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FEASIBILITY STUDY OF VARIANCE REDUCTION IN THE THUNDER CAMPAIGN-LEVEL MODEL

## THESIS

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# FEASIBILITY STUDY OF VARIANCE REDUCTION IN THE THUNDER CAMPAIGN-LEVEL MODEL 

THESIS

Presented to the Faculty<br>Department of Operational Sciences<br>Graduate School of Engineering and Management<br>Air Force Institute of Technology<br>Air University<br>Air Education and Training Command<br>In Partial Fulfillment of the Requirements for the<br>Degree of Master of Science in Operations Research

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March 2005

# FEASIBILITY STUDY OF VARIANCE REDUCTION IN THE THUNDER CAMPAIGN-LEVEL MODEL 

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#### Abstract

As an Air Force Chief of Staff endorsed topic, Air Force Studies and Analyses Agency (AFSAA) requested an effective and efficient way to reduce the variance in analysis results from THUNDER. THUNDER is a large-scale discrete event simulation of campaign-level military operations and is used to examine issues involving the utility and effectiveness of air and space power in a theater-level, joint warfare context. Given the large number of stochastic components within THUNDER, results are produced with highly variable measures of effectiveness (MOEs), causing difficulties in evaluating alternative force structures, weapon systems, etc.

This work responds to AFSAA's request by examining the application of Common Random Numbers (CRN), Antithetic Variates (AV), Control Variates (CV), and a combination of AVs and CVs and a combination of CRNs and CVs. The difference between the normal and variance reduction technique halfwidths for $95 \%$ confidence intervals were examined. Analysis of the correlation between MOEs and the random inputs in the CV technique gave insight into the workings of THUNDER. A new, state of the art combined multiple recursive generator was incorporated into THUNDER to synchronize the random inputs for CRN and AV. The result is methodology for implementing all four variance reduction techniques.


For My Wife and Son

## Acknowledgments

I have dedicated this thesis to my wife and my son. For without them I surely would be lost. My wife keeps me on track and nourishes me with encouragement and love. My son sustains my youth and enthusiasm and is a constant reminder of what life is all about. To them I am eternally grateful.

This thesis could not have been completed without the effort from a number people to whom I would like to express my sincere appreciation. First, I like to thank my advisor Dr. Kenneth Bauer for your words of encouragement and thoughtful insight. Next, I would like to thank my reader Dr. John O. Miller for keeping me working and always expecting more. I would also like to thank Dr. Phillip Amburn and Jamie Moyers from the ASC MSRC for their tireless effort in getting this version of THUNDER running at the ASC MSRC and setting up the divide and conquer web interface to work with the new version of THUNDER. Another thanks goes out to CACI for their part in setting up the new THUNDER on high performance computing resources at the ASC MSRC. Last but definitely not least; I would like to thank all of my friends at AFIT with whom I have shared the highs and lows with here at AFIT. Thank you for just being there.

Earl M. Bednar

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## Acronyms and Useful Definitions

| Acronym |  | Definition |
| :--- | :--- | :--- |
| AFIT |  | Air Force Institute of Technology |
| AFSAA |  | Air Force Systems and Analysis Agency |
| AFSAT |  | Air Force Standard Analysis Toolkit |
| ASC |  | Aeronautical Systems Center |
| AV | Antithetic Variate |  |
| AW | Air War |  |
| CC | Ground Combat Cycle |  |
| CMGR | Congruential Multiple Recursive Generator |  |
| CRN | Common Random Number |  |
| CV | Control Variate |  |
| DAC | Divide and Conquer |  |
| DoD | Department of Defense |  |
| FLOT | Forward Line of Troops |  |
| FPS | Flown per Planned Sortie |  |
| GUI | Graphical User Interface |  |
| HPC | High Performance Computing |  |
| LCG | Linear Congruential Generator |  |
| LPS | Losses per Sortie |  |
| MATLAB | Matrix Laboratory |  |
| MCA | All Pairwise Multiple Comparisons |  |
| AP |  |  |


| Acronym | $\underline{\text { Definition }}$ |
| :--- | :--- |
| MCB | Multiple Comparisons with the Best |
| MCC | Multiple Comparisons with a Control |
| ME | Refers to the Middle East THUNDER scenario |
| MOE | Measure of Effectiveness |
| MRG32k3a | The name of the congruential multiple recursive generator used in <br> L'Ecuyers random number generation package |
| MSRC | Major Shared Resource Center |
| RNG | Random Number Generator |
| SMG | Square Miles Gained |
| VRT | Variance Reduction Technique |

# FEASIBILITY STUDY OF VARIANCE REDUCTION IN THE THUNDER CAMPAIGN-LEVEL MODEL 

## 1. Introduction

### 1.1. Background

Over the years, the Department of Defense (DoD) has been tasked to continue to be the leader in military strength and technology, with less and less funding. One way the DoD has done this is through the use of stochastic combat simulation. Stochastic combat simulation is a way to model how the input parameters affect a specified measure of effectiveness (MOE) in a combat situation. To do this, an analyst uses simulation software to construct a combat simulation by assigning probabilities to weapons systems and logical flow in order to determine the interaction of the simulated combat forces. The results from a stochastic combat simulation are point estimates because they rely on a series of pseudo-random numbers. These pseudo-random numbers are used to model the randomness of a system. This randomness can greatly affect the simulation results, leading to a range of estimates.

There are many known ways to reduce the variance in the MOEs. The most common method is to increase the number of replications completed. This method can prove to be inefficient due to the size of the simulation. Even with the speed of today's computers, some of the larger simulations can take hours or even days to complete one replication. Therefore, an increase in replications may be unacceptable. Other methods
of reducing the variance are called variance reduction techniques (VRTs). These techniques use knowledge of the workings of computer simulations and mathematical techniques to reduce the variance without increasing the number of replications (20:581).

The DoD uses a variety of large, stochastic combat simulations to conduct various analyses of interest. These interests vary from acquisition to planning to training. One of the large, stochastic combat simulations used by the Air Force (AF) is THUNDER. THUNDER is a stochastic, two-sided, analytical simulation of campaign-level military operations developed in the 1980s under the guidance of the Air Force Studies and Analyses Agency (AFSAA). The simulation was designed and built expressly to examine issues involving the utility and effectiveness of air and space power in a theaterlevel, joint warfare context.

### 1.2. Problem Statement

Given the large number of stochastic components within THUNDER, the model often produces MOEs with significant variance. This variance often causes difficulties in evaluating alternative force structures, weapon systems, etc. With time sensitive analysis and long replication times, AFSAA is looking for an effective and efficient way to reduce the variance in the results from the THUNDER campaign-level model.

### 1.3. Research Objective

The objective of this research will be to study the use of variance reduction techniques on THUNDER. The study of existing variance reduction techniques will determine which, if any, are effective and efficient.

### 1.4. Research Focus

This research examines the use of several classic VRTs. The primary focus of this research identified potential control variates (CV), within the model, that can be used to reduce the variance in the output. Then using these potential control variates, determine how much each potential control variate influences the MOE variance. The areas are also checked to find out if there is any other information that can be gleaned from the relationship between the identified control and the simulation MOE.

A secondary focus of this research examines the use of two other classic VRTs. The second VRT applied is common random numbers (CRN). CRN is probably the second most popular way of reducing variance, following increasing the number of replications. It is primarily used when trying to identify differences between simulation configurations. The basic idea is to reduce noise, generated by the random inputs, in the variation of the system. The third VRT applied is antithetic variates (AV). AV attempts balance out any radically low or high random input. Also, the application of combining AV with CV is reviewed.

### 1.5. Methodology

In the control variate technique, the primary focus is in the generation of random variates throughout a simulation. Initially, the random variate averages are generated and stored. The differences in the simulated mean and expected value are regressed against the simulation MOEs. This identifies which random variates are influential to the selected MOEs.

To use the CRN technique, the pseudo-random number calls need to have their own random number stream. THUNDER already has a way to assign different random
number streams. The problem with THUNDERs current RNG is that there are only 10 streams to choose from and 245 different random number calls. Therefore, a new random number generator is needed to allow for all the random number calls to have its own stream. In this research, the random number generator suggested by Pierre L'Ecuyer, Richard Simard, E. Jack Chen, and W. David Kelton (18) was added to THUNDER.

### 1.6. Aeronautical Systems Center Major Shared Resource Center

Simulation replications for this research were performed with cooperation with the Aeronautical Systems Center (ASC) Majro Shared Resource Center (MSRC). The ASC MSRC is a computational science facility. It supports various DoD research initiatives including development, test, and evaluation. As part of the DoD's High Performance Computing (HPC) Modernization Program, the ASC MSRC uses high performance computing research and visualization. Access to the ASC MSRC HPC resources was facilitated by use of the Divide and Conquer (DAC) system. The DAC system is fronted by a web interface that assists in configuring experiments and has direct access to HPC resources.

### 1.7. Assumptions and Limitations

The major limitation of this research occurs due to the classification of the THUNDER model databases. Since most THUNDER model databases represent a real battle plan, implement new military technology, or contain specific data on current military systems, most all THUNDER model databases are classified to some level. Therefore, the only unclassified model databases available are the three notional model databases distributed with THUNDER. The model database used in this research is the

Middle East (ME) scenario. Assuming the VRT affects THUNDER and does not rely on the scenario, the VRT can be used on any other THUNDER scenario study.

### 1.8. Overview

There are five chapters in this research. Chapter 1, the Introduction, contains background information, problem statement, research objective, a synopsis of the methodology, and the assumptions and limitations of the research. Chapter 2, the Literature Review, contains descriptions of THUNDER, other THUNDER work, CVs, CRNs, AVs, other VRTs, and analysis techniques. Chapter 3, the Methodology, discusses how CVs, CRNs, and AVs are applied in THUNDER. Chapter 4, Results and Analysis, presents the output of the selected ME scenario MOEs and the results from the application of CVs, CRNs, and AVs. Chapter 5, Conclusions, is a look at the analysis done in the previous chapter with recommendations for future work in the application of VRTs and THUNDER.

## 2. Literature Review

This chapter contains background information on software, models, techniques and services used in completing this research. It begins with a discussion on combat simulations, specializing in the THUNDER campaign level combat simulation. It continues with a description of THUNDER's computer language, SIMSCRIPT II.5. Next, there is a lengthy section on variance reduction techniques. This section reviews Control Variates, Common Random Numbers, and Antithetic Variates in depth, while giving a brief overview of other techniques. To close this section, there is a review of other variance reduction studies. Next, the random number generator added to THUNDER for use with CRN is described. Following this, is a short description of the ASC MSRC's Divide and Conquer system used to replicate the runs. This chapter ends with an overview of variance and confidence intervals.

### 2.1. Combat Simulation

"Simulation of large-scale combat models contribute useful insights for many military decision problems."(14:1-1) There are many reasons why a system is modeled by computer simulation. For instance, a system may be too difficult or expensive to alter, the system may not exist, or even unacceptable to simulate in real life. In combat simulations, the latter is probably the case. It would be unethical to start real combat merely to test a new weapon system. The goal in the design of combat models is to represent warfare as accurately as possible, because results from combat simulation can guide national defense and other policy decisions.

There are many different levels of combat modeling. They range from engineering to campaign level simulations, see Figure 2.1 (26). A campaign level simulation encompasses a whole campaign and covers many different elements. The sheer volume of entities would make it difficult to simulate each entity interaction. Therefore, they are performed with low resolution and aggregation. This aggregation means that each entity is not simulated but is replaced by probabilities and percentages. This allows a large campaign level simulation to run in an acceptable time frame, at the cost of losing output resolution.


Figure 2.1: Combat Modeling Hierarchy (26)

### 2.2. THUNDER

This research is a study on THUNDER, a large stochastic, two-sided, constructive simulation that has been operational since 1986. It was developed to examine the utility and effectiveness of air and space power in at a campaign-level and models joint military operations at the campaign-level. The Air Force Studies and Analyses Agency (AFSAA)
sponsored the development of THUNDER. The following material is obtained from the THUNDER web pages (13) and Volume One of the THUNDER documentation (35). THUNDER is designed for analytical flexibility, accessibility, and ease of use. It operates in a UNIX environment and is written in SIMSCRIPT II.5, a simulation programming language. SIMSCRIPT II. 5 is ideal for the creation of large scale, discreteevent simulations. To aid the analyst, the distribution of THUNDER comes with indepth documentation and source code, allowing modification at the programming level. The source code of THUNDER, alone, consists of around 360,000 lines of codes spread across over 1,500 routines.

THUNDER operates at multiple levels of detail. It models air operations at a high level of detail, with the ability to simulate 27 different air missions. While it can simulate air operation at a high level of detail, it models ground operations at a medium level of detail, focusing on ground combat, logistics, terrain, C3I, and mobility.

To assist the user and speed up replications, THUNDER automatically generates its own Air Tasking Orders (ATOs) and Intelligence Tasking Orders (ITOs), based on theater-level apportionment and target priorities. It also uses a deterministic time stepped ground operations model based on the Center for Army Analysis Concept Evaluation Model (CEM) and its Attrition Calibration Methodology (ATCAL).

THUNDER can be run in two modes: analytical and wargame. The analytical mode focuses on the contribution of systems, capabilities, forces and employment concepts. It is run in a batch mode and delivers results that can be identified by replication or as a mean across replications with confidence intervals. When run in batch mode, THUNDER starts the following random number stream where the previous stream
stops. The wargame mode supports a user interaction in a game like atmosphere, where a player dynamically influences the outcome of a run.

As a member of the Air Force Standard Analysis Toolkit (AFSAT), THUNDER supports senior decision makers (Figure 2.2) in matters of policy, acquisition and operations. Typically, these customers require precise and timely analysis. The analysis applications include: exploration of readiness, modernization, force structure, and sustainability issues; comparison of alternative courses of action (ACA); assessments of evolving capabilities, alternative strategies and potential operational concepts; and the facilitation of senior staff training through wargaming.

| THUNDER user base |  |  |  |
| :---: | :---: | :---: | :---: |
| US Air Force | Joint/ Other Services | Allied Militaries | Industry |
| AFSAAISAAC AFSAAISAAI | Joint Strike Fighter Program Office | French Air Force | Boeing |
| ACCIXP-SAS | Naval Air Warfare Center [Aircraft Division] | RAF Air Warfare College | Boeing North American |
| HQ AFIXPY |  | RAF Air Warfare Center | Lockheed Martin I Atlanta |
| ASCIEN |  | ROK Air Force | Lockheed Martin I Fort Worth |
| Air Force Wargaming Institute |  |  | Lockheed Martin I Dallas |
| ESC/DIS |  |  | Northrop Grumman Corporation |
| SMCIXRE |  |  | Analysis International Ltd. |
| SWCIAEWF |  |  | SPA |
| AFIT/ENS |  |  | Veridian |

Figure 2.2: THUNDER User Base (13)

### 2.2.1. Input.

THUNDER is a data driven simulation. Therefore, the individual model parameters are held in a data structure or database and can be changed. THUNDER uses 99 text files as a model database. These files contain all the information relating to the
entities, map, and course of action. THUNDER also allows the user to use toggles to tailor how a model is run. The use of the databases allows for sharing of data and minimizing storage space. It also aids in portability between systems and even sharing between users.

### 2.2.2. Output.

THUNDER comes with three types of post-processors to handle the hundreds of different MOEs available. The first is a comprehensive suite of text post-processors. They provide the user with a means to extract information from a simulation. THUNDER uses an input file to toggle, which postprocessors are run and which output files are requested. An example of one of the output files is in Appendix (A). The second type is a graphical post processor that allows the user to step through a single replication graphically. It contains a situation map and it depicts the locations of military units for both sides over half day increments. Third, THUNDER comes with a post processor that creates user specified graphs.

THUNDER contains post processors that can produce many different MOEs. To obtain the data for a MOE, the user selects one or more reports of interest to produce. Three reports were chosen for this project. The three reports are CC-4 (FLOT Movement), AW-3 (Aircraft Loss Summary), and AW-5 (Sortie Generation). All three are contained in a single output file named reports.std located in the REPORTS subdirectory. When running multiple replications of a thunder model, there will be a number following the reports name. The number references the replication number from which the file was created. The reports.std file combines all three reports and organizes
them in time order, then report name order, and then order of side. Please refer to Appendix (A). It contains the output from the entire first day of a replication.

The CC-4 report is a Ground Combat Cycle report. It displays FLOT movement in square miles gained or lost by command and FLOT location in latitudes and longitudes by 12-hour increments. The AW-3 report is an Air War report. It contains losses and sorties by aircraft for each day. The AW-5 report also is an Air War report. It contains planned and flown sortie data for each mission type by base and squadron for each day.

### 2.2.3. Previous THUNDER Studies.

THUNDER is a large stochastic combat simulation. The size and complexity of THUNDER has made it a prime simulation for study. Over the years there have been efforts to analyze the inner workings of THUNDER. Much of this work has been done here at the Air Force Institute of Technology (AFIT). Below is a sampling of previous THUNDER research at AFIT.

Major David A. Davies performed a sensitivity analysis to command and control (C2) inputs in THUNDER. He employed design of experiments (DOE) to identify statistically significant variables to specified outputs. Then response surface methodology (RSM) was used to generate a response surface of the relationships between the input and output variables. Major Davies also demonstrated the ASC MSRCs capability to perform parallel processing, completing 1,560 THUNDER runs for the analysis. Normally this would have taken around 3 months, but it was accomplished at the ASC MSRC in around 3 days using parallel processing. (5)

Captain Michael R. Farmer studied how to quickly analyze the effects of changing force structure with THUNDER. The modifications Capt Farmer made
consisted of limiting the amount of primary tactical aircraft. Capt Farmer used RSM to create metamodels of THUNDER, which in turn could be used to provide a quick answer to a change in force structure. He also used factor analysis (FA) to reduce the dimensionality of data sets and determine relationships between MOEs. (8)

Captain Steven L. Forsythe applied RSM to a THUNDER scenario in order to optimize the air apportionment. Captain Forsythe created a response surface and used a steepest-gradient search to find the optimal aircraft apportionment for MOEs such as FLOT movement and attrition. In addition, he demonstrated how RSM could provide useful information as to how the algorithms used by TAC THUNDER, interact to simulate the combat environment. (9)

Captain Brian M. Godfrey compared five Air Expeditionary Force (AEF) packages. Captain Godfrey used THUNDER to obtain most of the MOEs used in the study. The study results in a methodology for modeling AEF packages using THUNDER and a methodology to analyze the results. (11)

Major James B. Grier developed a tool to capture the cost and capabilities of alternative force structures. The tool uses FA and RSM to link monetary cost to MOEs. USAF/XPY directed Major Grier to develop his tool using THUNDER. His research focused on the Southwest Asia scenario developed by HQ ASC/XR. (12)

Captain Francine N. Nelson conducted a validation assessment of THUNDER 6.5's Intelligence, Surveillance, and Reconnaissance (ISR) Module. This study provides an overview of the ISR process, level of aggregation, possible problem areas, and basis for ISR improvement within THUNDER. This study also presents a sensitivity analysis
of the ISR parameters relating to the Quality, Quantity, and Timeliness of ISR. DOE and linear regression were used to develop a response surface for the sensitivity analysis. (27)

Captain Timothy S. Webb examined the overall model variability, output interrelationships, and sensitivity to input parameters in THUNDER. The examination of the model variability focused on the replication requirements to obtain specified measures of outcome (MOOs). The application of principal component analysis (PCA) and FA were used to examine output interrelationships. The input parameters were tested for significance and consistency. (42)

### 2.3. SIMSCRIPT II. 5

To understand the inner workings of a simulation, one must learn the language it is programmed in. THUNDER is programmed in CACIs SIMSCRIPT II.5. According to the SIMSCRIPT II. 5 description web page at CACI, SIMSCRIPT II. 5 is an easy-to-use and powerful simulation language with English-like syntax (4). The English-like syntax facilitates a more intuitive understanding of the simulation.

As a simulation language, it provides a timing mechanism and concurrent process management in addition to all the standard features of a general-purpose high-level programming language. It has preprogrammed discrete event simulation features like entities, attributes, sets, events, processes and resources. These added features are key elements of an efficient and effective simulation. SIMSCRIPT II. 5 also has built in graphical capabilities, allows models to be stored as a database, is portable across a variety of platforms, and can readily interface between selected other programming languages.

There are three basic parts to the structure of a SIMSCRIPT II. 5 program. They are, the main program, routines, and the preamble. The main program is the essential part. It consists of either the whole program or the starting and ending part of a program that consists of calls to one or more subroutines. The main program is where SIMSCRIPT II. 5 looks for the start of the program. Routines are functions, subroutines, events or processes. They are generally used to compartmentalize the simulation code for better organization and ease of understanding the program flow, and are activated by either the main program or other routines. The preamble is where global variables are declared and constants are defined. THUNDER was developed with its own nomenclature; therefore its preamble is like a Rosetta stone. It provides the key to translate words and phrases in THUNDERs code back into a standard programming language.

SIMSCRIPT II. 5 is mainly used for discrete-event and combined discreteevent/continuous simulation models. It is applied world wide in building large-scale simulations and modeling applications. Some typical SIMSCRIPT II. 5 applications are seen in Telecommunications, Network analysis, Transportation, Manufacturing, Inventory control, Health care, Military operations, wargaming and logistics planning. Some organizations that use SIMSCRIPT II. 5 are the Federal Aviation Administration, Saudi Aramco, Raytheon, Rolls Royce, the U.S. Army and the U.S. Air Force. (4)

### 2.4. Variance Reduction Techniques

A stochastic simulation, such as THUNDER, relies on input that is random and therefore produces output that is also random. This means, that since there are inputs that vary randomly from replication to replication, there will also be random variation in the
output from replication to replication. Often this variation is so large it obscures the estimates of the models MOEs. Therefore analysts have developed techniques to reduce this random variation in the output. These techniques are called variance reduction techniques (VRTs). There are many different types of VRTs and the effectiveness of each technique is dependent on the particular model and application. Three common VRTs are Control Variates (CV), Common Random Numbers (CRN), and Antithetic Variates. CV and AV are single model techniques, meaning they are applied to a single configuration of a simulation, while CRN is a multiple model technique, meaning it is applied when two or more configurations are compared. These VRTs are explained in depth below and then are followed by a quick synopsis of some other VRTs.

### 2.4.1. Control Variates.

In essence, control variates attempt to take advantage of any correlation between random variates in a simulation and the selected output. It is best when talking about control variates to start out with a single response and a single control. The same theory can then be extended to multiple controls. The remainder of this section is derived from notes taken from Dr. Bauer. (1:64-70.5)

Consider the case were there is only a single control. Now, assume there is a mean response of interest from the simulation called $\mu_{Y}$ for which $Y$ is an estimator. Also, assume there is another output variable, $X$, that is correlated with the $Y$ response and has an expected value $\mu_{X}$ that is known. Since $X$ is correlated with the $Y$ variable, it is known as the control variable. Now consider the controlled estimator $Y(b)$ given in equation (2.1) where $b$ is a constant.

$$
\begin{equation*}
Y(b)=Y-b\left(X-\mu_{X}\right) \tag{2.1}
\end{equation*}
$$

Note that $Y(b)$ is an unbiased estimator of $\mu_{Y}$ by equation (2.2).

$$
\begin{align*}
& E[Y(b)]=E[Y]-E\left[b\left(X-\mu_{X}\right)\right]  \tag{2.2}\\
& E[Y(b)]=\mu_{Y}-b\left(\mu_{X}-\mu_{X}\right)=\mu_{Y} \tag{2.3}
\end{align*}
$$

The variance of $Y(b)$ is given in equation (2.4).

$$
\begin{equation*}
\operatorname{Var}(Y(b))=\operatorname{Var}(Y)+b^{2} \operatorname{Var}(X)-2 b \operatorname{Cov}(Y, X) \tag{2.4}
\end{equation*}
$$

With a little manipulation of equation (2.4) it can be shown that the variance of $Y(b)$ is smaller than the variance of $Y$ if equation (2.5) holds.

$$
\begin{equation*}
2 b \operatorname{Cov}(Y, X)>b^{2} \operatorname{Var}(X) \tag{2.5}
\end{equation*}
$$

In observing equation (2.5), it is apparent that if the variables $X$ and $Y$ are independent, then the $\operatorname{Cov}(X, Y)=0$ and it follows that there can be no reduction in variance of $Y$.

To find the value of $b$ that minimizes equation (2.4), the derivative with respect to $b$ is found.

$$
\begin{equation*}
\frac{\partial \operatorname{Var}(Y(b))}{\partial b}=2 b \operatorname{Var}(X)-2 \operatorname{Cov}(Y, X) \tag{2.6}
\end{equation*}
$$

From equation (2.6) the minimum point is found by setting the derivative to zero. Equation (2.7) is the candidate for $b$ that minimizes equation (2.4) thus labeled $\beta$.

$$
\begin{equation*}
\beta=\frac{\operatorname{Cov}(Y, X)}{\operatorname{Var}(X)} \tag{2.7}
\end{equation*}
$$

To verify this is a minimum, the second derivative is found.

$$
\begin{equation*}
\frac{\partial^{2} \operatorname{Var}(Y(b))}{\partial b^{2}}=2 \operatorname{Var}(X) \tag{2.8}
\end{equation*}
$$

Since the variance is always positive, then equation (2.7) is the value for $b$ that minimizes equation (2.4). Combining equation (2.4), equation (2.7), and using some simple algebra yields equation (2.9).

$$
\begin{equation*}
\operatorname{Var}(Y(\beta))=\left(1-\rho_{X Y}^{2}\right) \operatorname{Var}(Y) \tag{2.9}
\end{equation*}
$$

Where $\rho_{X Y}$ is the correlation coefficient between X and $\mathrm{Y} . \operatorname{Var}(Y(\beta))$ is the minimum variance. The controlled observations (2.10) are averaged (2.11) to obtain an unbiased estimator of $\mu_{Y}$.

$$
\begin{gather*}
Y_{i}(\beta)=Y_{i}-\beta\left(X_{i}-\mu_{X}\right), i=1, \ldots, K  \tag{2.10}\\
\bar{Y}(\beta)=\frac{1}{K} \sum_{i=1}^{K} Y_{i}(\beta) \tag{2.11}
\end{gather*}
$$

where $K$ is the sample size.
Since the value of $\beta$ is unknown, it must be estimated. An estimate of $\beta$ can be found by substituting the sample quantities into equation (2.7). This solution is the least squares solution for $\beta$. The least squares solution is also the maximum likelihood solution with the assumption of joint normality between $X$ and $Y$. Equation (2.12) estimates $\beta$.

$$
\begin{equation*}
\hat{\beta}=\frac{\sum_{i=1}^{K}\left(Y_{i}-\bar{Y}\right)\left(X_{i}-\bar{X}\right)}{\sum_{i=1}^{K}\left(X_{i}-\bar{X}\right)^{2}} \tag{2.12}
\end{equation*}
$$

The point estimator of $\mu_{Y}$ is estimated by equation (2.13).

$$
\begin{equation*}
\hat{\mu}_{Y}(\hat{\beta})=\frac{1}{K} \sum_{i=1}^{K} Y_{i}(\hat{\beta}) \tag{2.13}
\end{equation*}
$$

Using regression theory an interval estimate for $\mu_{Y}$ can be obtained. By making the assumption of joint normality for $X$ and $Y$, the conditional distribution of $Y$ given $X$ will be normal by equation (2.14)

$$
\begin{equation*}
Y \mid X=x \sim \mathrm{~N}\left(\mu_{Y}+\beta\left(x-\mu_{X}\right), \sigma_{E}^{2}\right) \tag{2.14}
\end{equation*}
$$

where

$$
\begin{equation*}
\sigma_{\varepsilon}^{2}=\sigma_{Y}^{2}\left(1-\rho_{X Y}^{2}\right) \tag{2.15}
\end{equation*}
$$

and

$$
\begin{equation*}
\sigma_{Y}^{2}=\operatorname{Var}(Y) \tag{2.16}
\end{equation*}
$$

Since the values of the control variable $X$ and it mean $\mu_{X}$ are known, then it can be seen that the conditional mean of $Y$ given $X$ has two terms. The terms are broken into the parameter to be estimated $\mu_{Y}$ and a correction term. To get the $\mu_{Y}$ term, the corrections need to be subtracted out as in equation (2.10). From equation (2.14), equation (2.17) can be formed.

$$
\begin{equation*}
Y_{i}=\mu_{Y}+\beta\left(X_{i}-\mu_{X}\right)+\varepsilon_{i}, 1 \leq i \leq K \tag{2.17}
\end{equation*}
$$

where $\varepsilon_{i}$ are the residuals and are of the form in equation (2.18).

$$
\begin{equation*}
\varepsilon_{i} \sim \mathrm{~N}\left(0, \sigma_{\varepsilon}^{2}\right) \tag{2.18}
\end{equation*}
$$

Since the values of $\mu_{Y}$ and $\beta$ are unknown, the method of least squares can be applied to solve for them. $\mu_{Y}$ will be the intercept and is normally distributed as in equation (2.19).

$$
\begin{equation*}
\hat{\mu}_{Y}(\hat{\beta})_{i} \sim \mathrm{~N}\left(\mu_{Y}, \sigma_{\varepsilon}^{2} s_{11}\right) \tag{2.19}
\end{equation*}
$$

The value of $s_{11}$ in equation (2.19) is the upper left hand entry in the matrix $\left(D^{T} D\right)^{-1}$ where $D$ is of the form in equation (2.20).

$$
D=\left[\begin{array}{cc}
1 & X_{1}-\mu_{X}  \tag{2.20}\\
1 & X_{2}-\mu_{X} \\
1 & X_{3}-\mu_{X} \\
\vdots & \vdots \\
1 & X_{K}-\mu_{X}
\end{array}\right]
$$

To generate a confidence interval about $\hat{\mu}_{Y}(\hat{\beta}), \sigma_{\varepsilon}^{2}$ must be estimated. Since $\sigma_{\varepsilon}^{2}$ represents the variability in $Y$ given $X$, the formula for the residual mean square error is used.

$$
\begin{equation*}
\sigma_{\varepsilon}^{2}=\frac{\sum_{i=1}^{K}\left(Y_{i}-\hat{Y}_{i}\right)^{2}}{K-2} \tag{2.21}
\end{equation*}
$$

where

$$
\begin{equation*}
\hat{Y}_{i}(\hat{\beta})=\hat{\mu}_{Y}(\hat{\beta})+\hat{\beta}\left(X_{i}-\mu_{X}\right), 1 \leq i \leq K \tag{2.22}
\end{equation*}
$$

It can be seen, from the above equations, that

$$
\begin{equation*}
\frac{\hat{\mu}_{Y}(\hat{\beta})-\mu_{Y}}{\left[\frac{\sigma_{\varepsilon}^{2} s_{11}}{K-2}\right]^{\frac{1}{2}}} \sim t_{K-2} \tag{2.23}
\end{equation*}
$$

has a Student-t distribution with $K-2$ degrees of freedom. Therefore, the confidence interval for $\mu_{Y}$ is given by

$$
\begin{equation*}
\hat{\mu}_{Y} \pm t_{K-2,\left(1-\frac{\alpha}{2}\right)} \sqrt{\sigma_{\varepsilon}^{2} s_{11}} \tag{2.24}
\end{equation*}
$$

Now in simulations there are possibly more than one control for a response.
Therefore, equation (2.17) is modified to be

$$
\begin{equation*}
Y_{i}=\mu_{Y}+\sum_{j=1}^{Q} \beta_{j}\left(X_{j i}-\mu_{X_{j}}\right)+\varepsilon_{i}, 1 \leq i \leq K \tag{2.25}
\end{equation*}
$$

where,

$$
\begin{equation*}
Q \leq K-1 \tag{2.26}
\end{equation*}
$$

Therefore, equation (2.21) is reformed into

$$
\begin{equation*}
\sigma_{\varepsilon}^{2}=\frac{\sum_{i=1}^{K}\left(Y_{i}-\left(\hat{\mu}_{Y}+\sum_{j=1}^{Q} \hat{\beta}_{j}\left(x_{j}-\mu_{X_{j}}\right)\right)\right)^{2}}{K-Q-1} \tag{2.27}
\end{equation*}
$$

and $s_{11}$ is the upper left hand entry in the matrix $\left(D^{T} D\right)^{-1}$ where $D$ is of the form

$$
D=\left[\begin{array}{ccccc}
1 & X_{11}-\mu_{1 X} & X_{21}-\mu_{2 X} & \cdots & X_{Q 1}-\mu_{Q X}  \tag{2.28}\\
1 & X_{12}-\mu_{1 X} & X_{21}-\mu_{2 X} & \cdots & X_{Q 1}-\mu_{Q X} \\
1 & X_{13}-\mu_{1 X} & X_{21}-\mu_{2 X} & \cdots & X_{Q 1}-\mu_{Q X} \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
1 & X_{1 K}-\mu_{1 X} & X_{21}-\mu_{2 X} & \cdots & X_{Q 1}-\mu_{Q X}
\end{array}\right]
$$

Then the $100(1-\alpha) \%$ confidence interval in equation (2.24) becomes

$$
\begin{equation*}
\hat{\mu}_{Y} \pm t_{K-Q-1,\left(1-\frac{\alpha}{2}\right)} \sqrt{\sigma_{\varepsilon}^{2} s_{11}} \tag{2.29}
\end{equation*}
$$

An important factor in the proper use of the CV technique is the selection of controls. In selecting potential controls, there are two things to look for. The first is an input of a random number or random variate. The input can be anything from a draw from a random distribution to a conditional switch. The second item of importance is that the expected value of the random input is known. There are many ways to determine what the expected value is, depending on the random input parameters. A table of expected values for distributions in THUNDER is presented in Appendix (B).

After all the potential candidates for controls are identified, the average of all the realizations of each potential candidate must be calculated and output in addition to the MOE of interest for each replication. Since the correlation between the control candidates and the MOE is unknown, a stepwise regression must be performed to identify which control candidates are significantly correlated to the selected MOE.

### 2.4.2. Common Random Numbers.

Common Random Numbers (CRN), also known as correlated sampling, is a relatively intuitive VRT. The rationale behind CRN is to induce a positive correlation in the random inputs between two or more configurations of a study. The positive correlation will diminish the noise generated by the difference in random inputs and in turn reduce the variance in the difference between the outputs. To create the positive correlation, the two configurations should use the same random variable inputs. The following theory behind CRN was adapted from Law and Kelton. (20:582-598)

To understand CRNs, consider two alternative configurations of a single simulation, where $X_{1 j}$ and $X_{2 j}$ are values from the first and second configuration on the $j^{\text {th }}$ independent replication. Also consider, the estimate in question is equation (2.30).

$$
\begin{equation*}
\zeta=\mu_{1}-\mu_{2}=\mathrm{E}\left[X_{1 j}\right]-\mathrm{E}\left[X_{2 j}\right] \tag{2.30}
\end{equation*}
$$

Let there be $n$ replications of each system and let

$$
\begin{equation*}
Z_{j}=X_{1 j}-X_{2 j}, j=1,2, \ldots, n \tag{2.31}
\end{equation*}
$$

then

$$
\begin{equation*}
\mathrm{E}\left[Z_{j}\right]=\zeta \tag{2.32}
\end{equation*}
$$

Therefore, equation (2.33) is an unbiased estimator of $\zeta$.

$$
\begin{equation*}
\bar{Z}(n)=\frac{1}{n} \sum_{j=1}^{n} Z_{j} \tag{2.33}
\end{equation*}
$$

Since the $Z_{j}$ 's are independent, identically distributed (IID) random variables,

$$
\begin{equation*}
\operatorname{Var}(\bar{Z}(n))=\frac{\operatorname{Var}\left(Z_{j}\right)}{n}=\frac{\operatorname{Var}\left(X_{1 j}\right)+\operatorname{Var}\left(X_{2 j}\right)-2 \operatorname{Cov}\left(X_{1 j}, X_{2 j}\right)}{n} \tag{2.34}
\end{equation*}
$$

If $X_{1 j}$ and $X_{2 j}$ are independent, then

$$
\begin{equation*}
\operatorname{Cov}\left(X_{1 j}, X_{2 j}\right)=0 \tag{2.35}
\end{equation*}
$$

If then $X_{1 j}$ and $X_{2 j}$ are not independent and are positively correlated

$$
\begin{equation*}
\operatorname{Cov}\left(X_{1 j}, X_{2 j}\right)>0 \tag{2.36}
\end{equation*}
$$

then $\operatorname{Var}(\bar{Z}(n))$ will be reduced.
The key to the implementation of CRN is synchronization. The random number streams should match up between the two configurations as much as possible. This means, that a random number drawn should be used for the same purpose in both configurations. Ideally, all the random variate draws should be applied in exactly the same way for both configurations. To synchronize the simulations, some initial coding is involved. First, all of the points in the simulation where a random number or variate is drawn must be identified. Then, each point is then assigned its own random number stream.

