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Relational Delegation*

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Abstract

We analyze a cheap talk game with partial commitment by the principal. We first treat the principal s commitment power as exogenous and then endogenize it in an infinitely repeated game. We characterize optimal decision making for any commitment power and show when it takes the form of threshold delegation - in which case the agent can make any decision below a threshold - and centralization - in which case the agent has no discretion. For small biases threshold delegation is optimal for any smooth distribution. Outsourcing can only be optimal if the principal s commitment power is sufficiently small.

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1. Introduction

The internal allocation of decision rights is a key determinant of the behavior of firms. While owners have the formal authority to make all decisions on behalf of their firms, they typically delegate at least some important decision rights to their employees. These employees, however, often have consistent biases and can be expected to make different decisions than the owners would (Jensen, 1986). An understanding of what determines the internal allocation of decision rights is therefore a prerequisite for understanding, and potentially being able to predict, the decisions that firms make, such as how much to invest and how many workers to hire and fire. In this paper we investigate the optimal allocation of decision rights within firms. In particular, we investigate how the owner of a firm should delegate decision rights to a biased employee.

While the formal authority to make decisions is concentrated at the top of firms, the information needed to make effective use of this authority is often dispersed throughout their ranks. The legal right to decide on the allocation of capital, for instance, resides with the owners of firms but CEOs, division managers, and other employees are often better informed about the profitability of different investment projects. The benefit of delegating decision rights is that it allows the owners to utilize the specific knowledge that their employees might have (Holmström, 1977, 1984; Jensen and Meckling, 1992).

There are two main difficulties in delegating decision rights, however. First, as mentioned above, there is ample evidence which suggests that employees have consistent biases and are therefore likely to make different decisions than the owners would want them to. Agency costs therefore place a limit on the ability of owners to delegate decision rights (Holmström, 1977, 1984; Jensen and Meckling, 1992). Second, delegated decision rights are always "loaned, not owned" (Baker, Gibbons, and Murphy, 1999, p. 56). In other words, while owners can delegate decision rights ex ante they can always overrule the decisions that employees make ex post. Anticipating the possibility of being overruled the employees in turn may act strategically and, as a result, their specific knowledge might not get used efficiently. Imperfect commitment therefore places a second limit on the ability of owners to delegate decision rights (Baker, Gibbons, and Murphy, 1999).

Due to the presence of agency costs and the lack of perfect commitment owners rarely engage in complete delegation, that is they rarely delegate decision rights without putting in place rules and regulations that constrain the decisions their employees can make. Consider, for instance, the decision over the allocation of capital which is often delegated to lower level managers and, in particular, to division managers. While in some firms these division managers have almost full discretion in deciding between different investment projects, in most they face a variety of constraints. In some firms, for instance, division managers are allowed to decide on investment projects that affect the daily operation of their divisions but not on those that are deemed to affect the future of the firm as a whole. In other firms division managers can decide on investment projects that do not exceed a certain threshold size and their superiors decide on larger projects.[1] In this paper we show that many of the organizational arrangements that we observe in practice arise optimally in a model in which a principal with imperfect commitment delegates decision rights to a better informed but biased agent.

Our analysis is based on a model with three main features: (i.) a firm that consists of a principal and an agent has to implement a project and the principal has the formal authority to decide which project is implemented. The potential projects differ on one dimension, for instance investment size, and the principal and the agents have different preferences over this dimension. (ii.) the agent is better informed about the projects' payoffs than the principal. In particular, only the agent observes the state of the world which determines the identity of his preferred project and that of the principal. Before making her decision the principal asks the agent for a recommendation. The principal then either rubber-stamps the recommendation or overrules it and implements another project. (iii.) the principal has some commitment power. In particular, before the agent makes his recommendation, the principal makes a promise about how she will respond to the agent's recommendation. In case the principal reneges on this promise, for instance by not rubber-stamping a recommendation that she promised to approve, she incurs a certain cost. This cost measures the principal's commitment power: the higher the cost, the more commitment power the principal has. We interpret this cost as the damage that an agent can impose on the principal through unproductive behavior in a repeated relationship. We first follow MacLeod (2003) in considering a static model in which the cost of conflict is exogenous and then develop a repeated game in which this cost is

[1] A large number of studies have described the capital budgeting rules that firms use. See, for instance, Marsheutz (1985), Taggart (1987) and, in particular, Bower (1970).

endogenously determined.

Although the principal always has the formal authority to decide on the projects, she can engage in many different types of relational delegation. In other words, she can implicitly commit to many different decision rules that map the agent's recommendations into decisions. For instance, she can engage in *complete delegation* by committing herself to always rubber-stamp the agent's recommendation. Other possibilities include *threshold delegation* – in which case the principal rubber-stamps the agent's recommendation up to a certain size and implements her preferred project if he recommends a project that is above the threshold – and *menu delegation* – in which case the principal rubber-stamps the agent's recommendation only if he proposes one of a finite number of projects. Of course the principal can also choose to ignore the agent's recommendation altogether and simply implement the project that maximizes her expected payoff given her prior. In other words, she can engage in *centralization*.

Should the principal centralize or delegate? And if she delegates, should she engage in complete delegation, threshold delegation, or some other form of delegation? The key trade-off that the principal faces when she considers the many different organizational arrangements is between the direct cost of biasing her decisions in favor of the agent and the indirect benefit of inducing the agent to reveal more information. Moreover, when optimizing this trade-off the principal must keep in mind that the extent to which she is able to bias her decisions is limited by her potentially imperfect commitment power. We show that in many cases the organizational arrangements that the principal chooses in our setting are commonly observed in the real world. In particular, we show that centralization, threshold delegation and menu delegation are often optimal and that which one of these arrangements is optimal depends only on the principal's commitment power, on the one hand, and a simple condition on the agents' bias and the distribution of the state space, on the other. Moreover, we show that for small biases threshold delegation is optimal for any smooth distribution. These results are consistent with the pervasive use of threshold delegation in organizations. Having derived our main characterization result we then investigate further implications, including the effects of changes in the bias and the amount of private information on the optimal organizational arrangement. Finally, we show that irrespective of the commitment power of the principal complete delegation is never optimal and that outsourcing can only be optimal if the principal's commitment power is sufficiently small.

In the next section we discuss the related literature. In Section 3 we then present our basic model in which the principal's commitment power is exogenously given. We analyze this model in Sections 4 and 5 and characterize the optimal organizational arrangements for any given level of commitment. In Section 6 we then embed our basic model in a repeated game in which the principal's commitment power is endogenously determined. There we show that the optimal relational contract corresponds to optimal organizational arrangements in the static model for an appropriately specified discount rate. The repeated game allows us to derive additional implications which we discuss in Section 7. Finally, we conclude in Section 8. All proofs are in the appendix.

2. Related literature

Suppose an organization, consisting of a principal and an agent, has to make a decision. The principal and the agent have different preferences over the decision and only the agent observes the state of the world which determines the principal's and the agent's preferred projects. A large number of papers have analyzed this basic problem and they can be categorized in two dimensions: (i.) whether or not they allow for transfers between the principal and the agent and (ii.) the extent of the principal's commitment power.

Our paper contributes to the strand of the literature which argues that in many environments transfers between the principal and the agent are difficult or impossible. Within this strand of the literature one can distinguish between delegation- and cheap talk models. In the cheap talk models that follow Crawford and Sobel (1982) principals cannot commit to arbitrary decision rules, that is they cannot commit to act on the information they receive in a pre-specified way. In contrast, in the delegation models that follow Holmström (1977, 1984) the principal can commit to a decision rule. Holmström (1977, 1984) considers a general version of the set up described above and proves the existence of an optimal delegation set or, equivalently, an optimal decision rule. He then characterizes optimal interval delegation sets, i.e. delegation sets in which the agent can choose any decision from a specific interval.[2] Arm-

[2] For a specific example he shows that interval delegation is optimal among all compact delegation sets (see p. 44 in Holmström, 1977).

strong (1995) considers a model similar to Holmström (1977, 1984) and allows for uncertainty over the agent's preferences. Like Holmström (1977, 1984) he focuses on interval delegation. In a setting in which the players' preferred decisions are linear functions of the state and the state is uniformly distributed, Melumad and Shibano (1991) characterize the optimum among all compact delegation sets. In a recent paper Alonso and Matouschek (2005) also solve for the optimal delegation set in a setting that allows for more general distributions and for arbitrary continuous state-dependent biases. Martimort and Semenov (2005) consider a setting with multiple agents and provide a sufficient condition for threshold delegation to be optimal. Since we allow for different degrees of commitment by the principal, varying from no commitment all the way to perfect commitment, our paper bridges the cheap talk and delegation literatures. Instead of making assumptions about what the principal can and cannot commit to, we endogenize her commitment power and characterize the optimal decision rule for any amount of commitment power.

The second strand of the literature that analyzes the principal-agent problem described above does allow for transfers. Ottaviani (2000) and Krishna and Morgan (2006), in particular, both allow for message-contingent transfers but make different assumption about the principal's commitment power. In particular, Krishna and Morgan (2006) focus on the case in which the principal can only commit to a transfer rule while Ottaviani (2000) allows the principal to commit to a transfer- and a decision rule.

Finally, our work is related to several recent papers that investigate the role of relational contracts within and between organizations. Baker, Gibbons, and Murphy (1994, 2002) investigate the use of objective and subjective performance measures and the ownership structures of firms in a repeated setting. Levin (2003) investigates relational incentive contracts in the presence of moral hazard and asymmetric information. MacLeod (2003) extends Levin (2003) to the case of a risk averse agent. We first follow MacLeod (2003) in treating the cost of conflict as exogenous and then follow the previous papers by endogenizing them in an infinitely repeated game.

3. The model with exogenous commitment

A firm needs to implement a project. A principal has the formal authority to decide what project is chosen but she needs to hire an agent to implement it.

Preferences: The projects are represented by a positive real number $y \in Y \subset R_+$. Although one can interpret y as measuring any one dimension on which the projects differ – for instance the number of workers to be hired for a new plant or the size of a new office building – we interpret it as the financial size of an investment. This interpretation facilitates the exposition and allows us to relate our findings to a number of papers that describe the capital budgeting rules which firms use to regulate the internal allocation of capital.[3] The principal and the agent have different preferences over the project. In particular, the principal's payoff from implementing project y is $U_P(y, \theta) = -(y - \theta)^2$, where $\theta \in \Theta = [0, 1]$ is the state of the world. In contrast, the agent's payoff is $U_A(y, \theta, b) = -(y - \theta - b)^2$, where the parameter $b > 0$ measures the congruence of the agent's and the principal's preferences. Given these preferences, the principal's preferred project is given by θ and the agent's is given by $(\theta + b)$. There is ample anecdotal evidence that documents the tendency of many managers to engage in empire building, i.e. to invest more than would be optimal from the perspective of their principals (see for instance Jensen 1986). For this reason we assume $b > 0$ so that the agent prefers a larger investment than the principal. The analysis can easily be adapted, however, to allow for negative biases. Since we are interpreting y as the financial size of an investment and since the agent's and the principal's preferred project sizes are increasing in the state θ, it is natural to think of low realizations of θ as bad states in which the business environment is unfavorable to new investments and large realizations of θ as good states in which the business environment is more favorable.

Information: The agent learns the realization of the state θ but the principal does not. It is commonly known, however, that θ is drawn from a cumulative distribution function $F(\theta)$. The corresponding probability density function $f(\theta)$ is absolutely continuous and strictly positive for all $\theta \in \Theta$.

Contracts and Communication: The principal has the legal right to decide on the projects. We adopt the incomplete contracting approach in assuming that projects cannot be contracted upon. The principal can therefore not rely on court-enforced contracts as a commitment device. We do, however, assume that the agent is able to impose a cost on the principal if

[3] For studies describing the capital budgeting rules that firms use see Footnote 3. Theoretical papers seeking to rationalize the observed rules include Harris and Raviv (1996) and Marino and Matsusaka (2005).

she reneges on a promise. This cost can be interpreted as the damage that an agent can impose on the principal by engaging in unproductive behavior in a repeated relationship. In our basic model we take this cost as exogenous but we endogenize it in Section 6. We follow the delegation literature in ruling out monetary transfers between the principal and the agent.

The timing is as follows. First the principal 'promises' to make her decision according to a decision rule $y(m) : M \to Y$ that maps the agent's message space M into projects. Second, the agent learns the state θ and sends a costless message $m \in M$. We assume that $M = Y$ and we say that the agent 'recommends' a project y if he sends a message $m = y$. Third, the principal decides what project to implement. We say that the principal 'rubber-stamps' the agent's recommendation if, in response to receiving the message $m = y$, she implements project y. If she does not renege on her promise to make the decision according to $y(m)$, then the principal and the agent realize $U_P(y(m), \theta)$ and $U_A(y(m), \theta, b)$ respectively. If she does renege by making a decision $y' \neq y(m)$ then the agent punishes her and she incurs a cost q^2. The principal's payoff is then $U_P(y', \theta) - q^2$ while the agent's is $U_A(y', \theta, b)$. The parameter $q \geq 0$ measures the principal's commitment power.

3. The cheap talk benchmark

We start the analysis by considering the cheap talk benchmark in which the principal does not have any commitment power, i.e. $q = 0$. Crawford and Sobel (1982) show that all equilibria of this game are interval equilibria in which the state space $[0, 1]$ is partitioned into intervals and the agent's recommendation only reveals which interval the state θ lies in. In this sense communication is noisy and information is lost. Having learned what interval the state lies in, the principal implements the project that maximizes her expected payoff, given her updated beliefs.

