are said to lack truth-value. This phenomenon is called presuppositional failure.

1.3 *Project Outline*

In this paper, in addition to the existence of objects like tables and chairs, I assume a realist view of two other types of entities. First, I take for granted a realist view towards the existence of time past and present, but antirealist view towards the existence of the future. I also take for granted a realist attitude toward the ontological status of events. Given these two assumptions, I contend that presupposition can plausibly account for why future contingent propositions are truth-valueless, and, using tools from free logic, build a system of semantics according to which future contingents lack truth-value, but EM is still valid. In particular, I argue that tensed propositions presuppose but do not assert the existence of the time during which the events asserted are said to take place, and these presuppositions fail when someone utters a contingent proposition about the future. My paper takes the following structure: In chapter II, I explain and motivate free logic, which plays a pivotal role in my semantics. In chapter III, I discuss the history of events-based semantics and sketch out my own system of eventuality semantics. In chapter IV, I formally combine events-based semantics with positive free logic to produce a system that (1) can express verb tense and aspect of English sentences and so capture the distinction between sentences that differ only in tense, (2) preserves the validity of EM, and (3) results in no truth-value for contingent propositions about the future.

CHAPTER 2

2.1 Defining and Motivating Free Logic

Classical predicate logic (henceforth CPL) assumes both that all terms (or names) in a language refer to some member of the domain, and that the domain is non-empty.[2] Systems of logic that dispense with the first assumption and sometimes the second are called free logics, because their terms are "free of existential assumptions" but their "quantifiers retain existential force."[3]

Consider an illustration of the problem of empty terms for CPL. The rule of existential generalization states roughly that if so-and-so has some property, then there exists something that has that property. More formally:

(EG) $$\Pi\alpha \rightarrow \exists\xi\, \Pi\, \xi$$

For instance, if it's true that "Steve is a brewer", then it must be true that "*Something* is a brewer" (or more elegantly in English, "Someone is a brewer"). The model-theoretic approach to logic captures this intuition by validating (EG)–that is, any formula matching the schema (EG) evaluates to true on all models of CPL. Given (EG), it's clear CPL assumes that all names refer: (EG) is valid if and only if there can be no occasions in which the inference from '$\Pi\alpha$' to '$\exists\xi\, \Pi\, \xi$' fails. Put another way, CPL assumes that for *any* name used in a formula in the place of 'α', there is some member of the domain that that formula is talking about.

2. Textbooks in classical predicate logic define the domain as a non-empty set.

3. Karel Lambert, *Philosophical Applications of Free Logic*. (New York: Oxford Univ. Press, 1991).

As a natural language, English contains names that are non-referring, names like 'the putative planet Vulcan' and 'Harry Potter'. Furthermore, the sentence "Harry Potter is a wizard" seems to be an instance of the schema $\Pi\alpha$. If so, then (EG) apparently licenses the inference from "Harry Potter is a wizard" to 'There exists something that is a wizard'–an inference that we surely want to avoid, since Harry Potter is fictional and there are no wizards, even though intuitively "Harry Potter is a wizard" is true.[4] Some philosophers may want to resist the claim that "Harry Potter is a wizard" is strictly true on the grounds that Harry Potter is fictional. But consider the sentence "The Greeks worshipped Zeus." Sentences like this seem to avoid the fiction-based objection. We may want to agree that "The Greeks worshipped Zeus" is true without being committed to there existing something that the Greeks worshipped.[5] Logicians and philosophers have proposed a variety of solutions to avoid consequences like this. One option involves challenging our understanding of the logical structure of terms, a la Russell and Quine. Another solution might be to deny that terms really are empty at all. Yet another option involves changes to existential assumptions and quantification. Selecting the third option, some logicians have proposed altering semantics and proof theory to make space for empty terms.

Along with assuming that all terms in the language refer, CPL also assumes a non-empty domain. This amounts to the assumption that *something exists*. On its face, it's plausible that the truth that anything exists at all is contingent. Even if there is some deep physical or metaphysical reason why something must exist, it may well be a physical or metaphysical truth

4. The ontology of fictional characters has been a robust source of controversy, particularly in the 20th century. See Betrand Russell, "On Denoting", *Mind* 14, No. 56 (1905): 473–493.

5. See Andrew Bacon, "Quantificational Logic and Empty Names," *Philosophers' Imprint* 13, no. 4 (2013): 1-21.

rather than a logical one. One might reasonably want logic to be silent on philosophical questions of existence rather than prejudge the issue. Logics that allow for an empty domain are sometimes called "inclusive" or "universally free" logics. Whereas all systems of inclusive logic are free logics, not all free logics are inclusive.

CPL's prohibition of both empty domains and empty names is not an accident. To see this, consider how CPL might evaluate the truth of a formula matching the schema Πα. A CPL model is an ordered pair $<\mathcal{D}, \mathcal{I}>$ where $\mathcal{D}$ is the domain (a non-empty set) and $\mathcal{I}$ is the interpretation function such that (1) if α is a constant, then $\mathcal{I}(\alpha) \in \mathcal{D}$, and (2) if Π is an n-place predicate, then $\mathcal{I}(\Pi)$ is an n-place relation over $\mathcal{D}$. It may further be stipulated that for every $d \in \mathcal{D}$, d has a name. This already generates a problem: from (1), there is an interpretation of a constant α, and $\mathcal{I}(\alpha) \in \mathcal{D}$. But if $\mathcal{D} = \{\}$, then for any object d, $d \notin \mathcal{D}$, so in particular $\mathcal{I}(\alpha) \notin \mathcal{D}$. Hence, such a system results in $\mathcal{I}(\alpha) \in \mathcal{D}$ and $\mathcal{I}(\alpha) \notin \mathcal{D}$. In systems like this, given any language containing individual constants, the combination of empty domains with the stipulation that for any constant α, $\mathcal{I}(\alpha) \in \mathcal{D}$ results in contradiction.

None of this is to say that free logic is the only or best solution to problems arising from empty names, nor that logic *mustn't* presume existence. However, free logic is a serious development in modern logic and constitutes a plausible solution to the problems illustrated above. Since Karel Lambert pioneered free logic in the middle of the 20th century, three distinct varieties have emerged. Positive free logics are those wherein *some* formulas containing at least one non-referring term–henceforth "empty-termed formulas"–are true. For instance, in a positive free logic, '*Πα'* might be truth-valueless if '*α*' is non-referring, yet '*α = α*' might still be true. Neutral free logics are those wherein all empty-termed atomic formulas are truth-valueless (with the possible exception of statements like '*α exists*'). Negative free logics are those in which all such empty-termed atomic formulas are false.

It is important to distinguish bivalent free logics which allow for truth-valueless formulas from trivalent logics whose third value is "undetermined", "middle", or some equivalent (where '0' corresponds to false, '1' to true, and '#' to the third value). In trivalent systems, a valuation function $\mathcal{V}_{\mathcal{M}}$ assigns truth-values to atomic formulas such that some formulas map to '0', other formulas to '1', and the remaining to '#'. In contrast, the free logic systems under discussion in this paper are bivalent. The valuation functions of positive and neutral systems are partial functions because some empty-termed atomic formulas are not mapped to a truth-value. The valuation functions of negative logics typically are total functions; all empty-termed atomic formulas are mapped to '0'.

2.2 Grammar of Free Logic

As seen above, formulas matching the schema (EG) are valid simpliciter in CPL, but not in free logic. The systems considered in this paper, then, are weaker than CPL because no formulas invalidated by CPL are validated by free logic.[6] Let us turn to the grammar of free logic, starting with a language $\mathcal{L}$, which consists of the following pieces of CPL:

- Primitive logical operators '→','¬', '=', and '∀', from which the other operators are defined in the usual way.
- Variables '*x*', '*y*', '*z*', with or without numerical subscripts.

6. Some forms of free logic constitute *extensions* of classical logic. See, for instance, Willard Van Orman Quine, *Word and Object.* (Cambridge: MIT Press, 1960). Such logics are not under consideration here.

- For each $n > 0$, n-place predicates 'P', 'Q', 'R', ... with or without numerical subscripts.
- Individual constants (terms) 'a', 'b', 'c', with or without numerical subscripts.