A problem that may arise, when coding for CRN, is the sufficiency of the RNG. A primary concern should be how to keep track of all the different random number streams. Some large simulations like THUNDER could require hundreds of streams. Ideally, each of the streams should be independent and not overlap. This could be
difficult, because these simulations could have millions or even billions of random number draws. Therefore, a large RNG with built in functions that keep track of random number streams is beneficial.

### 2.4.3. Antithetic Variates.

Antithetic variate (AV) VRT is similar to CRN. As in CRN, correlation is induced between separate runs, but this time the object is to induce a negative correlation. The main difference is that instead of having two configurations of one system, there only needs to be one system. The following is an excerpt from Law and Kelton(20:598604).

The basic idea is to make pairs of runs of a model where if the first run observes a "small" observation, then the second run observes a "large" observation. Therefore, the two observations are negatively correlated. To obtain the negative correlation, complementary numbers are used. For example, if $U_{k}$ is a particular random number used for a particular purpose in the first run, then $1-U_{k}$ is used for the same particular purpose on the second run. The use of $1-U_{k}$ is valid, since $U \sim \mathrm{U}(0,1)$ then $1-U \sim \mathrm{U}(0,1)$ as well. Note that like CRN, synchronization is extremely important to the successful use of AV.

To understand the basis for AV, suppose there are $n$ pairs of runs in a simulation resulting in observations $\left(X_{1}^{(1)}, X_{1}^{(2)}\right), \ldots,\left(X_{n}^{(1)}, X_{n}^{(2)}\right)$, where $X_{j}^{(1)}$ is from the normal run of the $j^{\text {th }}$ pair, and $X_{j}^{(2)}$ is from the antithetic run of the $j^{\text {th }}$ pair. Both $X_{j}^{(1)}$ and $X_{j}^{(2)}$ are legitimate observations of the simulation, therefore

$$
\begin{equation*}
\mathrm{E}\left[X_{j}^{(1)}\right]=\mathrm{E}\left[X_{j}^{(2)}\right]=\mu \tag{2.37}
\end{equation*}
$$

Since each run is independent, each pair of runs is independent. Now let

$$
\begin{equation*}
X_{j}=\frac{\left(X_{n}^{(1)}+X_{n}^{(2)}\right)}{2} \tag{2.38}
\end{equation*}
$$

and let

$$
\begin{equation*}
\bar{X}(n)=\frac{1}{n} \sum_{j=1}^{n} X_{j} \tag{2.39}
\end{equation*}
$$

be the unbiased point estimator of

$$
\begin{equation*}
\mu=\mathrm{E}\left[X_{j}^{(l)}\right]=\mathrm{E}\left[X_{j}\right]=\mathrm{E}[\bar{X}(n)] \tag{2.40}
\end{equation*}
$$

Since the $X_{j}$ 's are IID,

$$
\begin{equation*}
\operatorname{Var}[\bar{X}(n)]=\frac{\operatorname{Var}\left[X_{j}\right]}{n}=\frac{\operatorname{Var}\left[X_{j}^{(1)}\right]+\operatorname{Var}\left[X_{j}^{(2)}\right]+2 \operatorname{Cov}\left[X_{j}^{(1)}, X_{j}^{(2)}\right]}{4 n} \tag{2.41}
\end{equation*}
$$

If a negative correlation could be induced between $X_{j}^{(1)}$ and $X_{j}^{(2)}$, then

$$
\begin{equation*}
\operatorname{Cov}\left[X_{j}^{(1)}, X_{j}^{(2)}\right]<0 \tag{2.42}
\end{equation*}
$$

which reduces $\operatorname{Var}[\bar{X}(n)]$.

### 2.4.4. Control Variates and Antithetic Variates.

Kwon and Tew (15) compared three methods of combining Control Variates and Antithetic Variates. The first method consists of using CV on a simulation while using AV only on the random components not used in the CV technique. The second method consists of using CV on a simulation while using AV only on the random components used in the CV technique. The third method consists of using CV on a simulation while using AV on all of the random components in the simulation. Kwon and Tew concluded
that under general assumptions, the combined methods yield superior performance. They also found that the third method outperformed the other two. Method three is the method applied to THUNDER for this research.

The basic application behind method three of Kwon and Tew is to obtain the average response and the mean of the control variates across $h$ pairs of replicates. The theory follows from Kwon and Tew (15). Four properties are derived.

1. Since the random inputs into the simulation are essentially independent, there is Homogeneity of response variances across replicates,

$$
\begin{equation*}
\operatorname{Var}\left[y_{i}\right]=\sigma_{y}^{2} \text { for } i=1,2, \ldots, 2 h \tag{2.43}
\end{equation*}
$$

2. Antithetic effect: there is homogeneity of response correlations across replicate pairs and independence of responses between replicate pairs.

$$
\operatorname{Cov}\left[y_{i}, y_{k}\right]=\left\{\begin{array}{cl}
-\rho \sigma_{y}^{2} & \text { for } i=k+1(k=1,3, \ldots, 2 h-1)  \tag{2.44}\\
0 & \text { otherwise }
\end{array}\right\}
$$

Note: the correlation across replicate pairs is a negative correlation. Thus, $\rho$ is positive.
3. Control Variate and Antithetic effect: there is homogeneity of response-control variates covariance across replicates, homogeneity of response-control variates covariance across replicate pairs, and independence of response-control variates on different replicate pairs.

$$
\operatorname{Cov}\left[y_{i}, c_{k}\right]=\left\{\begin{array}{ll}
\sigma_{y c} & \text { for } i=k=1,2, \ldots, 2 h  \tag{2.45}\\
\sigma_{y c}^{(1)} & \text { for } i=k+1 \text { or } k=i+1(\min \{i, k\}=1,3, \ldots, 2 h-1) \\
0_{(s \times 1)} & \text { otherwise }
\end{array}\right\}
$$

4. Antithetic effect also gives us homogeneity of control variates covariances across replicates, homogeneity of control variates covariances across replicate pairs, and independence of control variates on different pairs of replicates.

$$
\operatorname{Cov}\left[c_{i}, c_{k}\right]=\left\{\begin{array}{ll}
\sum_{c} & \text { for } i=k=1,2, \ldots, 2 h  \tag{2.46}\\
\sum_{c}^{1} & \text { for } i=k+1(k=1,3, \ldots, 2 h-1) \\
0_{(s \times s)} & \text { otherwise }
\end{array}\right\}
$$

The following equations can be derived from the four properties above.

$$
\begin{align*}
& \operatorname{Var}\left[\bar{y}_{j}\right]=\frac{1}{4}\left(\operatorname{Var}\left[y_{2 j-1}\right]+\operatorname{Var}\left[y_{2 j}\right]+2 \operatorname{Cov}\left[y_{2 j-1}, y_{2 j}\right]\right)=\frac{1}{2}(1-\rho) \sigma_{y}^{2}  \tag{2.47}\\
& \operatorname{Cov}\left[\bar{c}_{j}\right]=\frac{1}{4}\left(\operatorname{Cov}\left[c_{2 j-1}\right]+\operatorname{Cov}\left[c_{2 j}\right]+2 \operatorname{Cov}\left[c_{2 j-1}, c_{2 j}\right]\right)=\frac{1}{2}\left(\sum_{c}+\sum_{c}^{(1)}\right) \tag{2.48}
\end{align*}
$$

and

$$
\begin{equation*}
\operatorname{Cov}\left[\bar{y}_{j}, \bar{c}_{j}\right]=\frac{1}{4} \operatorname{Cov}\left[y_{2 j-1}+y_{2 j}, c_{2 j-1}+c_{2 j}\right]=\frac{1}{2}\left(\sigma_{y c}+\sigma_{y c}^{(1)}\right) \tag{2.49}
\end{equation*}
$$

for $j=1,2, \ldots, h$. Under the joint normality assumption and properties from equations
(2.43), (2.44), (2.45), and (2.46), the joint distribution of $\bar{y}_{j}$ and $\bar{c}_{j}$ is given as

$$
\left[\begin{array}{c}
\bar{y}_{j}  \tag{2.50}\\
\bar{c}_{j}
\end{array}\right] \sim \mathrm{N}_{s+1}\left(\left[\begin{array}{c}
\mu_{y} \\
0_{(s \times 1)}
\end{array}\right], \frac{1}{2}\left[\begin{array}{cc}
(1-\rho) \sigma_{y}^{2} & \left(\sigma_{y c}+\sigma_{y c}^{(1)}\right)^{\mathrm{T}} \\
\sigma_{y c}+\sigma_{y c}^{(1)} & \sum_{c}+\sum_{c}^{(1)}
\end{array}\right]\right) \text { for } j=1,2, \ldots, h
$$

Since $\bar{c}_{j}(\mathrm{j}=1,2, \ldots, \mathrm{~h})$ are independent, the unconditional variance for $\hat{\mu}_{y}$ is

$$
\begin{equation*}
\operatorname{Var}\left[\hat{\mu}_{y}\right]=\frac{\tau^{2}}{2 h}\left(\frac{h-2}{h-s-2}\right) \tag{2.51}
\end{equation*}
$$

where

$$
\begin{equation*}
\tau^{2} \equiv(1-\rho) \sigma_{y}^{2}-\left(\sigma_{y c}+\sigma_{y c}^{(1)}\right)^{\mathrm{T}}\left(\sum_{c}+\sum_{c}^{(1)}\right)^{-1}\left(\sigma_{y c}+\sigma_{y c}^{(1)}\right) \tag{2.52}
\end{equation*}
$$

Therefore, a reduction in variance of $\hat{\mu}_{y}$ will occur if the effects due to AV and CV together compensate for the loss factor given as $((h-2) /(h-s-2))$. Where $s$ is the number of random number streams used to derive the control variate random numbers.

### 2.4.5. Control Variates and Common Random Numbers.

Yang and Nelson (44) developed a method that combines CVs and CRNs in applications of multiple comparisons. Multiple comparisons compare the expected values of univariate performance measures common to two or more systems by constructing simultaneous interval estimates of the differences in expected performance. Yang and Nelson address three types of multiple comparison procedures. They are Tukey's all-pairwise-multiple comparison (MCA) procedure, Hsu's multiple comparisons-with-the-best (MCB) procedure, and Dunnett's multiple-comparisons-with-a-control (MCC) procedure. MCA is used to determine the difference between the performance of each system design and every other system design. MCB is used to identify the system with the largest mean performance. MCC is used to determine the difference between the mean performance of a selected system design and every other system design. Their goal was to refine the three procedures, using the variance reduction techniques of CV and CRN , to increase sensitivity to differences in system performance.

The idea behind Yang and Nelson's approach is to use the following control variate estimators to estimate the mean performance for a system when applying multiple comparison procedures.

$$
\begin{equation*}
\hat{\theta}_{i}=\bar{Y}_{i}-\hat{\beta}_{i}^{\prime}(\overline{\mathbf{X}}-\boldsymbol{\mu}) \text { for } i=1,2, \ldots, r \tag{2.53}
\end{equation*}
$$

where $r$ is the total number of systems, $\bar{Y}_{i}$ is the mean performance measure, $\hat{\beta}_{i}$ is the control variate estimates generated though least squares estimates, $\mathbf{X}$ is a vector of independent random variables with known mean vector $\mu$. An assumption required to do this is that the random variables $\mathbf{X}_{\boldsymbol{j}}$, the control vectors, are available and identical across all systems. Therefore the control vecotors must be available in each system design so they can be forced to take identical values through the application of CRN. The other random inputs into the system need to be independent to develop the control variate estimation.

Yang and Nelson modify the interval estimates for all three multiple comparison procedures to use the control variate estimation which relies on the fact that the control vectors are identical across all systems. They then develop the following theorem

$$
\begin{equation*}
\frac{E\left[H_{C V} \mid \mathbf{X}\right]}{E[H]} \xrightarrow{P} \sqrt{1-R^{2}} \leq 1 \tag{2.54}
\end{equation*}
$$

where $E\left[H_{C V} \mid \mathbf{X}\right]$ is the expected width of the stadard control variate interval and $E[H]$ is the expected width of the multiple comparison interval. The equation calculates how the difference intervals developed by the control variates are smaller than the normally generated difference intervals.

### 2.4.6. Other Techniques.

There are many other VRTs than CV, CRN, and AV. In December of 1983, Lieutenant Colonel Mohamed Refat Elhefny (6) produced a thesis comparing many variance reduction techniques. He performed an extensive literature review to identify
and learn about the different VRTs. The following is a list of all the VRTs covered in LtCol Elhefny's research.

- Importance Sampling
- Russian Roulette and Splitting
- Systematic Sampling
- Stratified Sampling
- Expected Value
- Statistical Estimation
- Correlated Sampling
- History Reanalysis
- Control Variates
- Antithetic Variates
- Regression
- Sequential Sampling
- Orthogonal Functions
- Adjoint Method
- Transformation
- Conditional Monte Carlo

Within his research he created a table that outlines the characteristics of many VRTs. The table has been reproduced in Appendix (H). The table includes a description, criteria for application, advantages, disadvantages, and typical applications for each technique.

LtC Elhefny concluded, "there is no single technique which is the most suitable technique for every simulation problem."(6:VI-1) He also thought that research should be performed to identify VRTs that may be affective for specific simulations. This could aid the analysis by identifying VRTs that may work with a simulation.

### 2.4.7. Previous Variance Reduction Studies.

There have been many studies in variance reduction techniques. Curtis Smith from the Naval Postgraduate School (NPS) set out to identify VRTs that are applicable to a class of stochastic combat simulations and then exhibit their performance in the applications. He applied the Antithetic Variates and Stratified Sampling techniques to a simple stochastic combat simulation. He proposed that both of these techniques are capable of variance reduction in stochastic combat models. Since the work was on a simple stochastic combat simulation, it was recommended that further studies on larger, real world simulations be performed. (31)

### 2.5. Random Number Generator

The random number generator (RNG) is an integral part of a stochastic simulation. Therefore, it should be of great interest to the analyst as to what RNG is used for a simulation and how well it performs. Currently, most programming software use small linear congruential generators (LCGs) as the random number generators. It is recognized by simulation experts that small LCGs, with moduli around $2^{31}$, should not be used for most purposes. In fact, the period on most any LCG can be exhausted in a matter of minutes on a standard PC and LCGs are known to have poor point structure, both of which can bias simulation results. L'Ecuyer, proposed to replace the more common LCGs with a more robust RNG method called a combined multiple recursive generator (CMRG). L’Ecuyer, Simard, Chen, and Kelton have produced a CMRG RNG package, which is readily available on the Internet and is reviewed below. (18)

A parameter set for a CMRG, from a previous paper by L'Ecuyer, was selected as the backbone of the proposed RNG package. The parameter set is called MRG32k3a and was one of many sets that were discovered after a years worth of computer processing time (16). MRG32k3a has a long period of approximately $2^{191}$ or $3.1 \times 10^{57}$. RNGs with longer periods have been proposed, but MRG32k3a is more efficient with today's computers and the period is long enough not to be depleted by a years worth of computing. To remain efficient, a parameter set was chosen that had the ability to use floating-point arithmetic. Floating-point arithmetic is much faster to perform than regular arithmetic. MRG32k3a is also very sound statistically. The random number stream from MRG32k3a is so uniform; they pass the spectral test in at least 45 dimensions. This
means that when generating sets of $t$ numbers, they would cover a $t$-dimensional unit hypercube very uniformly.

The implementation of MRG32k3a is relatively simple. It maintains only 2 components, each of order 3. The components are stored in vectors. At step $n$, the vectors would be as follows.

$$
\begin{align*}
& s_{1, n}=\left(x_{1, n}, x_{1, n+1}, x_{1, n+2}\right)  \tag{2.55}\\
& s_{2, n}=\left(x_{2, n}, x_{2, n+1}, x_{2, n+2}\right) \tag{2.56}
\end{align*}
$$

The vectors evolve according to the equation (2.57) and (2.58).

$$
\begin{align*}
& x_{1, n}=\left(1403580 \times x_{1, n-2}-810728 \times x_{1, n-3}\right) \bmod 4294967087  \tag{2.57}\\
& x_{2, n}=\left(527612 \times x_{1, n-1}-1370589 \times x_{1, n-3}\right) \bmod 4294944443 \tag{2.58}
\end{align*}
$$

Then the output $u_{n}$ is defined by the following relationship.

$$
u_{n}=\left\{\begin{array}{cc}
z_{n} / 4294967088 & \text { if } z_{n}>0  \tag{2.59}\\
4294967087 / 4294967088 & \text { if } z_{n}=0
\end{array}\right.
$$

where

$$
\begin{equation*}
z_{n}=\left(x_{1, n}-x_{2, n}\right) \bmod 4294967087 \tag{2.60}
\end{equation*}
$$

To create multiple streams, the full period needed to be broken into smaller sections that are equal in length. The goal was to find a size for the streams and substreams that are relatively long and have good statistical properties. In essence each stream and substream should resemble a stand alone random number generator. To do this, the statistical structure of many different streams and substreams all ranging in length, between $2^{51}$ and $2^{150}$ were examined. The combination of each stream consisting of $2^{127}$ numbers, with $2^{51}$ substreams consisting of $2^{76}$ numbers was chosen. Since the
streams and substreams are of equal length, four $3 \times 3$ matrices were developed so that the software could advance the state of the RNG by either $2^{127}$ or $2^{76}$ numbers with a single matrix multiplication. This allows the software to immediately jump to the next stream or substream.

To maintain the proper synchronization within the RNG, the computer keeps track of the beginning of the current stream, the beginning of the current substream, the beginning of the next substream, and the current state of the stream for each stream. This allows the user to reset the stream, reset the substream, jump to the next stream or jump to the next substream without loosing continuity.

The speed of the MRG32k3a generator was compared with a commonly used LCG, lcgrand from Law and Kelton (16). A total of $10^{9}$ random numbers were generated from both RNGs and were timed. The RNGs were run on a 600 MHz PIII with 128 MB of RAM, running Windows 98. MRG32k3a completed the run in 12.9 minutes while lcgrand completed the run in 14.8 minutes. Even though the MRG32k3a has more than twice as many calculations as a LCG, it still runs faster. Primarily, this is because the code for MRG32k3a implements floating-point arithmetic. To implement floating-point arithmetic, no variable or intermediate variable can exceed the precision of the computer it is running on, which is generally 32 bits. To make sure the calculations do not exceed the precision of the computer, the backbone equations have been slightly modified to store the intermediate calculation variables in smaller variables.

The RNG package includes many extra utilities to aid the use. First, the seed values for the whole package or individual stream can be changed. Second, each stream can be set to produce Antithetic variables for variance reduction. Third, the stream can
be reset to the start of the stream or substream. Finally, the precision of the uniform random number can be increased from 32-bit precision to 53-bit precision. A list of C routines is located in Appendix (G).

### 2.6. Major Shared Resource Center and Divide and Conquer

The ASC MSRC is a computational science facility. It supports various DoD research including development, test, and evaluation. As part of the DoD's High Performance Computing (HPC) Modernization Program, the ASC MSRC uses high performance computing research and visualization. To provide an interface between the analyst and HPC computing resources, the Divide and Conquer (D\&C) system was developed. D\&C has a web based Graphical User Interface (GUI) that allows an analyst to specify parameters for a study. D\&C will take the study parameters and automatically set up all requested replications and directories. After setting up the replications, D\&C queues the replication requests in the specified HPC resource.

### 2.7. Matlab and Regression

Matrix Laboratory (MATLAB®) is a high-performance language for technical computing. It was developed to provide easy access to matrix software, embedding state of the art software for matrix computation. The basic data element is an array that does not require dimensioning, giving the ability to solve many technical computing problems, specifically those with matrix and vector formulations, without the need to write a program in a scalar, noninteractive language such as C, Visual Basic, or Fortran.

MATLAB features a family of add-on application-specific solutions called toolboxes. Toolboxes are collections of MATLAB functions that extend the MATLAB environment to solve particular classes of problems. One of the MATLAB toolboxes is
the statistics toolbox (24). The statistics toolbox is a collection of statistical tools built on the MATLAB computing environment. The toolbox supports a wide range of common statistical tasks, from random number generation, to curve fitting, to design of experiments, statistical process control, and regression (25).

A useful function from the statistical toolbox is stepwisefit. The following information about stepwisefit was taken from the MATLAB statistics documentation. Stepwisefit uses stepwise regression to model the response variable $y$ as a function of the predictor variables represented by the columns of the matrix $X$. A standard call to the stepwisefit function is [b, se, pval, inmodel, stats, nextstep, history] = stepwisefit(X, y, 'Param1', value1, 'Param2', value2, ...). Where $X, y$, 'Param1', value1, 'Param2', value2, etc. are function parameters and $b$, se, pval, inmodel, stats, nextstep, and history are function results. Table 2.1 contains a description of the stepwisefit parameters. $\boldsymbol{b}$ is a vector of estimated coefficient values for all columns of $X$. The $b$ value for a column not included in the final model is the coefficient that you would obtain by adding that column to the model. se is a vector of standard errors for $b$. pval is a vector of p -values for testing whether $b$ is 0 . inmodel is a logical vector, whose length equals the number of columns in $X$, specifying which predictors are in the final model. A 1 in position $j$ indicates that the $j$ th predictor is in the final model; a 0 indicates that the corresponding predictor in not in the final model. stats is a structure containing additional statistics. nextstep is the recommended next step, either the index of the next predictor to move in or out, or 0 if no further steps are recommended. history is a structure containing information about the history of steps
taken. The following table specifies one or more of the name/value pairs used as parameters (23).

Table 2.1: Parameter Specifications for Stepwisefit (23)

| Parameter Name | Parameter Value |
| :--- | :--- |
| 'inmodel' | Logical vector specifying the predictors to include in the initial fit. The <br> default is a vector of all zeros, specifying no predictors. |
| 'penter' | Maximum p-value for a predictor to be added. The default is 0.05. |
| 'premove' | Minimum p-value for a predictor to be removed. The default is 0.10. |
| 'display' | 'on' displays information about each step. <br> 'off' omits the information. |
| 'maxiter' | Maximum number of steps to take (default is no maximum) |
| 'keep' | Logical vector specifying the predictors to keep in their initial state. <br> The default is a vector of all zeros, specifying no predictors. |
| 'scale' | 'on' scales each column of X by its standard deviation before fitting. <br> 'off' does not scale (the default). |

### 2.8. Analysis

The following subsections contain calculations used in the analysis presented in chapter 4. The first subsection describes how to generate a halfwidth and a confidence interval. The second subsection gives the calculations for determining the number of replications needed to achieve a specified halfwidth. The third subsection outlines the calculations used to determine a paired-t CI. The last subsection describes the Bonferroni inequality and how to calculate it.

### 2.8.1. Confidence Intervals.

Confidence intervals (CIs) are used to determine the $100(1-\alpha) \%$ bound on an estimated mean. The CI is based heavily on the variance of the mean estimate. The standard equation for a CI is below.

$$
\begin{equation*}
\mathrm{CI}=\mu_{X} \pm \delta(n, \alpha) \tag{2.61}
\end{equation*}
$$

where

$$
\begin{equation*}
\mu_{X}=\frac{1}{n} \sum_{i=1}^{n} X_{i} \tag{2.62}
\end{equation*}
$$

and

$$
\begin{equation*}
\delta(n, \alpha)=t_{n-1,1-\alpha / 2} \sqrt{\frac{S^{2}(n)}{n}} \tag{2.63}
\end{equation*}
$$

and $n$ is the number of sample points, $S^{2}(n)$ is the variance of the sample points, and $t_{n-1,1-\alpha / 2}$ is the upper 1- $\alpha / 2$ critical point for the $t$-distribution with $n-1$ degrees of freedom. Equation (2.63) is called the halfwidth (20:511-513).

### 2.8.2. Determining the Number of Replications.

To calculate the estimated number of replications needed to achieve a specified halfwidth $\beta$, the following equation would be used.

$$
\begin{equation*}
n_{a}^{*}(\beta)=\min \left\{i \geq n: t_{i-1,1-\alpha / 2} \sqrt{\frac{S^{2}(n)}{i}} \leq \beta\right\} \tag{2.64}
\end{equation*}
$$

To use the above equation, it is assumed that $S^{2}(n)$ is constant for all $n(20: 512)$.

### 2.8.3. Paired-t Confidence Interval.

The paired-t CI is a method of comparing two systems on the basis of a performance measure. The paired-t CI calculates a CI about the difference between the performance measures. If the CI contains zero then the performance measures from both systems can be assumed equivalent.

Lets say there are $n_{1}$ replications of a performance measure of interest one $X_{1 j}$ where $j$ ranges from 1 to $n_{1}$ and there are $n_{2}$ replications of a performance measure of interest one $X_{2 k}$ where $k$ ranges from 1 to $n_{2}$. If $n_{1}=n_{2}=n$ then we can define

$$
\begin{equation*}
Z_{j}=X_{1 j}-X_{2 j}, j=1,2, \ldots, n \tag{2.65}
\end{equation*}
$$

Since the $X_{i j}$ are taken from replications, the $Z_{j}$ 's are IID random variables and the estimator for expected value of $Z$ is

$$
\begin{equation*}
\bar{Z}(n)=\frac{\sum_{j=1}^{n} Z_{j}}{n} \tag{2.66}
\end{equation*}
$$

and the estimator for the variance is

$$
\begin{equation*}
\hat{\operatorname{Var}}[\overline{\mathrm{Z}}(n)]=\frac{\sum_{j=1}^{n}\left[Z_{j}-\overline{\mathrm{Z}}(n)\right]^{2}}{n(n-1)} \tag{2.67}
\end{equation*}
$$

Now, to form the $100(1-\alpha) \%$ CI

$$
\begin{equation*}
\mathrm{CI}[\overline{\mathrm{Z}}(n)]=\bar{Z}(n) \pm t_{n-1,1-\alpha / 2} \sqrt{\hat{\operatorname{Var}}[\overline{\mathrm{Z}}(n)]} \tag{2.68}
\end{equation*}
$$

Where the halfwidth is defined as

$$
\begin{equation*}
\delta(n, \alpha)=t_{n-1,1-\alpha / 2} \sqrt{\hat{\operatorname{Var}}[\overline{\mathrm{Z}}(n)]} \tag{2.69}
\end{equation*}
$$

### 2.8.4. Bonferroni Inequality.

In most simulations, an analyst is interested in several simultaneous performance measures. Suppose that $I_{s}$ is a $100(1-\alpha) \%$ CI for the measure of performance $\mu_{s}$ where s $=1,2, \ldots, k$. The probability that all $k$ CIs simultaneously contain their respective true measures is

$$
\begin{equation*}
\mathrm{P}\left(\mu_{\mathrm{s}} \in I_{s} \forall s=1,2, \ldots, k\right) \geq 1-\sum_{s=1}^{k} \alpha_{s} \tag{2.70}
\end{equation*}
$$

Whether or not the $I_{s}$ 's are independent. This is known as the Bonferroni inequality.
Therefore, to maintain a simulation level of $\alpha$, choose $\alpha_{s}$ 's such that

$$
\begin{equation*}
\alpha=\sum_{s=1}^{k} \alpha_{s}, s=1,2, \ldots, k \tag{2.71}
\end{equation*}
$$

Chapter 3 uses the basic understanding of the different tools and techniques
developed in this chapter. It describes how the techniques are applied and which tools are used in the application.

## 3. Methodology

This chapter begins with the research objective. It then transitions into choice of THUNDER model databases and MOEs, describing how the MOEs are collected for analysis. Next, there is a description of how the VRTs are performed. Finally, it finishes with a discussion of how the analysis replications are run at the Major Shared Resource Center (MSRC) through the Divide and Conquer (DAC) web interface.

### 3.1. Research Objective

The objective of this research is to study the use of three variance reduction techniques on THUNDER. The three techniques are Control Variates, Common Random Numbers and Antithetic Variates. There are three basic steps to obtain the research objective. First, identify a set of MOEs that are commonly used by THUNDER users. This will guarantee that the results will be of some interest to the users. Second, modify the THUNDER source code to implement the three techniques. This consists of identifying areas where random variates are drawn, creating an output file of the mean value for the random inputs and implementing a new random number generator. Third, analyze the results from the variance reduction techniques to determine which, if any, are effective and if effective how efficient the technique is when applied to the model.

### 3.2. Model Selection for THUNDER Analysis

There are three notional model databases that are used as examples and come with THUNDER. They are called Databig, Datasmall, and ME. Most THUNDER model databases are classified due to the amount and the nature of the data in the model.

Therefore, one of the notional models was selected for the THUNDER analysis so that this research would remain unclassified. There is little documentation about any of the three example models, except that one of the three data sets is a European scenario, while the other two data sets are Middle Eastern scenarios. Since the THUNDER introductory course uses the ME, a Middle Eastern scenario, database (7), it was selected to be the representative scenario for this research. It is a 5-day scenario consisting of blue forces from Saudi Arabia and red forces from Iraq.

Running the standard package ME scenario is applicable for both CV and AV , since they both are single-model VRTs. On the other hand, the standard application of CRN requires at least two configurations or models to be compared. Therefore, a modification to the original ME configuration is needed. Because little is known about the scenario, a file that should have a significant effect on the outcome of the MOEs was chosen. That file selected was squadron.dat. It contains information pertaining to squadrons in the scenario. To modify it significantly, $204 \mathrm{~F}-111$ aircrafts were taken from the Blue forces. This eliminated all of the F-111 from the scenario. No modifications were made to any other ME input files. A section of Appendix C of the THUNDER Analyst Manual (33), pertaining to the squadron.dat file is located in Appendix (I).

### 3.3. Measure of Effectiveness Selection

The MOEs for the ME scenario were selected after discussion with expert THUNDER users (32). Three types of MOEs that seem to interest the interviewed users are forward line of troops (FLOT) movement, sortie generation and aircraft attrition. These MOEs can be extracted from standard THUNDER reports. The two types of
reports needed are Ground Combat Cycle reports (CC) and the Air War reports (AW).
The output data for the FLOT movement is located in the CC-4 (FLOT Movement) reports, the data for sorties generated is located in the AW-5 (Sortie Generation) reports and the data for aircraft attrition is located in the AW-3 (Aircraft Loss Summary) reports. All three reports are contained within a single output file named reports.std. A one-day sample of these reports is located in Appendix (A). A set of 21 specific MOEs was selected from the three reports. The following table identifies all MOEs, aggregation level, force, and the reports they are calculated from.

Table 3.1: Measure of Effectiveness Table

| MOE | AGGREGATION | FORCE | REPORT |
| :--- | :--- | :--- | :--- |
| A-10 Losses per Sortie (LPS) | System Level | BLUE | AW-3 |
| A-6 Losses per Sortie (LPS) | System Level | BLUE | AW-3 |
| BLUE Flown per Planned Sortie (FPS) | Force Level | BLUE | AW-5 |
| BLUE Losses per Sortie (LPS) | Force Level | BLUE | AW-3 |
| BLUE Square Miles Gained (SMG) | Force Level | BLUE | CC-4 |
| DHAHRAN Flown per Planned Sortie (FPS) | Base Level | BLUE | AW-5 |
| F-111 Losses per Sortie (LPS) | System Level | BLUE | AW-3 |
| F-15 Losses per Sortie (LPS) | System Level | BLUE | AW-3 |
| FA-18 Flown per Planned Sortie total (FPS total) | Squadron Level | BLUE | AW-5 |
| FA-18 Flown per Planned Sortie DCA (FPS DCA) | Squadron Level | BLUE | AW-5 |
| MIG-23 Losses per Sortie (LPS) | System Level | RED | AW-3 |
| MIG-29 Losses per Sortie (LPS) | System Level | RED | AW-3 |
| MIRAGE Flown per Planned Sortie total (FPS total) | Squadron Level | RED | AW-5 |
| MIRAGE Flown per Planned Sortie INT (FPS INT) | Squadron Level | RED | AW-5 |
| MIRAGE Losses per Sortie (LPS) | System Level | RED | AW-3 |
| MUDAYSIS Flown per Planned Sortie (FPS) | Base Level | RED | AW-5 |
| RED Flown per Planned Sortie (FPS) | Force Level | RED | AW-5 |
| RED Losses per Sortie (LPS) | Force Level | RED | AW-3 |
| RIYADH Flown per Planned Sortie (FPS) | Base Level | BLUE | AW-5 |
| SHAIBAH Flown per Planned Sortie (FPS) | Base Level | RED | AW-5 |

For this research, all of the selected MOEs were calculated. To calculate the Losses per Sortie (LPS) MOEs for the specific system, squadron, base or force, the total number of losses were divided by the total number of sorties. To calculate the Flown per Planned Sortie (FPS) MOEs for the specific system, squadron, base or force, the total
flown sorties were divided by the total planned sorties. The SMG was the only MOE that only needed to be summed over the total number of replication days.

### 3.4. Parsing THUNDER reports

The format for the reports in reports.std is good for a single printout, but it is not a good format for manipulating the data. reports.std is a text file. This means, that the format of the file is for a nice display, which is generally not conducive to data manipulation. In addition to the text format, report CC-4 is produced every 12simulation hours, while AW-3 and AW-5 are reported every 24 -simulation hours.

Therefore, there are twice as many CC-4 reports as there are AW-3 or AW-5 reports. Fortunately, the reports are relatively uniform and can be manipulated with a parsing script. The following chart describes the flow of the parsing script.


Figure 3.1: Parsing Script Flow Chart
Using a combination of Microsoft Excel, Microsoft Access and VBA, the reports.std is transformed from an ungainly text file into easy to manipulate Access tables. The tables produced are easily combined into a table that is readily transferable to software with regression capabilities such as JMP or MATLAB. THUNDER has been modified to produce a file named earltally.txt that contains the control variate
information and earltime.txt that contains the start and ending times for each replication.
A small example of the earltally.txt file is located below.

| 160, | 7, | $0 .$, | $0 .$, | 0, | -.00986138176387563802, | 210 |
| :--- | :--- | :--- | :--- | :--- | :--- | ---: |
| 104, | 11, | .1, | .33, | 0, | 45.43071582689116816311, | 210 |
| 105, | 11, | .1, | .33, | 0, | 43.01011811890460556639, | 210 |
| 71, | 9, | $0 .$, | $0 .$, | 0, | 601.25462303729726443179, | 1179 |
| 47, | 9, | $0 .$, | $0 .$, | 0, | 975.32737950515002722795, | 1955 |
| 70, | 9, | $0 .$, | $0 .$, | 0, | 258.31346595147626210575, | 535 |
| 114, | 9, | $0 .$, | $0 .$, | 0, | 4523.83226559217987983174, | 9025 |

Figure 3.2: Example of earltally.txt File
The earltally.txt file is a comma separated text file. This means, that the file contains data separated into columns by commas. It has a total of 7 columns. The first column indicates which random variate the draw is from. The second column indicates what distribution the draw is from. Table 3.2 decodes all 13 of the distributions used by THUNDER. The third, fourth, and fifth columns are parameters 1,2 , and 3 respectively, of the distribution. The sixth column is the sum of the responses for the specific random variate draw. The final column is a count of the random variate draws.

Table 3.2: Random Distribution Decodes

| code | decode |
| ---: | :--- |
| 1 | Ieta |
| 2 | linomial |
| 3 | Erlang |
| 4 | Exponential |
| 5 | lamma |
| 6 | Logorithmic Normal |
| 7 | Normal |
| 8 | Poisson |
| 9 | Random |
| 10 | Triangle |
| 11 | Uniform |
| 12 | Weibull |
| 13 | Randi |

The diagram in Figure 3.1 shows that the two output files, earltally.txt and reports.std, are handled separately and could be performed in parallel to speed up post
processing. Starting with the earltally.txt file, all replications are first loaded directly into Access. Then, Access singles out the random variates with only one entry per replication. Some random variates have distributions with parameters that change through out a THUNDER replication and therefore would have multiple entries in the earltally.txt file. Next, Access then filters out the random variates that were not used in all replications. Access then, calculates the expected value for each of the remaining data point and then subtracts the expected value from its corresponding data point. Finally, Access eliminates the resulting random variates that are zero or extremely close to zero. The final product is a table of random variates with deviations from the expected value that can be used as potential control variates. The table below has the random variates labeled across the top and identifies each replication as a row.

