Tchebichef Moment Transform on Image Dithering for Mobile Applications

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ABSTRACT

Currently, mobile image applications spend a lot of computing process to display images. A true color raw image contains billions of colors and it consumes high computational power in most mobile image applications. At the same time, mobile devices are only expected to be equipped with lower computing process and minimum storage space. Image dithering is a popular technique to reduce the numbers of bit per pixel at the expense of lower quality image displays. This paper proposes a novel approach on image dithering using 2x2 Tchebichef moment transform (TMT). TMT integrates a simple mathematical framework technique using matrices. TMT coefficients consist of real rational numbers. An image dithering based on TMT has the potential to provide better efficiency and simplicity. The preliminary experiment shows a promising result in term of error reconstructions and image visual textures.

Keyword-Image Dithering; Discrete Wavelet Transform; Tchebichef Moment Transform.

1. INTRODUCTION

Image dithering is a natural technique while reducing the color depth on the mobile image display. The aim of dithering is to distribute errors among pixels to exploit perceptually of the color images display with a limited color palette. Image dithering based on discrete Wavelet transform has been introduced by [1]. The lowest resolution image 2x2 pixel blocks is used to get the detail texture image. A 2x2 pixel block has been applied to enhance the image contours [2]. Image dithering based on Discrete Wavelet Transform (DWT) produces better natural image than image dithering based on Floyd Steinberg method. In addition, wavelet transform requires higher computing power and a special wavelet filter to analyze and reconstruct the signal. The use of wavelet filter or DWT basis function produces blurring and ringing noise near edge region in the output image. In order to overcome the complexity of DWT, this paper proposes Tchebichef moment transform on image dithering. 2x2 TMT is proposed instead of the complexity DWT on image dithering. TMT has been introduced and utilized in several computer vision applications. TMT has a lower computational complexity which does not require complex transform on full image as such in DWT. TMT does not involve any numerical approximation on friendly domain. The Tchebichef polynomials have unit weight and algebraic recurrence relations involving real rational coefficients. TMT has been widely used in several computer vision and image processing application. For examples, they are used in image analysis [3], texture segmentation, multispectral texture, template matching, pose estimation, image reconstruction [4], image projection, pattern recognition, monitoring crowds and image compression [5]-[8].

The organization of this paper is as follows. The definition of the discrete wavelet transform and Tchebichef moment transform are given in the next section. Section 3 presents the proposed Tchebichef moment transform on image dithering. The comparison of Floyd Steinberg between 2x2 discrete wavelet transform and 2x2 Tchebichef moment transform on image dithering are discussed in Section 4 and Section 5 shall concludes this paper.

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2. DISCRETE WAVELET TRANSFORM AND TCHEBICHEF MOMENT TRANSFORM

2.1 Discrete Wavelet Transform

The wavelet transform is computed separately for different segment of the time domain signal at different frequencies. DWT uses multi resolution filter banks and special wavelet filters for the analysis and reconstruction of signals. Filtering the image with 2-D DWT increases the phase distortion. Most DWT implementations use separable filtering with real coefficient filters associated with real wavelets resulting in real valued approximations and details. This domain contains more complicated basis functions called wavelets basis or mother wavelets [9]. DWT has shown to perform well in image processing applications. For examples, particularly has been used in image compression, image watermarking, texture analysis and image dithering [1].

2.2 Tchebichef Moment Transform

For a given set $\{t_n(x)\}$ of input value (image intensity values) of size $N = 2$, the forward discrete orthogonal Tchebichef Moments of order $m + n$ is given as follows [3]:

$$T_{mn} = \frac{1}{\rho(m,N)\rho(n,N)} \sum_{x=0}^{1} \sum_{y=0}^{1} t_m(x) t_n(y) f(x,y) \tag{1}$$

for $m = 0, 1$ and $n = 0,1$

Where $f(x,y)$ denote the intensity value at the pixel position (x,y) in the image. The $t_n(x)$ are defined using the following recursive relation:

$$t_0(x) = 1\,, \tag{2}$$

$$t_1(x) = \frac{2x + 1 - N}{N}, \tag{3}$$

The set $\{t_n(x)\}$ has a squared-norm given by

$$\rho(n,N) = \sum_{i=0}^{1} \{t_i(x)\}^2 = \frac{N.\left(1 - \frac{1^2}{N^2}\right).\left(1 - \frac{2^2}{N^2}\right)}{2n + 1} \tag{4}$$