Given the operators, predicates, constants, and variables, we can now define well-formed formulas (WFFs):

- If Π is an n-place predicate and $\alpha_1 \ldots \alpha_n$ are terms, then $\Pi\alpha_1 \ldots \alpha_n$ is a formula
- If ϕ is a formula, then ¬ϕ is a formula
- if ϕ and ψ are formulas, then ϕ → ψ is a formula
- If α and β are terms or variables, α = β is a formula
- If α is a variable and ϕ is a formula, then ∀αϕ is a formula
- Nothing else is a formula[7]

2.3 Semantics for a Neutral and Negative Free Logic

Now we look at a model theory for neutral free logic in $\mathcal{L}$. NUS_1 consists of an ordered pair $<\mathcal{D}, \mathcal{I}>$. $\mathcal{D}$ is a (possibly empty) set of existent objects. $\mathcal{I}$ is the interpretation function such that for every individual constant α in $\mathcal{L}$, $\mathcal{I}(\alpha) \in \mathcal{D}$ when $\mathcal{I}(\alpha)$ is defined, for every n-place predicate Π in $\mathcal{L}$, $\mathcal{I}(\Pi)$ is a set of n-tuples of $d \in \mathcal{D}$, and every member of $\mathcal{D}$ has a name. For any given model $<\mathcal{D}, \mathcal{I}>$, a valuation function $\mathcal{V}_{\mathcal{M}}$ assigns truth-values to formulas as follows:

I. $\mathcal{V}_{\mathcal{M}}(\Pi\alpha_1 \ldots \alpha_n)$ = true if each of $\mathcal{I}(\alpha_1), \ldots, \mathcal{I}(\alpha_n)$ is defined and $<\mathcal{I}(\alpha_1), \ldots, \mathcal{I}(\alpha_n)> \in \mathcal{I}(\Pi)$; false if each of $\mathcal{I}(\alpha_1), \ldots, \mathcal{I}(\alpha_n)$ is defined and $<\mathcal{I}(\alpha_1), \ldots, \mathcal{I}(\alpha_n)> \notin \mathcal{I}(\Pi)$; undefined (i.e. $\Pi\alpha_1 \ldots \alpha_n$ is truth-valueless) if any of $\mathcal{I}(\alpha_1), \ldots, \mathcal{I}(\alpha_n)$ is undefined.

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7. Formally, the '→' operator is base operator from which all other operators are defined. Formulas containing other operators are used informally but are understood to be shorthand for much longer formulas containing only '→'.

II. $\mathcal{V}_{\mathcal{M}}(\neg\phi)$ = true if $\mathcal{V}(\phi)$ = false, false if $\mathcal{V}_{\mathcal{M}}(\phi)$ = true, and truth-valueless if $\mathcal{V}_{\mathcal{M}}(\phi)$ is truth-valueless.

III. $\mathcal{V}(\phi\rightarrow\psi)$ = true if either $\mathcal{V}_{\mathcal{M}}(\phi)$ = false or $\mathcal{V}_{\mathcal{M}}(\psi)$ = true, false if $\mathcal{V}_{\mathcal{M}}(\phi)$ = true and $\mathcal{V}_{\mathcal{M}}(\psi)$ = false, and truth-valueless otherwise.

IV. $\mathcal{V}_{\mathcal{M}}(\forall\alpha\ \phi)$ = true if for all d ∈ $\mathcal{D}$, $\mathcal{V}_{\mathcal{M}(\beta,d)}(\phi(\beta/\alpha))$ = true (where β does not appear in ϕ).

V. $\mathcal{V}_{\mathcal{M}}(\alpha = \beta)$ = true if $\mathcal{I}(\alpha)$ and $\mathcal{I}(\beta)$ are both defined, and $\mathcal{I}(\alpha) = \mathcal{I}(\beta)$; false if $\mathcal{I}(\alpha)$ and $\mathcal{I}(\beta)$ are both defined, and $\mathcal{I}(\alpha) \neq \mathcal{I}(\beta)$; truth-valueless if either $\mathcal{I}(\alpha)$ or $\mathcal{I}(\beta)$ is undefined.

That NUS_1 is a neutral logic is evident given clauses I and V, since empty-termed atomic formulas are truth-valueless. A negative logic (NES_1) would differ from NUS_1 by replacing clauses I, II, and V as follows:

I'. $\mathcal{V}_{\mathcal{M}}(\Pi\alpha_1 \ldots \alpha_n)$ = true if each of $\mathcal{I}(\alpha_1), \ldots, \mathcal{I}(\alpha_n)$ is defined and $<\mathcal{I}(\alpha_1), \ldots, \mathcal{I}(\alpha_n)> \in \mathcal{I}(\Pi)$; and false otherwise.

II'. $\mathcal{V}_{\mathcal{M}}(\neg\phi)$ = true if $\mathcal{V}(\phi)$ = false, and false if $\mathcal{V}_{\mathcal{M}}(\phi)$ = true.

III'. $\mathcal{V}_{\mathcal{M}}(\phi\rightarrow\psi)$ = true if either $\mathcal{V}_{\mathcal{M}}(\phi)$ = false or $\mathcal{V}_{\mathcal{M}}(\psi)$ = true, and false otherwise.

V'. $\mathcal{V}_{\mathcal{M}}(\alpha = \beta)$ = true if $\mathcal{I}(\alpha)$ and $\mathcal{I}(\beta)$ are both defined, and $\mathcal{I}(\alpha) = \mathcal{I}(\beta)$; and false otherwise.

One noteworthy consequence of NUS_1 and NES_1 is that identity theory is non-classical, since α = α is valid in neither. Furthermore, instances of NC and EM (e.g. ¬(Πα ∧ ¬Πα) and Πα v ¬Πα), which are classically valid, are not validated in NUS_1.

To see this, consider an instance of excluded middle: ‘P*t* v ¬P*t*’. On a classic model, a valuation assigns one of two truth-values to all atomic formulas. On any model, if, for the first disjunct 'Pt', $\mathcal{V}_{\mathcal{M}}$ (Pt) = false, then for the second disjunct '¬Pt', $\mathcal{V}_{\mathcal{M}}$ (¬Pt) = true, thus for the whole disjunction 'Pt v ¬Pt', $\mathcal{V}_{\mathcal{M}}$(P$t$ v ¬Pt) = true. Likewise, if $\mathcal{V}_{\mathcal{M}}$ (Pt) = true and $\mathcal{V}_{\mathcal{M}}$ (¬Pt) = false, then $\mathcal{V}_{\mathcal{M}}$(P$t$ v ¬Pt) = true. $\mathcal{V}_{\mathcal{M}}$(P$t$ v ¬Pt) = true no matter the truth-value of 'Pt', so $\mathcal{V}_{\mathcal{M}}$(P$t$ v ¬Pt) = true on every valuation. However on NUS_1, excluded middle is not preserved. Given the same formula 'Pt v ¬Pt', when 't' is undefined, $\mathcal{V}_{\mathcal{M}}$ (Pt) = undefined (per clause I), so likewise $\mathcal{V}_{\mathcal{M}}$ (¬Pt) = undefined (per clause II). Since the operator 'v' is defined from '→' in the usual way, and both 'Pt' and '¬Pt' are truth-valueless, the whole formula 'Pt v ¬Pt' is truth-valueless, so in particular *is not true*. Thus, EM is not valid on NUS_1. The same holds for NC.

2.4 Semantics for a Positive Free Logic

The positive semantics POS_1 consists of NUS_1 augmented with supervaluations. Like NUS_1, POS_1 is an ordered pair $<\mathcal{D}, \mathcal{I}>$. For any given model, $\mathcal{V}_{\mathcal{M}}$ assigns truth-values to formulas as follows:

I. $\mathcal{V}_{\mathcal{M}}(\Pi\alpha_1 \dots \alpha_n)$ = true if each of $\mathcal{I}(\alpha_1), \dots, \mathcal{I}(\alpha_n)$ is defined and $<\mathcal{I}(\alpha_1), \dots, \mathcal{I}(\alpha_n)> \in \mathcal{I}(\Pi)$; false if each of $\mathcal{I}(\alpha_1), \dots, \mathcal{I}(\alpha_n)$ is defined and $<\mathcal{I}(\alpha_1), \dots, \mathcal{I}(\alpha_n)> \notin \mathcal{I}(\Pi)$; truth-valueless if any of $\mathcal{I}(\alpha_1), \dots, \mathcal{I}(\alpha_n)$ is undefined.

