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Resource Allocation in the Brain *

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Abstract

When an individual performs several tasks simultaneously, processing resources must be allocated to different brain systems to produce energy for neurons to fire. Following the evidence from neuroscience, we model the brain as an organization in which a coordinator allocates limited resources to the brain systems responsible for the different tasks. Systems are privately informed about the amount of resources necessary to perform their task and compete to obtain the resources. The coordinator arbitrates the demands while satisfying the resource constraint. We show that the optimal mechanism is to impose to each system with privately known needs a cap in resources that depends negatively on the amount of resources requested by the other system. This allocation can be implemented using a biologically plausible mechanism. Finally, we provide some implications of our theory: (i) performance is inversely related to the difficulty of the task and can be flawless for sufficiently simple tasks, (ii) the dynamic allocation rule exhibits inertia (current allocations are increasing in past needs), and (iii) different cognitive tasks are performed by different systems only if the tasks are sufficiently important.

Keywords: mechanism design, revelation principle, neuroeconomic theory, resource allocation, multiple brain systems, task inertia, neural Darwinism.

JEL Classification: D71, D82, D87.

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1 Introduction

Our ability to handle multiple tasks simultaneously depends on the coordination of various brain mechanisms. Research in the brain sciences has established that individual decision making requires the allocation of scarce processing resources to the brain systems involved in understanding tasks, planning responses, and implementing actions. The objective of this paper is to study the relationship between the mechanisms for allocating resources in the brain and the quality of the resulting decisions.

To this purpose, we develop a parsimonious theory of constrained optimal behavior based on resource allocation under neurophysiological limitations. This approach affords a new perspective on decision-making which is different from traditional bounded rationality models, as it provides foundations for "mistakes" and "biases" in decision-making that do not rely on the ad-hoc imposition of imperfections. The fundamental features of brain processes that will constitute the building blocks of our theory are briefly introduced here (the supporting evidence is reviewed more thoroughly in section 2.1).[1] First, there is brain specialization. Different brain systems are recruited to perform different tasks and neurons in a given system respond exclusively to features of that particular task. These neurons remain active as long as they receive resources and the task is not completed. The behavior of neurons in a system is therefore consistent with the maximization of task performance.[2] Second, there is "communication" of needs. The consumption of resources in a brain system triggers a signal which results in more resources being allocated to that system. Third, the resource allocation process is centralized. Some areas of the lateral prefrontal cortex (LPFC) play an active role when attention is divided, for instance when two tasks have to be completed at the same time. This points to the existence of what has been called a 'Central Executive System' (CES) whose role is to coordinate the systems involved in the different tasks. Fourth, resources are scarce. The brain has a limited capacity to deal with concurrent tasks and, as a result, it must allocate scarce resources efficiently.

In the paper, we build an agency model based on these four fundamental brain architecture principles. In our model, CES (the 'principal') is responsible for allocating resources to systems with privately known needs (the 'agents') given a resource constraint. More precisely, we consider the case of an individual who must perform three tasks $(0, 1, 2)$ at

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[1] Notice that the paper takes the brain architecture as given. It does not address important questions related to its evolutionary rationale (see Robson (2001) and Robson and Samuelson (2009) for formal models of the biological basis of economic behavior).

[2] Although this is sometimes surprising for economists, there are strong physiological and evolutionary arguments supporting the idea that brain systems compete for resources (see section 7 for a brief review and discussion).

the same time, each carried out by a different system $(0, 1, 2)$. The amount of resources necessary to perform a task is a function of its complexity, and performance decreases with the difference between resources needed and resources obtained. Total resources are available in a fixed amount. CES seeks to maximize the sum of performances in the three tasks. It knows the complexity of task 0 and extracts information from systems 1 and 2 about the complexity of tasks 1 and 2 vía a communication mechanism.

We first conduct a normative analysis where we assume that CES can resort to any communication mechanism. This allows us to restrict attention to direct revelation mechanisms that are incentive compatible in dominant strategies. We characterize the optimal mechanism and show that the allocation is such that each system is guaranteed a minimum level of resources. A system can obtain resources above that minimum if and only if at least one of the other systems chooses not to exhaust its guaranteed level of resources (Proposition 1). We then perform some comparative statics and find that a resource monotonicity principle holds generally: (i) if one system becomes less important from the viewpoint of CES then it receives fewer resources whereas all other systems receive (weakly) more resources, and (ii) if the total amount of resources available increases then all systems (weakly) benefit (Proposition 2).

The normative analysis is important in that it sets an upper bound on the attainable performance. We then show that the optimal mechanism can be implemented using a simple and neurophysiologically plausible process: systems receive resources at different rates, they choose whether to deplete them and, if they do, CES decides whether to provide more resources (Proposition 3). This finding is critical. Indeed, when we observe a simple allocation rule, one is tempted to conclude that it is because individuals are subject to ad-hoc limitations. Instead, our results shows that, for our problem, nothing would be gained by resorting to more complicated mechanisms: the constrained optimal choice can be implemented with a simple 'grab until satiated' procedure.

Next, we derive behavioral implications of our mechanism and confront them with the experimental results obtained in neuroscience studies. Most notably, our theory predicts that performance will be flawless if and only if tasks are sufficiently simple (Corollary 1). It also predicts performance improvements over time and task-inertia: if needs at every date are independently drawn from the same (unknown) distribution, the allocation of resources at a certain date will depend positively on the needs experienced in the past (Proposition 4 and Corollaries 2 and 3). These results match the experimental neuroscience evidence and arise in our framework only when needs are private information. Also, we show that in the biologically plausible implementation mechanism, the time required to complete an easy task is shorter the more difficult that same task was in the past (Corollary 4).

Last, we propose a novel testable implication regarding the architecture of the brain: from an informational viewpoint it is efficient to concentrate cognitive tasks in one system whenever the importance of the tasks is relatively low and to separate them into different systems otherwise (Proposition 5 and Corollary 5).

Finally, we believe our results can explain recent behavioral findings on self-control. According to Vohs et al. (2008) and Pocheptsova et al. (2009), the exercise of self-control impacts the performance in unrelated but effortful deliberative activities and vice versa. These findings are attributed to the depletion of glucose by both tasks, resulting in a shortage of resources to complete both of them efficiently (see e.g. Gaillot et al., 2007; Masicampo and Baumeister, 2008). This echoes our theory and exemplifies the mechanism we outline. Glucose, the critical metabolic resource, is present in the bloodstream in limited quantities. When the individual must both exercise self-control and complete an unrelated deliberative activity, both tasks compete for the scarce resource resulting in decreased performance.

The plan of the paper is as follows. In section 2, we present two literature reviews: the neuroscience evidence supporting our theory and the related research in economics. In section 3, we describe the formal model and solve for the benchmark case with full information. In section 4, we characterize the allocation rule under asymmetric information, perform some comparative statics, and discuss a biologically plausible implementation procedure. In sections 5 and 6, we discuss two behavioral implications of our theory: task inertia in dynamic allocation problems and the incentives for specialization vs. integration of tasks within systems. In section 7, we provide some concluding comments. Proofs of the propositions and corollaries can be found in Appendix A2.

2 Literature in neuroscience and economics

2.1 Evidence from neuroscience

This section reviews in detail the neuroscience evidence underlying our theory.[3] We will refer to it when we introduce the formal elements of our model. We are interested in the brain mechanisms governing decision-making when an individual is presented with two tasks to be performed concurrently.

Tasks and systems. When a decision-maker is facing a task, populations of neurons specialized in different features relevant for that task are recruited. These constitute a

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[3] It can be skipped by readers who either have a background in neuroscience or are not interested in the details of the brain architecture. Readers interested in a yet more detailed introduction to these physiological processes are referred to Brocas (2012).

system. To understand the 'objective' of a system, it is useful to look at its components: the neurons. Neurons fire in response to certain inputs. For instance, the spiking activity of a neuron in the visual system represents a small part of the visual environment, as the neuron is sensitive to the presence of a few specific features only.[4] As such, neurons are only concerned about transmitting information regarding the features they are tuned to detect. Given such a construct, a system only transmits information detected by its components, that is, information relevant for that task. Neurons keep firing as long as they detect relevant information. A system can therefore be represented as an entity that cares exclusively about transmitting information to perform its own task.[5]

Processing resources. A task is performed through a communication process between neurons used to detect features of the environment (in the sensory system), make choices and send orders to act accordingly (in the motor system). Neurons use electrical impulses and chemical signals to transmit information which requires energy delivered by the oxidation of glucose extracted from arterial blood. This energy is used for propagating signals and returning the membrane to its resting potential after firing (Attwell and Laughlin, 2001). Firing therefore relies on metabolic resources (oxygen and glucose) carried by the bloodstream. Enhanced firing in a system indicates the system is active. Given the relationship between firing and metabolic resources, the latter are commonly used as a proxy for neural activity[6] in a series of methods that record differentials in consumption of metabolic resources (Fox et al., 1988; Hyder et al., 2002) or differentials in blood flow.[7] This body of evidence suggests that task performance is related to the consumption of metabolic resources[8] which are, in principle, available in the bloodstream to be grabbed by neurons.[9] However, the availability of metabolic resources is only a necessary condition for

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[4] However, neurons respond not only to the presence or absence of features but also to their values by producing graded responses. They do so by controlling the number of spikes they fire.

[5] A system is related to a task. That is, neurons active in one task are part of the system performing that task, but they can also be active in a different task involving a different system. In other words, two systems do not need to be two physically different areas of the brain.

[6] The literature studying cerebral blood flow has established the existence of a functional coupling between neural activity and brain metabolism. Cerebral activation processes are accompanied by a dynamic adjustment of cerebral blood flow. Blood flow is correlated with oxygen delivery to the brain. The increase in blood flow following the presentation of a task is positively related to the performance in that task (Duschek and Schandry (2004, 2006)).

[7] In particular, PET monitors detect changes in blood flow, glucose usage or oxygen consumption. fMRI signals reflect the degree of blood oxygenation and flow, and measure the blood-oxygen-level dependent (BOLD) response.

[8] The *joint* observation that one system receives more oxygen when a certain task is performed *and* that subjects with a lesion in that system are unable to perform the task provides yet another indirect support for the idea that the system utilizes resources to perform the task.

[9] Some medical conditions are characterized by the inability to regulate the amount of resources in the brain. For instance, too much or too little glucose in insulin-dependent diabetes patients have detrimental

task performance. The metabolic cost of brain activity is high, which may be the limiting factor for both the number of neurons that can be active at any given point in time as well as the maximum frequency of firing of individual neurons (Lennie, 2003; Attwell and Gibb, 2005). This evidence suggests that the metabolic resources that can be used at any given point in time are limited. Recent studies have also shown that enhanced firing is correlated with increased attention to a stimulus, and several processes involving working memory have been found to be fundamental to attention (see Knudsen (2007) for a review). Those mechanisms are believed to modulate the signals sent along communication channels. In particular only some signals gain access to working memory (competitive selection, see Desimone and Duncan (1995)) and the strengths of the competing signals is regulated (top down sensitivity control, see Egeth and Yantis (1997)). This literature indicates that firing rates in a system result from a controlled usage of metabolic resources. Synaptic plasticity is thought to be the mechanism through which such regulation occurs. We will refer to the resources that can be used to transmit information efficiently as *processing resources*, but the reader may keep in mind their relationship to other terminologies such as attentional resources or computational resources. Processing resources are scarce and their allocation is constrained.

Asymmetric information. Typically, neurons in a system detect information contained in a stimulus before neurons in other systems, creating a time lag during which only part of the brain possesses relevant information about the stimulus. This information becomes available to other interested brain areas if and when it is transmitted. Information is not transmitted uniformly along all existing pathways but rather selectively, so that not all systems are aware of the information. Besides, some brain areas are either unconnected or unidirectionally connected to the other areas. This feature of the brain anatomy is the result of evolution, which optimizes the number and location of the highly scarce and energetically demanding neural connections. Delayed transmission and limited neural connectivity immediately implies a restricted flow of information or, in the economics language, asymmetric information.

Centralization of the resource allocation process. A number of fMRI studies have found that certain regions of the LPFC exhibit enhanced activation when two tasks are performed simultaneously. These regions do not exhibit such enhanced activation (i) when only one task is presented to the subject, (ii) when both tasks are presented but the subject is instructed to selectively focus on only one of them, or (iii) when both tasks are presented and performed sequentially (D'Esposito et al., 1995; Herath et al., 2001; Szameitat et al., 2002; Jiang, 2004). The same phenomenon is observed for branching, that is, when

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effects on cognitive functions (Cox et al., 2005).

subjects must keep in mind a main goal while performing concurrent subgoals (Koechlin et al., 1999). In parallel, the literature on task switching has found that several regions of the PFC are activated when a switch occurs (Monsell, 2003). Similar findings have been obtained with other techniques. In particular patients with brain lesions in the left DLPFC have problems switching between the attributes they are instructed to attend to (Rogers et al., 1998; Keele and Rafal, 2000) and subjects in a TMS study whose DLPFC has been disrupted exhibit an impaired ability to divide attention between tasks (Johnson et al., 2007). These results point to the plausible hypothesis of the existence of a *Central Executive System* (CES) whose role is to "coordinate the concurrent processing of the different streams of information" (Szameitat et al. (2002, p. 1184)). The CES is a construct that has long been invoked in theoretical models of human cognition, in particular to represent the allocation of attentional resources within working memory (Baddeley and Hitch, 1974; Norman and Shallice, 1986). Even though the evidence reported before is consistent with this construct, both the neuroanatomy and the specific role of the CES are still under study. Some findings suggest a distributed CES neuroanatomy (including regions of the LPFC) rather than a specific and unique region (Baddeley, 1998; Garavan et al., 2000). Said differently, CES is not a physical organ and LPFC may not be the only region showing differentiated activation in dual-tasks experiments. Moreover, the function of the regions involved specifically in dual tasks experiments cannot be inferred with certainty from activation patterns. However, LPFC is recurrently implicated in top-down control (Miller and Cohen (2001)), working memory (Romo et al., 1999; Romo and Salinas, 2003) and attention processes (Pessoa et al., 2003; Lau et al., 2004). Therefore, its involvement in resource management seems a reasonable hypothesis.

Behavioral interferences and neural activity patterns. Single- vs. dual-task experiments have established some interesting results on neural activation and behavioral patterns. Studies have shown that the volume of activation is smaller in the dual-task condition than in the sum of the two related single-task conditions (Just et al., 2001; Loose et al., 2003; Johnson and Zatorre, 2006; Newman et al., 2007).[10] Sub-additivity suggests the existence of "biological mechanisms that place an upper bound on the amount of cortical tissue that can be activated at any given time" (Just et al., 2001, p. 424)).[11] Some other studies highlight a significant behavioral interference when subjects perform the dual-

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[10] These studies measure activity in the sensory and association areas that are active in one (and only one) of the tasks. They are designed to minimize overlapping areas by choosing tasks that are known to recruit different brain systems (e.g., mental rotation of visually depicted objects and auditory comprehension).

[11] A puzzling result in Just et al. (2001) is that, contrary to the other papers reviewed above, LPFC activation does not change between the single- and dual-task treatments. A possible explanation is that subjects are requested to perform high-level cognitive tasks so that the single-task treatment may already be producing significant activation in the LPFC. This suggests evidence should be interpreted cautiously.

task. In particular, reaction times (Jiang, 2004) and error rates (Szameitat et al., 2002) increase, suggesting that the two tasks compete for attention. Behaviorally, performance in the dual-task is lower than in the sum of the two single tasks (Just et al., 2001), which is consistent with the above mentioned activation patterns, and with the scarcity hypothesis of processing resources.

Combining the evidence just reviewed, we will build a theory in line with the CES hypothesis and endow it with the ability to allocate scarce processing resources. We will then derive some behavioral implications. Yet, our theory is abstract; the reader shall keep in mind that the role of CES could be performed by a different brain system or process. As such, any controversy arising over the specific role of CES should not apply to our theoretical argument.

2.2 Related literature in economics

From a theoretical viewpoint, the problem is related to three strands of the economics literature. First, it is related to the research on mechanism design without transfers (see e.g., Holmström, 1977; Melumad and Shibano, 1991; Alonso and Matouschek, 2007, 2008; Martimort and Semenov, 2008; Carrasco and Fuchs, 2009; Koessler and Martimort, 2012). While the absence of transfers typically requires justification in the literature on organizations, in our setting the neurobiology evidence suggests the lack of other means of 'compensation'. With respect to this literature, our paper combines aspects of multiple agents and multiple actions in a novel setting with capacity constraints. Second, it also bears some commonalities with the axiomatic social choice literature that studies rationing problems (see e.g., Sprumont, 1991; Barbera, Jackson and Neme, 1997; Moulin, 2000). This literature has provided characterizations of rationing mechanisms that satisfy efficiency, strategy-proofness and some additional properties. We depart from it in two respects: we consider (weakly) monotone preferences rather than single-peaked preferences and we focus on mechanisms that maximize the expected performance of systems. As we will show, however, the optimal mechanism does satisfy their main properties. Finally, it is also related to the team theory literature that studies the decomposition and decentralization of resource allocations when systems with a common goal are unable to fully share all available information (as in e.g., Geanakoplos and Milgrom, 1991; see also Garicano and Van Zandt (2013) for a recent survey). Interestingly, the optimal mechanism that we derive also affords a decentralized implementation. In our setup, however, allocative imperfections in the decision rules result from the systems' self-interest rather than from exogenous constraints on communication.