Formally, an equilibrium of the stage game is characterized by (i.) the agent's communication rule $\mu(\theta) : \Theta \to \Delta M$ which specifies the probability of sending message $m \in M$ conditional on observing state θ, (ii.) the principal's decision rule $y(m) : M \to Y$ which maps messages into projects and (iii.) the principal's belief function $g(\theta \mid m) : M \to \Delta\Theta$ which states the probability of state θ conditional on observing message m. In a Perfect Bayesian Equilibrium the communication rule is optimal for the agent given the decision rule, the decision rule is optimal for the principal given the belief function and the belief function is derived from the communication rule using Bayes' rule whenever possible.

Since all equilibria are interval equilibria, we denote by $a \equiv (a_0, ..., a_N)$ the partitioning of $[0, 1]$ into N steps, with the dividing points between steps satisfying $0 \equiv a_0 < a_1 < ... < a_N \equiv 1$. Moreover, we denote by $\widehat{y}_i \equiv \arg\max_y \int_{a_{i-1}}^{a_i} U_P(y, \theta) dF(\theta) / (F(a_i) - F(a_{i-1}))$, for all a_{i-1}, $a_i \in [0, 1]$, the principal's preferred project if she believes the state lies in the interval (a_{i-1}, a_i). Finally, we denote by y_i the project that the principal implements if she receives a recommendation from interval i, i.e. $y_i \equiv y(m)$ for $m \in (a_{i-1}, a_i)$. We can now state the following proposition which follows directly from Theorem 1 in Crawford and Sobel (1982).

Proposition 1. If $b > 0$, then there exists a positive integer $N(b)$ such that for every N with $1 \leq N \leq N(b)$, there exists at least one equilibrium $(\mu(\cdot),\ y(\cdot),\ g(\cdot))$, where

i. $\mu(\theta) = \widehat{y}_i$ if $\theta \in (a_{i-1},\ a_i)$,

ii. $y_i = \widehat{y}_i$ if $m \in (a_{i-1},\ a_i)$,

iii. $g(\theta \mid m) = f(\theta) / (F(a_i) - F(a_{i-1}))$ if $m \in (a_{i-1},\ a_i)$,

iv. $a_i = \frac{1}{2}(\widehat{y}_i + \widehat{y}_{i+1} - 2b)$ for $i = 1, ...,\ N - 1$.

All other equilibria have relationships between m and the principal's induced choice of y that are the same as those in this class for some value of N with $1 \leq N \leq N(b)$; they are therefore economically equivalent.

Thus, when θ lies in an interval (a_{i-1}, a_i), the agent recommends project $\widehat{y}_i$, the principal's preferred project conditional on the state being in that interval. Given her updated beliefs, it is then optimal for the principal to rubber-stamp the agent's recommendation. If the agent recommends a project that lies in an interval (a_{i-1}, a_i) but is not equal to the principal's preferred project $\widehat{y}_i$, then the principal believes that θ is distributed on (a_{i-1}, a_i) according to part iii. of the proposition. Given these off-the-equilibrium path beliefs, it is then optimal for the principal to reject the agent's recommendation and implement $\widehat{y}_i$ instead. The dividing point a_i between the partitions is derived from the indifference condition $U_A(\widehat{y}_i, a_i) = U_A(\widehat{y}_{i+1}, a_i)$ which ensures that in state a_i the agent is indifferent between projects $\widehat{y}_i$ and $\widehat{y}_{i+1}$. As an example, suppose that θ is uniformly distributed. It then follows from part iv. of the proposition that

$$a_{i+1} - a_i = a_i - a_{i-1} + 4b. \quad (1)$$

The lengths of the intervals therefore increase by $4b > 0$ as i increases. Thus, less information gets communicated by the agent, the larger his recommendation.[4]

Crawford and Sobel (1982) provide sufficient conditions under which the expected payoffs of the principal and the agent are increasing in the number of intervals N. When these conditions are satisfied, as they are in our specification, one may therefore expect the players to coordinate on the equilibrium in which the number of intervals is maximized, i.e. in which $N = N(b)$. We denote this equilibrium by $(\mu^{CS}, y^{CS}, g^{CS})$ and the corresponding payoffs by U_A^{CS} and U_P^{CS}, where the superscript 'CS' stands for 'Crawford and Sobel.'

In this paper we interpret interval equilibria of the type described in the first proposition as a form of 'menu delegation,' as defined next.

Definition 1 (Menu Delegation). Under 'menu delegation' the principal offers a menu with a finite number of projects and rubber-stamps any project on the menu. If the agent recommends a project that is not on the menu, the principal overrules him and implements one of the projects that is on the menu.

Under menu delegation, therefore, the agent can choose between a finite number of projects.

5. Delegation with exogenous commitment

Suppose now that the principal does have some commitment power, i.e. that $q > 0$. Suppose further that she has promised to use a specific decision rule $y(m)$. For this promise to be credible, it must be the case that her expected payoff from keeping the promise is always higher than her expected payoff from reneging and implementing $\widehat{y}(m) \equiv \arg\max \mathrm{E}_\theta [U_P(y, \theta) \mid m]$. Thus, it must be that

[4] The specification of the communication equilibria in Proposition 1 is economically equivalent to the one in Crawford and Sobel (1982). There is, however, a technical difference between their specification and ours: in their specification an agent who observes $\theta \in (a_{i-1}, a_i)$ sends a message that is uniformly distributed on (a_{i-1}, a_i). All possible messages $M = [0, 1]$ are therefore used with positive probability so that off-the-equilibrium path beliefs do not need to be specified. In contrast, we need to specify off-the-equilibrium path beliefs since we assume that an agent who observes $\theta \in (a_{i-1}, a_i)$ sends a single message (see, for instance, Gibbons 1992, pp.216-217). We adopt our specification solely for expositional convenience.

$$q^2 \geq \mathrm{E}_\theta \left[U_P(\widehat{y}(m), \theta) \mid m \right] - \mathrm{E}_\theta \left[U_P(y(m), \theta) \mid m \right] = \left(\widehat{y}(m) - y(m) \right)^2, \tag{2}$$

where the equality is due to the quadratic loss function. The *optimal delegation scheme* $(y^*(m;q),\ \mu^*(\theta;q))$ that maximizes the principal's expected payoff therefore solves

$$\max_{y(m),\mu(\theta)} \mathrm{E}_\theta \left[U_P(y(m), \theta) \right] \tag{3}$$

subject to the agent's incentive compatibility constraint

$$\mu(\theta) \in \arg\max_{m \in M} U_A(y(m), \theta) \tag{4}$$

and the reneging constraint

$$\left(\widehat{y}(m) - y(m) \right)^2 \leq q^2. \tag{5}$$

The characterization of the optimal delegation scheme is greatly facilitated by the fact that it has to be *monotonic.*

Definition 2 (Monotonicity). A delegation scheme $(y(m),\ \mu(\theta))$ is monotonic if, for any two states θ' and $\theta'' > \theta'$, the chosen projects satisfy $y\left(\mu\left(\theta''\right)\right) \geq y\left(\mu\left(\theta'\right)\right)$.

The fact that the optimal delegation scheme is monotonic is shown in the next proposition.

Proposition 2. Every optimal delegation scheme is monotonic.

The characteristics of the optimal delegation scheme depend critically on whether the principal can credibly commit to implement the agent's preferred project. Suppose that the principal knows the state and has promised to implement the agent's preferred project. This promise is only credible if the punishment for reneging, q^2, is more than the benefit b^2 of implementing the principal's preferred project rather than that of the agent. Whether or not the principal can credibly commit to implement the agents's preferred project therefore depends on whether the commitment power q is larger or smaller than the agent's bias b.

In the next sub-section we characterize the optimal delegation scheme for high commitment power, i.e. for $q \geq b$, and in the subsequent sub-section we characterize it for low commitment power $q < b$.

High commitment power

In this subsection we show that when the principal's commitment power is high, i.e. $q \geq b$, then the solution to the contracting problem (3) - (5) often resembles commonly observed organizational arrangements. In particular, we show that the optimal delegation scheme can take the form of either *centralization* or *threshold delegation*, as defined next. To understand these definitions, recall that we say that the principal 'rubber-stamps' the agents recommendation if, in response to receiving a message $m = y$, she implements project y.

Definition 3 (Centralization). Under centralization the only project the principal rubber-stamps is $y = E[\theta]$, i.e. her preferred project given her prior beliefs. If the agent recommends any other project she overrules him and implements $y = E[\theta]$.

Given this decision making by the principal, it is optimal for the agent to always recommend $y = \mathrm{E}[\theta]$. The agent's information is therefore not used under this delegation scheme.

Definition 4 (Threshold Delegation). Under threshold delegation the principal rubber-stamps any recommendation below a threshold project $(a_1 + b)$ and she overrules the agent and implements $(a_1 + b)$ if he recommends a project above the threshold.

A graphical illustration of threshold delegation is given in Figure 1. The lower diagonal line plots the principal's preferred project θ for any state and the higher diagonal line $\theta + b$ plots the preferred projects for the agent. Given the decision making by the principal, it is optimal for the agent to recommend his preferred project if $\theta \leq a_1$ and to recommend the biggest permissible project $(a_1 + b)$ if $\theta > a_1$. The bold line in Figure 1 therefore graphs the implemented projects as a function of the state. For a threshold delegation scheme to maximize the principal's expected payoff, the threshold project $(a_1 + b)$ must be chosen such that $\mathrm{E}(\theta \mid \theta \geq a_1) = (a_1 + b)$. Threshold delegation schemes are widely observed in organizations and, in particular, capital budgeting rules often take this form. Threshold

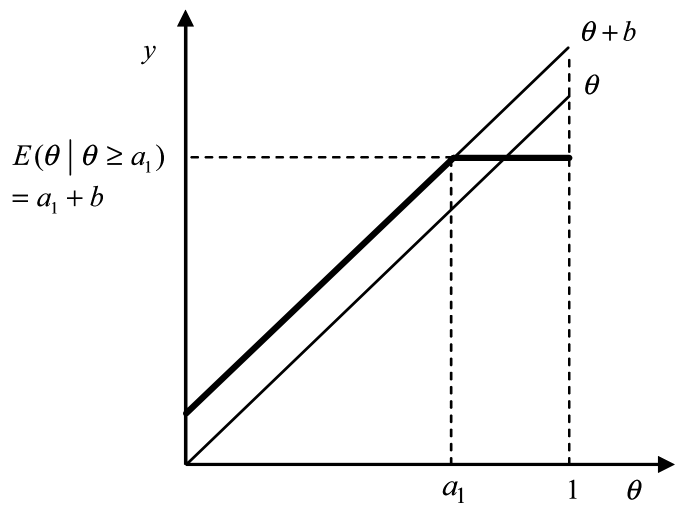

Figure 1: Threshold Delegation

delegation is also consistent with the observation in Ross (1986) that in many firms lower level managers can decide on small investments while senior managers can decide on larger investments.

The next proposition shows that in many cases threshold delegation is in fact the optimal delegation scheme.

Proposition 3. Suppose that $q \geq b$ and that $G(\theta) \equiv F(\theta) + bf(\theta)$ is strictly increasing in θ for all $\theta \in \Theta$. Then threshold delegation is optimal.

The distributional assumption stated in the proposition is satisfied for a large number of distributions and a wide range of biases. For instance, for any distribution that has a continuously differentiable density there exists a $b' > 0$ such that the condition is satisfied for all $b \leq b'$. Thus, it is satisfied for most common distributions when the bias is small.

To get an intuition for why, among the very many possible delegation schemes, threshold delegation often does best for the principal, we first need to think about the trade-off that she faces when deciding what projects to implement. The key question for the principal is how much she should bias her decision-making in favor of the agent. On the one hand, the principal clearly incurs a direct cost when she biases her decisions in favor of the agent. On the other hand, however, the agent is more willing to give precise recommendations, the more he expects his interests to be taken into account by the principal. Thus, the key trade-off that the principal faces is between the direct cost of biased decision making and the indirect benefit of better information. A feature of threshold delegation is that, conditional on the information the principal receives, decision making is biased entirely in favor of the agent when the state is below the threshold a_1 and it is not biased at all when the state is above the threshold. To see this, note that when the principal receives a recommendation $m = \theta \leq a_1$ she knows exactly the state but instead of using this information to implement her preferred project θ she uses it to implement the agent's preferred project $(\theta + b)$. In contrast, when the principal gets a recommendation $m = \theta > a_1$, she does not know the exact state and only knows that it is above the threshold. In this case it is optimal for her to implement the project $\mathrm{E}(\theta \mid \theta \geq a_1)$ that maximizes her expected payoff and not bias the decision at all in favor of the agent. As a result of this decision rule, the agent is willing to communicate all information when the state is below the threshold and very limited information when it is above the threshold.

To get an intuition for Proposition 3 it is therefore key to understand why it is optimal to bias the decisions entirely in favor of the agent in low states and not at all in high states. For this purpose, it is instructive to compare threshold delegation to two benchmarks. In the first benchmark the principal always implements her preferred projects and in the second she always implements the agent's preferred project.

When the principal always implements her preferred project, the agent is not willing to reveal the state and instead only reveals the intervals that it lies in. An example of such an equilibrium is illustrated in Figure 2 in which the principal implements the project $\widehat{y}_1 \equiv \mathrm{E}(\theta \mid \theta \leq a_1) = b$ if she receives a recommendation which is smaller than the threshold $(a_1 + b)$ and she implements a project $\widehat{y}_2 \equiv \mathrm{E}(\theta \mid \theta \geq a_1)$ if she receives a larger recommendation.[5] In this equilibrium the agent then only reveals whether the state is above or below a_1. If $G(\theta)$ is everywhere increasing in θ, then the principal can do better by rubber-stamping the agent's recommendation whenever he proposes a project that is smaller than $\widehat{y}_2$ and by implementing $\widehat{y}_2$ otherwise. In other words, she can do better by entirely biasing her decisions in favor of the agent for low states. On the one hand, doing so is costly for the principal since, for small θ, she now implements a project that is worse for her. In the example in Figure 3 the principal

[5] The assumption that $\widehat{y}_1 = b$ is not important and only facilitates the exposition.