II. $\mathcal{V}_{\mathcal{M}}(\neg\phi)$ = true if $\mathcal{V}(\phi)$ = false; false if $\mathcal{V}(\phi)$ = true; truth-valueless if $\mathcal{V}_{\mathcal{M}}(\phi)$ is truth-valueless.

III. $\mathcal{V}_{\mathcal{M}}(\phi\psi)$ = true if either $\mathcal{V}(\phi)$ = false or $\mathcal{V}(\psi)$ = true; false if $\mathcal{V}_{\mathcal{M}}(\phi)$ = true and $\mathcal{V}_{\mathcal{M}}(\psi)$ = false; truth-valueless otherwise.

IV. $\mathcal{V}_{\mathcal{M}}(\forall\alpha\ \phi)$ = true if for all d ∈ $\mathcal{D}$, $\mathcal{V}_{(\beta,d)}(\phi\ (\beta/\alpha))$ = true (where β does not appear in ϕ).

V. $\mathcal{V}_{\mathcal{M}}(\alpha = \beta)$ = true if $\mathcal{I}(\alpha)$ and $\mathcal{I}(\beta)$ are both defined and $\mathcal{I}(\alpha) = \mathcal{I}(\beta)$; false if $\mathcal{I}(\alpha)$ and $\mathcal{I}(\beta)$ are both defined, and $\mathcal{I}(\alpha) \neq \mathcal{I}(\beta)$; truth-valueless if either $\mathcal{I}(\alpha)$ or $\mathcal{I}(\beta)$ is undefined.

A supervaluation $\mathcal{S}$ is a set of every completion c of POS_1. Each completion is a dual-domain model <$\mathcal{D}_i$, $\mathcal{D}_o$, I_c>: $\mathcal{D}_i$ is the (possibly empty) inner domain, which is the domain $\mathcal{D}$ from POS_1; $\mathcal{D}_o$ is the (non-empty) outer domain, which provides referents for all empty terms; and I_c is an interpretation function such that, for every individual constant α in $\mathcal{L}$, $I(\alpha) \in \mathcal{D}_o$, for every n-place predicate Π in $\mathcal{L}$, $I(\Pi)$ is a set of n-tuples of $d \in \mathcal{D}_o$, and every member of $\mathcal{D}_o$ has a name. It can be helpful to think of the outer domain as representing the set of possible objects, and the inner domain the set of existing objects. Each completion c assigns referents in $\mathcal{D}_o$ to all and only empty terms in the language; all referring terms retain their original referents. A supervaluation then assigns truth-values to each formula ϕ containing at least one empty term α:

SI. $\mathcal{V}_{\mathcal{S}}(\phi)$ = supertrue if ϕ is true in every $c \in \mathcal{S}$; superfalse if ϕ is false in every $c \in \mathcal{S}$, and truth-valueless otherwise.

Given two formulas (1) $\Pi\alpha$ and (2) $\Pi\alpha \vee \neg\Pi\alpha$, if α has no referent, then both (1) and (2) are truth-valueless on NUS_1. But POS_1 yields different results. (1) will remain truth-valueless and (2) will evaluate as true. This is because each completion assigns as a referent for α some $d \in \mathcal{D}_o$. This referent either will or will not be in the extension of Π (which is an n-place relation over $\mathcal{D}_o$ in each completion). In short, an empty-termed formula is true if it *would* be true for every possible referent of the empty term, and false if it *would* be false for every possible referent. To illustrate, suppose in (1) and (2) '*Π*' means 'is a quadriped and *α* means 'Pegasus', an empty name. On POS_1 the completions $\in \mathcal{S}$ will disagree on the truth-values of (1): some referents will make (1) true, whereas some will make (1) false. But for any assigned referent, that referent will either be a quadruped or not. Since the completions will agree on the truth of (2), $\mathcal{V}_{\mathcal{S}}((2))$ = true. Thus, POS_1 will validate classically valid formulas invalidated by NUS_1 and NES_1.

The semantics above do not represent all free logic systems, nor do I intend to endorse any of the foregoing systems outright. However, they should suffice to show how systems of free logic tend to behave, the distinction between positive, neutral, and negative free logics, and how supervaluations can preserve the validity of classically valid schemas like EM and NC.

CHAPTER 3

3.1 History of Events-Based Semantics

Standard first-order logic involves a domain, $\mathcal{D}$, which is a set of objects–tables, chairs, people, dogs, etc. Predicates in a language are defined by their extensions: ‘Fido is furry,’ translated as ‘Furry(Fido),’ is true iff $\mathcal{I}(\text{Fido}) \in \mathcal{I}(\text{furry})$. Predicates, including action verbs, are defined as relations over sets. For instance, ‘Steve drinks coffee’ translates to ‘Drink(Steve, coffee)’ and is true iff $<\mathcal{I}(\text{Steve}), \mathcal{I}(\text{coffee})> \in \mathcal{I}(\text{Drink})$. This reflects much of the actual world, which contains people, animals, household objects, minds, and (perhaps) sets, abstract objects, and the like. But arguably, such things do not *exclusively* populate the world–there are also the things that such objects do: they sleep, laugh, run, drink, dance, think, argue, fight, grow, and play. There is reason to think the world is populated by more than just ordinary objects, but also by things like events such as sea battles, conversations, the breaking of glasses, etc. In what follows, I describe a brief history of and motivate events-based semantics and illustrate its usefulness.

The development of events-based semantics in the 20th century laid the groundwork for breakthroughs verb semantics and the analysis of adverbial modification. The basis for this progress was set in the late 1960s when Donald Davidson proposed that sentences with action verbs expressly refer to events.[8] Consider the sentences in group A:

A. (i) Tom buttered the bread.

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8. Donald Davidson, “The Individuation of Events,” in *Essays in Honor of Carl G. Hempel*, ed. Nicholas Rescher (Pittsburgh, PA: University of Pittsburgh Press. 1967). See also Donald Davidson, “The Logical Form of Action Sentences,” in *The Logic of Decision and Action* (Pittsburgh, PA: University of Pittsburgh Press. 1967).

(ii) Tom buttered the bread with the knife.

(iii) Tom buttered the bread with the knife in the kitchen.

The pre-Davidsonian analysis of sentences in A consists of the following (ignoring tense):

B. (i) *butter*(Tom, the bread)

(ii) *butter*(Tom, the bread, with the knife)

(iii) *butter*(Tom, the bread, with the knife, in the kitchen)

This analysis is problematic for a couple reasons. The first has to do with verbal arguments. In B(i) the verb *butter* takes two arguments, 'Tom' and 'bread'. Each sentence in A contains the same verb, 'butter,' but their corresponding analyses in group B contain verbs of irreducibly different arities. An ordered *n*-tuple, being an ordered set with *n* elements, just consists of its members and their ordering. If verbs are to be analyzed as relations over sets, which just are ordered *n*-tuples, then verbs of irreducibly different arities must be different verbs. Thus, the pre-Davidsonian approach apparently entails that the sentences in A do not actually contain the same verb. Moreover, on this analysis, a verb like *butter* could take an indefinite number of arguments since one can always add more adverbial modifiers (Tom buttered the bread *sleepily, with the knife*, *in the kitchen*, *in the dark*, *under the broken light*, *at midnight*, …).