Table 3.3: Example of earltally.txt Final Product

| REPLICATION | Total Of COUNT RESPONSE | ADF105 1 | ADF121 1 | ADF121 2 | ADF121 3 | ADF140 3 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 125583 | 0.002579423 | -0.000329738 | -0.000770258 | -0.000232268 | 0.004212851 |
| 2 | 132150 | -0.009506824 | -0.003220924 | -8.88252E-05 | 0.00125997 | 0.003561292 |
| 3 | 133485 | 0.001791816 | -0.001039787 | -0.001135727 | 0.001023826 | -0.002670896 |
| 4 | 125951 | 0.003829476 | 0.001651921 | 0.000130294 | 0.000752148 | -0.003725937 |
| 5 | 133589 | 0.00681057 | 0.00051887 | 0.000205273 | 0.000267634 | -0.001303033 |
| 6 | 122488 | 0.000387382 | 0.001067072 | 0.00013618 | 0.000565525 | 0.010540051 |
| 7 | 122536 | -0.000581091 | -0.000107913 | -9.22039E-05 | -0.001099367 | -0.001221987 |
| 8 | 123779 | 0.003961604 | -0.000126377 | 0.000277925 | -0.000589475 | 0.000663163 |
| 9 | 134503 | 0.004555212 | 0.000114831 | 0.000903415 | 0.001249704 | 0.007879616 |

The file reports.std undertakes a more convoluted path. First Access deletes all of the old output tables and opens a template Excel file called reportsifter.xls. This Excel file contains macros and sheets designed to decode the three reports contained in reports.std. Then Excel opens up the reports.std file and separates each report page into its own Excel worksheet. After separating the report pages, Excel looks through each worksheet for report AW-3 and combines important data from AW-3 onto a new
worksheet. After all the AW-3 reports are found, Excel repeats the combining process for reports AW-5 and CC-4. It then saves the three combined worksheets into a new Excel workbook. Excel then repeats the entire process for each replication, until all of the reports.std files in the directory have been loaded and labeled with its corresponding replication. Finally, Access looks in the directory and transfers all three worksheets, AW-3, AW-5, and CC-4, from each of the new Excel workbooks into Access and combines them. A partial example of the AW-3 finished sheet is below.

Table 3.4: Example of AW-3 Final Product

| REPLICATION | DAY | SIDE | ID | TYPE AIRCRAFT NAME | NUMBER OF SORTIES | LOSSES AIR TO AIR | LOSSES SURFACE TO AIR | LOSSES GROUND | LOSSES OTHER |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 4:0000 | BLUE | 1002 | F-16 | 150 | 0 | 1 | 0 | 0 |
| 1 | 4:0000 | BLUE | 1099 | ABL | 22 | 0 | 1 | 0 | 0 |
| 1 | 4:0000 | BLUE | 1014 | C-130H | 4 | 0 | 0 | 0 | 0 |
| 1 | 4:0000 | BLUE | 1013 | C-17 | 2 | 0 | 0 | 0 | 0 |
| 1 | 4:0000 | BLUE | 1012 | KC-135 | 58 | 0 | 0 | 0 | 0 |
| 1 | 4:0000 | BLUE | 1011 | E-8 | 4 | 0 | 0 | 0 | 0 |
| 1 | 4:0000 | BLUE | 1010 | E-3 | 6 | 0 | 0 | 0 | 0 |
| 1 | 4:0000 | BLUE | 1009 | F-14 | 108 | 1 | 0 | 0 | 0 |
| 1 | 4:0000 | BLUE | 1008 | A-6 | 101 | 0 | 5 | 0 | 0 |
| 1 | 4:0000 | BLUE | 1007 | F/A-18 | 48 | 0 | 0 | 0 | 0 |
| 1 | 4:0000 | BLUE | 1006 | AV-8B | 66 | 0 | 0 | 0 | 0 |
| 1 | 4:0000 | BLUE | 1005 | F-15 | 296 | 0 | 0 | 0 | 0 |
| 1 | 3:0000 | BLUE | 1012 | KC-135 | 55 | 0 | 0 | 0 | 0 |
|  | 4:0000 | BLUE | 1003 | RF-4 | 44 | 1 | 1 | 0 | 0 |
| 1 | 4:0000 | RED | 2005 | SU-25 | 50 | 5 | 3 | 0 | 0 |
|  | 4:0000 | BLUE | 1001 | A-10 | 254 | 3 | 1 | 0 | 0 |
| 1 | 3:0000 | RED | 2099 | SCUD | 43 | 3 | 0 | 2 | 0 |
|  | 3:0000 | RED | 2006 | MAINSTAY | 3 | 0 | 0 | 0 | 0 |
| 1 | 3:0000 | RED | 2005 | SU-25 | 61 | 5 | 10 | 0 | 0 |
|  | 3:0000 | RED | 2004 | MIG-29 | 8 | 0 | 0 | 0 | 0 |
|  | 3:0000 | RED | 2003 | MIG-21 | 96 | 13 | 0 | 0 | 0 |
|  | 3:0000 | RED | 2002 | MIRAGE F-1 | 46 | 5 | 2 | 0 | 0 |
|  | 3:0000 | RED | 2001 | MIG-23 | 61 | 11 | 8 | 0 | 0 |
|  | 3:0000 | BLUE | 1099 | ABL | 22 | 0 | 0 | 0 | 0 |
|  | 3:0000 | BLUE | 1014 | C-130H | 2 | 0 | 0 | 0 | 0 |

When the scripts are finished, the Access database contains four output tables.
They are AW-3, AW-5, CC-4, and the potential control variates. Using Access, the report tables can be modified to obtain the selected MOEs. Then the selected MOEs can
be combined with the potential control variates to create regression ready tables. A copy of the scripts outlined above is available from the Center for Operations Analysis (COA) at AFIT. Also, a list of the modified files is located in Appendix (C).

### 3.5. Control Variates

The method of Control Variates relies on the output of a random variate where the expected value is known. In simulations, the random variates generated throughout a replication are effectively random and have a known expected value. Therefore, all the random variates generated in a simulation are potential control variates and should be identified and output from the simulation. When creating an output file of random variates, the variates should be labeled and produced with the label, type, parameters, and result. Therefore, the variates can be identified by the labels, the mean can be calculated from the result, and the expected value can be determined by the type and parameters.

In the late 1980's, a very limited version of the CRN technique was added to THUNDER. Reviewing this work, 245 of the 254 points where random variates are drawn were identified by following the nomenclature of CRN streams. The control.dat file, see Appendix (D), contains the list of random inputs and allows the user to assign random number streams to each of the random inputs. The nomenclature of the random input identifies which module the random input is from and also labels for more than one random input from a module. For example, ..AIR070.2 is a random input from module AIR070.SIM and it is the second random input from that module.

Since the random variate draws had been identified, the problem was, understanding how the variates are drawn, what they are used for, and how to output them. There are two methods used in THUNDER to draw a random variate. The first
method is where a random variate is drawn in a logical expression. An example is shown below.

```
''determine result of shot
IF .PK > 0.0
        AND RANDOM.F(RN.STREAM(..AIR070.2)) <= .PK
        LET .NUM.KILLED = 1
ELSE
    LET .NUM.KILLED = 0
ALWAYS
```

Figure 3.3: Logical Expression Example (Original)
The bolded section in Figure 3.3 is the random variate draw. The code RANDOM.F( ) is the SIMSCRIPT II. 5 function for a uniform $(0,1)$ draw. RN.STREAM is a vector that contains the stream number for each random number draw. ..AIR070.2 is the label of the random variate draw. To capture the result, the random variate is drawn immediately prior to its use in THUNDER and placed in a variable called EARL.RESPONSE. Following the random number draw, the parameters are placed in variables EB.P1, EB.P2, and EB.P3. When a parameter is not used it is set to zero as in Figure 3.4. EARL.RESPONSE is then sent to an output routine EARL.STORE, along with the variate name, variate type, and all three parameters. EARL.STORE was specifically built to organize the random variate output for ease of use. Creating a single routine for all variate output allows for the ease of changing the output format if needed. After calling the output routine, THUNDER uses EARL.RESPONSE in place of the original random variate draw. Figure 3.4 is the modified version of logical expression in Figure 3.3. The added code to export the results is in bold.
''determine result of shot

''* Earl stuff
EARL.RESPONSE = RANDOM.F(RN.STREAM(..AIR070.2))
EB.P1 = 0.0
$E B . P 2=0.0$
$E B . P 3=0.0$
CALL EARL.STORE
GIVEN
. AIR070.2,
. .RANDOM, EARL.RESPONSE, EB.P1, EB.P2, EB.P3
' '* END STUFF
IF .PK > 0.0
IF .PK > 0.0
AND EARL.RESPONSE <= .PK
AND EARL.RESPONSE <= .PK
LET .NUM.KILLED = 1
LET .NUM.KILLED = 1
ELSE
ELSE
LET .NUM.KILLED = 0
LET .NUM.KILLED = 0
ALWAYS
ALWAYS

Figure 3.4: Logical Expression Example (Modified)
The second method to draw a random variate in THUNDER is through a subroutine. The subroutine is called because for the specific random variate, the user can set the variate distribution in an input file. Therefore the subroutine identifies the parameters and which distributions are needed and then draws from that distribution. The results from these random variate draws are returned from the subroutine and stored in a variable to be used by THUNDER. The example code in Figure 3.5 demonstrates a random variate subroutine call. The name of subroutine called is

UTL.720.CALCULATE.USER.FUNCTION. As in the first method, it has its own random variate identifier, which is ..AIR070.1. The subroutine returns the value and places it in .SHOT.DURATION.SEC, which is then used in the lines following the subroutine call.

```
IF .DTBM.WPN.TYPE = ..AIRBORNE.LASER
    OR .DTBM.WPN.TYPE = ..SPACE.BASED.LASER
    CALL UTL.720.CALCULATE.USER.FUNCTION
        GIVEN
            DWP.SHOT.DURATION.FN(.DWP),
            RN.STREAM(. . AIR070.1)
        YIELDING
            .SHOT.DURATION.SEC
    LET .SHOT.DURATION.SEC =
    MAX.F(.SHOT.DURATION.SEC, 0.0)
    LET .TOTAL.ELAPSED.SECONDS
        = .DETECT.SECONDS.DELAY + .SHOT.DURATION.SEC
    LET .INTERCEPT.FLAG = ..YES
ELSE
```

Figure 3.5: Random Variate Subroutine Call Example (Original)
To collect the results, code is added after the subroutine returns the random variate, but before it is used. The variate result, along with the variate name, type and parameters are sent to the same EARL.STORE as in the first method. Below is the modified version of Figure 3.5.

```
    IF .DTBM.WPN.TYPE = ..AIRBORNE.LASER
        OR .DTBM.WPN.TYPE = ..SPACE.BASED.LASER
        CALL UTL.720.CALCULATE.USER.FUNCTION
            GIVEN
                DWP.SHOT.DURATION.FN(.DWP),
                RN.STREAM(. . AIR070.1)
            YIELDING
                .SHOT.DURATION.SEC
```

```
''* Earl stuff
```

