

The General Theory of Space Time, Mass, Energy, Quantum Gravity, Perception, Four Fundamental Forces, Vacuum Energy, Quantum Field

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Abstract

Essentially GUT and Vacuum Field are related to Quantum field where Quantum entanglement takes place. Mass energy equivalence and its relationship with Quantum Computing are discussed in various papers by the author. Here we finalize a paper on the relationship of GUT on one hand and space-time, mass-energy, Quantum Gravity and Vacuum field with Quantum Field. In fact, noise, discordant notes also are all related to subjective theory of Quantum Mechanics which is related to Quantum Entanglement and Quantum computing.

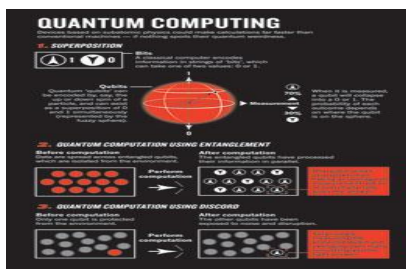
Key words: Quantum Mechanics, Quantum computing, Quantum entanglement, vacuum energy

Introduction:

Physicists have always thought quantum computing is hard because quantum states are incredibly fragile. But could noise and messiness actually help things along? (Zeeya Merali) Quantum computation, attempting to exploit subatomic physics to create a device with the potential to outperform its best macroscopic counterparts IS A Gordian knot with the Physicists. . Quantum systems are fragile, vulnerable and susceptible both in its thematic and discursive form and demand immaculate laboratory conditions to survive long enough to be of any use. Now White was setting out to test an unorthodox quantum algorithm that seemed to turn that lesson on its head. Energetic franticness, ensorcelled frenzy, entropic entrepotishness, Ergodic erythrism messiness and disorder would be virtues, not vices — and perturbations in the quantum system would drive computation, not disrupt it.

Conventional view is that such devices should get their computational power from quantum entanglement — a phenomenon through which particles can share information even when they are separated by arbitrarily large distances. But the latest experiments suggest that entanglement might not be needed after all. Algorithms could instead tap into a quantum resource called discord, which would be far cheaper and easier to maintain in the lab.

Classical computers have to encode their data in an either/or fashion: each bit of information takes a value of 0 or 1, and nothing else. But the quantum world is the realm of both/and. Particles can exist in 'superposition's' — occupying many locations at the same time, say, or simultaneously (e&eb)spinning clockwise and anticlockwise.



So, Feynman argued, computing in that realm could use quantum bits of information — qubits — that exist as superpositions of 0 and 1 simultaneously. A string of 10 such qubits could represent all 1,024 10-bit numbers simultaneously. And if all the qubits shared information through entanglement, they could race through myriad calculations in parallel — calculations that their classical counterparts would have to plod through in a languorous, lugubrious and lachrymoshish manner sequentially (see 'Quantum computing').

The notion that quantum computing can be done only through entanglement was cemented in 1994, when Peter Shor, a mathematician at the Massachusetts Institute of Technology in Cambridge, devised an entanglement-based algorithm that could factorize large numbers at lightning speed — potentially requiring only seconds to break the encryption currently used to send secure online communications, instead of the years required by ordinary computers. In 1996, Lov Grover at Bell Labs in Murray Hill, New Jersey, proposed an entanglement-based algorithm that could search rapidly through an unsorted database; a classical algorithm, by contrast, would have to laboriously search the items one by one.

But entanglement has been the bane of many a quantum experimenter's life, because the slightest interaction of the entangled particles with the outside world — even with a stray low-energy photon emitted by the warm walls of the laboratory — can destroy it. Experiments with entanglement demand ultra-low temperatures and careful handling. "Entanglement is hard to prepare, hard to maintain and hard to manipulate," says Xiaosong Ma, a physicist at the Institute for Quantum Optics and Quantum Information in Vienna. Current entanglement record-holder intertwines just 14 qubits, yet a large-scale quantum computer would need several thousand. Any scheme that bypasses entanglement would be warmly welcomed, without any hesitation, reservation, regret, remorse, compunction or contrition. Says Ma.

Clues that entanglement isn't essential after all began to trickle in about a decade ago, with the first examples of rudimentary regimentation and seriological sermonisations and padagouellogical pontifications quantum computation. In 2001, for instance, physicists at IBM's Almaden Research Center in San Jose and Stanford University, both in California, used a 7-qubit system to implement Shor's algorithm, factorizing the number 15 into 5 and 3. But controversy erupted over whether the experiments deserved to be called quantum computing, says Carlton Caves, a quantum physicist at the University of New Mexico (UNM) in Albuquerque.

The trouble was that the computations were done at room temperature, using liquid-based nuclear magnetic resonance (NMR) systems, in which information is encoded in atomic nuclei using(e) an internal quantum property known as spin. Caves and his colleagues had already shown that entanglement could not be sustained in these conditions. "The nuclear spins would be jostled about too much for them to stay lined up neatly," says Caves. According to the orthodoxy, no entanglement meant any quantum computation. The NMR community gradually accepted that they had no entanglement, yet the computations were producing real results. Experiments were explicitly performed for a quantum search without (e)e)xploiting entanglement. These experiments really called into question what gives quantum computing its power.

Order Out of Disorder

Discord, an obscure measure of quantum correlations. Discord quantifies (=) how much a system can be disrupted when people observe it to gather information. Macroscopic systems are not (e&eb)affected by observation, and so have zero discord. But quantum systems are unavoidably (e&eb) affected because measurement forces them to settle on one of their many superposition values, so any possible quantum correlations, including entanglement, give (eb) a positive value for discord. Discord is connected (e&eb)to quantum computing."An algorithm challenged the idea that quantum computing requires (e) to painstakingly prepare(eb) a set of pristine qubits in the lab.

In a typical optical experiment, the pure qubits might (e) consist of horizontally polarized photons representing 1 and vertically polarized photons representing 0. Physicists can entangle a stream of such pure qubits by passing them through a (e&eb) processing gate such as a crystal that alters (e&eb) the polarization of the light, and then read off the state of the qubits as they exit. In the real world, unfortunately, qubits rarely stay pure. They are far more likely to become messy, or 'mixed' — the equivalent of unpolarized photons. The conventional wisdom is that mixed qubits are(e) useless for computation because they e(e&eb) cannot be entangled, and any measurement of a mixed qubit will yield a random result, providing little or no useful information.

If a mixed qubit was sent through an entangling gate with a pure qubit. The two could not become entangled but, the physicists argued, their interaction might be enough to carry (eb)out a quantum computation, with the result read from the pure qubit. If it worked, experimenters could get away with using just one tightly controlled qubit, and letting the others be badly battered sadly shattered by environmental noise and disorder. "It was not at all clear why that should work," says White. "It sounded as strange as saying they wanted to measure someone's speed by measuring the distance run with a perfectly metered ruler and measuring the time with a stopwatch that spits out a random answer."

Datta supplied an explanation he calculated that the computation could be(eb) driven by the quantum correlation between the pure and mixed qubits — a correlation given mathematical expression by the discord."It's true that you must have entanglement to compute with idealized pure qubits," "But when you include mixed states, the calculations look very different."Quantum computation without (e) the hassle of entanglement," seems to have become a point where the anecdote of life had met the aphorism of thought. Discord could be like sunlight, which is plentiful but has to be harnessed in a certain way to be useful.

The team confirmed that the qubits were not entangled at any point. Intriguingly, when the researchers tuned down the polarization quality of the one pure qubit, making (eb) it almost mixed, the computation still worked. "Even when you have a system with just a tiny fraction of purity, that is (=) vanishingly close to classical, it still has power," says White. "That just blew our minds." The computational power only disappeared when the amount of discord in the system reached zero. "It's counter-intuitive, but it seems that putting noise and disorder in your system gives you power," says White. "Plus, it's easier to achieve."For Ma, White's results provided the "wow! Moment" that made him take discord seriously. He was keen to test discord-based algorithms that used more than the two qubits used by White, and that could perform more glamorous tasks, but he had none to test. "Before I can carry out any experiments, I need the recipe of what to prepare from theoreticians," he explains, and those instructions were not forthcoming.

Although it is easier for experimenters to handle noisy real-world systems than pristinely glorified ones, it is a lot harder for theoretical physicists to analyse them mathematically. "We're talking about messy physical systems, and the equations are even messier," says Modi. For the past few years, theoretical physicists interested in discord have been trying to formulate prescriptions for new tests. It is not proved that discord is (eb) essential to computation — just that it is there. Rather than being the engine behind computational power, it could just be along for the ride, he argues. Last year, Ac í and his colleagues calculated that almost every quantum system contains discord. "It's basically everywhere," he says. "That makes it difficult to explain why it causes power in specific situations and not others." It is almost like we can perform our official tasks amidst all noise, subordination pressure, superordinational scatological pontification, coordination dialectic deliberation, but when asked to do something different we want "peace". "Silence", "No disturbance". Personally, one thinks it is a force of habit. And habits die hard. Modi shares the concern. "Discord could be like sunlight, which is plentiful but has to be harnessed in a certain way to be useful. We need to identify what that way is," he says. Du and Ma are independently

conducting experiments to address these points. Both are attempting to measure the amount of discord at each stage of a computation — Du using liquid NMR and electron-spin resonance systems, and Ma using photons. The very ‘importance giving’, attitude itself acts as an anathema, a misnomer.

A finding that quantifies how and where discord acts would strengthen the case for its importance, says Acín. We suspect it acts only in cases where there is ‘speciality’ like in quantum level. Other ‘mundane ‘world’ happenings take place amidst all discord and noise. Nobody bothers because it is ‘run of the mill’ But for ‘selective and important issues’ one needs ‘calm’ and ‘non disturbance’ and doing’ all ‘things’ amidst this worldly chaos we portend is ‘Khuda’ ‘Allah’ or ‘Brahman’ And we feel that Quantum Mechanics is a subjective science and teaches this philosophy much better than others. But if these tests find discord wanting, the mystery of how entanglement-free computation works will be reopened. "The search would have to begin for yet another quantum property," he adds. Vedral notes that even if Du and Ma's latest experiments are a success, the real game-changer will be discord-based algorithms for factorization and search tasks, similar to the functions devised by Shor and Grover that originally ignited the field of quantum computing. "My gut feeling is that tasks such as these will ultimately need entanglement," says Vedral. "Though as yet there is no proof that they can't be done with discord alone."

Zurek says that discord can be thought of as a complement to entanglement, rather than as a usurper. "There is no longer a question that discord works," he declares. "The important thing now is to find out when discord without entanglement can be (e)xploited most usefully, and when entanglement is essential. ,and produces ‘Quantum Computation’"

Notation :

Space And Time

G_{13} : Category One Of Time

G_{14} :Category Two Of Time

G_{15} : Category Three Of Time

T_{13} : Category One Of Space

T_{14} : Category Two Of Space

T_{15} : Category Three Of Space

1

Mass And Energy

G_{16} : Category One Of Energy

G_{17} : Category Two Of Energy

G_{18} : Category Three Of Energy

T_{16} : Category One Of Matter

T_{17} : Category Two Of Matter

T_{18} : Category Three Of Matter

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Quantum Gravity And Perception:

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 G_{20} :Category One Of Perception

G_{21} :Category Two Of Perception

G_{22} : Category Three Of Perception

T_{20} : Category One Of Quantum Gravity

T_{21} : Category Two Of Quantum Gravity

T_{22} : Category Three Of Quantum Gravity

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Strong Nuclear Force And Weak Nuclear Force:

G_{24} : Category One Of Weak Nuclear Force	4
G_{25} : Category Two Of Weak Nuclear Force	
G_{26} : Category Three Of Weak Nuclear Force	
T_{24} :Category One Of Strong Nuclear Force	
T_{25} : Category Two Of Strong Nuclear Force	
T_{26} : Category Three Of Strong Nuclear Force	

Electromagnetism And Gravity:

G_{28} : Category One Of Gravity	
G_{29} : Category Two Of Gravity	5
G_{30} : Category Three Of Gravity	
T_{28} : Category One Of Electromagnetism	
T_{29} : Category Two Of Electromagnetism	
T_{30} : Category Three Of Electromagnetism	

Vacuum Energy And Quantum Field:

G_{32} : Category One Of Quantum Field	
G_{33} : Category Two Of Quantum Field	
G_{34} : Category Three Of Quantum Field	
T_{32} : Category One Of Vacuum Energy	6
T_{33} : Category Two Of Vacuum Energy	
T_{34} : Category Three Of Vacuum Energy	

Accentuation Coefficients:

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}, (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}$ $(b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}, (a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}, (b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$ $(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)},$ $(a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}, (a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$	7
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Dissipation Coefficients