From a conceptual viewpoint, the paper is related to the behavioral economics literature that studies decision-making when individuals have imperfect self-knowledge (see e.g, Carrillo and Mariotti, 2000; Bénabou and Tirole, 2004, 2011; Bodner and Prelec, 2003; Brocas and Carrillo, 2005, 2008; Dal Bo and Terviö, 2007; Ali, 2011). Our model focuses on a novel set of issues, namely performance in a multi-tasking environment. It also departs significantly in the methodological approach: rather than building a model of boundedly rational behavior based on introspection, empirical or experimental data, we take the neuroscience findings about the brain architecture as inputs for modeling the constraints in the optimization problem.[12]

3 The model

3.1 Systems and objectives

Based on the evidence described in section 2.1, we build the following resource allocation model. First, there is a set of systems. Each system is responsible for a task. Systems are composed of neurons, which demand resources. Resource deficits imply a decrease in performance. Second, there is a Central Executive System (CES) which is responsible for the optimal allocation of the scarce resources between systems and whose objective is to maximize an overall performance function.

Formally, we assume there are three tasks, and system l ($\in L = \{0, 1, 2\}$) is responsible for task l. As reviewed in section 2.1, system l can be represented as a selfish entity focused exclusively on the performance in its own task. Let $\Theta_l = [0, \overline{\theta}_l]$ be the set of possible resources that task l may require. If $\theta_l \in \Theta_l$ is the actual amount of resources necessary to carry out task l flawlessly and x_l are the resources allocated to system l, the system seeks $x_l = \theta_l$. Without loss of generality, a system can be endowed with a performance function $\Pi_l(x_l; \theta_l)$ that is maximized at $x_l = \theta_l$. There is a loss whenever $x_l < \theta_l$. The effect of too many resources is less clear. Indeed, the system may in some cases be able to costlessly discard resources above θ_l, which formally means that $\Pi_l(x_l; \theta_l) = \Pi_l(\theta_l; \theta_l)$ for all $x_l \geq \theta_l$. In some other cases, too many resources can have counter effects on performance, which formally means that $\Pi_l(x_l; \theta_l) < \Pi_l(x'_l; \theta_l)$ for all $x_l > x'_l \geq \theta_l$. For example, excessive attention may be counterproductive. Either way, the performance function is increasing in x_l up to θ_l and non-increasing above it. For the rest of the paper, we assume the following functional form:

$$\Pi_l(x_l; \theta_l) = \begin{cases} \alpha_l \, u_l(x_l - \theta_l) & \text{if} \quad x_l \leq \theta_l \\ 0 & \text{if} \quad x_l > \theta_l \end{cases} \tag{1}$$

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[12] In that respect, the paper is closer to Brocas and Carrillo (2008) which studies the dynamic choices of an individual when brain systems have different mental representations of current vs. distant prospects.

with $u_l(0)=0$, $u_l'(0)=0$, $u_l'(z)>0$ and $u_l''(z)\leq 0$ for all $z<0$. Under this formalization, a more complex task (higher θ_l) requires more resources.[13] As the difference between needs and resources granted $(\theta_l - x_l)$ increases, performance deteriorates. Resources above needs do not hinder performance because they can be discarded by the systems at no cost.[14]

3.2 Optimization under full information

The optimization problem of CES consists in distributing a fixed amount of resources k among the three systems so as to maximize an overall performance function. We formally represent it as:

$$\begin{aligned} \max_{\{x_0,x_1,x_2\}} \quad & \Pi_0(x_0;\theta_0)+\Pi_1(x_1;\theta_1)+\Pi_2(x_2;\theta_2) & \\ \text{s.t.} \quad & x_0+x_1+x_2\leq k & \text{(R)} \\ & x_0\geq 0,\ x_1\geq 0,\ x_2\geq 0 & \text{(F)} \end{aligned}$$

The objective function is the sum of the systems' performances, where the parameter α_l in (1) can capture either the weight of task l on the objective function of CES or a reward system imposed externally to complete each particular task. This objective function is, admittedly, restrictive (for example, in some settings one could expect complementarities or substitutabilities among tasks). It seems, however, a reasonable first approximation especially when the tasks are imposed externally. The *resource constraint* (R) reflects the maximum resources k available to perform the three tasks. The *feasibility constraint* (F) captures the minimum resources that can be allocated to each system. The analysis can be trivially extended to a positive minimum amount of resources necessary for a system to operate. The problem also presupposes that CES does not necessitate resources to coordinate the needs of systems. This goes largely against the evidence presented in section 2.1 but it is imposed only for simplicity.[15]

The problem is trivial when $\theta_0+\theta_1+\theta_2\leq k$: each system receives the resources it needs, (i.e., $x_l=\theta_l$), and the excess resources are discarded. In this case, each system performs flawlessly and performance is then maximized at zero. The problem becomes interesting when $\theta_0+\theta_1+\theta_2>k$, which immediately implies that the resource constraint is binding. Let x_l^F be the solution to the problem under full information in this case.

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[13] Note that the comparison over levels of complexity is defined within tasks not between tasks. For example, spelling an 8-letter word is more complex than spelling a 3-letter word.

[14] In a previous version (Alonso, Brocas and Carrillo 2011) we showed that similar results are obtained with a single-peaked quadratic performance function: $\Pi_l(x_l-\theta_l)=-\alpha_l\,(x_l-\theta_l)^2$ for all x_l.

[15] Indeed, one could trivially extend the model and assume that CES requires $\tilde{k}$ resources for coordinating activities and that only $k-\tilde{k}$ resources are available for the systems.

Given $u'(0) = 0$, the individual underperforms in *all* tasks ($x_l^F(\theta_0, \theta_1, \theta_2) < \theta_l$ for all l) except in the trivial limit case where no resources are needed ($\theta_l = 0$).

4 Incomplete knowledge of needs

The more realistic and interesting situation arises when CES does not know how many resources are required by some of the systems. As motivated in section 2.1, information asymmetry matches the physiological evidence on brain connectivity. It introduces an *endogenous cost* of resource allocation and information processing.

In the rest of the paper, we will consider two classes of systems. System 0 is responsible for a *basic motor skill* task 0 which corresponds, for example, to lifting an object or looking in a certain direction. The needs to perform this task, θ_0, are known. Systems 1 and 2 are responsible for *higher order cognitive* tasks. These include vision, hearing, abstract projection and language, among others. We use subscripts i and j for systems 1 and 2 with $i \neq j$. The needs of system i, θ_i, are unknown to CES, and depend crucially on the type and difficulty of the cognitive task to be performed (face identification, auditory comprehension, mental representation of shapes, word recognition, etc.). CES only knows that θ_1 and θ_2 are independently drawn from continuous distributions with c.d.f. $F^1(\theta_1)$ and $F^2(\theta_2)$ and densities $f^1(\theta_1)$ and $f^2(\theta_2)$.[16] Let $h_i(\theta_i) = \frac{f^i(\theta_i)}{1-F^i(\theta_i)}$ be the hazard rate of θ_i. We assume that the distribution of needs of system i has an increasing hazard rate (IHR) which, as is well known, rules out thick tails in the distribution. This condition is imposed to ensure certain regularity properties of the solution.

Assumption 1 (IHR) *$h_i(\theta_i') \geq h_i(\theta_i)$ for all $\theta_i' \geq \theta_i$.*

Our objective is to determine the resource allocation mechanism which is optimal from the viewpoint of CES given its imperfect knowledge of needs. As in the previous section, we maintain the assumption that the resource constraint is always binding which, given that θ_1 and θ_2 are unknown, can now be stated as follows.

Assumption 2 (shortage) *$\theta_0 \geq k$.*

The first step of our analysis consists in adopting a *normative approach* and determining the optimal allocation when CES can use any conceivable communication mechanism: each system sends a message requesting resources and CES responds with an allocation as a function of the messages received. Applying the revelation principle, we can without

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[16] The results can be trivially extended to more than one system (hence, more than one task) with known needs. By contrast, extensions to three or more systems with unknown needs would be more involved.

loss of generality restrict attention to direct revelation mechanisms where each system i 'announces' its needs $\tilde{\theta}_i \in \Theta_i$. Based on the announcements, CES 'commits' to a resource allocation rule D:

$$D(\tilde{\theta}_1, \tilde{\theta}_2) = \Big(x_0(\tilde{\theta}_1, \tilde{\theta}_2), x_1(\tilde{\theta}_1, \tilde{\theta}_2), x_2(\tilde{\theta}_1, \tilde{\theta}_2)\Big) \text{ for } (\tilde{\theta}_1, \tilde{\theta}_2) \in \Theta_1 \times \Theta_2.$$

We restrict attention to mechanisms that can be implemented in dominant strategies. From a neuroeconomic viewpoint, implementation in dominant strategies seems most natural as it ensures that a system does not have to 'form beliefs' about the objectives, needs, demands or even the 'existence' of other systems.[17] We also assume that when a system is indifferent between several allocations, it chooses the one that is optimal for CES. This rules out uninteresting equilibria such as, for example, one where system i always reports the highest needs and therefore, in equilibrium, the allocation is insensitive to system i's true needs.

The allocation rule is constructed in a way that, for system i, announcing $\tilde{\theta}_i = \theta_i$ is incentive compatible in dominant strategies. Formally:

$$\Pi_i(x_i(\theta_i, \theta_j); \theta_i) \geq \Pi_i(x_i(\tilde{\theta}_i, \theta_j); \theta_i) \qquad \forall\ i,\ \theta_i,\ \tilde{\theta}_i,\ \theta_j. \tag{DSIC}$$

Notice that the assumptions imposed on the behavior of systems are minimal. Their sole concern is to obtain the resources necessary to complete their tasks. Each system realizes that resources are scarce (simply by noticing that needs are not always fulfilled) and that their availability may depend on external factors. However, awareness of the needs or even the existence of other systems and other tasks is not required.

The normative analysis immediately raises a question: Is it realistic to think in these terms? The answer is yes and no. On the one hand, our entire research rests on the fact that the brain has some well-documented physiological limitations in the availability, transmission and processing of information. Putting no restrictions on the type of communication allowed contradicts that view. On the other hand, we show in Appendix A1 that a reasonable two-stage mechanism where CES allocates some initial resources, systems choose whether to consume them and, as a function of their choice, CES decides whether to grant more resources is formally equivalent to a static incentive compatible mechanism where systems (truthfully) report their needs. In any case, a crucial advantage of the normative analysis is that it provides an upper bound on the attainable performance of CES. A main contribution of the paper will rest on the subsequent *positive approach*, where

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[17] From a theory viewpoint it would be interesting to determine the optimal mechanism in Bayesian strategies, as Carrasco and Fuchs (2009) do in a somewhat related theoretical setting. However, we would have a hard time interpreting this type of mechanisms in our context.

we investigate if the optimal allocation described in the normative analysis can indeed be implemented using a simple and physiologically plausible mechanism.

4.1 The optimization problem

Given the imperfect knowledge of needs, we will assume that CES maximizes the expected performance of the tasks. Among all the possible direct mechanisms, let $\mathcal{M}$ be the class that admits the following representation: a mechanism $D \in \mathcal{M}$ if and only if there exist functions $\bar{x}_1(\theta_2)$ and $\bar{x}_2(\theta_1)$ such that

$$D(\theta_1, \theta_2) = \begin{cases} x_1(\theta_1, \theta_2) &= \min\{\theta_1, \bar{x}_1(\theta_2)\}, \\ x_2(\theta_1, \theta_2) &= \min\{\theta_2, \bar{x}_2(\theta_1)\}, \\ x_0(\theta_1, \theta_2) &= k - x_1(\theta_1, \theta_2) - x_2(\theta_1, \theta_2). \end{cases} \tag{2}$$

In other words, $\mathcal{M}$ is the class of direct mechanisms that simply impose an upper bound on the resources granted to each system, where this upper bound depends on the reports of other systems. Importantly, any $D \in \mathcal{M}$ is also dominant strategy incentive compatible. The following lemma allows us to narrow down the class of direct mechanisms that we must consider when studying the problem faced by CES.

Lemma 1 *Any feasible and dominant strategy incentive compatible mechanism that maximizes the expected performance of tasks must belong to $\mathcal{M}$, that is, takes the form (2).*

With this lemma, the problem under asymmetric information reduces to:

$$\begin{aligned} \max_{D\in\mathcal{M}} \iint & \Big[\Pi_0(x_0(\theta_1,\theta_2);\theta_0) + \Pi_1(x_1(\theta_1,\theta_2);\theta_1) + \Pi_2(x_2(\theta_1,\theta_2);\theta_2)\Big] dF^1(\theta_1)\, dF^2(\theta_2) \\ \text{s.t.} \quad & x_0(\theta_1,\theta_2) + x_1(\theta_1,\theta_2) + x_2(\theta_1,\theta_2) \le k \qquad \forall\, \theta_1,\, \theta_2 \quad \text{(R)} \\ & x_0(\theta_1,\theta_2) \ge 0,\ x_1(\theta_1,\theta_2) \ge 0,\ x_2(\theta_1,\theta_2) \ge 0 \qquad \forall\, \theta_1,\, \theta_2 \quad \text{(F)} \end{aligned}$$

where the *dominant strategy incentive compatibility constraint* (DSIC) is automatically satisfied by the mechanism D and therefore ignored, and (R) and (F) are the resource and feasibility constraints introduced previously. Given Assumption 2, (R) always binds at the optimum, that is, resources are always exhausted. Using (R) to express x_0 as a function of x_1 and x_2, inserting this expression in $\Pi_0(\cdot)$ and using (1) and (2), we can rewrite the problem as:

$$\begin{aligned} \mathcal{P}: \quad \max_{D\in\mathcal{M}} \iint & \Big[\alpha_1\, u_1\Big(x_1(\theta_1,\theta_2) - \theta_1\Big) + \alpha_2\, u_2\Big(x_2(\theta_1,\theta_2) - \theta_2\Big) \\ & + \alpha_0\, u_0\Big(k - x_1(\theta_1,\theta_2) - x_2(\theta_1,\theta_2) - \theta_0\Big)\Big]\, dF^1(\theta_1) dF^2(\theta_2) \\ \text{s.t.} \quad & x_1(\theta_1,\theta_2) \ge 0,\ x_2(\theta_1,\theta_2) \ge 0,\ x_1(\theta_1,\theta_2) + x_2(\theta_1,\theta_2) \le k \quad \forall\, \theta_1,\, \theta_2 \qquad \text{(F)} \end{aligned}$$

In the next section, we determine the optimal caps $\bar{x}_1(\theta_2)$ and $\bar{x}_2(\theta_1)$.

4.2 Optimal resource allocation with unknown needs

Consider an allocation rule where system i receives an allocation equal to its needs θ_i. Denote by $y_j(\theta_i)$ the optimal cap on the resources allocated to system j in this case. The cap $y_j(\theta_i)$ is continuous, non-increasing in θ_i and for $0 < y_j(\theta_i) < k - \theta_i$ solves the following equation (see Lemma 3 in Appendix A2):

$$\alpha_j \, E\Big[u_j'\Big(y_j(\theta_i) - \theta_j\Big) \;\Big|\; \theta_j \geq y_j(\theta_i)\Big] = \alpha_0 \, u_0'\Big((k - \theta_i - y_j(\theta_i)) - \theta_0\Big). \tag{3}$$

This equation has an intuitive interpretation. Suppose that system i receives all the resources it needs (θ_i). We simply have to determine how to optimally distribute the remaining $k - \theta_i$ resources between systems 0 and j. The left hand side of (3) represents the *expected* marginal benefit of assigning resources to system j conditional on those resources being desirable, that is, on system j having needs exceeding the cap $y_j(\theta_i)$. The right hand side of (3) represents the marginal benefit of assigning them to system 0. Optimal distribution of $k - \theta_i$ equates both marginal benefits. Moreover, it can be shown that $y_j'(\theta_i) \in (-1, 0)$: a one-unit increase in the needs of (and therefore in the resources allocated to) system i reduces the cap on system j by less than one unit, thereby reducing also the resources allocated to system 0.

Overall, (3) describes the cap on system j when system i receives its needs. Suppose now that both systems are constrained and denote by k_1 and k_2 the optimal amount of resources allocated to systems 1 and 2 in that case, with $k_0 = k - k_1 - k_2$. Whenever they are all positive, the values (k_0, k_1, k_2) solve the following system of equations

$$\alpha_1 E\Big[u_1'\Big(k_1 - \theta_1\Big) \;\Big|\; \theta_1 \geq k_1\Big] = \alpha_2 E\Big[u_2'\Big(k_2 - \theta_2\Big) \;\Big|\; \theta_2 \geq k_2\Big] = \alpha_0 u_0'\Big(k_0 - \theta_0\Big), \tag{4}$$

which has the same interpretation as before: the marginal benefit of allocating resources to system 0 equals the expected marginal benefit of allocating resources to either of the constrained systems 1 and 2. Note from (3) and (4) that $y_1(k_2) = k_1$ and $y_2(k_1) = k_2$.

With these premises in mind, we are in a position to characterize $\mathbf{M}$, the mechanism that solves problem $\mathcal{P}$.

