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Determining Future Phenotype Proportions

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DETERMINING FUTURE PHENOTYPE PROPORTIONS

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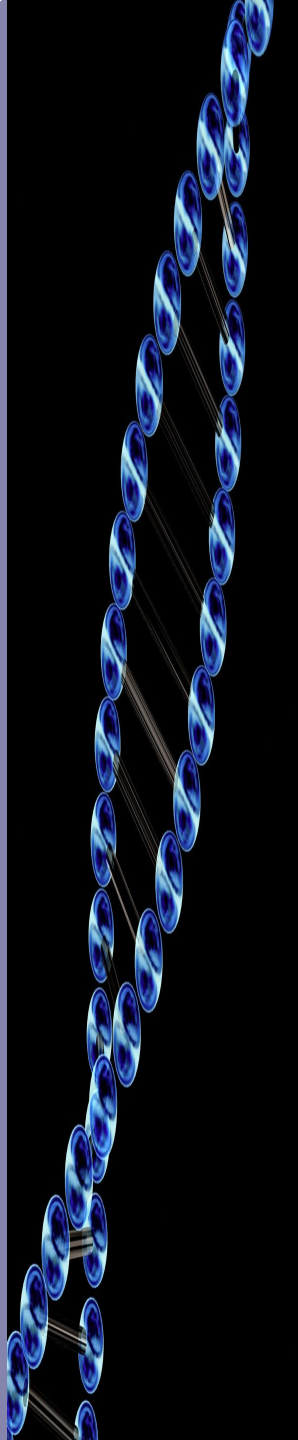

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AIMS OF THE PROJECT

- Use Markov Chains to describe forms of genetic inheritance
- Explain the use of an Autosomal Inheritance model
- Find a Matrix that can be used for X-linked inheritance
- Explain the uses of an X-linked inheritance model

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MARKOV CHAINS

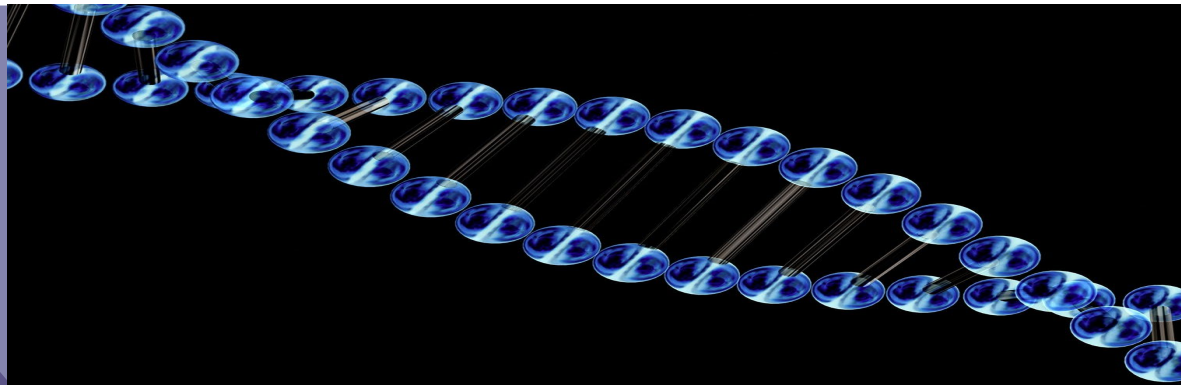
- A.A. Markov first presented Markov chains in the early 1900's [8]
- Historyless
  - The next state depends only on the current state [8]
    - Not any prior states

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} p_0(n) \\ p_1(n) \\ p_2(n) \end{bmatrix} = \begin{bmatrix} p_0(n+1) \\ p_1(n+1) \\ p_2(n+1) \end{bmatrix}$$

Example 0.1: Markov Chain

Transition Matrix

Probability vector (current state)



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MARKOV CHAINS IN GENETICS (AUTOSOMAL)

- Transition Matrix
  - Created by the use of *Punnett squares* [1]
    - *Predict the likelyhood that a genetic cross will produce an offspring with a particular genotype of phenotype* [2]

$$\begin{array}{c} \\ XX \\ Xx \\ xx \end{array} \begin{array}{c} \begin{array}{ccc} XX & XX & XX \\ XX & Xx & xx \end{array} \\ \begin{bmatrix} 1 & 1/2 & 0 \\ 0 & 1/2 & 1 \\ 0 & 0 & 0 \end{bmatrix} \end{array}$$