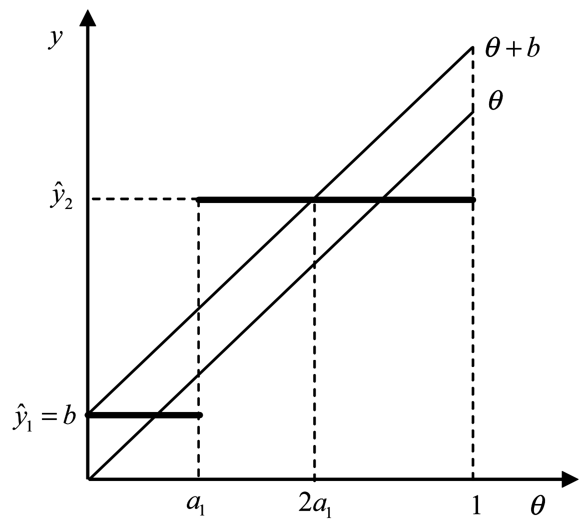

Figure 2: Implementing the Principal's Preferred Projects

implements a worse project for all $\theta \in [0, a_1]$ and the corresponding loss is indicated by triangle A. On the other hand, precisely because she is implementing a project that is worse for her when θ is small, she is able to implement a project that is better for her when θ is large. In the example in Figure 3 this is the case when $\theta \in [a_1, 2a_1]$ and the corresponding gain is indicated by triangle B. Essentially, biasing her decision in favor of the agent for low states relaxes the incentive constraint for higher states which in turn allows the principal to implement projects that are better for her. As long as the probability of being in the loss making interval $[0, a_1]$ is not too large compared to the probability of being in profiting interval $[a_1, 2a_1]$, the gain of biasing the decision in favor of the agent outweighs the costs and the principal is made better off. The condition that $G(\theta)$ is always increasing ensures that this is indeed the case.

To get a more formal intuition for the condition $G'(\theta) > 0$, consider

$$\Delta(a_1, t) = -\int_{a_1-t}^{a_1+t} b^2 \mathrm{d}F(\theta) + \int_{a_1-t}^{a_1} (a_1 + b - t - \theta)^2 \mathrm{d}F(\theta) + \int_{a_1}^{a_1+t} (a_1 + b + t - \theta)^2 \mathrm{d}F(\theta), \tag{6}$$

for $t \in [0, a_1]$. For $t = a_1$ this function is equal to the principal's expected utility under threshold delegation minus her expected payoff if only the two projects $\widehat{y}_1$ and $\widehat{y}_2$ get implemented. More generally, for $t \in [0, a_1]$ this function gives the difference between two delegation schemes

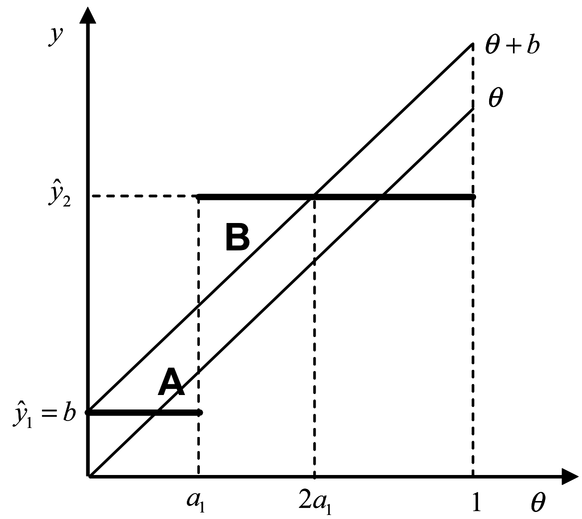

Figure 3: Comparison of Delegation Schemes

which only differ in the projects that get implemented if $\theta \in [a_1 - t, a_1 + t]$: the first delegation scheme implements the agent's preferred project $(\theta + b)$ for all $\theta \in [a_1 - t, a_1 + t]$ and the second implements $y_1 = (a_1 - t + b)$ for $\theta \in [a_1 - t, a_1]$ and $y_1 = (a_1 + t + b)$ for $\theta \in [a_1, a_1 + t]$. Taking derivatives gives $\mathrm{d}\Delta(a_1, 0)/\mathrm{d}t = 0$ and

$$\frac{\mathrm{d}^2\Delta(a_1, t)}{\mathrm{d}t^2} = 2\left[G(a_1 + t) - G(a_1 - t)\right]. \tag{7}$$

Thus, if $G(\theta)$ is always increasing, then $\Delta(a_1, t)$ is convex in t. Since $\Delta(a_1, 0) = \mathrm{d}\Delta(a_1, 0)/\mathrm{d}t = 0$ this implies that if $G(\theta)$ is increasing, then $\Delta(a_1, t) > 0$ for all $t > 0$ and, in particular, for $t = a_1$.

In the second benchmark, the principal biases her decision entirely in favor of the agent who in turn always reveals the state. While this arrangement allows the principal to elicit all available information, it also commits her to implement projects $y > 1$ that cannot be optimal for her in any state. This suggests an alternative arrangement in which the principal implements the agent's preferred project below a threshold $a_1 \leq 1$ and implements a single project $(a_1 + b)$ above the threshold. If a_1 is sufficiently high, the principal is made better off under the alternative scheme since she can realize the benefit of less biased decision making without the cost of tightening the incentive constraint for any higher states.

A key question we are interested in is what form delegation takes when a principal's ability to commit is limited. From our analysis above it follows that the optimal threshold delegation scheme can be implemented for any $q \geq b$ and not just as $q \to \infty$. This is the case since, under threshold delegation, the principal never biases her decision by more than b and thus never faces a reneging temptation of more than b^2. Thus, when $G(\theta)$ is everywhere increasing, a principal with high commitment power $q' \geq b$ behaves in exactly the same way as a principal with very high commitment power $q'' > q'$.

Proposition 3 has shown that in many cases threshold delegation is optimal. In the next proposition we show that when the conditions of that proposition are not satisfied, it is often optimal for the principal to centralize, that is to implement the project $y = \mathrm{E}(\theta)$ that she expects to maximize her payoff, given her prior.

Proposition 4. Suppose that $q \geq b$ and that $G(\theta) \equiv F(\theta) + bf(\theta)$ is strictly decreasing in θ for all $\theta \in \Theta$. Then centralization is optimal.

A necessary condition for $G(\theta)$ to be decreasing for all $\theta \in \Theta$ is that $f(\theta)$ is everywhere decreasing. In this sense, the condition is satisfied if bad states are more likely than better states. This condition is satisfied, for instance, for exponential distributions with sufficiently low means.

The formal proof of this proposition has two key parts. The first shows that if $G(\theta)$ is strictly decreasing, then separation can never be optimal, that is it can never be optimal to induce the agent to reveal the true state. For a sketch of this part of the proof, consider two delegation sets which only differ in the projects they implement if θ lies in some interval $[a_1 - t, a_1 + t]$. In particular, the first implements the agent's preferred project in this range, and therefore induces him to reveal the true state, while the second implements $y_1 = (a_1 - t + b)$ for $\theta \in [a_1 - t, a_1]$ and $y_2 = (a_1 + t + b)$ for $\theta \in [a_1, a_1 + t]$, inducing him to only reveal what interval the state lies in. The principal's expected payoff under the first scheme minus that under the second scheme is given by $\Delta(a_1, t)$ as defined in (6). Equation (7) shows that $\Delta(a_1, t)$ is concave in t if $G(\theta)$ is everywhere decreasing. Since $\Delta(a_1, 0) = \mathrm{d}\Delta(a_1, 0)/\mathrm{d}t = 0$ this implies that if $G(\theta)$ is decreasing, then $\Delta(a_1, t) < 0$ for all $t > 0$. Thus, the principal can improve on any delegation scheme that involves separation.

Having established this, the second part of the proof then shows that if $G(\theta)$ is always decreasing, centralization dominates any menu delegation scheme that offers two or more projects.

The proposition implies that in the absence of sophisticated monetary incentive schemes, it is often optimal for a principal to forgo the information that her agent possesses and to simply impose an uninformed decision. Essentially, when the principal is limited to delegation schemes, the cost of extracting information from the agent can be so high that the principal is better off making an ignorant but unbiased decision than to try to bias decisions in favor of her subordinates to elicit more information. Business history and newspapers are abound with descriptions of monolithic firms in which bureaucratic rules and regulations stifle the creativity and flexibility of their employees.[6] The proposition suggests that such bureaucracy may simply be a symptom of the firms' optimal responses to the agency problems they face.

We have seen above that when $G(\theta)$ is everywhere increasing, a principal with limited ability to commit $q \geq b$ implements the same delegation scheme as a principal with unlimited commitment power. The same is true when $G(\theta)$ is everywhere decreasing. This is so since the principal is always able to implement centralization, independent of the commitment power q that she possesses.

From the two previous propositions it is clear that the key condition that determines the optimal delegation scheme when commitment power is high is whether $G(\theta)$ is increasing or decreasing. To get a better sense for this condition and its implications we next consider an example. In particular, suppose that θ is drawn from a truncated exponential distribution with cumulative density

$$F(\theta) = \frac{1}{1 - e^{-1/\beta}} \left(1 - e^{-\theta/\beta}\right),$$

where $\beta > 0$ is the scale parameter. An increase in β causes a first order stochastic increase of the distribution and thus increases the mean $\mathrm{E}(\theta)$. Moreover, as $\beta \to \infty$, the distribution approaches the uniform distribution. It can be verified that for this exponential distribution $G(\theta)$ is everywhere increasing if $b \leq \beta$ and it is everywhere decreasing otherwise. Thus, if the bias is smaller than the scale parameter, threshold delegation is optimal and if the bias is larger than the scale parameter, centralization is optimal. To get some sense for the comparative

[6] For a colorful historical example see the case of The Hudson Bay Company in Milgrom and Roberts (1992).

statics, which we analyze more generally in Section 7, suppose that initially $\beta > b$ and consider the effect of an increase in the bias. Initially, such an increase leads to a reduction of the threshold below which the principal rubber-stamps the agent's recommendation. Eventually, $b > \beta$ and the principal centralizes, i.e. she simply implements $\mathrm{E}(\theta)$. At this point further increases in the bias do not affect the optimal delegation scheme or the decision that is made. Similarly, suppose that initially $\beta < b$ and consider the effect of an increase in β. Such an increase moves probability mass from low- to high states, making it less and less costly for the principal to implement the agent's preferred project when his recommendation is small. When β is sufficiently high, i.e. when $\beta \geq b$, it then becomes optimal for the principal to switch to threshold delegation and implement the agent's preferred projects for low states. Further increases in β then simply increase the threshold up to the maximum value of $a_1 = 1 - 2b$.

While for any exponential- and many other distributions, $G(\theta)$ is either everywhere increasing or decreasing, this is, of course, not always the case. For instance, for normal distributions with a sufficiently small variance, $G(\theta)$ is first increasing and then decreasing. For such distributions we can use a similar proof strategy as described above by dividing the support of this distributions into intervals in which $G(\theta)$ is monotonic. For an analysis of such distributions in the full commitment limit see Alonso and Matouschek (2005).

Low commitment power

In this subsection we characterize the solution to the contracting problem (3) - (5) when the principal's commitment power is low, i.e. $q < b$. The key difference between the high- and the low commitment power cases is that in the former the principal can credibly commit to decision rules that induce the agent to reveal the true states for some $\Theta' \subseteq \Theta$ while in the latter this is not possible. In other words, separation can be supported when $q \geq b$ but it cannot be supported when $q < b$. Together with the fact that optimal delegation schemes are monotonic, as established in Proposition 2, this implies that when commitment power is low, the optimal delegation scheme takes the form of menu delegation. We make this point formally in the next proposition.

Proposition 5. Suppose that $q < b$. Then menu delegation is optimal.

Thus, when commitment power is low, the principal cannot do better than to let the agent choose between a finite number of projects. Having established that for $q < b$ menu delegation is optimal, the only remaining question is what projects the principal should put on the menu. To address this question it is useful to restate the original contracting problem (3) - (5) as

$$\max_{N, y_1, \ldots, y_N} \mathrm{E}_\theta \left[U_P \right] = -\sum_{i=1}^{N} \int_{a_{i-1}}^{a_i} (y_i - \theta)^2 \mathrm{d}F(\theta) \tag{8}$$

subject to $a_0 = 0$, $a_N = 1$,

$$a_i = \frac{1}{2} \left(y_i + y_{i+1} - 2b \right) \text{ for } i = 1, ..., N-1 \tag{9}$$

and

$$\Delta y_i^2 \leq q^2 \text{ for } i = 1, ..., N, \tag{10}$$

where $\Delta y_i \equiv y_i - \widehat{y}_i$ is the difference between the project y_i that the agent recommends if the state lies in interval i and project $\widehat{y}_i$, the project that maximizes the principal's expected payoff in this case.

Just as in the case with high commitment power, the key trade-off that the principal faces is between the extent to which decision making is biased in favor of the agent, given her information, and the amount of information that is communicated by the agent. To see this, suppose that θ is uniformly distributed and recall that in the cheap talk benchmark in which $q = 0$, the intervals grow by $4b$, as shown in (1). When $q < b$, then it follows from the incentive constraints (9) that

$$(a_{i+1} - a_i) = (a_i - a_{i-1}) + \left(4b - 2\Delta y_{i+1} - 2\Delta y_i \right). \tag{11}$$

The lengths of the intervals therefore increase by $4b - 2\Delta y_{i+1} - 2\Delta y_i > 0$ as i increases. Thus, just as in the cheap talk benchmark, less information gets communicated by the agent, the larger his recommendation. The above expression, however, shows that when $q > 0$ the principal can reduce the loss of information by committing to bias her decision in favor of the agent, i.e. by setting $\Delta y_i > 0$ for $i = 1, ..., N-1$. Intuitively, the agent is more willing to communicate information if the principal is committed to take his interests into account when making a decision. It is because of the improved communication that the principal may be willing to incur the direct cost of biasing her decisions in favor of the agent.