Proposition 1 (Characterization) *The optimal mechanism* $\mathbf{M}$ *is characterized by the following caps* $\bar{x}_1^*(\theta_2)$ *and* $\bar{x}_2^*(\theta_1)$*:*

$$\bar{x}_1^*(\theta_2) = \begin{cases} y_1(\theta_2) & \textit{if} \quad \theta_2 < k_2 \\ k_1 & \textit{if} \quad \theta_2 \geq k_2 \end{cases} \quad \textit{and} \quad \bar{x}_2^*(\theta_1) = \begin{cases} y_2(\theta_1) & \textit{if} \quad \theta_1 < k_1 \\ k_2 & \textit{if} \quad \theta_1 \geq k_1 \end{cases}$$

Let us first understand what mechanism $\mathbf{M}$ implies for the *equilibrium allocations* to systems 1 and 2 which, from now on, are denoted by $x_1^*(\theta_1, \theta_2)$ and $x_2^*(\theta_1, \theta_2)$. According to Proposition 1, system j has a guaranteed minimum level of resources k_j. This means that, if its needs are $\theta_j \leq k_j$, they are fully satisfied and the resulting performance is flawless. If, on the other hand, its needs are $\theta_j > k_j$, then system j receives extra resources only if system i is satiated. The amount by which the resources to j are increased depends on the marginal benefit of allocating them to 0 vs. j, as expressed in (3). Notice that this allocation rule implies that when the needs of both 1 and 2 are above k_1 and k_2, none of them receives resources above those levels. Finally, the resources that are not allocated to systems 1 and 2 (if any) go to system 0.

Figure 1 provides a graphical representation of the resources $(x_1^*(\theta_1, \theta_2), x_2^*(\theta_1, \theta_2))$ allocated to systems 1 and 2 under mechanism $\mathbf{M}$ for every pair of needs (θ_1, θ_2). The dotted and bold lines represent the optimal caps on systems 1 and 2 as a function of the needs of systems 2 and 1 respectively. The resulting final allocations are such that, in the lower left quadrant (systems 1 and 2 have low needs), both systems receive all the resources they need. In the upper right quadrant (systems 1 and 2 have high needs), both systems receive fixed amounts. In the remaining two quadrants, the system with low needs is unconstrained and the system with high needs is constrained by an amount that depends negatively on the needs of the other system. The remaining resources $x_0^*(\theta_1, \theta_2) = k - x_1^*(\theta_1, \theta_2) - x_2^*(\theta_1, \theta_2)$ are then allocated to system 0.

The intuition for the optimality of mechanism $\mathbf{M}$ is as follows. First, CES has for each system i only one instrument at its disposal– the allocation x_i– which dramatically limits its scope for intervention.[18] As established in Lemma 1, the best CES can do is to set a cap $\bar{x}_i$ on each system i. Because system i's performance is (weakly) increasing in the resources obtained, imposing a cap that depends non-trivially on i's own report cannot be incentive compatible. By contrast, as the needs of system j increase, so does the opportunity cost of granting resources to system i. Therefore, the cap on system i must be non-increasing in the report made by system j about its own needs. After a certain threshold, however, system j is also capped and higher reports do not translate into higher resources. At that point, the opportunity cost of allocating resources to system i becomes constant and so does the cap on i. Combining these properties naturally leads to the mechanism described in Proposition 1.

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[18] Trading-off two instruments as in the traditional mechanism design literature is not possible in our setting due to the absence of monetary transfers.

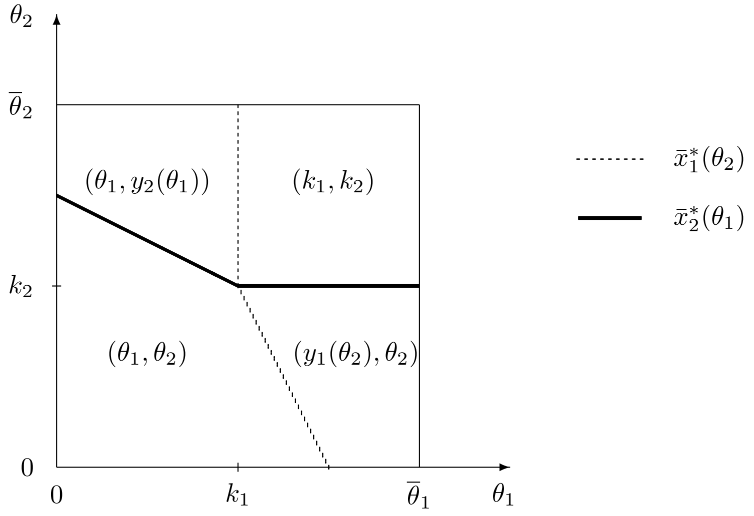

Figure 1. Allocation (x_1^*, x_2^*) as a function of needs (θ_1, θ_2) in the optimal mechanism **M**.

From a technical viewpoint, we can establish a connection between our mechanism and a prominent class of mechanisms studied in the social choice literature in a different but related context (with single peaked rather than weakly-increasing preferences). Moulin (2000) introduced the notion of "priority mechanism", a rule that consists first in ranking agents lexicographically and then sequentially allocating resources according to their needs and the pre-specified priority order. The author characterizes some axiomatic properties of this simple allocation rule. Interestingly, Proposition 1 shows that the mechanism which *maximizes* social welfare (the sum of expected utilities of all agents) has a priority-type format: if $\theta_i < k_i$ then give priority to system i and divide the remaining resources optimally between systems j and 0, and if both $\theta_1 > k_1$ and $\theta_2 > k_2$ then give a fixed allocation k_l to each system l.

The optimal mechanism **M** has an interesting property that we describe below.

Corollary 1 *Under full information, the individual will always under-perform in the cognitive tasks. Under incomplete information, the individual will perform flawlessly in simple cognitive tasks and severely under-perform in difficult cognitive tasks.*

Recall that under complete information, a system with positive needs always receives fewer resources than desired. This implies that the individual always under-performs in tasks 1 and 2. Under incomplete information, on the contrary, the individual performs cognitive tasks flawlessly as long as they are simple enough. The result is illustrated in

Figure 2. It represents the final allocation of resources to system i under complete information (dotted line) and incomplete information (full line) as a function of its needs θ_i and for a given announcement θ_j by system j. As the needs of system i increase, the allocation obtained by that system under **M** is fully responsive up to a level $(dx_i^*/d\theta_i = 1)$ and non-responsive afterwards $(dx_i^*/d\theta_i = 0)$. This is to be contrasted with the full information case in which the allocation is always below optimal and strictly increases with the needs. The result has two immediate implications. First, only under incomplete information a multi-tasking individual may perform both cognitive tasks flawlessly. Second, performance in a cognitive task under complete information exceeds performance under incomplete information if and only if needs are above a certain threshold, that is, the cognitive task is sufficiently difficult.

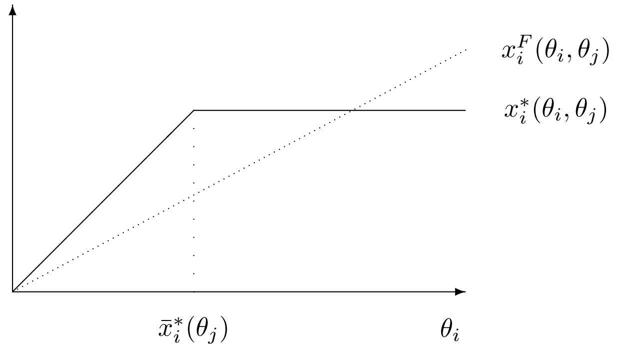

Figure 2. Allocations x_i^F and x_i^* as a function of the needs θ_i.

4.3 Comparative statics

We now study how the optimal allocation rule is affected by changes in the resources available and the relative importance of the performance of systems. Consider the optimal mechanism **M** given the parameters $(\alpha_0, \alpha_1, \alpha_2, \theta_0, k)$. We have the following result.

Proposition 2 (Comparative statics) *The resources $x_l^*(\theta_1, \theta_2)$ allocated to system l (weakly) increase if α_l or k increase or if α_{-l} decreases. Also, $x_i^*(\theta_1, \theta_2)$ decreases and $x_0^*(\theta_1, \theta_2)$ increases if θ_0 increases.*

The comparative statics follow a general resource monotonicity principle which can be summarized as *"abundance is shared and relative importance is compensated."* If a system becomes more valuable for CES (due, for example, to an increase in the marginal cost of under-performance), it receives more resources at the expense of *both* the other systems.

Conversely, if new resources become available (k increases), then *all* systems benefit from this surplus.[19]

Changes in α_0 are interesting. If shortages in the amount granted to the motor skill task become less and less costly (α_0 decreases), systems 1 and 2 receive more resources ($y_i(\theta_j)$ increases for all θ_j). They also become less sensitive to each other's demands ($|y_i'(\theta_j)|$ decreases for all θ_j), because higher needs of system j come more at the expense of system 0 and less at the expense of system i. Eventually, x_0^* hits the non-negativity constraint. Once this occurs, system 0 receives no resources and the problem reduces to allocating a fixed amount k between systems 1 and 2.

The optimal mechanism $\mathbf{M}$ and the comparative statics can be illustrated with the following stylized analytical example.[20]

Example 1 (Uniform-Quadratic) Suppose that performance is quadratic for all $x_l \leq \theta_l$ and needs in the cognitive tasks are uniformly distributed: $u_l(a) = -a^2$ and $\theta_i \sim \mathrm{U}[0, \overline{\theta}_i]$. To reduce the number of parameters, let $\theta_0 = k$. Using (3)-(4), we get:

$$y_i(\theta_j) = \frac{\alpha_i \overline{\theta}_i}{\alpha_i + 2\alpha_0} - \frac{2\alpha_0}{\alpha_i + 2\alpha_0}\theta_j \quad \text{and} \quad k_i = \frac{\alpha_i \alpha_j \overline{\theta}_i + 2\alpha_0 \alpha_i \overline{\theta}_i - 2\alpha_0 \alpha_j \overline{\theta}_j}{\alpha_i \alpha_j + 2\alpha_0 \alpha_i + 2\alpha_0 \alpha_j}$$

where the slope of the cap function is constant: $y_i'(\theta_j) = -\frac{2\alpha_0}{\alpha_i + 2\alpha_0} \in (-1, 0)$.

One may want to give a quantitative assessment of the efficiency of our optimal second-best mechanism $\mathbf{M}$ relative to some alternatives. To this end, we compare the performance of $\mathbf{M}$ in Example 1 to the first-best full information mechanism and to two third-best simpler mechanisms: one where system i receives priority and the remaining resources are optimally distributed between 0 and j, and another where system 0 receives priority (given Assumption 2, in this last case no resources are left to systems 1 or 2).

Example 2 (Performance Comparison) Suppose that $\alpha_l = 1$ for all l, $u_l(a) = -a^2$, $\theta_i \sim \mathrm{U}[0, 1]$, and $\theta_0 - k \equiv r \in [0, 1/2]$. Figure 3 represents the expected utility of CES as a function of r under four different mechanisms: (i) First-best (Full Information), (ii) Second-best (Mechanism $\mathbf{M}$), (iii) Priority to i, and (iv) Priority to 0.[21]

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[19] This comparative statics is consistent with experiments in which subjects have to exercise self control and make effortful choices after drinking lemonade containing either glucose or a substitute (Masicampo and Baumeister (2008)). Performance was significantly higher for subjects who drank glucose suggesting the positive effect of extra resources in the bloodstream.

[20] The algebraic details in Examples 1 and 2 are omitted for brevity but are available from the authors.

[21] We restrict attention to $r \leq 1/2$ so that system 0 is not overwhelmingly important (for instance it is trivial that when $r = 1$ Full Information, Mechanism $\mathbf{M}$ and Priority to 0 perform identically simply because all three mechanisms allocate all the resources to system 0, the system with greatest needs).

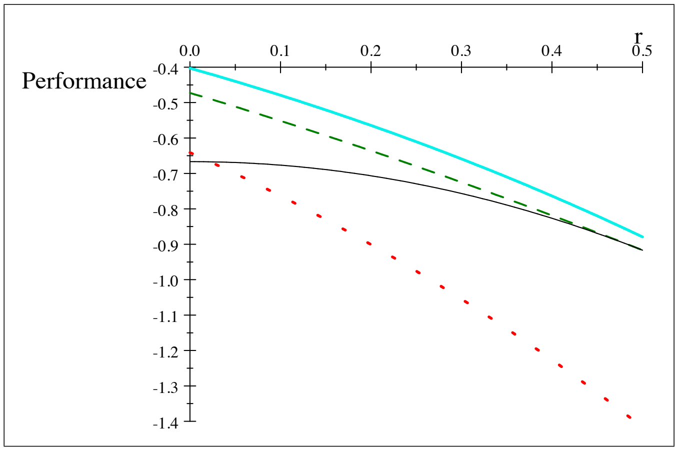

Figure 3. CES utility under full information (light thick line), mechanism **M** (dashed line), Priority to i (dotted line) and Priority to 0 (dark thin line).

When $r = 0$, **M** performs substantially better than other simple mechanisms: utility under **M** is about 20% lower than under first-best whereas utility under priority to i or priority to 0 is about 60% lower than under first-best. As r increases, the inefficiency of Priority to i increases whereas the inefficiency of Priority to 0 decreases relative to **M** simply because, other things being equal, higher needs of system 0 (i.e., higher r) implies that more resources should be granted to that system.

4.4 Implementation

The solution described in Proposition 1 represents a normative upper bound on the efficiency of the resource allocation problem. A direct revelation mechanism where each system 'communicates' its needs truthfully given the 'commitment' by CES to split resources following a pre-determined rule is nothing but an abstract formalization of the problem. Indeed, although systems may not be able to literally send messages to CES, they can signal their needs through the usage of the processing resources made available to them. As discussed in Appendix A1, the direct revelation and signal-through-consumption mechanisms can be formally equivalent. However, that approach is still fairly abstract. The purpose of this section is to determine whether the efficient allocation rule can be reached using a simple and biologically plausible process. Assume that the tasks must be completed between time 0 and time k, and only one unit of processing resources is delivered at each instant. We may think of processing resources as metabolic resources that can be used to efficiently transmit a signal.

Proposition 3 (Implementation) $\mathbf{M}$ *can be implemented with the following mechanism* $\mathbf{M}'$*:*

(i) CES sends resources to system l *at a constant rate* $r_l = k_l/k$.

(ii) As long as i *and* j *consume resources, the flow rates* (r_0, r_1, r_2) *are maintained. If* i *stops consuming at time* $\tilde{t}$*, then resources are redirected to* j *and 0 at revised rates* ν_j *and* $1 - \nu_j$*, such that* $r_j\tilde{t} + \nu_j(k - \tilde{t}) = y_j(r_i\tilde{t})$.

(iii) If both i *and* j *stop consuming, all the remaining resources are redirected to 0.*

Mechanism $\mathbf{M}'$ follows the biological principles highlighted in the introduction. CES sends resources simultaneously to the systems in charge of performing tasks. The systems deplete the resources, and depletion is (correctly) interpreted by CES as a signal that more resources are needed. The process is dynamic but extremely fast. If one system stops consuming, no further resources are sent to it. Mechanism $\mathbf{M}'$ is extraordinarily simple for systems: it just requires them to grab any incoming resources until they are satiated. It means in particular that, for the optimal mechanism to work, each system's knowledge about the existence and needs of other systems is virtually nil. In fact, systems do not even need to know their own needs at any point in time, only whether an extra unit of resources is valuable or not. On the other hand, the mechanism requires a certain degree of sophistication by CES, which must be able to select different flow rates for different systems and be ready to redirect resources when some needs are satiated. We conjecture that the activity measured in the LPFC in the dual-task experiments reviewed in section 2.1 (D'Esposito et al., 1995; Szameitat et al., 2002, and others) captures this extra top-down involvement of CES in the coordination and allocation of "attentional resources." However, this is somewhat speculative since the precise implications of increased levels of blood oxygenation in LPFC are not known with certainty.

5 Task inertia and performance improvements

In this section we study the sequential allocation of resources. To this purpose consider the following extension of the basic model. Suppose that CES has imperfect knowledge of the distribution $F^i(\cdot)$ from which the needs of system $i \in \{1, 2\}$ are drawn. More precisely, there is an underlying state $s_i \in S_i = [\underline{s}_i, \overline{s}_i]$ that determines the distribution of needs for system i. For example, suppose the individual performs an auditory comprehension task which is often (though not always) complex. Then, the auditory system will often (though, again, not always) require substantial resources. This is formally captured by an underlying state s_i that places high probability on auditory needs being large.

We order the states from highest likelihood of small needs to highest likelihood of large needs, and assume that a (strict) Monotone Likelihood Ratio Property (MLRP) holds.[22]

Assumption 3 (MLRP) $\dfrac{d}{d\theta_i}\left(\dfrac{f^i_{s_i}(\theta_i|s_i)}{f^i(\theta_i|s_i)}\right) > 0 \quad \forall\ i,\ \theta_i,\ s_i.$

According to this assumption, needs increase (in a stochastic sense) as we move towards a higher state. Stated differently, the state s_i is a parameter that captures how complex the task is likely to be, and therefore how important the needs are likely to be. When the individual performs tasks only once, the problem is identical to the one studied previously, as CES is not interested in states *per se* but only as a way to identify more accurately the needs of systems. To see this, suppose the state s_i is drawn from a known distribution $P^i(s_i)$ with density $p^i(s_i)$, and that states are independent across systems ($p^i(s_i|s_j) = p^i(s_i)$ for all s_j).[23] The probability that system i has needs θ_i is:

$$g^i(\theta_i) = \int_{\underline{s}_i}^{\overline{s}_i} f^i(\theta_i|s_i)p^i(s_i)ds_i \tag{5}$$

We can then perform the very same analysis as before where $f^i(\cdot)$ is replaced by $g^i(\cdot)$.