Matrix 1.1: Autosomal Inheritance

Traits	X	X
X	XX	XX
x	Xx	Xx

Table 1.1: Punnett Square of XX-Xx pairing

½ outcomes in Table 1.1 are XX and ½ outcomes are Xx

- Uses
  - Scientists multiply matrix 1.1 by a probability vector to represent one generation of inheritance
  - Converges to all XX genotypes

X-LINKED INHERITANCE MODEL

- X-Linked Inheritance
  - An inheritance pattern in certain species that involves genes that are located only on the X chromosome [3]
- Same method used for Autosomal inheritance (Punnett Squares)

	XY- XX	XY – Xx	XY – xx	xY – XX	xY - Xx
XX	½	¼	0	0	0
Xx	0	¼	½	½	¼
xx	0	0	0	0	¼
XY	½	¼	0	½	¼
xY	0	¼	½	0	¼

(Rows) Offspring genotypes

(Columns) Parent's genotype

Table 2.1: X-linked inheritance proportions of phenotypes

- Y chromosome
  - Not involved in the passing of genes [2]
    - Not displayed in an organisms phenotype

Photo from genegeek.ca

X-LINKED INHERITANCE MATRIX

- Uses elements in Table 2.1 to construct a transition matrix
  - Copies the pattern of X-linked inheritance

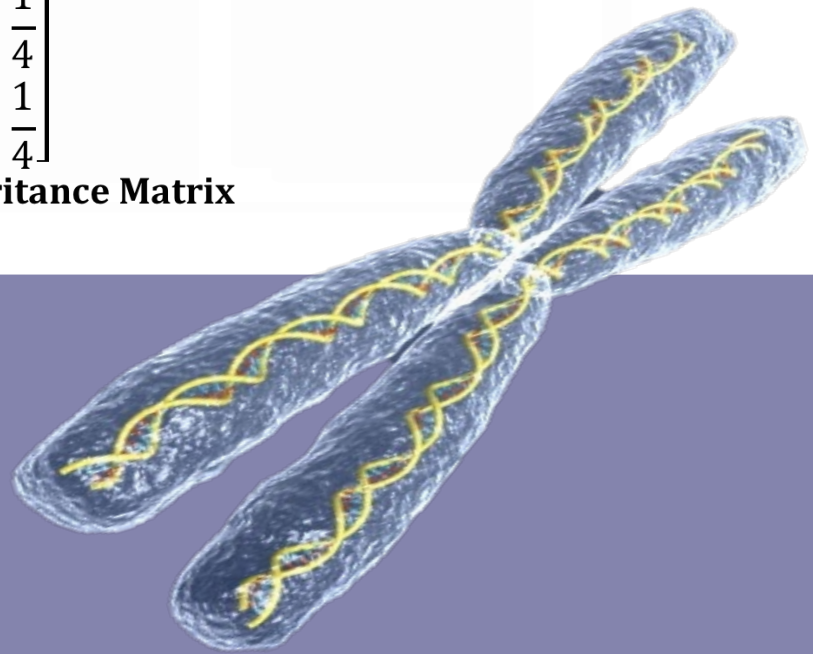
Elements from Table 2.1

$$\begin{bmatrix} \frac{1}{2} & \frac{1}{4} & 0 & 0 & 0 \\ 0 & \frac{1}{4} & \frac{1}{2} & \frac{1}{2} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{4} & 0 & \frac{1}{2} & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{2} & 0 & \frac{1}{4} \end{bmatrix}$$

Transition Matrix (Does not change)