$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)},$ $(b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$ $(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}$ $(a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}, (a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$

Governing Equations: For The System Space And Time:

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The differential system of this model is now

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - [(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)]G_{13} \quad 9$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - [(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)]G_{14} \quad 10$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - [(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)]G_{15} \quad 11$$

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)]T_{13} \quad 12$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)]T_{14} \quad 13$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)]T_{15} \quad 14$$

$$+(a''_{13})^{(1)}(T_{14}, t) = \text{First augmentation factor} \quad 15$$

$$-(b''_{13})^{(1)}(G, t) = \text{First detritions factor} \quad 16$$

Governing Equations: Of The System Mass (Matter) And Energy

The differential system of this model is now

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - [(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)]G_{16} \quad 18$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - [(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)]G_{17} \quad 19$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - [(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)]G_{18} \quad 20$$

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19}), t)]T_{16} \quad 21$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}((G_{19}), t)]T_{17} \quad 22$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19}), t)]T_{18} \quad 23$$

$$+(a''_{16})^{(2)}(T_{17}, t) = \text{First augmentation factor} \quad 24$$

$$-(b''_{16})^{(2)}((G_{19}), t) = \text{First detritions factor} \quad 25$$

Governing Equations: Of The System Quantum Gravity And Perception

The differential system of this model is now

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - [(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)]G_{20} \quad 26$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - [(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}, t)]G_{21} \quad 27$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - [(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}, t)]G_{22} \quad 28$$

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}, t)]T_{20} \quad 29$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}, t)]T_{21} \quad 30$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}, t)]T_{22} \quad 31$$

$$+(a''_{20})^{(3)}(T_{21}, t) = \text{First augmentation factor} \quad 32$$

$$-(b''_{20})^{(3)}(G_{23}, t) = \text{First detritions factor}$$

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Governing Equations: Of The System Strong Nuclear Force And Weak Nuclear Force:

The differential system of this model is now

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - [(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)]G_{24} \quad 34$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - [(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}, t)]G_{25} \quad 35$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - [(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}, t)]G_{26} \quad 36$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27}, t))]T_{24} \quad 37$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}((G_{27}, t))]T_{25} \quad 38$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27}, t))]T_{26} \quad 39$$

$$+(a''_{24})^{(4)}(T_{25}, t) = \text{First augmentation factor} \quad 40$$

$$-(b''_{24})^{(4)}((G_{27}, t)) = \text{First detritions factor} \quad 41$$

Governing Equations: Of The System Electromagnetism And Gravity:

The differential system of this model is now

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - [(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)]G_{28} \quad 42$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - [(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}, t)]G_{29} \quad 43$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - [(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}, t)]G_{30} \quad 44$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31}, t))]T_{28} \quad 45$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}((G_{31}, t))]T_{29} \quad 46$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31}, t))]T_{30} \quad 47$$

$$+(a''_{28})^{(5)}(T_{29}, t) = \text{First augmentation factor} \quad 48$$

$$-(b''_{28})^{(5)}((G_{31}, t)) = \text{First detritions factor} \quad 49$$

Governing Equations: Of The System Vacuum Energy And Quantum Field:

The differential system of this model is now

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - [(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)]G_{32} \quad 50$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - [(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}, t)]G_{33} \quad 51$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - [(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}, t)]G_{34} \quad 52$$

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35}, t))]T_{32} \quad 53$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}((G_{35}, t))]T_{33} \quad 54$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35}, t))]T_{34} \quad 55$$

$$+(a''_{32})^{(6)}(T_{33}, t) = \text{First augmentation factor} \quad 56$$

$$-(b''_{32})^{(6)}(G_{35}, t) = \text{First detritions factor} \quad 57$$

Concatenated Governing Equations Of The Global System Space-Time-Mass-Energy-Quantum Gravity-Perception-Strong Nuclear Force –Weak Nuclear Force-Electromagnetism-Gravity-Vacuum Energy And Quantum Field

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[\begin{array}{|c|c|c|c|} \hline (a'_{13})^{(1)} & +(a''_{13})^{(1)}(T_{14}, t) & +(a''_{16})^{(2,2)}(T_{17}, t) & +(a''_{20})^{(3,3)}(T_{21}, t) \\ \hline \end{array} \right] G_{13} \quad 58$$

$$\left[\begin{array}{|c|c|c|c|} \hline +(a''_{24})^{(4,4,4,4)}(T_{25}, t) & +(a''_{28})^{(5,5,5,5)}(T_{29}, t) & +(a''_{32})^{(6,6,6,6)}(T_{33}, t) & \\ \hline \end{array} \right] \quad 59$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[\begin{array}{|c|c|c|c|} \hline (a'_{14})^{(1)} & +(a''_{14})^{(1)}(T_{14}, t) & +(a''_{17})^{(2,2)}(T_{17}, t) & +(a''_{21})^{(3,3)}(T_{21}, t) \\ \hline \end{array} \right] G_{14} \quad 60$$

$$\left[\begin{array}{|c|c|c|c|} \hline +(a''_{25})^{(4,4,4,4)}(T_{25}, t) & +(a''_{29})^{(5,5,5,5)}(T_{29}, t) & +(a''_{33})^{(6,6,6,6)}(T_{33}, t) & \\ \hline \end{array} \right] \quad 61$$

Where $(a''_{13})^{(1)}(T_{14}, t)$, $(a''_{14})^{(1)}(T_{14}, t)$, $(a''_{15})^{(1)}(T_{14}, t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2)}(T_{17}, t)$, $+(a''_{17})^{(2,2)}(T_{17}, t)$, $+(a''_{18})^{(2,2)}(T_{17}, t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{20})^{(3,3)}(T_{21}, t)$, $+(a''_{21})^{(3,3)}(T_{21}, t)$, $+(a''_{22})^{(3,3)}(T_{21}, t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4)}(T_{25}, t)$, $+(a''_{25})^{(4,4,4,4)}(T_{25}, t)$, $+(a''_{26})^{(4,4,4,4)}(T_{25}, t)$ are fourth augmentation coefficient for category 1, 2 and 3

and 3

$+(a''_{28})^{(5,5,5,5)}(T_{29}, t)$, $+(a''_{29})^{(5,5,5,5)}(T_{29}, t)$, $+(a''_{30})^{(5,5,5,5)}(T_{29}, t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6)}(T_{33}, t)$, $+(a''_{33})^{(6,6,6,6)}(T_{33}, t)$, $+(a''_{34})^{(6,6,6,6)}(T_{33}, t)$ are sixth augmentation coefficient for category 1, 2 and 3

3

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[\begin{array}{|c|c|c|} \hline (b'_{13})^{(1)} & -(b''_{13})^{(1)}(G, t) & -(b''_{16})^{(2,2)}(G_{19}, t) & -(b''_{20})^{(3,3)}(G_{23}, t) \\ \hline \end{array} \right] T_{13} \quad 62$$

$$\left[\begin{array}{|c|c|c|} \hline -(b''_{24})^{(4,4,4,4)}(G_{27}, t) & -(b''_{28})^{(5,5,5,5)}(G_{31}, t) & -(b''_{32})^{(6,6,6,6)}(G_{35}, t) \\ \hline \end{array} \right] \quad 63$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[\begin{array}{|c|c|c|} \hline (b'_{14})^{(1)} & -(b''_{14})^{(1)}(G, t) & -(b''_{17})^{(2,2)}(G_{19}, t) & -(b''_{21})^{(3,3)}(G_{23}, t) \\ \hline \end{array} \right] T_{14} \quad 64$$

$$\left[\begin{array}{|c|c|c|} \hline -(b''_{25})^{(4,4,4,4)}(G_{27}, t) & -(b''_{29})^{(5,5,5,5)}(G_{31}, t) & -(b''_{33})^{(6,6,6,6)}(G_{35}, t) \\ \hline \end{array} \right] \quad 65$$

Where $-(b''_{13})^{(1)}(G, t)$, $-(b''_{14})^{(1)}(G, t)$, $-(b''_{15})^{(1)}(G, t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2)}(G_{19}, t)$, $-(b''_{17})^{(2,2)}(G_{19}, t)$, $-(b''_{18})^{(2,2)}(G_{19}, t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3)}(G_{23}, t)$, $-(b''_{21})^{(3,3)}(G_{23}, t)$, $-(b''_{22})^{(3,3)}(G_{23}, t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4,4)}(G_{27}, t)$, $-(b''_{25})^{(4,4,4,4)}(G_{27}, t)$, $-(b''_{26})^{(4,4,4,4)}(G_{27}, t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5)}(G_{31}, t)$, $-(b''_{29})^{(5,5,5,5)}(G_{31}, t)$, $-(b''_{30})^{(5,5,5,5)}(G_{31}, t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6)}(G_{35}, t)$, $-(b''_{33})^{(6,6,6,6)}(G_{35}, t)$, $-(b''_{34})^{(6,6,6,6)}(G_{35}, t)$ are sixth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[\begin{array}{|c|c|c|c|} \hline (a'_{16})^{(2)} & +(a''_{16})^{(2)}(T_{17}, t) & +(a''_{13})^{(1,1)}(T_{14}, t) & +(a''_{20})^{(3,3,3)}(T_{21}, t) \\ \hline \end{array} \right] G_{16} \quad 66$$

$$\left[\begin{array}{|c|c|c|} \hline +(a''_{24})^{(4,4,4,4,4)}(T_{25}, t) & +(a''_{28})^{(5,5,5,5,5)}(T_{29}, t) & +(a''_{32})^{(6,6,6,6,6)}(T_{33}, t) \\ \hline \end{array} \right] \quad 66$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)} G_{16} - \left[\begin{array}{|c|c|c|} \hline (a'_{17})^{(2)} & + (a''_{17})^{(2)}(T_{17}, t) & + (a''_{14})^{(1,1)}(T_{14}, t) & + (a''_{21})^{(3,3,3)}(T_{21}, t) \\ \hline + (a''_{25})^{(4,4,4,4,4)}(T_{25}, t) & + (a''_{29})^{(5,5,5,5,5)}(T_{29}, t) & + (a''_{33})^{(6,6,6,6,6)}(T_{33}, t) & \\ \hline \end{array} \right] G_{17} \quad 67$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)} G_{17} - \left[\begin{array}{|c|c|c|} \hline (a'_{18})^{(2)} & + (a''_{18})^{(2)}(T_{17}, t) & + (a''_{15})^{(1,1)}(T_{14}, t) & + (a''_{22})^{(3,3,3)}(T_{21}, t) \\ \hline + (a''_{26})^{(4,4,4,4,4)}(T_{25}, t) & + (a''_{30})^{(5,5,5,5,5)}(T_{29}, t) & + (a''_{34})^{(6,6,6,6,6)}(T_{33}, t) & \\ \hline \end{array} \right] G_{18} \quad 68$$

Where $+ (a''_{16})^{(2)}(T_{17}, t)$, $+ (a''_{17})^{(2)}(T_{17}, t)$, $+ (a''_{18})^{(2)}(T_{17}, t)$ are first augmentation coefficients for category 1, 2 and 3 69

$+ (a''_{13})^{(1,1)}(T_{14}, t)$, $+ (a''_{14})^{(1,1)}(T_{14}, t)$, $+ (a''_{15})^{(1,1)}(T_{14}, t)$ are second augmentation coefficient for category 1, 2 and 3

$+ (a''_{20})^{(3,3,3)}(T_{21}, t)$, $+ (a''_{21})^{(3,3,3)}(T_{21}, t)$, $+ (a''_{22})^{(3,3,3)}(T_{21}, t)$ are third augmentation coefficient for category 1, 2 and 3

$+ (a''_{24})^{(4,4,4,4,4)}(T_{25}, t)$, $+ (a''_{25})^{(4,4,4,4,4)}(T_{25}, t)$, $+ (a''_{26})^{(4,4,4,4,4)}(T_{25}, t)$ are fourth augmentation coefficient for category 1, 2 and 3

$+ (a''_{28})^{(5,5,5,5,5)}(T_{29}, t)$, $+ (a''_{29})^{(5,5,5,5,5)}(T_{29}, t)$, $+ (a''_{30})^{(5,5,5,5,5)}(T_{29}, t)$ are fifth augmentation coefficient for category 1, 2 and 3

$+ (a''_{32})^{(6,6,6,6,6)}(T_{33}, t)$, $+ (a''_{33})^{(6,6,6,6,6)}(T_{33}, t)$, $+ (a''_{34})^{(6,6,6,6,6)}(T_{33}, t)$ are sixth augmentation coefficient for category 1, 2 and 3