The problem becomes more interesting when the individual performs the same set of tasks in consecutive periods. We assume that s_i remains constant over time. At each date t and conditional on the state s_i, the needs of system i are drawn independently from $F^i(\theta_i|s_i)$. The past realization of needs then conveys information about the state, which itself informs about the distribution of present needs. Formally and applying Bayes rule, the probability that the needs of system i at date t are θ_i^t given that its needs at date $t-1$ were θ_i^{t-1} is:

$$g^i(\theta_i^t|\theta_i^{t-1}) = \int_{\underline{s}_i}^{\overline{s}_i} f^i(\theta_i^t|s_i)p^i(s_i|\theta_i^{t-1})ds_i = \frac{\displaystyle\int_{\underline{s}_i}^{\overline{s}_i} f^i(\theta_i^t|s_i)f^i(\theta_i^{t-1}|s_i)p^i(s_i)ds_i}{\displaystyle\int_{\underline{s}_i}^{\overline{s}_i} f^i(\theta_i^{t-1}|s_i)p^i(s_i)ds_i} \tag{6}$$

The following lemma is a key step for our subsequent analysis.

Lemma 2 *Under Assumption 3,* $\dfrac{d}{d\theta_i^t}\left(\dfrac{g^i_{\theta_i^{t-1}}(\theta_i^t \mid \theta_i^{t-1})}{g^i(\theta_i^t|\theta_i^{t-1})}\right) > 0 \quad \forall\ i,\ \theta_i^t,\ \theta_i^{t-1}.$

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[22] Subscripts in c.d.f. or density functions denote partial derivatives with respect to that argument.

[23] If states are not independent, then θ_i and θ_j are correlated. The optimal mechanism must then exploit this correlation, as it is well known in the mechanism design literature.

According to Lemma 2, *MLRP begets MLRP*: if the individual experiences high needs at some date, it means that the state is likely to be high (in an MLRP sense), and therefore that needs are likely to be high also in the future (again, in an MLRP sense).

The dynamic allocation of needs in the framework developed above has some new features. Suppose that, at the end of each date t, CES learns what the needs of each system were at that date. This occurs for example if the performance $\Pi_i(\cdot)$ of system i is observed after system i has performed the task: the individual receives feedback about its performance and this is interpreted by CES. Then, the needs reported by systems at some date affect current allocations but not future allocations. Hence, independently of whether system i is myopic (most likely) or forward-looking (least likely), it will 'communicate' its needs in order to optimize exclusively its present allocation. The mechanism $\mathbf{M}$ developed in Proposition 1 as well as the implementation procedure $\mathbf{M}'$ described in Proposition 3 remain optimal at each date t, where $f^i(\theta_i)$ is simply replaced by $g^i(\theta_i^t \,|\, \theta_i^{t-1})$ updated using the posterior $p^i(s_i \,|\, \theta_i^{t-1})$. This mechanism, however, has new interesting properties.

Proposition 4 (Task inertia) *The resources $x_{it}^*(\theta_1^t, \theta_2^t)$ allocated to system i at date t (weakly) increase if θ_i^{t-1} increases or if θ_j^{t-1} decreases.*

The idea is simple. If CES realizes that the needs of system i in the previous period were high, it concludes that state s_i is likely to be high which, other things being equal, shifts the updated distribution of system i's future needs towards high values. As a result, it becomes optimal to grant more resources to system i in the current period, that is, to set a higher cap. Given our resource monotonicity property, a more generous allocation to system i comes necessarily at the expense of both systems 0 and j.

Using Proposition 1, we can then compare two models. In the first one, CES knows at the beginning of each date t the needs (θ_1^t, θ_2^t) of systems 1 and 2 (the 'full information' case, as in section 3). In the second one, CES does not know at the beginning of date t the needs (θ_1^t, θ_2^t) of systems 1 and 2 (the 'incomplete information' case, as in section 4). In both models, however, needs are revealed to CES at the end of date t. Also, since the underlying state is unknown, there is learning over time about s_i in both models, and therefore about the distribution $f^i(\theta_i|s_i)$. These two models yield two different implications for observed behavior that are summarized below.

Corollary 2 *In a dynamic multi-task setting, the expected future performance at any time $t' > t$ evaluated at time t is constant under full information while it improves over time under incomplete information.*

With full information, the resource allocation rule of CES does not depend on his assessment of the state. Still, higher states lead to (stochastically) higher needs and therefore lower expected performance whereas lower states lead to lower needs and therefore higher expected performance. From the perspective of date t, however, learning about the state may go in either direction, hence the constant expected performance under full information. With incomplete information, a new effect appears. Over time, CES learns about s_i through the realization of θ_i. This reduces the information asymmetry between CES and system i, which results in an improved expected performance. The conclusion is in line with experimental evidence. Subjects tend to adapt their behavior and obtain better outcomes in the presence of feedback about performance even when there is nothing to 'learn' about the characteristics of the task. In a sense, the result rationalizes performance improvements purely through practice or task repetition.

Proposition 4 has also an immediate but important implication regarding the existence or non-existence of a link between the past needs of a system and its current allocation.

Corollary 3 *Under full information, the allocation rule at each date t depends exclusively on the present needs. Under asymmetric information it depends on the present needs and also on the history of needs.*

With full information, present needs are a sufficient statistic to determine the optimal allocation. It then follows that learning about s_i allows CES to better predict future performance but does not vary the way it distributes resources at future dates. In other words, the allocation rule at any time is, conditional on the current needs, independent of the history of needs. With incomplete information, however, the optimal allocation mechanism depends on the distribution from which needs are drawn. Learning about s_i thus leads to a history-dependent allocation rule: higher past needs of system i reflects a higher likelihood of present needs inducing a more favorable treatment by CES through a higher consumption cap. This more favorable treatment translates into a higher performance of system i at the expense of systems j and 0.

Task inertia and the resulting history-dependent allocation and performance is a particularly interesting result in the light of the recent neuroscience research. Indeed, suppose that for the first few periods the task performed by system i is more complex than the task performed by system j. Not surprisingly, resources are primarily directed to system i. Suppose now that, at some point, there is a reversion in complexity. There is substantial fMRI evidence of residual activity right after the change in the previously crucial but now unimportant system i. Conversely too few resources are allocated to the previously unimportant but now crucial system j following the reversion in complexity. This misallocation vanishes after a few periods. Behaviorally, it translates into a short-term lowered performance (slower response and more mistakes) in the task for which system j is responsible (Wylie and Allport (2000), Monsell (2003), Yeung et al. (2006)). Neuroscientists argue that this phenomenon is due to what they call a "task inertia" or a "task switching cost." However, the evidence on the existence of such switching cost is not accompanied by an understanding of where it comes from and why it vanishes rapidly. In order to generate this effect in a model with full information, we would need to impose some ad-hoc cost of adaptation. Perhaps more satisfactorily, our model shows that inertia arises naturally under incomplete information. The model thus proposes an explanation for why adaptation to a changing environment may take a few iterations, and therefore offers a foundation for theories that take this result as an assumption.

Last, the biologically plausible mechanism $\mathbf{M}'$ discussed in Proposition 3 has a natural implication in the multi-period framework.

Corollary 4 *In a dynamic multi-task setting, mechanism* $\mathbf{M}'$ *predicts that the initial rate of resources sent to system* i *at date* t *increases when* θ_i^{t-1} *increases. If, in equilibrium, the needs of system* i *are satiated* ($\theta_i^t < \bar{x}_{it}^*(\theta_j)$)*, the individual completes that task faster the higher the past needs.*

It suffices to apply Proposition 3 to each period. Interestingly, the rates r_i and ν_i will change over time as a function of past needs. It comes immediately from Proposition 4 that the initial rates allocated to task i at date t increase if θ_i^{t-1} increases and if θ_j^{t-1} decreases: more resources are sent if the task is expected to be more difficult given the feedback obtained. This also implies that the task should be completed *faster* at date t as processing resources are available more rapidly, providing another testable implication of the theory.

6 The architecture of brain systems: integration vs. specialization

So far, we have assumed that each system performs exactly one task. In reality, systems are responsible for multiple tasks and tasks require the coordination of multiple systems. There are numerous reasons for such an organization of the brain. In this section, we focus on one specific aspect that builds on the core premise of our theory: restricted channels of communication. More precisely, we study from a purely informational viewpoint the trade-off between *integrating* the two cognitive tasks into one system vs. *specializing* systems into performing one cognitive task each.

Specialization corresponds to the case already analyzed in section 4, where each cognitive task is performed by a different system with private knowledge of needs. Integration is modeled as follows. There is one system, denoted by I, which encompasses systems 1 and 2: it knows the needs θ_1 and θ_2, undertakes tasks 1 and 2, and cares about the sum of performances in those tasks. Formally:

$$\Pi_I(x_1, x_2; \theta_1, \theta_2) \equiv \Pi_1(x_1; \theta_1) + \Pi_2(x_2; \theta_2)$$

We impose the following assumptions. First, under integration, CES can only choose which resources are allocated to system I and which are allocated to system 0. System I, who knows the relative needs in tasks 1 and 2, then decides how to split its resources between tasks 1 and 2. Second, for analytic tractability we restrict attention to quadratic performance functions:

$$u_0(x_0 - \theta_0) = -(1-\gamma)(x_0 - \theta_0)^2 \quad \text{and} \quad u_i(x_i - \theta_i) = -\gamma\,(x_i - \theta_i)^2$$

where $\gamma \in [0,1]$ captures the importance of the cognitive tasks relative to the motor skill task in the overall performance function. As we will develop below, the main objective of this section is to determine which brain architecture is more efficient as a function of this parameter. Finally, we also focus on the case where $\overline{\theta}_1 + \overline{\theta}_2 = k$. We impose this assumption to be in the interesting situation where integration and specialization yield identical performance if either $\gamma = 0$ or $\gamma = 1$. In the former case, only the motor skill task matters and optimality requires $x_0 = k$. In the latter case, only the cognitive tasks matter and first best can be achieved by setting $x_1 = \theta_1$ and $x_2 = \theta_2$.[24] Proposition 5 discusses which brain architecture dominates when $\gamma \in (0,1)$.

Proposition 5 (Brain architecture) *Assume* $\overline{\theta}_1 + \overline{\theta}_2 = k$*,* $f_1(\overline{\theta}_1) f_2(\overline{\theta}_2) > 0$ *and quadratic performance functions. There exist* $\underline{\gamma}$ *and* $\overline{\gamma}$ *with* $0 < \underline{\gamma} \leq \overline{\gamma} < 1$ *such that, from the viewpoint of CES, integration dominates specialization for all* $\gamma \in (0, \underline{\gamma})$ *and specialization dominates integration for all* $\gamma \in (\overline{\gamma}, 1)$*.*

From a purely informational perspective, integration has both benefits and costs. On the one hand, system I knows θ_1 and θ_2, so it can compute the relative value of allocating resources to task 1 vs. task 2. Moreover, there is congruence with the interests of CES on this relative value. Therefore, for a given amount of resources allocated to the cognitive

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[24] If we assume $\overline{\theta}_1 + \overline{\theta}_2 > k$, then integration dominates specialization when $\gamma = 1$ (and therefore also when $\gamma \to 1$) but for ad-hoc reasons. Indeed, when $\gamma = 1$, *all* the resources are directed to tasks 1 and 2. Since resources are sometimes scarce ($\theta_1 + \theta_2 > k$ for some (θ_1, θ_2)), it is efficient to have system I (which by assumption knows the relative needs in those tasks) deciding how to split k between the two.

tasks, the integrated system I performs a split between the two which is optimal from the viewpoint of CES. On the other hand, system I does not care about the performance in task 0, therefore all the resources granted to I are allocated to task 1 or task 2 independently of the opportunity cost of allocating them to system 0. This contrasts with the separation case where the marginal benefits of granting resources to each system are jointly taken into consideration when determining the optimal caps of systems 1 and 2.

According to Proposition 5, specialization is preferred when the importance of the cognitive tasks is high relative to the motor skill task ($\gamma > \overline{\gamma}$) and integration is preferred when the importance of the cognitive tasks is low relative to the motor skill task ($\gamma < \underline{\gamma}$).[25] Intuitively, when the cognitive tasks are important, the majority of resources are allocated to tasks 1 and 2. It then becomes relatively more valuable to get extra information about the needs in each of these tasks in order to determine how much to grant to task 0. This is obtained through specialization. Conversely, when the motor skill task is very valuable, most of the resources are allocated to system 0 anyway. At the margin, it is then important to optimize the (few) resources granted to the cognitive tasks, and this is achieved through integration.

Figure 4 depicts the equilibrium allocation as a function of needs under integration (bold line) assuming $\hat{k}$ resources are optimally allocated to system I. The allocation under specialization is the same as mechanism **M** in Figure 1, and it is superimposed in the graph (dotted lines). Two differences between the integration and specialization mechanisms deserve emphasis. First, under integration the amount of resources consumed in task i are θ_i if $\theta_1 + \theta_2 \leq \hat{k}$ and $\hat{x}_i(\theta_1, \theta_2) < \theta_i$ if $\theta_1 + \theta_2 > \hat{k}$. It means that, in equilibrium, the needs in either none or both cognitive tasks are constrained. This contrasts with specialization which has four regions so that, for some parameters, the needs of one and only one task are constrained. Second and by construction, under integration the cap is set on total resources for the cognitive tasks, so lower needs in task i do not result in spillovers for task 0 unless system j is satiated. Again, this contrasts with specialization, where lower needs by system i always result in more resources for both system j and system 0.

An immediate implication of Proposition 5 with intuitive appeal is summarized below.

Corollary 5 *Different cognitive tasks should be performed by different systems if cognitive tasks are crucial for CES and by the same system if they are not.*

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[25] We show for a parametric example ($\theta_i \sim U[0,1]$ and $\alpha_1 = \alpha_2$) that $\underline{\gamma} = \overline{\gamma}$. Unfortunately, the uniqueness of the cutoff does not extend to other cases so, in general, we cannot determine what happens when $\gamma \in [\underline{\gamma}, \overline{\gamma}]$.

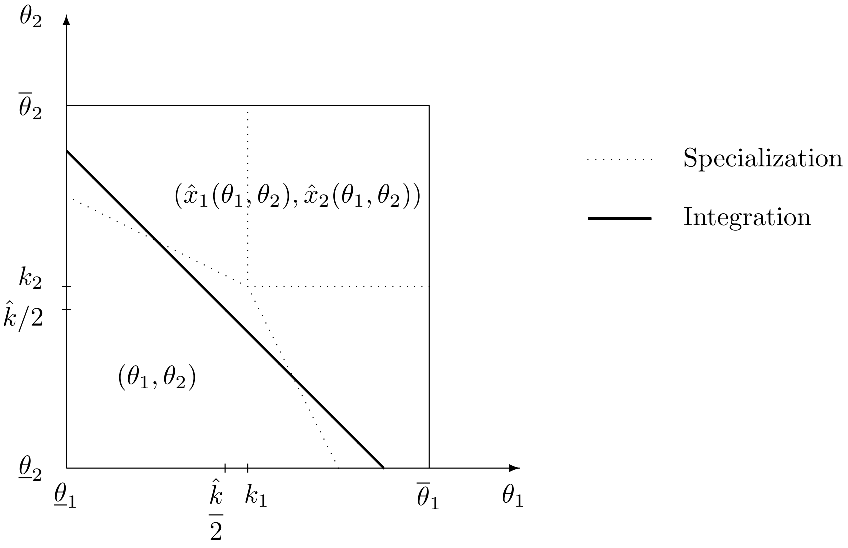

Figure 4. Optimal allocation under integration (**I**) and specialization (**M**).

7 Discussion and conclusions

Incorporating choice imperfections in the decisions of agents has become central to behavioral economics. Observed behaviors or "outputs" such as empirical evidence, experimental data or sometimes mere introspection have been the main source of inspiration for models of bounded rationality. The premise of the present research is that "inputs" such as physiological constraints in our ability to perceive events, process information, and select between options should also be used as building blocks for new theories of decision making. This paper follows this alternative route. It determines the constrained optimal allocation of resources to brain systems when multiple tasks are performed simultaneously. It shows that the optimal mechanism takes a resource cap structure and that it can be implemented using a simple and biologically plausible procedure. Some implications of the theory are discussed, most notably the inverse relation between task difficulty and performance, the endogenous emergence of task inertia and the conditions for the optimality of task integration. A natural next step would be to test this theory in a controlled laboratory setting. Given the existing research by Baumeister and co-authors on self-control and multi-tasking, this area seems a good candidate for such test. For example, our theory predicts that subjects who find it more difficult to exert self-control should perform worse in unrelated tasks.

Although our theory is motivated by the neurobiology of the brain, the model can also be applied to more traditional areas of economics. For example, it can be straightforwardly reinterpreted as a manager in a firm whose objective is to allocate scarce funds between self-interested units (research, production, marketing, etc.) given private information of needs.[26] It can also capture the decision problem of colluding firms who decide how to split the market without using side transfers that would provide compromising evidence of their illegal activities. Other natural applications include provision of private goods to a group of individuals and lobbying activities in political contexts.

As a final point, economists often express reservations about the idea that brain systems may have competing goals. In particular, wouldn't it be more efficient if every subpart pursued the common good? There are evolutionary, physiological and empirical arguments against a common interest approach. First, the well-known neural Darwinism (Edelman, 1987) and neuronal selectionism (Changeux, 1985) theories provide models where neuronal groups within the brain compete with each other for stimulus and reward resources.[27] Second, and paradoxically, a cooperative approach would require a greater degree of brain connectivity and sophistication. Indeed, each system would have to be able either to 'communicate' its needs back to the central decision-maker or to perform a non-trivial marginal analysis and give up worthy resources whenever these are more valuable to other systems. Instead, the physiological evidence reviewed in section 2.1 points towards a lack of information flowing from systems to CES (possibly due to the scarcity of the energetically costly neural connections) and a simplistic 'deplete-until-satiation' behavior of neurons in the decision systems. These two features are consistent with our mechanism $\mathbf{M}'$. Third, some of the empirical regularities discussed in the paper (for example, the possibility of flawless behavior and the prevalence of task inertia) arise naturally in our non-cooperative model with private information but would not be present in a model with common objectives. In any case, we believe that system competition provides at the very least a plausible alternative to the cooperative 'team theoretic' approach.