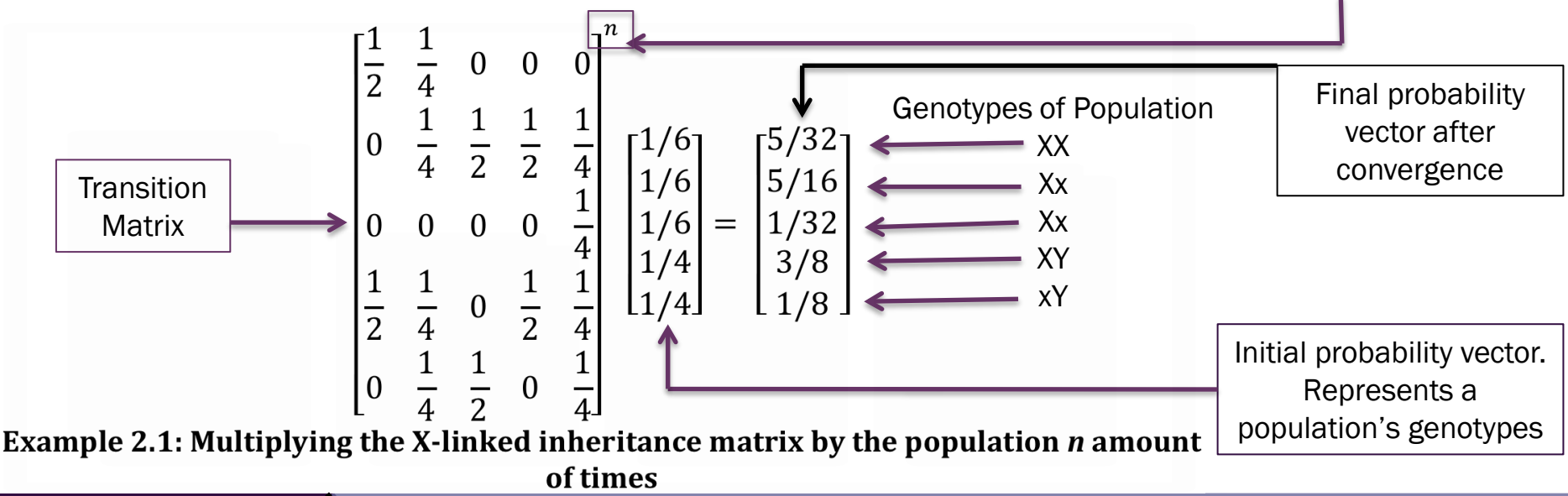
Matrix 2.1: X-linked inheritance Matrix

Photos from: axsoris.com
www.123rf.com



X-LINKED INHERITANCE MARKOV CHAIN

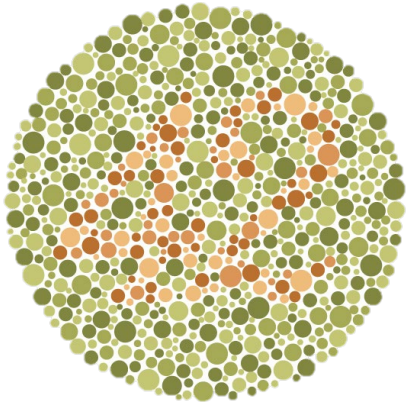
- Just as in autosomal inheritance:
  - Multiplying the transition matrix with a probability matrix
    - Leads to a convergence
    - Probability Matrix represents a population of genotypes



Example 2.1: Multiplying the X-linked inheritance matrix by the population *n* amount of times

- The Probability Matrix eventually converges
  - 1/8 of the population are infected males
  - 1/32 of the population are infected females
  - Expected outcomes: males are always more likely than females to be infected by X-Linked trait [?]

COLORBLINDNESS



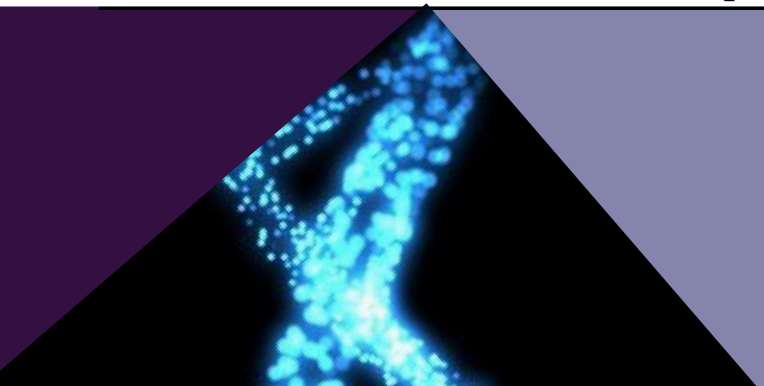
- Colorblindness
  - X-linked inheritable trait
    - 1942 study indicates 1/2500 women were colorblind [5]
    - 2012 study indicates 1/20 women were colorblind [6]
      - Drastic increase?