The solution to the above contracting problem again depends crucially on the distribution of θ and the bias b. It follows immediately from Proposition 4 that, when $G(\theta) \equiv F(\theta) + bf(\theta)$ is decreasing in θ, centralization is optimal. This is the case since, under centralization, the temptation to renege is equal to zero and can therefore be implemented for any level of commitment power q. This result is stated formally in the next proposition.

Proposition 6. Suppose that $q < b$ and that $G(\theta) \equiv F(\theta) + bf(\theta)$ is decreasing for all $\theta \in \Theta$. Then centralization is optimal.

When $G(\theta)$ is not everywhere decreasing, the optimal menu delegation scheme does depend on the level of commitment power q. To get a better understanding of how changes in q affect the optimal menu delegation scheme in this case, the next proposition provides a characterization for an example in which θ is uniformly distributed on $[0, 1]$.

Proposition 7. Suppose that $q < b$ and that $F(\theta) = \theta$. Then there exists a $\overline{q} \in (0, b)$ such that

i. for all $q \leq \overline{q}$, $\Delta y_i = q$ for all i and the number of intervals N is maximized.

ii. for all $q > \overline{q}$, $\Delta y_1 \leq q$, $\Delta y_i = q$ for $i = 2, ..., N-1$, and $\Delta y_N \leq q$.

Thus, when the principal has very little commitment power, i.e. when $q \leq \overline{q}$, the benefit of additional information is so large that the gain of biasing decisions dominates the costs. As a result, it is optimal for her to bias her decision up to the maximum credible level. Note that in this case the number of intervals is maximized and that intervals grow by $4(b - q)$, as can been from (11). Thus, the amount of information that is being communicated is exactly the same as the one that would be communicated in the best Crawford and Sobel equilibrium of the static game when the agent has a bias of $(b - q)$. In terms of information transmission, therefore, commitment power is a perfect substitute for a reduction in the agent's bias.

When the amount of commitment power grows beyond the threshold $\overline{q}$, it is still the case that the principal wants to extract more information by biasing all intermediate decisions

	$G(\cdot)$ increasing	$G(\cdot)$ decreasing
High Relational Capital	Threshold Delegation	Centralization
Low Relational Capital	Menu Delegation	Centralization

Figure 4: Summary of Optimal Organizational Arrangements

$y_2, ...y_{N-1}$ as much as possible. However, it can now be optimal to reduce Δy_1 and Δy_N so as to economize on the cost of biased decision making. In fact, we know from Proposition 3 that when $q = b$, the bias of the last and largest interval is optimally set to zero. Thus, although the principal could extract as much information as in a static game with bias $(b - q)$, it is not always optimal for her to do so when $q > \overline{q}$.

In summary, the analysis so far has shown that commonly observed organizational arrangements are often optimal in our basic model. Moreover, we have seen that exactly what arrangement is optimal depends crucially on two factors, namely the principal's commitment power and the interplay between the bias and the distribution of the state, as summarized in the simple condition $G(\theta) = F(\theta) + bf(\theta)$. In particular, Figure 4, which summarizes some of the key results that we derived so far, shows that when $G(\theta)$ is always increasing, threshold delegation is optimal when commitment power is high and menu delegation is optimal when commitment power is low while centralization is always optimal when $G(\theta)$ is decreasing. Also, we have seen that in many cases changes in the commitment power do not affect the optimal delegation scheme. In particular, when either $G(\theta)$ is decreasing or $G(\theta)$ is increasing and commitment power is high, increases in q have no effect on the optimal delegation scheme. Only when commitment power is small and $G(\theta)$ is not everywhere decreasing can changes in q lead to changes in the optimal delegation scheme.

6. Delegation with endogenous commitment

So far we assumed that the agent is able to impose some exogenous cost q^2 on the principal whenever she reneges on a promise. This cost is meant to capture the damage that an agent can impose on the principal through unproductive behavior in a repeated relationship. In this section we endogenize this cost in an infinitely repeated version of the above model. We characterize the relational contract that maximizes the principal's expected payoff and show that it is closely related to the optimal delegation schemes described above. In the next section we then describe additional implications of the model.

To endogenize the principal's commitment power, consider an infinitely repeated version of the static game in which a long-lived principal faces a series of short-lived agents. In particular, there are infinitely many periods $t = 1, 2, 3, ...$ and in every period the principal and an agent play the same stage game. This stage game is identical to that described in Section 3 except that the agent is not able to impose an exogenous cost on the principal. The principal is infinitely long lived and in every period she aims to maximize the present discounted value of her stage game payoffs, where the discount rate is given by $\delta \in [0, 1)$. At the beginning of every period t the principal is matched with a new, randomly drawn agent who only interacts with her for one period. An agent who is matched with the principal in period t aims to maximize his stage game payoff $U_A(y_t, \theta_t, b)$, where y_t and θ_t are, respectively, the project and the state in period t. Note that all the agents have the same payoff function and, in particular, the same bias b. We assume that the states θ_t are i.i.d. over time and that they become publicly known at the end of each stage game. The history of the game up to date t is denoted by $h^t = (\theta_0,\, m_0,\, y_0,\, ...,\, \theta_{t-1},\, m_{t-1},\, y_{t-1)}$, the null history is denoted by h^0, and the set of all possible date t histories is denoted by H^t.

A *relational contract* describes the behavior over time in the repeated game, both on the equilibrium path and following a deviation. Formally, a relational contract specifies for any date t and any history $h^t \in H^t$, (i.) a communication rule $\mu_t(\cdot) : \Theta \times H^t \to \Delta(M)$ which assigns a probability distribution over M for any state θ_t; (ii.) a decision rule $y_t(\cdot) : M \times H^t \to \mathbb{R}$ which assigns a project y_t for every message m_t; (iii.) a belief function $g_t(\cdot) : M \times H^t \to \Delta(\Theta)$ which assigns a probability distribution over the states θ_t for every message m_t. Note, in particular, that histories are public. The belief function g_t is derived from μ_t using Bayes' rule wherever possible.

A relational contact is *self-enforcing* if it describes a sub-game perfect equilibrium of the repeated game. We focus on self-enforcing relational contracts with two properties. First, they are *optimal*, in the sense that they maximize the principal's expected present discounted payoff. Second, the most severe punishment that can be implemented off the equilibrium path calls for the agents and the principal to revert to the best static equilibrium (μ^{CS} , y^{CS}, g^{CS}). In other words, in the punishment phase the principal and the agents play the strategies that maximize their expected stage game payoffs. This assumption captures our belief that, when relational contracts break down, members of the same firm are likely to coordinate on the equilibrium that maximizes their respective payoffs in the absence of trust.[7] We discuss this assumption further at the end of this section.

We start the analysis of the repeated game by showing that the search for the optimal relational contract can be greatly simplified by focusing on *stationary* contracts.

Definition 5 (Stationarity). A relational contract is stationary if on-the-equilibrium path $\mu_t(\cdot) = \mu(\cdot)$ and $y_t(\cdot) = y(\cdot)$ for every date t, where $\mu(\cdot)$ is some communication rule and $y(\cdot)$ is some decision rule.

Under a stationary relational contract, each agent uses the same communication rule $\mu(\cdot)$ and in every period the principal uses the same decision rule $y(\cdot)$. We can now establish the following proposition.

Proposition 8. There always exists an optimal relational contract that is stationary.

To characterize the optimal relational contract we therefore only need to characterize the *optimal relational delegation scheme* ($y^{**}(m;\delta)$, $\mu^{**}(\theta;\delta)$) which solves

$$\max_{y(m),\mu(\theta)} \mathrm{E}_\theta \left[U_P(y(m),\theta)\right] \tag{12}$$

subject to the agent's incentive compatibility constraint

[7] Baker, Gibbons, Murphy (1994) make a similar assumption for the same reason.

$$\mu(\theta) \in \arg\max_{m \in M} U_A(y(m), \theta) \tag{13}$$

and the reneging constraint

$$\left(\widehat{y}\left(m\right) - y\left(m\right)\right)^2 \leq \frac{\delta}{1-\delta} \mathrm{E}_\theta \left[U_P\left(y\left(\cdot\right), \theta\right) - U_P^{CS}\right]. \tag{14}$$

The LHS of the reneging constraint is the principal's one period benefit from making decision $\widehat{y}(m) \equiv \arg\max \mathrm{E}_\theta \left[U_P(y, \theta) \mid m\right]$ rather than decision $y(m)$. The RHS is the maximum punishment that the agents can impose on the principal for reneging: by reverting to the best static equilibrium, the agents ensure that the principal's expected payoff in every post-reneging period is $\mathrm{E}_\theta \left[U_P^{CS}\right]$ rather than $\mathrm{E}_\theta \left[U_P\left(y\left(\cdot\right), \theta\right)\right]$. The expression on the RHS therefore corresponds to (the square of) the exogenously given commitment power q in the previous section.

We can use Propositions 3 to 7 to characterize the optimal relational delegation scheme $(y^{**}(m; \delta), \mu^{**}(\theta; \delta))$. To see this, note that the only difference between the static contacting problem (3) - (5) and the contracting problem in the repeated game (12) - (14) is the RHS of the reneging constraint: in the static problem the commitment power is exogenously given while it is endogenously determined in the repeated game problem. It then follows that the solution to the static problem is equivalent to that of the repeated game problem for an appropriately specified discount rate. In particular, if $\left(y^*\left(m; q\right), \mu^*\left(\theta; q\right)\right)$ are optimal for a given q, then the optimal relational delegation scheme is given by $y^{**}\left(m; \delta'\right) = y^*\left(m; q\right)$ and $\mu^{**}\left(\theta; \delta'\right) = \mu^*\left(\theta; q\right)$, where δ' is the unique discount rate $\delta \in [0, \infty)$ that solves

$$\frac{\delta}{1-\delta} \mathrm{E}_\theta \left[U_P\left(y^*\left(m; q\right), \theta\right) - U_P^{CS}\right] = q^2. \tag{15}$$

We make this point formally in the next proposition.

Proposition 9. Let $\left(y^*\left(m; q\right), \mu^*\left(\theta; q\right)\right)$ be the optimal delegation scheme for a given q. The optimal relational delegation scheme is then given by $\mu^{**}\left(\theta; \delta'\right) = \mu^*\left(\theta; q\right)$ and $y^{**}\left(m; \delta'\right) = y^*\left(m; q\right)$, where δ' solves (15).

The insights of Propositions 3 to 7 can therefore be directly applied to the repeated game. Thus, for instance, Propositions 4 and 6 imply that centralization is optimal for any discount rate if $G(\theta)$ is always decreasing. If, instead, $G(\theta)$ is always increasing then it follows from Propositions 3 and 5 that threshold delegation is optimal if the discount rate is sufficiently high and menu delegation is optimal otherwise.

To conclude this section it should be noted that our qualitative results do not depend on what assumption is made about the off-the-equilibrium path punishment. Above we have assumed that the worst punishment that can be imposed on the principal is to revert to the best static equilibrium. More severe punishments would merely reduce the principal's off-the-equilibrium path payoff and therefore increase the principal's commitment power for any discount rate. The only effect of allowing for a more severe punishment would therefore be to lower the critical discount rate above which threshold delegation can be implemented.

7. Implications

In this section we explore further implications of our analysis of optimal relational contracts.

Relational delegation for small biases

In the previous sections we have seen that in many cases three commonly observed organizational arrangements are optimal. It turns out that for small biases only one organizational arrangement is optimal. Specifically, as the next proposition shows, threshold delegation is optimal for almost all distributions when the bias is sufficiently small.

Proposition 10. Suppose that $f(\theta)$ is twice continuously differentiable. Then threshold delegation is optimal for sufficiently small biases.

Recall that when $f(\theta)$ is continuously differentiable, $G(\theta)$ is increasing for a sufficiently small bias b. To prove Proposition 10 we therefore only need to show that threshold delegation can be credibly implemented when b is small enough. To see that this is indeed the case, consider the reneging constraint $b^2 \leq \delta/(1-\delta)\mathrm{E}_\theta\left[U_P^{TD} - U_P^{CS}\right]$, where the LHS is the maximum reneging temptation under threshold delegation, the RHS is the punishment for reneging and U_P^{TD} is the principal's stage game payoff under the optimal threshold delegation scheme. Note that a reduction in the bias increases the payoff U_P^{CS} that the principal can realize in the absence of a relational contract. Thus, a reduction in the bias not only reduces the benefit of reneging – the LHS of the inequality – but also, potentially, the punishment of doing so – the RHS. It is therefore not immediate that a reduction in the bias makes the reneging constraint less binding. In the formal proof we show, however, that when $f(\theta)$ is twice continuously differentiable, then, as b goes to zero, the benefit of reneging goes to zero faster than the punishment. Thus, for sufficiently small b the reneging constraint is satisfied and threshold delegation can be credibly implemented.

The effects of changes in the bias and the amount of private information

Since threshold delegation and centralization play such prominent roles in our model we next investigate how they are affected by changes in the economic environment.

Suppose first that threshold delegation is optimal and consider the maximization problem that determines the optimal threshold a_1 below which the principal implements the agents' preferred project and above which she implements her own preferred project:

$$\max_{a_1} \mathrm{E}_\theta\left[U_P\right] = -\left(\int_0^{a_1} b^2 \mathrm{d}F(\theta) + \int_{a_1}^1 (a_1 + b - \theta)^2 \mathrm{d}F(\theta)\right).$$

The optimal threshold level then solves the necessary first order condition

$$(\mathrm{E}(\theta \mid \theta \geq a_1) - (a_1 + b)) \quad \begin{cases} \leq 0 & \text{if } a_1 = 0 \\ = 0 & \text{if } a_1 > 0. \end{cases}$$

Comparative statics can now be easily performed using the graphical representation of the first order condition in Figure 5.