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[26] As mentioned in section 2.2, the literature on organizations has studied related questions. However, to our knowledge a problem with two actions and two agents with private information and where the organization cannot price resources has not been addressed before. The implications for inertia in organizations and the trade-off integration vs. specialization of units within an organization are also potentially important for the theory of the internal organization of the firm.

[27] Under this approach, biological evolution encourages fitness of the neuronal system, rather than fitness at a higher level (the individual) or a lower level (the gene). See Tooby and Cosmides (1992) for an evolutionary theory of internal conflicts in changing environments. See also Livnat and Pippenger (2006) and Bisin and Iantchev (2010) for models highlighting the evolutionary advantages of having modules with non-congruent objectives.

One could also build evolutionary theories where Nature shapes the performance functions of systems (as in Robson (2001) or Rayo and Robson (2013) for example) to internalize the opportunity cost of resources. Whether such approach would result in selfish, fully cooperative or partially cooperative systems is an open question of considerable interest.

Appendix

A1. Interpreting the direct revelation mechanism

Suppose there are two stages. In stage $\tau \in \{1,2\}$, CES allocates a local budget to each system (y_1^τ, y_2^τ). A budget is a function of past "messages", which in our case corresponds to past "consumptions". We denote by m_i^τ the consumption of system i in stage τ.

In stage 2, the resources needed by system i are r_i^2 and its allocation is $y_i^2(m_1^1, m_2^1)$. It is optimal to consume exactly what is needed or, if this is not possible, to deplete the local budget:

$$m_i^2\left(y_i^2(m_1^1, m_2^1), r_i^2\right) = \min\left\{y_i^2(m_1^1, m_2^1), r_i^2\right\}$$

In stage 1, the resources needed by system i are θ_i and its allocation is y_i^1. Consumption cannot exceed the allocation ($m_i^1 \leqslant y_i^1$). If $\theta_i < y_i^1$, then it is (weakly) optimal to consume $m_i^1 = \theta_i$. If $\theta_i > y_i^1$, system i chooses m_i^1 and the ex-post utility is:

$$\alpha_i\, u_i\left(m_i^1 + \min\left\{y_i^2(m_1^1, m_1^2), \theta_i - m_i^1\right\} - \theta_i\right)$$

where stage 2 needs are replaced by total needs minus stage 1 consumption. We look for a solution in dominant strategies, that is:

$$\alpha_i\, u_i\left(m_i^1 + \min\left\{y_i^2(m_i^1, m_j^1), \theta_i - m_i^1\right\} - \theta_i\right) \geqslant \alpha_i\, u_i\left(\tilde{m}_i^1 + \min\left\{y_i^2(\tilde{m}_i^1, m_j^1), \theta_i - \tilde{m}_i^1\right\} - \theta_i\right)$$

for all $m_i^1 \leqslant y_i^1$ and $\tilde{m}_i^1 \leqslant y_i^1$, yielding a solution $m_i^{1*}(\theta_i, y_i^1)$. At equilibrium, system i consumes $m_i^{1*}(\theta_i, y_i^1)$ in stage 1. He receives $y_i^2(m_i^{1*}(\theta_i, y_i^1), m_j^{1*}(\theta_j, y_j^1))$ in stage 2 and consumes $m_i^{2*}(\theta_i, \theta_j, y_i^1, y_j^1) = \min\left\{y_i^2(m_i^{1*}(\theta_i, y_i^1), m_j^{1*}(\theta_j, y_j^1)), \theta_i - y_i^1\right\}$. Total consumption is $m_i^{1*}(\theta_i, y_i^1) + m_i^{2*}(\theta_i, \theta_j, y_i^1, y_j^1)$ which, by construction, is less than $y_i^1 + y_i^2(m_i^{1*}(\theta_i, y_i^1), m_j^{1*}(\theta_j, y_j^1))$.

Let $x_i(\theta_i, \theta_j, y_i^1, y_j^1) = m_i^{1*}(\theta_i, y_i^1) + m_i^{2*}(\theta_i, \theta_j, y_i^1, y_j^1)$. Again by construction, for all θ_j and for all y_i^1, y_j^1, we have:

$$\alpha_i\, u_i\left(x_i(\theta_i, \theta_j, y_i^1, y_j^1) - \theta_i\right) \geqslant \alpha_i\, u_i\left(x_i(\theta_i', \theta_j, y_i^1, y_j^1) - \theta_i\right)$$

which means that the two-stage mechanism where stage 2 budget depends on stage 1 consumption is formally equivalent to a direct mechanism where, for any initial local budgets (y_1^1, y_2^1), each system i is asked to report its total needs θ_i and receives a final allocation x_i that is divided among the two stages. The mechanism is direct and incentive compatible in dominant strategies. Moreover, for any such mechanisms with initial budgets (y_1^1, y_2^1), there exists an equivalent mechanism with no budget in stage 1. That mechanism is itself equivalent to a static mechanism in which all resources are allocated in stage 2. We can thus restrict to such mechanisms.

A2. Proof of Lemmas and Propositions

A. Proof of Lemma 1

We proceed in two steps. We first show that any mechanism that solves CES problem must be ex-post efficient. We then show that, given Assumption 2, the class of strategy-proof, ex-post efficient mechanisms is contained in $\mathcal{M}$.

Consider an arbitrary mechanism D,

$$D(\theta_1, \theta_2) = (x_0(\theta_1, \theta_2), x_1(\theta_1, \theta_2), x_2(\theta_1, \theta_2)),$$

which is feasible, strategy-proof but not ex-post efficient. We can then construct a feasible, strategy-proof mechanism D' that increases the CES performance, implying that D does not solve the problem of CES. It follows from Assumption 2 that a feasible mechanism is not ex-post efficient if and only if for some $i \in \{1, 2\}$ there exists a profile (θ_1, θ_2) such that $x_i(\theta_1, \theta_2) > \theta_i$, that is system i obtains excess resources. In particular, feasibility and Assumption 2 imply that system 0 can never obtain excess resources. Denote by Ψ_i the set of states in which system $i \in \{1, 2\}$ obtains resources above needs, and define the mechanism D' that follows D except for $(\theta_1, \theta_2) \in \Psi_i$, $i \in \{1, 2\}$ in which case we have:

$$D'(\theta_1, \theta_2) = \begin{cases} x'_0(\theta_1, \theta_2) = x_0(\theta_1, \theta_2) + (x_i(\theta_1, \theta_2) - \theta_i), \\ x'_i(\theta_1, \theta_2) = \theta_i, & \text{if } (\theta_1, \theta_2) \in \Psi_i. \\ x'_j(\theta_1, \theta_2) = x_j(\theta_1, \theta_2), \end{cases}$$

That is, D' assigns to system 0 the excess resources of other systems. It is clear that the expected utility of CES increases if the equilibrium allocation rule follows D' instead of D. It remains to show that D' is strategy-proof. First, for $(\theta_1, \theta_2) \in \Psi_i$, system i achieves the same (maximum) performance with the new mechanism D' as with the previous mechanism D. Second, the allocation to system i does not become more desirable to any other type θ'_i. Third, system j's allocation does not change in Ψ_i. These three observations imply that if D is strategy-proof, so is D'.

We next show that the set of allocations induced by system i when system j reports θ_j, $\mathcal{X}_i(\theta_j) = \{x_i(\theta_i, \theta_j) : \theta_i \in \Theta_i\}$ for any strategy-proof, ex-post efficient mechanism must be an interval. Strategy-proofness then implies a "cap structure" as in (2). Suppose, by way of contradiction, that for some θ_j, $\mathcal{X}_i(\theta_j)$ is not an interval. This implies that there exist two elements $x_i, x'_i \in \mathcal{X}_i(\theta_j)$, $x_i < x'_i$, with $(x_i, x'_i) \cap \mathcal{X}_i(\theta_j) = \varnothing$. Let $\underline{\theta}_i = \inf\{\theta_i \in \Theta_i : x_i(\theta_i, \theta_j) = x'_i\}$ be the minimum type that induces allocation x'_i. First, ex-post efficiency implies that $\underline{\theta}_i \geq x'_i$. Second, letting $\theta''_i \in (x_i, x'_i)$, incentive compatibility requires that $x_i(\theta''_i, \theta_j) = x'_i$, implying that $\underline{\theta}_i \leq \theta''_i$ thus violating $\underline{\theta}_i \geq x'_i$. Having reached a contradiction it thus follows that $\mathcal{X}_i(\theta_j)$ is an interval. □

B. Proof of Proposition 1

The proof of Proposition 1 will be based on Lemmas 3, 4 and 5.

Lemma 3 *Define a* $\succeq_i$ *−priority mechanism to be a mechanism where CES assigns resources* θ_i *to system* i*. The optimal strategy-proof* $\succeq_i$ *−priority mechanism* P_i *allocates resources according to:*

$$\begin{aligned} x_i(\theta_1,\theta_2) &= \min\{\theta_i,k\}, \\ x_j(\theta_1,\theta_2) &= \min\{\theta_j,y_j(\theta_i)\}, \\ x_0(\theta_1,\theta_2) &= k - x_i(\theta_1,\theta_2) - x_j(\theta_1,\theta_2), \end{aligned}$$

where $y_j(\theta_i)$ *is continuous, non increasing and at each point of differentiability we have* $y_j'(\theta_i) \in [-1,0]$*. If* $0 < y_j(\theta_i) < k - \theta_i$ *then* $y_j(\theta_i)$ *is the unique solution to:*

$$\alpha_j E\left[u_j'\left(y_j(\theta_i)-\theta_j\right) \mid \theta_j \geq y_j(\theta_i)\right] = \alpha_0 u_0'\left(k-\theta_i-y_j(\theta_i)-\theta_0\right),$$

and $y_j'(\theta_i) \in (-1,0)$*.*

Proof. Under an $\succeq_i$ −priority mechanism CES assigns resources θ_i to system i as long as it does not violate the resource constraint $\theta_i \leq k$. Therefore $x_i(\theta_1,\theta_2) = \min\{\theta_i,k\}$. We first restrict attention to $\succeq_i$ −priority mechanisms that set a cap on the resources allocated to system j, i.e. $x_j(\theta_1,\theta_2) = \min\{\theta_j, y_j(\theta_i)\}$, and we characterize the mechanism $\mathbf{P}_i$ with an optimal cap $y_j(\theta_i)$. We then show that $\mathbf{P}_i$ is optimal in the general class of strategy-proof $\succeq_i$ −priority mechanisms.

(a) **Optimal cap** $y_j(\theta_i)$.
The expected performance when system i announces θ_i and resources $k' = k - \theta_i$ are distributed between systems 0 and j by imposing a resource cap $y_j(\theta_i)$ on system j is (omitting the dependence of $y_j(\theta_i)$ on θ_i to avoid clutter)

$$J_j(y_j) = \int_{y_j}^{\bar{\theta}_j} \alpha_j u_j\left(y_j-\theta_j\right) dF^j(\theta_j) + \int_{\Theta_j} \alpha_0 u_0\left(x_0\left(\theta_j\right)-\theta_0\right) dF^j(\theta_j),$$

where $x_0(\theta_j) = k' - \theta_j$ if $\theta_j \leq y_j$ and $x_0(\theta_j) = k' - y_j$ if $\theta_j > y_j$. The optimal cap y_j solves

$$\max J_j(y_j) \quad \text{s.t.} \quad 0 \leq y_j \leq k'. \tag{7}$$

We first show that J_j is quasiconcave in $[0,k']$. Define the function H_j as

$$H_j(y_j) = E\left[u_j'\left(y_j-\theta_j\right) \mid \theta_j \geq y_j\right]. \tag{8}$$

By differentiating J_j we have

$$\begin{aligned} J_j'(y_j) &= \left(1-F^j\left(y_j\right)\right)\left(\alpha_j E\left[u_j'\left(y_j-\theta_j\right) \mid \theta_j \geq y_j\right]-\alpha_0 u_0'\left(k'-y_j-\theta_0\right)\right)= \\ &= \left(1-F^j\left(y_j\right)\right)\left(\alpha_j H_j(y_j)-\alpha_0 u_0'\left(k'-y_j-\theta_0\right)\right). \qquad (9) \end{aligned}$$

We now establish that $\alpha_j H_j(y_j)-\alpha_0 u_0'\left(k'-y_j-\theta_0\right)$ is non-increasing in y_j by showing that $H_j'(y_j) \leq 0$. Concavity of u_0 would then complete the proof of this claim. To ease notation define

$$\Lambda(y_j)=\int_{y_j}^{\bar{\theta}_j}\left(u_j'\left(y_j-\theta_j\right)+\frac{u_j''\left(y_j-\theta_j\right)}{h(y_j)}\right) dF^j(\theta_j),$$

so that totally differentiating (8) we have

$$H_j'(y_j)=\frac{\Lambda(y_j)h(y_j)}{1-F^j(y_j)}.$$

We now show that $\Lambda(y_j) \leq 0$. As $u_j'' \leq 0$, Assumption 1 implies that $h(\theta_j) \geq h(y_j)$, for $\theta_j \geq y_j$ so that

$$\frac{u_j''\left(y_j-\theta_j\right)}{h(y_j)} \leq \frac{u_j''\left(y_j-\theta_j\right)}{h(\theta_j)} \text{ for } \theta_j \geq y_j.$$

Integration by parts provides the following identity

$$\int_{y_j}^{\overline{\theta}_j} u_j'\left(y_j-\theta_j\right) dF^j(\theta_j)=-\int_{y_j}^{\overline{\theta}_j} u_j''\left(y_j-\theta_j\right)(1-F^j(\theta_j))d\theta_j. \qquad (10)$$

Therefore

$$\begin{aligned} \Lambda(y_j) &\leq \int_{y_j}^{\overline{\theta}_j} u_j'\left(y_j-\theta_j\right) dF^j(\theta_j)+\int_{y_j}^{\bar{\theta}_j} \frac{u_j''\left(y_j-\theta_j\right)}{h(\theta_j)} dF^j(\theta_j)= \\ &= -\int_{y_j}^{\overline{\theta}_j} u_j''\left(y_j-\theta_j\right)(1-F^j(\theta_j))d\theta_j+\int_{y_j}^{\overline{\theta}_j} u_j''\left(y_j-\theta_j\right)(1-F^j(\theta_j))d\theta_j=0. \end{aligned}$$

As $\Lambda(y_j) \leq 0$ then $H_j'(y_j) \leq 0$ implying that $J_j'(y_j)$ changes sign at most once (from positive to negative) and thus J_j is quasiconcave.

With these insights we can now solve (7). First, for $y_j=0$ to be a solution of (7) it is necessary and sufficient that $J_j'(0) \leq 0$ which is equivalent to $\alpha_j H_j(0) \leq \alpha_0 u_0'\left(k'-\theta_0\right)$. For $y_j=k'$ to be a solution of (7) it is necessary and sufficient that $\alpha_j H_j(k') \geq \alpha_0 u_0'\left(-\theta_0\right)$. In all other cases the maximizer of (7) is the unique solution to $J_j'(y_j)=0$ thus satisfying:

$$\alpha_j H_j(y_j)=\alpha_0 u_0'\left(k'-y_j-\theta_0\right).$$

Given the uniqueness of maximizer, the Maximum Theorem establishes the continuity of $y_j(\theta_i)$. Summarizing, the optimal threshold $y_j(\theta_i)$ under $\mathbf{P}_i$ satisfies:

$$\begin{cases} y_j(\theta_i) = 0 & \text{if } \alpha_j H_j(0) \leq \alpha_0 u_0'(k - \theta_i - \theta_0), \\ y_j(\theta_i) = k - \theta_i & \text{if } \alpha_j H_j(k - \theta_i) \geq \alpha_0 u_0'(-\theta_0), \\ \alpha_j H_j(y_j) = \alpha_0 u_0'(k - \theta_i - y_j - \theta_0) & \text{otherwise.} \end{cases} \tag{11}$$

At any θ_i at which $y_j(\theta_i') = k - \theta_i'$ for θ_i' in a neighborhood of θ_i we have $y_j'(\theta_i) = -1$, while at any point θ_i at which $y_j(\theta_i') = 0$ for θ_i' in a neighborhood of θ_i we have $y_j'(\theta_i) = 0$. We can implicitly differentiate (11) for any interior solution to find

$$y_j'(\theta_i) = -\frac{\alpha_0 u_0''(k' - y_j - \theta_0)}{\alpha_0 u_0''(k' - y_j - \theta_0) + \alpha_j H_j'(y_j)} \in [-1, 0).$$

In particular, if $h_j'(\theta_j) > 0$ then $H_j'(y_j) < 0$ and $y_j'(\theta_j) \in (-1, 0)$.