Probability Vector based on proportions described in 1942 study

$$\begin{bmatrix} 1999/5000 \\ 1/10 \\ \boxed{1/5000} \\ 12/25 \\ 1/50 \end{bmatrix}$$

xx Genotype
1/2500 females colorblind
(1/5000 of the population is a colorblind female)

Matrix 2.2: Population Matrix during 1942



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EXPERIMENT WITH X-LINKED MODEL

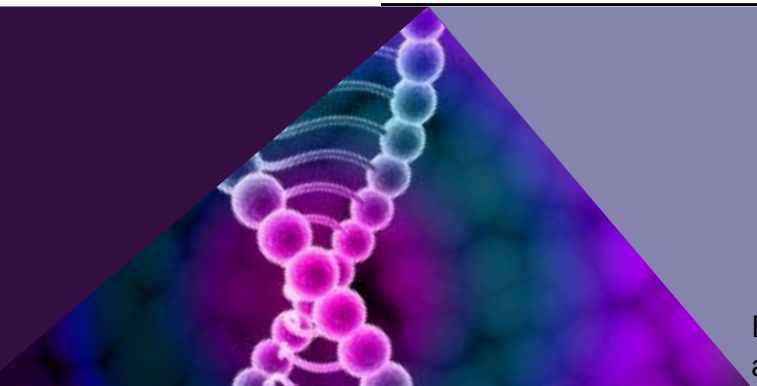
- X-linked transition matrix is multiplied by the probability vector
  - Represents one generation after each multiplication
  - Average age a mother gave birth during this time is 21 years old [7]
    - Each multiplication process represents 21 years
    - In this experiment:
      - The Transition matrix is multiplied by a probability vector 4 times (84 years)

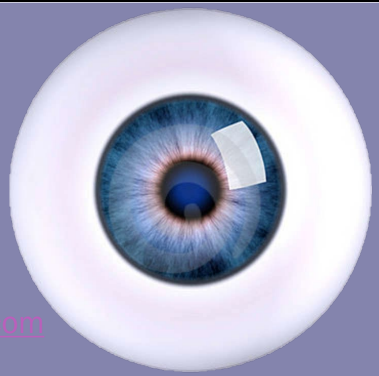
$$\begin{bmatrix} \frac{1}{2} & \frac{1}{4} & 0 & 0 & 0 \\ 0 & \frac{1}{4} & \frac{1}{2} & \frac{1}{2} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{4} & 0 & \frac{1}{2} & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{2} & 0 & \frac{1}{4} \end{bmatrix}^{n} \begin{bmatrix} 1999/5000 \\ 1/10 \\ 1/5000 \\ 12/25 \\ 1/50 \end{bmatrix}$$