For instance, suppose that threshold delegation is optimal for a given b and consider the effect of a reduction in the bias. Note that if $G(\theta)$ is increasing for a given b then it is also increasing for any $b' < b$; thus threshold delegation remains optimal after the reduction in the bias. It can be seen in Figure 5 that a reduction in b shifts down $(a_1 + b)$ but does not affect $\mathrm{E}(\theta \mid \theta \geq a_1)$. Thus, a reduction in the bias increases the optimal threshold, i.e. it leads to

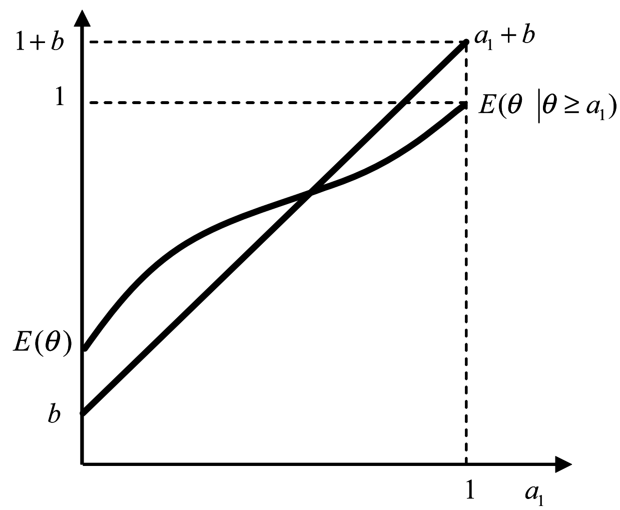

Figure 5: Comparative Statics

more delegation. This result is in line with Jensen and Meckling (1992) who argue that a reduction in agency costs should generally lead to more delegation.

Suppose next that threshold delegation is optimal for a given distribution and consider the effect of an increase in the amount of private information, as formalized by a mean preserving spread of the distribution. At first glance one may think that such a change makes the agents' information 'more important' and should thus lead to more delegation. In our model, however, there are two reasons why this is not necessarily the case. First, a mean preserving spread can affect the sign of $G'(\theta)$. Thus, it is quite possible that after an increase in the amount of private information threshold delegation is no longer optimal. Second, even if $G(\theta)$ is still increasing after the mean preserving spread, it has an ambiguous effect on the optimal threshold a_1. To see this, consider Figure 5 and note that while a mean preserving spread does not affect $(a_1 + b)$, it has an ambiguous effect on conditional mean $\mathrm{E}(\theta \mid \theta \geq a_1)$. Thus, in our model, a change in the amount of private information can lead to more or less delegation, depending on the exact parameter values and distributional assumptions.

Finally, consider the effects of changes in the economic environment on centralization. Suppose that centralization is optimal for a given bias b' and consider the effect an increase in the bias to $b'' > b'$. If $G(\theta)$ is everywhere decreasing for b' then it is also everywhere decreasing for $b'' > b'$. Thus, after the increase in the bias centralization is still optimal. Moreover, since an increase in the bias does not affect $\mathrm{E}(\theta)$, the principal implements the same decision after the increase in b as she did before the increase. While the effect of an increase in the bias on centralization is unambiguous, the effect of an increase in the amount of private information is less clear-cut. This is again the case since a mean preserving spread can change the sign of $G(\theta)$ so that centralization may no longer be optimal after the increase in the amount of private information. If it does not change the sign of $G(\theta)$ an increase in the amount of private information does not affect the optimal delegation scheme and the decision that is implemented by the principal remains $\mathrm{E}(\theta)$.

Threshold delegation and investment inefficiencies

Whenever the principal chooses threshold delegation, she optimally induces overinvestment in low states and underinvestment in high states.

Corollary. Under the conditions in Proposition 3, it is optimal for the principal to induce underinvestment if $\theta \geq E(\theta \mid \theta \geq a_1)$ and to induce overinvestment otherwise.

To see this, consider Figure 1 which gives an example of a threshold delegation scheme. From the principal's perspective, the efficient investment level in state θ is simply θ. In Figure 1, however, it can be seen that this efficient investment level is almost never achieved. Instead, it is optimal for the principal to induce investments that are larger than θ when the states are low, i.e. below $\mathrm{E}(\theta \mid \theta \geq a_1)$, and to induce investments that are smaller than θ when states are high. In other words, given the informational asymmetry, the principal cannot do better than to allow the agents to spend too much on small projects and too little on large projects.

Complete delegation and outsourcing

An organizational arrangement that has received a lot of attention in the literature (see in particular Dessein 2002) and is notably absent from our discussion up to this point is *complete delegation*, as defined next.

Definition 6 (Complete Delegation). Under complete delegation the principal always rubber-stamps the agent's recommendation.

Faced with this decision rule, an agent always recommends his preferred project.

Proposition 11. Complete delegation is never optimal.

To see this, suppose that the principal does engage in complete delegation and note that to do so she must be able to resist a maximum reneging temptation of b^2. When she has enough commitment power to implement complete delegation, however, she also has enough commitment power to implement an alternative scheme in which she rubber-stamps the agents' proposals when they are small and implements a threshold project when they are large. Such a scheme increases the principal's expected payoff but does not increase the maximum reneging temptation, which remains to be b^2. Thus, whenever complete delegation is feasible, it is not payoff maximizing for the principal.

So far we have ruled out the possibility of *outsourcing*, by which we mean the transfer of formal authority to the agents. If the principal could outsource, then agents would always choose their preferred project. Thus, outsourcing implements the same decision rule as complete delegation. In contrast to complete delegation, however, it does not require any commitment power by the principal. We have seen above that when commitment power is high, the principal can implement complete delegation but does not find it optimal to do so. It is then immediate that outsourcing cannot be optimal for a principal with high commitment power. However, a principal with low commitment power cannot implement complete delegation and may find it optimal to outsource since doing so allows her to credibly commit to having the agents' preferred projects being implemented. We can therefore state the following proposition.

Proposition 12. There exists a critical level of commitment power $q' < b$ such that outsourcing does better than relational delegation only if $q < q'$.

This proposition is related to a key result in Dessein (2002). He considers a static game that is very similar to our basic model and compares outsourcing to 'communication,' i.e. the best equilibrium without any commitment.[8] The key result in his paper is that, in a static setting, the principal is often better off outsourcing than relying on communication. The above proposition shows that this can only be the case if the principal is sufficiently impatient.

[8] Dessein (2002) uses the term 'delegation' to refer to what we call 'outsourcing.'

8. Conclusions

In this paper we investigated the allocation of decision rights within firms. In particular, we analyzed a principal-agent problem in which an uninformed principal can elicit information from an informed agent by implicitly committing herself to act on the information she receives in a particular manner. We showed that commonly observed organizational arrangements arise optimally in this setting. Specifically, we showed that centralization, threshold delegation and menu delegation are often optimal. Which one of these organizational arrangements is optimal depends only on the principal's commitment power, on the one hand, and a simple condition on the agents' bias and the distribution of the state space, on the other. Moreover, we showed that for small biases threshold delegation is optimal for any smooth distribution. Finally, we showed that complete delegation is never optimal and that outsourcing can only be optimal if the principal is sufficiently impatient.

The analysis in this paper can be extended in at least two directions. First, to take a step towards investigating delegation in a setting with imperfect commitment power, we have focused on the principal's commitment problem and have abstracted from that of agents. We believe that it would be interesting to investigate delegation when either only the agents or the agents and the principal have some commitment power. Second, in the repeated game we have assumed that the state is publicly observed at the end of each period. This assumption ensures that histories are public and thereby facilitates the analysis. Relaxing this assumption would surely be interesting and would shed more light on the internal organization of firms. We leave the investigation of these issues for future research.

Appendix

This appendix contains the proofs of all propositions. We first present the proofs of Propositions 3 and 4 and then the proofs of all remaining propositions.

Proofs of Propositions 3 and 4

For the proofs of Propositions 3 and 4 it is useful to introduce some new notation. In particular, let $\widetilde{F}(\theta) \equiv 1-F(\theta)$, $S(\theta) \equiv \widetilde{F}(\theta)\left[(\theta+b) - \mathrm{E}\left[s\,|s \geq \theta\right]\right]$ and $T(\theta) \equiv F(\theta)\left[(\theta+b) - \mathrm{E}\left[s\,|s \leq \theta\right]\right]$. It is also useful to introduce two lemmas. To do so, let

$$\Delta(p,t) \equiv -\int_{p-b-t}^{p-b+t} b^2 \mathrm{d}F(\theta) - \left(-\int_{p-b-t}^{p-b} \left[p-t-\theta\right]^2 \mathrm{d}F(\theta) - \int_{p-b}^{p-b+t} \left[p+t-\theta\right]^2 \mathrm{d}F(\theta)\right),$$

where $(p-b-t)$ and $(p-b+t)$ belong to $[0,1]$ and $t \geq 0$. To understand the economic meaning of this function, suppose that there are two decision rules which only differ in the projects that are induced for $\theta \in [p-b-t, p+t-b]$. In particular, the first decision rule induces $y = \theta + b$ for $\theta \in [p-b-t, p+t-b]$ while the second decision rule implements $y = p-t$ for all $\theta \in [p-b-t, p-b)$ and $y = p+t$ for all $\theta \in [p-b, p+t-b]$. The function $\Delta(p,t)$ captures the difference in the principal's expected payoff from these decision rules.

Lemma A1. Suppose that $G(\theta)$ is strictly monotone in $[\underline{\theta}, \overline{\theta}] \subset [0,1]$. If $G(\theta)$ is strictly increasing in $[\underline{\theta}, \overline{\theta}]$ then $\Delta(p,t) > 0$. If $G(\theta)$ is strictly decreasing in $[\underline{\theta}, \overline{\theta}]$ then $\Delta(p,t) < 0$, with $p = \left(\underline{\theta} + \overline{\theta}\right)/2 + b$ and $t = \left(\underline{\theta} - \overline{\theta}\right)/2$.

Proof. We first note that,

$$\frac{\partial \Delta(p,t)}{\partial t} = \int_{p-b-t}^{p-b} 2\left[-p+\theta\right] \mathrm{d}F(\theta) + \int_{p-b}^{p-b+t} 2\left[p-\theta\right] \mathrm{d}F(\theta) + 2t[F(p-b+t) - F(p-b-t)]$$

and

$$\frac{\partial^2 \Delta(p,t)}{\partial t^2} = 2\left[F(p-b+t) + bf(p-b+t) - F(p-b-t) - bf(p-b-t)\right].$$

Thus, if $G(\theta)$ is strictly increasing in $\theta \in [\underline{\theta}, \overline{\theta}]$, then $\partial^2 \Delta(\left(\underline{\theta} + \overline{\theta}\right)/2 + b, t)/\ \partial t^2 > 0$ for all $0 < t \leq \left(\underline{\theta} - \overline{\theta}\right)/2$. Since $\partial\Delta(\left(\underline{\theta} + \overline{\theta}\right)/2 + b, 0)/\partial t = 0$ it follows that for all $t > 0$, $\partial\Delta(p,t)/\ \partial t > 0$ and $\Delta(\left(\underline{\theta} + \overline{\theta}\right)/2 + b, \left(\underline{\theta} + \overline{\theta}\right)/2) = \int\limits_0^{\left(\underline{\theta}+\overline{\theta}\right)/2} \frac{\partial\Delta(\left(\underline{\theta}+\overline{\theta}\right)/2+b,t')}{\partial t} \mathrm{d}t' > 0$. By a similar reasoning, if $G(\theta)$ is decreasing in θ, we have that $\Delta(\left(\underline{\theta} + \overline{\theta}\right)/2 + b, \left(\underline{\theta} + \overline{\theta}\right)/2) < 0$. *Q.E.D.*

Lemma A2. Suppose that $G(\theta)$ is strictly monotone in $[0,1]$, then (i.) if $G(\theta)$ is strictly decreasing in $[0,1]$ then $E\left[\theta \left| \theta \geq a_1\right.\right] < a_1 + b$ for all $a_1 \in [0,1]$ (ii.) if $G(\theta)$ is strictly increasing in $[0,1]$ then the equation $E\left[\theta \left| \theta \geq a_1\right.\right] = a_1 + b$, $a_1 \in (0,1)$ has a solution if and only if $E[\theta] > b$. Furthermore, this solution is unique.

Proof. (i.) Since $G(\theta)$ is strictly decreasing we have that $1 - F(\theta) < 1 - F(1) + b\left(f(\theta) - f(1)\right) = b\left(f(\theta) - f(1)\right)$. Integrating both sides between a_1 and 1 yields the inequality $\int\limits_{a_1}^{1} \theta f(\theta)\mathrm{d}\theta - \widetilde{F}(a_1)a_1 < \widetilde{F}(a_1)b - bf(1)(1-a_1) < \widetilde{F}(a_1)b$. It then follows that $\mathrm{E}\left[\theta \left| \theta \geq a_1\right.\right] < a_1 + b$.

(ii.) Recall the definition of $S(\theta)$ and note that $\mathrm{d}S(\theta)/\mathrm{d}\theta = 1 - G(\theta)$ for $\theta \in (0,1)$. Thus, $S(\theta)$ is strictly concave from the assumptions on $G(\theta)$. Since $S(1) = 0$, strict concavity implies that there can be at most one point $\theta \in [0,1)$ at which $S(\theta) = 0$. The existence of this point in turn requires $S(0)$ to be non-positive, i.e. $S(0) = b - \mathrm{E}[\theta] \leq 0$ and thus establishes necessity. Now suppose that $\mathrm{E}[\theta] > b$. Since for $0 < \varepsilon < b$ we have that $(1 - \varepsilon + b) - \mathrm{E}\left[s \left| s \geq 1 - \varepsilon\right.\right] \geq b - \varepsilon > 0$. Therefore $S(0) < 0$ and $S(1 - \varepsilon) > 0$ which guarantees the existence of a solution to $\mathrm{E}\left[s \left| s \geq \theta\right.\right] = \theta + b$ thus proving sufficiency. *Q.E.D.*

PROOF OF PROPOSITION 3

To establish Proposition 3 we need to introduce two more lemmas.