''* Earl stuff
r********************
r********************
EARL.RESPONSE = .SHOT.DURATION.SEC
EARL.RESPONSE = .SHOT.DURATION.SEC
EB.P1 = UF.PARM1(DWP.SHOT.DURATION.FN(.DWP))
EB.P1 = UF.PARM1(DWP.SHOT.DURATION.FN(.DWP))
EB.P2 = UF.PARM2(DWP.SHOT.DURATION.FN(.DWP))
EB.P2 = UF.PARM2(DWP.SHOT.DURATION.FN(.DWP))
EB.P3 = 0.0
EB.P3 = 0.0
CALL EARL.STORE
GIVEN
..AIR070.1,
UF.FUNCTION(DWP.SHOT.DURATION.FN(.DWP)),
EARL.RESPONSE,
EB.P1,
EB.P2,
EB.P3
''* END STUFF
LET .SHOT.DURATION.SEC =
MAX.F(.SHOT.DURATION.SEC, 0.0)
LET .TOTAL.ELAPSED.SECONDS
= .DETECT.SECONDS.DELAY + .SHOT.DURATION.SEC
LET .INTERCEPT.FLAG = ..YES
ELSE

```

Figure 3.6: Random Variate Subroutine Call Example (Modified)
Note the code added to THUNDER is identified at the beginning of a section with Earl stuff and at an ending of a section with END STUFF. For a full comparison, Appendix
(E) and Appendix (F) contain the full unmodified and modified code respectively of Figure 3.3 through Figure 3.6.

A single five day run of the ME scenario in THUNDER can result in over 500,000 random variate draws. Outputting every random variate can create a very large output file. To save time and space, the random variates are stored as a table within THUNDER. Therefore, instead of storing the responses individually, the table keeps a running sum of the responses for repeated random variate draws and keeps track of the number of draws. THUNDER outputs this matrix in to the earltally.txt file upon completion of a replication. Figure 3.2 is an example of how the data is stored.

Using the script outlined above, earltally.txt can be combined with MOEs and displayed as a table that can be entered into a regression software package. Regression requires the amount of replications to equal or exceed the final number of control variates. Since there is a possibility of having 254 control variates, at least 254 replications should be performed to allow for the maximum number of control variates. Stepwise regression is preferred to single out the control variates that have statistically significant correlation with the selected MOE. There are many ways to perform a stepwise regression; the stepwisefit function in Matlab was used for this research. Three hundred replications of the scenario were used. Then a matrix of potential controls and MOEs were imported into a Matlab script. The matrix of potential controls contained the potential control variates left from the parsing script minus the known mean. Then stepwisefit with a parameter of max p-value to enter of 0.06 was used to identify the controls that are statistically significant to each MOE. After the controls were identified for each MOE, the MOEs are regressed with their corresponding controls and output into
an Excel workbook. A halfwidth for each MOE can be calculated with the known controls. This new halfwidth should be smaller than a halfwidth calculated from replications.

\subsection*{3.6. Common Random Numbers}

As stated before, THUNDER had previously been coded for CRNs. The coding identifies the points where random numbers are drawn. Therefore, potential synchronization of the random number draws was already in place. Also, the control.dat file contains a method of assigning points where random variates are drawn from a specific random number stream. The problem with using CRNs with THUNDER is that the random number generator (RNG) only allows for 10 streams, while there are 254 random inputs where streams could be used. To solve this problem, a new random number generator was added to THUNDER. The random number generator proposed by Pierre L'Ecuyer (18) has \(1.8 \times 10^{19}\) streams, each of length \(1.7 \times 10^{38}\) elements. To put this in perspective, a standard software package RNG normally does not have streams and its entire length is generally between \(1.6 \times 10^{7}\) and \(2.8 \times 10^{14}\) elements. As an added benefit, the random number generator is already coded in C with a support package. SIMSCRIPT II. 5 will readily compile routines programmed in C and incorporates methods to call the compiled routines. Therefore, the proposed random number generation package could be compiled with THUNDER.

The current version of THUNDER tallies how many random number calls are made during a simulation run. It does this by rerouting a random number draw to a subroutine that increments a counter, draws a uniform \([0,1]\) random number, and returns the uniform \([0,1]\) random number. Therefore to direct THUNDER to use the new RNG
all that is needed to be done is replace the uniform \([0,1]\) random number draw from the standard function with a call to the new random number routines.

Not all random inputs are uniform [0, 1] random numbers. When THUNDER was coded to tally the number of random number draws, a series of subroutines were built to transform uniform \([0,1]\) random numbers into random variates from specified distributions with given parameters.

A new RNG designator has been placed in the control.dat file, Appendix (D), used to run a THUNDER replication. If this is set to YES, THUNDER will use the new RNG. When the new RNG is used, it reads a new seed file that contains the number of the first stream. If CRN designator in control.dat is also set to YES, THUNDER will then create a separate stream for each of the 254 points where random variates are drawn. At the end of a replication, THUNDER records the next starting seed in the seed file for each replication.

\subsection*{3.7. Antithetic Variates}

Antithetic variates, like CRN, rely on synchronization. Since THUNDER has been setup with CRN, AV can utilize the synchronization. To implement AV, a replication of the simulation should be run normally with the new RNG and CRN in control.dat set to YES. Then a second replication of the simulation with the same streams should be run with the new RNG, CRN, and AV in control.dat set to YES. This will cause the simulation to be run with all the random numbers to be \(1-\) the random numbers of the previous replication. Then the two replications are averaged to obtain the variance reduction. This may seem tedious, but if all the random input are synchronized as in CRN, then a set of replications can be run normally and then followed by a set of
antithetic runs synchronized to the previous runs. Upon completion, the MOEs can be averaged.

The new RNG package compiled with THUNDER comes complete with the ability to use antithetic variates. The way this is implemented is by designating a stream as antithetic upon creation. The modified control.dat file has an antithetic designator. Setting the designator to YES, will assign all random number streams to antithetic. In addition, to perform AV effectively, the common random number designator should also be set to YES for both the normal and antithetic replications.

\subsection*{3.8. Integrated Control and Antithetic Variates}

The third method outlined by Kwon and Tew (15) was used to analyze the variance reduction gained by integrating Control Variates and Antithetic Variates. Their method is relatively simple. First, run a simulation as you would for AVs with \(n\) replications, while producing the random variate output file. Then, combine each of the \(n / 2\) antithetic pairs and average them. This includes averaging both corresponding random variates and corresponding MOEs. The result is \(n / 2\) independent antithetic pairs. Then, use the antithetic pairs to perform a normal Control Variate analysis.

\subsection*{3.9. Integrated Control Variates and Common Random Numbers}

To analyze the variance reduction by integrating Control Variates and Common Random Numbers a modification of the method employed by Yang and Nelson (44) was used because Yang and Nelson's multiple comparisons relies on identical control variates and THUNDER does not produce identical control variates, even with synchronization. The modified method is the application of the control variate estimators to develop
paired-t confidence intervals. First, independent THUNDER runs were performed using the base model database and the modified model database. These runs were used as a base paired-t confidence interval and to identify the random variates that are significantly correlated with the MOEs. These random variates were then synchronized while the rest were left independent for the analysis runs. Then, the control variate estimators identified in the base runns estimate the response and are used in the paired-t confidence intervals. The normal distribution is used instead of the studentized \(t\) distribution of calculating the halfwidth. This is because the studentized-t distribution resembles the normal distribution when the degrees of freedom become greater than 150 .

\subsection*{3.10. Divide and Conquer and the Major Shared Resource Center}

The first 2400 THUNDER replications were performed on the ASC MSRC HPC resources, while the last were performed on the AFIT Linux cluster. To perform the different types of analysis for this research, THUNDER needed to be run under several different settings. To organize and set up all of the replications with the different settings can be cumbersome. To make this practical, the ASC MSRC Divide and Conquer (DAC) web interface was used. The following diagram depicts the DAC process.


Figure 3.7: Divide and Conquer Flow Diagram (29)
The DAC has a graphical user interface (GUI) that loads a simulation template and allows the user to set up and modify select parameters for analysis. It even allows the user to select multiple levels of a parameter. After the parameters are set, the DAC web interface will automatically organize the runs and create scripts to perform the runs. These scripts are compressed into a .tar file and delivered to the proper HPC resource, unpacked and run.

\subsection*{3.11. Analysis}

To analyze the data, the confidence interval halfwidths of the base case, no VRT, are compared with the confidence interval halfwidths generated by the variance reduction technique. Access databases were built to manipulate the data into the MOEs for each technique and then exported to Excel for ease of comparison. Then the variance reduction is calculated by dividing the difference between the base halfwidth and the VRT halfwith by the base halfwidth.

A paired-t confidence interval is used, to determine if a pair of values is the same.
This test determines the pair of values to be equal if the confidence interval includes zero. All statistical tests were calculated at the \(\alpha=0.05\) level.

Since there were three types of variance reduction techniques to analyze, different experiment settings were needed. A total of eight different run settings were identified for all of the experiments, with 300 replications at each setting for 2400 total THUNDER replications. The following table identifies the different setting used for the VRT analysis.

Table 3.5: Experiment Settings
\begin{tabular}{|c|c|c|c|c|c|}
\hline & RNG & SYNCHRONIZED & ANTITHETIC & COLLECT CV & CONFIGURATION \\
\hline SET & (NEWIOLD) & \((\) YES/NO \()\) & \((\) YES/NO & \((\) YES/NO) & (ORIGINAL/MODIFIED) \\
\hline A & OLD & NO & NO & YES & ORIGINAL \\
\hline B & NEW & NO & NO & NO & ORIGINAL \\
\hline C & NEW & NO & NO & YES & MODIFIED \\
\hline D & NEW & YES & NO & YES & ORIGINAL \\
\hline E & NEW & YES & NO & YES & MODIFIED \\
\hline F & NEW & YES & YES & YES & ORIGINAL \\
\hline G & OLD & NO & NO & NO & ORIGINAL \\
\hline\(H\) & NEW & YES & NO & NO & ORIGINAL \\
\hline M & NEW & NO* & NO & YES & ORIGINAL \\
\hline O & NEW & NO* & NO & YES & MODIFIED \\
\hline S & NEW & YES* \(^{*}\) & NO & YES & ORIGINAL \\
\hline T & NEW & YES* & NO & YES & MODIFIED \\
\hline
\end{tabular}

RNG identifies use of the current (OLD) or MRG32k3a (NEW) random number generator. SYNCHRONIZED indicates wheter all of the random inputs are synchronized or not. The values of (YES*) indicates that only some of the random variates are synchronized and the values (NO*) indicates a different stream than 1 was used to generate the independent random variates. ANTITHETIC identifies if the random number draws are drawn normally (NO) or using 1- \(U\) (YES). COLLECT CV identifies whether the replications output a controls file. CONFIGURATION inidicates
the use of the standard ME model database (ORIGINAL) or the use of the ME model database with the 209 F-111's eliminated (MODIFIED).

\section*{4. Analysis}

This section contains results and examples of analysis performed to determine the effectiveness of the variance reduction techniques applied to THUNDER. The first three sections cover the results from applying the control variate technique, common random numbers, and antithetic variates respectively. The fourth section details the combination of both control variates and antithetic variates. The fifth sections reports on the combination of control variates with common random numbers. The last section covers the comparison between the new random number generator and the original random number generator.

\subsection*{4.1. Control Variates}

A total of 600 replications of the ME scenario were performed to analyze the effectiveness of the control variate technique. Table 4.1 details the experiment settings; remember that each set consists of 300 replications. All of the replications used the original random number generator, with no common random numbers, with no antithetic variates and used the original configuration of the ME scenario. The first 300 replications, set A, generated control variate information. They were used to determine the halfwidth for each MOE with and without control variates. The second 300 replications, set G, were generated without control variates. They were used, in conjunction with the set A , for comparing the difference in speed between generating control variates and not generating control variates.

Table 4.1: Control Variate Experiment Settings
\begin{tabular}{|c|c|c|c|c|c|}
\hline & RNG & SYNCHRONIZED & ANTITHETIC & COLLECT CV & CONFIGURATION \\
\hline SET & (NEWIOLD) & (YES/NO) & (YES/NO) & (YES/NO) & (ORIGINAL/MODIFIED) \\
\hline A & OLD & NO & NO & YES & ORIGINAL \\
\hline G & OLD & NO & NO & NO & ORIGINAL \\
\hline
\end{tabular}

The following analysis compares the halfwidths generated by standard replication and control variates. The second subsection estimates the number of extra replications needed to achieve a similar reduction as the reduction achieved through control variates. The following subsection demonstrates how control variates may be used in gaining insight into the workings of the simulation. The last subsection provides an analysis of the simulation speed, comparing time needed to produce a control variate output file and the time needed to run a simulation without producing a control variate file.

\subsection*{4.1.1. Replications vs. Variance Reduction Technique.}

The ME scenario was replicated 300 times, generating the control variate file and in turn, creating a potential control variate table. The control variate table, along with the response table was processed through the control variate script to produce the control variate halfwidths. Then the halfwidths for each MOE were calculated at the \(\alpha=0.05\) level and compared to the control variate halfwidths. The results are below.

Table 4.2: Control Variate Halfwidth Reduction
\begin{tabular}{|l|r|r|r|r|r|}
\hline \multicolumn{1}{|c|}{ MOE } & \multicolumn{2}{c|}{ REPLICATION } & \multicolumn{2}{c|}{ CONTROL VARIATE } & PERCENT \\
& \multicolumn{1}{c|}{ MEAN } & \multicolumn{1}{c|}{ HALFWIDTH } & \multicolumn{1}{c|}{ MEAN } & \multicolumn{1}{c|}{ HALFWIDTH } & CHANGE \\
\hline A-10 LPS & 0.004928615 & 0.000271598 & 0.004947585 & 0.000270219 & \(-0.51 \%\) \\
\hline A-6 LPS & 0.018843645 & 0.001023321 & 0.018829341 & 0.001014611 & \(-0.85 \%\) \\
\hline BLUE FPS & 0.581540025 & 0.002144585 & 0.581287551 & 0.002034784 & \(-5.12 \%\) \\
\hline BLUE LPS & 0.010229418 & 0.00015922 & 0.010225498 & 0.000157836 & \(-0.87 \%\) \\
\hline BLUE SMG & 34487.47 & 81.58752506 & 34501.15753 & 79.42601286 & \(-2.65 \%\) \\
\hline DHAHRAN FPS & 0.374067144 & 0.007240438 & 0.373871063 & 0.007070694 & \(-2.34 \%\) \\
\hline F-111 LPS & 0.017016367 & 0.000422881 & 0.016995175 & 0.00041728 & \(-1.32 \%\) \\
\hline F-15 LPS & 0.00642758 & 0.000296031 & 0.006433521 & 0.000293957 & \(-0.70 \%\) \\
\hline FA-18 FPS total & 0.232338281 & 0.003721562 & 0.232495993 & 0.003661179 & \(-1.62 \%\) \\
\hline FA-18 FPS DCA & 0.043787767 & 0.001197699 & 0.043683747 & 0.001181571 & \(-1.35 \%\) \\
\hline MIG-23 LPS & 0.253296028 & 0.006221384 & 0.252744641 & 0.00619634 & \(-0.40 \%\) \\
\hline MIG-29 LPS & 0.195555155 & 0.007370947 & 0.19570513 & 0.007264416 & \(-1.45 \%\) \\
\hline MIRAGE FPS total & 0.598071397 & 0.004573737 & 0.598463965 & 0.004517323 & \(-1.23 \%\) \\
\hline MIRAGE FPS INT & 0.702633484 & 0.005673337 & 0.703846015 & 0.005482963 & \(-3.36 \%\) \\
\hline MIRAGE LPS & 0.274969404 & 0.00452846 & 0.275087315 & 0.004501166 & \(-0.60 \%\) \\
\hline MUDAYSIS FPS & 0.19221417 & 0.005066926 & 0.192327882 & 0.004982654 & \(-1.66 \%\) \\
\hline RED FPS & 0.391637103 & 0.001710069 & 0.39176041 & 0.001693386 & \(-0.98 \%\) \\
\hline RED LPS & 0.192105922 & 0.001578977 & 0.19217664 & 0.001531504 & \(-3.01 \%\) \\
\hline RIYADH FPS & 0.855569083 & 0.001545936 & 0.855668121 & 0.001516207 & \(-1.92 \%\) \\
\hline SHAIBAH FPS & 0.499191886 & 0.006482834 & 0.499297882 & 0.006409984 & \(-1.12 \%\) \\
\hline
\end{tabular}

A reduction in all the halfwidths is realized. The reduction may be small, but this is expected because as we can see in Table 4.3, \(\mathrm{R}^{2}\) from each model is also small.

Table 4.3: Control Variate RSquared
\begin{tabular}{|l|r|r|}
\hline \multicolumn{1}{|c|}{ MOE } & RSQUARED & \multicolumn{1}{r|}{\begin{tabular}{r} 
RDJUSTED \\
RSQUARED
\end{tabular}} \\
\hline A-10 LPS & 0.017990681 & 0.017990681 \\
\hline A-6 LPS & 0.026694585 & 0.02342846 \\
\hline BLUE FPS & 0.12506954 & 0.11018977 \\
\hline BLUE LPS & 0.021109417 & 0.021109417 \\
\hline BLUE SMG & 0.100168792 & 0.081742215 \\
\hline DHAHRAN FPS & 0.059778386 & 0.053446928 \\
\hline F-111 LPS & 0.040550054 & 0.034089112 \\
\hline F-15 LPS & 0.024657888 & 0.021384928 \\
\hline FA-18 FPS total & 0.046169914 & 0.039746816 \\
\hline FA-18 FPS DCA & 0.038015219 & 0.034787082 \\
\hline MIG-23 LPS & 0.025816687 & 0.022547616 \\
\hline MIG-29 LPS & 0.060690505 & 0.051170476 \\
\hline MIRAGE FPS total & 0.044797447 & 0.038365106 \\
\hline MIRAGE FPS INT & 0.095539229 & 0.083275355 \\
\hline MIRAGE LPS & 0.016059425 & 0.016059425 \\
\hline MUDAYSIS FPS & 0.052473379 & 0.046092729 \\
\hline RED FPS & 0.026028565 & 0.026028565 \\
\hline RED LPS & 0.06941087 & 0.063144276 \\
\hline RIYADH FPS & 0.066740758 & 0.05728205 \\
\hline SHABAH FPS & 0.029355585 & 0.026098389 \\
\hline
\end{tabular}

A graphical example of the variance reduction can be seen in the following examples.
They are the largest reduction, BLUE Flown per Planned Sortie, and the smallest reduction, MIG-23 Losses per Sortie.


Figure 4.1: BLUE Flown per Planned Sortie Confidence Intervals


Figure 4.2: MIG-23 Losses per Sortie Confidence Intervals
There is a shift in the means between the replication and control varates. This is because the calculations for the replication mean is calculated by using the arithmetic average and the CV mean is calculated from regression using least squares estimates. Using the means from Table 4.2, the following tabe was genereated and the absolute percent change was calculated.

Table 4.4: Absolute Percent Change In Means
\begin{tabular}{|l|r|r|r|}
\hline \multicolumn{1}{|c|}{ MOE } & \begin{tabular}{r} 
REPLICATION \\
MEAN
\end{tabular} & \begin{tabular}{c} 
CV \\
MEAN
\end{tabular} & \begin{tabular}{r} 
PERCENT \\
CHANGE
\end{tabular} \\
\hline A-10 LPS & 0.004928615 & 0.004947585 & \(0.38 \%\) \\
\hline A-6 LPS & 0.018843645 & 0.018829341 & \(0.08 \%\) \\
\hline BLUE FPS & 0.581540025 & 0.581287551 & \(0.04 \%\) \\
\hline BLUE LPS & 0.010229418 & 0.010225498 & \(0.04 \%\) \\
\hline BLUE SMG & 34487.47 & 34501.15753 & \(0.04 \%\) \\
\hline DHAHRAN FPS & 0.374067144 & 0.373871063 & \(0.05 \%\) \\
\hline F-111 LPS & 0.017016367 & 0.016995175 & \(0.12 \%\) \\
\hline F-15 LPS & 0.00642758 & 0.006433521 & \(0.09 \%\) \\
\hline FA-18 FPS total & 0.232338281 & 0.232495993 & \(0.07 \%\) \\
\hline FA-18 FPS DCA & 0.043787767 & 0.043683747 & \(0.24 \%\) \\
\hline MIG-23 LPS & 0.253296028 & 0.252744641 & \(0.22 \%\) \\
\hline MIG-29LPS & 0.195555155 & 0.19570513 & \(0.08 \%\) \\
\hline MIRAGE FPS total & 0.598071397 & 0.598463965 & \(0.07 \%\) \\
\hline MIRAGE FPS INT & 0.702633484 & 0.703846015 & \(0.17 \%\) \\
\hline MIRAGE LPS & 0.274969404 & 0.275087315 & \(0.04 \%\) \\
\hline MUDAYSIS FPS & 0.19221417 & 0.192327882 & \(0.06 \%\) \\
\hline RED FPS & 0.391637103 & 0.39176041 & \(0.03 \%\) \\
\hline RED LPS & 0.192105922 & 0.19217664 & \(0.04 \%\) \\
\hline RIYADH FPS & 0.855569083 & 0.855668121 & \(0.01 \%\) \\
\hline SHAIBAH FPS & 0.499191886 & 0.499297882 & \(0.02 \%\) \\
\hline
\end{tabular}

As can be seen above, all of the absolute percent changes in means are positive, therefore the shift in means is relatively insignificant.

\subsection*{4.1.2. Variance Reduction Technique to Number of Replications.}

Another way of looking at the reduction gained by using control variates it to calculate the number of replications needed to achieve the control variate halfwidth. To do this, the halfwidth calculation is iterated by incrementing the number of replications in the equation until the halfwidth is smaller than the control variate halfwidth. A major assumption to this method is the variance remains constant. The following table displays the number of replications needed to achieve the variance reduction.

Table 4.5: Number of Replications Required to Achieve Control Variate Reduction
\begin{tabular}{|l|r|r|r|}
\hline \multicolumn{1}{|c|}{ MOE } & \begin{tabular}{c} 
CHANGE IN
\end{tabular} & \begin{tabular}{r} 
PERCENT \\
CHANGE
\end{tabular} \\
\hline A-10 LPS & 304 & 4 & \(1.33 \%\) \\
\hline A-6 LPS & 306 & 6 & \(2.00 \%\) \\
\hline BLUE FPS & 333 & 33 & \(11.00 \%\) \\
\hline BLUE LPS & 306 & 6 & \(2.00 \%\) \\
\hline BLUE SMG & 317 & 17 & \(5.67 \%\) \\
\hline DHAHRAN FPS & 315 & 15 & \(5.00 \%\) \\
\hline F-111 LPS & 309 & 9 & \(3.00 \%\) \\
\hline F-15 LPS & 305 & 5 & \(1.67 \%\) \\
\hline FA-18 FPS total & 310 & 10 & \(3.33 \%\) \\
\hline FA-18 FPS DCA & 309 & 9 & \(3.00 \%\) \\
\hline MIG-23 LPS & 303 & 3 & \(1.00 \%\) \\
\hline MIG-29 LPS & 309 & 9 & \(3.00 \%\) \\
\hline MIRAGE FPS total & 308 & 8 & \(2.67 \%\) \\
\hline MIRAGE FPS INT & 321 & 21 & \(7.00 \%\) \\
\hline MIRAGE LPS & 304 & 4 & \(1.33 \%\) \\
\hline MUDAYSIS FPS & 311 & 11 & \(3.67 \%\) \\
\hline RED FPS & 306 & 6 & \(2.00 \%\) \\
\hline RED LPS & 319 & 19 & \(6.33 \%\) \\
\hline RIYADH FPS & 312 & 12 & \(4.00 \%\) \\
\hline SHAIBAH FPS & 307 & 7 & \(2.33 \%\) \\
\hline
\end{tabular}

\subsection*{4.1.3. Control Variate Insight.}

The method of control variates has an added benefit to the analyst. It generates a series of random inputs that are statistically significant in their correlation to the selected MOE. Knowledge about these random inputs can give insight into how the simulation works. First, knowing the weight and sign of the coefficient can inform the analyst how much and in which direction a shift in the control can alter the selected MOE. The following table identifies the significant controls for each MOE and gives the weight and sign of the corresponding control.

Table 4.6: Control Variate Statistically Significant Controls
\begin{tabular}{|c|c|c|c|c|c|}
\hline MOE & WEIGHT & CONTROL & MOE & WEIGHT & CONTROL \\
\hline A-10 LPS & -0.0002 & AIR060_2 & \multirow[t]{2}{*}{MIG-23 LPS} & -6.6526 & ADF121_2 \\
\hline \multirow[t]{2}{*}{A-6 LPS} & -0.0071 & AIR050_2 & & 0.1433 & AIR101_1 \\
\hline & -0.0759 & ISR000_4 & \multirow[t]{4}{*}{MIG-29 LPS} & 0.6471 & AIR528_1 \\
\hline \multirow[t]{6}{*}{BLUE FPS} & -0.0722 & AIR050_1 & & -1.4314 & BSE050_1 \\
\hline & -0.0659 & AIR070_2 & & -0.2311 & GRD095_1 \\
\hline & -0.0035 & AIR800_3 & & -0.7147 & ISR000_4 \\
\hline & 0.3282 & BSE400_1 & \multirow[t]{3}{*}{MIRAGE FPS total} & -0.3242 & ADF150_1 \\
\hline & 46.8237 & PLA443_1 & & 0.5198 & AlR561_1 \\
\hline & 0.6659 & UTL104_1 & & 0.0855 & AIR602_1 \\
\hline BLUE LPS & 0.0101 & ADF150_2 & \multirow[t]{5}{*}{MIRAGE FPS INT} & -3.3544 & ADF121_1 \\
\hline \multirow[t]{7}{*}{BLUE SMG} & -23825.1823 & ADF105_1 & & 7.9362 & ADF121_2 \\
\hline & 40.5388 & AIR060 2 & & 0.0101 & AIR800_3 \\
\hline & 3649.5981 & AIR101_2 & & -1.0679 & BSE003_2 \\
\hline & -1408.6753 & AIR561_2 & & 106.4336 & PLA443_1 \\
\hline & -20765.0668 & AIR840_1 & MIRAGE LPS & -2.3615 & ADF121_1 \\
\hline & -27766.1967 & AIR840_2 & \multirow[t]{3}{*}{MUDAYSIS FPS} & -0.2438 & ADF150_2 \\
\hline & -8446.2005 & BSE400_1 & & -0.0089 & AIR800_3 \\
\hline \multirow[t]{3}{*}{DHAHRAN FPS} & -0.1779 & AIR050_1 & & 0.1621 & GRD095_2 \\
\hline & -0.2230 & AIR070_2 & RED FPS & 0.6949 & BSE200 5 \\
\hline & 0.9677 & BSE400_1 & \multirow[t]{3}{*}{RED LPS} & 0.0922 & ADF150_1 \\
\hline \multirow[t]{3}{*}{F-111 LPS} & 0.0213 & ADF150_2 & & -0.0629 & AIR070_2 \\
\hline & -0.0120 & AIR060_1 & & 0.6117 & UUTL104_1 \\
\hline & -0.1221 & BSE200_5 & \multirow[t]{4}{*}{RIYADH FPS} & -0.2501 & AIR007_1 \\
\hline \multirow[t]{2}{*}{F-15 LPS} & 0.0069 & AIR050_1 & & -0.0395 & AIR101_1 \\
\hline & 0.0019 & AIR050_2 & & -0.0621 & AIR101_2 \\
\hline \multirow[t]{3}{*}{FA-18 FPS total} & 0.0252 & AIR050_2 & & -0.0505 & GRD095_1 \\
\hline & -0.0889 & AIR070_2 & \multirow[t]{2}{*}{SHAIBAH FPS} & -0.1675 & AIR101_1 \\
\hline & -1.3057 & AIR810_2 & & 1.3115 & BSE003_1 \\
\hline \multirow[t]{2}{*}{FA-18 FPS DCA} & 1.4195 & ADF121_3 & \multicolumn{3}{|l|}{\multirow[t]{2}{*}{}} \\
\hline & 0.0006 & AIR060_2 & & & \\
\hline
\end{tabular}

The table above can also be used in conjunction with knowledge of the simulation code. If the analyst looks at how the controls are used, he may be able to identify systems or actions that affect the MOE. For example, take MOE BLUE SMG (Square Miles Gained). The controls ADF105_1, AIR060_2, AIR101_2, AIR561_2, AIR840_1, AIR840_2, and BSE400_1 are all significant. By looking at the code, the use of each of the random variates can be identified by examination of the module purpose, the
comments in the code and the nomenclature of the interacting variables. The following table outlines information gleaned from the code.

Table 4.7: Control Variate Purpose of Statistically Significant Controls
\begin{tabular}{|l|c|l|l|}
\hline CONTROL & WEIGHT & \multicolumn{1}{c|}{ MODULEPURPOSE } & \multicolumn{1}{c|}{ SPECIFIC DRAW PURPOSE } \\
\hline ADF105_1 & -23825.18 & \begin{tabular}{l} 
Used to calculate the flight groups \\
position, altitude, speed, average \\
location, and delivery profile.
\end{tabular} & \begin{tabular}{l} 
Used to check for point on battlefield. \\
(Distribution)
\end{tabular} \\
\hline AIR060_2 & 40.54 & \begin{tabular}{l} 
Manages defensive anti tactical \\
ballistic missile mission detections and \\
engagements.
\end{tabular} & \begin{tabular}{l} 
Used in setting saturation delay on \\
shooter. (Distribution)
\end{tabular} \\
\hline AIR101_2 & 3649.60 & \begin{tabular}{l} 
Sets up and air engagement by creating \\
and air-to-air engagement for each \\
flight to be used for storing \\
computations.
\end{tabular} & \begin{tabular}{l} 
Used in determining the engagement \\
probability. (prob.engage \(<\) U( 0,1\())\)
\end{tabular} \\
\hline AIR561_2 & -1408.68 & \begin{tabular}{l} 
Determines if, having arrived at the \\
target's estimated coordinate, the flight \\
group can find the target.
\end{tabular} & \begin{tabular}{l} 
Determines the success of a mid- \\
course update. (U( 0,1\()<=\).best.prob)
\end{tabular} \\
\hline AIR840_1 & -20765.07 & \begin{tabular}{l} 
Determines the number of target \\
elements that are destroyed when a \\
given weapons area of effect fully or \\
partially covers a target.
\end{tabular} & \begin{tabular}{l} 
Determine whether a live target element \\
is hit. (U(0,1)<.prob.hit.live)
\end{tabular} \\
\hline AIR840_2 & -27766.20 & \begin{tabular}{l} 
Determines the number of target \\
elements that are destroyed when a \\
given weapons area of effect fully or \\
partially covers a target.
\end{tabular} & \begin{tabular}{l} 
Determine whether a live target element \\
is hit. (U(0,1)<.exp.live.hits)
\end{tabular} \\
\hline BSE400_1 & -8446.20 & \begin{tabular}{l} 
Determine s the availability of aircraft, \\
establishes the munition \\
configurations, allocates fuels and \\
determines whether to cancel or run the \\
mission as scheduled.
\end{tabular} & \begin{tabular}{l} 
Determines the cancellation of a sortie \\
due to degrade. \\
\((\) U(0,1)<eawp.sortie.degrade)
\end{tabular} \\
\hline
\end{tabular}

Using the chart above, a number of system characteristics can be determined.
First, by increasing in the saturation delay on the shooter, there will be a small increase in
BLUE SMG. Second, it can be determined that increasing the number of random draws greater than the engagement probability will greatly increase the BLUE SMG. Another way to look at this is decreasing the air-to-air engagements will increase the BLUE SMG.

Third, by icreasing the success of a mid-course update will increase the BLUE SMG.
Fourth, increasing the probability of hitting a live target will greatly increase the BLUE

SMG. Fifth, the greatest increase in BLUE SMG is related to the number of sorties. If the degrade in the sorties are large, then the number of sorties are reduced and the BLUE SMG is also reduced. An overall look at this implies that the more sorties that are generated and the fewer enemy engagements along with better strikes will lead to a more BLUE Square Miles Gained. This same analysis can be applied to the control variates related to other MOEs.

\subsection*{4.1.4. Replication Speed With And Without Generating Control Variates.}

To test the difference in running times, the start and completion times for each replication is recorded and the difference is calculated in seconds. Then a paired-t CI is calculated and if the CI contains zero, it is determined that the times are not statistically different. The 300 replications performed while producing control variate information, set A, are compared to the 300 replications performed while not producing control variate information, set G, by subtracting the replication times of set G from set A . The confidence interval on the difference is \((0.01284,0.18049)\). A graphical representation is below.


Figure 4.3: Replication Completion Difference Confidence Interval
Since the confidence interval does not contain zero, the completion times are statistically different at the \(\alpha=0.05\) level. Since the times for replications not generating the control
variates are subtracted from the times for replications generating the control variates, the CI implies that the times for generating the control variates are larger.

\subsection*{4.2. Common Random Numbers}

To analyze the effectiveness of the CRN technique, a total of 1200 replications were performed. Table 4.8 details the experiment settings; remember that each set consists of 300 replications. The sets can be separated into two groups. The first group is the standard non-synchronized data that makes up the base data and the second group is the synchronized data. All replications were of the standard ME scenario, using the new RNG. The first 300 replications, set B, contained no changes to the database and with all random inputs using a single stream for each replication. The second 300 replications, set C , were performed with all of the \(\mathrm{F}-111 \mathrm{~s}\) removed from the database and with all random inputs using the same single streams applied to set B. The third 300 replications, set D , contained no changes to the database and were performed with all of the random inputs synchronized therefore, receiving their own random number stream in each replication. The last 300 replications, set E, were performed with all of the F-111s removed from the database and with the random inputs synchronized with set D .

Table 4.8: Common Random Number Experiment Settings
\begin{tabular}{|c|c|c|c|c|c|}
\hline & RNG & SYNCHRONIZED & ANTITHETIC & COLLECT CV & CONFIGURATION \\
\hline SET & (NEWIOLD) & \((\) YES/NO \()\) & \((\) YES/NO \()\) & \((\) YES/NO) & (ORIGINAL/MODIFIED) \\
\hline B & NEW & NO & NO & NO & ORIGINAL \\
\hline C & NEW & NO & NO & YES & MODIFIED \\
\hline D & NEW & YES & NO & YES & ORIGINAL \\
\hline E & NEW & YES & NO & YES & MODIFIED \\
\hline
\end{tabular}

The following analysis compares the halfwidths generated by calculating the difference between scenario configurations. The first comparison is by the standard
method, not synchronized, and the second comparison is by CRN with synchronization.
The second section highlights insights gleaned from using CRN.

\subsection*{4.2.1. Normal to Variance Reduction Technique.}

The paired-t CI at the \(\alpha=0.05\) level was used to determine whether the MOEs
generated from a configuration change in the ME scenario resulted in statistically
different values. Then the paired-t halfwidths were compared to determine the reduction in variance. The following table displays the results of this comparison.

Table 4.9: Common Random Number Halfwidth Reduction
\begin{tabular}{|l|r|r|r|r|r|}
\hline & \multicolumn{2}{|c|}{ BASE } & \multicolumn{2}{c|}{ CRN } & PERCENT \\
\multicolumn{1}{|c|}{ MOE } & \multicolumn{1}{|c|}{ MEAN } & \multicolumn{1}{c|}{ HALFWIDTH } & \multicolumn{1}{c|}{ MEAN } & HALFWIDTH & CHANGE \\
\hline A-10 LPS & -0.004978203 & 0.00053875 & -0.005019134 & 0.000533584 & \(-0.96 \%\) \\
\hline A-6 LPS & -0.003968323 & 0.001541612 & -0.003959031 & 0.001382776 & \(-10.30 \%\) \\
\hline BLUE FPS & 0.07850412 & 0.00280076 & 0.079916562 & 0.002443302 & \(-12.76 \%\) \\
\hline BLUE LPS & -0.000834882 & 0.000253594 & -0.00110882 & 0.000234114 & \(-7.68 \%\) \\
\hline BLUE SMG & 323.05 & 109.3136609 & 265.22 & 115.3944755 & \(5.56 \%\) \\
\hline DHAHRAN FPS & 0.047589307 & 0.008795164 & 0.046170201 & 0.007928092 & \(-9.86 \%\) \\
\hline F-15 LPS & -0.000518151 & 0.000438013 & -0.000617566 & 0.000422892 & \(-3.45 \%\) \\
\hline FA-18 FPS total & 0.023195554 & 0.004674708 & 0.023448124 & 0.004512142 & \(-3.48 \%\) \\
\hline FA-18 FPS DCA & 0.000753974 & 0.001907906 & -0.000751392 & 0.001799667 & \(-5.67 \%\) \\
\hline MIG-23 LPS & 0.010507245 & 0.010701722 & 0.016051307 & 0.008963076 & \(-16.25 \%\) \\
\hline MIG-29 LPS & 0.052738368 & 0.00954175 & 0.0621385 & 0.010117406 & \(6.03 \%\) \\
\hline MIRAGE FPS total & 0.01092006 & 0.006458806 & 0.008898713 & 0.005904645 & \(-8.58 \%\) \\
\hline MIRAGE FPS INT & -0.002057046 & 0.007664981 & -0.002582203 & 0.007259603 & \(-5.29 \%\) \\
\hline MIRAGE LPS & -0.001408885 & 0.006220598 & 0.005438013 & 0.005672349 & \(-8.81 \%\) \\
\hline MUDAYSIS FPS & 0.073151537 & 0.005964334 & 0.067373498 & 0.005894866 & \(-1.16 \%\) \\
\hline RED FPS & 0.034304208 & 0.002341687 & 0.032198334 & 0.002392857 & \(2.19 \%\) \\
\hline RED LPS & 0.02384662 & 0.002443123 & 0.023647381 & 0.002339682 & \(-4.23 \%\) \\
\hline RIYADH FPS & -0.072238335 & 0.001832508 & -0.071748036 & 0.001917861 & \(4.66 \%\) \\
\hline SHAIBAH FPS & 0.090617611 & 0.009656621 & 0.089769928 & 0.010008527 & \(3.64 \%\) \\
\hline
\end{tabular}

Looking above, we can see that the halfwidth is not always smaller. The changes highlighted above are the halfwidths that are reduced. This implies that there is not always a reduction in the variance when using CRN. The following two charts are a graphical representation of the largest reduction, MIG-23 LPS and the largest increase, MIG-29 LPS.


Figure 4.4: MIG-23 Losses per Sortie Value Difference Confidence Intervals The confidence interval generated for the base \((-0.00019,0.021209)\) and for the synchronized ( \(0.007088,0.025014\) ).


Figure 4.5: MIG-29 Losses per Sortie Value Difference Confidence Intervals The confidence interval generated for the base \((0.043197,0.06228)\) and for the synchronized ( \(0.052021,0.072256\) ).

\subsection*{4.2.2. Common Random Numbers Insights.}

As can be seen above in Figure 4.4, the decision of whether the MOE results between configuration are statistically different or not, may change between the base non-synchronized and the synchronized, as a result from both the change in halfwidth and a shift in the mean value. The following table displays the decision results among the MOEs and identifies any changes in the decisions between the base and synchronized
results. Each of the CIs developed for the MOEs below are calculated at the \(\alpha=0.05\) level and are calculated separately.

Table 4.10: Common Random Number Configuration Decision
\begin{tabular}{|l|l|l|}
\hline MOE & \multicolumn{1}{|c|}{\begin{tabular}{c} 
BASE \\
DECISION
\end{tabular}} & \multicolumn{1}{c|}{\begin{tabular}{c} 
CRN \\
DECISION
\end{tabular}} \\
\hline A-10 LPS & DIFFERENT & DIFFERENT \\
\hline A-6 LPS & DIFFERENT & DIFFERENT \\
\hline BLUE FPS & DIFFERENT & DIFFERENT \\
\hline BLUE LPS & DIFFERENT & DIFFERENT \\
\hline BLUE SMG & DIFFERENT & DIFFERENT \\
\hline DHAHRAN FPS & DIFFERENT & DIFFERENT \\
\hline F-15 LPS & DIFFERENT & DIFFERENT \\
\hline FA-18 FPS total & DIFFERENT & DIFFERENT \\
\hline FA-18 FPS DCA & NOT DIFFERENT & NOT DIFFERENT \\
\hline MIG-23 LPS & NOT DIFFERENT & DIFFERENT \\
\hline MIG-29 LPS & DIFFERENT & DIFFERENT \\
\hline MIRAGE FPS total & DIFFERENT & DIFFERENT \\
\hline MIRAGE FPS INT & NOT DIFFERENT & NOT DIFFERENT \\
\hline MIRAGE LPS & NOT DIFFERENT & NOT DIFFERENT \\
\hline MUDAYSIS FPS & DIFFERENT & DIFFERENT \\
\hline RED FPS & DIFFERENT & DIFFERENT \\
\hline RED LPS & DIFFERENT & DIFFERENT \\
\hline RIYADH FPS & DIFFERENT & DIFFERENT \\
\hline SHAIBAH FPS & DIFFERENT & DIFFERENT \\
\hline
\end{tabular}

Looking at the table above we can see that the decision changes only once, and it is highlighted in gray.

Since the change is not consistently positive or negative and the MOEs were selected to reflect a range of aggregation, it seems logical to see if there are any commonalities between the aggregation levels and reduction. When sorted into the different aggregation levels no pattern emerges. The only item of possible note is that the data collected at the squadron level are all positive. When the data is sorted by force, RED and BLUE, there is also no notable pattern.

\subsection*{4.3. Antithetic Variates}

A total of 600 replications were performed to analyze the affects of antithetic variates on THUNDER. Table 4.11 details the experiment settings; remember that each set consists of 300 replications. The first 300 replications, set D , were used to generate base confidence intervals for comparison and the first half of the antithetic pairs. The replications were performed with the new RNG , with each random input synchronized with its own stream, and antithetic variates not selected. The second 300 replications, set F, were the second half of the antithetic pairs. The replications were performed with the new RNG, with each random number input synchronized with the replications from set D , and antithetic variates turned on. Turning on antithetic variates forces all of the random numbers to be drawn as 1 - the original random number. The 300 antithetic pairs are generated from all 600 replications. The halfwidths were calculated at the \(\alpha=0.05\) level.

Table 4.11: Antithetic Variate Experiment Settings
\begin{tabular}{|c|c|c|c|c|c|}
\hline & RNG & SYNCHRONIZED & ANTITHETIC & COLLECT CV & CONFIGURATION \\
\hline SET & (NEWIOLD) & (YES/NO) & (YES/NO) & (YES/NO) & (ORIGINAL/MODIFIED) \\
\hline D & NEW & YES & NO & YES & ORIGINAL \\
\hline F & NEW & YES & YES & YES & ORIGINAL \\
\hline
\end{tabular}

The first subsection contains the comparison of the antithetic variates halfwidths with the regular halfwidths. The second subsection contains the calculations to determine the number of replications needed to obtain the variance reduction obtained with antithetic variates. The last subsection highlights insights seen in antithetic analysis.

\subsection*{4.3.1. Normal to Variance Reduction Technique.}

To test the reduction in variance, the standard generated halfwidths of the MOEs are compared with the antithetic variate halfwidths of the MOEs. The following table reports the variance reduction due to antithetic variates.

Table 4.12: Antithetic Variate (300 Replication) vs. Replication Reduction
\begin{tabular}{|l|r|r|r|r|r|}
\hline \multicolumn{1}{|c|}{ MOE } & \multicolumn{2}{c|}{ BASE } & \multicolumn{1}{c|}{ ANTITHETIC VARIATES } & PERCENT \\
& \multicolumn{1}{|c|}{ MEAN } & \multicolumn{1}{c|}{ HALFWIDTH } & \multicolumn{1}{c|}{ MEAN } & HALFWIDTH & CHANGE \\
\hline A-10 LPS & 0.005145212 & 0.00028569 & 0.004980108 & 0.000214215 & \(-25.02 \%\) \\
\hline A-6 LPS & 0.019104498 & 0.000928243 & 0.018614329 & 0.000708626 & \(-23.66 \%\) \\
\hline BLUE FPS & 0.582000237 & 0.002036313 & 0.581054499 & 0.001481472 & \(-27.25 \%\) \\
\hline BLUE LPS & 0.010170101 & 0.000174156 & 0.010111304 & 0.000112451 & \(-35.43 \%\) \\
\hline BLUE SMG & 34474.62 & 84.52936072 & 34526.50667 & 57.67080159 & \(-31.77 \%\) \\
\hline DHAHRAN FPS & 0.372019758 & 0.006733921 & 0.372211006 & 0.004820585 & \(-28.41 \%\) \\
\hline F-111 LPS & 0.016760015 & 0.000416308 & 0.016685204 & 0.000253913 & \(-39.01 \%\) \\
\hline F-15 LPS & 0.006207278 & 0.00031009 & 0.006242106 & 0.000208184 & \(-32.86 \%\) \\
\hline FA-18 FPS TOTAL & 0.233627904 & 0.003723047 & 0.232053597 & 0.002729652 & \(-26.68 \%\) \\
\hline FA-18 FPS DCA & 0.044337541 & 0.001389283 & 0.043733405 & 0.000830637 & \(-40.21 \%\) \\
\hline MIG-23 LPS & 0.257329967 & 0.006037934 & 0.257671361 & 0.004167409 & \(-30.98 \%\) \\
\hline MIG-29 LPS & 0.200408126 & 0.007942862 & 0.197127827 & 0.006042087 & \(-23.93 \%\) \\
\hline MIRAGE FPS TOTAL & 0.600422202 & 0.00449042 & 0.601209538 & 0.003406191 & \(-24.15 \%\) \\
\hline MIRAGE FPS INT & 0.699855667 & 0.005731393 & 0.702536288 & 0.003813333 & \(-33.47 \%\) \\
\hline MIRAGE LPS & 0.276394858 & 0.004501578 & 0.275381579 & 0.003051761 & \(-32.21 \%\) \\
\hline MUDAYSIS FPS & 0.189549709 & 0.00488572 & 0.1887675 & 0.003383585 & \(-30.75 \%\) \\
\hline RED FPS & 0.391872732 & 0.001772589 & 0.392056037 & 0.001094404 & \(-38.26 \%\) \\
\hline RED LPS & 0.192619582 & 0.001692612 & 0.191275098 & 0.001079866 & \(-36.20 \%\) \\
\hline RIYADH FPS & 0.854947018 & 0.001535138 & 0.854861142 & 0.00111826 & \(-27.16 \%\) \\
\hline SHAIBAH FPS & 0.504592164 & 0.006312332 & 0.50172746 & 0.004368135 & \(-30.80 \%\) \\
\hline
\end{tabular}

The reduction in halfwidths in the above table initially looks great, but the fact that the antithetic variates are generated with twice the number of replications may inflate the actual reduction. Therefore, the following antithetic variate vs. replication table was created using only the first 150 antithetic pairs.

Table 4.13: Antithetic Variate (150 Replication) vs. Replication Reduction
\begin{tabular}{|l|r|r|r|r|r|}
\hline \multicolumn{1}{|c|}{ MOE } & \multicolumn{2}{c|}{ BASE } & \multicolumn{1}{c|}{ ANTITHETIC VARIATES } & PERCENT \\
& \multicolumn{1}{c|}{ MEAN } & \multicolumn{1}{c|}{ HALFWIDTH } & \multicolumn{1}{c|}{ MEAN } & HALFWIDTH & CHANGE \\
\hline A-10 LPS & 0.005145212 & 0.00028569 & 0.004980108 & 0.000304497 & \(6.58 \%\) \\
\hline A-6 LPS & 0.019104498 & 0.000928243 & 0.018614329 & 0.001007279 & \(8.51 \%\) \\
\hline BLUE FPS & 0.582000237 & 0.002036313 & 0.581054499 & 0.002105844 & \(3.41 \%\) \\
\hline BLUE LPS & 0.010170101 & 0.000174156 & 0.010111304 & 0.000159844 & \(-8.22 \%\) \\
\hline BLUE SMG & 34474.62 & 84.52936072 & 34526.50667 & 81.9763507 & \(-3.02 \%\) \\
\hline DHAHRAN FPS & 0.372019758 & 0.006733921 & 0.372211006 & 0.006852237 & \(1.76 \%\) \\
\hline F-111 LPS & 0.016760015 & 0.000416308 & 0.016685204 & 0.000360925 & \(-13.30 \%\) \\
\hline F-15 LPS & 0.006207278 & 0.00031009 & 0.006242106 & 0.000295924 & \(-4.57 \%\) \\
\hline FA-18 FPS TOTAL & 0.233627904 & 0.003723047 & 0.232053597 & 0.003880073 & \(4.22 \%\) \\
\hline FA-18 FPS DCA & 0.044337541 & 0.001389283 & 0.043733405 & 0.001180712 & \(-15.01 \%\) \\
\hline MIG-23 LPS & 0.257329967 & 0.006037934 & 0.257671361 & 0.005923777 & \(-1.89 \%\) \\
\hline MIG-29 LPS & 0.200408126 & 0.007942862 & 0.197127827 & 0.008588544 & \(8.13 \%\) \\
\hline MIRAGE FPS TOTAL & 0.600422202 & 0.00449042 & 0.601209538 & 0.004841742 & \(7.82 \%\) \\
\hline MIRAGE FPS INT & 0.699855667 & 0.005731393 & 0.702536288 & 0.005420474 & \(-5.42 \%\) \\
\hline MIRAGE LPS & 0.276394858 & 0.004501578 & 0.275381579 & 0.004337935 & \(-3.64 \%\) \\
\hline MUDAYSIS FPS & 0.189549709 & 0.00488572 & 0.1887675 & 0.004809609 & \(-1.56 \%\) \\
\hline RED FPS & 0.391872732 & 0.001772589 & 0.392056037 & 0.001555645 & \(-12.24 \%\) \\
\hline RED LPS & 0.192619582 & 0.001692612 & 0.191275098 & 0.001534979 & \(-9.31 \%\) \\
\hline RIYADH FPS & 0.854947018 & 0.001535138 & 0.854861142 & 0.001589555 & \(3.54 \%\) \\
\hline SHAIBAH FPS & 0.504592164 & 0.006312332 & 0.50172746 & 0.006209099 & \(-1.64 \%\) \\
\hline
\end{tabular}

Now, with the number of replications used to generate the base and antithetic pairs being equal, the reduction becomes mixed. The halfwidth reductions are highlighted above.

\subsection*{4.3.2. Variance Reduction Technique to Number of Replications.}

The reduction gained can also be seen by calculating the number of replications needed to achieve the antithetic variate halfwidth. To do this, the halfwidth calculation is iterated by incrementing the number of replications in the equation until the halfwidth is smaller than the antithetic variate halfwidth. A major assumption to this method is the variance remains constant. The following table displays the number of replications needed to achieve the variance reduction with the 300 antithetic pairs.

Table 4.14: Replications Needed to Achieve Antithetic Variate Reduction (300 Replications)
\begin{tabular}{|c|c|c|c|}
\hline MOE & REPLICATIONS & CHANGE IN REPLICATIONS & PERCENT CHANGE \\
\hline A-10 LPS & 532 & 232 & 77.33\% \\
\hline A-6 LPS & 513 & 213 & 71.00\% \\
\hline BLUE FPS & 565 & 265 & 88.33\% \\
\hline BLUE LPS & 716 & 416 & 138.67\% \\
\hline BLUE SMG & 642 & 342 & 114.00\% \\
\hline DHAHRAN FPS & 583 & 283 & 94.33\% \\
\hline F-111 LPS & 802 & 502 & 167.33\% \\
\hline F-15 LPS & 662 & 362 & 120.67\% \\
\hline FA-18 FPS total & 556 & 256 & 85.33\% \\
\hline FA-18 FPS DCA & 834 & 534 & 178.00\% \\
\hline MIG-23 LPS & 627 & 327 & 109.00\% \\
\hline MIG-29 LPS & 517 & 217 & 72.33\% \\
\hline MIRAGE FPS total & 520 & 220 & 73.33\% \\
\hline MIRAGE FPS INT & 674 & 374 & 124.67\% \\
\hline MIRAGE LPS & 650 & 350 & 116.67\% \\
\hline MUDAYSIS FPS & 623 & 323 & 107.67\% \\
\hline RED FPS & 783 & 483 & 161.00\% \\
\hline RED LPS & 733 & 433 & 144.33\% \\
\hline RIYADH FPS & 563 & 263 & 87.67\% \\
\hline SHAIBAH FPS & 624 & 324 & 108.00\% \\
\hline
\end{tabular}

Note above, that not all of the replication increases are greater than or equal to the 300 extra replications needed to create the antithetic pairs. The following table is a better representation of the number of the number of replications needed to achieve the variance reduction. It uses the antithetic variates with the 150 replications.

Table 4.15: Replications Needed to Achieve Antithetic Variate Reduction (150 Replications)
\begin{tabular}{|c|c|c|c|}
\hline MOE & REPLICATIONS & CHANGE IN REPLICATIONS & \begin{tabular}{l}
PERCENT \\
CHANGE
\end{tabular} \\
\hline A-10 LPS & 265 & -35 & -11.67\% \\
\hline A-6 LPS & 256 & -44 & -14.67\% \\
\hline BLUE FPS & 281 & -19 & -6.33\% \\
\hline BLUE LPS & 356 & 56 & 18.67\% \\
\hline BLUE SMG & 319 & 19 & 6.33\% \\
\hline DHAHRAN FPS & 290 & -10 & -3.33\% \\
\hline F-111 LPS & 399 & 99 & 33.00\% \\
\hline F-15 LPS & 330 & 30 & 10.00\% \\
\hline FA-18 FPS total & 277 & -23 & -7.67\% \\
\hline FA-18 FPS DCA & 415 & 115 & 38.33\% \\
\hline MIG-23 LPS & 312 & 12 & 4.00\% \\
\hline MIG-29 LPS & 258 & -42 & -14.00\% \\
\hline MIRAGE FPS total & 259 & -41 & -13.67\% \\
\hline MIRAGE FPS INT & 336 & 36 & 12.00\% \\
\hline MIRAGE LPS & 323 & 23 & 7.67\% \\
\hline MUDAYSIS FPS & 310 & 10 & 3.33\% \\
\hline RED FPS & 389 & 89 & 29.67\% \\
\hline RED LPS & 365 & 65 & 21.67\% \\
\hline RIYADH FPS & 281 & -19 & -6.33\% \\
\hline SHAIBAH FPS & 310 & 10 & 3.33\% \\
\hline
\end{tabular}

As seen in the previous section, not all halfwidths are reduced. Also, note that the MOEs in Table 4.14, that have an increase less than 300, are the same MOEs in the table above that have increases.

\subsection*{4.3.3. Antithetic Variates Insights.}

As with common random numbers, since the reduction is not always positive, a search for commonalities between the aggregation levels and reduction could be fruitful. When sorted into the different aggregation levels no pattern emerges. Also, when sorted by force, RED and BLUE, no pattern emerges.

\subsection*{4.4. Control Variates and Antithetic Variates}

The same 600 replications used for antithetic variates were also used for the combination of control variates and antithetic variates. Table 4.16 details the experiment
settings; remember that each set consists of 300 replications. The first 300 replications, set \(D\), were used to generate base confidence intervals for comparison and the first half of the antithetic pairs. The replications were performed with the new RNG, with each random input synchronized with its own stream, and antithetic variates not selected. The second 300 replications, set F, were the second half of the antithetic pairs. The replications were performed with the new RNG, with each random number input synchronized with the replications from set D , and antithetic variates turned on. Turning on antithetic variates forces all of the random numbers to be drawn as 1 - the original random number. The only difference is that control variates are used to reduce the variance. The halfwidths were calculated with at the \(\alpha=0.05\) level.

Table 4.16: Antithetic Variate / Control Variate Experiment Settings
\begin{tabular}{|c|c|c|c|c|c|}
\hline & RNG & SYNCHRONIZED & ANTITHETIC & COLLECT CV & CONFIGURATION \\
\hline SET & (NEWIOLD) & (YES/NO) & (YES/NO) & (YES/NO) & (ORIGINAL/MODIFIED) \\
\hline D & NEW & YES & NO & YES & ORIGINAL \\
\hline F & NEW & YES & YES & YES & ORIGINAL \\
\hline
\end{tabular}

The first subsection contains the comparison of the base halfwidths to the control variate and antithetic variate reduction. The second subsection contains an analysis of the number of replications needed to achieve the control variate and antithetic variate reduction. The last subsection contains insights obtained by using antithetic variates in combination with control variates.

\subsection*{4.4.1. Normal to Variance Reduction Technique.}

To test the reduction in variance, the standard generated halfwidths of the MOEs are compared with the antithetic variate halfwidths of the MOEs reduced even further by also applying the control variate technique. The following table reports the variance reduction due to antithetic variates and control variates.

Table 4.17: Control Variate and Antithetic Variate (300 Replication) vs. Replication Reduction
\begin{tabular}{|l|r|r|r|r|r|}
\hline \multicolumn{1}{|c|}{ MOE } & \multicolumn{2}{c|}{ BASE } & \multicolumn{2}{c|}{ AV I CV } & PERCENT \\
& \multicolumn{1}{|c|}{ MEAN } & \multicolumn{1}{c|}{ HALFWIDTH } & \multicolumn{1}{c|}{ MEAN } & HALFWIDTH & CHANGE \\
\hline A-10 LPS & 0.005145212 & 0.00028569 & 0.004999208 & 0.000195998 & \(-31.40 \%\) \\
\hline A-6 LPS & 0.019104498 & 0.000928243 & 0.01877966 & 0.000687489 & \(-25.94 \%\) \\
\hline BLUE FPS & 0.582000237 & 0.002036313 & 0.580978958 & 0.001410833 & \(-30.72 \%\) \\
\hline BLUE LPS & 0.010170101 & 0.000174156 & 0.010142479 & 0.000115121 & \(-33.90 \%\) \\
\hline BLUE SMG & 34474.62 & 84.52936072 & 34473.68258 & 61.43489655 & \(-27.32 \%\) \\
\hline DHAHRAN FPS & 0.372019758 & 0.006733921 & 0.369991922 & 0.004627463 & \(-31.28 \%\) \\
\hline F-111 LPS & 0.016760015 & 0.000416308 & 0.016805487 & 0.000266501 & \(-35.98 \%\) \\
\hline F-15 LPS & 0.006207278 & 0.00031009 & 0.006329536 & 0.00020287 & \(-34.58 \%\) \\
\hline FA-18 FPS TOTAL & 0.233627904 & 0.003723047 & 0.232572805 & 0.002590169 & \(-30.43 \%\) \\
\hline FA-18 FPS DCA & 0.044337541 & 0.001389283 & 0.044025386 & 0.000931742 & \(-32.93 \%\) \\
\hline MIG-23 LPS & 0.257329967 & 0.006037934 & 0.256304079 & 0.004466771 & \(-26.02 \%\) \\
\hline MIG-29 LPS & 0.200408126 & 0.007942862 & 0.198779022 & 0.005634799 & \(-29.06 \%\) \\
\hline MIRAGE FPS TOTAL & 0.600422202 & 0.00449042 & 0.600109645 & 0.003195737 & \(-28.83 \%\) \\
\hline MIRAGE FPS INT & 0.699855667 & 0.005731393 & 0.700002069 & 0.003816317 & \(-33.41 \%\) \\
\hline MIRAGE LPS & 0.276394858 & 0.004501578 & 0.276342171 & 0.002949469 & \(-34.48 \%\) \\
\hline MUDAYSIS FPS & 0.189549709 & 0.00488572 & 0.189966945 & 0.003417543 & \(-30.05 \%\) \\
\hline RED FPS & 0.391872732 & 0.001772589 & 0.391871895 & 0.001191619 & \(-32.78 \%\) \\
\hline RED LPS & 0.192619582 & 0.001692612 & 0.192330512 & 0.001167319 & \(-31.03 \%\) \\
\hline RIYADH FPS & 0.854947018 & 0.001535138 & 0.854828925 & 0.001071476 & \(-30.20 \%\) \\
\hline SHAIBAH FPS & 0.504592164 & 0.006312332 & 0.505066145 & 0.004503055 & \(-28.66 \%\) \\
\hline
\end{tabular}

Again, this reduction is created with a total of 600 replications, double the number of replications used for the base case. The next table is the reduction due to antithetic variates using only the first 150 antithetic pairs and control variates.

Table 4.18: Control Variate and Antithetic Variate (150 Replication) vs. Replication Reduction
\begin{tabular}{|l|r|r|r|r|r|}
\hline \multicolumn{1}{|c|}{ MOE } & \multicolumn{2}{c|}{ BASE } & \multicolumn{2}{c|}{ AVICV } & PERCENT \\
& \multicolumn{1}{|c|}{ MEAN } & HALFWIDTH & MEAN & HALFWIDTH & CHANGE \\
\hline A-10 LPS & 0.005145212 & 0.00028569 & 0.005038231 & 0.000292638 & \(2.43 \%\) \\
\hline A-6 LPS & 0.019104498 & 0.000928243 & 0.018696752 & 0.00100282 & \(8.03 \%\) \\
\hline BLUE FPS & 0.582000237 & 0.002036313 & 0.581084169 & 0.002088085 & \(2.54 \%\) \\
\hline BLUE LPS & 0.010170101 & 0.000174156 & 0.010110917 & 0.000156743 & \(-10.00 \%\) \\
\hline BLUE SMG & 34474.62 & 84.52936072 & 34547.7152 & 80.62975389 & \(-4.61 \%\) \\
\hline DHAHRAN FPS & 0.372019758 & 0.006733921 & 0.370927039 & 0.006942557 & \(3.10 \%\) \\
\hline F-111 LPS & 0.016760015 & 0.000416308 & 0.016744672 & 0.00034699 & \(-16.65 \%\) \\
\hline F-15 LPS & 0.006207278 & 0.00031009 & 0.006303071 & 0.000289469 & \(-6.65 \%\) \\
\hline FA-18 FPS TOTAL & 0.233627904 & 0.003723047 & 0.231209234 & 0.003892977 & \(4.56 \%\) \\
\hline FA-18 FPS DCA & 0.044337541 & 0.001389283 & 0.043528432 & 0.001156129 & \(-16.78 \%\) \\
\hline MIG-23 LPS & 0.257329967 & 0.006037934 & 0.257478208 & 0.005841578 & \(-3.25 \%\) \\
\hline MIG-29 LPS & 0.200408126 & 0.007942862 & 0.194782302 & 0.007954705 & \(0.15 \%\) \\
\hline MIRAGE FPS TOTAL & 0.600422202 & 0.00449042 & 0.600955714 & 0.004651799 & \(3.59 \%\) \\
\hline MIRAGE FPS INT & 0.699855667 & 0.005731393 & 0.702804931 & 0.005358865 & \(-6.50 \%\) \\
\hline MIRAGE LPS & 0.276394858 & 0.004501578 & 0.27581175 & 0.004245828 & \(-5.68 \%\) \\
\hline MUDAYSIS FPS & 0.189549709 & 0.00488572 & 0.189203986 & 0.00448441 & \(-8.21 \%\) \\
\hline RED FPS & 0.391872732 & 0.001772589 & 0.392220779 & 0.001535464 & \(-13.38 \%\) \\
\hline RED LPS & 0.192619582 & 0.001692612 & 0.191270973 & 0.001510308 & \(-10.77 \%\) \\
\hline RIYADH FPS & 0.854947018 & 0.001535138 & 0.854997708 & 0.001574458 & \(2.56 \%\) \\
\hline SHAIBAH FPS & 0.504592164 & 0.006312332 & 0.501744195 & 0.006108584 & \(-3.23 \%\) \\
\hline
\end{tabular}

Again, the reduction is mixed and the reduced halfwidths are highlighted. When compared to the regular antithetic variates, the halfwidth reduction is larger and the halfwidth increase is smaller.

\subsection*{4.4.2. Variance Reduction Technique to Number of Replications.}

Anther way to represent the reduction realized from the combination of control
variates and antithetic variates is seen by calculating the number of replications needed to achieve the control variate and antithetic variate halfwidth. To do this, the halfwidth calculation is iterated by incrementing the number of replications in the equation until the halfwidth is smaller than the antithetic variate halfwidth. A major assumption to this method is the variance remains constant. The following table displays the number of
replications needed to achieve the variance reduction created by the 300 replication antithetic variates and control variates.

Table 4.19: Replications Needed to Achieve Control Variate and Antithetic Variate Reduction ( \(\mathbf{3 0 0}\) replications)
\begin{tabular}{|c|c|c|c|}
\hline MOE & REPLICATIONS & CHANGE IN REPLICATIONS & PERCENT CHANGE \\
\hline A-10 LPS & 635 & 335 & 111.67\% \\
\hline A-6 LPS & 545 & 245 & 81.67\% \\
\hline BLUE FPS & 622 & 322 & 107.33\% \\
\hline BLUE LPS & 683 & 383 & 127.67\% \\
\hline BLUE SMG & 566 & 266 & 88.67\% \\
\hline DHAHRAN FPS & 632 & 332 & 110.67\% \\
\hline F-111 LPS & 728 & 428 & 142.67\% \\
\hline F-15 LPS & 697 & 397 & 132.33\% \\
\hline FA-18 FPS total & 617 & 317 & 105.67\% \\
\hline FA-18 FPS DCA & 664 & 364 & 121.33\% \\
\hline MIG-23 LPS & 546 & 246 & 82.00\% \\
\hline MIG-29 LPS & 594 & 294 & 98.00\% \\
\hline MIRAGE FPS total & 590 & 290 & 96.67\% \\
\hline MIRAGE FPS INT & 673 & 373 & 124.33\% \\
\hline MIRAGE LPS & 695 & 395 & 131.67\% \\
\hline MUDAYSIS FPS & 610 & 310 & 103.33\% \\
\hline RED FPS & 661 & 361 & 120.33\% \\
\hline RED LPS & 628 & 328 & 109.33\% \\
\hline RIYADH FPS & 613 & 313 & 104.33\% \\
\hline SHAIBAH FPS & 587 & 287 & 95.67\% \\
\hline
\end{tabular}

Note again, that not all of the replication increases are greater than or equal to the 300 extra replications needed to create the antithetic variates. The following table displays the number of replications needed to achieve the halfwidth generated by the combination of control variates and antithetic variates with the 150 antithetic pairs.

Table 4.20: Replications Needed to Achieve Control Variate and Antithetic Variate Reduction (150 replications)
\begin{tabular}{|c|c|c|c|}
\hline MOE & REPLICATIONS & CHANGE IN REPLICATIONS & PERCENT CHANGE \\
\hline A-10 LPS & 287 & -13 & -4.33\% \\
\hline A-6 LPS & 258 & -42 & -14.00\% \\
\hline BLUE FPS & 286 & -14 & -4.67\% \\
\hline BLUE LPS & 370 & 70 & 23.33\% \\
\hline BLUE SMG & 330 & 30 & 10.00\% \\
\hline DHAHRAN FPS & 283 & -17 & -5.67\% \\
\hline F-111 LPS & 431 & 131 & 43.67\% \\
\hline F-15 LPS & 344 & 44 & 14.67\% \\
\hline FA-18 FPS total & 275 & -25 & -8.33\% \\
\hline FA-18 FPS DCA & 432 & 132 & 44.00\% \\
\hline MIG-23 LPS & 321 & 21 & 7.00\% \\
\hline MIG-29 LPS & 300 & 0 & 0.00\% \\
\hline MIRAGE FPS total & 280 & -20 & -6.67\% \\
\hline MIRAGE FPS INT & 343 & 43 & 14.33\% \\
\hline MIRAGE LPS & 337 & 37 & 12.33\% \\
\hline MUDAYSIS FPS & 356 & 56 & 18.67\% \\
\hline RED FPS & 399 & 99 & 33.00\% \\
\hline RED LPS & 377 & 77 & 25.67\% \\
\hline RIYADH FPS & 286 & -14 & -4.67\% \\
\hline SHAIBAH FPS & 321 & 21 & 7.00\% \\
\hline
\end{tabular}

\subsection*{4.4.3. Combined Antithetic Variate and Control Variate Insight.}

Since the control variate technique is applied to the antithetic variates,
information can be gleaned as in section 4.1.3 Control Variate Insight. A list of statistically significant controls related to the MOEs from the 150-replication antithetic pairs is below.

Table 4.21: Antithetic Variate and Control Variate Statistically Significant Controls Table (150 Replications)


As seen with the CV, these weights and controls can be used to give the analyst a better
understanding of how the simulation works. The following table, Table 4.22, examines the MOE MUDAYSIS FPS (Flown per Planned Sortie). It can be seen that the statistically significant controls are ADF140.3, AIR007.1, AIR070.1, AIR567.2, AIR840.2, and BSE200.5. Note, AIR007.1 is a random input that is miss labeled. It should be in AIR007.sim, but it is located in AIR014.1.

Table 4.22: Antithetic Variate and Control Variate Purpose of Statistically Significant Controls
\begin{tabular}{|c|c|c|c|}
\hline CONTROL & WEIGHT & MODULE PURPOSE & SPECIFIC DRAW PURPOSE \\
\hline ADF140_3 & -10.8821 & Calculates, for each air defense type, the number of engaged sites and rounds fired per site for primaries. & Used determine the actual number of sites engaging as a function of FRAC.INTEGRATED (used to measure command and control). \((\mathrm{U}(0,1)<\) frac.f(.engaged.shooters)) \\
\hline AIR007_1 & 10.0131 & Continue a flight after the air-to-air refueling decision has been made. & Used to determine the launching of interceptors. \((\mathrm{U}(0,1)<=\) TA.PROB.ENEMY.INT.LAUN CH) \\
\hline AIR070_1 & 0.0405 & Adjudicate a defensive anti tactical ballistic missile mission VS. tactical ballistic missile (TBM) air-to-air engagement. & Used to determine determine TBM flight phase (and pK ) at which to adjudicate. (distribution) Used to determine duration of aiborne laser shot duration. \\
\hline AIR567_2 & 0.0007 & Compute target location error for an offensive anti tactical ballistic missile mission (OTBM) flight group which has arrived at its target. & Generate meters error from a normal distribution. (distribution) \\
\hline AIR840_2 & -14.8256 & Determine the number of target elements that are destroyed when a given weapons area of effect fully or partially covers a target. & Determine how many live target elements were hit. ( \(\mathrm{U}(0,1)<\).PROB.HIT.LIVE) \\
\hline BSE200_5 & 24.6841 & Determine maintenance tasks to be performed on aircraft in the given flight and schedule those for which resources are currently available. Print warning when required non-consumable resource is not available at squadron's mob. & \begin{tabular}{l}
Used to build list of maintenance tasks required. \\
\((\mathrm{U}(0,1)<=\) TA.PROB.LONG.TERM.REPA IR
\end{tabular} \\
\hline
\end{tabular}

Using the chart above, a number of system characteristics can be determined.
First, an increase in the number of engaged air defense shooters would decrease
MUDAYSIS FPS. Second, a decrease in the launching of interceptors would decrease
MUDAYSIS FPS. Third, a decrease in the airborne laser shot duration required will
slightly decrease MUDAYSIS FPS. Fourth, a decrease in the normal error of an OTBM
(Offensive Anti Tactical Ballistic Missile Mission) will slightly decrese MUDAYSIS
FPS. Fifth, an increse in the number of live targets hit will decrease MUDAYSIS FPS.

Sixth, the increase in the long-term repairs, will greatly decrese MUDAYSIS FPS. Overall, it seems that an increse in engagement will decrese the number of flown per planned sorties. This same analysis can be applied to the control variates related to other MOEs.

As with common random numbers, since the reduction is not always positive, a search for commonalities between the aggregation levels and reduction could be fruitful. When sorted into the different aggregation levels no pattern emerges. Also, when sorted by force, no pattern emerges.

\subsection*{4.5. Control Variates and Common Random Numbers}

To analyze the effectiveness of combining CV and CRN techniques, a total of 1200 replications were performed. Table 4.8 details the experiment settings; remember that each set consists of 300 replications. The sets can be separated into two groups. The first group is the standard independent data that makes up the base data and the second group is the partially synchronized data. All replications were of the standard ME scenario, using the new RNG. The first 300 replications, set M, contained no changes to the database and with all random inputs using a single stream for each replication. The second 300 replications, set O , were performed with all of the F -111s removed from the database and with all random inputs using the same single streams applied to set B . The third 300 replications, set S , contained no changes to the database and were performed with selected random inputs synchronized therefore, receiving their own random number stream in each replication and the rest independent. The last 300 replications, set T, were performed with all of the F-111s removed from the database and with the selected random inputs synchronized with set S and the rest independent.

Table 4.23: Control Variate and Common Random Number Experiment Settings
\begin{tabular}{|c|c|c|c|c|c|}
\hline & RNG & SYNCHRONIZED & ANTITHETIC & COLLECT CV & CONFIGURATION \\
\hline SET & (NEWIOLD) & (YES/NO) & (YES/NO) & (YES/NO) & (ORIGINALMODIFIED) \\
\hline\(M\) & NEW & NO* \(^{*}\) & NO & YES & ORIGINAL \\
\hline\(O\) & NEW & NO \(^{*}\) & NO & YES & MODIFIED \\
\hline\(S\) & NEW & YES* \(^{*}\) & NO & YES & ORIGINAL \\
\hline T & NEW & YES* \(^{*}\) & NO & YES & MODIFIED \\
\hline
\end{tabular}

The following analysis compares the halfwidths generated by calculating the difference between scenario configurations. The first comparison is by the standard method, not synchronized, and the second comparison is by combining CRN with selected synchronization and CV. The second section highlights insights gleaned from using CRN.

\subsection*{4.5.1. Normal to Variance Reduction Technique.}

The paired-t CI at the \(\alpha=0.05\) level was used to determine whether the MOEs generated from a configuration change in the ME scenario resulted in statistically different values. Then the paired-t halfwidths were compared to determine the reduction in variance. The following table displays the results of this comparison.

Table 4.24: Control Variate and Common Random Number Halfwidth Reduction
\begin{tabular}{|l|r|r|r|r|r|}
\hline \multicolumn{1}{|c|}{ MOE } & \multicolumn{2}{c|}{ BASE } & \multicolumn{2}{c|}{ CVICRN } & PERCENT \\
& \multicolumn{1}{|c|}{ MEAN } & HALFWIDTH & \multicolumn{1}{c|}{ MEAN } & HALFWIDTH & CHANGE \\
\hline A-10 LPS & -0.005074435 & 0.000471166 & -0.005420376 & 0.000495875 & \(5.24 \%\) \\
\hline A-6 LPS & -0.003709347 & 0.001243332 & -0.003862475 & 0.001142089 & \(-8.14 \%\) \\
\hline BLUE FPS & 0.080204272 & 0.002488093 & 0.078021187 & 0.002276083 & \(-8.52 \%\) \\
\hline BLUE LPS & -0.000980203 & 0.000224242 & -0.001228896 & 0.000216219 & \(-3.58 \%\) \\
\hline BLUE SMG & 321.6566667 & 92.75065974 & 283.8370489 & 94.46705704 & \(1.85 \%\) \\
\hline DHAHRAN FPS & 0.047497651 & 0.00785785 & 0.043664487 & 0.007499356 & \(-4.56 \%\) \\
\hline F-15 LPS & -0.00029301 & 0.000348631 & -0.000778342 & 0.00039424 & \(13.08 \%\) \\
\hline FA-18 FPS total & 0.023885856 & 0.004348857 & 0.021403974 & 0.004115626 & \(-5.36 \%\) \\
\hline FA-18 FPS DCA & \(9.37616 E-05\) & 0.001776789 & -0.002378309 & 0.001642749 & \(-7.54 \%\) \\
\hline MIG-23 LPS & 0.00720557 & 0.009116531 & 0.002253995 & 0.008420355 & \(-7.64 \%\) \\
\hline MIG-29 LPS & 0.058508654 & 0.008627634 & 0.058072515 & 0.008322543 & \(-3.54 \%\) \\
\hline MIRAGE FPS total & 0.008655135 & 0.005768579 & 0.001181363 & 0.00581596 & \(0.82 \%\) \\
\hline MIRAGE FPS INT & -0.004831629 & 0.006898662 & -0.001808304 & 0.007049026 & \(2.18 \%\) \\
\hline MIRAGE LPS & -0.000117038 & 0.00564862 & 0.005595717 & 0.005615011 & \(-0.60 \%\) \\
\hline MUDAYSIS FPS & 0.071070466 & 0.00554528 & 0.066758129 & 0.005276546 & \(-4.85 \%\) \\
\hline RED FPS & 0.034613556 & 0.002097873 & 0.030483796 & 0.002043576 & \(-2.59 \%\) \\
\hline RED LPS & 0.022702029 & 0.002120665 & 0.02316245 & 0.002015068 & \(-4.98 \%\) \\
\hline RIYADH FPS & -0.072184072 & 0.001689959 & -0.072668778 & 0.001737694 & \(2.82 \%\) \\
\hline SHAIBAH FPS & 0.093599432 & 0.008845515 & 0.095513973 & 0.008256205 & \(-6.66 \%\) \\
\hline
\end{tabular}

Looking above, we can see that the halfwidth is not always smaller. The changes
highlighted above are the halfwidths that are reduced. This implies that there is not
always a reduction in the variance when combining CV and CRN. Knowing from section
4.2, that CRN does not always reduce the halfwidth, the control variate estimator is
applied to the independent base data set to determine the effect due to CV. The following
table is the result.

Table 4.25: Control Variate Only Halfwidth Reduction
\begin{tabular}{|l|r|r|r|r|r|}
\hline \multicolumn{1}{|c|}{ MOE } & \multicolumn{2}{c|}{ BASE } & \multicolumn{2}{c|}{ CV } & PERCENT \\
& \multicolumn{1}{c|}{ MEAN } & HALFWIDTH & \multicolumn{1}{c|}{ MEAN } & HALFWIDTH & CHANGE \\
\hline A-10 LPS & -0.005074435 & 0.000471166 & -0.00506369 & 0.000464018 & \(-1.52 \%\) \\
\hline A-6 LPS & -0.003709347 & 0.001243332 & -0.003673866 & 0.001221513 & \(-1.75 \%\) \\
\hline BLUE FPS & 0.080204272 & 0.002488093 & 0.080081841 & 0.002427363 & \(-2.44 \%\) \\
\hline BLUE LPS & -0.000980203 & 0.000224242 & -0.000971024 & 0.000219306 & \(-2.20 \%\) \\
\hline BLUE SMG & 321.6566667 & 92.75065974 & 322.6088152 & 92.30977839 & \(-0.48 \%\) \\
\hline DHAHRAN FPS & 0.047497651 & 0.00785785 & 0.047189217 & 0.007795751 & \(-0.79 \%\) \\
\hline F-15 LPS & -0.00029301 & 0.000348631 & -0.000292464 & 0.000345254 & \(-0.97 \%\) \\
\hline FA-18 FPS total & 0.023885856 & 0.004348857 & 0.023798247 & 0.004325066 & \(-0.55 \%\) \\
\hline FA-18 FPS DCA & \(9.37616 E-05\) & 0.001776789 & 0.000161301 & 0.001764302 & \(-0.70 \%\) \\
\hline MIG-23 LPS & 0.00720557 & 0.009116531 & 0.006046667 & 0.008687175 & \(-4.71 \%\) \\
\hline MIG-29 LPS & 0.058508654 & 0.008627634 & 0.058064528 & 0.008499025 & \(-1.49 \%\) \\
\hline MIRAGE FPS total & 0.008655135 & 0.005768579 & 0.009424086 & 0.005625573 & \(-2.48 \%\) \\
\hline MIRAGE FPS INT & -0.004831629 & 0.006898662 & -0.005030783 & 0.006825481 & \(-1.06 \%\) \\
\hline MIRAGE LPS & -0.000117038 & 0.00564862 & -0.000991526 & 0.005563242 & \(-1.51 \%\) \\
\hline MUDAYSIS FPS & 0.071070466 & 0.00554528 & 0.071135554 & 0.005548611 & \(0.06 \%\) \\
\hline RED FPS & 0.034613556 & 0.002097873 & 0.034764827 & 0.002032008 & \(-3.14 \%\) \\
\hline RED LPS & 0.022702029 & 0.002120665 & 0.02226502 & 0.002083652 & \(-1.75 \%\) \\
\hline RIYADH FPS & -0.072184072 & 0.001689959 & -0.072522867 & 0.001643588 & \(-2.74 \%\) \\
\hline SHAIBAH FPS & 0.093599432 & 0.008845515 & 0.096093296 & 0.008691198 & \(-1.74 \%\) \\
\hline
\end{tabular}

Note, that almost all of the halfwidths are reduced. This implies that the majority variance increases are due to the application of CRN.

\subsection*{4.5.2. Combined Control Variate and Common Random Number Insight.}

As can be seen with CRN, the decision of whether the MOE results between configurations are statistically different or not, may change between the base nonsynchronized and the synchronized, as a result from both the change in halfwidth and a shift in the mean value. The following table displays the decision results among the MOEs and identifies any changes in the decisions between the base and synchronized results. Each of the CIs developed for the MOEs below are calculated at the \(\alpha=0.05\) level and are calculated separately.

Table 4.26: Control Variate and Common Random Number Configuration Decision
\begin{tabular}{|l|l|l|}
\hline MOE & \multicolumn{1}{|c|}{\begin{tabular}{c} 
BASE \\
DECISION
\end{tabular}} & \multicolumn{1}{|c|}{\begin{tabular}{c} 
CVICRN \\
DECISION
\end{tabular}} \\
\hline A-10 LPS & DIFFERENT & DIFFERENT \\
\hline A-6 LPS & DIFFERENT & DIFFERENT \\
\hline BLUE FPS & DIFFERENT & DIFFERENT \\
\hline BLUE LPS & DIFFERENT & DIFFERENT \\
\hline BLUE SMG & DIFFERENT & DIFFERENT \\
\hline DHAHRAN FPS & DIFFERENT & DIFFERENT \\
\hline F-15 LPS & NOT DIFFERENT & DIFFERENT \\
\hline FA-18 FPS total & DIFFERENT & DIFFERENT \\
\hline FA-18 FPS DCA & NOT DIFFERENT & DIFFERENT \\
\hline MIG-23 LPS & NOT DIFFERENT & NOT DIFFERENT \\
\hline MIG-29 LPS & DIFFERENT & DIFFERENT \\
\hline MIRAGE FPS total & DIFFERENT & NOT DIFFERENT \\
\hline MIRAGE FPS INT & NOT DIFFERENT & NOT DIFFERENT \\
\hline MIRAGE LPS & NOT DIFFERENT & NOT DIFFERENT \\
\hline MUDAYSIS FPS & DIFFERENT & DIFFERENT \\
\hline RED FPS & DIFFERENT & DIFFERENT \\
\hline RED LPS & DIFFERENT & DIFFERENT \\
\hline RIYADH FPS & DIFFERENT & DIFFERENT \\
\hline SHAIBAH FPS & DIFFERENT & DIFFERENT \\
\hline
\end{tabular}

Looking at the table above, we can see that the decision changes three times and they are highlighted in gray.

Since the change is not consistently positive or negative and the MOEs were selected to reflect a range of aggregation, it seems logical to see if there are any commonalities between the aggregation levels and reduction. When sorted into the different aggregation levels no pattern emerges. The only item of possible note is that the data collected at the squadron level are all positive. When the data is sorted by force, RED and BLUE, there is also no notable pattern.

\subsection*{4.6. Random Number Generator Comparison}

To perform the CRN analysis a new RNG needed to be implemented. The research in generating this new RNG proclaims that it has better statistical properties and its speed is faster (16). For the application in THUNDER, a more important question is
not how good the statistical properties are, but are the results statistically different from the original RNG and do replications using the new RNG take longer to run. To test these questions, three different comparisons were conducted. To perform the comparisons, paired-t CIs were developed for all MOEs at the \(\alpha=0.05\) level. Since all MOEs and time are compared simultaneously, application of the Bonferroni inequality is used to maintain the \(\alpha=0.05\) level throughout all MOEs and time comparisons.

\section*{Table 4.27: Random Number Experiment Settings}
\begin{tabular}{|c|c|c|c|c|c|}
\hline & RNG & SYNCHRONIZED & ANTITHETIC & COLLECT CV & CONFIGURATION \\
\hline SET & (NEW/OLD) & (YES/NO) & (YES/NO) & (YES/NO) & (ORIGINAL/MODIFIED) \\
\hline B & NEW & NO & NO & NO & ORIGINAL \\
\hline G & OLD & NO & NO & NO & ORIGINAL \\
\hline H & NEW & YES & NO & NO & ORIGINAL \\
\hline
\end{tabular}

The table above displays the experiment settings used for the comparisons; remember that each set was performed with 300 replications. The first subsection contains a comparison between the original RNG and the new RNG. This comparison uses sets B and G from the table above. The next subsection contains a comparison between the original RNG and the new RNG with synchronized random inputs. Generating a unique stream for each random input. This comparison uses sets G and H from the above table. The last subsection contains a comparison between the new RNG, not synchronized, and the new RNG with synchronized random inputs. This comparison uses sets B and H from the table above.

\subsection*{4.6.1. Original vs. New (not synchronized) Random Number Generator.}

Confidence intervals were created for a comparison to decide if the new RNG is statistically different from the original RNG. The following table and graph depicts the confidence intervals and determines which, if any, are statistically different.

Table 4.28: Original vs. New (not synchronized) Random Number Generator
\begin{tabular}{|l|l|l|l|}
\hline \multicolumn{1}{|c|}{ MOE } & \multicolumn{1}{c|}{\begin{tabular}{c} 
LOWER \\
BOUND
\end{tabular}} & \multicolumn{1}{c|}{\begin{tabular}{c} 
UPPER \\
BOUND
\end{tabular}} & \begin{tabular}{c} 
CONTAINS \\
ZERO
\end{tabular} \\
\hline A-10 LPS & -0.000673185 & 0.000498496 & YES \\
\hline A-6 LPS & -0.002034542 & 0.001989532 & YES \\
\hline BLUE FPS & -0.004493169 & 0.00452297 & YES \\
\hline BLUE LPS & -0.000380992 & 0.000266721 & YES \\
\hline BLUE SMG & -206.0933947 & 143.946728 & YES \\
\hline DHAHRAN FPS & -0.013936772 & 0.016032878 & YES \\
\hline F-111 LPS & -0.000871293 & 0.000863446 & YES \\
\hline F-15 LPS & -0.000541279 & 0.00066419 & YES \\
\hline FA-18 FPS total & -0.009697343 & 0.006429048 & YES \\
\hline FA-18 FPS DCA & -0.004787346 & 0.000840694 & YES \\
\hline MIG-23 LPS & -0.01238682 & 0.013461924 & YES \\
\hline MIG-29 LPS & -0.013436245 & 0.018160475 & YES \\
\hline MIRAGE FPS total & -0.011752971 & 0.007611029 & YES \\
\hline MIRAGE FPS INT & -0.009436666 & 0.015837388 & YES \\
\hline MIRAGE LPS & -0.006410013 & 0.011816126 & YES \\
\hline MUDAYSIS FPS & -0.012006375 & 0.009409523 & YES \\
\hline RED FPS & -0.005130225 & 0.002138004 & YES \\
\hline RED LPS & -0.002755065 & 0.004516344 & YES \\
\hline RIYADH FPS & -0.002162778 & 0.004127981 & YES \\
\hline SHAIBAH FPS & -0.016117409 & 0.009644726 & YES \\
\hline TIME & -0.184111452 & 0.070778119 & YES \\
\hline
\end{tabular}


Figure 4.6: Original vs. New (not synchronized) Random Number Generator Confidence Intervals

Notice how all of the confidence intervals contain zero. Statistically, there is no difference in the output form the original RNG and the new (not synchronized) RNG.

\subsection*{4.6.2. Original vs. New (synchronized) Random Number Generators.}

Confidence intervals were created for a comparison to decide if the new RNG, when synchronized for CRN, is statistically different from the original RNG. The
following table and graph depicts the confidence intervals and determines which, if any, are statistically different.

Table 4.29: Original vs. New (synchronized) Random Number Generator
\begin{tabular}{|l|r|r|r|}
\hline \multicolumn{1}{|c|}{ MOE } & \multicolumn{1}{c|}{\begin{tabular}{c} 
LOWER \\
BOUND
\end{tabular}} & \multicolumn{1}{c|}{\begin{tabular}{c} 
UPPER \\
BOUND
\end{tabular}} & \begin{tabular}{r} 
CONTAINS \\
ZERO
\end{tabular} \\
\hline A-10 LPS & -0.000776407 & 0.000343213 & YES \\
\hline A-6 LPS & -0.002242076 & 0.00172037 & YES \\
\hline BLUE FPS & -0.004762879 & 0.003842456 & YES \\
\hline BLUE LPS & -0.000279397 & 0.000398031 & YES \\
\hline BLUE SMG & -160.3646513 & 186.0646513 & YES \\
\hline DHAHRAN FPS & -0.012801006 & 0.016895777 & YES \\
\hline F-111 LPS & -0.000626852 & 0.001139557 & YES \\
\hline F-15 LPS & -0.000401774 & 0.000842377 & YES \\
\hline FA-18 FPS total & -0.009003699 & 0.006424452 & YES \\
\hline FA-18 FPS DCA & -0.003217184 & 0.002117636 & YES \\
\hline MIG-23 LPS & -0.016939633 & 0.008871754 & YES \\
\hline MIG-29 LPS & -0.01958925 & 0.009883308 & YES \\
\hline MIRAGE FPS total & -0.011330577 & 0.006628967 & YES \\
\hline MIRAGE FPS INT & -0.008677052 & 0.014232686 & YES \\
\hline MIRAGE LPS & -0.010454198 & 0.00760329 & YES \\
\hline MUDAYSIS FPS & -0.007141542 & 0.012470464 & YES \\
\hline RED FPS & -0.003830793 & 0.003359536 & YES \\
\hline RED LPS & -0.003990189 & 0.002962869 & YES \\
\hline RIYADH FPS & -0.002425742 & 0.003669871 & YES \\
\hline SHAIBAH FPS & -0.017991997 & 0.007191441 & YES \\
\hline TIME & -0.219334662 & 0.052667995 & YES \\
\hline
\end{tabular}


Figure 4.7: Original vs. New (synchronized) Random Number Generator Confidence Intervals

Notice how all of the confidence intervals contain zero. Statistically, there is no difference in the output form the original RNG and the new (synchronized) RNG.

\subsection*{4.6.3. New (not synchronized) vs. New (synchronized) Random Number}

Generators.
Confidence intervals were created for a comparison to decide if the new RNG, when synchronized for CRN, is statistically different from the new RNG, when not
synchronized for CRN. The following table and graph depicts the confidence intervals
and determines which, if any, are statistically different.
Table 4.30: New (not synchronized) vs. New (synchronized) Random Number Generator
\begin{tabular}{|l|r|r|l|}
\hline \multicolumn{1}{|c|}{ MOE } & \multicolumn{1}{c|}{\begin{tabular}{c} 
LOWER \\
BOUND
\end{tabular}} & \begin{tabular}{c} 
UPPER \\
BOUND
\end{tabular} & \begin{tabular}{c} 
CONTAINS \\
ZERO
\end{tabular} \\
\hline A-10 LPS & -0.00074143 & 0.000482925 & YES \\
\hline A-6 LPS & -0.002085688 & 0.001608992 & YES \\
\hline BLUE FPS & -0.004700458 & 0.003750234 & YES \\
\hline BLUE LPS & -0.000235073 & 0.000467977 & YES \\
\hline BLUE SMG & -122.4487384 & 210.295405 & YES \\
\hline DHAHRAN FPS & -0.01372776 & 0.015726424 & YES \\
\hline F-111 LPS & -0.000617935 & 0.001138486 & YES \\
\hline F-15 LPS & -0.000473583 & 0.000791275 & YES \\
\hline FA-18 FPS total & -0.007783706 & 0.008472755 & YES \\
\hline FA-18 FPS DCA & -0.001374783 & 0.004221887 & YES \\
\hline MIG-23 LPS & -0.016885089 & 0.007742106 & YES \\
\hline MIG-29 LPS & -0.022430556 & 0.008000384 & YES \\
\hline MIRAGE FPS total & -0.010063659 & 0.009503992 & YES \\
\hline MIRAGE FPS INT & -0.012372388 & 0.011527301 & YES \\
\hline MIRAGE LPS & -0.013056713 & 0.004799694 & YES \\
\hline MUDAYSIS FPS & -0.006391227 & 0.014317001 & YES \\
\hline RED FPS & -0.002442988 & 0.004963952 & YES \\
\hline RED LPS & -0.00493556 & 0.00214696 & YES \\
\hline RIYADH FPS & -0.003442907 & 0.002721834 & YES \\
\hline SHAIBAH FPS & -0.015273159 & 0.010945286 & YES \\
\hline TIME & -0.146257618 & 0.092924284 & YES \\
\hline
\end{tabular}


Figure 4.8 New (not synchronized) vs. New (synchronized) Random Number Generator Confidence Intervals

Notice how all of the confidence intervals contain zero. Statistically, there is no difference in the output form the new RNG (not synchronized) and the new (synchronized) RNG.

The following chapter concludes the analysis from this chapter. It develops usable results from this analysis and presents future work in these areas.

\section*{5. Conclusion}

This section contains all of the conclusions and recommendations from this research. It begins with conclusions derived from the analysis of chapter 4. It then covers some recommendations for further work or study.

\subsection*{5.1. Conclusions}

\subsection*{5.1.1. Control Variates.}

Of all the variance reduction techniques applied to THUNDER, control variates have performed the best. It consistently realized a reduction in the halfwidths for all MOEs studied, is relatively easy to implement, can be used to reduce the required number of replications, and give valuable insight into the model. The consistency in the halfwidth reduction allows a postprocessor to automatically apply control variates and give a reduced halfwidth without worry of possibly increasing the halfwidth. The ease of implementation also allows a postprocessor to apply control variates without the direct monitoring of the analyst. With a halfwidth goal, use of control variates will reduce the number of replication needed and in turn reduce the time required to analyze the model. Use of the insight given by the weight and sign of the controls can help identify inconsistencies of the code or methodology. It can also aid in understanding how elements of the model affect desired MOEs.

The analysis in chapter 4 identified a statistically significant increase in the replication generation time for control variates. This difference in time ranges between 0.01 and 0.18 second. The size of the difference is a fraction of a second and is not
practically significant. Also, the model \(\mathrm{R}^{2} \mathrm{~s}\) are small and the reduction in the halfwidths are small, but the effects of the reduction may be significant to the analysis. Therefore, consistent use of control variates is practical and effective in light of the drawbacks.

\subsection*{5.1.2. Common Random Numbers.}

When comparing different configurations of a system, a form of common random numbers is generally used, either voluntarily or nonvoluntarily. When two or more configurations start with the same seeds and only one stream is accessed, common random numbers are used. This could potentially cause unintentional problems. Since common random numbers are most likely being used, they should be used properly where all of the random inputs are synchronized.

In this research, common random numbers did not necessarily reduce all of the halfwidths. In the 19 MOEs analyzed, there were 14 halfwidth reductions and 5 halfwidth increases. When looking through the changes in the halfwidths, the largest reduction is \(16.25 \%\) with a total of 7 MOEs with reductions greater than \(7 \%\), while the largest increase in the halfwidths was \(6.03 \%\). Therefore, it is more likely to get a reduction in a halfwidth than getting an increase in a halfwidth. Also, the size of a reduction in a halfwidth is generally larger than an increase in a halfwidth.

\subsection*{5.1.3. Antithetic Variates.}

When looking at the results from antithetic variates, the implementation does not seem very promising. The amount of extra effort in generating the antithetic pair runs results in only 11 of the 19 MOEs analyzed, \(57.89 \%\), receiving a reduction in the halfwidth.

\subsection*{5.1.4. Control Variates and Antithetic Variates.}

This method is a little more promising than the standard antithetic variates. It still requires the same amount of effort in generating the antithetic pair runs and only \(57.89 \%\) of the MOEs analyzed received a reduction in the halfwidths. The major difference is that the 11 reductions in halfwidths are larger than either antithetic variates or control variates independently. Also the 8 increases in halfwidths are smaller than the regular antithetic variates. This implies that there still may be some benefit in using this method if the MOE is determined to be reducible by antithetic variates. Also, since control variates are applied in this method, insight gained from analyzing the weight and direction of the controls can be beneficial.

\subsection*{5.1.5. Control Variates and Common Random Numbers.}

This method did not prove very successful. Thirteen of the 19 or \(68 \%\) of the MOEs received a reduction in variance, and the decision changed 3 times. The CRN method has produced mixed results in the past, so an application of just the CVs was examined. The analysis of the CV by itself did prove successful, 18 of the 19 or \(95 \%\) of MOEs received a reduction in the halfwidth. Therefore, some sort of application of only the CV method may be beneficial to implement in two system comparisons.

\subsection*{5.1.6. Random Number Generator.}

To properly implement common random numbers, a new random number generator needed to be implemented. With the implementation of a new random number generator, we needed to make sure the new random number generator would not significantly alter the THUNDER results. Three different comparisons were performed. They were the new random number generator without synchronizing all random inputs
against the original random number generator, the new random number generator synchronizing all random inputs against the original random number generator, and the new random number generator without synchronizing all random inputs against the new random number generator synchronizing all random inputs. The results for all three analyses were the same. At an \(\alpha=0.05\) level, all MOEs were statistically equal. An important part of the test is that the replication times were also statistically equal.

\subsection*{5.2. Recommendations for Further Work or Study}

While working on this research and examining the THUNDER source code some topics for further work or study were discovered. Some of them are small fixes and some are areas where an analysis is needed.

\subsection*{5.2.1. Further THUNDER Work.}

First, examination of the random variate modules of THUNDER, it was realized that THUNDER does not allow the analyst to assign all three parameters to the triangular distribution. Fixing this problem would require a through search through the code to identify all sections of code that are associated with the triangular distribution.

Second, more random variate distributions should be added to THUNDER. Currently THUNDER only allows the 13 distributions in Table 3.2. More distributions would allow more flexibility for the analyst.

Third, incorporate control variates and antithetic variates into the post processors. Application should be simple, by following the methodology used in this research. An output of control variate weights should also be incorporated into the post processors. Common random numbers does not need to be added to a postprocessor since it is a passive variance reduction technique.

Fourth, a table identifying the random inputs and what they are used for should be made for THUNDER users. This could be added as an appendix to the THUNDER documentation and used in conjunction to control variate weights after it is incorporated in a post processor.

Fifth, some of the random inputs in THUNDER are mislabeled. While using the name structucre to locate the random inputs, the are ..BSE030.1, ..BSE001.1, ..BSE001.2, and ..AIR007.1 were not in the proper source files. After a thurogh search through every source files, the correct source files were identified. Therefore, they should be changed to ..UTL260.1, ..BSE225.1, ..BSE225.2, and ..AIR014.1 respectively.

Sixth, permanently implement a new random number generator in THUNDER. The random number generator used for this research is authorized for student use and authorization from Dr. L'Ecuyer is needed to implement MRG32k3a for commercial purposes. It is also possible, with a little research, that any of the other parameters from Dr. L'Ecuyers article (16) could be used to build a combined multiple recursive random number generator for the Air Force.

\subsection*{5.2.2. Further THUNDER Study.}

First, find a way to utilize random inputs that use more than one distribution. These variates are not used in the current analysis. A possible direction to go from here is to apply control variates at the central random number draw and not on the random inputs.

Second, test control variates, common random numbers, and antithetic variates with larger THUNDER databases. The ME database is relatively small and this may affect the response of the variance reduction technique. Also, a test to see if test pilot
runs of a small amount of days or only a couple of replications has the same affect as a full test with a large amount of days or replications.

Third, try applying antithetic variates only to random inputs that are not branches and common random numbers to branches. This method may make antithetic variates more effective. Then, combined with control variates, this method may have a lot of variance reduction potential.

\section*{Appendix A: THUNDER Report Example (1 Full Day)}

A. 1
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|}
\hline \multicolumn{8}{|l|}{THUNDER 6.808: ME} & \multicolumn{2}{|l|}{11/08/2004 08:55 P} & PAGE \\
\hline C 0 M & B A T C Y & C LE & \multicolumn{5}{|l|}{TIME: 1:1200} & \multicolumn{3}{|l|}{\multirow[t]{2}{*}{REPORT: CC-4}} \\
\hline & & & & & R E P OR & & & & & \\
\hline & * & & \multicolumn{5}{|l|}{FLOT MOVEMENT} & \multicolumn{3}{|l|}{\multirow[t]{2}{*}{PART 1}} \\
\hline * BLUE & * & & & & & & & & & \\
\hline & * & & & & & & & & & \\
\hline \multicolumn{2}{|l|}{\multirow[t]{2}{*}{--OPPOSING CMD--}} & \multicolumn{3}{|l|}{\multirow[t]{2}{*}{-------------FLOT SEGMENT}} & \multicolumn{3}{|l|}{\multirow[t]{2}{*}{FROM}} & \multicolumn{3}{|l|}{\multirow[t]{2}{*}{то}} \\
\hline & & & & & & & & & & \\
\hline ID & NAME & GAINED (LOST) & MILES & ID & NAME & LATITUDE & LONGITUDE & LATITUDE & LONGITUDE & \\
\hline 1013 & PERSIAN GU & 0 & 118 & 2009 & PERSIAN GU & 27D43.0M-N & 52D15.1M-E & 28D50.9M-N & 50D46.0M-E & \\
\hline 1008 & PERSIAN GU & 0 & 118 & 2010 & PERSIAN GU & 28D50.9M-N & 50D46.0M-E & 28D34.6M-N & 48D19.9M-E & \\
\hline 1009 & MARINES & 401 & 62 & 2011 & 1 IRAQI AR & 28D34.6M-N & 48D19.9M-E & 29D06.8M-N & 47D30.0M-E & \\
\hline 1010 & ARAB CORPS & 391 & 50 & 2012 & 2 IRAQI AR & 29D06.8M-N & 47D30.0M-E & 29D14.8M-N & 46 D 38.6 M - & \\
\hline 1020 & VII CORPS & 1087 & 75 & 2013 & 3 IRAQI AR & 29D14.8M-N & 46D38.6M-E & 29D44.9M-N & 45D32.8M-E & \\
\hline 1021 & XVIII AIRB & 1767 & 121 & 2016 & BAGHDAD FO & 29D44.9M-N & 45D32.8M-E & 29D50.4M-N & 43D50.5M-E & \\
\hline & & & & 2014 & DESERT FOR & 29D50.4M-N & 43D50.5M-E & 30D09.0M-N & 43D30.2M-E & \\
\hline 1012 & DESERT FOR & 0 & 109 & 2014 & DESERT FOR & 30D09.0M-N & 43D30.2M-E & 31D02.0M-N & 42D16.3M-E & \\
\hline & & & & 2015 & DESERT FOR & 31D02.0M-N & 42D16.3M-E & 31D09.0M-N & 42D02.8M-E & \\
\hline 1014 & DESERT FOR & 0 & 109 & 2015 & DESERT FOR & 31D09.0M-N & 42D02.8M-E & \(31 \mathrm{D} 48.9 \mathrm{M}-\mathrm{N}\) & 40D33.2M-E & \\
\hline & & & & 2017 & DESERT FOR & 31D48.9M-N & 40D33.2M-E & 31D50.5M-N & 40 D 23.5 M - & \\
\hline 1015 & DESERT FOR & 0 & 109 & 2017 & DESERT FOR & 31D50.5M-N & 40D23.5M-E & 32D32.0M-N & \(38 \mathrm{D} 43.4 \mathrm{M}-\mathrm{E}\) & \\
\hline
\end{tabular}

\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|}
\hline THUND & ER 6.808: ME & & & & & & & 11/08/2004 & 08:55 PAGE & \\
\hline C 0 M & B A T C Y & C L E & & & TIME: & 1:1200 & & & & \\
\hline & & & & & R E P O R & & & REPO & T: CC-4 & \\
\hline & * & & & & FLOT MOVEME & & & & & \\
\hline * RED & * & & & & & & & PART & & \\
\hline \[
{ }^{*} * * *
\] & & & & & & & & & & \\
\hline --OPP & Sing CMD-- & & -FLOT & EgMENT & & & & & & \\
\hline & COMMAND & SQ. MILES & WIDTH & & & & OM & & то & \\
\hline ID & NAME & GAINED (LOST) & MILES & ID & NAME & LATITUDE & LONGITUDE & LATITUDE & LONGITUDE & \\
\hline 2009 & PERSIAN GU & 0 & 118 & 1013 & PERSIAN GU & 27D43.0M-N & 52D15.1M-E & 28D50.9M-N & 50D46.0M-E & \\
\hline 2010 & PERSIAN GU & 0 & 118 & 1008 & PERSIAN GU & 28D50.9M-N & 50D46.0M-E & 28D34.6M-N & 48D19.9M-E & \\
\hline 2011 & 1 IRAQI AR & 401) & 62 & 1009 & MARINES & 28D34.6M-N & 48D19.9M-E & 29D06.8M-N & 47D30.0M-E & \\
\hline 2012 & 2 IRAQI AR & 391) & 50 & 1010 & ARAB CORPS & 29D06.8M-N & 47D30.0M-E & 29D14.8M-N & 46D38.6M-E & \\
\hline 2013 & 3 IRAQI AR & 1087) & 75 & 1020 & VII CORPS & 29D14.8M-N & 46D38.6M-E & 29D44.9M-N & 45D32.8M-E & \\
\hline 2016 & BAGHDAD FO & 1359) & 93 & 1021 & XVIII AIRB & 29D44.9M-N & 45D32.8M-E & 29D50.4M-N & 43D50.5M-E & \\
\hline 2014 & DESERT FOR & 408) & 121 & 1021 & XVIII AIRB & 29D50.4M-N & 43D50.5M-E & 30D09.0M-N & 43D30.2M-E & \\
\hline & & & & 1012 & DESERT FOR & 30D09.0M-N & 43D30.2M-E & 31D02.0M-N & 42D16.3M-E & \\
\hline 2015 & DESERT FOR & 0 & 115 & 1012 & DESERT FOR & 31D02.0M-N & 42D16.3M-E & 31D09.0M-N & 42D02.8M-E & \\
\hline & & & & 1014 & DESERT FOR & 31D09.0M-N & 42D02.8M-E & \(31 \mathrm{D} 48.9 \mathrm{M}-\mathrm{N}\) & 40D33.2M-E & \\
\hline 2017 & DESERT FOR & 0 & 118 & 1014 & DESERT FOR & 31D48.9M-N & 40D33.2M-E & \(31050.5 \mathrm{M}-\mathrm{N}\) & 40D23.5M-E & \\
\hline & & & & 1015 & DESERT FOR & 31D50.5M-N & 40D23.5M-E & \(32 \mathrm{D} 32.0 \mathrm{M}-\mathrm{N}\) & \(38 \mathrm{D} 43.4 \mathrm{M}-\mathrm{E}\) & \\
\hline
\end{tabular}
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|}
\hline THunde & R 6.808: ME & & & & & & & 11/08/2004 & 08:55 P & PAGE \\
\hline c 0 M & B A T C Y & C L E & & & TIME: & 2:0000 & & & & \\
\hline & & & & & REPOR & & & REPOR & T: CC-4 & \\
\hline & * & & & & FLot moveme & & & & & \\
\hline * BLUE & * & & & & & & & PART & & \\
\hline & * & & & & & & & & & \\
\hline --OPPO & SING CMD-- & & -FLot & gGMEnt & & & & & & \\
\hline & COMMAND & SQ. MILES & WIDTH & & & & FROM & & о & \\
\hline ID & NAME & GAINED (LOST) & MILES & ID & NAME & LATITUDE & LONGITUDE & LATITUDE & LONGITUDE & \\
\hline 1013 & PERSIAN GU & 0 & 118 & 2009 & PERSIAN GU & 27D43.0M-N & 52D15.1M-E & 28D50.9M-N & 50D46.0M-E & \\
\hline 1008 & PERSIAN GU & 0 & 118 & 2010 & PERSIAN GU & 28D50.9M-N & 50D46.0M-E & 28D34.6M-N & 48D19.9M-E & \\
\hline 1009 & MARINES & 0 & 62 & 2011 & 1 IRAQI AR & 28D34.6M-N & 48D19.9M-E & 29D12.3M-N & 47D33.6M-E & \\
\hline 1010 & ARAB CORPS & 384 & 50 & 2012 & 2 IRAQI AR & 29D12.3M-N & 47D33.6M-E & 29D25.5M-N & 46 D 45.4 M - & \\
\hline 1020 & VII CORPS & 1043 & 75 & 2013 & 3 IRAQI AR & 29D25.5M-N & 46D45.4M-E & 29D55.5M-N & 45D39.5M-E & \\
\hline 1021 & XVIII AIRB & 1701 & 121 & 2016 & BAGHDAD FO & 29D55.5M-N & 45D39.5M-E & 30D00.4M-N & 43 D 56.6 M - & \\
\hline & & & & 2014 & DESERT FOR & 30D00.4M-N & 43D56.6M-E & 30D09.0M-N & 43D30.2M-E & \\
\hline 1012 & DESERT FOR & 0 & 109 & 2014 & DESERT FOR & 30D09.0M-N & 43D30.2M-E & \(31 \mathrm{D} 02.0 \mathrm{M}-\mathrm{N}\) & 42D16.3M-E & \\
\hline & & & & 2015 & DESERT FOR & 31D02.0M-N & 42D16.3M-E & 31D09.0M-N & \(42 \mathrm{D02.8M-E}\) & \\
\hline 1014 & DESERT FOR & 0 & 109 & 2015 & DESERT FOR & 31D09.0M-N & 42D02.8M-E & \(31 \mathrm{D} 48.9 \mathrm{M}-\mathrm{N}\) & 40D33.2M-E & \\
\hline & & & & 2017 & DESERT FOR & 31D48.9M-N & 40D33.2M-E & 31D50.5M-N & \(40 \mathrm{D} 23.5 \mathrm{M}-\mathrm{E}\) & \\
\hline 1015 & DESERT FOR & 0 & 109 & 2017 & DESERT FOR & 31D50.5M-N & 40D23.5M-E & \(32 \mathrm{D} 32.0 \mathrm{M}-\mathrm{N}\) & \(38 \mathrm{D} 43.4 \mathrm{M}-\mathrm{E}\) & \\
\hline
\end{tabular}



A. 8


















Appendix B: Table of Distribution Expected Values
\begin{tabular}{|c|c|c|}
\hline Distribution & Probability Function & Expected Value \\
\hline Beta & \[
\begin{gathered}
f(y)=\left[\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha) \Gamma(\beta)}\right] y^{\alpha-1}(1-y)^{\beta-1} \\
0<y<1
\end{gathered}
\] & \[
\frac{\alpha}{\alpha+\beta}
\] \\
\hline Binomial & \[
\begin{gathered}
p(y)=\binom{n}{y} p^{y}(1-p)^{n-y} \\
y=0,1, \ldots, n
\end{gathered}
\] & \(n p\) \\
\hline Erlang & \[
\begin{aligned}
f(y)= & \frac{1}{\lambda} e^{-\frac{y}{\lambda} \frac{\left(\frac{y}{\lambda}\right)^{n-1}}{(n-1)!}} \\
& 0<y
\end{aligned}
\] & \(k \beta\) \\
\hline Exponential & \[
\begin{aligned}
& f(y)=\frac{1}{\beta} e^{-y / \beta} \\
& 0<\beta ; 0<y<\infty
\end{aligned}
\] & \(\beta\) \\
\hline Gamma & \[
\begin{gathered}
f(y)=\left[\frac{1}{\Gamma(\alpha) \beta^{\alpha}}\right] y^{\alpha-1} e^{\frac{-y}{\beta}} \\
0<y<\infty
\end{gathered}
\] & \(\alpha \beta\) \\
\hline Logorithmic Normal & \[
\begin{gathered}
f(y)=\frac{1}{y \sigma \sqrt{2 \pi}} e^{\left[-\left(\frac{1}{2 \sigma^{2}}\right)(\ln y-\mu)^{2}\right]} \\
0<y
\end{gathered}
\] & \(e^{\frac{\mu+\sigma^{2}}{2}}\) \\
\hline Normal & \[
\begin{gathered}
f(y)=\frac{1}{\sigma \sqrt{2 \pi}} e^{\left[-\left(\frac{1}{2 \sigma^{2}}\right)(y-\mu)^{2}\right]} \\
-\infty<y<+\infty
\end{gathered}
\] & \(\mu\) \\
\hline Poisson & \[
\begin{gathered}
p(y)=\frac{\lambda^{y} e^{-\lambda}}{y!} \\
y=0,1,2, \ldots
\end{gathered}
\] & \(\lambda\) \\
\hline \begin{tabular}{l}
Randi \\
Discrete Uniform (a, b)
\end{tabular} & \[
\begin{gathered}
p(y)=\frac{1}{(b-a+1)} \\
y=a, a+1, \ldots, b-1, b
\end{gathered}
\] & \[
\frac{\sum_{i=a}^{b} i}{(b-a+1)}
\] \\
\hline
\end{tabular}
\begin{tabular}{|c|c|c|}
\hline Distribution & Probability Function & Expected Value \\
\hline \begin{tabular}{l}
Random \\
Uniform (0, 1)
\end{tabular} & \[
\begin{aligned}
& f(y)=\frac{1}{1} \\
& 0 \leq y \leq 1
\end{aligned}
\] & 0.5 \\
\hline Triangle & \[
f(y)= \begin{cases}\frac{2(y-a)}{(b-a)(c-a)} & a \leq y \leq c \\ \frac{2(b-y)}{(b-a)(b-c)} & c<y \leq b \\ a \leq y \leq b\end{cases}
\] & \(\frac{a+b+c}{3}\) \\
\hline Uniform & \[
\begin{gathered}
f(y)=\frac{1}{\theta_{2}-\theta_{1}} \\
\theta_{1} \leq y \leq \theta_{2}
\end{gathered}
\] & \(\frac{\theta_{1}+\theta_{2}}{2}\) \\
\hline Weibull & \[
\begin{gathered}
f(y)=\alpha \beta^{-\alpha} y^{\alpha-1} e^{-\left(\frac{y}{\beta}\right)^{\alpha}} \\
0 \leq y
\end{gathered}
\] & \(\beta \Gamma\left(1+\frac{1}{\alpha}\right)\) \\
\hline
\end{tabular}

\section*{Appendix C: The Modified and Added Files}

The modified SIMSCRIPT II. 5 files located in the Source directory:
\begin{tabular}{lllll} 
ADF040.sim & AIR042.sim & AIR830.sim & GRD300.sim & REC012.sim \\
ADF041.sim & AIR050.sim & AIR840.sim & GRD302.sim & REC013.sim \\
ADF101.sim & AIR051.sim & AIR911.sim & GRD309.sim & REC014.sim \\
ADF105.sim & AIR060.sim & AIR920.sim & GRD871.sim & REC015.sim \\
ADF121.sim & AIR070.sim & ANT211.sim & ISR000.sim & REC016.sim \\
ADF140.sim & AIR101.sim & BSE001.sim & ISR001.sim & REC017.sim \\
ADF150.sim & AIR110.sim & BSE002.sim & ISR004.sim & REC018.sim \\
ADF160.sim & AIR120.sim & BSE003.sim & ISR005.sim & REC310.sim \\
ADF161.sim & AIR150.sim & BSE020.sim & ISR115.sim & REC320.sim \\
ADF200.sim & AIR241.sim & BSE050.sim & ISR250.sim & REC330.sim \\
ADF420.sim & AIR526.sim & BSE200.sim & ISR251.sim & REC341.sim \\
ADF430.sim & AIR528.sim & BSE201.sim & ISR360.sim & REC900.sim \\
ADF440.sim & AIR540.sim & BSE225.sim & ISR365.sim & UTL104.sim \\
ADF452.sim & AIR550.sim & BSE251.sim & ISR400.sim & UTL260.sim \\
ADF460.sim & AIR561.sim & BSE260.sim & ISR500.sim & UTL305.sim \\
ADF500.sim & AIR564.sim & BSE270.sim & LOG700.sim & UTL520.sim \\
AIR014.sim & AIR567.sim & BSE400.sim & PLA010.sim & UTL550.sim \\
AIR017.sim & AIR602.sim & BSE800.sim & PLA443.sim & UTL750.sim \\
AIR021.sim & AIR800.sim & GRD095.sim & PLA557.sim & UTL920.sim \\
AIR041.sim & AIR810.sim & GRD100.sim & REC011.sim & STA900.sim
\end{tabular}

The added SIMSCRIPT II. 5 files located in the Source directory:
ZEARL1.sim
ZEARL2.sim

The modified THUNDER scripts located in the bin directory:
ttrun
ttrep
ttcompile
ttuser.each

The modified ME data files located in the Data/ME directory:
control.dat

The added ME data files located in the Data/ME directory:
newseed.val

The added C files located in the Source directory:
RngStream.c
RngStream.h

\section*{Appendix D: CONTROL.DAT File}

CONTROLS. 020
```