Secondly, there is an entailment relation among sentences A(i), A(ii), and A(iii), but pre-Davidsonian analysis loses this relation entirely. According to Davidson, this relation resembles the dropping of conjuncts by simplification:

C. (i) p

(ii) $p \wedge q$

(iii) $p \wedge q \wedge$ r

The entailment relation among the sentences in C–namely, C(iii) ⊨ C(ii) ⊨ C(i)–strongly resembles the entailment relation among the sentences in A, which an adequate semantic analysis should capture. Hence, sentences in B should read *something* like the sentences in C. To this end, the Davidsonian analysis construes adverbial modifiers as conjuncts predicated of a hidden event argument rather than arguments of a verb. By predicating adverbs to events, the analysis can both capture the entailment relation and maintain stable verb arity. The Davidsonian approach captures A(i)-A(iii) as follows:

D. (i) ∃e(*butter*(e, Tom, the bread))

(ii) ∃e(*butter*(e, Tom, the bread) ∧ *instr*(e, the knife))

(iii) ∃e(*butter*(e, Tom, the bread) ∧ *instr*(e, the knife) ∧ *in*(e, the kitchen))

On this approach, adverbs and adverbial modifiers are analyzed as first-order predicates, which modify "not verbs but the events that certain verbs introduce."[9] There are other reasons to think a satisfactory semantic analysis should deal directly with events:

E. (i) Matt visited his grandmother twice more than did his brother Mike.

(ii) Matt saw the glass hit the floor.

(iii) It happened suddenly.

In E(i), what entities are being counted? Ordinary objects will not do. There are only two brothers, and one grandmother. In E(ii), what did Matt see? "…saw the glass hit the floor" is not the same as "…saw the glass … and … the glass hit the floor." In E(iii), what does "it" refer to? Introducing events as real, discrete entities that can be counted and witnessed allows for a straightforward and parsimonious analysis of the sentences in E. The Davidsonian program has

9. Donald Davidson. "The Individuation of Events," 1967.

produced substantial breakthroughs and advantages in analysis of adverbs, numerals, and the anaphoric pronoun “it”, and captures entailment relations among sentences that earlier analyses lost, all within first-order logic. The introduction of an event variable to a formal system also allows that system to capture sentences that make use of 0-place predicates, such as “It snows” with ease: ∃e(*snow*(e)).

Since Davidson introduced events-based semantics, the paradigm has evolved into what is now coined the “Neo-Davidsonian” approach and has been developed in two distinct ways. The first has to do with the arity of verbs and their predicates: Davidson introduced events as additional hidden arguments to some, but not all, verbs, whereas Neo-Davidsonian semantics reduce verbal predicates to a single argument–the event–and use thematic roles to connect an event to its participants.[10] The Davidsonian analysis F(ii) and it’s Neo-Davidsonian counterpart F(iii) of sentence A(i) are as follows:

F. (i) Tom buttered the bread.

(ii) ∃e(butter(e, Tom, the bread))

(iii) ∃e(butter(e) ∧ Agent(e, Tom) ∧ Patient(e, the bread))

The list of thematic relations such as Agent (the person performing an action) and Patient (the object the action is performed on) is extensive, and includes location, instrument, and a host of other roles capturing various modifiers.

I will favor the Davidsonian approach to analyzing events for the purposes of this paper for its simplicity. Unlike Davidson, I analyze all verbs as having an event argument for reasons I

10. For motivation and defense of this move, see Terence Parsons, *Events in the Semantics of English: A Study in Subatomic Semantics* (Cambridge, MA: MIT Press, 1994). For criticism, see Manfred Bierwisch, “The Event Structure of Cause and Become,” in *Event Arguments: Foundations and Applications* (Berlin: Mouton De Gruyter, 2005).

explain below. This is not to say that a Neo-Davidsonian analysis of sentences is an incorrect analysis–there may not be a single correct analysis. Indeed, the progressed enjoyed by the field in recent decades suggests that, if the field ever arrives at a consensus, it may be due to developments in the Neo-Davidsonian paradigm.

3.2 Which Sentences Introduce Events?

Events are useful for analyzing sentences like those in groups A and E, but not all sentences appear to introduce events. Sentences (1) and (2) illustrate the point:

(1) Steve is bald.

(2) Steve is tired.

Both 'bald' and 'tired' are one-place predicates and would receive the same treatment in standard first-order logic. But they appear to differ in their candidacy for events-based analysis. Many 20th century commentators noticed this and sought to apply event arguments to broader situations, which Vendler classifies as states, activities, accomplishments, and achievements.[11] Likewise, Jaegwon Kim suggests that, "when we talk of explaining an event, we are not excluding what, in a narrower sense of the term, is not an event but rather a state or a process."[12] Emmon Bach introduced the term "eventuality" to cover this broad notion, reserving "event" for its narrower use.[13] I will adopt the term "eventuality" in the broader sense hereafter.

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11. Zeno Vendler, "Linguistics in Philosophy" *Philosophy* 45, no. 171 (1970):71-.

12. Jaegwon Kim, "Events and their descriptions: Some considerations," in *Essays in Honor of Carl G. Hempel* (*Dordrecht: D. Reidel, 1969).*

13. Emmon Bach, "The Algebra of Events," *Linguistics and Philosophy* 9, No. 1 (1986), 5-16.

The difference between sentences (1) and (2) is captured by what is known as the 'stage-level/individual-level' distinction, which was first introduced into events-based semantics by Kratzer in 1995.[14] Speaking very loosely, stage-level predicates (or SLPs) are those which are temporary or accidental (adjectives like 'drunk' or 'tired,' and verbs like 'speak,' 'see,' or 'eat') whereas individual-level predicates (or ILPs) are more permanent or essential–for example, adjectives like 'tall,' 'intelligent,' or kindhearted,' and verbs like 'know' or 'resemble'. Although a thorough discussion of the merits, criticisms, and evidence of this distinction are outside the scope of this paper, its relevance is worth flagging. The ontology of events-based semantics assumes that events are concrete spatiotemporal entities, and hence must have some location and occur at some time. Kratzer argues that SLPs can be located in space and time in ways that ILPs cannot. Compare sentences (3) and (4) to (5) and (6):

(3) Steve was tired in the office yesterday.

(4) Steve ate in the office yesterday.

(5) Steve was tall in the office yesterday.

(6) Steve loved in the office yesterday.

The modifiers 'in the office and 'yesterday' contribute sensibly to sentences (3) and (4), but not to (5) or (6). According to Kratzer, the sensibility of the modifiers to sentences (3) and (4) as opposed to (5) and (6) has to do with the predicates' non-location in space, resulting in the presence or absence of an event argument in each sentence's analysis. However, Kratzer's view is not without its critics. Gennaro Chierchia, for instance, has proposed that all predicates

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14. See Angelika Kratzer, "Individual-Level and Stage-Level Predicates," in *The Generic Book* (Chicago, IL: University of Chicago Press, 1997).

introduce event arguments, and distinguishes between events that are tethered to a location and those that are not.[15]

My approach more closely aligns with Chierchia's analysis in that, on my view, any predicate that expresses an action, process, or state introduces an eventuality argument. It is relatively uncontroversial in the philosophical literature that events have vague spatial boundaries but crisp temporal ones. I contend that Kratzer is correct that the modifiers in sentences (5) and (6) do not contribute to the sentences' meanings, but it does not follow that 'being tall' and 'loving' do not introduce eventualities or are not located in space. ILPs like 'being tall' and 'loving' take place over long time intervals and therefore stretch across many locations, whereas SLPs like 'being tired' and 'eating' occur over short time intervals and are likely to be confined to a single location. Consider the ILP 'is tall' for illustration, along with the fact that Steve is tall. If Steve is tall, then as long as Steve is an adult in ordinary circumstances, Steve is always tall. Thus, Steve's being tall and Steve's being in the office yesterday together entail that Steve was tall in the office yesterday, so the addition of the modifier is redundant. This, it seems to me, plausibly accounts for why 'individual-level' predicates like 'is tall' are not sensibly modified by phrases like "yesterday". For these reasons, my system will not observe Kratzer's stage-level/individual-level distinction, instead analyzing all such predicates as introducing eventuality arguments. Sentences (3)-(6), then, will each be analyzed as introducing eventuality arguments (ignoring tense and adverbial modifiers):

(3') $\exists e(\text{tired}(e, \text{Steve}))$

(4') $\exists e(\text{ate}(e, \text{Steve}))$

15. Gennaro Chierchia, "Individual Level Predicates as Inherent Generics," in *The Generic Book*. See also Louise Mcnally, "Stativity and Theticity," in *Events and Grammar.*

(5’) ∃e(tall(e, Steve))

(6’) ∃e(loved(e, Steve))

3.3 Tensed Eventuality-Based Semantics

Now I augment the system sketched above to capture time by introducing time-intervals, allowing the system to capture English tense and aspect. But there may be many ways a system could express temporal relations. Why express time with intervals rather than, say, instants? Instant-based temporal models are generally constituted by a set of primitive entities–time instants–with the binary precedence relation ‘<’ over the set of instances. Interval-based models enjoy a richer ontology along with a broader stock of relations over the set of intervals such as the precedence, inclusion, and overlap relations. The nature of time is a hotly debated subject of philosophy, which is far outside the scope of this paper. I’ll just note that interval- and instant-based models are formally reducible to one another, and that I’ve opted to incorporate intervals in the present system because of the convenience it affords for expressing temporal relations.