Lemma A3.. Suppose that $G(\theta)$ is strictly increasing in $[0,1]$. Then if $y(\cdot)$ and $\mu(\cdot)$ is an optimal delegation scheme there cannot be two consecutive pooling regions, i.e. there cannot be two intervals $[\theta_i, \theta_{i+1}]$ and $[\theta_{i+1}, \theta_{i+2}]$ with $0 \leq \theta_i < \theta_{i+1} < \theta_{i+2} \leq 1$ such that $y(\mu(\theta)) = y_i$ for all $\theta \in (\theta_i, \theta_{i+1})$ and $y(\mu(\theta)) = y_{i+1}\,(\neq y_i)$ for all $\theta \in (\theta_{i+1}, \theta_{i+2})$.

Proof. We consider three cases: (i.) both consecutive projects $y_i, y_{i+1} \in [b, 1+b]$, (ii.) $y_i < b$ and $b < y_{i+1} < 1+b$ (iii.) $b < y_i < 1+b$ and $y_{i+1} > 1+b$.

CASE I: Let $[\theta_i, \theta_{i+1}]$ and $[\theta_{i+1}, \theta_{i+2}]$ be the two pooling regions that, for all interior points, induce the projects y_i and y_{i+1}, respectively. Then from incentive compatibility, it must be that $y_i - b \in [\theta_i, \theta_{i+1}]$ and $y_{i+1} - b \in [\theta_{i+1}, \theta_{i+2}]$. Now consider an alternative ($\widehat{y}(\cdot)$, $\widehat{\mu}(\cdot)$) such that $\widehat{y}(\widehat{\mu}(\theta)) = \theta + b$ if $\theta \in [y_i - b, y_{i+1} - b]$ and $y(\mu(\theta))$ otherwise. It is immediate that $(\widehat{y}(\cdot), \widehat{\mu}(\cdot))$ is incentive compatible if $y(\cdot)$ and $\mu(\cdot)$ is. Since $G(\theta)$ is strictly increasing in $[0,1]$ by Lemma A1 we infer that $\Delta((y_i + y_{i+1})/2 - b, (y_{i+1} - y_i)/2) > 0$ and therefore the principal strictly prefers $(\widehat{y}(\cdot), \widehat{\mu}(\cdot))$ to $(y(\cdot), \mu(\cdot))$. Hence $(y(\cdot), \mu(\cdot))$ cannot be optimal.

CASE II: If both y_i and $y_{i+1}$are to be induced with positive probability it must be that $b < (y_i + y_{i+1})/2$. From incentive compatibility of the agent, y_i is induced in $[0, (y_i + y_{i+1})/2 - b)$ and for $((y_i + y_{i+1})/2 - b, y_{i+1} - b]$ we have $y(\mu(\theta)) = y_{i+1}$. Now consider the alternative incentive compatible $(\widehat{y}(\cdot), \widehat{\mu}(\cdot))$ such that

$$\widehat{y}(\widehat{\mu}(\theta)) = \begin{cases} b & \text{if} \quad \theta \in [0, (y_i + y_{i+1})/2 - b] \\ y_i + y_{i+1} - b & \text{if} \quad \theta \in ((y_i + y_{i+1})/2 - b, y_i + y_{i+1} - 2b] \\ \theta + b & \text{if} \quad \theta \in [y_i + y_{i+1} - 2b, y_{i+1} - b] \\ y(\mu(\theta)) & \text{otherwise.} \end{cases}$$

Let $a \equiv (y_i + y_{i+1})/2 - b$. Then the increment in the principal's expected payoff of switching from $(y(\cdot), \mu(\cdot))$to $(\widehat{y}(\cdot), \widehat{\mu}(\cdot))$ is

$$\begin{aligned} \Delta \quad \equiv \quad & \int_0^a \left([y_i - \theta]^2 - [b - \theta]^2 \right) \mathrm{d}F(\theta) + \int_a^{2a} \left(\left[y_{i+1} - \theta\right]^2 - [y_i + y_{i+1} - b - \theta]^2 \right) \mathrm{d}F(\theta) \\ & + \int_{2a}^{y_{i+1} - b} \left(\left[y_{i+1} - \theta\right]^2 - b^2 \right) \mathrm{d}F(\theta). \end{aligned}$$

Note that

$$\Delta > \int_0^a 2 \left([y_i - b] \left[\frac{y_i + b}{2} - \theta \right] \right) \mathrm{d}F(\theta) + \int_a^{2a} 2 \left([b - y_i] \left[\frac{y_i + 2y_{i+1} - b}{2} - \theta \right] \right) \mathrm{d}F(\theta).$$

Using $T(\theta)$ as defined at the beginning of this section, the above inequality can then be rewritten as $\Delta > [b - y_i] [2T(2a) - 4T(a)] + [b - y_i]^2 F(2a)$. Since $G(\theta)$ is strictly increasing,

$T(\theta)$ is strictly convex which in turn implies that $T(2a) > 2T(a)$. This establishes that $\Delta > 0$. Thus, $(y(\cdot), \mu(\cdot))$ cannot be optimal.

CASE III: Suppose $y_{i+1} > 1 + b, b < y_i < 1 + b$. In this case if both y_i and $y_{i+1}$are to be induced with positive probability it must be that $(y_i + y_{i+1})/2 < 1 + b$. From incentive compatibility of the agent, y_i is induced in $[y_i - b, (y_i + y_{i+1})/2 - b)$ and for $((y_i + y_{i+1})/2 - b, 1]$ we have $y(\mu(\theta)) = y_{i+1}$. Now consider the alternative incentive compatible $(\widehat{y}(\cdot), \widehat{\mu}(\cdot))$ such that $\widehat{y}(\widehat{\mu}(\theta)) = y_i$ if $\theta \in [y_i - b, 1]$ and $y(\mu(\theta))$ otherwise. Note that $(y(\cdot), \mu(\cdot))$ and $(\widehat{y}(\cdot), \widehat{\mu}(\cdot))$ only differ for $\theta \in ((y_i + y_{i+1})/2 - b, 1]$. Thus, the increment in the principal's expected payoff of switching to $(\widehat{y}(\cdot), \widehat{\mu}(\cdot))$ is

$$\Delta \equiv \int_{(y_i+y_{i+1})/2-b}^{1} \left(\left[y_{_{i+1}} - \theta \right]^2 - [y_i - \theta]^2 \right) \mathrm{d}F(\theta) = 2 \left[y_{_{i+1}} - y_i \right] S((y_i + y_{i+1})/2 - b).$$

If $\mathrm{E}[\theta] < b$, then by the proof of Lemma A2 (ii.) we have $S(\theta) > 0$ for $\theta \in [0, 1)$. Hence, $\Delta > 0$. If $\mathrm{E}[\theta] \geq b$, then let θ^* be the unique solution to $\mathrm{E}[s \,|s \geq \theta] = \theta + b$. If $(y_i + y_{i+1})/2 - b > \theta^*$ then again $S((y_i + y_{i+1})/2 - b) > 0$ and $\Delta > 0$.

For the case that $\mathrm{E}[\theta] \geq b$ and $(y_i + y_{i+1})/2 - b < \theta^*$ consider the alternative incentive compatible $(\widehat{y}(\cdot), \widehat{\mu}(\cdot))$ which is derived from $(y(\cdot), \mu(\cdot))$ by replacing the project y_{i+1} with $\widehat{y}$, such that $y_i < \widehat{y} < y_{i+1}$. Then $(\widehat{y}(\cdot), \widehat{\mu}(\cdot))$ is defined by $\widehat{y}(\widehat{\mu}(\theta)) = y_i$ if $\theta \in [y_i - b, (y_i + \widehat{y})/2 - b)$, $\widehat{y}(\widehat{\mu}(\theta)) = \widehat{y}$ if $\theta \in [(y_i + \widehat{y})/2 - b, 1]$ and $y(\mu(\theta))$ otherwise. Defining $a = (y_i + \widehat{y})/2 - b$ and $c = (y_i + y_{i+1})/2 - b$ the increment in the principal's expected payoff of switching to $(\widehat{y}(\cdot), \widehat{\mu}(\cdot))$ is

$$\Delta \equiv \int_a^c \left([y_i - \theta]^2 - [\widehat{y} - \theta]^2 \right) \mathrm{d}F(\theta) + \int_c^1 \left(\left[y_{_{i+1}} - \theta \right]^2 - [\widehat{y} - \theta]^2 \right) \mathrm{d}F(\theta).$$

Noting that $\int_a^c \left([y_i - \theta]^2 - [\widehat{y} - \theta]^2 \right) dF(\theta) = 2(y_i - \widehat{y})[S(a) - S(c) + ((y_{i+1} - \widehat{y})/2)\, \widetilde{F}(c)]$ and $\int_c^1 \left(\left[y_{_{i+1}} - \theta \right]^2 - [\widehat{y} - \theta]^2 \right) dF(\theta) = 2(y_{_{i+1}} - \widehat{y})[S(c) - ((y_i - \widehat{y})/2)\, \widetilde{F}(c)]$ we can express the increment Δ as

$$\Delta \equiv 2(y_i - \widehat{y}) S(a) + 2(y_{i+1} - \widehat{y}) S(c). \tag{A1}$$

From $a < c < \theta^*$ and Lemma A2 (ii.) we have that $S(a) < 0$, which implies that the first term on the RHS of (A1) is positive and the second term is negative. By selecting a project $\widehat{y}$ close enough to y_{i+1} we have that $\Delta > 0$ and $(y(\cdot), \mu(\cdot))$ cannot be optimal. *Q.E.D.*

Lemma A4. Suppose that $G(\theta)$ is strictly increasing in $[0,1]$. Then if $y(\cdot)$ and $\mu(\cdot)$ is an optimal delegation scheme it must be that (i.) if $E[\theta] < b$ then $y(\mu(\theta)) = E[\theta]$ for all $\theta \in [0,1]$ (ii.) if $E[\theta] > b$ then there exists a threshold level $\overline{\theta}$, such that $y(\mu(\theta)) = \theta + b$ if $\theta \in [0, \overline{\theta}]$ and $y(\mu(\theta)) = \overline{\theta} + b$ if $\theta \in [\overline{\theta}, 1]$. Moreover the threshold level $\overline{\theta}$ satisfies $\overline{\theta} + b = E\left[\theta \left| \theta \geq \overline{\theta} \right.\right]$.

Proof. From the previous lemma the optimal delegation scheme is characterized by two threshold levels $\left\{\underline{\theta}, \overline{\theta}\right\}$ with (a.)

$$y(\mu(\theta)) = \begin{cases} \underline{\theta} + b & \text{if} \quad \theta \in [0, \underline{\theta}) \\ \theta + b & \text{if} \quad \theta \in [\underline{\theta}, \overline{\theta}] \\ \overline{\theta} + b & \text{if} \quad \theta \in (\underline{\theta}, 1]. \end{cases}$$

if $0 < \underline{\theta} < \overline{\theta} < 1$, and (b.) $y(\mu(\theta))$ constant over $[0,1]$ if $\underline{\theta} = \overline{\theta}$. The expected utility of the principal is given by

$$-\int_0^{\underline{\theta}} [\underline{\theta} + b - \theta]^2 \, \mathrm{d}F(\theta) - \int_{\underline{\theta}}^{\overline{\theta}} b^2 \mathrm{d}F(\theta) - \int_{\overline{\theta}}^1 \left[\overline{\theta} + b - \theta\right]^2 \mathrm{d}F(\theta).$$

Optimality of $y(\cdot)$ and $\mu(\cdot)$ requires that $\underline{\theta}$ and $\overline{\theta}$ satisfy the first order necessary conditions $2F(\underline{\theta})\left[(\underline{\theta} + b) - \mathrm{E}\left[\theta \left| \theta \leq \underline{\theta} \right.\right]\right] = \lambda - \nu$ and $2\widetilde{F}(\overline{\theta})\left[(\overline{\theta} + b) - \mathrm{E}\left[\theta \left| \theta \geq \overline{\theta} \right.\right]\right] = \nu$, where λ, ν are nonnegative multipliers associated with the constraints $\underline{\theta} \geq 0$, and $\overline{\theta} \geq \underline{\theta}$, respectively. First we establish that that $\underline{\theta} = 0$ is necessary. If at an optimum $\underline{\theta} > 0$ then, since $\underline{\theta} + b > \mathrm{E}\left[\theta \left| \theta \leq \underline{\theta} \right.\right]$, we must have $\lambda - \nu > 0$. This necessarily implies that $\lambda > 0$ and the constraint $\underline{\theta} \geq 0$ binds, reaching a contradiction.

Next suppose that $\mathrm{E}[\theta] < b$. By Lemma A2 (ii.) there is no interior point at which $S(\theta) = \widetilde{F}(\theta)\left[\theta + b - \mathrm{E}\left[s \left| s \geq \theta \right.\right]\right]$ and therefore $\nu > 0$. But then $\overline{\theta} \geq \underline{\theta}$ binds, i.e. $\overline{\theta} = \underline{\theta} = 0$. In this case the principal selects a single project for all states. Since no information from the agent is used in this case it must be that the principal implements $y(\mu(\theta)) = \mathrm{E}[\theta]$ for all $\theta \in [0,1]$.