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)} T_{17} - \left[\begin{array}{|c|c|c|} \hline (b'_{16})^{(2)} & - (b''_{16})^{(2)}(G_{19}, t) & - (b''_{13})^{(1,1)}(G, t) & - (b''_{20})^{(3,3,3)}(G_{23}, t) \\ \hline - (b''_{24})^{(4,4,4,4,4)}(G_{27}, t) & - (b''_{28})^{(5,5,5,5,5)}(G_{31}, t) & - (b''_{32})^{(6,6,6,6,6)}(G_{35}, t) & \\ \hline \end{array} \right] T_{16} \quad 70$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)} T_{16} - \left[\begin{array}{|c|c|c|} \hline (b'_{17})^{(2)} & - (b''_{17})^{(2)}(G_{19}, t) & - (b''_{14})^{(1,1)}(G, t) & - (b''_{21})^{(3,3,3)}(G_{23}, t) \\ \hline - (b''_{25})^{(4,4,4,4,4)}(G_{27}, t) & - (b''_{29})^{(5,5,5,5,5)}(G_{31}, t) & - (b''_{33})^{(6,6,6,6,6)}(G_{35}, t) & \\ \hline \end{array} \right] T_{17} \quad 71$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)} T_{17} - \left[\begin{array}{|c|c|c|} \hline (b'_{18})^{(2)} & - (b''_{18})^{(2)}(G_{19}, t) & - (b''_{15})^{(1,1)}(G, t) & - (b''_{22})^{(3,3,3)}(G_{23}, t) \\ \hline - (b''_{26})^{(4,4,4,4,4)}(G_{27}, t) & - (b''_{30})^{(5,5,5,5,5)}(G_{31}, t) & - (b''_{34})^{(6,6,6,6,6)}(G_{35}, t) & \\ \hline \end{array} \right] T_{18} \quad 72$$

where $- (b''_{16})^{(2)}(G_{19}, t)$, $- (b''_{17})^{(2)}(G_{19}, t)$, $- (b''_{18})^{(2)}(G_{19}, t)$ are first detrition coefficients for category 1, 2 and 3 73

$- (b''_{13})^{(1,1)}(G, t)$, $- (b''_{14})^{(1,1)}(G, t)$, $- (b''_{15})^{(1,1)}(G, t)$ are second detrition coefficients for category 1,2 and 3

$- (b''_{20})^{(3,3,3)}(G_{23}, t)$, $- (b''_{21})^{(3,3,3)}(G_{23}, t)$, $- (b''_{22})^{(3,3,3)}(G_{23}, t)$ are third detrition coefficients for category 1,2 and 3

$- (b''_{24})^{(4,4,4,4,4)}(G_{27}, t)$, $- (b''_{25})^{(4,4,4,4,4)}(G_{27}, t)$, $- (b''_{26})^{(4,4,4,4,4)}(G_{27}, t)$ are fourth detrition coefficients for category 1,2 and 3

$- (b''_{28})^{(5,5,5,5,5)}(G_{31}, t)$, $- (b''_{29})^{(5,5,5,5,5)}(G_{31}, t)$, $- (b''_{30})^{(5,5,5,5,5)}(G_{31}, t)$ are fifth detrition coefficients for category 1,2 and 3

$- (b''_{32})^{(6,6,6,6,6)}(G_{35}, t)$, $- (b''_{33})^{(6,6,6,6,6)}(G_{35}, t)$, $- (b''_{34})^{(6,6,6,6,6)}(G_{35}, t)$ are sixth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)} G_{21} - \left[\begin{array}{|c|c|c|} \hline (a'_{20})^{(3)} & + (a''_{20})^{(3)}(T_{21}, t) & + (a''_{16})^{(2,2,2)}(T_{17}, t) & + (a''_{13})^{(1,1,1)}(T_{14}, t) \\ \hline + (a''_{24})^{(4,4,4,4,4,4)}(T_{25}, t) & + (a''_{28})^{(5,5,5,5,5,5)}(T_{29}, t) & + (a''_{32})^{(6,6,6,6,6,6)}(T_{33}, t) & \\ \hline \end{array} \right] G_{20} \quad 74$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)} G_{20} - \left[\begin{array}{|c|c|c|} \hline (a'_{21})^{(3)} & + (a''_{21})^{(3)}(T_{21}, t) & + (a''_{17})^{(2,2,2)}(T_{17}, t) & + (a''_{14})^{(1,1,1)}(T_{14}, t) \\ \hline + (a''_{25})^{(4,4,4,4,4,4)}(T_{25}, t) & + (a''_{29})^{(5,5,5,5,5,5)}(T_{29}, t) & + (a''_{33})^{(6,6,6,6,6,6)}(T_{33}, t) & \\ \hline \end{array} \right] G_{21} \quad 75$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)} G_{21} - \left[\begin{array}{|c|c|c|} \hline (a'_{22})^{(3)} & + (a''_{22})^{(3)}(T_{21}, t) & + (a''_{18})^{(2,2,2)}(T_{17}, t) & + (a''_{15})^{(1,1,1)}(T_{14}, t) \\ \hline + (a''_{26})^{(4,4,4,4,4,4)}(T_{25}, t) & + (a''_{30})^{(5,5,5,5,5,5)}(T_{29}, t) & + (a''_{34})^{(6,6,6,6,6,6)}(T_{33}, t) & \\ \hline \end{array} \right] G_{22} \quad 76$$

$+ (a''_{20})^{(3)}(T_{21}, t)$, $+ (a''_{21})^{(3)}(T_{21}, t)$, $+ (a''_{22})^{(3)}(T_{21}, t)$ are first augmentation coefficients for category 1, 2 and 3 77

$+ (a''_{16})^{(2,2,2)}(T_{17}, t)$, $+ (a''_{17})^{(2,2,2)}(T_{17}, t)$, $+ (a''_{18})^{(2,2,2)}(T_{17}, t)$ are second augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1)}(T_{14}, t)}$, $\boxed{+(a''_{14})^{(1,1,1)}(T_{14}, t)}$, $\boxed{+(a''_{15})^{(1,1,1)}(T_{14}, t)}$ are third augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25}, t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25}, t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25}, t)}$ are fourth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29}, t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29}, t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29}, t)}$ are fifth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33}, t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33}, t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33}, t)}$ are sixth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[\begin{array}{ccc} \boxed{(b'_{20})^{(3)}} \boxed{-(b''_{20})^{(3)}(G_{23}, t)} \boxed{-(b'_{16})^{(2,2,2)}(G_{19}, t)} \boxed{-(b'_{13})^{(1,1,1)}(G, t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27}, t)} \boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31}, t)} \boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35}, t)} \end{array} \right] T_{20} \quad 78$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[\begin{array}{ccc} \boxed{(b'_{21})^{(3)}} \boxed{-(b''_{21})^{(3)}(G_{23}, t)} \boxed{-(b'_{17})^{(2,2,2)}(G_{19}, t)} \boxed{-(b'_{14})^{(1,1,1)}(G, t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27}, t)} \boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31}, t)} \boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35}, t)} \end{array} \right] T_{21} \quad 79$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[\begin{array}{ccc} \boxed{(b'_{22})^{(3)}} \boxed{-(b''_{22})^{(3)}(G_{23}, t)} \boxed{-(b'_{18})^{(2,2,2)}(G_{19}, t)} \boxed{-(b'_{15})^{(1,1,1)}(G, t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27}, t)} \boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31}, t)} \boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35}, t)} \end{array} \right] T_{22} \quad 80$$

$\boxed{-(b''_{20})^{(3)}(G_{23}, t)}$, $\boxed{-(b''_{21})^{(3)}(G_{23}, t)}$, $\boxed{-(b''_{22})^{(3)}(G_{23}, t)}$ are first detrition coefficients for category 1, 2 and 3 81

$\boxed{-(b'_{16})^{(2,2,2)}(G_{19}, t)}$, $\boxed{-(b'_{17})^{(2,2,2)}(G_{19}, t)}$, $\boxed{-(b'_{18})^{(2,2,2)}(G_{19}, t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b'_{13})^{(1,1,1)}(G, t)}$, $\boxed{-(b'_{14})^{(1,1,1)}(G, t)}$, $\boxed{-(b'_{15})^{(1,1,1)}(G, t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b'_{24})^{(4,4,4,4,4)}(G_{27}, t)}$, $\boxed{-(b'_{25})^{(4,4,4,4,4)}(G_{27}, t)}$, $\boxed{-(b'_{26})^{(4,4,4,4,4)}(G_{27}, t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31}, t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31}, t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31}, t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35}, t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35}, t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35}, t)}$ are sixth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[\begin{array}{ccc} \boxed{(a'_{24})^{(4)}} \boxed{+(a''_{24})^{(4)}(T_{25}, t)} \boxed{+(a''_{28})^{(5,5)}(T_{29}, t)} \boxed{+(a''_{32})^{(6,6)}(T_{33}, t)} \\ \boxed{+(a''_{13})^{(1,1,1,1)}(T_{14}, t)} \boxed{+(a''_{16})^{(2,2,2,2)}(T_{17}, t)} \boxed{+(a''_{20})^{(3,3,3,3)}(T_{21}, t)} \end{array} \right] G_{24} \quad 82$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[\begin{array}{ccc} \boxed{(a'_{25})^{(4)}} \boxed{+(a''_{25})^{(4)}(T_{25}, t)} \boxed{+(a''_{29})^{(5,5)}(T_{29}, t)} \boxed{+(a''_{33})^{(6,6)}(T_{33}, t)} \\ \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14}, t)} \boxed{+(a''_{17})^{(2,2,2,2)}(T_{17}, t)} \boxed{+(a''_{21})^{(3,3,3,3)}(T_{21}, t)} \end{array} \right] G_{25} \quad 83$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[\begin{array}{ccc} \boxed{(a'_{26})^{(4)}} \boxed{+(a''_{26})^{(4)}(T_{25}, t)} \boxed{+(a''_{30})^{(5,5)}(T_{29}, t)} \boxed{+(a''_{34})^{(6,6)}(T_{33}, t)} \\ \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14}, t)} \boxed{+(a''_{18})^{(2,2,2,2)}(T_{17}, t)} \boxed{+(a''_{22})^{(3,3,3,3)}(T_{21}, t)} \end{array} \right] G_{26} \quad 84$$

Where $\boxed{(a'_{24})^{(4)}(T_{25}, t)}$, $\boxed{(a'_{25})^{(4)}(T_{25}, t)}$, $\boxed{(a'_{26})^{(4)}(T_{25}, t)}$ are first augmentation coefficients for category 1, 2 and 3 85

$\boxed{+(a''_{28})^{(5,5)}(T_{29}, t)}$, $\boxed{+(a''_{29})^{(5,5)}(T_{29}, t)}$, $\boxed{+(a''_{30})^{(5,5)}(T_{29}, t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6)}(T_{33}, t)}$, $\boxed{+(a''_{33})^{(6,6)}(T_{33}, t)}$, $\boxed{+(a''_{34})^{(6,6)}(T_{33}, t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1,1)}(T_{14}, t)}$, $\boxed{+(a''_{14})^{(1,1,1,1)}(T_{14}, t)}$, $\boxed{+(a''_{15})^{(1,1,1,1)}(T_{14}, t)}$ are fourth augmentation coefficients for category 1, 2, and 3

$\boxed{+(a''_{16})^{(2,2,2,2)}(T_{17}, t)}$, $\boxed{+(a''_{17})^{(2,2,2,2)}(T_{17}, t)}$, $\boxed{+(a''_{18})^{(2,2,2,2)}(T_{17}, t)}$ are fifth augmentation coefficients for category 1, 2, and 3

$\boxed{+(a''_{20})^{(3,3,3,3)}(T_{21}, t)}$, $\boxed{+(a''_{21})^{(3,3,3,3)}(T_{21}, t)}$, $\boxed{+(a''_{22})^{(3,3,3,3)}(T_{21}, t)}$ are sixth augmentation coefficients for category 1, 2, and 3