Consider the following sentences:

3. (i) Steve is tired.

(ii) Steve was tired.

3(i) and 3(ii) attribute the same predicate to Steve but do not express the same proposition, since the two events occur over different times. But without a way to account for this, the system sketched above fails to capture the difference in meaning, yielding the same analysis:

4. (i) ∃e(tired(e, Steve))

(ii) ∃e(tired(e, Steve))

Since temporal relations are important features of English sentences, time needs to somehow be built into the notation if the system is to capture the difference illustrated by the sentences group 3, where otherwise identical events differ primarily in the times when they occur.[16] I accomplish this by subscripting time intervals to the event variable. Whereas "Steve is tired" indicates that the time interval during which Steve is tired is the present moment, the sentence "Steve was tired" indicates that Steve was tired at some time in the past without indicating which time specifically. The expression of two potentially distinct eventualities can be captured with the use of variables t_1 and t_2 subscripted to the event variable:

5. (i) $\exists e_{t1}(\text{tired}(e_{t1}, \text{Steve}))$

   (ii) $\exists e_{t2}(\text{tired}(e_{t2}, \text{Steve}))$

Though two (potentially) distinct propositions are expressed now, it's unclear what relation 5(i) and 5(ii) stand toward each other. Perhaps t_1 precedes t_2, or vice versa, or perhaps $t_1 = t_2$ and 5(i) and 5(ii) express the same proposition. I resolve this by introducing a temporal relation superscripted to the event variables. If t_1 refers to the present moment (represented by the term 'p'), then the relation between t_1 and *p* can be captured with the identity relation '='. If t_2 refers to a time interval prior to the present moment, then the relation between t_2 and p is captured with the precedence relation '<'. With these relations, the tense of 3(i) and 3(ii) can be expressed:

6. (i) $\exists e_{t1}(\text{tired}(e^{=p}_{t1}, \text{Steve}))$

   (ii) $\exists e_{t2}(\text{tired}(e^{<p}_{t2}, \text{Steve}))$

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16 Although it is surely possible to use apply temporal relations to events to say one event precedes or overlaps another, it's not clear that this can be done when there is only one event in question. Adding time intervals not only accomplishes both of these, but does so in a way that I think more accurately models how time is built into English grammar.

This addition of notation to the system is sufficient to capture not only simple present and past English tenses, but even continuous aspect. Consider the following sentences:

7. (i) Steve is running.

(ii) Steve ran.

(iii) Steve was running.

(iv) Steve was running when Joshua jumped.

7(i)-7(iii) employ the same subject and verb–'Steve' and 'run'–but differ in meaning. This difference has to do with the interval over which the running event takes place. 7(i) expresses that a running-event was occurring at the moment of utterance or writing of 7(i); 7(ii) expresses that a running-event took place and terminated before the utterance or writing of 7(ii); 7(iii) expresses a running-event that was *in progress* at some time prior to the moment of utterance or writing of 7(iii), and 7(iv) expresses that a running event was in progress when jumping event occurred at some time prior to the present moment. The intervals and relations of 7(i, ii, and iv) (I will turn to 7(iii) momentarily) are represented as follows:

8. (i) $t_1 = p$

(ii) $t_2 < p$

(iv) $t_1 \sqsupseteq t_2 \wedge t_2 < p$

8(i) expresses that the interval denoted by t_1 is the present moment. 8(ii) expresses that the interval denoted by t_2 precedes the present moment. 8(iv) expresses that the interval denoted by t_1 includes the interval denoted by t_2, which in turn precedes the present moment. A few comments about 7(iii), which is past tense and continuous aspect: the past continuous is not used in English without reference to some other past event, either explicitly or implicitly,[9] so “Steve was running” would not be uttered in isolation, but in the context of some other past event, such as “when Joshua jumped.” This is why I have omitted an analysis of 7(iii) in group 8. Sentence 7(iv) is a more natural sentence in English that makes use of 7(iii). The combination of tense and aspect in 7(iv) expresses that there was a running-event by Steve in the past, and there was a jumping-event by Joshua in the past, and the running-event was in progress when the jumping-event took place.

But why introduce intervals as variables subscripted to the event variable? When we refer to past events, we indicate that the event in question occurred at some time in the past without necessarily specifying (or even knowing) the exact time. This behavior is best captured by a variable, so that we can say the event occurred during *some* past interval without having to name the specific interval. However, introducing an interval variable subscripted to the eventuality variable is more than just pragmatically useful. Consider the sentence “Somebody ate my porridge” for illustration. When someone utters that sentence, she is asserting the existence of somebody, and an eating of her porridge by that somebody. But the reference to a past time interval during which the event occurred is quite different: the time interval is not asserted lexically, but instead is built in to the structure of the verb itself. Indeed, the fact that times are referred to grammatically rather than lexically in ordinary language is a strong indication that the reference to times differs importantly from references to individuals, places, or events.[10] Whereas “Somebody ate my porridge” asserts the existence of somebody and of a porridge-eating event, it structurally presupposes the existence of some past time interval. Assertions about temporal relations enter the picture on the back of temporal adverbs (e.g. “He drank my beer *in* the morning.”) and subordinating conjunctions (e.g. “Someone had drunk my beer *before* I woke up."). These phrases and clauses constitute assertions about relations between intervals rather than the existence of any one interval. For instance, the sentence "Someone ate my porridge before I returned home" asserts, among other things, the existence of a porridge-eating event and a returning event, that the porridge-eating event and the returning event both precede the present moment, and that the time interval during which the porridge-eating event took place precedes the time interval during which the returning event took place.[17] But notice that it does not assert the existence of the time-interval during which alleged porridge-eating event takes place, because it's contradiction, "Someone did not eat my porridge before I returned home" does not deny the existence of the time interval.[18] Notice also that claims regarding the relations indicated by the words "before" and "in" are still made lexically, whereas the existence of the time intervals in question are not among the entities asserted. In short, I contend that when tensed English sentences are used to make assertions about tables, chairs, events, and time relations, the existence of time intervals is presupposed in the structure of the verb and not asserted outright.

For these reasons, subscripting the interval variable to the event variable accomplishes quite a lot at once: it allows us to refer to some interval without explicitly stating *which* interval, capturing the way time is referred to in ordinary language. It also allows the quantifier to range

17. Plausibly, "Someone ate my porridge before I returned home" presupposes rather than asserts the existence of a returning event, but merely asserts the existence of a porridge-eating event, that the presupposed porridge-eating event precedes the present moment, and that the porridge-eating event that this porridge-eating event precedes the presupposed returning event.

18 . It may be objected that "It's not the case that someone ate my porridge before I returned home" is the strict contradiction of "Someone ate my porridge before I returned home." However, no English speaker would infer that "It's not the case that someone at my porridge before I returned home" denies the existence of the time interval preceding the time when I returned home.

over the interval variable secondarily, without existential force: the quantifier asserts the existence of an eventuality without asserting the existence of the time interval.