Initial probability vector

Element under study

Example 2.2: Multiplying X-Linked Matrix by 1943 Population Matrix





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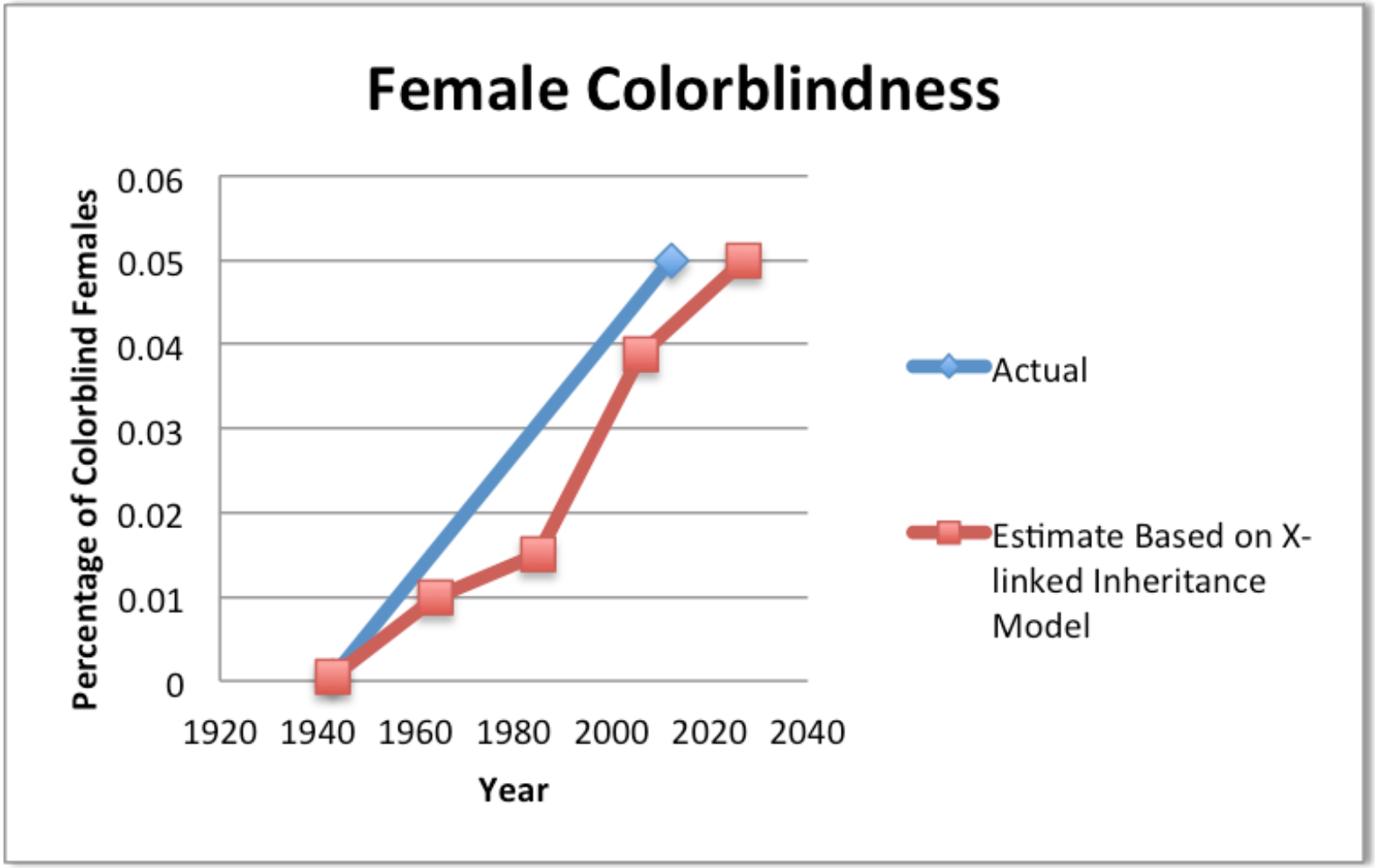

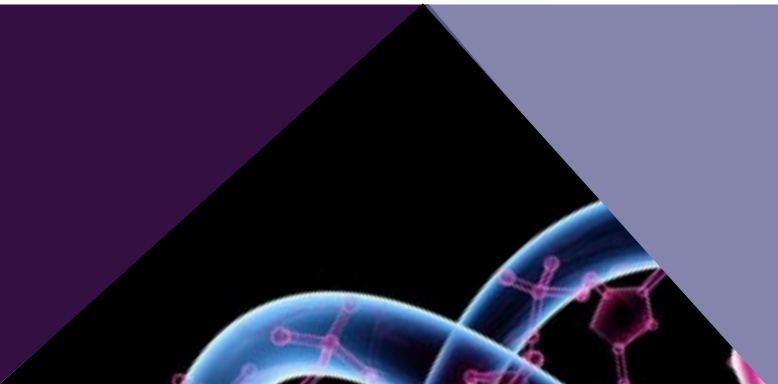

Photo from: extremebodyfit.com

RESOURCES

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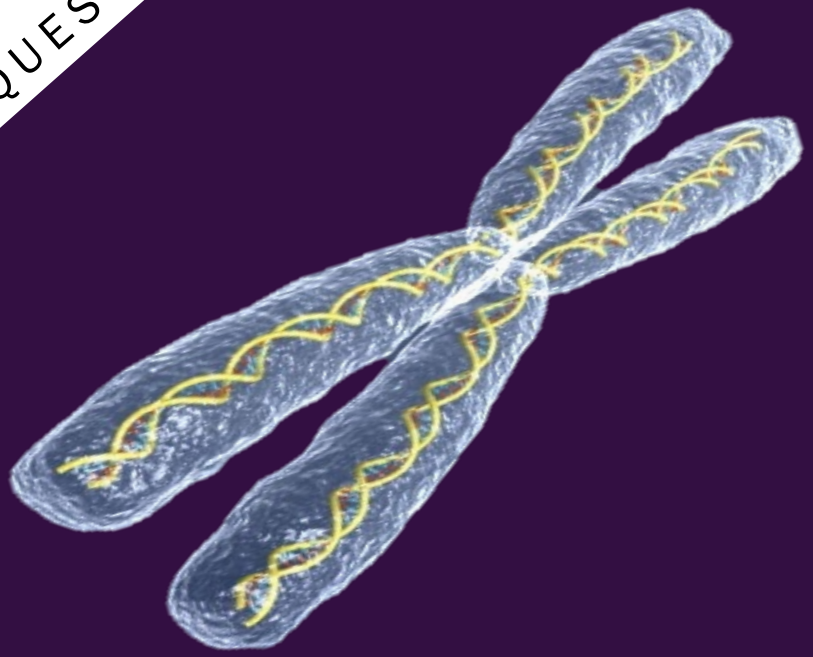
THANK YOU

ANY QUESTIONS??