(b) **Optimality of resource-cap priority mechanisms.**
We prove the optimality of (11) in the class of strategy-proof $\succeq_i$ −priority mechanisms following the same proof strategy as in Lemma 1: we show that, under an optimal mechanism, the set of resources awarded to system j is indeed an interval for any θ_i. To reach a contradiction, suppose that this set is not an interval. That is, letting $x_j^i(\theta_j)$ be the resources awarded to system j when system i announces θ_i, then there exist $\theta_j' < \theta_j''$ such that $x_j^i(\theta_j) \notin (\theta_j', \theta_j'')$ with $x_j^i(\theta_j'') = \theta_j''$. As system j's preferences are monotone, system j would never demand resources θ_j' when its needs exceed it, i.e. when $\theta_j > \theta_j'$. If resources (θ_j', θ_j'') are ruled out, system j would obtain $x_j^i(\theta_j) \geq \theta_j''$ when $\theta_j > \theta_j'$. We now show that allowing all points in (θ_j', θ_j'') increases the CES performance. By awarding system j the amount θ_j when $\theta_j \in (\theta_j', \theta_j'')$, the CES can allocate the remaining $x_j^i(\theta_j) - \theta_j \geq \theta_j'' - \theta_j > 0$ to system 0, thus increasing its performance. Therefore, the mechanism that rules outs (θ_j', θ_j'') cannot be optimal. This implies that the optimal mechanism is continuous. It is also immediate that the CES would never impose a binding lower bound on the resources granted to system j, as eliminating such lower bound would free resources to be allocated to system 0. Therefore the optimal mechanism $\mathbf{P}_i$ in the class of $\succeq_i$ −priority mechanisms is given by (11). □

Lemma 4 *Let $H_j(y_j) = E\left[u_j'(y_j - \theta_j) \mid \theta_j \geq y_j\right]$. The thresholds $y_2(\theta_1)$ and $y_1(\theta_2)$ in the priority mechanisms $\mathbf{P}_1$ and $\mathbf{P}_2$ defined in Lemma 3 intersect if and only if there exist $\tilde{\theta}_1$ and $\tilde{\theta}_2$ such that the following two conditions are satisfied:*

$$\begin{aligned} \alpha_1 H_1(\tilde{\theta}_1) - \alpha_2 H_2(0) \geq 0 \quad \textit{with} \quad \alpha_2 H_2(0) = \alpha_0 u_0'\left(k - \tilde{\theta}_1 - \theta_0\right), \\ \alpha_2 H_2(\tilde{\theta}_2) - \alpha_1 H_1(0) \geq 0 \quad \textit{with} \quad \alpha_1 H_1(0) = \alpha_0 u_0'\left(k - \tilde{\theta}_2 - \theta_0\right). \end{aligned} \tag{12}$$

Furthermore, if $h_1(\theta_1)$ and $h_2(\theta_2)$ are strictly increasing and $y_i(\theta_j) < k - \theta_j$ then $y_2(\theta_1)$ and $y_1(\theta_2)$ intersect at a single point.

Proof. We first show that if at least one of the conditions in (12) is not satisfied then $y_1(\theta_2)$ and $y_2(\theta_1)$ never intersect. The functions $y_1(\theta_2) - y_2^{-1}(\theta_2)$ and $y_2(\theta_1) - y_1^{-1}(\theta_1)$ are weakly increasing for $\theta_i \in [y_i(0), y_i(\overline{\theta}_j)] \cap [0, \overline{\theta}_i]$, since at any point of differentiability $d(y_i(\theta_j) - y_j^{-1}(\theta_j))/d\theta_j = y_i'(\theta_j) - (1/y_j'(y_j^{-1}(\theta_j))) \geq 0$.[28] Therefore, a necessary and sufficient condition for $y_1(\theta_2)$ and $y_2(\theta_1)$ to never intersect is that either $y_1(0) - y_2^{-1}(0) > 0$ or $y_2(0) - y_1^{-1}(0) > 0$.

Consider first the case $y_1(0) - y_2^{-1}(0) > 0$. In other words, under a $\succeq_1$ $-$priority mechanism $\mathbf{P}_1$ there is a θ_1 such that whenever system 1 requests at least θ_1 system 2 obtains zero resources ($y_2(\theta_1) = 0$) and $\theta_1 < y_1(0) \leq k$. By (11), the minimum value $\tilde{\theta}_1$ at which $y_2(\tilde{\theta}_1) = 0$ satisfies $\alpha_2 H_2(0) = \alpha_0 u_0'(k - \tilde{\theta}_1 - \theta_0)$. The condition $y_1(0) > \tilde{\theta}_1$ can be restated as requiring that the marginal effect on overall performance of increasing the threshold to system 1 at $\tilde{\theta}_1$ must be positive, which from Lemma 3 implies:

$$\alpha_1 H_1(\tilde{\theta}_1) - \alpha_0 u_0'\left(k - \tilde{\theta}_1 - \theta_0\right) \geq 0.$$

Substituting the value of $\tilde{\theta}_1$ this requires:

$$\alpha_1 H_1(\tilde{\theta}_1) - \alpha_2 H_2(0) \geq 0 \text{ with } \alpha_2 H_2(0) = \alpha_0 u_0'\left(k - \tilde{\theta}_1 - \theta_0\right).$$

Following a similar analysis, $y_2(0) - y_1^{-1}(0) > 0$ if and only if:

$$\alpha_2 H_2(\tilde{\theta}_2) - \alpha_1 H_1(0) \geq 0 \text{ with } \alpha_1 H_1(0) = \alpha_0 u_0'\left(k - \tilde{\theta}_2 - \theta_0\right).$$

Second, suppose that $y_1(\theta_2) < k - \theta_2$ and $y_2(\theta_1) < k - \theta_1$, i.e. both $y_i(\theta_j)$ are interior solutions of (11) and suppose that the hazard rates are strictly increasing. We now show that, given (12) is satisfied, then $y_2(\theta_1) - y_1^{-1}(\theta_1) = 0$ has a unique solution. From $h_2'(\theta_2) > 0$, $h_1'(\theta_1) > 0$ and Lemma 3, it follows that $0 > y_2'(\theta_1) > -1$ and $d(y_1^{-1}(\theta_1))/d\theta_1 < -1$. Taking both implications together, we have that the difference $y_2(\theta_1) - y_1^{-1}(\theta_1)$ is *strictly* increasing in θ_1 and thus changes sign at most once. Therefore if the curves $y_1(\theta_2)$ and $y_2(\theta_1)$ intersect at an interior point, then they intersect only once. □

Lemma 5 *Let $\Theta_i^+ = \left\{\theta_i : \theta_j > \overline{x}_j^*(\theta_i) \Rightarrow \theta_i > \overline{x}_i^*(\theta_j)\right\}$ be the set of values θ_i such that under an optimal mechanism $\mathbf{M}$ and for any (θ_i, θ_j) in which system j receives less than θ_j, system i receives less than θ_i. If θ_1 and θ_2 are independent, then $\overline{x}_j^*(\theta_i)$ is constant in Θ_i^+.*

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[28] It can be readily shown that any other point must entail a binding constraint $y_i(\theta_j) = k - \theta_j$ in which case the functions $y_i(\theta_j) - y_j^{-1}(\theta_j)$ are continuous and weakly increasing.

Proof. Let $\theta_i \in \Theta_i^+$. Then the optimal cap on system j, $\overline{x}_j^*(\theta_i)$, must satisfy the first order condition:

$$\int_{\overline{x}_j^*}^{\overline{\theta}_j} \left[\alpha_j u_j'\left(\overline{x}_j^* - \theta_j\right) - \alpha_0 u_0'\left(k - \theta_0 - \left(\overline{x}_j^* + x_i\left(\theta_i, \theta_j\right)\right)\right)\right] dF^j(\theta_j) = 0.$$

By definition, if $\theta_j \geq \overline{x}_j^*$ system i receives less than its needs when $\theta_i \in \Theta_i^+$. Therefore its allocation $x_i\left(\theta_i, \theta_j\right)$ is independent of θ_i, $x_i\left(\theta_i, \theta_j\right) = \overline{x}_i^*\left(\theta_j\right)$. Then the FOC is independent of θ_i and thus for all $\theta_i \in \Theta_i^+$, the optimum $\overline{x}_j^*$ is independent of θ_i. □

Using Lemmas 3, 4 and 5, we proceed to the characterization of the optimal mechanism **M**.

Proof of Proposition 1: The first-order condition for $\overline{x}_i^*\left(\theta_j\right)$ when system j reports θ_j is

$$\int_{\overline{x}_i^*}^{\overline{\theta}_j} \alpha_i u_i'\left(\overline{x}_i^* - \theta_i\right) dF^i(\theta_i) = \int_{\overline{x}_i^*}^{\overline{\theta}_i} \alpha_0 u_0'\left(k - \theta_0 - \left(\overline{x}_i^* + x_j\left(\theta_i, \theta_j\right)\right)\right) dF^i(\theta_i). \tag{13}$$

From the proof of Lemma 3 the optimal cap on a system must be lower if other systems are given more resources. This observation allows us to establish the following facts regarding the allocation rule under an optimal mechanism **M** : (i) system i obtains at least the same resources as under the priority mechanism $\mathbf{P}_j$, and (ii) system i's resources never exceed those obtained under $\mathbf{P}_j$ when system j demands zero resources. Indeed, since $x_j\left(\theta_i, \theta_j\right) = \min\left\{\theta_j, \overline{x}_j\left(\theta_i\right)\right\} \leq \theta_j$ and the right hand side of (13) is increasing in $x_j\left(\theta_i, \theta_j\right)$, we then have:

$$y_i\left(\theta_j\right) \leq \overline{x}_i^*\left(\theta_j\right) \leq y_i\left(0\right). \tag{14}$$

We consider first the case in which $y_1\left(\theta_2\right)$ and $y_2\left(\theta_1\right)$ intersect at an interior point (Lemma 4). We characterize **M** by proving a series of properties implied by optimality.

(i) *The sets* Θ_i^+ *are non-empty.*

Suppose that $\bar{\theta}_i > y_i(0)$. From Lemma 4 we have that $y_2(0) < y_1^{-1}(0)$, that is the maximum resources granted to system 2 in a priority mechanism $\mathbf{P}_1$ (which occurs when system 1 demands zero) are less than the needs of system 2 that would lead system 1 to obtain zero resources under $\mathbf{P}_2$. From (14), $y_2(0)$ represents an upper bound on the resources that system 2 would obtain under **M**. Therefore, for every $\theta_2 \geq y_2(0)$ we have $\theta_2 \in \Theta_2^+$ as system 2 is necessarily constrained. Lemma 4 also implies that $y_1(0) < y_2^{-1}(0)$. Therefore, for every $\theta_1 \geq y_1(0)$ we have $\theta_1 \in \Theta_1^+$.

(ii) *Mechanism* M *behaves like a priority mechanism* P_i *for* $\theta_i \leq k_i$

From Lemma 5, define $k_i = \overline{x}_i^*\left(\theta_j\right)$ to be the constant cap for $\theta_j \in \Theta_j^+$. Then k_i represents the minimum resources guaranteed to system i in the sense that $\overline{x}_i^*\left(\theta_j\right) \geq k_i$ for all θ_j. This is easy to see as $x_j\left(\theta_i, \theta_j\right)$ is monotone in θ_j so that the left hand side of (13) increases in θ_j. Therefore $k_i = \overline{x}_i^* \left(\bar{\theta}_j\right) \leq \min \overline{x}_i^* \left(\bar{\theta}_j\right)$. This implies that for $\theta_i \leq k_i$ system i always obtains its resource needs $x_i\left(\theta_i, \theta_j\right) = \theta_i$. Therefore, for $\theta_i \leq k_i$ the optimality condition (13) is satisfied by $\overline{x}_j^*\left(\theta_i\right) = y_j\left(\theta_i\right)$.

(iii) *Optimal guaranteed resources satisfy* $k_j = y_j\left(k_i\right)$.
Define k_i^* as the point of intersection of $y_1\left(\theta_2\right)$ and $y_2\left(\theta_1\right)$, i.e. $k_j^* = y_j\left(k_i^*\right)$. We now show that $k_i = k_i^*$.

First, system 1 obtains at least resources k_1 when $\theta_1 > k_1$. This implies that the resources obtained by system 2 cannot exceed those obtained under a priority mechanism $\mathbf{P}_1$ when system 1 demands resources k_1, i.e. $x_2\left(\theta_1, \theta_2\right) \leq \overline{x}_2^*\left(\theta_1\right) \leq y_2\left(k_1\right)$ for $\theta_1 > k_1$. Therefore the optimal k_1 that satisfies (13) is (weakly) higher than the cap under a priority mechanism $\mathbf{P}_2$ when system 2 demanded resources $y_2\left(k_1\right)$, i.e. $k_1 \geq y_1\left(y_2\left(k_1\right)\right)$, or $y_2(k_1) - y_1^{-1}(k_1) \geq 0$. By Lemma 4, $y_2(\theta_1) - y_1^{-1}(\theta_1)$ is an increasing function implying that

$$k_1 \geq k_1^*. \tag{15}$$

Analogously we obtain that $k_2 \geq k_2^*$.

Second, system 2 always obtains at least resources k_2 when $\theta_2 \geq k_2$. Therefore the optimal k_1 that satisfies (13) cannot exceed the cap under a priority mechanism $\mathbf{P}_2$ when system 2 demands resources k_2 , i.e.

$$k_1 \leq y_1\left(k_2\right). \tag{16}$$

Combining (15) and (16) and $k_2 \geq k_2^*$ we have

$$k_1 \leq y_1\left(k_2\right) \leq y_1\left(k_2^*\right) = k_1^* \leq k_1.$$

A similar reasoning yields $k_2 = k_2^*$.

Finally, we also consider the case in which $y_1\left(\theta_2\right)$ and $y_2\left(\theta_1\right)$ never intersect. If $y_i\left(0\right) - y_j^{-1}\left(0\right) > 0$ then by Lemma 4 we have $y_j(\theta_i) < y_i^{-1}(\theta_i)$ for all θ_i which implies that $k_i = y_i(0)$. Therefore $\mathbf{M}$ is a priority mechanism $\mathbf{P}_i$ for $\theta_i \leq y_i(0)$ while it implements the allocation $x_i\left(\theta_1, \theta_2\right) = k_i$ and $x_j\left(\theta_1, \theta_2\right) = 0$ for $\theta_i > y_i\left(0\right)$. □

C. Proof of Proposition 2

The thresholds $y_i(\theta_j)$ are defined in (11). An increase in α_i to $\alpha_i' > \alpha_i$ relaxes the conditions $\alpha_i H_i(0) > \alpha_0 u_0'\left(k - \theta_i - \theta_0\right)$ and $\alpha_i H_i(k - \theta_j) \geq \alpha_0 u_0'\left(-\theta_0\right)$ implying that if $y_i(\theta_j)(\alpha_i) \geq 0$ then $y_i(\theta_j)(\alpha_i') \geq 0$, and if $y_i(\theta_j)(\alpha_i) = k - \theta_j$ then $y_i(\theta_j)(\alpha_j') = k - \theta_j$. If $y_i(\theta_j)(\alpha i)$ satisfies $\alpha_i H_i(y_i(\theta_j)) = \alpha_0 u_0'\left(k - \theta_j - y_i(\theta_j) - \theta_0\right)$, then implicitly differentiating we have that $\partial y_i(\theta_j)/\partial \alpha_i > 0$. In summary, if $\alpha_i' > \alpha_i$ then $y_i\left(\theta_j\right)\left(\alpha_i'\right) \geq y_i\left(\theta_j\right)\left(\alpha_i\right)$ for all $\theta_j \in [\underline{\theta}_j, \overline{\theta}_j]$. Since $y_j(\theta_i)$ does not depend on α_i, we have $\bar{x}_i^*(\theta_j)(\alpha_i') \geq \bar{x}_i^*(\theta_j)(\alpha_i)$, $\bar{x}_j^*(\theta_i)(\alpha_i') \leq \bar{x}_j^*(\theta_i)(\alpha_i)$, and $k_i(\alpha_i') \geq k_i(\alpha_i)$, $k_j(\alpha_i') \leq k_j(\alpha_i)$.