REPORT.TITLE "ME"
BASIC.SIMULATION.CONTROLS
NUMBER.OF.DAYS.OF.WAR(INT.DAYS) 5
BEGIN.DAYLIGHT(DEC.HOURS) 6.00
END.DAYLIGHT(DEC.HOURS) 18.00
ADX.PROVIDE.TERM.AD.FOR.FIXED.TGTS YES
IGNORE.RANGE.CONSTRAINTS NO
COUNT.ALERT.MSN.AS.COMPLETE NO
END.BASIC.SIMULATION.CONTROLS
GAME.CONTROLS
GAME.FILE.MOD.FLAG YES
GAME.ENABLE NO
GAME.CYCLE
24
FIRST.GAME.STOP(DAYS) 2.0
END.GAME.CONTROLS
ADF.CALIBRATION.CONTROLS
ADF.CALIBRATION.MODE.ENABLE NO
if enabled
only preplanned ATOs fly
all ADvsAC Pks = 0.0
unlimited AD ammo reloads
lethal SEAD disabled
air-to-ground disabled
special ADF calibration transactions enabled
ADF results averaged over multiple reps:
REPLICATIONS.PER.ADF.CALCULATION 500
USE.MANUAL.IADS.VALUES(YES,NO) NO
MANUAL.IADS.INTEGRATION.LEVEL(0.0-1.0) 0.7
MANUAL.IADS.SECONDS.DELAY 15.0
MANUAL.IADS.INTIMIDATE 0.0
END.ADF.CALIBRATION.CONTROLS
COMPUTATIONAL.RESOLUTION. LEVELS(LOW, HIGH, VERY. HIGH)
BLUE.AIR.DEFENSE HIGH
RED.AIR.DEFENSE HIGH
AIR.WAR HIGH
BLUE.RECONNAISSANCE HIGH
RED.RECONNAISSANCE HIGH
WEATHER HIGH
GROUND.MOVEMENT HIGH
CRITICAL.RESOURCES HIGH
END.COMPUTATIONAL.RESOLUTION.LEVELS
GRAPHIC.OUTPUT.CONTROLS
WRITE.GRAPHIC.TO.TRANS
NO

```
```