Now suppose that $\mathrm{E}[\theta] > b$. Then by Lemma A2 (ii.) there is a unique interior point $\overline{\theta} \in (0,1)$ at which $S(\overline{\theta}) = 0$. Then the quadruple $\left\{0, \overline{\theta}, 0, 0\right\}$ satisfies the necessary conditions where $\overline{\theta}$ solves $\overline{\theta} + b = \mathrm{E}\left[\theta \left| \theta \geq \overline{\theta}\right.\right]$. *Q.E.D.*

Proof of Proposition 3. Follows from Lemmas A1-A4. *Q.E.D.*

PROOF OF PROPOSITION 4

The proof of Proposition 4 is carried out through a sequence of lemmas that gradually reduce the class of potential optimal delegation schemes when $G(\theta)$ is strictly decreasing in $[0,1]$.

Lemma A5. Suppose that $G(\theta)$ is strictly decreasing in $[0,1]$. Then if $(y(\cdot),\mu(\cdot))$ is an optimal delegation scheme, there cannot be a non-degenerate interval $[\underline{\theta}, \overline{\theta}] \subset [0,1]$ where $y(\mu(\theta)) = \theta + b$ for $\theta \in [\underline{\theta}, \overline{\theta}]$.

Proof. Suppose on the contrary that $y(\mu(\theta)) = \theta + b$ for $\theta \in [\underline{\theta}, \overline{\theta}]$. Now consider the alternative $(\widehat{y}(\cdot), \widehat{\mu}(\cdot))$ such that

$$\widehat{y}(\widehat{\mu}(\theta)) = \begin{cases} \underline{\theta} + b & \text{if} \qquad \theta \in [\underline{\theta}, \left(\underline{\theta} + \overline{\theta}\right)/2] \\ \overline{\theta} + b & \text{if} \qquad \theta \in (\left(\underline{\theta} + \overline{\theta}\right)/2, \overline{\theta}] \\ y(\mu(\theta)) & \text{otherwise.} \end{cases}$$

It is immediate that $\widehat{y}(\widehat{\mu}(\cdot))$ is incentive compatible if $y(\mu(\cdot))$ is. Furthermore, under $\widehat{y}(\widehat{\mu}(\theta))$ only the extreme projects $\left\{\underline{\theta} + b, \overline{\theta} + b\right\}$ are implemented in $[\underline{\theta}, \overline{\theta}]$. Since $G(\theta)$ is strictly decreasing in $[0,1]$ by Lemma A1 we infer that $\Delta(\left(\underline{\theta} + \overline{\theta}\right)/2, \left(\underline{\theta} - \overline{\theta}\right)/2) < 0$ and therefore the principal strictly prefers $(\widehat{y}(\cdot), \widehat{\mu}(\cdot))$ to $(y(\cdot), \mu(\cdot))$. Hence $y(\cdot)$ and $\mu(\cdot)$ cannot be optimal. *Q.E.D.*

Lemma A6. Suppose that $G(\theta)$ is strictly decreasing in $[0,1]$. Then if $(y(\cdot),\mu(\cdot))$ is an optimal menu delegation scheme then $y(\mu(\cdot))$ induces at most two projects, i.e. $y(\mu(\theta)) \in \{y_1, y_2\}$ for $\theta \in [0,1]$.

Proof. Let $D_A = \{y \in \mathbb{R} : y(\mu(\theta)) = y, \theta \in [0,1]\}$ be the set of projects induced by $y(\cdot)$ and $\mu(\cdot)$ and suppose that D_A contains more than two projects. We consider in turn two cases: (i.)

there are three projects $y_1 < y_2 < y_3$, $\{y_1, y_2, y_3\} \subset D_A$, that are consecutive in the sense that no other project is induced between them (i.e. such that $(y_1, y_2) \cap D_A = (y_2, y_3) \cap D_A = \varnothing$), (ii.) there do not exist three consecutive projects in D_A.

i.) Consider then three consecutive projects $y_1 < y_2 < y_3$. Incentive compatibility implies that (a.) $y(\mu(\theta)) = y_1$ for $\theta \in [y_1 - b, (y_1 + y_2)/2 - b)$, (b.) $y(\mu(\theta)) = y_2$ for $\theta \in ((y_1 + y_2)/2 - b, (y_2 + y_3)/2 - b)$, and (c.) $y(\mu(\theta)) = y_3$ for $\theta \in ((y_2 + y_3)/2 - b, y_3 - b]$. We now propose an alternative delegation scheme in which the project y_2 is not implemented by the principal, i.e. consider $(\widehat{y}(\cdot), \widehat{\mu}(\cdot))$ such that

$$\widehat{y}(\widehat{\mu}(\theta)) = \begin{cases} y_1 & \text{if} \qquad \theta \in [y_1 - b, (y_1 + y_3)/2 - b] \\ y_3 & \text{if} \qquad \theta \in ((y_1 + y_3)/2 - b, y_3 - b] \\ y(\mu(\theta)) & \text{otherwise.} \end{cases}$$

Suppose that $y_2 \leq (y_1 + y_3)/2$ (the analysis if $y_2 \geq (y_1 + y_3)/2$ would follow the same argument). Letting $r = (y_1 + y_2)/2 - b$, $s = (y_2 + y_3)/2 - b$, $t = (y_2 + y_3)/2 - b$, the increment in the principal's expected payoff when switching to $(\widehat{y}(\cdot), \widehat{\mu}(\cdot))$ is

$$\begin{aligned} \Delta U &\equiv \int_r^s \left([y_2 - \theta]^2 - [y_1 - \theta]^2 \right) \mathrm{d}F(\theta) + \int_s^t \left([y_2 - \theta]^2 - [y_3 - \theta]^2 \right) \mathrm{d}F(\theta) \\ &= 2(y_2 - y_1) \int_r^s \left[\frac{y_2 + y_1}{2} - \theta \right] \mathrm{d}F(\theta) + 2(y_2 - y_3) \int_s^t \left[\frac{y_2 + y_3}{2} - \theta \right] \mathrm{d}F(\theta). \end{aligned}$$

Making use of $T(\cdot)$ as defined above, we obtain

$$\Delta U = 2\left[(y_3 - y_1)T(s) - (y_2 - y_1)T(r) - (y_3 - y_2)T(t) \right].$$

If we express $s = (y_2 + y_3)/2 - b$ as a convex combination of r and t, $s = \lambda r + (1 - \lambda)t$, and noting that $y_3 - y_2 = (1 - \lambda)(y_3 - y_1)$ and $y_2 - y_1 = \lambda(y_3 - y_1)$, we can write ΔU in the more transparent form $\Delta U = 2(y_3 - y_1)\left[T(\lambda r + (1 - \lambda)t) - \lambda T(r) - (1 - \lambda)T(t) \right]$.

Since $G(\theta)$ is strictly decreasing, $T(\theta)$ is strictly concave and hence $\Delta U > 0$. This establishes that the original delegation scheme $(y(\cdot), \mu(\cdot))$ where more than two projects are implemented cannot be optimal.

ii.) Suppose that D_A does not contain three consecutive projects. From Proposition 2 $y(\mu(\theta))$ is weakly increasing and therefore continuous except in a countable set of points

$\{\alpha_i\}, i \in \mathbb{N}$. [9] We will now introduce some notation pertinent to this part of the proof. Let $\overline{d} = \max D_A$ and $\underline{d} = \min D_A$ be the maximum and minimum projects induced under $y(\mu(\theta))$ and, for each $i \in \mathbb{N}$, let $y_i^+ = \lim_{\theta \to \alpha_i^+} y(\mu(\theta))$ and $y_i^- = \lim_{\theta \to \alpha_i^-} y(\mu(\theta))$ be the two projects induced to the left and to the right of the point of discontinuity α_i, respectively . Finally let $A_i = \{\theta \in [0,1] : y(\mu(\theta)) \in \{y_i^+, y_i^-\}\}$ be the set of states under which the projects $\{y_i^+, y_i^-\}$ are induced. For convenience we will order the set of discontinuity points $\{\alpha_i\}, i \in \mathbb{N}$ in such a way that the probability of the projects y_i^+ or y_i^- being induced is (weakly) larger than the probability of inducing y_{i+1}^+ or y_{i+1}^-, i.e. such that $\text{Prob}[\theta \in A_i] \geq \text{Prob}[\theta \in A_{i+1}]$. By the assumptions on $y(\mu(\theta))$ we have that $\lim_{i\to\infty} \text{Prob}\left[\theta \in \cup_1^i A_j\right] = 1$.

Consider now a sequence of incentive compatible delegation schemes $(y_i(\cdot),\mu_i(\cdot))$ $i = 0, 1, 2....$to be defined momentarily. Denoting by $D_A^i = \{y \in \mathbb{R} : y_i(\mu_i(\theta)) = y, \theta \in [0,1]\}$ the set of projects induced by $(y_i(\cdot),\mu_i(\cdot))$, we define D_A^i inductively as follows: $D_A^0 = \{\overline{d}, \underline{d}\}$, $D_A^{i+1} = D_A^i \cup \{y_{i+1}^+, y_{i+1}^-\}$. We note that this scheme fully identifies $y_i(\mu_i(\cdot))$ except possibly at its points of discontinuity. For completeness we define $y_i(\cdot), \mu_i(\cdot)$ such that $y_i(\mu_i(\theta))$ is left continuous at its points of discontinuity.

Since $G(\theta)$ is strictly decreasing, the analysis of the case with three consecutive projects establishes that $\text{E}_\theta\left[U_P(y_i(\mu_i(\theta)), \theta)\right] > \text{E}_\theta\left[U_P(y_{i+1}(\mu_{i+1}(\theta)), \theta)\right]$. Next we show that $\text{E}_\theta\left[U_P(y_i(\mu_i(\theta)), \theta)\right]$ converges to $\text{E}_\theta[U_P(y(\mu(\theta)), \theta)]$ as $i \to \infty$.For $\varepsilon > 0$ there exist an i such that $\text{Prob}\left[\theta \in \cup_0^i A_j\right] > 1 - \frac{\varepsilon}{2(\overline{d}-\underline{d})\overline{d}}$. Therefore we have that for $k > i$, with $S_k = \left(\cup_1^k A_j\right)^c$.

$$|\text{E}_\theta\left[U_P(y_k(\mu_k(\theta)), \theta)\right] - \text{E}_\theta[U_P(y(\mu(\theta)), \theta)]| < \int_{S_k} \left|[y(\mu(\theta)) - \theta]^2 - [y_i(\mu_i(\theta)) - \theta]^2\right| \text{d}F(\theta) =$$

$$= \int_{S_k} 2\,|y(\mu(\theta)) - y_i(\mu_i(\theta))| \left|\frac{y(\mu(\theta)) + y_i(\mu_i(\theta)}{2} - \theta\right| \text{d}F(\theta) \leq 2\left(\overline{d} - \underline{d}\right)\overline{d}\,\text{Pr ob}\left[\theta \in S_k\right] < \varepsilon.$$

Thus $\text{E}_\theta\left[U_P(y_i(\mu_i(\theta)), \theta)\right] \longrightarrow \text{E}_\theta[U_P(y(\mu(\theta)), \theta)]$ for $i \to \infty$. Therefore $\text{E}_\theta\left[U_P(y_i(\mu_i(\theta)), \theta)\right] > \text{E}_\theta[U_P(y(\mu(\theta)), \theta)]$ for all i which implies that $y(\cdot)$ and $\mu(\cdot)$ cannot be optimal. *Q.E.D.*

Lemma A7. Suppose that $G(\theta)$ is strictly decreasing in $[0,1]$. Then any two-project delegation scheme is dominated by centralization, i.e. by a decision rule that implements $E[\theta]$ for all messages the agent selects.

[9] We note that since D_A does not contain three consecutive projects and does not contain any nondegenerate interval the set of discontinuity points of $y(\mu(\theta))$ must indeed be infinite.

Proof. Let $y(\mu(\theta)) \in \{y_1, y_2\}$ be an optimal two-project equilibrium with $y_1 < y_2$. Since both projects are selected with positive probability there must exist a state a_1, with $0 < a_1 < 1$, at which the agent is indifferent between y_1 and y_2, i.e. $(y_1 + y_2)/2 = a_1 + b$. Since, for fixed a_1, the projects $\{y_1, y_2\}$ are optimal they must satisfy the first order condition $(y_1 - \mathrm{E}[\theta \,|\theta \le a_1])\, F(a_1) = (y_2 - \mathrm{E}[\theta \,|\theta \ge a_1])\, \widetilde{F}(a_1)$. Equivalently, these can be expressed as

$$S(a_1) - T(a_1) + \frac{y_2 - y_1}{2} = 0. \tag{A2}$$

Now consider the difference in expected utility to the principal between the best centralized decision $\mathrm{E}[\theta]$ and $y(\mu(\theta))$

$$\Delta U = \int_0^1 - (\mathrm{E}[\theta] - \theta)^2 \mathrm{d}F(\theta) + \int_0^{a_1} (y_1 - \theta)^2 \mathrm{d}F(\theta) + \int_{a_1}^1 (y_2 - \theta)^2 \mathrm{d}F(\theta).$$

Using the fact that $S(a_1) + T(a_1) = (y_1 + y_2)/2 - \mathrm{E}[\theta]$ this expression can be rewritten as

$$\Delta U = 2\,(y_2 - \mathrm{E}[\theta])\, S(a_1) + (\mathrm{E}[\theta] - y_1)\left(S(a_1) - T(a_1) + \frac{y_2 - y_1}{2}\right). \tag{A3}$$

Using Lemma A2 (i.) we have that $S(a_1) > 0$ for $0 \le a_1 < 1$, and, in particular, $S(0) = b - \mathrm{E}[\theta] > 0$. Since $y_2 > b$ the first term in (A3) is positive and the second term is zero from the first order condition (A2) implying that $\Delta U > 0$. Thus any two-project optimal delegation scheme is dominated by centralization. *Q.E.D.*

Proof of Proposition 4. Follows from Lemmas A1, A2 and A5-A7. *Q.E.D.*

Proofs of All Remaining Propositions

Proof of Proposition 1. We will follow Theorem 1 in Crawford and Sobel (1982) for the proof of Proposition 1. In particular we will derive a well-defined second order difference equation that the elements of the partition a must satisfy for any equilibrium. This difference equation captures the idea that the agent of type a_i must be indifferent between the decisions selected by the principal when $m \in (a_{i-1}, a_i)$ and $m \in (a_i, a_{i+1})$, and is a necessary and sufficient condition for the communication rule of the agent to be a best response to the principal's decision rule (given in part ii.). Turning our attention to the principal it is immediate that the beliefs in part iii are consistent with the agent's equilibrium communication rule (given by part i.). Also given these beliefs the principal's optimal response would be to "rubberstamp" $\widehat{y}_i$. We need only specify the beliefs of the principal when she observes a recommendation $y \neq \widehat{y}_i,\ i \in \{1, ..., N\}$. As stated in the text we assume that off-the-equilibrium if the principal observes $y \neq \widehat{y}_i$ she updates her beliefs according to part iii. This will lead her to overrule the agent and select $\widehat{y}_i$ instead.