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[\begin{array}{ccc} (b'_{24})^{(4)} - (b''_{24})^{(4)}(G_{27}, t) & - (b''_{28})^{(5,5)}(G_{31}, t) & - (b''_{32})^{(6,6)}(G_{35}, t) \\ - (b''_{13})^{(1,1,1,1)}(G, t) & - (b''_{16})^{(2,2,2,2)}(G_{19}, t) & - (b''_{20})^{(3,3,3,3)}(G_{23}, t) \end{array} \right] T_{24} \quad 86$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[\begin{array}{ccc} (b'_{25})^{(4)} - (b''_{25})^{(4)}(G_{27}, t) & - (b''_{29})^{(5,5)}(G_{31}, t) & - (b''_{33})^{(6,6)}(G_{35}, t) \\ - (b''_{14})^{(1,1,1,1)}(G, t) & - (b''_{17})^{(2,2,2,2)}(G_{19}, t) & - (b''_{21})^{(3,3,3,3)}(G_{23}, t) \end{array} \right] T_{25} \quad 87$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[\begin{array}{ccc} (b'_{26})^{(4)} - (b''_{26})^{(4)}(G_{27}, t) & - (b''_{30})^{(5,5)}(G_{31}, t) & - (b''_{34})^{(6,6)}(G_{35}, t) \\ - (b''_{15})^{(1,1,1,1)}(G, t) & - (b''_{18})^{(2,2,2,2)}(G_{19}, t) & - (b''_{22})^{(3,3,3,3)}(G_{23}, t) \end{array} \right] T_{26} \quad 88$$

Where $-(b''_{24})^{(4)}(G_{27}, t)$, $-(b''_{25})^{(4)}(G_{27}, t)$, $-(b''_{26})^{(4)}(G_{27}, t)$ are first detrition coefficients for category 1, 2 and 3
 $-(b''_{28})^{(5,5)}(G_{31}, t)$, $-(b''_{29})^{(5,5)}(G_{31}, t)$, $-(b''_{30})^{(5,5)}(G_{31}, t)$ are second detrition coefficients for category 1, 2 and 3
 $-(b''_{32})^{(6,6)}(G_{35}, t)$, $-(b''_{33})^{(6,6)}(G_{35}, t)$, $-(b''_{34})^{(6,6)}(G_{35}, t)$ are third detrition coefficients for category 1, 2 and 3
 $-(b''_{13})^{(1,1,1,1)}(G, t)$, $-(b''_{14})^{(1,1,1,1)}(G, t)$, $-(b''_{15})^{(1,1,1,1)}(G, t)$ are fourth detrition coefficients for category 1, 2 and 3
 $-(b''_{16})^{(2,2,2,2)}(G_{19}, t)$, $-(b''_{17})^{(2,2,2,2)}(G_{19}, t)$, $-(b''_{18})^{(2,2,2,2)}(G_{19}, t)$ are fifth detrition coefficients for category 1, 2, 3
 $-(b''_{20})^{(3,3,3,3)}(G_{23}, t)$, $-(b''_{21})^{(3,3,3,3)}(G_{23}, t)$, $-(b''_{22})^{(3,3,3,3)}(G_{23}, t)$ are sixth detrition coefficients for category 1, 2, 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[\begin{array}{ccc} (a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}, t) & + (a''_{24})^{(4,4)}(T_{25}, t) & + (a''_{32})^{(6,6,6)}(T_{33}, t) \\ + (a''_{13})^{(1,1,1,1,1)}(T_{14}, t) & + (a''_{16})^{(2,2,2,2,2)}(T_{17}, t) & + (a''_{20})^{(3,3,3,3,3)}(T_{21}, t) \end{array} \right] G_{28} \quad 90$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[\begin{array}{ccc} (a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}, t) & + (a''_{25})^{(4,4)}(T_{25}, t) & + (a''_{33})^{(6,6,6)}(T_{33}, t) \\ + (a''_{14})^{(1,1,1,1,1)}(T_{14}, t) & + (a''_{17})^{(2,2,2,2,2)}(T_{17}, t) & + (a''_{21})^{(3,3,3,3,3)}(T_{21}, t) \end{array} \right] G_{29} \quad 91$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[\begin{array}{ccc} (a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}, t) & + (a''_{26})^{(4,4)}(T_{25}, t) & + (a''_{34})^{(6,6,6)}(T_{33}, t) \\ + (a''_{15})^{(1,1,1,1,1)}(T_{14}, t) & + (a''_{18})^{(2,2,2,2,2)}(T_{17}, t) & + (a''_{22})^{(3,3,3,3,3)}(T_{21}, t) \end{array} \right] G_{30} \quad 92$$

Where $+(a''_{28})^{(5)}(T_{29}, t)$, $+(a''_{29})^{(5)}(T_{29}, t)$, $+(a''_{30})^{(5)}(T_{29}, t)$ are first augmentation coefficients for category 1, 2 and 3

And $+(a''_{24})^{(4,4)}(T_{25}, t)$, $+(a''_{25})^{(4,4)}(T_{25}, t)$, $+(a''_{26})^{(4,4)}(T_{25}, t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6)}(T_{33}, t)$, $+(a''_{33})^{(6,6,6)}(T_{33}, t)$, $+(a''_{34})^{(6,6,6)}(T_{33}, t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,1,1)}(T_{14}, t)$, $+(a''_{14})^{(1,1,1,1,1)}(T_{14}, t)$, $+(a''_{15})^{(1,1,1,1,1)}(T_{14}, t)$ are fourth augmentation coefficients for category 1, 2, and 3

$+(a''_{16})^{(2,2,2,2,2)}(T_{17}, t)$, $+(a''_{17})^{(2,2,2,2,2)}(T_{17}, t)$, $+(a''_{18})^{(2,2,2,2,2)}(T_{17}, t)$ are fifth augmentation coefficients for category 1, 2, and 3

$+(a''_{20})^{(3,3,3,3,3)}(T_{21}, t)$, $+(a''_{21})^{(3,3,3,3,3)}(T_{21}, t)$, $+(a''_{22})^{(3,3,3,3,3)}(T_{21}, t)$ are sixth augmentation coefficients for category 1, 2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[\begin{array}{ccc} (b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31}, t) & - (b''_{24})^{(4,4)}(G_{27}, t) & - (b''_{32})^{(6,6,6)}(G_{35}, t) \\ - (b''_{13})^{(1,1,1,1,1)}(G, t) & - (b''_{16})^{(2,2,2,2,2)}(G_{19}, t) & - (b''_{20})^{(3,3,3,3,3)}(G_{23}, t) \end{array} \right] T_{28} \quad 94$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[\begin{array}{ccc} (b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31}, t) & - (b''_{25})^{(4,4)}(G_{27}, t) & - (b''_{33})^{(6,6,6)}(G_{35}, t) \\ - (b''_{14})^{(1,1,1,1,1)}(G, t) & - (b''_{17})^{(2,2,2,2,2)}(G_{19}, t) & - (b''_{21})^{(3,3,3,3,3)}(G_{23}, t) \end{array} \right] T_{29} \quad 95$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[\begin{array}{|c|c|c|} \hline (b'_{30})^{(5)} & -(b''_{30})^{(5)}(G_{31}, t) & -(b''_{26})^{(4,4)}(G_{27}, t) & -(b''_{34})^{(6,6,6)}(G_{35}, t) \\ \hline \hline -(b''_{15})^{(1,1,1,1,1)}(G, t) & -(b''_{18})^{(2,2,2,2,2)}(G_{19}, t) & -(b''_{22})^{(3,3,3,3,3)}(G_{23}, t) \\ \hline \end{array} \right] T_{30} \quad 96$$

where $-(b''_{28})^{(5)}(G_{31}, t)$, $-(b''_{29})^{(5)}(G_{31}, t)$, $-(b''_{30})^{(5)}(G_{31}, t)$ are first detrition coefficients
 for category 1, 2 and 3

$-(b''_{24})^{(4,4)}(G_{27}, t)$, $-(b''_{25})^{(4,4)}(G_{27}, t)$, $-(b''_{26})^{(4,4)}(G_{27}, t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6)}(G_{35}, t)$, $-(b''_{33})^{(6,6,6)}(G_{35}, t)$, $-(b''_{34})^{(6,6,6)}(G_{35}, t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{13})^{(1,1,1,1,1)}(G, t)$, $-(b''_{14})^{(1,1,1,1,1)}(G, t)$, $-(b''_{15})^{(1,1,1,1,1)}(G, t)$ are fourth detrition coefficients for category 1, 2, and 3

$-(b''_{16})^{(2,2,2,2,2)}(G_{19}, t)$, $-(b''_{17})^{(2,2,2,2,2)}(G_{19}, t)$, $-(b''_{18})^{(2,2,2,2,2)}(G_{19}, t)$ are fifth detrition coefficients for category 1, 2, and 3

$-(b''_{20})^{(3,3,3,3,3)}(G_{23}, t)$, $-(b''_{21})^{(3,3,3,3,3)}(G_{23}, t)$, $-(b''_{22})^{(3,3,3,3,3)}(G_{23}, t)$ are sixth detrition coefficients for category 1, 2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[\begin{array}{|c|c|c|c|} \hline (a'_{32})^{(6)} & +(a''_{32})^{(6)}(T_{33}, t) & +(a''_{28})^{(5,5,5)}(T_{29}, t) & +(a''_{24})^{(4,4,4)}(T_{25}, t) \\ \hline \hline \hline \hline +(a''_{13})^{(1,1,1,1,1)}(T_{14}, t) & +(a''_{16})^{(2,2,2,2,2)}(T_{17}, t) & +(a''_{20})^{(3,3,3,3,3)}(T_{21}, t) \\ \hline \end{array} \right] G_{32} \quad 98$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[\begin{array}{|c|c|c|c|} \hline (a'_{33})^{(6)} & +(a''_{33})^{(6)}(T_{33}, t) & +(a''_{29})^{(5,5,5)}(T_{29}, t) & +(a''_{25})^{(4,4,4)}(T_{25}, t) \\ \hline \hline \hline \hline +(a''_{14})^{(1,1,1,1,1)}(T_{14}, t) & +(a''_{17})^{(2,2,2,2,2)}(T_{17}, t) & +(a''_{21})^{(3,3,3,3,3)}(T_{21}, t) \\ \hline \end{array} \right] G_{33} \quad 99$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[\begin{array}{|c|c|c|c|} \hline (a'_{34})^{(6)} & +(a''_{34})^{(6)}(T_{33}, t) & +(a''_{30})^{(5,5,5)}(T_{29}, t) & +(a''_{26})^{(4,4,4)}(T_{25}, t) \\ \hline \hline \hline \hline +(a''_{15})^{(1,1,1,1,1)}(T_{14}, t) & +(a''_{18})^{(2,2,2,2,2)}(T_{17}, t) & +(a''_{22})^{(3,3,3,3,3)}(T_{21}, t) \\ \hline \end{array} \right] G_{34} \quad 100$$

$+(a''_{32})^{(6)}(T_{33}, t)$, $+(a''_{33})^{(6)}(T_{33}, t)$, $+(a''_{34})^{(6)}(T_{33}, t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{28})^{(5,5,5)}(T_{29}, t)$, $+(a''_{29})^{(5,5,5)}(T_{29}, t)$, $+(a''_{30})^{(5,5,5)}(T_{29}, t)$ are second augmentation coefficients for category 1, 2, 3

$+(a''_{24})^{(4,4,4)}(T_{25}, t)$, $+(a''_{25})^{(4,4,4)}(T_{25}, t)$, $+(a''_{26})^{(4,4,4)}(T_{25}, t)$ are third augmentation coefficients for category 1, 2, 3

$+(a''_{13})^{(1,1,1,1,1)}(T_{14}, t)$, $+(a''_{14})^{(1,1,1,1,1)}(T_{14}, t)$, $+(a''_{15})^{(1,1,1,1,1)}(T_{14}, t)$ - are fourth augmentation coefficients

$+(a''_{16})^{(2,2,2,2,2)}(T_{17}, t)$, $+(a''_{17})^{(2,2,2,2,2)}(T_{17}, t)$, $+(a''_{18})^{(2,2,2,2,2)}(T_{17}, t)$ - fifth augmentation coefficients