    POST.PROCESS.GRAPHICS(BOTH, CHART,MAP,NONE) BOTH
    AD.COMPLEX.STATUS.UPDATE.CYCLE(HOURS) 6
    AIRBASE.STATUS.UPDATE.CYCLE(HOURS) 3
    CHOKE.POINT.STATUS.UPDATE.CYCLE(HOURS) 6
    COMMAND.STATUS.UPDATE.CYCLE(HOURS) 6
    LOG.FAC.STATUS.UPDATE.CYCLE(HOURS) 6
    STRATEGIC.TARGET.STATUS.UPDATE.CYCLE(HOURS) }
    SUPPLY.TRAIN.STATUS.UPDATE.CYCLE(HOURS) 6
    TBM.UNIT.STATUS.UPDATE.CYCLE(HOURS) 6
    UNIT.STATUS.UPDATE.CYCLE(HOURS) 6
    PERCEPT.STATUS.UPDATE.CYCLE(HOURS) 6
    AIR.REPORT.CYCLE(DAY,GAME,BLUE,RED) DAY
    END.OUTPUT.CONTROLS
OVERALL.REPORT.CONTROLS
REPORT.MODE(STANDARD,METRIC,BOTH) STANDARD
END.OVERALL.REPORT.CONTROLS
DATA.REPORT.CONTROLS
NUMBER.OF.DATA.REPORTS 91
CONTROL.FILE DR-2 NO
RANDOM.SEEDS DR-3 NO
STANDARD.TARGETS DR-5 NO
STANDARD.TARGET.REPAIR.PROBS DR-6 NO
TYPE.RADAR
TYPE.JAMMER
AIR.DEFENSE.DATA.REPORTS
TYPE.AIR.DEFENSE
POSTURES.\&.CLASSES
THREAT.CLASSES
INTEGRATED.AD.SYSTEM
ADA.ENGAGEMENT.PROBS
AD.VS.AIRCRAFT.PKS
AD. COMPLEXES
PLANNING.DATA.REPORTS
AIR.RULES
AIR.PLANNING.RULES
AIR.ALLOCATION
AIR.PLANNING.FACTORS
OCA.TARGETS
PREPLANNED.ATOS
PREPLANNED.LIFT.ATOS
ZONE.SECTOR.TGT.PRIORITY
INT.DEPTH.FACTOR.CURVES
STI.TARGETS
AIR.TO.AIR.DATA.REPORTS
AIR.TO.AIR.PKS
RELEASE.RANGE.ADVANTAGE
CONFIG.DETECT.PROBS
AIR.TO.GROUND.PKS
HARM.PKS
MINE.PKS
TAKEOFF.LANDING.PKS
WMD.EFFECTS
TYPE.AIR.MUNITIONS
DR-2 NO
DR-10
NO
DR-11 NO
****
DR-12 NO
DR-13 NO
DR-16 NO
DR-14 NO
DR-15 NO
DR-27 NO
DR-66 NO
****
DR-20 NO
DR-79 NO
DR-21 NO
DR-23 NO
DR-29 NO
DR-22 NO
DR-65 NO
DR-68 NO
DR-70 NO
****
DR-24 NO
DR-25 NO
DR-26 NO
DR-28 NO
DR-40 NO
DR-41 NO
DR-42 NO
DR-43 NO
DR-31 NO

```
\begin{tabular}{|c|c|c|}
\hline MUNITION.SURVIVABILITY.CURVES & DR-44 & NO \\
\hline TYPE.AIRCRAFT & DR-32 & NO \\
\hline AIRCRAFT.MAINTENANCE & DR-39 & NO \\
\hline TBM. UNITS & DR-75 & NO \\
\hline TBM. DETECTION & DR-76 & NO \\
\hline AIRBASE & DR-33 & NO \\
\hline CARRIER.BATTLE.GROUP & DR-30 & NO \\
\hline SUPPLY.KIT & DR-34 & NO \\
\hline SQUADRON & DR-35 & NO \\
\hline AIR.TO.GRD.MIN.CEIL.VIS & DR-36 & NO \\
\hline SUPPLY.URGENCY. CURVES & DR-37 & NO \\
\hline CRITICAL.RESOURCES & DR-38 & NO \\
\hline NODES & DR-45 & NO \\
\hline TRANS.SHIPMENT.POINTS & DR-46 & NO \\
\hline ARCS & DR-47 & NO \\
\hline CHOKE.POINTS & DR-48 & NO \\
\hline MINE. DELAY. CURVES & DR-49 & NO \\
\hline BATTLEFIELD & DR-50 & NO \\
\hline AIR.NETWORK & DR-73 & NO \\
\hline GROUND. TARGET. ACQUISITION & DR-51 & NO \\
\hline TARGET.ACQUISITION. CURVES & DR-17 & NO \\
\hline TERRAIN & DR-52 & NO \\
\hline INTERVISIBILITY.CURVES & DR-53 & NO \\
\hline TYPE.C3.FACILITY & DR-55 & NO \\
\hline COMMAND & DR-56 & NO \\
\hline GROUND. RULES & DR-57 & NO \\
\hline SUPPLY.TRAIN.TARGETS & DR-54 & NO \\
\hline LOGISTIC.FACILITY & DR-58 & NO \\
\hline TYPE.UNIT & DR-59 & NO \\
\hline TYPE.EQUIPMENT & DR-60 & NO \\
\hline EQUIPMENT.SIZE.CATEGORY & DR-61 & NO \\
\hline WEAPON.VS.EQUIPMENT.EFFECTS & DR-62 & NO \\
\hline UNIT & DR-63 & NO \\
\hline LIFT.EVENTS & DR-82 & NO \\
\hline LOG.RULES & DR-81 & NO \\
\hline SPACE.LOGISTICS & DR-78 & NO \\
\hline PERCEPTIONS & DR-64 & NO \\
\hline STRATEGIC.TARGETS & DR-69 & NO \\
\hline AIRCRAFT. FACTORS & DR-71 & NO \\
\hline GROUND.WAR.SUPPORT.EFFECTS & DR-72 & NO \\
\hline BOOST. PHASE. INTERCEPT & DR-77 & NO \\
\hline TARGET. AVAILABILITY & DR-74 & NO \\
\hline WEATHER.STATIONS & DR-80 & NO \\
\hline SATELLITE.NAV.JAM.STATES & DR-92 & NO \\
\hline SATELLITE.NAV.MUNT.EFFECTS & DR-93 & NO \\
\hline SATELLITE.PASSES & DR-94 & NO \\
\hline ISR. DATA.REPORTS & * & \\
\hline ISR.SATELLITES & DR-83 & NO \\
\hline ISR.PLANNING & DR-84 & NO \\
\hline ISR.TARGETS & DR-85 & NO \\
\hline ISR.SENSORS.AND. EMITTERS & DR-86 & NO \\
\hline ISR.EFFECTS & DR-87 & NO \\
\hline ISR.INITIALIZATION & DR-88 & NO \\
\hline ISR.EVENTS & DR-89 & NO \\
\hline ISR. NODES & DR-90 & NO \\
\hline
\end{tabular}
\begin{tabular}{|c|c|c|c|c|c|}
\hline ISR.SENSOR.VS.EMITTER.PROBS & DR-91 & \multicolumn{2}{|r|}{NO} & & \\
\hline MUNITION. MANAGEMENT. FACTORS & DR-95 & & NO & & \\
\hline CHEMICAL.WEAPON.EFFECTS & DR-96 & & NO & & \\
\hline CHEMICAL. AGENT. DELIVERY & DR-97 & & NO & & \\
\hline CAMPAIGN.COST. DATA & DR-99 & & NO & & \\
\hline \multicolumn{6}{|l|}{END. DATA.REPORT.CONTROLS} \\
\hline \multicolumn{6}{|l|}{OUTPUT.REPORT.CONTROLS} \\
\hline COMPLETED.MISSION. INFO & AW-1 & NO & BOTH & 0.00 & 300.00 \\
\hline \multicolumn{6}{|l|}{DAILY.AIR.WAR.RESULTS.SUMMARIES} \\
\hline MISSION.\&.SORTIE.SUMMARY & AW-2 & NO & BOTH & 0.00 & 300.00 \\
\hline AIRCRAFT.LOSS.SUMMARY & AW-3 & YES & BOTH & 0.00 & 300.00 \\
\hline AIRBASE. ACTIVITY & AW-4 & NO & BOTH & 0.00 & 300.00 \\
\hline SORTIE.GENERATION & AW-5 & YES & BOTH & 0.00 & 300.00 \\
\hline CANCELLED.MISSION. INFO & AW-6 & NO & BOTH & 0.00 & 300.00 \\
\hline \multicolumn{6}{|l|}{INDIVIDUAL.MISSION.KILL.RESULTS} \\
\hline DCA.MISSION.KILLS & AW-7 & NO & BOTH & 0.00 & 300.00 \\
\hline ODCA.MISSION.KILLS & AW-8 & NO & BOTH & 0.00 & 300.00 \\
\hline HVAA.MISSION.KILLS & AW-9 & NO & BOTH & 0.00 & 300.00 \\
\hline BARCAP.MISSION.KILLS & AW-10 & NO & BOTH & 0.00 & 300.00 \\
\hline FSWP.MISSION.KILLS & AW-11 & NO & BOTH & 0.00 & 300.00 \\
\hline DTBM.MISSION.KILLS & AW-12 & NO & BOTH & 0.00 & 300.00 \\
\hline STI.MISSION.KILLS & AW-13 & NO & BOTH & 0.00 & 300.00 \\
\hline CAS.MISSION.KILLS & AW-14 & NO & BOTH & 0.00 & 300.00 \\
\hline BAI.MISSION.KILLS & AW-15 & NO & BOTH & 0.00 & 300.00 \\
\hline INT.MISSION.KILLS & AW-16 & NO & BOTH & 0.00 & 300.00 \\
\hline OCA.MISSION.KILLS & AW-17 & NO & BOTH & 0.00 & 300.00 \\
\hline DSEAD.MISSION.KILLS & AW-18 & NO & BOTH & 0.00 & 300.00 \\
\hline SUPPRESSION.MISSION.KILLS & AW-19 & NO & BOTH & 0.00 & 300.00 \\
\hline OTBM.MISSION.KILLS & AW-20 & NO & BOTH & 0.00 & 300.00 \\
\hline AIR.ESCORT.MISSION.KILLS & AW-21 & NO & BOTH & 0.00 & 300.00 \\
\hline HIGH.VALUE.ASSET. ACTIVITY & AW-22 & NO & BOTH & 0.00 & 300.00 \\
\hline DEFENDING.AIRCRAFT.KILLS & AW-23 & NO & BOTH & 0.00 & 300.00 \\
\hline LIFT.MISSION & AW-24 & NO & BOTH & 0.00 & 300.00 \\
\hline \multicolumn{6}{|l|}{AIRCRAFT.MAINTENANCE} \\
\hline TYPE.AIRCRAFT. MX. ACTIVITY & MX-1 & NO & BOTH & 0.00 & 300.00 \\
\hline CONSUM. RESOURCE. ACTIVITY & MX-2 & NO & BOTH & 0.00 & 300.00 \\
\hline NON. CONSUM. RES. ACTIVITY & MX-3 & NO & BOTH & 0.00 & 300.00 \\
\hline \multicolumn{6}{|l|}{GROUND. COMBAT. CYCLE. REPORTS} \\
\hline UNIT.STATUS & CC-1 & NO & BOTH & 0.00 & 300.00 \\
\hline C3.FACILITY.STATUS & CC-2 & NO & BOTH & 0.00 & 300.00 \\
\hline CHOKE.POINT.STATUS & CC-3 & NO & BOTH & 0.00 & 300.00 \\
\hline FLOT.MOVEMENT & CC-4 & NO & BOTH & 0.00 & 300.00 \\
\hline AIR.GRID.AD.STATUS & CC-5 & NO & BOTH & 0.00 & 300.00 \\
\hline TYPE.AD.SITE.STATUS & CC-6 & NO & BOTH & 0.00 & 300.00 \\
\hline STRATEGIC.TARGET.STATUS & CC-7 & NO & BOTH & 0.00 & 300.00 \\
\hline IADS. OBJECT. STATUS & CC-8 & NO & BOTH & 0.00 & 300.00 \\
\hline \multicolumn{6}{|l|}{AIR.MISSION.PLANNING.REPORTS} \\
\hline APPORTIONMENT. ALLOCATION & MP-1 & NO & BOTH & 0.00 & 300.00 \\
\hline AIR.TASKING.ORDERS & MP-2 & NO & BOTH & 0.00 & 300.00 \\
\hline SORTIES.SCHEDULED. SUMMARY & MP-3 & NO & BOTH & 0.00 & 300.00 \\
\hline TARGET.PRIORITIES & MP-4 & NO & BOTH & 0.00 & 300.00 \\
\hline AIR.REFUELING.ALLOCATION & MP-5 & NO & BOTH & 0.00 & 300.00 \\
\hline AIR.PLANNING.RULE.STATUS & MP-6 & NO & BOTH & 0.00 & 300.00 \\
\hline GROUND.SUPPLY.CYCLE.REPORTS & & ** & & & \\
\hline
\end{tabular}
```

            LOGISTIC.FACILITY.STATUS SC-1 NO BOTH 0.00 300.00
            SUPPLY.TRAIN.STATUS
            CRITICAL.RESOURCE.STATUS
            SPACE.LOGISTICS.REPORT
    END.OUTPUT.REPORT.CONTROLS
    GAME.REPORT.SELECTION
    GC-1 NO BOTH "AIR COMMAND MISSION ALLOCATION REPORT"
    GC-2 NO BOTH "AIR COMMAND RESOURCE ALLOCATION REPORT"
    GC-3 NO BOTH "TARGET PRIORITY REPORT"
    GC-4 NO BOTH "GROUND COMMAND STRUCTURE WITH SUPPORTING AIR
    COMMANDS"
GC-5 NO BOTH "AIR COMMAND STRUCTURE WITH SUPPORTED GROUND COMMANDS"
GC-6 NO BOTH "AIR BASE ASSETS REPORT"
GC-7 NO BOTH "LOGISTICS FACILITY ASSETS REPORT"
GC-8 NO BOTH "AIRCRAFT CHARACTERISTICS REPORT"
GC-9 NO BOTH "ZONE SECTOR PERCEPTION HISTORY"
GC-10 NO BOTH "FLOT MOVEMENT REPORT"
GC-11 NO BOTH "MISSION ALLOCATION PROJECTION"
GC-12 NO BOTH "PROJECTED AIR TASKING ORDERS"
GC-13 NO BOTH "PROJECTED SORTIES SCHEDULED"
GC-14 NO BOTH "AIRCRAFT PLANNING FACTORS"
GC-15 NO BOTH "PREPLANNED ATO REPORT"
GC-16 NO BOTH "AIR GRID AIR DEFENSE SITE STATUS"
GC-17 NO BOTH "AIR COMBAT RULES"
GC-18 NO BOTH "AIR PLANNING RULES"
GC-19 NO BOTH "PREPLANNED LIFT ATO REPORT"
GC-20 NO BOTH "MUNITION MANAGEMENT FACTORS"
END.GAME.REPORT.SELECTION
TRANSACTION.CONTROLS
@DEBUG.TRANSACTIONS
ADFL 0 0.000 300.00
ADFH 0 0.000 300.00
AIRA 0 0.000 300.00
AIRF 0 0.000 300.00
AIRG 0 0.000 300.00
AIRP 0 0.000 300.00
ANTC 0 0.000 300.00
ANTI 0 0.000 300.00
ANTP 0 0.000 300.00
BSEL 0 0.000 300.00
BSEM 0 0.000 300.00
BSEO 0 0.000 300.00
CONA 0 0.000 300.00
GRDC 0 0.000 300.00
GRDM 0 0.000 300.00
GRDO 0 0.000 300.00
ISRE 0 0.000 300.00
ISRP 0 0.000 300.00
ISRR 0 0.000 300.00
ISRS 0 0.000 300.00
ISRT 0 0.000 300.00
LOGS 0 0.000 300.00
LOGE 0 0.000 300.00

```


VARIANCE.REDUCTION.CONTROLS USE.NEW.RANDOM. NUMBER.GENERATOR YES USE. COMMON.RANDOM.NUMBERS YES USE.ANTITHETIC.VARIABLES NO USE.CONTROL.VARIATES

RANDOM.NUMBER.STREAM.DATA READ. RANDOM. SEEDS
```