We now consider the comparative statics on α_0, θ_0 and k. Following a similar argument as before, we can show that if $\alpha_0' > \alpha_0$ or $\theta_0' > \theta_0$ then optimal caps satisfy $y_i(\theta_j)(\alpha_0') \leq y_i(\theta_j)(\alpha_0)$ and $y_i(\theta_j)(\theta_0') \leq y_i(\theta_j)(\theta_0)$ for $\theta_j \in [0, \overline{\theta}_j]$. Now, let $H_i(\theta_i)$ as given by (8). If $k_i(\alpha_0) > 0$ and $k_i(\theta_0) > 0$, from Proposition 1 we have that the guaranteed levels k_i satisfy:

$$\alpha_1 H_1(k_1) = \alpha_2 H_2(k_2) \qquad \text{if} \quad k_1 + k_2 = k, \tag{17}$$

$$\alpha_1 H_1(k_1) = \alpha_2 H_2(k_2) = \alpha_0 u_0' \left(k - k_1 - k_2 - \theta_0\right) \quad \text{if} \quad k_1 + k_2 < k. \tag{18}$$

We now show that both guaranteed levels are reduced for any of the following changes: (i) $\alpha_0' > \alpha_0$, (ii) $\theta_0' > \theta_0$, or (iii) $k' < k$. First consider the case (17). Then for any $\alpha_0' > \alpha_0$ or $\theta_0' > \theta_0$ such that we still have $k_1\left(\alpha_0'\right) + k_2\left(\alpha_0'\right) = k$ or $k_1\left(\theta_0'\right) + k_2\left(\theta_0'\right) = k$ the guaranteed levels do not change as (17) does not depend on α_0 or θ_0. Next suppose that $k_1 + k_2 < k$. Then by the implicit function theorem applied to (18):

$$\frac{\partial k_i}{\partial \alpha_0} = -\frac{\alpha_j H_j'(k_j) u_0''(\tilde{k} - \theta_0)}{\Delta} \qquad \text{and} \qquad \frac{\partial k_i}{\partial \theta_0} = -\frac{\partial k_i}{\partial k} = -\frac{\alpha_0 \alpha_j H_j'(k_j) u_0''(\tilde{k} - \theta_0)}{\Delta},$$

where $\tilde{k} = k_1 + k_2$ and $\Delta = \alpha_1 \alpha_2 H_1'(k_1) H_2'(k_2) + \alpha_0 u_0''(\tilde{k} - \theta_0) \left(\sum_{j=1,2} \alpha_j H_j'(k_j)\right) > 0$. Therefore $\partial k_i / \partial \alpha_0 < 0$, $\partial k_i / \partial k > 0$ and $\partial k_i / \partial \theta_0 < 0$. □

D. Proof of Proposition 3

Trivial to check once we note that if system i stops consuming at $\tilde{t}$, then it implies that $r_i \tilde{t} = \theta_i$. □

E. Proof of Lemma 2

The condition in the statement of the lemma is equivalent to log-supermodularity of $g^i(\theta_i^t | \theta_i^{t-1})$, which requires that for each $\tilde{\theta}_i^t > \theta_i^t$ and $\tilde{\theta}_i^{t-1} > \theta_i^{t-1}$ we must have

$$g^i(\tilde{\theta}_i^t | \tilde{\theta}_i^{t-1}) g^i(\theta_i^t | \theta_i^{t-1}) - g^i(\tilde{\theta}_i^t | \theta_i^{t-1}) g^i(\theta_i^t | \tilde{\theta}_i^{t-1}) \geq 0.$$

where $g^i(\theta_i^t | \theta_i^{t-1})$ is given by (6). Therefore, we simply need

$$h(\theta_i^t, \theta_i^{t-1}) = \int_{\underline{s}_i}^{\overline{s}_i} f^i(\theta_i^t | s_i) f^i(\theta_i^{t-1} | s_i) p^i(s_i) ds_i$$

to be log-supermodular to guarantee that $g^i(\theta_i^t | \theta_i^{t-1})$ is log-supermodular. A theorem of Karlin (1968) establishes that, for a measure μ, whenever $i(x, s)$ and $j(y, s)$ are log-supermodular, then $k(x, y) = \int_S i(x, s) j(y, s) d\mu(s)$ is also log-supermodular. Since, by Assumption 3, $f^i(\theta_i^t | s_i)$ is log-supermodular for any t, this completes the proof. □

F. Proof of Proposition 4

Given Proposition 1 and Lemma 2 it suffices to show that if $\overline{x}_i^*(\theta_j)(G_1)$ is the optimal resource cap when θ_1 is distributed according to G_1, then for $G_1' \succ_{LR} G_1$ we have $\overline{x}_1^*(\theta_2)(G_1') \geq \overline{x}_1^*(\theta_2)(G_1)$ and $\overline{x}_2^*(\theta_1)(G_1') \leq \overline{x}_2^*(\theta_1)(G_1)$.

If $G_1' \succ_{LR} G_1$, then the hazard rates satisfy $h_{G_1'}(\theta_1) \leq h_{G_1}(\theta_1)$ and $\frac{1-G_1'(\theta_1)}{1-G_1(\theta_1)}$ increases in θ_1. Since (10) implies that

$$E_{G_1'}\left[u_1'\left(y_1-\theta_1\right) \mid \theta_1 \geq y_1\right] = -\int_{y_1}^{\bar{\theta}_1} u_1''\left(y_1-\theta_1\right)\frac{1-G_1(\theta_1)}{1-G_1(y_1)}d\theta_1,$$

it follows from (10) that

$$E_{G_1'}\left[u_1'\left(y_1-\theta_1\right) \mid \theta_1 \geq y_1\right] > E_{G_1}\left[u_1'\left(y_1-\theta_1\right) \mid \theta_1 \geq y_1\right] \tag{19}$$

From (19) and the definition of $y_1(\theta_2)$ in (11) we have: (i) if $y_1(\theta_2)(G_1) = 0$ then $y_1(\theta_2)(G_1') \geq 0$; (ii) if $y_1(\theta_2)(G_1) = k-\theta_2$ then $y_1(\theta_2)(G_1') = k-\theta_2$; and (iii) if $y_1(\theta_2)(G_1)$ satisfies

$$\alpha_1 H_1(y_1(\theta_2)(G_1)) = \alpha_0 u_0'\left(k-\theta_2-y_1(\theta_2)(G_1)-\theta_0\right),$$

then $y_1(\theta_2)(G_1) < y_1(\theta_2)(G_1')$. In summary, if $G_1' \succ_{LR} G_1$ then $y_1(\theta_2)(G_1') \geq y_1(\theta_2)(G_1)$ for all $\theta_2 \in [\underline{\theta}_2, \overline{\theta}_2]$. Since $y_2(\theta_1)(G_1') = y_2(\theta_1)(G_1)$, we can immediately conclude that (i) $k_1(G_1') > k_1(G)$, $k_2(G_1') < k_2(G_1)$ and (ii) $\overline{x}_1^*(\theta_2)(G_1') \geq \overline{x}_1^*(\theta_2)(G_1)$, $\overline{x}_2^*(\theta_1)(G_1') \leq \overline{x}_2^*(\theta_1)(G_1)$. $\square$

G. Proof of Corollary 2

Let I_t denote the history of needs prior to time t and let $E_t[\cdot] = E[\cdot \mid I_t]$ be the conditional expectation given history I_t. First, the full information optimal allocation rule at any time t' is independent of the state s_i given the needs $(\theta_1^{t'}, \theta_2^{t'})$. Therefore, the law of iterated expectations implies that for any $t' > t$

$$E_t\left[\Pi_{CES}^{t'}\right] = E_t\left[E\left[\Pi_{CES}^{t'} \mid I_{t'}\right]\right] = E_t\left[\Pi_{CES}^{t}\right].$$

This follows as knowing $I_{t'}$ does not change the allocation rule given that the CES will first observe $(\theta_1^{t'}, \theta_2^{t'})$. Second, improved expected performance under incomplete information follows from two observations. First, the constraints on dominant strategy incentive compatibility do not depend on the common beliefs on the distribution of needs. Thus, every mechanism that is feasible and DSIC remains so after the CES updates his belief over the state s_i. Second, as the set of direct incentive compatible mechanisms remains unchanged with the CES' beliefs, the CES cannot do worse with additional information on s_i if systems act myopically. Formally, letting D be the set of feasible and DSIC mechanisms we have that for $t'' > t' > t$,

$$E_t\left[E\left[\Pi^{t''}_{CES}\,|\,I_{t''}\right]\right] = E_t\left[\max_D E\left[\Pi^{t''}_{CES}\,|\,I_{t''}\right]\right] \geq E_t\left[E\left[\Pi^{t''}_{CES}\left(D'\right)|\,I_{t''}\right]\right] = E_t\left[\Pi^{t'}_{CES}\right].$$

where D' is the optimal mechanism given history $I_{t'}$. Therefore the expected performance at $t'' > t'$ exceeds the expected performance at t'. □

H. Proof of Corollary 3

In the text.

I. Proof of Corollary 4

In the text.

J. Proof of Proposition 5

The proof of Proposition 5 is divided into two parts. First, Lemma 6 provides a characterization of the optimal mechanism **I** under integration of tasks 1 and 2 by system I. We then proceed to prove Proposition 5. To streamline the exposition in Lemma 6, let $\alpha'_0 = (1-\gamma)\alpha_0$ and $\alpha'_i = \gamma\alpha_i$ for $i = 1, 2$.

Lemma 6 *The optimal integration mechanism* **I** *is such that CES sets a fixed cap* $\hat{k}$ *on the total resources allocated to system* I*. The optimal cap* $\hat{k}$ *satisfies:*

$$\frac{\alpha'_1\alpha'_2}{\alpha'_1+\alpha'_2}\Big(E[\theta_1+\theta_2\,|\,\theta_1+\theta_2 \geq \hat{k}] - \hat{k}\Big) = \alpha'_0\Big(\theta_0 - [k-\hat{k}]\Big) \tag{20}$$

Proof. For fixed resources k' (≥ 0) and needs (θ_1, θ_2), system I will choose to distribute them according to

$$\begin{array}{lll} \text{1. If } k' \geq \max_i\left\{\theta_i - \frac{\alpha'_j}{\alpha'_i}\theta_j\right\} & \text{then} & \left\{\begin{array}{l} x_1(k';\theta_1,\theta_2) = \theta_1 - \alpha'_2(\theta_1+\theta_2-k')/(\alpha'_1+\alpha'_2) \\ x_2(k';\theta_1,\theta_2) = \theta_2 - \alpha'_1(\theta_1+\theta_2-k')/(\alpha'_1+\alpha'_2) \end{array}\right. , \\ \text{2. If } k' < \theta_1 - \frac{\alpha'_2}{\alpha'_1}\theta_2 & \text{then} & \left\{\begin{array}{lcl} x_1(k';\theta_1,\theta_2) & = & k' \\ x_2(k';\theta_1,\theta_2) & = & 0 \end{array}\right. , \\ \text{3. If } k' < \theta_2 - \frac{\alpha'_1}{\alpha'_2}\theta_1 & \text{then} & \left\{\begin{array}{lcl} x_1(k';\theta_1,\theta_2) & = & 0 \\ x_2(k';\theta_1,\theta_2) & = & k' \end{array}\right. , \end{array} \tag{21}$$

and the ex-post performance of system I is

$$\begin{array}{lcll} \Pi_I(\theta_1,\theta_2,k') & = & -\frac{\alpha'_1\alpha'_2}{\alpha'_1+\alpha'_2}\left(\min\{k'-\theta_1-\theta_2,0\}\right)^2 & \text{if} \quad k' \geq \max_i\{\theta_i - \frac{\alpha'_j}{\alpha'_i}\theta_j\}, \\ \Pi_I(\theta_1,\theta_2,k') & = & -\alpha'_1\left(\min\{k'-\theta_1,0\}\right)^2 - \alpha'_2\theta_2^2 & \text{if} \quad k' < \theta_1 - \frac{\alpha'_2}{\alpha'_1}\theta_2, \\ \Pi_I(\theta_1,\theta_2,k') & = & -\alpha'_1\theta_1^2 - \alpha'_2\left(\min\{k'-\theta_2,0\}\right)^2 & \text{if} \quad k' < \theta_2 - \frac{\alpha'_1}{\alpha'_2}\theta_1. \end{array}$$

The fact that resources allocated to each task must be non-negative accounts for the allocation rules whenever $k' < \max_i \left\{\theta_i - \frac{\alpha'_j}{\alpha'_i}\theta_j\right\}$. In fact, non-negativity implies that system I cannot "borrow" negative resources from tasks with lower needs and redirect them to tasks with higher needs. This insight provides an upper bound on the ex-post performance of system I given optimal distribution rules (21):

$$\Pi_I(\theta_1, \theta_2, k') \leq -\frac{\alpha'_1\alpha'_2}{\alpha'_1+\alpha'_2}\left(\theta_1 + \theta_2 - k'\right)^2. \tag{22}$$

Clearly, for any k' and needs (θ_1, θ_2), expected overall performance is

$$\Pi_{CES} = \Pi_I(\theta_1, \theta_2, k') - \alpha'_0(\min\left\{\left(k - k' - \theta_0\right), 0\right\})^2.$$

To study the optimal mechanism under integration we consider a related optimization problem (problem $\mathcal{P}'$) where the performances of CES and system I are given by:

$$\begin{aligned} \Pi'_I(\theta_1, \theta_2, k') &= -\frac{\alpha'_1\alpha'_2}{\alpha'_1+\alpha'_2}\left(\theta_1 + \theta_2 - k'\right)^2, \\ \Pi'_{CES} &= \min\left\{-\alpha'_0\Big(\theta_0 - (k - k')\Big)^2, 0\right\} + \Pi'_I(\theta_1, \theta_2, k'). \end{aligned} \tag{23}$$

We then show that the optimal mechanism for problem $\mathcal{P}'$ is also the optimal mechanism for our original specification.

(i) *Optimal Mechanism for Problem $\mathcal{P}'$.*
The overall performance from the viewpoint of CES in (23) can be written as

$$\Pi'_{CES} = -\left(\frac{\alpha'_1\alpha'_2}{\alpha'_1+\alpha'_2} + \alpha_0\right)\left(k' - k'_{CES}\left(\theta_1, \theta_2\right)\right)^2 - \frac{\alpha'_0\alpha'_1\alpha'_2}{\alpha'_1\alpha'_2 + \alpha'_1\alpha'_0 + \alpha'_2\alpha'_0}\left(k - \theta_1 - \theta_2 - \theta_0\right)^2,$$

where

$$k'_{CES}\left(\theta_1, \theta_2\right) = \max\left\{\frac{\alpha'_1\alpha'_2}{\alpha'_1\alpha'_2 + \alpha'_1\alpha'_0 + \alpha'_2\alpha'_0}(\theta_1 + \theta_2) - \frac{\alpha'_0\left(\alpha'_1+\alpha'_2\right)}{\alpha'_1\alpha'_2 + \alpha'_1\alpha'_0 + \alpha'_2\alpha'_0}(\theta_0 - k), 0\right\},$$

is the optimal total amount of resources to system I if the CES knew (θ_1, θ_2). We first consider the optimal mechanism of the form $k\left(z\right) = \min\{z, \hat{k}\}$, and then argue that this is the optimal mechanism for problem $\mathcal{P}'$.

(ii) *Optimal cap $\hat{k}$.*
First, with $z = \theta_1 + \theta_2$ we have

$$G(z) = \int_{\Theta_1}\int_{\underline{\theta}_2}^{\theta_2 = z - \theta_1} f\left(\theta_1, \theta_2\right) d\theta_1 d\theta_2 \quad \text{and} \quad g(z) = \int_{\Theta_1} f_1\left(\theta_1\right) f_2\left(z - \theta_1\right) d\theta_1$$

which follows from independence of θ_1 and θ_2. The distribution function $G(z)$ is the convolution of two distributions with an increasing hazard rate, and therefore has also an increasing hazard rate (Barlow et al 1963). Moreover, the expected performance of CES with a resource allocation rule $k^{\mathbf{I}}(z) = \min\{z,\, k'\}$ is

$$\begin{aligned} J^I_{CES}\left(k'\right) &= \int_0^{z=k'} -\alpha_0'\left(k-\theta_0-z\right)^2 dG(z) \\ &\quad + \int_{z=k'}^{\overline{z}} \left(-\alpha_0'\left(k-\theta_0-k'\right)^2 - \frac{\alpha_1'\alpha_2'}{\alpha_1'+\alpha_2'}\left(z-k'\right)^2\right) dG(z), \end{aligned}$$

where $\overline{z} = \overline{\theta}_1 + \overline{\theta}_2$. The optimal cap $\hat{k}$ satisfies the first order condition

$$2\int_{z=\hat{k}}^{\overline{z}} \left(\alpha_0'\left(k-\theta_0-\hat{k}\right) + \frac{\alpha_1'\alpha_2'}{\alpha_1'+\alpha_2'}\left(z-\hat{k}\right)\right) dG(z) = 0,$$

which translates into

$$\frac{\alpha_1'\alpha_2'}{\alpha_1'+\alpha_2'}\left(E\left[z\,\middle|\,z \geq \hat{k}\right] - \hat{k}\right) = \alpha_0'\left(\theta_0 - \left(k-\hat{k}\right)\right). \tag{24}$$

The second order condition is satisfied as z has an increasing hazard rate. Note also that there would be no gain to CES when solving problem $\mathcal{P}'$ from banning intermediate decisions. Indeed suppose that CES offers a mechanism $k'(z)$ of the form

$$k'(z) = \begin{cases} z & \text{if} \quad z \leq k_1 \\ \min\{\max\{z, k_2\}, \hat{k}\} & \text{if} \quad z > k_1 \end{cases},$$

where resources $k' \in (k_1, k_2)$ are not available to system I. If system I's needs are $z \in (k_1, k_2)$ as his preferences are monotone it would demand at least k_2. The CES would then be better off allowing all resources in (k_1, k_2), thus reducing the resources allocated to system I (without changing its performance) and thus improving the performance of system 0. Therefore ruling out intermediate resources is not optimal.

(iii) *Optimal Mechanism under integration.*
First we observe that under a mechanism of the form $k(z) = \min\{z,\, \hat{k}\}$ the allocation for each task whenever $z \leq \hat{k}$ is the same for a system I with preferences as in (23) or in our original setup. Moreover, the fact that the solution to problem $\mathcal{P}'$ is of the form $k'(z) = \min\{z,\, \hat{k}\}$ implies that it is never optimal to "rule out" intermediate resources in the original problem. Thus the optimal mechanism in the original problem is of the form $k(z) = \min\{z,\, \hat{k}\}$ with $\hat{k}$ as in (24).

Proof of Proposition 5: Denote by $J^I_{CES}(\gamma)$ the maximum expected performance under integration and by $J^M_{CES}(\gamma)$ the maximum expected performance under specialization.