$+(a''_{20})^{(3,3,3,3,3)}(T_{21}, t)$, $+(a''_{21})^{(3,3,3,3,3)}(T_{21}, t)$, $+(a''_{22})^{(3,3,3,3,3)}(T_{21}, t)$ sixth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[\begin{array}{|c|c|c|} \hline (b'_{32})^{(6)} & -(b''_{32})^{(6)}(G_{35}, t) & -(b''_{28})^{(5,5,5)}(G_{31}, t) & -(b''_{24})^{(4,4,4)}(G_{27}, t) \\ \hline \hline \hline \hline -(b''_{13})^{(1,1,1,1,1)}(G, t) & -(b''_{16})^{(2,2,2,2,2)}(G_{19}, t) & -(b''_{20})^{(3,3,3,3,3)}(G_{23}, t) \\ \hline \end{array} \right] T_{32} \quad 102$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[\begin{array}{|c|c|c|} \hline (b'_{33})^{(6)} & -(b''_{33})^{(6)}(G_{35}, t) & -(b''_{29})^{(5,5,5)}(G_{31}, t) & -(b''_{25})^{(4,4,4)}(G_{27}, t) \\ \hline \hline \hline \hline -(b''_{14})^{(1,1,1,1,1)}(G, t) & -(b''_{17})^{(2,2,2,2,2)}(G_{19}, t) & -(b''_{21})^{(3,3,3,3,3)}(G_{23}, t) \\ \hline \end{array} \right] T_{33} \quad 103$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[\begin{array}{|c|c|c|} \hline (b'_{34})^{(6)} & -(b''_{34})^{(6)}(G_{35}, t) & -(b''_{30})^{(5,5,5)}(G_{31}, t) & -(b''_{26})^{(4,4,4)}(G_{27}, t) \\ \hline \hline \hline \hline -(b''_{15})^{(1,1,1,1,1)}(G, t) & -(b''_{18})^{(2,2,2,2,2)}(G_{19}, t) & -(b''_{22})^{(3,3,3,3,3)}(G_{23}, t) \\ \hline \end{array} \right] T_{34} \quad 104$$

$-(b''_{32})^{(6)}(G_{35}, t)$, $-(b''_{33})^{(6)}(G_{35}, t)$, $-(b''_{34})^{(6)}(G_{35}, t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5)}(G_{31}, t)$, $-(b''_{29})^{(5,5,5)}(G_{31}, t)$, $-(b''_{30})^{(5,5,5)}(G_{31}, t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{24})^{(4,4,4)}(G_{27}, t)$, $-(b''_{25})^{(4,4,4)}(G_{27}, t)$, $-(b''_{26})^{(4,4,4)}(G_{27}, t)$ are third detrition coefficients for category 1, 2 and 3

$-(b''_{13})^{(1,1,1,1,1)}(G, t)$, $-(b''_{14})^{(1,1,1,1,1)}(G, t)$, $-(b''_{15})^{(1,1,1,1,1)}(G, t)$ are fourth detrition coefficients for category 1, 2, and 3

$-(b''_{16})^{(2,2,2,2,2)}(G_{19}, t)$, $-(b''_{17})^{(2,2,2,2,2)}(G_{19}, t)$, $-(b''_{18})^{(2,2,2,2,2)}(G_{19}, t)$ are fifth detrition coefficients for category 1, 2,

and 3

$-(b''_{20})^{(3,3,3,3,3,3)}(G_{23}, t)$, $-(b''_{21})^{(3,3,3,3,3,3)}(G_{23}, t)$, $-(b''_{22})^{(3,3,3,3,3,3)}(G_{23}, t)$ are sixth detrition coefficients for category 1, 2,

and 3

Where we suppose

(A) $(a_i)^{(1)}, (a'_i)^{(1)}, (a''_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (b''_i)^{(1)} > 0$, 106
 $i, j = 13, 14, 15$

(B) The functions $(a''_i)^{(1)}, (b''_i)^{(1)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(1)}, (r_i)^{(1)}$:

$$(a''_i)^{(1)}(T_{14}, t) \leq (p_i)^{(1)} \leq (\hat{A}_{13})^{(1)}$$

$$(b''_i)^{(1)}(G, t) \leq (r_i)^{(1)} \leq (b'_i)^{(1)} \leq (\hat{B}_{13})^{(1)}$$

(C) $\lim_{T_2 \rightarrow \infty} (a''_i)^{(1)}(T_{14}, t) = (p_i)^{(1)}$ 107
 $\lim_{G \rightarrow \infty} (b''_i)^{(1)}(G, t) = (r_i)^{(1)}$

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}$ are positive constants and $i = 13, 14, 15$

They satisfy Lipschitz condition: 108

$$|(a''_i)^{(1)}(T'_{14}, t) - (a''_i)^{(1)}(T_{14}, t)| \leq (\hat{k}_{13})^{(1)} |T'_{14} - T_{14}| e^{-(\hat{M}_{13})^{(1)}t}$$

$$|(b''_i)^{(1)}(G', t) - (b''_i)^{(1)}(G, t)| < (\hat{k}_{13})^{(1)} \|G - G'\| e^{-(\hat{M}_{13})^{(1)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions 109

$(a''_i)^{(1)}(T'_{14}, t)$ and $(a''_i)^{(1)}(T_{14}, t)$. (T'_{14}, t) and (T_{14}, t) are points belonging to the interval $[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}]$. It is to be noted that $(a''_i)^{(1)}(T_{14}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a''_i)^{(1)}(T_{14}, t)$, the first augmentation coefficient would be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$: 110

(D) $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$: 111

(E) There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ which together with $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a'_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}, i = 13, 14, 15$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}} [(a_i)^{(1)} + (a'_i)^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}} [(b_i)^{(1)} + (b'_i)^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

(F) $(a_i)^{(2)}, (a'_i)^{(2)}, (a''_i)^{(2)}, (b_i)^{(2)}, (b'_i)^{(2)}, (b''_i)^{(2)} > 0$, $i, j = 16, 17, 18$

(G) The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(2)}, (r_i)^{(2)}$: 112

$$(a_i'')^{(2)}(T_{17}, t) \leq (p_i)^{(2)} \leq (\hat{A}_{16})^{(2)} \quad 113$$

$$(b_i'')^{(2)}(G_{19}, t) \leq (r_i)^{(2)} \leq (b_i'')^{(2)} \leq (\hat{B}_{16})^{(2)} \quad 114$$

(H) $\lim_{T_2 \rightarrow \infty} (a_i'')^{(2)}(T_{17}, t) = (p_i)^{(2)}$ 115

$$\lim_{G \rightarrow \infty} (b_i'')^{(2)}(G_{19}, t) = (r_i)^{(2)} \quad 116$$

Definition of $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$: 117

Where $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}$ are positive constants and $i = 16, 17, 18$

They satisfy Lipschitz condition:

$$|(a_i'')^{(2)}(T'_{17}, t) - (a_i'')^{(2)}(T_{17}, t)| \leq (\hat{k}_{16})^{(2)} |T'_{17} - T_{17}| e^{-(\hat{M}_{16})^{(2)}t} \quad 118$$

$$|(b_i'')^{(2)}((G'_{19}), t) - (b_i'')^{(2)}((G_{19}), t)| < (\hat{k}_{16})^{(2)} |(G'_{19}) - (G_{19})'| e^{-(\hat{M}_{16})^{(2)}t} \quad 119$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T'_{17}, t)$ 120

and $(a_i'')^{(2)}(T_{17}, t)$. (T'_{17}, t) and (T_{17}, t) are points belonging to the interval $[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}]$. It is

to be noted that $(a_i'')^{(2)}(T_{17}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17}, t)$, the SECOND first augmentation coefficient would be absolutely continuous.

Definition of $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$:

(I) $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 121

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}} + \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

Definition of $(\hat{P}_{16})^{(2)}, (\hat{Q}_{16})^{(2)}$: 122

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)}$ and $(\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}, i = 16, 17, 18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}} [(a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad 123$$

$$\frac{1}{(\hat{M}_{16})^{(2)}} [(b_i)^{(2)} + (b_i')^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad 124$$

Where we suppose

(J) $(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i, j = 20, 21, 22$ 125

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(3)}, (r_i)^{(3)}$:

$$(a_i'')^{(3)}(T_{21}, t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23}, t) \leq (r_i)^{(3)} \leq (b_i'')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$$\lim_{T_2 \rightarrow \infty} (a_i'')^{(3)}(T_{21}, t) = (p_i)^{(3)} \quad 126$$

$$\lim_{G \rightarrow \infty} (b_i'')^{(3)}(G_{23}, t) = (r_i)^{(3)}$$

Definition of $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$:

Where $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}$ are positive constants and $i = 20, 21, 22$

They satisfy Lipschitz condition:

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$$|(a_i'')^{(3)}(T_{21}', t) - (a_i'')^{(3)}(T_{21}, t)| \leq (\hat{k}_{20})^{(3)} |T_{21}' - T_{21}| e^{-(\hat{M}_{20})^{(3)}t}$$

$$|(b_i'')^{(3)}(G_{23}', t) - (b_i'')^{(3)}(G_{23}, t)| < (\hat{k}_{20})^{(3)} ||G_{23}' - G_{23}|| e^{-(\hat{M}_{20})^{(3)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}', t)$ and $(a_i'')^{(3)}(T_{21}, t)$. (T_{21}', t) and (T_{21}, t) are points belonging to the interval $[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}]$. It is to be noted that $(a_i'')^{(3)}(T_{21}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21}, t)$, the THIRD augmentation coefficient attributable would be absolutely continuous.

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Definition of $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$:

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(K) $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$$

There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)}$ and $(\hat{B}_{20})^{(3)}$ and the constants $(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20, 21, 22$, satisfy the inequalities

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$$\frac{1}{(\hat{M}_{20})^{(3)}} [(a_i)^{(3)} + (a_i')^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)} (\hat{k}_{20})^{(3)}] < 1$$

$$\frac{1}{(\hat{M}_{20})^{(3)}} [(b_i)^{(3)} + (b_i')^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)} (\hat{k}_{20})^{(3)}] < 1$$

Where we suppose

(L) $(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, i, j = 24, 25, 26$

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(M) The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(4)}, (r_i)^{(4)}$:

$$(a_i'')^{(4)}(T_{25}, t) \leq (p_i)^{(4)} \leq (\hat{A}_{24})^{(4)}$$

$$(b_i'')^{(4)}((G_{27}), t) \leq (r_i)^{(4)} \leq (b_i')^{(4)} \leq (\hat{B}_{24})^{(4)}$$

$$\lim_{T_2 \rightarrow \infty} (a_i'')^{(4)}(T_{25}, t) = (p_i)^{(4)}$$

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$$\lim_{G \rightarrow \infty} (b_i'')^{(4)}((G_{27}), t) = (r_i)^{(4)}$$

Definition of $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$:

Where $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}$ are positive constants and $i = 24, 25, 26$

They satisfy Lipschitz condition:

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$$|(a_i'')^{(4)}(T_{25}', t) - (a_i'')^{(4)}(T_{25}, t)| \leq (\hat{k}_{24})^{(4)} |T_{25}' - T_{25}| e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b_i'')^{(4)}((G_{27})', t) - (b_i'')^{(4)}((G_{27}), t)| < (\hat{k}_{24})^{(4)} ||(G_{27})' - (G_{27})|| e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}', t)$ and $(a_i'')^{(4)}(T_{25}, t)$. (T_{25}', t) and (T_{25}, t) are points belonging to the interval $[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}]$. It is to be noted that $(a_i'')^{(4)}(T_{25}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 1$

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then the function $(a_i'')^{(4)}(T_{25}, t)$, the **FOURTH augmentation coefficient** would be absolutely continuous.

Definition of $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$: 135

(N) $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}, \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$$

Definition of $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$: 136

(O) There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with

$(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)}$ and $(\hat{B}_{24})^{(4)}$ and the constants

$(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}, i = 24, 25, 26,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}} [(a_i)^{(4)} + (a_i')^{(4)} + (\hat{A}_{24})^{(4)} + (\hat{P}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}} [(b_i)^{(4)} + (b_i')^{(4)} + (\hat{B}_{24})^{(4)} + (\hat{Q}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

(P) $(a_i)^{(5)}, (a_i')^{(5)}, (a_i'')^{(5)}, (b_i)^{(5)}, (b_i')^{(5)}, (b_i'')^{(5)} > 0, i, j = 28, 29, 30$ 137

(Q) The functions $(a_i'')^{(5)}, (b_i'')^{(5)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(5)}, (r_i)^{(5)}$:

$$(a_i'')^{(5)}(T_{29}, t) \leq (p_i)^{(5)} \leq (\hat{A}_{28})^{(5)}$$

$$(b_i'')^{(5)}(G_{31}, t) \leq (r_i)^{(5)} \leq (b_i')^{(5)} \leq (\hat{B}_{28})^{(5)}$$

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(R) $\lim_{T_2 \rightarrow \infty} (a_i'')^{(5)}(T_{29}, t) = (p_i)^{(5)}$

$$\lim_{G \rightarrow \infty} (b_i'')^{(5)}(G_{31}, t) = (r_i)^{(5)}$$

Definition of $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$:

Where $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}$ are positive constants and $i = 28, 29, 30$

They satisfy Lipschitz condition:

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$$|(a_i'')^{(5)}(T_{29}', t) - (a_i'')^{(5)}(T_{29}, t)| \leq (\hat{k}_{28})^{(5)} |T_{29}' - T_{29}| e^{-(\hat{M}_{28})^{(5)} t}$$

$$|(b_i'')^{(5)}(G_{31}', t) - (b_i'')^{(5)}(G_{31}, t)| < (\hat{k}_{28})^{(5)} |(G_{31}') - (G_{31})| e^{-(\hat{M}_{28})^{(5)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(5)}(T_{29}', t)$ 140

and $(a_i'')^{(5)}(T_{29}, t)$. (T_{29}', t) and (T_{29}, t) are points belonging to the interval $[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}]$. It is

to be noted that $(a_i'')^{(5)}(T_{29}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} =$

1 then the function $(a_i'')^{(5)}(T_{29}, t)$, the **FIFTH augmentation coefficient** would be absolutely

continuous.