WRITE.RANDOM.SEEDS
YES
SINGLE.SEED.ALL.NUMBERS
NO
SINGLE.SEED.IS(INT)
1
RN.STREAMS
ADF040.1 1
ADF041.1 1
ADF041.2 1
ADF041.3 1
ADF101.1 1
ADF101.2 1
ADF105.1 1
ADF121.1 1
ADF121.2 1
ADF121.3 1
ADF140.1 1
ADF140.2 1
ADF140.3 1
ADF150.1 1
ADF150.2 1
ADF150.3 1
ADF150.4 1
ADF150.5 1
ADF150.6 1
ADF150.7 1
ADF160.1 1
ADF161.1 1
ADF161.2 1
ADF161.3 1
ADF161.4 1
ADF161.5 1
ADF200.1 1
ADF420.1 1
ADF420.2 1
ADF420.3 1
ADF430.1 1
ADF430.2 1
ADF430.3 1
ADF440.1 1
ADF440.2 1
ADF440.3 1
ADF452.1 1
ADF452.2 1
ADF452.3 1
ADF452.4 1
ADF452.5 1
ADF452.6 1
ADF452.7 1
ADF460.1 1
ADF460.2 1
ADF500.1 1
AIR007.1 1
AIR017.1 1
AIR041.1 1
AIR041.2 1

```
\begin{tabular}{|c|c|}
\hline AIR041.3 & 1 \\
\hline AIR042.1 & 1 \\
\hline AIR042.2 & 1 \\
\hline AIR050.1 & 1 \\
\hline AIR050.2 & 1 \\
\hline AIR051.1 & 1 \\
\hline AIR060.1 & 1 \\
\hline AIR060.2 & 1 \\
\hline AIR060.3 & 1 \\
\hline AIR070.1 & 1 \\
\hline AIR070. 2 & 1 \\
\hline AIR021.1 & 1 \\
\hline AIR101.1 & 1 \\
\hline AIR101.2 & 1 \\
\hline AIR110.1 & 1 \\
\hline AIR120.1 & 1 \\
\hline AIR150.1 & 1 \\
\hline AIR150.2 & 1 \\
\hline AIR241.1 & 1 \\
\hline AIR526.1 & 1 \\
\hline AIR528.1 & 1 \\
\hline AIR540.1 & 1 \\
\hline AIR550.1 & 1 \\
\hline AIR550. 2 & 1 \\
\hline AIR561.1 & 1 \\
\hline AIR561.2 & 1 \\
\hline AIR564.1 & 1 \\
\hline AIR567.1 & 1 \\
\hline AIR567.2 & 1 \\
\hline AIR602.1 & 1 \\
\hline AIR800. 1 & 1 \\
\hline AIR800. 2 & 1 \\
\hline AIR800.3 & 1 \\
\hline AIR810.1 & 1 \\
\hline AIR810.2 & 1 \\
\hline AIR830.1 & 1 \\
\hline AIR830.2 & 1 \\
\hline AIR830. 3 & 1 \\
\hline AIR830.4 & 1 \\
\hline AIR840.1 & 1 \\
\hline AIR840.2 & 1 \\
\hline AIR840.3 & 1 \\
\hline AIR840.4 & 1 \\
\hline AIR911.1 & 1 \\
\hline AIR911. 2 & 1 \\
\hline AIR920.1 & 1 \\
\hline ANT211.1 & 1 \\
\hline ANT211. 2 & 1 \\
\hline ANT211.3 & 1 \\
\hline BSE001.1 & 1 \\
\hline BSE001.2 & 1 \\
\hline BSE002.1 & 1 \\
\hline BSE002.2 & 1 \\
\hline BSE003.1 & 1 \\
\hline BSE003.2 & 1 \\
\hline
\end{tabular}
```

BSE020.1 1
BSE030.1 1
BSE050.1 1
BSE050.2 1
BSE200.1 1
BSE200.2 1
BSE200.3 1
BSE200.4 1
BSE200.5 1
BSE201.1 1
BSE201.2 1
BSE251.1 1
BSE260.1 1
BSE260.2 1
BSE270.1 1
BSE400.1 1
BSE800.1 1
GRD095.1 1
GRD095.2 1
GRD095.3 1
GRD100.1 1
GRD300.1 1
GRD302.1 1
GRD309.1 1
GRD871.1 1
GRD871.2 1
ISR000.1 1
ISR000.2 1
ISR000.3 1
ISR000.4 1
ISR001.1 1
ISR001.2 1
ISR001.3 1
ISR004.1 1
ISR005.1 1
ISR115.1 1
ISR115.2 1
ISR250.1 1
ISR250.2 1
ISR251.1 1
ISR251.2 1
ISR251.3 1
ISR251.4 1
ISR360.1 1
ISR365.1 1
ISR400.1 1
ISR500.1 1
ISR500.2 1
ISR500.3 1
LOG700.1 1
LOG700.2 1
LOG700.3 1
PLA010.1 1
PLA010.2 1
PLA443.1 1

```
```

PLA557.1 1
REC011.1 1
REC011.2 1
REC011.3 1
REC012.1 1
REC012.2 1
REC012.3 1
REC012.4 1
REC012.5 1
REC012.6 1
REC013.1 1
REC013.2 1
REC013.3 1
REC013.4 1
REC013.5 1
REC013.6 1
REC014.1 1
REC014.2 1
REC014.3 1
REC014.4 1
REC014.5 1
REC014.6 1
REC014.7 1
REC014.8 1
REC014.9 1
REC015.1 1
REC015.2 1
REC015.3 1
REC015.4 1
REC015.5 1
REC015.6 1
REC015.7 1
REC015.8 1
REC016.1 1
REC016.2 1
REC016.3 1
REC016.4 1
REC016.5 1
REC016.6 1
REC016.7 1
REC016.8 1
REC016.9 1
REC016.10 1
REC016.11 1
REC017.1 1
REC017.2 1
REC017.3 1
REC017.4 1
REC017.5 1
REC017.6 1
REC017.7 1
REC017.8 1
REC017.9 1
REC017.10 1
REC018.1 1

```
```

    REC018.2 1
    REC018.3 1
    REC018.4 1
    REC310.1 1
    REC310.2 1
    REC320.1 1
    REC320.2 1
    REC320.3 1
    REC330.1 1
    REC330.2 1
    REC330.3 1
    REC330.4 1
    REC330.5 1
    REC330.6 1
    REC330.7 1
    REC330.8 1
    REC341.1 1
    REC341.2 1
    REC341.3 1
    REC341.4 1
    REC900.1 1
    REC900.2 1
    UTL104.1 1
    UTL305.1 1
    UTL520.1 1
    UTL520.2 1
    UTL550.1 1
    UTL550.2 1
    UTL750.1 1
    UTL920.1 1
    ISR000.5 1
    ADF140.4 1
    ADF440.4 1
    AIR830.5 1
    AIR830.6 1
    ISR251.5 1
    ISR251.6 1
    ISR251.7 1
    LOG700.4 1
    END.RN.STREAMS
    END.RANDOM.NUMBER.STREAM.DATA
END.CONTROL

```

\title{
Appendix E: Example SIMSCRIPT II. 5 Code AIR070.SIM (Original)
}
```

ROUTINE AIR.070.ADJUDICATE.DTBM
''\$Id: air070.sim,v 2.10 2000/09/20 20:12:38 steveh Exp \$
GIVEN
.DTBM.PLATFORM, ''PTR TO DTBM PLATFORM FIRING AT TBM
.DTBM.WPN.TYPE, ''TYPE OF SHOOTER ENGAGING TBM
.TBU, ''INDEX OF TBM.UNIT LAUNCHING TBM
.TBM.FLT, ''POINTER TO TBM FLIGHT BEING LAUNCHED
.DETECT.SECONDS.DELAY ''BETWEEN TBM LAUNCH AND DTBM RESPONSE
''YIELDING
'' NONE
''PURPOSE
''-------
'' ADJUDICATE A DTBM VS. TBM AIR-TO-AIR ENGAGEMENT
''PROCESSING
''----------
''
' ' ============================================================
''CALLED BY
''--------
'' AIR.060.BOOST.PHASE.INTERCEPT.MANAGER
''CHANGE LOG
''---------
'' DATE
''----------
1-11-96 MORGA
'' 1-23-96 EMERT
'' 2-6-96 MORGA
'' 2-9-96 MORGAN
'' 6-12-96 MORGAN
'' 8-09-99 HADLEY
''
'' 08-16-99 LENNOX CALL GRA.730, GRA.740 FOR GRAPHER2K
'' 2-11-00 COSGROVE ADD SBL
'' 07-10-00 HADLEY UP FLIGHT GROUP ID IN TRANS TO I 7
NORMALLY MODE IS INTEGER
DEFINE .DETECT.SECONDS.DELAY, .PK, .SHOT.DURATION.SEC, .SUM.SECONDS,
.TOTAL.ELAPSED.SECONDS
AS DOUBLE VARIABLES
IF .DTBM.WPN.TYPE = ..SPACE.BASED.LASER
LET .DTBM.SAT = .DTBM.PLATFORM
ELSE

```
```

    LET .DTBM.FG = .DTBM.PLATFORM
    LET .DTBM.FLT = F.FG.FLIGHT.SET(.DTBM.FG)
    IF FL.MISSION(.DTBM.FLT) <> ..DTBM
        TRACE
        STOP
    OTHERWISE
    ALWAYS
LET .TBM.FG = FL.FLT.GP(.TBM.FLT)
LET .TBM.LAUNCH.X = TBU.X.COORD(.TBU)
LET .TBM.LAUNCH.Y = TBU.Y.COORD(.TBU)
LET .TBM.TAC = TBU.TYPE.AC(.TBU)
LET .TTT = TA.TBM.TARGET.TYPE(.TBM.TAC)
LET .TBM.TGT = FG.TARGET(.TBM.FG)
LET .TBM.TGT.X = TG.X.COORD(.TBM.TGT)
LET .TBM.TGT.Y = TG.Y.COORD(.TBM.TGT)
IF .DTBM.WPN.TYPE = ..SPACE.BASED.LASER
LET .TSW = SAT.TYPE.SPACE.WPN(.DTBM.SAT)
LET .DWP = TSW.DTBM.WEAPON(.TSW)
LET .PLATFORM.ID = SAT.ID(.DTBM.SAT)
ELSE
FOR EVERY .AM IN FL.MUNITION.SET(.DTBM.FLT)
WITH (AM.FUNCTION(.AM) = ..LASER
OR AM.FUNCTION(.AM) = ..KKV)
AND AM.RNDS.REMAINING(.AM) > 0
FIND THE FIRST CASE
IF NONE
TRACE
STOP
OTHERWISE
LET .TAM = AM.TYPE(.AM)
LET .DWP = TAM.DTBM.WEAPON(.TAM)
LET .PLATFORM.ID = FG.ID(.DTBM.FG)
ALWAYS
''determine TBM flight phase (and pK) at which to adjudicate
IF .DTBM.WPN.TYPE = ..AIRBORNE.LASER
OR .DTBM.WPN.TYPE = ..SPACE.BASED.LASER
CALL UTL.720.CALCULATE.USER.FUNCTION
GIVEN
DWP.SHOT.DURATION.FN(.DWP),
RN.STREAM(..AIR070.1)
YIELDING
.SHOT.DURATION.SEC
LET .SHOT.DURATION.SEC = MAX.F(.SHOT.DURATION.SEC, 0.0)
LET .TOTAL.ELAPSED.SECONDS
= .DETECT.SECONDS.DELAY + .SHOT.DURATION.SEC
LET .INTERCEPT.FLAG = ..YES
ELSE
IF .DTBM.WPN.TYPE <> ..KKV

```

TRACE
STOP
OTHERWISE
LET .DTBM. \(\mathrm{X}=\mathrm{FG} . \mathrm{X} . \operatorname{COORD}(. \mathrm{DTBM} . \mathrm{FG})\)
LET .DTBM.Y = FG.Y.COORD(.DTBM.FG)
LET . DTBM. Z = PTH.ALTITUDE(FG.CURR.PATH(.DTBM.FG))
CALL AIR.071.COMPUTE.KKV.INTERCEPT.TIME GIVEN .TTT, ''INDEX OF TBM.TARGET.TYPE
.TBM. LAUNCH.X, .TBM. LAUNCH.Y, .TBM.TGT.X, .TBM.TGT.Y, .DWP, ''INDEX OF DTBM.WEAPON .DTBM.X, . DTBM.Y, . DTBM.Z, .DETECT.SECONDS.DELAY ''FROM TBM LAUNCH TO KKV LAUNCH YIELDING .INTERCEPT.FLAG, ''..YES OR ..NO SUCCESSFUL INTERCEPT .TOTAL.ELAPSED.SECONDS ''FROM TBM LAUNCH TO KKV INTERCEPT
ALWAYS
IF .INTERCEPT.FLAG = ..YES
FOR EVERY TBM.FLIGHT.PHASE CALLED .TFP
DO
ADD TPT.DURATION.SEC(.TFP, .TTT) TO .SUM.SECONDS
IF .TOTAL.ELAPSED.SECONDS <= .SUM.SECONDS LEAVE
OTHERWISE
LOOP
IF .TFP > N.TBM.FLIGHT.PHASE LET .PK = 0.0
ELSE
LET .PK = DPT.PK(.DWP, .TFP, .TTT)
ALWAYS
ELSE ''KKV failed to intercept
LET .PK = 0.0
ALWAYS
```

''determine result of shot
IF .PK > 0.0
AND RANDOM.F(RN.STREAM(..AIR070.2)) <= .PK
LET .NUM.KILLED = 1
ELSE
LET .NUM.KILLED = 0
ALWAYS

```
IF (DT.PRINT.FLAG(..AIRF) >= ..LV3 AND
    DT.START.TIME(..AIRF) \(<=\) TIME.V \(<=\) DT.STOP.TIME(..AIRF))
```

    OR (DT.PRINT.FLAG(..BPIPP) > ..LV0 AND
    DT.START.TIME(..BPIPP) <= TIME.V <= DT.STOP.TIME(..BPIPP))
    ''Result of DTBM engagement with a TBM
    WRITE TIME.V,
        TBU.ID(.TBU),
        FG.ID(FL.FLT.GP(.TBM.FLT)),
        TA.ID(.TBM.TAC),
        .PLATFORM.ID,
        DWP.ID(.DWP),
        .TOTAL.ELAPSED.SECONDS,
        .INTERCEPT.FLAG,
        .PK,
        .NUM.KILLED
    AS D(8,4), " AIR070013 ", I 5, S 1, I 7, S 1, I 5, S 1, I 7, S 1,
        S 1, I 5, S 1, D(6,1), S 1, I 1, S 1, D(5,3), S 1, I 1, /
    ALWAYS
''expend weapon
IF .DTBM.WPN.TYPE = ..SPACE.BASED.LASER
SUBTRACT 1 FROM SAT.SPACE.WPN.SHOTS.REMAINING(.DTBM.SAT)
IF SAT.SPACE.WPN.SHOTS.REMAINING(.DTBM.SAT) < 0
TRACE
STOP
OTHERWISE
ELSE
IF DT.PRINT.FLAG(..DB.EXEC) >= ..LV1 AND
DT.START.TIME(..DB.EXEC) <= TIME.V <= DT.STOP.TIME(..DB.EXEC)
CALL DBS.220.BEGIN.AIR.TO.AIR.ENGAGEMENT
GIVEN
.DTBM.X,
.DTBM.Y
ALWAYS
SUBTRACT 1 FROM AM.RNDS.REMAINING(.AM)
LET .TMI = TAM.INSTANCE(.TAM)
LET .EXPENDED = TAM.NUMBER(.TAM)
IF TMI.CRITICAL.RESOURCE(TAM.INSTANCE(.TAM)) > 0
SUBTRACT .EXPENDED FROM TCR.LOADED(TMI.CRITICAL.RESOURCE(.TMI))
ADD .EXPENDED TO TCR.EXPENDED(TMI.CRITICAL.RESOURCE(.TMI))
ALWAYS
IF DT.PRINT.FLAG(..DB.EXEC) >= ..LV1 AND
DT.START.TIME(..DB.EXEC) <= TIME.V <= DT.STOP.TIME(..DB.EXEC)
CALL DBS.222.MUNITION.EXPEND.AIR.TO.AIR
GIVEN
.DTBM.FLT,
.DTBM.FG,
.TMI,
.EXPENDED
ALWAYS
IF DT.PRINT.FLAG(..GRAPHER2K) >= ..LV1 AND
DT.START.TIME(..GRAPHER2K) <= TIME.V <=

```

DT.STOP.TIME(..GRAPHER2K)
CALL GRA.730.AIR.MUNITION.EXPENDITURE GIVEN
..AA.MUNT.LAUNCHED, ''expenditure type
.DTBM.FLT, .TAM,
1, ''\# of munt sticks
0, ''not used here
0, ''not used here
0, ''not used here
0 ''not used here
ALWAYS
```

IF DT.PRINT.FLAG(..GRAPHER2K) >= ..LV1 AND
DT.START.TIME(..GRAPHER2K) <= TIME.V <=
DT.STOP.TIME(..GRAPHER2K)
IF .NUM.KILLED > 0
CALL GRA.740.AIR.TO.AIR.KILLS
GIVEN
.TBM.FLT, ''victim flight
.DTBM.FLT, ''killer flight
.NUM.KILLED ''losses to victim flight

```
    ALWAYS
ALWAYS
''set up DTBM flight group statistics for air reports
IF FG.MSN.STAT(.DTBM.FG) <> 0
    TRACE
    STOP
OTHERWISE
CREATE MISSION.STATISTIC CALLED .MS
LET MS.FLIGHT.GROUP.ID(.MS) = FG.ID(.DTBM.FG)
LET MS.TIME(.MS) = TIME.V
LET MS.TARGET.TYPE(.MS) = ..INTERCEPT.POINT
LET MS.TARGET.ID(.MS) = FG.ID(.TBM.FG)
LET MS.TARGET.NAME(.MS) = ""
LET MS.KILLS(.MS) = .NUM.KILLED
LET FG.MSN.STAT(.DTBM.FG) = .MS
CREATE FLIGHT.STAT CALLED .FS
LET FLS.FLIGHT.ID(.FS) = FL.ID(.DTBM.FLT)
LET FLS.TYPE.AC(.FS) = FL.TYPE.AC(.DTBM.FLT)
LET FLS.NUM.AC(.FS) = FL.NUM.AC(.DTBM.FLT)
LET FLS.LOSSES(.FS) \(=0\)
FILE .FS IN MS.FLIGHT.STAT.SET(.MS)
CREATE TARGET. COMPONENT.STAT CALLED .TCS
LET TCS.TYPE.COMPONENT(.TCS) = FL.TYPE.AC(.TBM.FLT)
LET TCS.COMPONENT.ID(.TCS) = FL.ID(.TBM.FLT)
LET TCS.QTY.PCT.KILLED(.TCS) = .NUM.KILLED
FILE .TCS IN MS.TARGET.COMPONENT.SET(.MS)
```

CREATE AIR.MUNITION.STAT CALLED .AMS
LET AMS.TYPE(.AMS) = .TAM
LET AMS.LAUNCHES(.AMS) = 1
LET AMS.TGT.TYPE.COMPONENT(.AMS) = ..AIRCRAFT
LET AMS.TGT.COMPONENT.ID(.AMS) = FL.TYPE.AC(.TBM.FLT)
FILE .AMS IN FLS.MUNITION.SET(.FS)
IF FG.MSN.STAT(.TBM.FG) = 0
CREATE A MISSION.STATISTIC CALLED .MS
LET FG.MSN.STAT(.TBM.FG) = .MS
ELSE
LET .MS = FG.MSN.STAT(.TBM.FG)
ALWAYS
LET MS.FLIGHT.GROUP.ID(.MS) = FG.ID(.TBM.FG)
LET MS.TIME(.MS) = TIME.V
LET MS.TARGET.TYPE(.MS) = ..INTERCEPT.POINT
LET MS.TARGET.ID(.MS) = FG.ID(.DTBM.FG)
LET MS.TARGET.NAME(.MS) = ""
LET MS.KILLS(.MS) = 0
CREATE FLIGHT.STAT CALLED .FS
LET FLS.FLIGHT.ID(.FS) = FL.ID(.TBM.FLT)
LET FLS.TYPE.AC(.FS) = FL.TYPE.AC(.TBM.FLT)
LET FLS.NUM.AC(.FS) = FL.NUM.AC(.TBM.FLT)
LET FLS.LOSSES(.FS) = .NUM.KILLED
FILE .FS IN MS.FLIGHT.STAT.SET(.MS)
CREATE TARGET.COMPONENT.STAT CALLED .TCS
LET TCS.TYPE.COMPONENT(.TCS) = FL.TYPE.AC(.DTBM.FLT)
LET TCS.COMPONENT.ID(.TCS) = FL.ID(.DTBM.FLT)
LET TCS.QTY.PCT.KILLED(.TCS) = 0
FILE .TCS IN MS.TARGET.COMPONENT.SET(.MS)
IF DT.PRINT.FLAG(..AA.ENC) >= ..LV1 AND
DT.START.TIME(..AA.ENC) <= TIME.V <= DT.STOP.TIME(..AA.ENC)
CALL UTL.503.LOCATE.SECTOR.FLOT.X
GIVEN
.TBM. LAUNCH.X,
.TBM.LAUNCH.Y,
..BLUE
YIELDING
.FLOT.X
LET .BLUE.DIST.TO.FLOT = .TBM.LAUNCH.X - .FLOT.X
CALL UTL.503.LOCATE.SECTOR.FLOT.X
GIVEN
.TBM.LAUNCH.X,
.TBM.LAUNCH.Y,
..RED
YIELDING
.FLOT.X
LET .RED.DIST.TO.FLOT = .FLOT.X - .TBM.LAUNCH.X

```
```

    ''Number of flights in the air engagement and the distance
    ''from the sector FLOT for each side
    WRITE TIME.V,
        1,
        .BLUE.DIST.TO.FLOT,
        1, ''red flight
        .RED.DIST.TO.FLOT
    AS D(8,4)," AIR070091 ",I 3,S 1,I 9,S 1,I 3,S 1,I 9, /
    ''Describes a DTBM flight in an air engagement.
    WRITE TIME.V,
        TA.USED.INDEX(FL.TYPE.AC(.DTBM.FLT)),
        FL.MISSION(.DTBM.FLT),
        FL.NUM.AC(.DTBM.FLT),
        FG.STATUS(FL.FLT.GP(.DTBM.FLT))
    AS D(8,4)," AIR070101 ",I 5,S 1,I 5,S 1,I 5,S 1,I 5, /
    ''Describes a TBM flight in an air engagement.
    WRITE TIME.V,
        TA.USED.INDEX(FL.TYPE.AC(.TBM.FLT)),
        FL.MISSION(.TBM.FLT),
        FL.NUM.AC(.TBM.FLT),
        FG.STATUS(FL.FLT.GP(.TBM.FLT))
    AS D(8,4)," AIR070101 ",I 5,S 1,I 5,S 1,I 5,S 1,I 5, /
    ALWAYS
IF DT.PRINT.FLAG(..AA.KILLS) >= ..LV1 AND
DT.START.TIME(..AA.KILLS) <= TIME.V <= DT.STOP.TIME(..AA.KILLS)
''Air-to-Air kills for post processor
WRITE TIME.V,
TA.USED.INDEX(FL.TYPE.AC(.DTBM.FLT)),
FL.MISSION(.DTBM.FLT),
FL.NUM.AC(.DTBM.FLT),
TA.USED.INDEX(FL.TYPE.AC(.TBM.FLT)),
FL.MISSION(.TBM.FLT),
FL.NUM.AC(.TBM.FLT),
0 ''num lost by DTBM
AS D(8,4), " AIR070081", S 1, I 5, S 1, I 5, S 1,
I 5, S 1, I 5, S 1, I 5, S 1, I 5, S 1, I 5, /
''Air-to-Air kills for post processor
WRITE TIME.V,
TA.USED.INDEX(FL.TYPE.AC(.TBM.FLT)),
FL.MISSION(.TBM.FLT),
FL.NUM.AC(.TBM.FLT),
TA.USED.INDEX(FL.TYPE.AC(.DTBM.FLT)),
FL.MISSION(.DTBM.FLT),
FL.NUM.AC(.DTBM.FLT),
.NUM.KILLED
AS D(8,4), " AIR070081", S 1, I 5, S 1, I 5, S 1,
I 5, S 1, I 5, S 1, I 5, S 1, I 5, S 1, I 5, /
ALWAYS
''report results

```

CALL UTL.270.MISSION.STAT.REPORT GIVEN
. DTBM.FG, . . DTBM

CALL UTL.270.MISSION.STAT.REPORT GIVEN
.TBM.FG, . . ALL.FLIGHTS
ALWAYS
```

IF .NUM.KILLED = 1
IF .DTBM.WPN.TYPE <> ..SPACE.BASED.LASER
IF DT.PRINT.FLAG(..DB.EXEC) >= ..LV1 AND
DT.START.TIME(..DB.EXEC) <= TIME.V <=
DT.STOP.TIME(..DB.EXEC)
CALL DBS.221.FLT.LOSS.AIR.TO.AIR
GIVEN
.TBM.FLT,
.TBM.FG,
1, ''loss
..YES, ''before target
TA.ID(.TBM.TAC),
FL.MISSION(.TBM.FLT)
ALWAYS
IF (POST.PROCESS.GRAPHICS = ..MAP OR
POST.PROCESS.GRAPHICS = ..BOTH) AND
DT.PRINT.FLAG(..GRAA) GE ..LV2 AND
DT.START.TIME(..GRAA) LE TIME.V LE DT.STOP.TIME(..GRAA)
IF FG.SIDE(.DTBM.FG) = ..BLUE
LET .BLUE.FG = .DTBM.FG
LET .RED.FG = .TBM.FG
ELSE
LET .BLUE.FG = .TBM.FG
LET .RED.FG = .DTBM.FG
ALWAYS
''FOR NOW
LET .ENGAGE.X = .5 * (.TBM.LAUNCH.X + .TBM.TGT.X)
LET .ENGAGE.Y = .5 * (.TBM.LAUNCH.Y + .TBM.TGT.Y)
LET .SAVE.TBM.X = FG.X.COORD(.TBM.FG)
LET .SAVE.TBM.Y = FG.Y.COORD(.TBM.FG)
LET FG.X.COORD(.TBM.FG) = .ENGAGE.X
LET FG.Y.COORD(.TBM.FG) = .ENGAGE.Y
IF DT.PRINT.FLAG(..GRAA) >= ..LV3
LET FG.CURR.PATH(.TBM.FG) = F.FG.PATH.SET(.TBM.FG)
CALL GRA.600.PRINT.MISSION.MOVE.EVENT
GIVEN
.TBM.FG,

```
.TBM.FLT

LET FG.CURR.PATH(.TBM.FG) \(=0\)
ALWAYS
LET FL.NUM.AC.LOST.LAST.ATTACK (.TBM.FLT) = .NUM.KILLED
LET FL.ENGAGE.FLAG(.TBM.FLT) = ..YES
LET FL.ENGAGE.FLAG(.DTBM.FLT) = ..YES
''TEMPORARILY MOVE DTBM FLIGHT GROUP SO GRAPHICS WILL SHOW '' ENGAGEMENT ALONG TBM FLIGHT PATH
LET .SAVE.DTBM.X = FG.X.COORD(.DTBM.FG)
LET .SAVE.DTBM.Y = FG.Y.COORD(.DTBM.FG)
LET FG.X.COORD(.DTBM.FG) = .ENGAGE.X
LET FG.Y.COORD(.DTBM.FG) = .ENGAGE.Y
CALL GRA.540.AIR.TO.AIR.LOSS
GIVING
.BLUE.FG, .RED.FG

LET FG.X.COORD (.DTBM.FG) = .SAVE.DTBM.X
LET FG.Y.COORD(.DTBM.FG) = .SAVE.DTBM.Y
LET FG.X.COORD (.TBM.FG) = .SAVE.TBM.X
LET FG.Y.COORD(.TBM.FG) = .SAVE.TBM.Y
LET FL.NUM.AC. LOST. LAST.ATTACK(.TBM.FLT) \(=0\)
LET FL.ENGAGE.FLAG(.TBM.FLT) = ..NO
LET FL.ENGAGE.FLAG(.DTBM.FLT) = ..NO
ALWAYS
ALWAYS
''clean up tbm flight \& flight group
CALL UTL. 090.DESTROY.AIRCRAFT
GIVEN
. TBM.FLT,
.NUM.KILLED,
. . AIRCRAFT

CALL AIR.510.DECISION.POINT
GIVEN
.TBM.FG,
..YES ''aircraft lost
YIELDING
. CONTINUE

IF .CONTINUE <> ..NO
TRACE
STOP
OTHERWISE
FOR EVERY .EVENT4 IN EV.S(I.AIR.004.MOVE.FLT.GP)
WITH MFG.FLT.GP(.EVENT4) = .TBM.FG
FIND THE FIRST CASE

IF NONE TRACE STOP OTHERWISE

CANCEL AIR.004.MOVE.FLT.GP CALLED .EVENT4 DESTROY AIR.004.MOVE.FLT.GP CALLED .EVENT4
ALWAYS

IF .DTBM.WPN.TYPE <> ..SPACE.BASED. LASER

IF DT.PRINT.FLAG(..DB.EXEC) >= ..LV1 AND DT.START.TIME(..DB.EXEC) <= TIME.V <= DT.STOP.TIME(..DB.EXEC) CALL DBS.229.END.AIR.TO.AIR.ENGAGEMENT
ALWAYS
''continuation decision of no will cause DTBM to leave station CALL AIR.510.DECISION.POINT GIVEN
. DTBM.FG, ..NO ''aircraft lost YIELDING
. CONTINUE
ALWAYS
EXITROUTINE
ENDROUTINE

\section*{Appendix F: Example SIMSCRIPT II. 5 Code AIR070.SIM (Modified)}
```

ROUTINE AIR.070.ADJUDICATE.DTBM
''\$Id: air070.sim,v 2.10 2000/09/20 20:12:38 steveh Exp \$
GIVEN
.DTBM.PLATFORM, ''PTR TO DTBM PLATFORM FIRING AT TBM
.DTBM.WPN.TYPE, ''TYPE OF SHOOTER ENGAGING TBM
.TBU, ''INDEX OF TBM.UNIT LAUNCHING TBM
.TBM.FLT, ''POINTER TO TBM FLIGHT BEING LAUNCHED
.DETECT.SECONDS.DELAY ''BETWEEN TBM LAUNCH AND DTBM RESPONSE
''YIELDING
'' NONE
''PURPOSE
''-------
'' ADJUDICATE A DTBM VS. TBM AIR-TO-AIR ENGAGEMENT
''PROCESSING
''----------
''
' '============================================================
''CALLED BY
''---------
'' AIR.060.BOOST.PHASE.INTERCEPT.MANAGER
''CHANGE LOG
''----------
'' DATE
''----------
1-11-96 MORGA
'' 1-23-96 EMERT
'' 2-6-96 MORGA
'' 2-9-96 MORGAN
'' 6-12-96 MORGAN
'' 8-09-99 HADLEY
''
'' 08-16-99 LENNOX CALL GRA.730, GRA.740 FOR GRAPHER2K
'' 2-11-00 COSGROVE ADD SBL
'' 07-10-00 HADLEY UP FLIGHT GROUP ID IN TRANS TO I 7
NORMALLY MODE IS INTEGER
DEFINE .DETECT.SECONDS.DELAY, .PK, .SHOT.DURATION.SEC, .SUM.SECONDS,
.TOTAL.ELAPSED.SECONDS
AS DOUBLE VARIABLES
IF .DTBM.WPN.TYPE = ..SPACE.BASED.LASER
LET .DTBM.SAT = .DTBM.PLATFORM
ELSE

```
```

    LET .DTBM.FG = .DTBM.PLATFORM
    LET .DTBM.FLT = F.FG.FLIGHT.SET(.DTBM.FG)
    IF FL.MISSION(.DTBM.FLT) <> ..DTBM
        TRACE
        STOP
    OTHERWISE
    ALWAYS
LET .TBM.FG = FL.FLT.GP(.TBM.FLT)
LET .TBM.LAUNCH.X = TBU.X.COORD(.TBU)
LET .TBM.LAUNCH.Y = TBU.Y.COORD(.TBU)
LET .TBM.TAC = TBU.TYPE.AC(.TBU)
LET .TTT = TA.TBM.TARGET.TYPE(.TBM.TAC)
LET .TBM.TGT = FG.TARGET(.TBM.FG)
LET .TBM.TGT.X = TG.X.COORD(.TBM.TGT)
LET .TBM.TGT.Y = TG.Y.COORD(.TBM.TGT)
IF .DTBM.WPN.TYPE = ..SPACE.BASED.LASER
LET .TSW = SAT.TYPE.SPACE.WPN(.DTBM.SAT)
LET .DWP = TSW.DTBM.WEAPON(.TSW)
LET .PLATFORM.ID = SAT.ID(.DTBM.SAT)
ELSE
FOR EVERY .AM IN FL.MUNITION.SET(.DTBM.FLT)
WITH (AM.FUNCTION(.AM) = ..LASER
OR AM.FUNCTION(.AM) = ..KKV)
AND AM.RNDS.REMAINING(.AM) > 0
FIND THE FIRST CASE
IF NONE
TRACE
STOP
OTHERWISE
LET .TAM = AM.TYPE(.AM)
LET .DWP = TAM.DTBM.WEAPON(.TAM)
LET .PLATFORM.ID = FG.ID(.DTBM.FG)
ALWAYS
''determine TBM flight phase (and pK) at which to adjudicate
IF .DTBM.WPN.TYPE = ..AIRBORNE.LASER
OR .DTBM.WPN.TYPE = ..SPACE.BASED.LASER
CALL UTL.720.CALCULATE.USER.FUNCTION
GIVEN
DWP.SHOT.DURATION.FN(.DWP),
RN.STREAM(..AIR070.1)
YIELDING
.SHOT.DURATION.SEC
''*******************
''* Earl stuff
!।*******************
EARL.RESPONSE = .SHOT.DURATION.SEC
EB.P1 = UF.PARM1(DWP.SHOT.DURATION.FN(.DWP))
EB.P2 = UF.PARM2(DWP.SHOT.DURATION.FN(.DWP))

```
\(E B \cdot P 3=0.0\)
CALL EARL.STORE
GIVEN
. AIR070.1, UF. FUNCTION(DWP. SHOT. DURATION. FN(.DWP)), EARL.RESPONSE, EB.P1, EB.P2, EB.P3
```

''* END STUFF

```
    LET .SHOT.DURATION.SEC = MAX.F(.SHOT.DURATION.SEC, 0.0)
    LET .TOTAL.ELAPSED.SECONDS
        = .DETECT.SECONDS.DELAY + .SHOT.DURATION.SEC
    LET .INTERCEPT.FLAG = ..YES
ELSE
    IF .DTBM.WPN.TYPE <> ..KKV
        TRACE
        STOP
    OTHERWISE
    LET .DTBM.X = FG.X.COORD(.DTBM.FG)
    LET .DTBM.Y = FG.Y.COORD(.DTBM.FG)
    LET .DTBM.Z = PTH.ALTITUDE(FG.CURR.PATH(.DTBM.FG))
    CALL AIR.071.COMPUTE.KKV.INTERCEPT.TIME
        GIVEN
            .TTT, ''INDEX OF TBM.TARGET.TYPE
            .TBM. LAUNCH.X,
            .TBM. LAUNCH.Y,
            .TBM.TGT. \(X\),
            .TBM.TGT.Y,
            .DWP, ''INDEX OF DTBM.WEAPON
            .DTBM.X,
            .DTBM.Y,
            . DTBM.Z,
            .DETECT. SECONDS.DELAY ''FROM TBM LAUNCH TO KKV LAUNCH
        YIELDING
            .INTERCEPT.FLAG, ''..YES OR ..NO SUCCESSFUL INTERCEPT
                .TOTAL.ELAPSED.SECONDS ''FROM TBM LAUNCH TO KKV INTERCEPT
ALWAYS
IF .INTERCEPT.FLAG = ..YES
    FOR EVERY TBM.FLIGHT.PHASE CALLED .TFP
    DO
        ADD TPT.DURATION.SEC(.TFP, .TTT) TO .SUM.SECONDS
        IF . TOTAL.ELAPSED.SECONDS <= .SUM.SECONDS
            LEAVE
        OTHERWISE
    LOOP
    IF .TFP > N.TBM.FLIGHT.PHASE
```