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MathFest 2013

MathFest was an excellent experience to have during my undergraduate career. At the convention, I was able to learn a tremendous amount of mathematics in a short period of time, make new friends while experiencing a city I have never been to, and I began to understand what the future holds for me.

At MathFest in Hartford Connecticut, professors from around the country gave presentations on everything from the excellence of pentagons to teaching Mathematics using poetry. Every presentation had something beneficial to offer. All of the presentations taught me to look at Mathematics from a different prospective to better my understanding of each topic.

The presentations given by the professors were great, but I greatly enjoyed the presentations given by undergraduate students. It was very interesting to learn about all the work in mathematics that people of my age were doing. Most of the student's work was made original, as they attempted to discover or study something that has never been done before. Some studies by undergraduates could greatly benefit society. One student gave a presentation on watermarking audio files in order to protect against piracy and another gave a presentation on cutting the time it takes for passengers to board a plane. Every single undergraduate presentation that I attended was very impressive. I had never realized what someone of my age was capable of doing.

To my surprise, my presentation was liked by many. I remember being asked about my presentation hours after having given it. Students seemed to be as interested in my project as I was theirs. My presentation was about Determining Future Phenotype Proportions. This presentation used linear algebra in the field of genetics. I never gave a presentation outside of my speech classes so this was a great experience. Professor Joyati Debnath had rehearsed with me many times before the presentation and this helped me become more comfortable speaking in front of a crowd. There were four judges and I could tell they had enjoyed my

presentation as they were very curious after, asking many questions pertaining to my project. My project was ranked 4th out of 385 undergraduate presentations. This was a huge achievement for me and I was very surprised after hearing that my presentation was that highly ranked.

I didn't know what to expect when my plane arrived in Hartford Connecticut. But soon after arriving, I made several friends. Some of these friends were from Minnesota, and others were from Boston or even Toronto, Canada. It was easy to make new friends as everyone there shared the same common trait, an interest in mathematics. After talking to some of my new friends, I noticed they had many other interests that matched mine. Some of these included of a liking for basketball, biking, and certain television shows. Together, we explored Hartford and discovered it was actually a neat city. Besides having a Dunk'n Donuts on every block, Hartford had a terrific library, capital, and Science Center. I spent around 6 hours at the Science Center and had fun with science experiments that were conducted there. In Connecticut, I never ran out of something to do. My new friends and I were either going out to eat, or finding something fun to do in Connecticut. I particularly remember a certain restaurant named Russel's where I had the best Quesadilla in my life.

At MathFest, I realized that graduate school was the right choice for me. I want an opportunity to constantly better my mathematical knowledge and graduate school will be an excellent start. Many people I met in Hartford were, in fact, graduate students. All of them shared their experiences and the topics they were researching. It all sounded very exciting to me. I hope to soon go to graduate school so I can have new experiences that I am able to share with those who are interested in my field. MathFest presented the field of mathematics in an exciting way and this encourages my decision to better myself by studying math.

My experience at MathFest was very beneficial and will continue to benefit me as I search for a graduate school. I hope to attend other mathematics organizations like MathFest. In January there is another convention where I hope to

have more great experiences like I did at MathFest. In conclusion, MathFest was an excellent experience that expanded my knowledge in mathematics and also greatened my interest in the field.

In conclusion, I have many people and organizations to thank. I wanted to thank the MAA and the organizers of MathFest for giving me the opportunity to present the research that I have done this summer. The event was greatly organized and a very fun atmosphere. I would also like to thank Winona State University for helping me finance the trip to Hartford Connecticut. I would not have been able to take part in MathFest without the financial aid given by both Winona State University and the MAA. And last but certainly not least; I would like to thank my professor/advisor Joyati Debnath. She worked with me every step of the way to make my trip to MathFest very successful and memorable. I understand all the work that was done by my professor to make certain that my trip was funded, safe, and very well organized. I appreciate all the assistance that was given to me for this event.