The system sketched above, though adequate to capture a wide range of meaning from the tense and aspect of English verbs, will not exhaustively capture English tense and aspect. Take, for instance, the present simple sentence, “Arthur plays basketball.” Unlike the present continuous tense, which denotes an action taking place at the time of utterance (e.g., “Arthur is playing basketball”), the present simple expresses actions which are habitual or repeated. Notably, present-simple English sentences like “Arthur plays basketball” could be analyzed as meaning “Arthur is a basketball player” and therefore as candidates for analysis as first-order predicates ($\exists e_t$(*isabasketballplayer*($e^{=p}{}_t$, Arthur)). Although there may be some English constructions outside the reach of the system sketched in this chapter, the foregoing gives reason to think that this system not only wields substantial expressive power, but may be a very plausible analysis of English sentences.

CHAPTER 4

4.1 Free Tensed Eventuality-Based Semantics

We begin with a language, $\mathcal{L}$, consisting of the following primitive vocabulary:

- Logical operators '→','¬', '=', and '∀', from which '∧', '∨', and '↔', and '∃' are defined in the usual ways.
- Three kinds of variables:
  - Eventuality variable '*e*', '*e1*', '*e2*', ..., '*en*' with alphanumerical subscripts and superscripts.[19]
  - Interval variables '*t*', '*t1*', '*t2*', ... ,'*tn*'.
  - Individual variables '*x*', '*y*', '*z*', with or without numerical subscripts.
- For each *n* > 0, *n*-place predicates 'P', 'Q', 'R', ... with or without numerical subscripts.
- Three kinds of constants:
  - Eventuality constants 'ə1', 'ə2', …, 'ən'.
  - Individual constants 'a', 'b', 'c', with or without numerical subscripts.
  - Interval constants '[p1, p2]', '[p3, p4]', ... '[p*n-1*, p*n*]', and 'p'.
- Four temporal relation operators:
  - The precedence relation '⋖'.
  - The antecedence relation '⋗'.
  - The inclusion relation '⊒'.

19. Notice that eventuality variables are augmented with numerical suffixes rather than subscripts or superscripts. This is because subscripts and superscripts are reserved for interval variables and temporal relation operators.

- The equality relation, ‘=’.

- Parentheses, brackets, and commas.

The primitive vocabulary above can be used to form WFFs in $\mathcal{L}$ in the following ways:

- If Π is an *n*-place predicate, ε is an eventuality constant, τ_1 and τ_2 are interval constants, $\alpha_1 \dots \alpha_n$ individual or eventuality constants, and # is a temporal relation operator, then $\Pi(\varepsilon^{\#\tau_2}_{\tau_1}, \alpha_1, \dots, \alpha_n)$ is a formula.
- If α and β are constants of the same type, α = β is a formula.
- If ξ is an eventuality variable, ζ is an interval variable, and ϕ is a formula containing eventuality constant ε and interval constant τ (where τ is subscripted to ε), then $\forall\xi_\zeta\phi$ (where ξ replaces every instance of ε in ϕ and ζ replaces every instance of τ in ϕ) is a formula.
- If ξ is an individual variable and ϕ is a formula containing individual constant α, then ∀ξϕ (where ξ replaces every instance of α in ϕ) is a formula.
- If ϕ is a formula, then ¬ϕ is a formula.
- If ϕ and ψ are formulas, then ϕ → ψ is a formula.
- Nothing else is a formula.

Note that all atomic well-formed formulas are either of the form α = β or $\Pi(\varepsilon^{\#\tau_2}_{\tau_1}, \alpha_1, \dots, \alpha_n)$. Call the former ‘identity formulas’ and the latter ‘predicate formulas.’ Note also that all predicates have an arity of 1 + *n*, and that the grammar above disallows free variables in formulas–that is, every variable that appears in a formula ϕ is bound by a quantifier. Since every well-formed atomic predicate formula contains two interval constants τ_1 and τ_2 where τ_1 is subscripted to the eventuality constant and τ_2 is superscripted to the eventuality constant, call τ_1 the ‘subinterval term’ and τ_2 the ‘superinterval term’. It’s also worth articulating that all predicate formulas contain interval terms or variables.

4.2 Positive Free Logic Semantics for $\mathcal{L}$

The semantics, *S*, for $\mathcal{L}$ will be a positive free logic augmented with supervaluations. A model in *S* is an ordered pair <$\mathcal{D}$, $\mathcal{I}$> consisting of a domain $\mathcal{D}$ and an interpretation function, $\mathcal{I}$. $\mathcal{D}$ is a (possibly-empty) set containing eventualities, intervals, and objects. To define it formally, $\mathcal{D}=\mathcal{D}_e \cup \mathcal{D}_t \cup \mathcal{D}_i$. $\mathcal{I}$ is a function such that for every $e \in \mathcal{D}_e$, there is some eventuality constant ε in $\mathcal{L}$ and $\mathcal{I}(\varepsilon) = e$; for every $t \in \mathcal{D}_t$, there is some interval constant τ in $\mathcal{L}$ and $\mathcal{I}(\tau) = t$; for every $i \in \mathcal{D}_i$, there is some individual constant α in $\mathcal{L}$ and $\mathcal{I}(\alpha) = i$; for any n-place predicate Π, $\mathcal{I}(\Pi)$ is an n-place relation over $\mathcal{D}$. Unlike in classical models, $\mathcal{I}$ is a partial function (provided $\mathcal{D}$ is non-empty) since for every $d \in \mathcal{D}$, $\mathcal{I}$ maps some term in $\mathcal{L}$ to d, but it need not be that, for any term t in $\mathcal{L}$, t is defined.

Now we turn to consider $\mathcal{D}_t$, the set of time intervals. An interval is an ordered pair [p1, p2] such that p1, p2 ∈ **R** (the line of real numbers) and p1 ≤ p2. $\mathcal{D}_t$ = {[p1, p2]: p1 ≤ p2 ∧ p2 ≤ 0}. Informally, the set of time intervals is every pair of real number such that the second number is neither less than the first nor greater than zero, with zero represented by the constant ‘p’ (shorthand for the ordered pair [0,0]) which refers to the present moment. Elements of $\mathcal{D}_t$ allow for definitions of the temporal relations above. Given any intervals [p1, p2] and [p3, p4], the relations are defined as follows:

i. [p1, p2] $\prec$ [p3, p4] iff p2 < p3.

ii. [p1, p2] $\succ$ [p3, p4] iff p1 > p4.[20]

iii. [p1, p2] $\sqsupseteq$ [p3, p4] iff p1 ≤ p3 and p4 ≤ p2.

iv. [p1, p2] = [p3, p4] iff p1 = p3 and p2 = p4.

Given a predicate formula ϕ of the form $\Pi(\varepsilon^{\#\tau_2}{}_{\tau_1}, \alpha_1, \ldots, \alpha_n)$, the 'temporal relation' of ϕ is '$\tau_1 \# \tau_2$'. The temporal relation of a predicate formula ϕ is 'satisfied' iff the relation # obtains between τ_1 and τ_2 as defined above.

A few bits of notation before defining truth in a model. If ξ is a variable, and β is a constant occurring in formula ϕ, then ϕ(β/ξ) is the result of substituting ξ for every occurrence of β in ϕ. An 'assignment', where, in a given model $\mathcal{M}$, some $d \in \mathcal{D}$ is temporarily assigned as the referent of a constant β, is denoted by the notation $\mathcal{V}_{\mathcal{M}(\beta,d)}$. For any given model $\mathcal{M}$, the valuation function $\mathcal{V}_{\mathcal{M}}$ assigns truth-values to WFFs as follows:

I. $\mathcal{V}_{\mathcal{M}}(\Pi(\varepsilon^{\#\tau_2}{}_{\tau_1}, \alpha_1, \ldots, \alpha_n))$ = true if $\mathcal{I}(\varepsilon) \in \mathcal{D}_e$, $\mathcal{I}(\tau_1) \in \mathcal{D}_t$, $\mathcal{I}(\alpha_1), \ldots, \mathcal{I}(\alpha_n) \in \mathcal{D}_i$, $<\mathcal{I}(\varepsilon), \mathcal{I}(\tau_1), \mathcal{I}(\alpha_1), \ldots \mathcal{I}(\alpha_n)> \in \mathcal{I}(\Pi)$, and the temporal relation $\tau_1 \# \tau_2$ obtains; false if $\mathcal{I}(\tau_1) \in \mathcal{D}_t$, $\mathcal{I}(\alpha_1), \ldots, \mathcal{I}(\alpha_n) \in \mathcal{D}_i$, and $<\mathcal{I}(\varepsilon), \mathcal{I}(\tau_1), \mathcal{I}(\alpha_1), \ldots \mathcal{I}(\alpha_n)> \notin \mathcal{I}(\Pi)$ or the temporal relation $\tau_1 \# \tau_2$ does not obtain; truth-valueless if $\mathcal{I}(\tau_1) \notin \mathcal{D}_t$ or $\mathcal{I}(\alpha_1), \ldots, \mathcal{I}(\alpha_n) \notin \mathcal{D}_i$.