            LET .PK = 0.0
    ELSE
            LET .PK = DPT.PK(.DWP, .TFP, .TTT)
        ALWAYS
    ELSE ''KKV failed to intercept
    LET .PK = 0.0
    ALWAYS
    ''determine result of shot
    ''*********************
''* Earl stuff
'ノ**********************
EARL.RESPONSE = RANDOM.F(RN.STREAM(..AIR070.2))
EB.P1 = 0.0
EB.P2 = 0.0
EB.P3 = 0.0
CALL EARL.STORE
GIVEN
..AIR070.2,
..RANDOM,
EARL.RESPONSE,
EB.P1,
EB.P2,
EB.P3
''* END STUFF
IF .PK > 0.0
AND EARL.RESPONSE <= .PK
LET .NUM.KILLED = 1
ELSE
LET .NUM.KILLED = 0
ALWAYS
IF (DT.PRINT.FLAG(..AIRF) >= ..LV3 AND
DT.START.TIME(..AIRF) <= TIME.V <= DT.STOP.TIME(..AIRF))
OR (DT.PRINT.FLAG(..BPIPP) > ..LV0 AND
DT.START.TIME(..BPIPP) <= TIME.V <= DT.STOP.TIME(..BPIPP))
''Result of DTBM engagement with a TBM
WRITE TIME.V,
TBU.ID(.TBU),
FG.ID(FL.FLT.GP(.TBM.FLT)),
TA.ID(.TBM.TAC),
.PLATFORM.ID,
DWP.ID(.DWP),
.TOTAL.ELAPSED.SECONDS,
.INTERCEPT.FLAG,
.PK,
.NUM.KILLED

```

AS \(D(8,4), " A I R 070013\) ", \(\mathrm{I} 5, \mathrm{~S} 1, \mathrm{I} 7, \mathrm{~S} 1, \mathrm{I} 5, \mathrm{~S} 1, \mathrm{I} 7, \mathrm{~S} 1\), S 1, I 5, S 1, \(D(6,1), S 1, I 1, S 1, D(5,3), S 1, I 1, /\) ALWAYS
```

''expend weapon
IF .DTBM.WPN.TYPE = ..SPACE.BASED.LASER
SUBTRACT 1 FROM SAT.SPACE.WPN.SHOTS.REMAINING(.DTBM.SAT)
IF SAT.SPACE.WPN.SHOTS.REMAINING(.DTBM.SAT) < 0
TRACE
STOP
OTHERWISE
ELSE
IF DT.PRINT.FLAG(..DB.EXEC) >= ..LV1 AND
DT.START.TIME(..DB.EXEC) <= TIME.V <= DT.STOP.TIME(..DB.EXEC)
CALL DBS.220.BEGIN.AIR.TO.AIR.ENGAGEMENT
GIVEN
.DTBM.X,
.DTBM.Y
ALWAYS
SUBTRACT 1 FROM AM.RNDS.REMAINING(.AM)
LET .TMI = TAM.INSTANCE(.TAM)
LET .EXPENDED = TAM.NUMBER(.TAM)
IF TMI.CRITICAL.RESOURCE(TAM.INSTANCE(.TAM)) > 0
SUBTRACT .EXPENDED FROM TCR.LOADED(TMI.CRITICAL.RESOURCE(.TMI))
ADD .EXPENDED TO TCR.EXPENDED(TMI.CRITICAL.RESOURCE(.TMI))
ALWAYS
IF DT.PRINT.FLAG(..DB.EXEC) >= ..LV1 AND
DT.START.TIME(..DB.EXEC) <= TIME.V <= DT.STOP.TIME(..DB.EXEC)
CALL DBS.222.MUNITION.EXPEND.AIR.TO.AIR
GIVEN
.DTBM.FLT,
.DTBM.FG,
.TMI,
.EXPENDED
ALWAYS
IF DT.PRINT.FLAG(..GRAPHER2K) >= ..LV1 AND
DT.START.TIME(..GRAPHER2K) <= TIME.V <=
DT.STOP.TIME(..GRAPHER2K)
CALL GRA.730.AIR.MUNITION.EXPENDITURE
GIVEN
..AA.MUNT.LAUNCHED, ''expenditure type
.DTBM.FLT,
.TAM,
1, ''\# of munt sticks
0, ''not used here
0, ''not used here
0, ''not used here
0 ''not used here
ALWAYS

```
```

IF DT.PRINT.FLAG(..GRAPHER2K) >= ..LV1 AND
DT.START.TIME(..GRAPHER2K) <= TIME.V <=
DT.STOP.TIME(..GRAPHER2K)
IF .NUM.KILLED > 0
CALL GRA.740.AIR.TO.AIR.KILLS
GIVEN
.TBM.FLT, ''victim flight
.DTBM.FLT, ''killer flight
.NUM.KILLED ''losses to victim flight
ALWAYS
ALWAYS
''set up DTBM flight group statistics for air reports
IF FG.MSN.STAT(.DTBM.FG) <> 0
TRACE
STOP
OTHERWISE
CREATE MISSION.STATISTIC CALLED .MS
LET MS.FLIGHT.GROUP.ID(.MS) = FG.ID(.DTBM.FG)
LET MS.TIME(.MS) = TIME.V
LET MS.TARGET.TYPE(.MS) = ..INTERCEPT.POINT
LET MS.TARGET.ID(.MS) = FG.ID(.TBM.FG)
LET MS.TARGET.NAME(.MS) = ""
LET MS.KILLS(.MS) = .NUM.KILLED
LET FG.MSN.STAT(.DTBM.FG) = .MS
CREATE FLIGHT.STAT CALLED .FS
LET FLS.FLIGHT.ID(.FS) = FL.ID(.DTBM.FLT)
LET FLS.TYPE.AC(.FS) = FL.TYPE.AC(.DTBM.FLT)
LET FLS.NUM.AC(.FS) = FL.NUM.AC(.DTBM.FLT)
LET FLS.LOSSES(.FS) = 0
FILE .FS IN MS.FLIGHT.STAT.SET(.MS)
CREATE TARGET.COMPONENT.STAT CALLED .TCS
LET TCS.TYPE.COMPONENT(.TCS) = FL.TYPE.AC(.TBM.FLT)
LET TCS.COMPONENT.ID(.TCS) = FL.ID(.TBM.FLT)
LET TCS.QTY.PCT.KILLED(.TCS) = .NUM.KILLED
FILE .TCS IN MS.TARGET.COMPONENT.SET(.MS)
CREATE AIR.MUNITION.STAT CALLED .AMS
LET AMS.TYPE(.AMS) = .TAM
LET AMS.LAUNCHES(.AMS) = 1
LET AMS.TGT.TYPE.COMPONENT(.AMS) = ..AIRCRAFT
LET AMS.TGT.COMPONENT.ID(.AMS) = FL.TYPE.AC(.TBM.FLT)
FILE .AMS IN FLS.MUNITION.SET(.FS)
IF FG.MSN.STAT(.TBM.FG) = 0
CREATE A MISSION.STATISTIC CALLED .MS
LET FG.MSN.STAT(.TBM.FG) = .MS
ELSE
LET .MS = FG.MSN.STAT(.TBM.FG)
ALWAYS

```
```

LET MS.FLIGHT.GROUP.ID(.MS) = FG.ID(.TBM.FG)
LET MS.TIME(.MS) = TIME.V
LET MS.TARGET.TYPE(.MS) = ..INTERCEPT.POINT
LET MS.TARGET.ID(.MS) = FG.ID(.DTBM.FG)
LET MS.TARGET.NAME(.MS) = ""
LET MS.KILLS(.MS) = 0
CREATE FLIGHT.STAT CALLED .FS
LET FLS.FLIGHT.ID(.FS) = FL.ID(.TBM.FLT)
LET FLS.TYPE.AC(.FS) = FL.TYPE.AC(.TBM.FLT)
LET FLS.NUM.AC(.FS) = FL.NUM.AC(.TBM.FLT)
LET FLS.LOSSES(.FS) = .NUM.KILLED
FILE .FS IN MS.FLIGHT.STAT.SET(.MS)
CREATE TARGET.COMPONENT.STAT CALLED .TCS
LET TCS.TYPE.COMPONENT(.TCS) = FL.TYPE.AC(.DTBM.FLT)
LET TCS.COMPONENT.ID(.TCS) = FL.ID(.DTBM.FLT)
LET TCS.QTY.PCT.KILLED(.TCS) = 0
FILE .TCS IN MS.TARGET.COMPONENT.SET(.MS)
IF DT.PRINT.FLAG(..AA.ENC) >= ..LV1 AND
DT.START.TIME(..AA.ENC) <= TIME.V <= DT.STOP.TIME(..AA.ENC)
CALL UTL.503.LOCATE.SECTOR.FLOT.X
GIVEN
.TBM. LAUNCH.X,
.TBM.LAUNCH.Y,
. .BLUE
YIELDING
.FLOT.X
LET .BLUE.DIST.TO.FLOT = .TBM.LAUNCH.X - .FLOT.X
CALL UTL.503.LOCATE.SECTOR.FLOT.X
GIVEN
.TBM.LAUNCH.X,
.TBM.LAUNCH.Y,
..RED
YIELDING
.FLOT.X
LET .RED.DIST.TO.FLOT = .FLOT.X - .TBM.LAUNCH.X
''Number of flights in the air engagement and the distance
''from the sector FLOT for each side
WRITE TIME.V,
1, ''blue flight
.BLUE.DIST.TO.FLOT,
1, ''red flight
.RED.DIST.TO.FLOT
AS D(8,4)," AIR070091 ",I 3,S 1,I 9,S 1,I 3,S 1,I 9, /
''Describes a DTBM flight in an air engagement.
WRITE TIME.V,
TA.USED.INDEX(FL.TYPE.AC(.DTBM.FLT)),

```
```

            FL.MISSION(.DTBM.FLT),
            FL.NUM.AC(.DTBM.FLT),
            FG.STATUS(FL.FLT.GP(.DTBM.FLT))
        AS D(8,4)," AIR070101 ",I 5,S 1,I 5,S 1,I 5,S 1,I 5, /
    ''Describes a TBM flight in an air engagement.
        WRITE TIME.V,
            TA.USED.INDEX(FL.TYPE.AC(.TBM.FLT)),
            FL.MISSION(.TBM.FLT),
            FL.NUM.AC(.TBM.FLT),
            FG.STATUS(FL.FLT.GP(.TBM.FLT))
            AS D(8,4)," AIR070101 ",I 5,S 1,I 5,S 1,I 5,S 1,I 5, /
    ALWAYS
IF DT.PRINT.FLAG(..AA.KILLS) >= ..LV1 AND
DT.START.TIME(..AA.KILLS) <= TIME.V <= DT.STOP.TIME(..AA.KILLS)
''Air-to-Air kills for post processor
WRITE TIME.V,
TA.USED.INDEX(FL.TYPE.AC(.DTBM.FLT)),
FL.MISSION(.DTBM.FLT),
FL.NUM.AC(.DTBM.FLT),
TA.USED.INDEX(FL.TYPE.AC(.TBM.FLT)),
FL.MISSION(.TBM.FLT),
FL.NUM.AC(.TBM.FLT),
0 ''num lost by DTBM
AS D(8,4), " AIR070081", S 1, I 5, S 1, I 5, S 1,
I 5, S 1, I 5, S 1, I 5, S 1, I 5, S 1, I 5, /
''Air-to-Air kills for post processor
WRITE TIME.V,
TA.USED.INDEX(FL.TYPE.AC(.TBM.FLT)),
FL.MISSION(.TBM.FLT),
FL.NUM.AC(.TBM.FLT),
TA.USED.INDEX(FL.TYPE.AC(.DTBM.FLT)),
FL.MISSION(.DTBM.FLT),
FL.NUM.AC(.DTBM.FLT),
.NUM.KILLED
AS D(8,4), " AIR070081", S 1, I 5, S 1, I 5, S 1,
I 5, S 1, I 5, S 1, I 5, S 1, I 5, S 1, I 5, /
ALWAYS
''report results
CALL UTL.270.MISSION.STAT.REPORT
GIVEN
.DTBM.FG,
. .DTBM
CALL UTL.270.MISSION.STAT.REPORT
GIVEN
.TBM.FG,
..ALL.FLIGHTS
ALWAYS
IF .NUM.KILLED = 1

```
```

IF .DTBM.WPN.TYPE <> ..SPACE.BASED.LASER
IF DT.PRINT.FLAG(..DB.EXEC) >= ..LV1 AND
DT.START.TIME(..DB.EXEC) <= TIME.V <=
DT.STOP.TIME(..DB.EXEC)
CALL DBS.221.FLT.LOSS.AIR.TO.AIR
GIVEN
.TBM.FLT,
.TBM.FG,
1, ''loss
..YES, ''before target
TA.ID(.TBM.TAC),
FL.MISSION(.TBM.FLT)
ALWAYS
IF (POST.PROCESS.GRAPHICS = ..MAP OR
POST.PROCESS.GRAPHICS = ..BOTH) AND
DT.PRINT.FLAG(..GRAA) GE ..LV2 AND
DT.START.TIME(..GRAA) LE TIME.V LE DT.STOP.TIME(..GRAA)
IF FG.SIDE(.DTBM.FG) = ..BLUE
LET .BLUE.FG = .DTBM.FG
LET .RED.FG = .TBM.FG
ELSE
LET .BLUE.FG = .TBM.FG
LET .RED.FG = .DTBM.FG
ALWAYS
''FOR NOW
LET .ENGAGE.X = .5 * (.TBM.LAUNCH.X + .TBM.TGT.X)
LET .ENGAGE.Y = .5 * (.TBM.LAUNCH.Y + .TBM.TGT.Y)
LET .SAVE.TBM.X = FG.X.COORD(.TBM.FG)
LET .SAVE.TBM.Y = FG.Y.COORD(.TBM.FG)
LET FG.X.COORD(.TBM.FG) = .ENGAGE.X
LET FG.Y.COORD(.TBM.FG) = .ENGAGE.Y
IF DT.PRINT.FLAG(..GRAA) >= ..LV3
LET FG.CURR.PATH(.TBM.FG) = F.FG.PATH.SET(.TBM.FG)
CALL GRA.600.PRINT.MISSION.MOVE.EVENT
GIVEN
.TBM.FG,
.TBM.FLT
LET FG.CURR.PATH(.TBM.FG) = 0
ALWAYS
LET FL.NUM.AC.LOST.LAST.ATTACK(.TBM.FLT) = .NUM.KILLED
LET FL.ENGAGE.FLAG(.TBM.FLT) = ..YES
LET FL.ENGAGE.FLAG(.DTBM.FLT) = ..YES
''TEMPORARILY MOVE DTBM FLIGHT GROUP SO GRAPHICS WILL SHOW
'' ENGAGEMENT ALONG TBM FLIGHT PATH
LET .SAVE.DTBM.X = FG.X.COORD(.DTBM.FG)
LET .SAVE.DTBM.Y = FG.Y.COORD(.DTBM.FG)

```
```

            LET FG.X.COORD(.DTBM.FG) = .ENGAGE.X
            LET FG.Y.COORD(.DTBM.FG) = .ENGAGE.Y
            CALL GRA.540.AIR.TO.AIR.LOSS
                GIVING
                .BLUE.FG,
                    .RED.FG
            LET FG.X.COORD(.DTBM.FG) = .SAVE.DTBM.X
            LET FG.Y.COORD(.DTBM.FG) = .SAVE.DTBM.Y
            LET FG.X.COORD(.TBM.FG) = .SAVE.TBM.X
            LET FG.Y.COORD(.TBM.FG) = .SAVE.TBM.Y
            LET FL.NUM.AC.LOST.LAST.ATTACK(.TBM.FLT) = 0
            LET FL.ENGAGE.FLAG(.TBM.FLT) = ..NO
            LET FL.ENGAGE.FLAG(.DTBM.FLT) = ..NO
        ALWAYS
    ALWAYS
    ''clean up tbm flight & flight group
    CALL UTL.090.DESTROY.AIRCRAFT
    GIVEN
        .TBM.FLT,
        .NUM.KILLED,
        ..AIRCRAFT
    CALL AIR.510.DECISION.POINT
    GIVEN
        .TBM.FG,
        ..YES ''aircraft lost
    YIELDING
        .CONTINUE
    IF .CONTINUE <> ..NO
    TRACE
    STOP
    OTHERWISE
FOR EVERY .EVENT4 IN EV.S(I.AIR.004.MOVE.FLT.GP)
WITH MFG.FLT.GP(.EVENT4) = .TBM.FG
FIND THE FIRST CASE
IF NONE
TRACE
STOP
OTHERWISE
CANCEL AIR.004.MOVE.FLT.GP CALLED .EVENT4
DESTROY AIR.004.MOVE.FLT.GP CALLED .EVENT4
ALWAYS

```

IF .DTBM.WPN.TYPE <> ..SPACE.BASED.LASER
```

IF DT.PRINT.FLAG(..DB.EXEC) >= ..LV1 AND
DT.START.TIME(..DB.EXEC) <= TIME.V <= DT.STOP.TIME(..DB.EXEC)
CALL DBS.229.END.AIR.TO.AIR.ENGAGEMENT
ALWAYS
''continuation decision of no will cause DTBM to leave station
CALL AIR.510.DECISION.POINT
GIVEN
.DTBM.FG,
..NO ''aircraft lost
YIELDING
.CONTINUE
ALWAYS
EXITROUTINE

```
ENDROUTINE

\section*{Appendix G: MRG32k3a Routines and Variables (18)}

\section*{Routines:}

RngStream (const char *name = "");
This constructor creates a new stream with (optional) descriptor name. It initializes its seed \(I g\), and sets \(B g\) and \(C g\) to \(I g\). It also sets its anti and incPrec switches to false. The seed \(I g\) is equal to the initial seed of the package if this is the first stream created; otherwise it is \(Z\) steps ahead of the seed of the most recently created stream.
void SetPackageSeed (const unsigned long seed[6]);
Sets the initial seed s 0 of the package to the six integers in the vector seed. The first 3 integers in the seed must all be less than \(\mathrm{ml}=4294967087\), and not all 0 ; and the last 3 integers must all be less than \(\mathrm{m} 2=4294944443\), and not all 0 . If this method is not called, the default initial seed is \((12345,12345,12345,12345,12345,12345)\).
void ResetStartStream ();
Reinitializes the stream to its initial state: Cg and Bg are set to Ig .
void ResetStartSubstream ();
Reinitializes the stream to the beginning of its current substream: Cg is set to Bg .
void ResetNextSubstream ();
Reinitializes the stream to the beginning of its next substream: Ng is computed, and Cg and Bg are set to Ng .
void SetAntithetic (bool a);
If \(a=\) true, the stream will start generating antithetic variates, i.e., \(1-U\) instead of \(U\), until this method is called again with \(\mathrm{a}=\) false.
void IncreasedPrecis (bool incp);
After calling this method with incp = true, each call to the generator (direct or indirect) for this stream will return a uniform random number with more bits of resolution ( 53 bits if machine follows IEEE 754 standard) instead of 32 bits, and will advance the state of the stream by 2 steps instead of 1 . More precisely, if \(s\) is a stream of the class RngStream, in the nonantithetic case, the instruction "u = s.RandU01()" will be equivalent to " \(u=(s . R a n d U 01()+s . R a n d U 01() *\) fact \() \% 1.0\) " where the constant fact is equal to \(2-24\). This also applies when calling RandU01 indirectly (e.g., via RandInt, etc.). By default, or if this method is called again with incp = false, each call to RandU01 for this stream advances the state by 1 step and returns a number with 32 bits of resolution.
void SetSeed (const unsigned long seed[6]);
Sets the initial seed Ig of the stream to the vector seed. The vector seed should contain
valid seed values as described in SetPackageSeed. The state of the stream is then reset to this initial seed. The states and seeds of the other streams are not modified. As a result, after calling this method, the initial seeds of the streams are no longer spaced Z values apart. We discourage the use of this method; proper use of the Reset* methods is preferable.
void AdvanceState (long e, long c);
Advances the state by \(n\) steps (see below for the meaning of \(n\) ), without modifying the states of other streams or the values of Bg and Ig in the current object. If \(\mathrm{e}>0\), then n \(=2 \mathrm{e}+\mathrm{c}\); if \(\mathrm{e}<0\), then \(\mathrm{n}=-2-\mathrm{e}+\mathrm{c}\); and if \(\mathrm{e}=0\), then \(\mathrm{n}=\mathrm{c}\). Note: c is allowed to take negative values. We discourage the use of this method.
void GetState (unsigned long seed[6]) const;
Returns in seed[0..5] the current state Cg of this stream. This is convenient if we want to save the state for subsequent use.
void WriteState () const;
Writes (to standard output) the current state Cg of this stream.
void WriteStateFull () const;
Writes (to standard output) the value of all the internal variables of this stream: name, anti, incPrec, \(\mathrm{Ig}, \mathrm{Bg}, \mathrm{Cg}\).
double RandU01 ();
Normally, returns a (pseudo)random number from the uniform distribution over the interval \((0,1)\), after advancing the state by one step. The returned number has 32 bits of precision in the sense that it is always a multiple of \(1 /(232-208)\). However, if IncreasedPrecis(true) has been called for this stream, the state is advanced by two steps and the returned number has bits of precision.
long RandInt (long i , long j );
Returns a (pseudo)random number from the discrete uniform distribution over the integers \(\{\mathrm{i}, \mathrm{i}+1, \ldots, \mathrm{j}\}\). Makes one call to RandU01.

\section*{Variables:}
double \(\mathrm{Cg}[6], \mathrm{Bg}[6], \operatorname{Ig}[6]\);
Vectors to store the current seed, the beginning of the current block (substream) and the beginning of the current stream.
bool anti, incPrec;
Variables to indicate whether to generate antithetic or increased precision random numbers.
std::string name;
String to store the optional name of the current RngStream object.
static double nextSeed[6];
Static vector to store the beginning state of the next RngStream to be created (instantiated).
double U01 ();
The backbone uniform random number generator.
double U01d ();
The backbone uniform random number generator with increased precision.

Appendix H: CHARACTERISTICS OF VRTs (6)
\begin{tabular}{|c|c|c|c|c|c|}
\hline \multicolumn{6}{|l|}{CHARACTERISTICS OF VARIANCE REDUCTION TECHNIQUES} \\
\hline VRT & DESCRIPTION OF TECHNIQUE & CRITERIA OF APPLICATION & ADVANTAGES & DISADVANTAGES & \begin{tabular}{l}
TYPICAL \\
APPLICATIONS
\end{tabular} \\
\hline \begin{tabular}{l}
Importance \\
Sampling
\end{tabular} & Sample procedure is modified such that the more important events are sampled more frequently & \begin{tabular}{l}
Certain events are known to be important \\
Estimated parameters are based on events with low probability of occurrence \\
Certain events are not of interest \\
Regions of importance can be defined
\end{tabular} & \begin{tabular}{l}
It can give great improvement with limited effort \\
Well developed procedures \\
Easy implementation
\end{tabular} & \begin{tabular}{l}
Can give worthless results if not properly applied \\
A priori knowledge is needed \\
Additional computer time is needed to implement the technique
\end{tabular} & \begin{tabular}{l}
Most commonly used method. Application in PERT, reliability fault tree analysis, queuing and radiation transport \\
Applicable to almost any problem having criteria indicated.
\end{tabular} \\
\hline \begin{tabular}{l}
Russian \\
Roulette and Splitting (RR\&S)
\end{tabular} & Use of probabilities to kill samples in an uninteresting portion of the simulation ( RR ) and to increase number of samples in regions of interest (S) & \begin{tabular}{l}
Sequential process \\
Events with low probabilities are involved \\
Relative importance of all outcomes is known \\
Useful where importance sampling or expected values are used
\end{tabular} & \begin{tabular}{l}
Very Limited computation \\
Convenient way to accomplish crude importance sampling
\end{tabular} & \begin{tabular}{l}
Requires a priori knowledge of importance of intermediate stages \\
May not be effective if used alone \\
Splitting may be difficult to implement
\end{tabular} & \begin{tabular}{l}
Particle transport, search problems, network analysis, traffic flow and random walk. \\
Usually when low probability events or unimportant events are involved.
\end{tabular} \\
\hline
\end{tabular}
\begin{tabular}{|c|c|c|c|c|c|}
\hline \multicolumn{6}{|l|}{CHARACTERISTICS OF VARIANCE REDUCTION TECHNIQUES (Continued)} \\
\hline VRT & DESCRIPTION OF TECHNIQUE & CRITERIA OF APPLICATION & ADVANTAGES & DISADVANTAGES & \begin{tabular}{l}
TYPICAL \\
APPLICATIONS
\end{tabular} \\
\hline Systematic Sampling & Selected random numbers are systematically distributed over the sample space & \begin{tabular}{l}
Random variables are to be generated at the beginning of the problem (Source or Initial Conditions) \\
Relative importance of ranges of sampled variables is unknown
\end{tabular} & \begin{tabular}{l}
Easy to implement \\
Little additional computation \\
No risk in application \\
Always gives variance reduction
\end{tabular} & \begin{tabular}{l}
Often gives marginal improvement \\
Effective only in limited number of situations
\end{tabular} & \begin{tabular}{l}
Could be useful in any problem where random inputs are of interest \\
Examples are in queuing and reliability
\end{tabular} \\
\hline \begin{tabular}{l}
Stratified \\
Sampling
\end{tabular} & Sample space is divided into sections. Sampling is accomplished from each section. More important areas are emphasized & \begin{tabular}{l}
Random variables are to be generated at the beginning of the problem \\
Certain ranges of the variables are more important than the other
\end{tabular} & \begin{tabular}{l}
Easy to implement Can give great improvement \\
Gives better results than systematic if properly applied \\
Can be optimally applied
\end{tabular} & \begin{tabular}{l}
Requires a priori knowledge of importance \\
Can lead to worse results if not properly applied \\
Sometimes difficult to define sampling ranges
\end{tabular} & \begin{tabular}{l}
Queuing \\
Any problem with random conditions such as reliability and network analysis
\end{tabular} \\
\hline
\end{tabular}
\begin{tabular}{|c|c|c|c|c|c|}
\hline \multicolumn{6}{|l|}{CHARACTERISTICS OF VARIANCE REDUCTION TECHNIQUES (Continued)} \\
\hline VRT & DESCRIPTION OF TECHNIQUE & CRITERIA OF APPLICATION & ADVANTAGES & DISADVANTAGES & \begin{tabular}{l}
TYPICAL \\
APPLICATIONS
\end{tabular} \\
\hline \begin{tabular}{l}
Expected \\
Value
\end{tabular} & Include known expected values in the process to replace a stochastic portion of a simulation & \begin{tabular}{l}
Analytical results, probabilities, or expected values of the problem are known \\
Higher moments of the above mentioned portions are not essential to overall simulation
\end{tabular} & \begin{tabular}{l}
Almost always gives improvement \\
Easy to implement
\end{tabular} & May require prohibitive analysis or computation & \begin{tabular}{l}
PERT, Radiation transport \\
Limited in other areas \\
Could be used where known results are available for parts of the problem
\end{tabular} \\
\hline \begin{tabular}{l}
Statistical \\
Estimation
\end{tabular} & Include known expected values or probabilities in estimated and not in simulation model & \begin{tabular}{l}
Analytical results, probabilities or expected values of portions of the problem are known \\
Stochastic nature is essential to overall simulation \\
Process is sequential in nature \\
Probability of final outcome is known but small at all intermediate stages
\end{tabular} & Always gives improvement Can get great improvement Could be the only way to get an answer & May require prohibitive analysis or computation & \begin{tabular}{l}
Primary application in radiation transport \\
Potential to use at any simulation
\end{tabular} \\
\hline
\end{tabular}
\begin{tabular}{|l|l|l|l|l|l|}
\hline \multicolumn{8}{|c|}{ CHARACTERISTICS OF VARIANCE REDUCTION TECHNIQUES (Continued) }
\end{tabular}
\begin{tabular}{|c|c|c|c|c|c|}
\hline \multicolumn{6}{|l|}{CHARACTERISTICS OF VARIANCE REDUCTION TECHNIQUES (Continued)} \\
\hline VRT & DESCRIPTION OF TECHNIQUE & CRITERIA OF APPLICATION & ADVANTAGES & DISADVANTAGES & \begin{tabular}{l}
TYPICAL \\
APPLICATIONS
\end{tabular} \\
\hline \begin{tabular}{l}
Control \\
Variates
\end{tabular} & Using simplified or approximate positively correlated representation for undetermined parts of simulation & \begin{tabular}{l}
Portions of the problem can be approximated with an analytical representation \\
A simple low variance approximation is known
\end{tabular} & \begin{tabular}{l}
Requires simple calculation \\
Provides considerable flexibility \\
Can give great improvement
\end{tabular} & \begin{tabular}{l}
Requires understanding of the process \\
Simple analytical approximation may not exist
\end{tabular} & \begin{tabular}{l}
Potentially can be used at any area where approximate representation of the random variable under estimation is known \\
Has been applied in queuing problems
\end{tabular} \\
\hline Antithetic Variates & Using simplified or approximate negatively correlated representation for undetermined parts of the problem & Analytical or approximate representation are known and negatively correlated with the problem variables & \begin{tabular}{l}
Requires simple calculation \\
Provides considerable flexibility \\
Can give great improvement
\end{tabular} & \begin{tabular}{l}
Requires understanding of the process \\
Costs more than control variates
\end{tabular} & Could be used in reliability, queuing, sensitivity analysis Has been applied to queuing problems \\
\hline
\end{tabular}
\begin{tabular}{|c|l|l|l|l|l|}
\hline \multicolumn{8}{|c|}{ CHARACTERISTICS OF VARIANCE REDUCTION TECHNIQUES (Continued) }
\end{tabular}
\begin{tabular}{|c|c|c|c|c|c|}
\hline \multicolumn{6}{|l|}{CHARACTERISTICS OF VARIANCE REDUCTION TECHNIQUES (Continued)} \\
\hline VRT & DESCRIPTION OF TECHNIQUE & CRITERIA OF APPLICATION & ADVANTAGES & DISADVANTAGES & \begin{tabular}{l}
TYPICAL \\
APPLICATIONS
\end{tabular} \\
\hline Orthogonal Functions & The orthogonal characteristics of certain othonormal functions are used to formulate a sampling scheme with reduced variance & \begin{tabular}{l}
Multidimensional variables in the problem \\
Multiple integral formulations can be determined
\end{tabular} & Great possibilities for problems with multidimensional integral & \begin{tabular}{l}
Not well developed for general applications \\
Difficult to implement efficiently \\
Requires considerable calculations \\
Involves sampling from joint random variables
\end{tabular} & \begin{tabular}{l}
Multiple integrals \\
Applied in nuclear physics \\
Applied when multiple integrals are possible in complex systems
\end{tabular} \\
\hline Adjoint Method & The problem is simulated in a backward or reverse sense (mathematical adjoint formulation) & \begin{tabular}{l}
Reverse process can be formulated \\
Starting positions for final outcomes can be defined \\
Mathematical adjoint formulation is known
\end{tabular} & \begin{tabular}{l}
Can give indication of what importance sampling is required \\
Very powerful when it can be applied \\
Useful when studying system input sensitivity
\end{tabular} & \begin{tabular}{l}
Not well developed for all applications \\
Difficult to implement \\
Adjoint concept is difficult to formulate for most problems
\end{tabular} & \begin{tabular}{l}
Radiation transport \\
Could be used in queuing, inventory, etc. If prom can be formulated as integral or differential equations
\end{tabular} \\
\hline
\end{tabular}
\begin{tabular}{|c|c|c|c|c|c|}
\hline \multicolumn{6}{|l|}{CHARACTERISTICS OF VARIANCE REDUCTION TECHNIQUES (Continued)} \\
\hline VRT & DESCRIPTION OF TECHNIQUE & CRITERIA OF APPLICATION & ADVANTAGES & DISADVANTAGES & TYPICAL APPLICATIONS \\
\hline Transformation & Use a transform of the process being simulated to give a problem with reduced variance when simulated & \begin{tabular}{l}
Approximations are known for parts of the problem \\
An analytical formulation is known \\
Transformed equation can be easily simulated
\end{tabular} & \begin{tabular}{l}
Permits parametric representation of knowledge about the system \\
Can lead to substantial improvements
\end{tabular} & \begin{tabular}{l}
Not well developed for most applications \\
May require complete analytical statement of the problem \\
Appears to be limited in scope of application
\end{tabular} & Used in radiation transport problems Can be used in queuing, inventory, etc. \\
\hline \begin{tabular}{l}
Conditional \\
Monte Carlo
\end{tabular} & Imbeds problem in a larger process using conditional probabilities in larger process & Process is complicated, but can be embedded in a simpler simulation & \begin{tabular}{l}
When applicable, can give great improvement \\
Some algorithms are available for network application
\end{tabular} & Sometimes difficult to recognize applicability & Stochastic PERT, CPM, and inventory \\
\hline
\end{tabular}

\section*{Appendix I: SQUADRON.DAT File Example (33)}
```

Squadrons (squadron.dat)
SQUADRONS. }30
NUMBER.OF.MISSION.CLASSES: 12
(N.MISSION.CLASS)
AIR.SUPERIORITY
DEEP.STRIKE
GROUND.SUPPORT
JAMMER
MULTI.ROLE
RECCE
WEASEL
AWACS
JSTARS
TANKER
TBM
BPI
(Mission class names (MC.NAME))
NUMBER.OF.SORTIE.PROFILES: 14
(N.SORTIE.RATE.PROFILE)
1001 "A-10"
(Sortie rate profile ID (SRP.ID) and Name (SRP.NAME))
DAY.IN.THEATER. . AUTH.QTY.SORT/DAY . . AC . MAX . SORT/DAY
1.00 4.00 4.00
(Start time from arrival in theater (SRE.START.TIME), Nominal sortie rate
(SRE.MAX.RATE), Maximum sortie rate (SRE.MAX.RATE))
END.PROFILE
1002 "F-16"
DAY.IN. THEATER. . AUTH . QTY.SORT/DAY . . AC . MAX .SORT/DAY
$1.00 \quad 3.60 \quad 6.00$
6.00
2.50
6.00
5.00
END.PROFILE
NUMBER.OF.SQUADRONS: 30
(Number of squandrons (will be added to N.TBM.UNIT to form N.SQUADRON))
1001 "1ST FTR"
(Squadron ID (SQN.ID) and Name (SQN.NAME))

| SIDE. . SUP . CMD . ID . TYPE . AC. ID . . AUTH . QTY . . AR. PRIORITY . . . RECCE . PRIORITY |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 1102 | 1005 | 24 | 0 |

(Side the squadron belongs to ( $1=\mathrm{BLUE}, 2=$ RED) (SQN.SIDE), Superior command ID (SQN.COMMAND), Aircraft type ID (SQN.TYPE.AC), Quantity of aircraft (SQN.QTY.AC), Priority for air refueling (must be at least 0 and at most 100) (SQN.REFUEL.PRIORITY), Squadron reconnaissance priority (SQN.RECCE.PRIORITY))

| MOB.ID. .DISP.AB.ID. .SERV.KIT.ID . SORT.PROF.ID . MISSION.CLASS |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| (Main operating base ID number (SQN.MOB), Dispersal base ID number (SQN.DISPERSAL.BASE), |  |  |  |  |
| Service Kit ID number (SQN.SERVICE.CONFIG), Sortie profile ID number |  |  |  |  |

```


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\section*{Vita}

1Lt Earl Bednar was born in Everett, Washington. He graduated from Marysville Pilchuck High School in 1990. He then spent the next six years working and attending community college. He graduated from Everett Community College with an Transfer Associated degree in 1996. Next, he attended Western Washington University and recived a Bachelors of Science in Applied Mathematics in 1998.

After spending three years working in the civilian sector, 1Lt Bednar attended Officer Training School in Maxwell AFB, Alabama in 2001. His first assignement was working as an Assignments Analysis Officer for Headquarters Air Combat Commands Department of Personnel. In 2003, he was selected to attend the Air Force Institute of Technology's Graduate of Operations Research program. After his completion of the program, 1Lt Bednar will be asigned to the Air Force Resarch Labs Information Directorate, Advanced Computing Technology branch in Rome, NY.


\section*{12. DISTRIBUTION/AVAILABILITY STATEMENT}

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\section*{13. SUPPLEMENTARY NOTES}

\section*{14. ABSTRACT}

As an Air Force Chief of Staff endorsed topic, Air Force Studies and Analyses Agency (AFSAA) requested an effective and efficient way to reduce the variance in analysis results from THUNDER. THUNDER is a large-scale discrete event simulation of campaign-level military operations and is used to examine issues involving the utility and effectiveness of air and space power in a theater-level, joint warfare context. Given the large number of stochastic components within THUNDER, results are produced with highly variable measures of effectiveness (MOEs), causing difficulties in evaluating alternative force structures, weapon systems, etc.
This work responds to AFSAA's request by examining the application of Common Random Numbers (CRN), Antithetic Variates (AV), Control Variates (CV), and a combination of AVs and CVs and a combination of CRNs and CVs. The difference between the normal and variance reduction technique halfwidths for \(95 \%\) confidence intervals were examined. Analysis of the correlation between MOEs and the random inputs in the CV technique gave insight into the workings of THUNDER. A new, state of the art combined multiple recursive generator was incorporated into THUNDER to synchronize the random inputs for CRN and AV. The result is methodology for implementing all four variance reduction techniques.

\section*{15. SUBJECT TERMS}

THUNDER, Control Variates, Common Random Numbers, Antithetic Variates, Random Number Generator, Combined Multiple Recursive Generator, variance reduction, confidence intervals, halfwidth
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