The description of squared-norm $\rho()$ and the properties of orthogonal Tchebichef polynomials are given in [3]. The process of image reconstruction from its moments, the inverse moment Tchebichef moments are given as follows:

$$f(x,y) = \sum_{m=0}^{1} \sum_{n=0}^{1} T_{mn} t_m(x) t_n(y) \tag{5}$$

for $m = 0, 1$ and $n = 0,1$

Where M denotes the maximum order of moments used and $f(x,y)$ the reconstructed intensity distribution.

3. THE PROPOSED TCHEBICHEF MOMENT TRANSFORM ON IMAGE DITHERING

There are many variants technique to reduce the depth color of the image. Floyd Steinberg is a popular classical algorithm based on error dispersion in image dithering. The Floyd Steinberg filter is one of the popular methods to distribute the error among the neighbouring pixels. The error dispersed to pixels right and below. The Floyd Steinberg error diffusion filter is presented in Fig. 1.

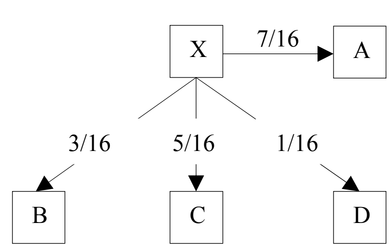

Figure 1. The Floyd Steinberg error diffusion filter.

Where X represents the current pixel, and A, B, C and D represent the neighbouring pixels that receive 7/16, 3/16, 5/16 and 1/16 of the error. In this experiment, the image is scanned in the normal left to right from top to bottom order. The original gray scale image and the experiment result of image dithering based Floyd Steinberg are shown in Fig. 2.

Figure 2. Original gray scale image (left) and original Floyd Steinberg (right).

In this paper, a new error diffusion scheme is proposed in image dithering based on TMT as presented in Fig. 3.

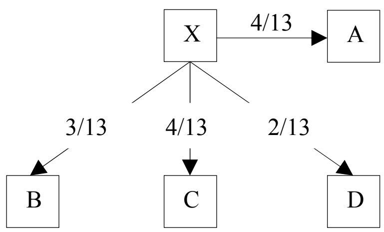

Figure 3. The proposed error diffusion filter.

3.1 TMT Coefficient

The image matrix was subdivided into 2x2 pixels where the orthogonal Tchebichef moments on each block are computed independently. The block size N is taken to be 2, and it is compared to the image dithering with $N = 2$ using DWT. Based on discrete orthogonal moments as defines in (1)-(4) above, a kernel matrix $K_{(2x2)}$ is given as follows:

$$K = \begin{bmatrix} t_0(0) & t_1(0) \\ t_0(1) & t_1(1) \end{bmatrix} \tag{6}$$

The image block matrix by $F_{(2x2)}$ with $f(x, y)$ denotes the intensity value of the pixel:

$$F = \begin{bmatrix} f(0,0) & f(0,1) \\ f(1,0) & f(1,1) \end{bmatrix} \tag{7}$$

The matrix $T_{(2x2)}$ of moments is defined based on (1) above as follows:

$$T = K^T F K \tag{8}$$

This process is repeated for every block in the original image to generate coefficient discrete orthogonal Tchebichef Moments. The inverse moments relation used to reconstruct the image block from the above moments is given as follow:

$$G = K T K^T \tag{9}$$

Where $G_{(2x2)}$ denotes the matrix image of the reconstructed intensity value. This process is repeated for every block of the coefficient Tchebichef Moments. In order to adjust the contrast of the image, the filtering process is applied for TMT coefficient. The filtering process is given as follows:

$$c_{(2x2)} = P\, c_{(2x2)} \tag{10}$$

Where $c_{(2x2)}$ is the 2x2 TMT coefficient and P is the weight on the filtering process. The weight used for filtering process on image dithering using TMT and DWT is given as follows:

$$P = \begin{bmatrix} 1.2 & 4.8 \\ 4.8 & 19.2 \end{bmatrix} \quad (11)$$

3.2 Evaluate Measurement Quality

The pixel value of binary image is represented by 255 and 0. In order to measure the quality of image dithering on binary image, the pixel value of 255 is represented by 192, otherwise the value is represented by 64. The image reconstruction error calculated for the differences between reconstructed image $g(i,j)$ and original image $f(i,j)$ is given as follows:

$$E(S) = \frac{1}{MN} \sum_{i=0}^{M-1} \sum_{j=0}^{N-1} |g(i,j) - f(i,j)| \quad (12)$$

where the original grayscale image size is MxN. Another convenient measurement is the Means Squared Error (MSE), it calculates the average of the square of the error. The MSE is defined as follows:

$$MSE = \sum_{i=0}^{M-1} \sum_{j=0}^{N-1} \|g(i,j) - f(i,j)\|^2 \quad (13)$$

For evaluation of the propose method, Peak Signal to Noise Ratio (PSNR) is used as an objectives measurement for the performance. PSNR is used to measure the quality of image reconstruction. A higher PSNR means that the reconstruction is higher in quality. The PSNR is defined as follows:

$$PSNR = 20 \log_{10}\left(\frac{Max_i}{\sqrt{MSE}}\right) = 10 \log_{10}\left(\frac{255^2}{\sqrt{MSE}}\right) \quad (14)$$

where Max_i is the maximum possible pixel value of the image. In this experiment, the sample grayscale image is represented by using 8 bits per sample, its means that the maximum intensity of sample image is 255. The comparison of quality image reconstruction and time taken among image dithering based on Floyd Steinberg, 2x2 DWT and 2x2 TMT is shown in Table 1 and Table 2.

Table 1. Average Error Score among Floyd Steinberg, 2x2 DWT and 2x2 TMT.

Evaluate Measurement	Floyd Steinberg	2x2 DWT	2x2 TMT
Full Error	51.7100	47.9877	47.9052
MSE	4057.92	3522.27	3509.83
PSNR	12.0478	12.6626	12.6779

Table 2. Time Taken Comparison among Floyd Steinberg, 2x2 DWT and 2x2 TMT.

Time Taken	Floyd Steinberg	2x2 DWT	2x2 TMT
Lena Image	1.773739 sec	129.398574 sec	4.339701 sec

The experiment result of image dithering based on 2x2 DWT and 2x2 TMT is shown in Fig. 4.

Figure 4. The comparison among image dithering based on TMT (left) and image dithering on zoomed in to 300% based on TMT (middle) and DWT (right).

4. DISCUSSION

Refer to the Lena image above, an experiment of the image dithering has been done. The experiment result of image dithering using Floyd Steinberg, 2x2 DWT and 2x2 TMT is compared and analyzed. In general, the result of image dithering based on 2x2 DWT and 2x2 TMT for gray scale image respectively gives similar output. The experiment result shows that TMT has significant advantage in term of quality measurement. Based on the experiment result as presented in Table 1, image dithering based on 2x2 TMT produces higher quality image and lower error reconstruction image. According to visual observation as presented in Fig. 4, image dithering based on TMT show similar identical texture image to image dithering based on DWT. In addition, image dithering based on DWT required longer time to dither as presented in Table 2. TMT performs efficiently computation than DWT and it produces more natural texture pixel image than classical Floyd Steinberg method on image dithering.

5. CONCLUSION

A mobile image application requires efficient image displays and transmissions. An efficient image dithering is useful on mobile devices. This paper proposes 2x2 TMT on image dithering instead of the complex DWT. A compact 2x2 image sub block has been chosen here instead of the 8x8 standard image sub block. A 2x2 TMT on image dithering produces a simpler and more computationally efficient operation than DWT. At the same time, TMT coefficients consist of only real rational numbers. TMT produces more natural and clearer texture pixels after dithering. Thus, Tchebichef moment is an ideal potential candidate for image dithering.

6. ACKNOWLEDGMENTS

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