Definition of $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$: 141

(S) $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}, \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

Definition of $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$: 142

(T) There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)}$ and $(\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a'_i)^{(5)}, (b_i)^{(5)}, (b'_i)^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}, i = 28, 29, 30$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}} [(a_i)^{(5)} + (a'_i)^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}} [(b_i)^{(5)} + (b'_i)^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

$$(a_i)^{(6)}, (a'_i)^{(6)}, (a''_i)^{(6)}, (b_i)^{(6)}, (b'_i)^{(6)}, (b''_i)^{(6)} > 0, \quad i, j = 32, 33, 34 \quad 143$$

(U) The functions $(a''_i)^{(6)}, (b''_i)^{(6)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(6)}, (r_i)^{(6)}$:

$$(a''_i)^{(6)}(T_{33}, t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$$

$$(b''_i)^{(6)}((G_{35}), t) \leq (r_i)^{(6)} \leq (b_i)^{(6)} \leq (\hat{B}_{32})^{(6)}$$

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$$(V) \quad \lim_{T_2 \rightarrow \infty} (a''_i)^{(6)}(T_{33}, t) = (p_i)^{(6)}$$

$$\lim_{G \rightarrow \infty} (b''_i)^{(6)}((G_{35}), t) = (r_i)^{(6)}$$

Definition of $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$:

Where $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}$ are positive constants and $i = 32, 33, 34$

They satisfy Lipschitz condition:

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$$|(a''_i)^{(6)}(T'_{33}, t) - (a''_i)^{(6)}(T_{33}, t)| \leq (\hat{k}_{32})^{(6)} |T'_{33} - T_{33}| e^{-(\hat{M}_{32})^{(6)}t}$$

$$|(b''_i)^{(6)}((G_{35})', t) - (b''_i)^{(6)}((G_{35}), t)| < (\hat{k}_{32})^{(6)} |(G_{35})' - (G_{35})| e^{-(\hat{M}_{32})^{(6)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a''_i)^{(6)}(T'_{33}, t)$ and $(a''_i)^{(6)}(T_{33}, t)$. (T'_{33}, t) and (T_{33}, t) are points belonging to the interval $[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}]$. It is

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to be noted that $(a''_i)^{(6)}(T_{33}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a''_i)^{(6)}(T_{33}, t)$, the **SIXTH augmentation coefficient** would be absolutely continuous.

Definition of $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$:

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$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$, are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

Definition of $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$:

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There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)}$ and $(\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a'_i)^{(6)}, (b_i)^{(6)}, (b'_i)^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32, 33, 34$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}} [(a_i)^{(6)} + (a'_i)^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}} [(b_i)^{(6)} + (b'_i)^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

Theorem 1: if the conditions (A)-(E) above are fulfilled, there exists a solution satisfying the conditions 149

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t} , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t} , \quad \boxed{T_i(0) = T_i^0 > 0}$$

If the conditions (F)-(J) above are fulfilled, there exists a solution satisfying the conditions 150

Definition of $G_i(0), T_i(0)$

$$G_i(t) \leq (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \leq (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} , \quad T_i(0) = T_i^0 > 0$$

If the conditions (K)-(O) above are fulfilled, there exists a solution satisfying the conditions 151

$$G_i(t) \leq (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t} , \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \leq (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t} , \quad T_i(0) = T_i^0 > 0$$

If the conditions (P)-(T) above are fulfilled, there exists a solution satisfying the conditions 152

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)}t} , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)}t} , \quad \boxed{T_i(0) = T_i^0 > 0}$$

If the conditions (U)-(Y) above are fulfilled, there exists a solution satisfying the conditions 153

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)}t} , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)}t} , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Theorem 1: if the conditions (Y)-(X₄) above are fulfilled, there exists a solution satisfying the conditions 154

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)}t} , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)}t} , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Proof: Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions $G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0, T_i(0) = T_i^0, G_i^0 \leq (\hat{P}_{13})^{(1)}, T_i^0 \leq (\hat{Q}_{13})^{(1)}, \quad 156$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 157$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 158$$

By 159

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}(s_{(13)}) - \left((a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}(s_{(13)}), s_{(13)}) \right) G_{13}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}(s_{(13)}), s_{(13)}) \right) G_{14}(s_{(13)}) \right] ds_{(13)} \quad 160$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}(s_{(13)}), s_{(13)}) \right) G_{15}(s_{(13)}) \right] ds_{(13)} \quad 161$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)}(G(s_{(13)}), s_{(13)}) \right) T_{13}(s_{(13)}) \right] ds_{(13)} \quad 162$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)} (G(s_{(13)}), s_{(13)}) \right) T_{14}(s_{(13)}) \right] ds_{(13)} \quad 163$$

$$\bar{T}_{15}(t) = T_{15}^0 + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)} (G(s_{(13)}), s_{(13)}) \right) T_{15}(s_{(13)}) \right] ds_{(13)} \quad 164$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions $G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0, T_i(0) = T_i^0, G_i^0 \leq (\hat{P}_{16})^{(2)}, T_i^0 \leq (\hat{Q}_{16})^{(2)}, \quad 165$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \quad 166$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t} \quad 167$$

By 168

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16}^{(2)} (T_{17}(s_{(16)}), s_{(16)}) \right) G_{16}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)} (T_{17}(s_{(16)}), s_{(17)}) \right) G_{17}(s_{(16)}) \right] ds_{(16)} \quad 169$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)} (T_{17}(s_{(16)}), s_{(16)}) \right) G_{18}(s_{(16)}) \right] ds_{(16)} \quad 170$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)} (G(s_{(16)}), s_{(16)}) \right) T_{16}(s_{(16)}) \right] ds_{(16)} \quad 171$$

$$\bar{T}_{17}(t) = T_{17}^0 + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)} (G(s_{(16)}), s_{(16)}) \right) T_{17}(s_{(16)}) \right] ds_{(16)} \quad 172$$

$$\bar{T}_{18}(t) = T_{18}^0 + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)} (G(s_{(16)}), s_{(16)}) \right) T_{18}(s_{(16)}) \right] ds_{(16)} \quad 173$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions $G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0, T_i(0) = T_i^0, G_i^0 \leq (\hat{P}_{20})^{(3)}, T_i^0 \leq (\hat{Q}_{20})^{(3)}, \quad 174$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t} \quad 175$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t} \quad 176$$

By 177

$$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20}^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{21}(s_{(20)}) \right] ds_{(20)} \quad 178$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{22}(s_{(20)}) \right] ds_{(20)} \quad 179$$

$$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)} (G(s_{(20)}), s_{(20)}) \right) T_{20}(s_{(20)}) \right] ds_{(20)} \quad 180$$

$$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)} (G(s_{(20)}), s_{(20)}) \right) T_{21}(s_{(20)}) \right] ds_{(20)} \quad 181$$

$$\bar{T}_{22}(t) = T_{22}^0 + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)} (G(s_{(20)}), s_{(20)}) \right) T_{22}(s_{(20)}) \right] ds_{(20)} \quad 182$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions $G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0, T_i(0) = T_i^0, G_i^0 \leq (\hat{P}_{24})^{(4)}, T_i^0 \leq (\hat{Q}_{24})^{(4)}, \quad 183$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)}t} \quad 184$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)}t} \quad 185$$

By 186

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + a''_{24}(T_{25}(s_{(24)}), s_{(24)})) G_{24}(s_{(24)}) \right) \right] ds_{(24)} \quad 187$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + a''_{25}(T_{25}(s_{(24)}), s_{(24)})) G_{25}(s_{(24)}) \right) \right] ds_{(24)} \quad 187$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + a''_{26}(T_{25}(s_{(24)}), s_{(24)})) G_{26}(s_{(24)}) \right) \right] ds_{(24)} \quad 188$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)}(G(s_{(24)}), s_{(24)})) T_{24}(s_{(24)}) \right) \right] ds_{(24)} \quad 189$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)}(G(s_{(24)}), s_{(24)})) T_{25}(s_{(24)}) \right) \right] ds_{(24)} \quad 190$$

$$\bar{T}_{26}(t) = T_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)}(G(s_{(24)}), s_{(24)})) T_{26}(s_{(24)}) \right) \right] ds_{(24)} \quad 191$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions $G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0, T_i(0) = T_i^0, G_i^0 \leq (\hat{P}_{28})^{(5)}, T_i^0 \leq (\hat{Q}_{28})^{(5)}, \quad 192$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)}t} \quad 193$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)}t} \quad 194$$

By 195

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28}(T_{29}(s_{(28)}), s_{(28)})) G_{28}(s_{(28)}) \right) \right] ds_{(28)} \quad 196$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + a''_{29}(T_{29}(s_{(28)}), s_{(28)})) G_{29}(s_{(28)}) \right) \right] ds_{(28)} \quad 196$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + a''_{30}(T_{29}(s_{(28)}), s_{(28)})) G_{30}(s_{(28)}) \right) \right] ds_{(28)} \quad 197$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)}(G(s_{(28)}), s_{(28)})) T_{28}(s_{(28)}) \right) \right] ds_{(28)} \quad 198$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)}(G(s_{(28)}), s_{(28)})) T_{29}(s_{(28)}) \right) \right] ds_{(28)} \quad 199$$

$$\bar{T}_{30}(t) = T_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)}(G(s_{(28)}), s_{(28)})) T_{30}(s_{(28)}) \right) \right] ds_{(28)} \quad 200$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

Proof:

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions $G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0, T_i(0) = T_i^0, G_i^0 \leq (\hat{P}_{32})^{(6)}, T_i^0 \leq (\hat{Q}_{32})^{(6)}, \quad 201$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)}t} \quad 202$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)}t} \quad 203$$

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{32}(s_{(32)}) \right] ds_{(32)} \quad 204$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{33}(s_{(32)}) \right] ds_{(32)} \quad 205$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{34}(s_{(32)}) \right] ds_{(32)} \quad 206$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)} (G(s_{(32)}), s_{(32)}) \right) T_{32}(s_{(32)}) \right] ds_{(32)} \quad 207$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)} (G(s_{(32)}), s_{(32)}) \right) T_{33}(s_{(32)}) \right] ds_{(32)} \quad 208$$

$$\bar{T}_{34}(t) = T_{34}^0 + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)} (G(s_{(32)}), s_{(32)}) \right) T_{34}(s_{(32)}) \right] ds_{(32)} \quad 209$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

(a) The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying 3 into itself. Indeed it is obvious that

$$G_{13}(t) \leq G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}} \right) \right] ds_{(13)} = \\ (1 + (a_{13})^{(1)} t) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left(e^{(\hat{M}_{13})^{(1)} t} - 1 \right)$$

From which it follows that

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$$(G_{13}(t) - G_{13}^0) e^{-(\hat{M}_{13})^{(1)} t} \leq \frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0 \right) e^{-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0}} + (\hat{P}_{13})^{(1)} \right]$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

(b) The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying into itself. Indeed it is obvious that

$$G_{16}(t) \leq G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} s_{(16)}} \right) \right] ds_{(16)} = (1 + (a_{16})^{(2)} t) G_{17}^0 + \\ \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1 \right)$$

From which it follows that

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \leq \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[\left((\hat{P}_{16})^{(2)} + G_{17}^0 \right) e^{-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0}} + (\hat{P}_{16})^{(2)} \right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

(a) The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying into itself. Indeed it is obvious that

$$G_{20}(t) \leq G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} \left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}} \right) \right] ds_{(20)} = \\ (1 + (a_{20})^{(3)} t) G_{21}^0 + \frac{(a_{20})^{(3)} (\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left(e^{(\hat{M}_{20})^{(3)} t} - 1 \right)$$

From which it follows that

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$$(G_{20}(t) - G_{20}^0) e^{-(\hat{M}_{20})^{(3)} t} \leq \frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left[\left((\hat{P}_{20})^{(3)} + G_{21}^0 \right) e^{-\frac{(\hat{P}_{20})^{(3)} + G_{21}^0}{G_{21}^0}} + (\hat{P}_{20})^{(3)} \right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