II. $\mathcal{V}_{\mathcal{M}}(\forall\xi\phi)$ = true, when ξ is in individual variable, if for all $i \in \mathcal{D}_i$, $\mathcal{V}_{\mathcal{M}(\beta,i)}(\phi(\beta/\xi))$ = true (where β does not appear in ϕ); false otherwise.

III. $\mathcal{V}_{\mathcal{M}}(\forall\xi_\zeta\phi)$ = true, when ξ is an eventuality variable, ζ is an interval variable, ε is an eventuality constant, and τ is an interval constant, if for all $e \in \mathcal{D}_e$ and for all $t \in \mathcal{D}_t$, $\mathcal{V}_{\mathcal{M}(\varepsilon,e)(\tau,t)}(\phi(\varepsilon/\xi, \tau/\zeta))$ = true (where ε and τ do not appear in ϕ); truth-valueless if the

20. Of course, the antecedence relation could be defined in terms of the precedence relation. But the addition of the antecedence relation allows for a simpler system of semantics that more straightforwardly captures certain tense relations in English, which I illustrate later.

temporal relation between τ and its superinterval is not satisfied for any $t \in \mathcal{D}_t$; and false otherwise.

IV. $\mathcal{V}_{\mathcal{M}}(\neg\phi)$ = true if $\mathcal{V}(\phi)$ = false; false if $\mathcal{V}(\phi)$ = true; truth-valueless if $\mathcal{V}_{\mathcal{M}}(\phi)$ is truth-valueless.

V. $\mathcal{V}_{\mathcal{M}}(\phi\rightarrow\psi)$ = true if either $\mathcal{V}(\phi)$ = false or $\mathcal{V}(\psi)$ = true; false if $\mathcal{V}_{\mathcal{M}}(\phi)$ = true and $\mathcal{V}_{\mathcal{M}}(\psi)$ = false; truth-valueless otherwise.

VI. $\mathcal{V}_{\mathcal{M}}(\alpha = \beta)$ = true if $I(\alpha) \in \mathcal{D}$, $I(\beta) \in \mathcal{D}$, and $I(\alpha) = I(\beta)$; false if $I(\alpha) \in \mathcal{D}$, $I(\beta) \in \mathcal{D}$, and $I(\alpha) \neq I(\beta)$; truth-valueless if $I(\alpha) \notin \mathcal{D}$ or $I(\beta) \notin \mathcal{D}$.

As defined above, *S* is a neutral free logic because formulas with nonreferring terms do not evaluate to a truth-value, per clauses I and VI. However, the system becomes a positive free logic when augmented with supervaluations, in such a way that classical tautologies remain true. A supervaluation $\mathcal{S}$ is a set of every completion c of P. Each completion c is an ordered triplet, $<\mathcal{D}^+, \mathcal{D}^-, I_c>$, where the outer domain $\mathcal{D}^+$ is a (non-empty) set which contains eventualities, times, and objects, and provides referents for all empty constants. The inner domain $\mathcal{D}^-$ is the original domain $\mathcal{D}$. Formally, $\mathcal{D}^- = \mathcal{D}^-_e \cup \mathcal{D}^-_t \cup \mathcal{D}^-_i$, $\mathcal{D}^+ = \mathcal{D}^+_e \cup \mathcal{D}^+_t \cup \mathcal{D}^+_i$, $\mathcal{D}^- \subseteq \mathcal{D}^+$, and for every $d \in \mathcal{D}^+$, d has a name.[21] Intuitively, the inner domain can be thought of as containing existents and the outer domain as containing possibilia. $\mathcal{D}^+_e$ is the set of all possible eventualities (the writing of *How Hamlet Stole Christmas*, the third presidential inauguration of Barack Obama, etc); $\mathcal{D}^+_i$ is the set of all possible individuals (the Lochness Monster, Santa Claus,

21. It is common practice in dual-domain logics to stipulate that the inner and outer domains are disjoint. I have opted to make the inner domain a subset of the outer domain to preserve the intuition that anything that is actual is possible, given the outer domain contains all possibilia.

etc.); $\mathcal{D}^+_t$ is the set of all intervals. Formally, $\mathcal{D}^+_t = \{[p1, p2]: p1, p2 \in \mathbf{R} \wedge p1 \leq p2\}$. $\mathcal{I}_c$ assigns objects from $\mathcal{D}^+$ as referents for all and only terms in $\mathcal{L}$ without referents in $\mathcal{D}^-$. Completions interpret predicates as relations over $\mathcal{D}^+$. Referents and truth-values are assigned in the following ways:

VII. For every truth-valueless atomic predicate formula ϕ, each c assigns some $e \in \mathcal{D}^+_e$ to each empty eventuality constant in ϕ; some $t \in \mathcal{D}^+_t$ as a referent for each empty interval constant in ϕ; some $i \in \mathcal{D}^+_i$ for each empty individual constant in ϕ.

VIII. $\mathcal{V}_c(\Pi(\varepsilon^{\#\tau_2}{}_{\tau_1}, \alpha_1, \ldots, \alpha_n))$ = true if $<\mathcal{I}_c(\varepsilon), \mathcal{I}_c(\tau), \mathcal{I}_c(\alpha_1), \ldots \mathcal{I}_c(\alpha_n)> \in \mathcal{I}(\Pi)$ and the temporal relation $\tau_1 \# \tau_2$ obtains; false otherwise.

IX. $\mathcal{V}_c(\forall\xi\phi)$ = true, when ξ is in individual variable, if for all $i \in \mathcal{D}^+_i$, $\mathcal{V}_{\mathcal{M}(\beta,i)}(\phi(\beta/\xi))$ = true (where β does not appear in ϕ); false otherwise.

X. $\mathcal{V}_c(\forall\xi_\zeta\phi)$ = true, when ξ is an eventuality variable, ζ is an interval variable, ε is an eventuality constant, and τ is an interval constant, if for all $e \in \mathcal{D}^+_e$ and for all $t \in \mathcal{D}^+_t$, $\mathcal{D}^+{}_{(\varepsilon,e)(\tau,t)}(\phi(\varepsilon/\xi, \tau/\zeta))$ = true (where ε and τ do not appear in ϕ); and false otherwise.

XI. $\mathcal{V}_c(\neg\phi)$ = true if $\mathcal{V}(\phi)$ = false; false if $\mathcal{V}(\phi)$ = true.

XII. $\mathcal{V}_c(\alpha = \beta)$ = true if $\mathcal{I}(\alpha) = \mathcal{I}(\beta)$; false if $\mathcal{I}(\alpha) \neq \mathcal{I}(\beta)$.

XIII. $\mathcal{V}_c(\phi\rightarrow\psi)$ = true if either $\mathcal{V}(\phi)$ = false or $\mathcal{V}(\psi)$ = true; false if $\mathcal{V}_c(\phi)$ = true and $\mathcal{V}_c(\psi)$ = false.

XIV. $\mathcal{V}_\mathcal{S}(\phi)$ = supertrue if ϕ is true in every $c \in \mathcal{S}$; superfalse if ϕ is false in every $c \in \mathcal{S}$; truth-valueless otherwise.

The addition of supervaluations renders *S* a positive semantics in which all and only classical tautologies and contradictions about the future have truth-value. Thus, *S* preserves excluded middle and non-contradiction. Take, for instance, the propositions A and B and their corresponding analyses as predicate formulas in *S*:

A) Arthur will see Ruth.

B) Arthur will not see Ruth.