To complete the proof we turn to the indifference condition determining the partition a. In particular given the principal's decision rule (in part i.) an agent of type a_i must be indifferent between recommending decision $\widehat{y}_i$ and $\widehat{y}_{i+1}$, i.e. $U_A(\widehat{y}_i, a_i, b) = U_A(\widehat{y}_{i+1}, a_i, b)$ which translates into

$$a_i + b = (\widehat{y}_i + \widehat{y}_{i+1})/2. \tag{B1}$$

Since $\widehat{y}_i$ and $\widehat{y}_{i+1}$ are functions of a_{i-1} and a_i , and a_i and a_{i+1}, respectively, we have that (B1) defines a (possibly non-linear) second order difference equation with the boundary conditions $a_i = 0$ and $a_i = N$. Also note that since $\widehat{y}_i < a_i + b$ and $U_A(y, \theta, b)$ is strictly concave w.r.t. y we must have $\widehat{y}_{i+1} > a_i + b$. This in turn implies that solutions to (B1) satisfy $a_{i+1} > a_i$. and thus the partition a that satisfies (B1) is well defined. We now argue that given the principal's decision rule (part ii.), which applies both to recommendations on and off the equilibrium path, recommending decision $\widehat{y}_i$ is a best response for the agent whenever $\theta \in (a_{i-1}, a_i)$. First we note that concavity of $U_A(y, \theta, b)$ w.r.t. y implies that $U_A(\widehat{y}_i, a_i, b) = \max_{1 \leq j \leq N} U_A(\widehat{y}_j, a_i, b)$. Next, since $\frac{\partial^2}{\partial y \partial \theta}[U_A(y, \theta, b)] > 0$ we have that for $1 \leq k \leq i \leq l \leq N$

$$\begin{aligned} U_A(\widehat{y}_i, \theta, b) - U_A(\widehat{y}_k, \theta, b) &\geq U_A(\widehat{y}_i, a_{i-1}, b) - U_A(\widehat{y}_k, a_{i-1}, b) \geq 0 \\ U_A(\widehat{y}_l, \theta, b) - U_A(\widehat{y}_i, \theta, b) &\leq U_A(\widehat{y}_l, a_{i+1}, b) - U_A(\widehat{y}_i, a_{i+1}, b) \leq 0 \end{aligned}$$

which establishes that recommending decision $\widehat{y}_i$ is a best response for an agent at state $\theta \in (a_{i-1}, a_i)$. For the claim that there is a finite $N(b)$ such that for every $N \leq N(b)$ there exists an equilibrium with a partition of N intervals we refer to the proof of Theorem 1 in Crawford and Sobel (1982). *Q.E.D.*

Proof of Proposition 2. This proposition follows immediately from Proposition 1 in Melumad and Shibano (1991) where it is shown that an optimal communication rule must induce decisions that are weakly monotonic in the state θ. *Q.E.D.*

Proof of Proposition 5. Follows from the discussion in the text. *Q.E.D.*

Proof of Proposition 6. Follows from Proposition 4. *Q.E.D.*

Proof of Proposition 7. We first show that for given N, the solution to (8) subject to (9) and (10) satisfies $\Delta y_i = q$ for $i = 2, ..., N-1$. To see this, note first that the reneging constraint (10) can be stated as $\Delta y_1 = (3y_1 - y_2 + 2b)/4 \leq q$, $\Delta y_N = (3y_N - y_{N-1} - 2(1-b)) \leq q$ and

$$\frac{1}{4}\left(2y_i - y_{i-1} - y_{i+1} + 4b\right) \leq q \quad \text{for } i = 2, ..., N-1.$$

Thus, an increase in y_j relaxes the reneging constraint for all $i \neq j$. Note second that

$$\frac{\mathrm{dE}_\theta\left[U_P\right]}{\mathrm{d}y_i} = \frac{1}{4}(y_{i+1} - y_{i-1})(y_{i+1} + y_{i-1} - 2y_i) \quad \text{for } i = 2, ..., N-1.$$

If $\Delta y_j < q$ for any $j \in \{2, ..., N-1\}$, then, from the above two equations, $\mathrm{dE}_\theta\left[U_P\right]/\mathrm{d}y_j > 0$. Since an increase in y_j relaxes all the other reneging constraints it follows that $\Delta y_j < q$ cannot be a solution. Thus, the solution must satisfy $\Delta y_i = q$ for $i = 2, ..., N-1$.

We can now prove part (i.) of the proposition. We first prove that if $q \leq b/4$, then $\Delta y_i = q$ for $i = 1, N$. Solving $\Delta y_i = q$ for $i = 2, ..., N-1$ we obtain

$$y_i = \frac{N-i}{N-1}y_1 + \frac{i-1}{N-1}y_N - 2(i-1)(N-i)(b-q) \text{ for } i = 2, ..., N-1.$$

Substituting into (8) and differentiating then gives

$$\frac{\mathrm{dE}_\theta\left[U_P\right]}{\mathrm{d}y_1} = b\left(y_2 - y_1\right) - 2\Delta y_1 a_1 + \sum_{i=2}^{N-1} \frac{\partial \mathrm{E}_\theta\left[U_P\right]}{\partial y_i}\frac{\mathrm{d}y_i}{\mathrm{d}y_1}.$$

Note that the third term on the RHS is positive and that $(y_2 - y_1) > a_1$. Thus, $q \leq b/2$ is a sufficient condition for $\Delta y_1 = q$.

Suppose that $q \leq b/2$ and $\Delta y_1 = q$. Note that $\Delta y_N \leq q$ if and only if

$$y_N \leq (1 + b - (2N-1)(b-q)) \equiv \widetilde{y}_N$$

and that $\widetilde{y}_N < 1$. Next, differentiating $\mathrm{E}_\theta [U_P]$ twice gives

$$\frac{\mathrm{d}^2 \mathrm{E}_\theta [U_P]}{\mathrm{d}y_N^2} = \frac{-2}{(2N-1)N} (1 + 4N(N-1)(1-y_N)).$$

Note that the second derivative is negative for all $y_N \leq \widetilde{y}_N$. Thus, it is optimal to set $y_N = \widetilde{y}_N$, and thus $\Delta y_N = q$, if and only if

$$\left.\frac{\mathrm{d}\mathrm{E}_\theta [U_P]}{\mathrm{d}y_N}\right|_{y_N = \widetilde{y}_N} \geq 0.$$

Differentiating $\mathrm{E}_\theta [U_P]$ once shows that this inequality is satisfied if and only if

$$q \leq \frac{(N-1)(2N(N-2)(b-q)+3)(b-q)}{3(1+2N(N-1)(b-q))} \equiv \widetilde{q}(N).$$

Since $\widetilde{q}(N)$ is increasing in N, $\Delta y_N = q$ for all $N \geq 2$ if and only if $q \leq \widetilde{q}(2) \equiv \overline{q}$. It is straightforward to verify that $\overline{q} \in (0,1)$.

We now know that for given N, the solution satisfies $\Delta y_i = q$ for $i = 1, ..., N$ if $q \leq \overline{q}$. We next argue that the optimal number of intervals is given by the maximum number of intervals $\widetilde{N}$ that can be supported in equilibrium. We do so in two steps. First, we show that $\widetilde{N}$ is increasing in Δy_i. Second, we show that if $\Delta y_i = q$ for $i = 1, ..., N$, then $\mathrm{E}_\theta [U_P]$ is increasing in N.

From (9) it follows that

$$(a_i - a_{i-1}) = a_1 + 4(i-1)b - 2\Delta y_1 - 2\Delta y_i - 4\sum_{j=2}^{i-1} \Delta y_j \text{ for all } i = 2, ..., N. \tag{B2}$$

In any equilibrium the intervals must add up to one, i.e. $a_1 + \sum_{i=2}^{N}(a_i - a_{i-1}) = 1$. Since it must be that $a_1 \geq 0$, $\widetilde{N}$ is given by the largest integer for which $\sum_{i=2}^{N}(a_i - a_{i-1}) \leq 1$. From (B2) it then follows that $\widetilde{N}$ is increasing in Δy_i for $i = 1, ..., N$.

Next, suppose that $\Delta y_i = q$ for $i = 1, ..., N$. Then

$$\mathrm{E}_\theta\left[U_P\right] = -\left(\left(4(b-q)^2 N^2\left(N-1\right)\left(N+1\right)+1\right)+q^2\right)/\left(12N^2\right).$$

The expression on the RHS is increasing in N for all $N \leq \widetilde{N}$. This proves part (i.).

For part (ii.) note that from the above $\Delta y_i = q$ for $i = 2, ..., N-1$ for any $q < b$. *Q.E.D.*

Proof of Proposition 8. The proof will follow in two steps: (i) first we establish that for any optimal relational contract it must be the case that along the equilibrium path the continuation utility of the principal is constant (almost everywhere), (ii.) this allows us to construct a new relational contract that is self-enforcing, stationary, and gives the principal the same expected utility as the previous equilibrium.

Consider an optimal relational contract $(H_t, \mu_t(\cdot), y_t(\cdot))$, $t \in \{0, 1, ...\}$, where $\mu_t(\cdot)$ is simply a best response to $y_t(\cdot)$, and any deviation by the principal along the equilibrium path reverts play to the equilibrium with the highest number of partitions of the static cheap talk benchmark. Let $V_P(h_t) = \sum_{\tau=t}^{\infty} \delta^{\tau-t} \mathrm{E}_{\theta_\tau}\left[U_P(y_\tau(\mu_\tau(\theta_\tau, h_\tau), h_\tau), \theta_\tau)\right]$ be the principal's expected discounted utility at time t after history h_t, with $V_P(h_0) = v$. Now define $\Psi = \{\theta_0 : V_P(\{\theta_0, \mu_0(\theta_0), y_0(\mu_0(\theta_0))\}) < v\}$ to be the set of states in the first period which generate continuation utilities on the equilibrium path less than v. If $\Pr\left[\theta_0 \in \Psi\right] > 0$ we can construct a new relational contract that after the first period history $\{\theta_0, \mu_0(\theta_0), y_0(\mu_0(\theta_0))\}, \theta_0 \in \Psi$ calls for play of the original relational contract. To see that this new contract is subgame perfect note that, since histories are common knowledge and any deviation reverts to the static cheap talk benchmark, the first period choice $y_0(\mu_0(\theta_0))$, remains optimal for the principal if $\theta_0 \notin \Psi$and she obtains a higher continuation utility by playing $y_0(\mu_0(\theta_0))$ whenever $\theta_0 \in \Psi$. Finally, since $V_P(h_0) = \delta \mathrm{E}_{\theta_0}\left[U_P(y_0(\mu_0(\theta_0)), \theta_0)\right] + (1-\delta)\mathrm{E}_{\theta_0}\left[V_P(h_1))\right] = v$ it follows that $\mathrm{E}_{\theta_0}\left[U_P(y_0(\mu_0(\theta_0)), \theta_0)\right] = v$.

Now consider a stationary contract $(H_t, \mu_t'(\cdot), y_t'(\cdot))$, $t \in \{0, 1, ...\}$ where along the equilibrium path $\mu_t'(\cdot) = \mu_0(\cdot), y_t'(\cdot) = y_0(\cdot)$. Note that on-the-equilibrium path the principal obtains the same continuation utility and thus his incentive to renege remains the same. Thus this new contract is subgame perfect, stationary, and $V_P(h_0) = v$. *Q.E.D.*

Proof of Proposition 9. Follows immediately from the discussion in the text. *Q.E.D.*

Proof of Proposition 10. Note that since $f(\theta)$ is continuously differentiable, there exists a $b' > 0$ such that for all $b \leq b'$, $G(\theta)$ is increasing in θ for all $\theta \in \Theta$. Thus, for sufficiently small b threshold delegation is optimal if it does not violate the reneging constraint. To see that for sufficiently small b threshold delegation does not violate the reneging constraint, let δ_{TD} be the δ for which the reneging constraint is binding under threshold delegation, i.e. $b^2 = \delta_{TD}/(1-\delta_{TD})\mathrm{E}_\theta\left[U_P^{TD} - U_P^{CS}\right]$. Similarly, let $\delta_{CD} \equiv -b^2/\mathrm{E}_\theta\left[U_P^{CS}\right]$ be the δ for which the reneging constraint binds under complete delegation. From Proposition 3 in Dessein (2002) it follows that if $f(\theta)$ is twice continuously differentiable, then $\lim_{b\to 0}\delta_{CD} = 0$. Note next that since $\mathrm{E}_\theta\left[U_P^{TD}\right] \geq \mathrm{E}_\theta\left[U_P^{CD}\right]$, $\delta_{CD} \geq \delta_{TD}$. Thus, $\lim_{b\to 0}\delta_{CD} = 0 \geq \lim_{b\to 0}\delta_{TD}$. *Q.E.D.*

Proof of Proposition 11. Follows immediately from the discussion in the text. *Q.E.D.*

Proof of Proposition 12. Follows immediately from the discussion in the text. *Q.E.D.*

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