(b) The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself. Indeed it is obvious that

$$G_{24}(t) \leq G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} \left(G_{25}^0 + (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} s_{(24)}} \right) \right] ds_{(24)} = \\ (1 + (a_{24})^{(4)} t) G_{25}^0 + \frac{(a_{24})^{(4)} (\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}} \left(e^{(\hat{M}_{24})^{(4)} t} - 1 \right)$$

From which it follows that

$$(G_{24}(t) - G_{24}^0)e^{-(\hat{M}_{24})^{(4)}t} \leq \frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}} \left[((\hat{P}_{24})^{(4)} + G_{25}^0)e^{-\frac{(\hat{P}_{24})^{(4)} + G_{25}^0}{G_{25}^0}} + (\hat{P}_{24})^{(4)} \right]$$

(G_i^0) is as defined in the statement of theorem NUMBERED ONE

(c) The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying into itself .Indeed it is obvious that

$$G_{28}(t) \leq G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} \left(G_{29}^0 + (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)}s(28)} \right) \right] ds_{(28)} = \\ (1 + (a_{28})^{(5)}t)G_{29}^0 + \frac{(a_{28})^{(5)}(\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}} \left(e^{(\hat{M}_{28})^{(5)}t} - 1 \right)$$

From which it follows that

$$(G_{28}(t) - G_{28}^0)e^{-(\hat{M}_{28})^{(5)}t} \leq \frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}} \left[((\hat{P}_{28})^{(5)} + G_{29}^0)e^{-\frac{(\hat{P}_{28})^{(5)} + G_{29}^0}{G_{29}^0}} + (\hat{P}_{28})^{(5)} \right]$$

(G_i^0) is as defined in the statement of theorem 1

(d) The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying into itself .Indeed it is obvious that

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$$G_{32}(t) \leq G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} \left(G_{33}^0 + (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)}s(32)} \right) \right] ds_{(32)} = \\ (1 + (a_{32})^{(6)}t)G_{33}^0 + \frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}} \left(e^{(\hat{M}_{32})^{(6)}t} - 1 \right)$$

From which it follows that

$$(G_{32}(t) - G_{32}^0)e^{-(\hat{M}_{32})^{(6)}t} \leq \frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}} \left[((\hat{P}_{32})^{(6)} + G_{33}^0)e^{-\frac{(\hat{P}_{32})^{(6)} + G_{33}^0}{G_{33}^0}} + (\hat{P}_{32})^{(6)} \right]$$

(G_i^0) is as defined in the statement of theorem ONE

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_j)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$ and to choose

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \left[(\hat{P}_{13})^{(1)} + ((\hat{P}_{13})^{(1)} + G_j^0)e^{-\frac{(\hat{P}_{13})^{(1)} + G_j^0}{G_j^0}} \right] \leq (\hat{P}_{13})^{(1)} \\ \frac{(b_j)^{(1)}}{(\hat{M}_{13})^{(1)}} \left[((\hat{Q}_{13})^{(1)} + T_j^0)e^{-\frac{(\hat{Q}_{13})^{(1)} + T_j^0}{T_j^0}} + (\hat{Q}_{13})^{(1)} \right] \leq (\hat{Q}_{13})^{(1)}$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i, T_i satisfying 34,35,36 into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric

$$d \left((G^{(1)}, T^{(1)}), (G^{(2)}, T^{(2)}) \right) = \\ \sup_i \left\{ \max_{t \in \mathbb{R}_+} |G_i^{(1)}(t) - G_i^{(2)}(t)| e^{-(\hat{M}_{13})^{(1)}t}, \max_{t \in \mathbb{R}_+} |T_i^{(1)}(t) - T_i^{(2)}(t)| e^{-(\hat{M}_{13})^{(1)}t} \right\}$$

Indeed if we denote

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Definition of \tilde{G}, \tilde{T} : $(\tilde{G}, \tilde{T}) = \mathcal{A}^{(1)}(G, T)$

It results

$$|\tilde{G}_{13}^{(1)} - \tilde{G}_i^{(2)}| \leq \int_0^t (a_{13})^{(1)} |G_{14}^{(1)} - G_{14}^{(2)}| e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} e^{(\widehat{M}_{13})^{(1)}s_{(13)}} ds_{(13)} +$$

$$\int_0^t \{ (a'_{13})^{(1)} |G_{13}^{(1)} - G_{13}^{(2)}| e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} +$$

$$(a''_{13})^{(1)} (T_{14}^{(1)}, s_{(13)}) |G_{13}^{(1)} - G_{13}^{(2)}| e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} e^{(\widehat{M}_{13})^{(1)}s_{(13)}} +$$

$$G_{13}^{(2)} | (a''_{13})^{(1)} (T_{14}^{(1)}, s_{(13)}) - (a''_{13})^{(1)} (T_{14}^{(2)}, s_{(13)}) | e^{-(\widehat{M}_{13})^{(1)}s_{(13)}} e^{(\widehat{M}_{13})^{(1)}s_{(13)}} \} ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$|G^{(1)} - G^{(2)}| e^{-(\widehat{M}_{13})^{(1)}t} \leq$$

$$\frac{1}{(\widehat{M}_{13})^{(1)}} \left((a_{13})^{(1)} + (a'_{13})^{(1)} + (\widehat{A}_{13})^{(1)} + (\widehat{P}_{13})^{(1)} (\widehat{k}_{13})^{(1)} \right) d \left((G^{(1)}, T^{(1)}; G^{(2)}, T^{(2)}) \right)$$

And analogous inequalities for G_i and T_i . Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a'_{13})^{(1)}$ and $(b'_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis, in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t}$ and $(\widehat{Q}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t}$ respectively of \mathbb{R}_+ .

If instead of proving the existence of the solution on \mathbb{R}_+ , we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(1)}$ and $(b''_i)^{(1)}$, $i = 13, 14, 15$ depend only on T_{14} and respectively on G (and not on t) and hypothesis can be replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$

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From 19 to 24 it results

$$G_i(t) \geq G_i^0 e^{-\int_0^t \{ (a'_i)^{(1)} - (a''_i)^{(1)} (T_{14}(s_{(13)}), s_{(13)}) \} ds_{(13)}} \geq 0$$

$$T_i(t) \geq T_i^0 e^{-(b'_i)^{(1)}t} > 0 \text{ for } t > 0$$

Definition of $((\widehat{M}_{13})^{(1)})_1$, $((\widehat{M}_{13})^{(1)})_2$ and $((\widehat{M}_{13})^{(1)})_3$:

Remark 3: if G_{13} is bounded, the same property have also G_{14} and G_{15} . indeed if

$$G_{13} < (\widehat{M}_{13})^{(1)} \text{ it follows } \frac{dG_{14}}{dt} \leq ((\widehat{M}_{13})^{(1)})_1 - (a'_{14})^{(1)} G_{14} \text{ and by integrating}$$

$$G_{14} \leq ((\widehat{M}_{13})^{(1)})_2 = G_{14}^0 + 2(a_{14})^{(1)} ((\widehat{M}_{13})^{(1)})_1 / (a'_{14})^{(1)}$$

In the same way, one can obtain

$$G_{15} \leq ((\widehat{M}_{13})^{(1)})_3 = G_{15}^0 + 2(a_{15})^{(1)} ((\widehat{M}_{13})^{(1)})_2 / (a'_{15})^{(1)}$$

If G_{14} or G_{15} is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If G_{13} is bounded, from below, the same property holds for G_{14} and G_{15} . The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below.

Remark 5: If T_{13} is bounded from below and $\lim_{t \rightarrow \infty} ((b''_i)^{(1)}(G(t), t)) = (b'_{14})^{(1)}$ then $T_{14} \rightarrow \infty$.

Definition of $(m)^{(1)}$ and ε_1 :

Indeed let t_1 be so that for $t > t_1$

$$(b_{14})^{(1)} - (b'_i)^{(1)}(G(t), t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right) (1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right)$, $t = \log \frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded.

The same property holds for T_{15} if $\lim_{t \rightarrow \infty} (b''_{15})^{(1)}(G(t), t) = (b'_{15})^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\bar{M}_{16})^{(2)}}$, $\frac{(b_j)^{(2)}}{(\bar{M}_{16})^{(2)}} < 1$ and to choose

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$(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\bar{M}_{16})^{(2)}} \left[(\hat{P}_{16})^{(2)} + ((\hat{P}_{16})^{(2)} + G_j^0) e^{-\left(\frac{(\hat{P}_{16})^{(2)} + G_j^0}{G_j^0}\right)} \right] \leq (\hat{P}_{16})^{(2)}$$

$$\frac{(b_j)^{(2)}}{(\bar{M}_{16})^{(2)}} \left[((\hat{Q}_{16})^{(2)} + T_j^0) e^{-\left(\frac{(\hat{Q}_{16})^{(2)} + T_j^0}{T_j^0}\right)} + (\hat{Q}_{16})^{(2)} \right] \leq (\hat{Q}_{16})^{(2)}$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i, T_i satisfying 34,35,36 into itself

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric

$$d\left(\left((G_{19})^{(1)}, (T_{19})^{(1)}\right), \left((G_{19})^{(2)}, (T_{19})^{(2)}\right)\right) =$$

$$\sup_i \left\{ \max_{t \in \mathbb{R}_+} |G_i^{(1)}(t) - G_i^{(2)}(t)| e^{-(\bar{M}_{16})^{(2)}t}, \max_{t \in \mathbb{R}_+} |T_i^{(1)}(t) - T_i^{(2)}(t)| e^{-(\bar{M}_{16})^{(2)}t} \right\}$$

Indeed if we denote

Definition of $\tilde{G}_{19}, \tilde{T}_{19} : (\tilde{G}_{19}, \tilde{T}_{19}) = \mathcal{A}^{(2)}(G_{19}, T_{19})$

It results

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$$|\tilde{G}_{16}^{(1)} - \tilde{G}_{16}^{(2)}| \leq \int_0^t (a_{16})^{(2)} |G_{17}^{(1)} - G_{17}^{(2)}| e^{-(\bar{M}_{16})^{(2)}s_{(16)}} e^{(\bar{M}_{16})^{(2)}s_{(16)}} ds_{(16)} +$$

$$\int_0^t \{ (a'_{16})^{(2)} |G_{16}^{(1)} - G_{16}^{(2)}| e^{-(\bar{M}_{16})^{(2)}s_{(16)}} e^{-(\bar{M}_{16})^{(2)}s_{(16)}} +$$

$$(a''_{16})^{(2)} (T_{17}^{(1)}, s_{(16)}) |G_{16}^{(1)} - G_{16}^{(2)}| e^{-(\bar{M}_{16})^{(2)}s_{(16)}} e^{(\bar{M}_{16})^{(2)}s_{(16)}} +$$

$$G_{16}^{(2)} | (a'_{16})^{(2)} (T_{17}^{(1)}, s_{(16)}) - (a''_{16})^{(2)} (T_{17}^{(2)}, s_{(16)}) | e^{-(\bar{M}_{16})^{(2)}s_{(16)}} e^{(\bar{M}_{16})^{(2)}s_{(16)}} \} ds_{(16)}$$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$|(G_{19})^{(1)} - (G_{19})^{(2)}| e^{-(\bar{M}_{16})^{(2)}t} \leq$$

$$\frac{1}{(\bar{M}_{16})^{(2)}} \left((a_{16})^{(2)} + (a'_{16})^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)} (\hat{K}_{16})^{(2)} \right) d\left(\left((G_{19})^{(1)}, (T_{19})^{(1)}\right); \left((G_{19})^{(2)}, (T_{19})^{(2)}\right)\right)$$

And analogous inequalities for G_i and T_i . Taking into account the hypothesis (34,35,36) the result follows

Remark 1: The fact that we supposed $(a''_{16})^{(2)}$ and $(b''_{16})^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis, in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\hat{P}_{16})^{(2)} e^{(\bar{M}_{16})^{(2)}t}$ and $(\hat{Q}_{16})^{(2)} e^{(\bar{M}_{16})^{(2)}t}$ respectively of \mathbb{R}_+ .