A_S) $\exists e_t(\text{see}(e^{>p}_t, \text{Arthur, Ruth}))$

B_S) $\neg\exists e_t(\text{see}(e^{>p}_t, \text{Arthur, Ruth}))$

The non-existence of the future is captured by $\mathcal{D}_t$ in that 0 (the present moment) is the greatest p$n \in \mathcal{D}_t$. A is truth-valueless in *S* because there is no interval $t \in \mathcal{D}_t$ such $t > p$.[22] B, the negation of A, is truth-valueless for the same reason. However, the disjunction of A and B, a classical tautology, retains its validity in *S*.

C) Either Arthur will see Ruth or Arthur will not see Ruth.

C_S) $\exists e_t(\text{see}(e^{>p}_t, \text{Arthur, Ruth})) \vee \neg\exists e_t(\text{see}(e^{>p}_t, \text{Arthur, Ruth}))$

In A_S and B_S, the atomic formulas over which the quantifiers range must contain empty terms. In each case, the atomic formulas are well-formed predicate formulas and therefore contain some interval constant. Since there is no interval $t \in \mathcal{D}_t$ such that $t > p$, whatever constant which is replaced by the variable 't' must be empty. Therefore, per clause VII, some $t \in \mathcal{D}^{+}_t$ is assigned as a referent for constant replaced by 't'. For the predicate 'see' (which is now interpreted as a

[22] This is why I define the antedence relation distinct from the precedence relation. In languages like English, the eventuality being talked about is primary, and its time is defined in relation to another time. The sentence "Shakespeare wrote *Hamlet*" is about Shakespeare's writing of *Hamlet*, and the interval during which Shakespeare's writing of *Hamlet* takes place precedes the present moment. Likewise, the sentence "Arthur will see Ruth" is about Arthur's seeing Ruth, and the interval during which Arthur's seeing of Ruth takes place antecedes the present moment. I prioritize keeping the relations and notation in this system uniform in order to preserve the 'about-ness' of sentences in English in formulas of the system.

relation over $\mathcal{D}^+$), either t is a member of some ordered tuple $\in$see, or not. Thus C_S evaluates to true in every completion, and therefore is supertrue in *S*. The same result holds for any proposition analyzed as an identity formula in *S*. The proposition "Arthur worshiped Thor" is truth-valueless in *S* since $\mathcal{I}$(Thor) $\notin \mathcal{D}_i$, but "Thor is identical to Thor," which is analyzed as an identity formula Thor = Thor, remains valid since for any possible referent $i \in \mathcal{D}^+_i$ for the empty term "Thor", $i = i$. Hence, identity theory is classical in *S*.

As defined above, *S* is adequate to capture the meaning of a variety of English sentences. The simple past sentence "Arthur saw Ruth" asserts a seeing-event of Ruth by Arthur, which took place over some interval that terminated in the past. This can be captured by the following analysis:

$$\exists e_t(\text{see}(e^{<p}_t, \text{Arthur}, \text{Ruth}))$$

Now consider a more complicated English sentence which makes use of the past continuous, "Arthur was running when Ruth jumped." The system yields the following analysis:

$$\exists e_t \exists e1_{t1}((\text{run}(e^{\sqsupseteq t1}_t, \text{Arthur})) \wedge (\text{jump}(e1^{<p}_{t1}, \text{Ruth})))$$

The past continuous tense denotes a past action that was in progress when another past action occurred. This is adequately captured by the inclusion relation. Moreover, the way *S* analyzes the sentence "Arthur was running when Ruth jumped" not only captures the truth-conditions of the sentence and its temporal relations, but it does so in a manner that is faithful to how the sentences are expressed in English. The past continuous is used in the context of other past events. Likewise, in the analysis of "Arthur was running when Ruth jumped," both quantifiers range over both atomic formulas, capturing the context-dependent nature of past continuous sentences in English.

The system is also adequate to express the perfect tenses in English, like the sentence "Arthur had run when Ruth jumped," which makes use of the past perfect simple and simple past tense:

$$\exists e_{t}\exists e1_{t1}((\mathrm{run}(e^{\lessdot t_1}_{t}, \mathrm{Arthur})) \wedge (\mathrm{jump}(e1^{\lessdot p}_{t1}, \mathrm{Ruth})))$$

The past perfect simple tense in English denotes a past action that took place and terminated before another past action took place, all of which can be captured by the precedence relation.

The system can also be augmented to preserve the entailment relation among propositions discussed in chapter 3 using thematic relations. For instance, "Tom buttered the bread" could be analyzed as "$\exists e_{t}(\mathrm{butter}(e^{\lessdot p}_{t}, \mathrm{Tom}, \mathrm{bread}))$", and "Tom buttered the bread with a knife" analyzed as $\exists e_{t}(\mathrm{butter}(e^{\lessdot p}_{t}, \mathrm{Tom}, \mathrm{bread}) \wedge \mathrm{instrument}(e^{\lessdot p}_{t}, \mathrm{Tom}, \mathrm{knife}))$. Thus, the system preserves the entailment relation which partially motived the development of events-based semantics.

CHAPTER 5

5.1 Conclusion

At this point in my paper, I have argued that the combination of realism about the past and present and antirealism about the future, a tensed events-based analysis of natural language, and a view about presupposition can lend itself as a solution to the problem of future contingent propositions. Since English language sentences are tensed, if the existence of time intervals is presupposed rather than asserted (a rather plausible view) in English sentences, then an analysis that captures that presupposition can yield a system in which future propositions are meaningful but lack truth-value. In chapter 4, I formally defined the system by combining Davidsonian events-based semantics, free logic, and interval logic. In addition to yielding truth-valueless future propositions, the system preserves the classical validity of EM and NC. It is also independently motivated because it can capture meaning built into English tense and aspect, allowing for more expressive power than standard first-order logic.

I close with some caveats and qualifications. *S* represents all sentences of English as one of two kinds of formulas, predicate formulas or identity formulas. This point demands more attention. All predicate formulas are tensed, but, as the system has been defined, identity formulas are not. Which sentences of English are best translated as predicate formulas and which are best translated as identity formulas will demand more clarity. Take, for instance, the sentence “Samuel Clemens was Mark Twain.” This sentence seems to express a straightforward identity relation. The sentence is also tensed, but identity formulas in *S* are not tensed. As it stands, the system cannot capture the difference between “Samuel Clemens is Mark Twain” and “Samuel Clemens was Mark Twain.”

Why, then, distinguish between identity and predicate formulas at all? After all, don't all sentences of English involve tense? Most sentences in English express events that occur at certain times. But the correct analysis for sentences that express mathematical truths—say, “Four squared is sixteen”—is not quite so obvious, and such sentences could constitute exceptions. In English, sentences like this are always expressed in the present simple. The simple past or simple future future (“Four squared was sixteen” or “Four squared will be sixteen”) wouldn’t be uttered in isolation, and the continuous aspect (“Four squared is equaling sixteen”) isn’t even sensible. Identity formulas are naturally suited to express logical and mathematical truths. The language could be expanded to include numbers, so that these truths can be expressed alongside predicate formulas. The challenge, then, would be to distinguish between the expression of mathematical truths like “Four squared is sixteen” and “Samuel Clemens is Mark Twain”, since both express identity but tense can be sensibly changed only in the latter.

It’s also worth noting again that the system formalizes the commitment to the positive ontological status of three types of entities: times, individuals, and events. The motivation for realism about events comes from their usefulness for making sense of much of natural language, a case which was made by Davidson and thinkers that followed him. At the beginning of this paper, I explicitly took for granted a realist view of the past and the present. Since first-order logic presupposes the existence of individuals, the combination of these three views results in a commitment to the existence of three distinct types of entities. Readers might ask what a semantics like *S* implies or assumes with respect to A- and B-theories of time. I do not intend to comment on the truth or evidence for either of these theories. *S* is intended largely to explicate sentences of the English language—positing the existence of time intervals (whatever *time* is) as entities offers utility in analyzing natural sentences of English. In closing, I want to suggest that a system that takes these types seriously, including time intervals, possessive useful, substantial expressive power, offers a plausible analysis of English sentences that preserves logical relations important to any system of logic, and remains friendly to indeterminism in its treatment of future contingent propositions.

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