(a) **Relative performance of Integration vs. Specialization as $\gamma \to 1$.**

(a-i) *Performance under integration.*

Given that $\overline{z} = \bar{\theta}_1 + \bar{\theta}_2 \leq k$, Lemma 6 establishes that for γ close to 1 the optimal integration mechanism $\mathbf{I}$ sets a cap on resources $\hat{k}$ where

$$\hat{k} = \frac{\gamma/\alpha_0}{\gamma/\alpha_0 + (1-\gamma)(\frac{\alpha_1\alpha_2}{\alpha_1+\alpha_2})} E\left[z \,\middle|\, z \geq \hat{k}\right] - \frac{(1-\gamma)(\frac{\alpha_1\alpha_2}{\alpha_1+\alpha_2})}{\gamma/\alpha_0 + (1-\gamma)(\frac{\alpha_1\alpha_2}{\alpha_1+\alpha_2})} \left(\theta_0 - k\right). \tag{25}$$

Since $\theta_0 > k$ it follows that $\hat{k} < \overline{z}$ whenever $\gamma < 1$ and $\hat{k} \to \overline{z}$ as $\gamma \to 1$. To study the performance of integration as $\gamma \to 1$ we first determine the rate at which the $\hat{k}$ increases. To this end, we make two preliminary observations. First, given the bounded support of θ_1 and θ_2, the p.d.f. of $g(\overline{z})$ satisfies

$$g(\overline{z}) = \int_{\Theta_1} f^1\left(\theta_1\right) f^2\left(\bar{\theta}_1 - \theta_1 + \bar{\theta}_2\right) d\theta_1 = 0,$$

as $f^2\left(\bar{\theta}_1 - \theta_1 + \bar{\theta}_2\right) = 0$ for $\theta_1 < \bar{\theta}_1$. Second,

$$g'_-(\overline{z}) = -f^1\left(\bar{\theta}_1\right) f^2\left(\bar{\theta}_2\right) < 0.$$

In order to compute $\partial\hat{k}/\partial\gamma|_{\gamma=1^-}$ as an application of the implicit function theorem to (25) we first compute $dE\left[z - \hat{k} \,\middle|\, z \geq \hat{k}\right]/d\hat{k}|_{\gamma=1^-}$

$$\left.\frac{dE\left[z - \hat{k} \,\middle|\, z \geq \hat{k}\right]}{d\hat{k}}\right|_{\gamma=1^-} = -1 + \lim_{\gamma=1^-} \frac{g(\hat{k})}{1 - G(\hat{k})} E\left[z - \hat{k} \,\middle|\, z \geq \hat{k}\right].$$

Defining the function $N(k) = \int_k^{\overline{z}} (z-k) dG(z) = \int_k^{\overline{z}} (1 - G(z)) dz$, we have

$$\frac{g(k)}{1-G(k)} E\left[z - k \,|\, z \geq k\right] = \frac{N''(k)N(k)}{(-N'(k))^2}.$$

The (leftward) Taylor series expansion of $N(k)$ around $\overline{z}$ is given by $N(k) = \frac{g'(\overline{z})}{3!}(k - \overline{z})^3 + O((k-\overline{z})^4)$ for $k \leq \overline{z}$. With this expression we readily obtain

$$\lim_{\gamma\to1^-} \frac{g(\hat{k})}{1-G(\hat{k})} E\left[z - \hat{k} \,\middle|\, z \geq \hat{k}\right] = \lim_{\gamma\to1^-} \frac{N''(\hat{k})N(\hat{k})}{(-N'(\hat{k}))^2} = \frac{\frac{g'(\overline{z})}{3!}\frac{g'(\overline{z})}{1!}}{\left(\frac{g'(\overline{z})}{2!}\right)^2} = \frac{2}{3}.$$

Thus

$$\left.\frac{dE\left[z - \hat{k} \,\middle|\, z \geq \hat{k}\right]}{d\hat{k}}\right|_{\gamma=1^-} = -1 + \lim_{\gamma=1^-} \frac{g(\hat{k})}{1-G(\hat{k})} E\left[z - \hat{k} \,\middle|\, z \geq \hat{k}\right] = -\frac{1}{3},$$

and by the implicit function theorem

$$\left.\frac{\partial \hat{k}}{\partial \gamma}\right|_{\gamma=1^-} = -\alpha_0 \frac{\left(\frac{\alpha_1\alpha_2}{\alpha_1+\alpha_2}\right)\left(\overline{z}+\theta_0-k\right)}{\left.\frac{dE\left[z-\hat{k}|z\geq\hat{k}\right]}{d\hat{k}}\right|_{\gamma=1^-}} = 3\alpha_0\left(\frac{\alpha_1\alpha_2}{\alpha_1+\alpha_2}\right)\left(\overline{z}+\theta_0-k\right) > 0. \tag{26}$$

(a-ii) *Performance under specialization.*
To study the case of specialization we will consider the mechanism $\mathbf{M}_D$ defined by

$$\begin{aligned} x_1^{\mathbf{M}_D}(\theta_1,\theta_2) &= \min\{\theta_1, y_1(\theta_2)\}, \\ x_2^{\mathbf{M}_D}(\theta_1,\theta_1) &= \min\{\theta_2, y_2(\theta_1)\}, \\ x_0^{\mathbf{M}_D}(\theta_1,\theta_2) &= k - x_1^{\mathbf{M}_D}(\theta_1,\theta_2) - x_2^{\mathbf{M}_D}(\theta_1,\theta_2), \end{aligned}$$

where

$$y_i(\theta_j) = \begin{cases} \widetilde{k}_i & \text{if} \quad \theta_j \leq \hat{k} - \overline{\theta}_i \\ \hat{k} - \theta_j & \text{if} \quad \theta_j \geq \hat{k} - \overline{\theta}_i \end{cases},$$

and

$$\widetilde{k}_i = \bar{\theta}_i - (1-\gamma)\frac{\alpha_0}{\alpha_i\left(\gamma + \frac{\delta_i}{2\alpha_i}\right)}\left(\bar{\theta}_i + \theta_0 - k\right), \tag{27}$$

where δ_i is a strictly positive parameter. The mechanism M_D always satisfies the resource constraint (given that $\bar{\theta}_1 + \bar{\theta}_2 \leq k$) and is dominant strategy incentive compatible. Clearly $\mathbf{M}_D$ is not necessarily optimal under specialization, i.e. $J_{CES}^{M}(\gamma) \geq J_{CES}^{M_D}(\gamma)$. Nevertheless $\mathbf{M}_D$ is simpler to analyze than the optimal mechanism under specialization and will suffice to show that specialization dominates integration as $\gamma \to 1$.

(a-iii) *Comparison integration-specialization.*
Define the sets

$$\begin{aligned} A_i &= \left\{(\theta_1,\theta_2) : \theta_i \geq \widetilde{k}_i, \theta_j \leq \hat{k} - \overline{\theta}_i\right\}, \quad i \in \{1,2\}, \\ B &= \left\{(\theta_1,\theta_2) : \theta_1 + \theta_2 \geq \hat{k}\right\}. \end{aligned}$$

As the allocation rule under the mechanisms $\mathbf{I}$ and $\mathbf{M}_D$ coincides outside the sets A_i and B we only need to estimate the difference $J_{CES}^{M_D}(\gamma) - J_{CES}^{I}(\gamma)$ in each of these three sets.

First, for $(\theta_1,\theta_2) \in A_i$ (i) the mechanism $\mathbf{M}_D$ restricts the allocation to system i to $\widetilde{k}_i$ while it grants its needs to system j and (ii) both tasks obtain their needs under the mechanism **I**. We thus have

$$
\begin{aligned}
&\left(J^{M_D}_{CES}(\gamma)-J^{I}_{CES}(\gamma)\right)\Big|_{(\theta_1,\theta_2)\in A_i}\\
&=\int_{A_i}\left(-\alpha_0\left(1-\gamma\right)\left(k-\widetilde{k}_i-\theta_j-\theta_0\right)^2-\gamma\alpha_i\left(\widetilde{k}_i-\theta_i\right)^2+\left(1-\gamma\right)\alpha_0\left(k-\theta_i-\theta_j-\theta_0\right)^2\right)dF^1dF^2\\
&=2\int_{A_i}\int_{\widetilde{k}_i}^{\theta_i}\left(\alpha_0\left(1-\gamma\right)\left(s+\theta_j+\theta_0-k\right)-\gamma\alpha_i\left(s-\widetilde{k}_i\right)\right)dsdF^1dF^2\\
&\geq 2\int_{A_i}\int_{\widetilde{k}_i}^{\theta_i}\left(\alpha_0\left(1-\gamma\right)\left(s+\theta_0-k\right)-\gamma\alpha_i\left(s-\widetilde{k}_i\right)\right)dsdF^1dF^2,
\end{aligned}
$$

which leads to the estimate

$$
2\int_{A_i}\int_{\widetilde{k}_i}^{\theta_i}\left(\alpha_0\left(1-\gamma\right)\left(s+\theta_0-k\right)-\gamma\alpha_i\left(s-\widetilde{k}_i\right)\right)dsdF^1dF^2\geq\delta_i\left(\bar{\theta}_i-\widetilde{k}_i\right)E\left[\theta_i-\widetilde{k}_i|A_i\right]\Pr\left[A_i\right],
$$

where $\Pr\left[A_i\right]=(1-F^i(\widetilde{k}_i))F^j(\hat{k}-\bar{\theta}_i)$ and the last inequality follows from the definition of $\widetilde{k}_i$ and that, for $1>\gamma>\frac{\alpha_0}{\alpha_0+\alpha_i}$,

$$
\alpha_0\left(1-\gamma\right)\left(s+\theta_0-k\right)-\gamma\alpha_i\left(s-\widetilde{k}_i\right)\geq\alpha_0\left(1-\gamma\right)\left(\bar{\theta}_i+\theta_0-k\right)-\gamma\alpha_i\left(\bar{\theta}_i-\widetilde{k}_i\right)=\left(\bar{\theta}_i-\widetilde{k}_i\right)\frac{\delta_i}{2}>0.
$$

Second, for $(\theta_1,\theta_2)\in B$ both tasks obtain less than their needs both under $\mathbf{M}_D$ and **I**.[29] Thus

$$
\begin{aligned}
&\left(J^{M_D}_{CES}(\gamma)-J^{I}_{CES}(\gamma)\right)\Big|_{(\theta_1,\theta_2)\in B}\\
&\qquad=\;-\int_B\left(\sum\nolimits_{i=1}^2\gamma\alpha_i\left(\hat{k}-\theta_1-\theta_2\right)^2-\gamma\frac{\alpha_1\alpha_2}{\alpha_1+\alpha_2}\left(\theta_1+\theta_2-\hat{k}\right)^2\right)dF^1dF^2\\
&\qquad=\;-\gamma\left(\bar{\theta}_1+\bar{\theta}_2-\hat{k}\right)^2\left(\alpha_1+\alpha_1-\frac{\alpha_1\alpha_1}{\alpha_1+\alpha_2}\right)\Pr\left[B\right].
\end{aligned}
$$

---

[29] This can be seen by observing that mechanism $\mathbf{M}_D$ restricts the resources to both tasks only if mechanism **I** restricts total resources to system I. This follows by the observation that $\widetilde{k}_1+\widetilde{k}_2>\hat{k}$ for γ close to 1. Indeed, since

$$
\frac{\partial\widetilde{k}_i}{\partial\gamma}\bigg|_{\gamma=1}=\frac{\alpha_0(\bar{\theta}_i+\theta_0-k)}{\alpha_i\left(1+\frac{\delta_i}{2\alpha_i}\right)}>0,
$$

comparing (26) and (27) we have that

$$
\sum\nolimits_{i=1}^2\frac{\partial\widetilde{k}_i}{\partial\gamma}\bigg|_{\gamma=1}=\sum\nolimits_{i=1}^2\frac{\alpha_0(\bar{\theta}_i+\theta_0-k)}{\alpha_i\left(1+\frac{\delta_i}{2\alpha_i}\right)}<\sum\nolimits_{i=1}^2\frac{\alpha_0(\bar{\theta}_i+\theta_0-k)}{\alpha_i}<\sum\nolimits_{i=1}^2\frac{\alpha_0(\overline{z}+\theta_0-k)}{\alpha_i}=\frac{1}{3}\frac{\partial\hat{k}}{\partial\gamma}\bigg|_{\gamma=1^-}
$$

Therefore $\frac{\partial}{\partial\gamma}\left(\sum_{i=1}^2\widetilde{k}_i-\hat{k}\right)\Big|_{\gamma=1}<0$ implying that for γ in a neighborhood of $\gamma=1$, $\widetilde{k}_1+\widetilde{k}_2>\hat{k}$.

Using both estimates we obtain the lower bound

$$J^{M_D}_{CES}(\gamma) - J^{I}_{CES}(\gamma) \geq \left(\sum\nolimits_{i=1}^{2}(1 - F^{i}(\widetilde{k}_i)) + \Pr\left[B\right]\right)\left(\bar{\theta}_1 + \bar{\theta}_2 - \hat{k}\right)^2 \left(C_1(\gamma) - C_2(\gamma)\right), \tag{28}$$

where

$$\begin{aligned} C_1(\gamma) &= \frac{\sum_{i=1}^{2}\delta_i\left(\bar{\theta}_i - \widetilde{k}_i\right)E\left[\theta_i - \widetilde{k}_i | A_i\right]\Pr\left[A_i\right]}{\left(\sum_{i=1}^{2}(1 - F^{i}(\widetilde{k}_i)) + \Pr\left[B\right]\right)\left(\bar{\theta}_1 + \bar{\theta}_2 - \hat{k}\right)^2}, \\ C_2(\gamma) &= \left(\alpha_1 + \alpha_1 - \frac{\alpha_1\alpha_1}{\alpha_1 + \alpha_2}\right)\frac{\gamma\Pr\left[B\right]}{\sum_{i=1}^{2}(1 - F^{i}(\widetilde{k}_i)) + \Pr\left[B\right]}. \end{aligned}$$

The following limits follow by application of L'Hôpital's rule

$$\lim_{\gamma=1^-}\frac{\bar{\theta}_i - \widetilde{k}_i}{\bar{\theta}_1 + \bar{\theta}_2 - \hat{k}} = \frac{-\left.\frac{d\widetilde{k}_i}{d\gamma}\right|_{\gamma=1^-}}{-\left.\frac{d\hat{k}}{d\gamma}\right|_{\gamma=1^-}} > 0, \quad \lim_{\gamma=1^-}\frac{\bar{\theta}_i - E\left[\theta_i|A_i\right]}{\bar{\theta}_1 + \bar{\theta}_2 - \hat{k}} = \frac{-\left.\frac{dE[\theta_i|A_i]}{d\gamma}\right|_{\gamma=1^-}}{-\left.\frac{d\hat{k}}{d\gamma}\right|_{\gamma=1^-}} > 0,$$

$$\lim_{\gamma=1^-}\frac{\Pr\left[B\right]}{\sum_{i=1}^{2}(1 - F^{i}(\widetilde{k}_i)) + \Pr\left[B\right]} = \frac{g\left(\bar{\theta}_1 + \bar{\theta}_2\right)\left.\frac{d\hat{k}}{d\gamma}\right|_{\gamma=1^-}}{-\sum_{i=1}^{2}f^{i}(\bar{\theta}_i)\left.\frac{d\widetilde{k}_i}{d\gamma}\right|_{\gamma=1^-} - g\left(\bar{\theta}_1 + \bar{\theta}_2\right)\left.\frac{d\hat{k}}{d\gamma}\right|_{\gamma=1^-}} = 0,$$

$$\lim_{\gamma=1^-}\frac{\sum_{i=1}^{2}\Pr\left[A_i\right]}{\sum_{i=1}^{2}(1 - F^{i}(\widetilde{k}_i)) + \Pr\left[B\right]} = 1.$$

which imply that

$$\lim_{\gamma\to 1^-}C_1(\gamma) > 0 \quad \text{and} \quad \lim_{\gamma\to 1^-}C_2(\gamma) = 0.$$

Therefore there exists a neighborhood M of $\gamma = 1$ where $\gamma > \max_i\left[\frac{\alpha_0}{\alpha_0+\alpha_i}\right]$ and $J^{M}_{CES}(\gamma) \geq J^{M_D}_{CES}(\gamma) > J^{I}_{CES}(\gamma)$ for $\gamma \in M, \gamma \neq 1$.

(b) **Relative performance of Integration vs. Specialization as** $\gamma \to 0$. Given $\theta_0 > k$, when $\gamma = 0$ the CES assigns all resources to system 0 both under aggregation and specialization implying $J^{I}_{CES}(0) = J^{M}_{CES}(0)$. By the Milgrom-Segal Envelope Theorem (Milgrom and Segal, 2002) we have

$$\left.\frac{\partial J^{I}_{CES}(\gamma)}{\partial\gamma}\right|_{\gamma=0^+} = -\frac{\alpha_1\alpha_2}{\alpha_1+\alpha_2}E\left[(\theta_1+\theta_2)^2\right], \quad \left.\frac{\partial J^{M}_{CES}(\gamma)}{\partial\gamma}\right|_{\gamma=0^+} = -\alpha_1 E\left[\theta_1^2\right] - \alpha_2 E\left[\theta_2^2\right].$$

Then

$$\left.\frac{\partial\left(J^{I}_{CES}(\gamma) - J^{M}_{CES}(\gamma)\right)}{\partial\gamma}\right|_{\gamma=0^+} = \frac{1}{\alpha_1+\alpha_2}E\left[(\alpha_1\theta_1 - \alpha_2\theta_2)^2\right] > 0.$$

where the strict inequality follows from independence of θ_1 and θ_2. Therefore, there exists a neighborhood N of $\gamma = 0$ where $J^I_{CES}(\gamma) > J^M_{CES}(\gamma)$ for $\gamma \in N, \gamma \neq 0$.

K. Proof of Corollary 5

In the text.

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