If instead of proving the existence of the solution on \mathbb{R}_+ , we have to prove it only on a compact then it

suffices to consider that $(a_i'')^{(2)}$ and $(b_i'')^{(2)}$, $i = 16, 17, 18$ depend only on T_{17} and respectively on (G_{19}) (and not on t) and hypothesis can be replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$

From global equations it results

$$G_i(t) \geq G_i^0 e^{-\int_0^t \{(a_i')^{(2)} - (T_{17}(s_{(16)}), s_{(16)})\} ds_{(16)}} \geq 0$$

$$T_i(t) \geq T_i^0 e^{-(b_i')^{(2)}t} > 0 \text{ for } t > 0$$

Definition of $((\widehat{M}_{16})^{(2)})_1$, $((\widehat{M}_{16})^{(2)})_2$ and $((\widehat{M}_{16})^{(2)})_3$:

Remark 3: if G_{16} is bounded, the same property have also G_{17} and G_{18} . indeed if

$$G_{16} < ((\widehat{M}_{16})^{(2)})_1 \text{ it follows } \frac{dG_{17}}{dt} \leq ((\widehat{M}_{16})^{(2)})_1 - (a_{17}')^{(2)}G_{17} \text{ and by integrating}$$

$$G_{17} \leq ((\widehat{M}_{16})^{(2)})_2 = G_{17}^0 + 2(a_{17}')^{(2)}((\widehat{M}_{16})^{(2)})_1 / (a_{17}')^{(2)}$$

In the same way, one can obtain

$$G_{18} \leq ((\widehat{M}_{16})^{(2)})_3 = G_{18}^0 + 2(a_{18}')^{(2)}((\widehat{M}_{16})^{(2)})_2 / (a_{18}')^{(2)}$$

If G_{17} or G_{18} is bounded, the same property follows for G_{16} , G_{18} and G_{16} , G_{17} respectively.

Remark 4: If G_{16} is bounded, from below, the same property holds for G_{17} and G_{18} . The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below.

Remark 5: If T_{16} is bounded from below and $\lim_{t \rightarrow \infty} ((b_i'')^{(2)}((G_{19})(t), t)) = (b_{17}')^{(2)}$ then $T_{17} \rightarrow \infty$.

Definition of $(m)^{(2)}$ and ε_2 :

Indeed let t_2 be so that for $t > t_2$

$$(b_{17}')^{(2)} - (b_i'')^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$$

Then $\frac{dT_{17}}{dt} \geq (a_{17}')^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to

$$T_{17} \geq \left(\frac{(a_{17}')^{(2)}(m)^{(2)}}{\varepsilon_2} \right) (1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t} \text{ If we take } t \text{ such that } e^{-\varepsilon_2 t} = \frac{1}{2} \text{ it results}$$

$$T_{17} \geq \left(\frac{(a_{17}')^{(2)}(m)^{(2)}}{2} \right), \quad t = \log \frac{2}{\varepsilon_2} \text{ By taking now } \varepsilon_2 \text{ sufficiently small one sees that } T_{17} \text{ is unbounded.}$$

The same property holds for T_{18} if $\lim_{t \rightarrow \infty} (b_{18}'')^{(2)}((G_{19})(t), t) = (b_{18}')^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

It is now sufficient to take $\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}, \frac{(b_j)^{(3)}}{(\widehat{M}_{20})^{(3)}} < 1$ and to choose

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$(\widehat{P}_{20})^{(3)}$ and $(\widehat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}} \left[(\widehat{P}_{20})^{(3)} + ((\widehat{P}_{20})^{(3)} + G_j^0) e^{-\left(\frac{(\widehat{P}_{20})^{(3)} + G_j^0}{G_j^0} \right)} \right] \leq (\widehat{P}_{20})^{(3)}$$

$$\frac{(b_j)^{(3)}}{(\widehat{M}_{20})^{(3)}} \left[((\widehat{Q}_{20})^{(3)} + T_j^0) e^{-\left(\frac{(\widehat{Q}_{20})^{(3)} + T_j^0}{T_j^0} \right)} + (\widehat{Q}_{20})^{(3)} \right] \leq (\widehat{Q}_{20})^{(3)}$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions G_i, T_i satisfying 34, 35, 36 into itself

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric

$$d\left(\left((G_{23})^{(1)}, (T_{23})^{(1)}\right), \left((G_{23})^{(2)}, (T_{23})^{(2)}\right)\right) = \sup_i \left\{ \max_{t \in \mathbb{R}_+} |G_i^{(1)}(t) - G_i^{(2)}(t)| e^{-(\widehat{M}_{20})^{(3)}t}, \max_{t \in \mathbb{R}_+} |T_i^{(1)}(t) - T_i^{(2)}(t)| e^{-(\widehat{M}_{20})^{(3)}t} \right\}$$

Indeed if we denote

Definition of $\widetilde{G}_{23}, \widetilde{T}_{23} : \left((\widetilde{G}_{23}), (\widetilde{T}_{23}) \right) = \mathcal{A}^{(3)}\left((G_{23}), (T_{23}) \right)$

It results

$$\begin{aligned} |\widetilde{G}_{20}^{(1)} - \widetilde{G}_i^{(2)}| &\leq \int_0^t (a_{20})^{(3)} |G_{21}^{(1)} - G_{21}^{(2)}| e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{(\widehat{M}_{20})^{(3)}s_{(20)}} ds_{(20)} + \\ &\int_0^t \{ (a'_{20})^{(3)} |G_{20}^{(1)} - G_{20}^{(2)}| e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} + \\ &(a''_{20})^{(3)} (T_{21}^{(1)}, s_{(20)}) |G_{20}^{(1)} - G_{20}^{(2)}| e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{(\widehat{M}_{20})^{(3)}s_{(20)}} + \\ &G_{20}^{(2)} | (a''_{20})^{(3)} (T_{21}^{(1)}, s_{(20)}) - (a''_{20})^{(3)} (T_{21}^{(2)}, s_{(20)}) | e^{-(\widehat{M}_{20})^{(3)}s_{(20)}} e^{(\widehat{M}_{20})^{(3)}s_{(20)}} \} ds_{(20)} \end{aligned}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$|G^{(1)} - G^{(2)}| e^{-(\widehat{M}_{20})^{(3)}t} \leq \frac{1}{(\widehat{M}_{20})^{(3)}} \left((a_{20})^{(3)} + (a'_{20})^{(3)} + (\widehat{A}_{20})^{(3)} + (\widehat{P}_{20})^{(3)} (\widehat{k}_{20})^{(3)} \right) d\left(\left((G_{23})^{(1)}, (T_{23})^{(1)}\right); (G_{23})^{(2)}, (T_{23})^{(2)}\right)$$

And analogous inequalities for G_i and T_i . Taking into account the hypothesis (34,35,36) the result follows

Remark 1: The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis, in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)} e^{(\widehat{M}_{20})^{(3)}t}$ and $(\widehat{Q}_{20})^{(3)} e^{(\widehat{M}_{20})^{(3)}t}$ respectively of \mathbb{R}_+ .

If instead of proving the existence of the solution on \mathbb{R}_+ , we have to prove it only on a compact then it suffices to consider that $(a'_i)^{(3)}$ and $(b'_i)^{(3)}$, $i = 20, 21, 22$ depend only on T_{21} and respectively on (G_{23}) (and not on t) and hypothesis can be replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$

From 19 to 24 it results

$$\begin{aligned} G_i(t) &\geq G_i^0 e^{-\int_0^t \{ (a'_i)^{(3)} - (a''_i)^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \} ds_{(20)}} \geq 0 \\ T_i(t) &\geq T_i^0 e^{-(b'_i)^{(3)}t} > 0 \text{ for } t > 0 \end{aligned}$$

Definition of $((\widehat{M}_{20})^{(3)})_1, ((\widehat{M}_{20})^{(3)})_2$ and $((\widehat{M}_{20})^{(3)})_3$:

Remark 3: if G_{20} is bounded, the same property have also G_{21} and G_{22} . indeed if

$$G_{20} < (\widehat{M}_{20})^{(3)} \text{ it follows } \frac{dG_{21}}{dt} \leq ((\widehat{M}_{20})^{(3)})_1 - (a'_{21})^{(3)} G_{21} \text{ and by integrating}$$

$$G_{21} \leq ((\widehat{M}_{20})^{(3)})_2 = G_{21}^0 + 2(a_{21})^{(3)} ((\widehat{M}_{20})^{(3)})_1 / (a'_{21})^{(3)}$$

In the same way, one can obtain

$$G_{22} \leq ((\widehat{M}_{20})^{(3)})_3 = G_{22}^0 + 2(a_{22})^{(3)} ((\widehat{M}_{20})^{(3)})_2 / (a'_{22})^{(3)}$$

If G_{21} or G_{22} is bounded, the same property follows for G_{20} , G_{22} and G_{20} , G_{21} respectively.

Remark 4: If G_{20} is bounded, from below, the same property holds for G_{21} and G_{22} . The proof is analogous with the preceding one. An analogous property is true if G_{21} is bounded from below.

Remark 5: If T_{20} is bounded from below and $\lim_{t \rightarrow \infty} ((b_i'')^{(3)}((G_{23})(t), t)) = (b_{21}')^{(3)}$ then $T_{21} \rightarrow \infty$.

Definition of $(m)^{(3)}$ and ε_3 :

Indeed let t_3 be so that for $t > t_3$

$$(b_{21})^{(3)} - (b_i'')^{(3)}((G_{23})(t), t) < \varepsilon_3, T_{20}(t) > (m)^{(3)}$$

Then $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$ which leads to

$$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3} \right) (1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t} \text{ If we take } t \text{ such that } e^{-\varepsilon_3 t} = \frac{1}{2} \text{ it results}$$

$$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2} \right), \quad t = \log \frac{2}{\varepsilon_3} \text{ By taking now } \varepsilon_3 \text{ sufficiently small one sees that } T_{21} \text{ is unbounded.}$$

The same property holds for T_{22} if $\lim_{t \rightarrow \infty} (b_{22}'')^{(3)}((G_{23})(t), t) = (b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions

It is now sufficient to take $\frac{(a_i)^{(4)}}{(\bar{M}_{24})^{(4)}}, \frac{(b_i)^{(4)}}{(\bar{M}_{24})^{(4)}} < 1$ and to choose

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$(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\bar{M}_{24})^{(4)}} \left[(\hat{P}_{24})^{(4)} + ((\hat{P}_{24})^{(4)} + G_j^0) e^{-\left(\frac{(\hat{P}_{24})^{(4)} + G_j^0}{G_j^0} \right)} \right] \leq (\hat{P}_{24})^{(4)}$$

$$\frac{(b_i)^{(4)}}{(\bar{M}_{24})^{(4)}} \left[((\hat{Q}_{24})^{(4)} + T_j^0) e^{-\left(\frac{(\hat{Q}_{24})^{(4)} + T_j^0}{T_j^0} \right)} + (\hat{Q}_{24})^{(4)} \right] \leq (\hat{Q}_{24})^{(4)}$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions G_i, T_i into itself

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric

$$d \left(((G_{27})^{(1)}, (T_{27})^{(1)}), ((G_{27})^{(2)}, (T_{27})^{(2)}) \right) = \sup_i \{ \max_{t \in \mathbb{R}_+} |G_i^{(1)}(t) - G_i^{(2)}(t)| e^{-(\bar{M}_{24})^{(4)}t}, \max_{t \in \mathbb{R}_+} |T_i^{(1)}(t) - T_i^{(2)}(t)| e^{-(\bar{M}_{24})^{(4)}t} \}$$

Indeed if we denote

$$\underline{\text{Definition of}} (\widehat{G_{27}}, \widehat{T_{27}}) : (\widehat{G_{27}}, \widehat{T_{27}}) = \mathcal{A}^{(4)}((G_{27}), (T_{27}))$$

It results

$$\begin{aligned} |\tilde{G}_{24}^{(1)} - \tilde{G}_{24}^{(2)}| &\leq \int_0^t (a_{24})^{(4)} |G_{25}^{(1)} - G_{25}^{(2)}| e^{-(\bar{M}_{24})^{(4)}s} e^{(\bar{M}_{24})^{(4)}s_{(24)}} ds_{(24)} + \\ &\int_0^t \{ (a'_{24})^{(4)} |G_{24}^{(1)} - G_{24}^{(2)}| e^{-(\bar{M}_{24})^{(4)}s_{(24)}} e^{-(\bar{M}_{24})^{(4)}s_{(24)}} + \\ &(a''_{24})^{(4)} (T_{25}^{(1)}, s_{(24)}) |G_{24}^{(1)} - G_{24}^{(2)}| e^{-(\bar{M}_{24})^{(4)}s_{(24)}} e^{(\bar{M}_{24})^{(4)}s_{(24)}} + \\ &G_{24}^{(2)} | (a''_{24})^{(4)} (T_{25}^{(1)}, s_{(24)}) - (a''_{24})^{(4)} (T_{25}^{(2)}, s_{(24)}) | e^{-(\bar{M}_{24})^{(4)}s_{(24)}} e^{(\bar{M}_{24})^{(4)}s_{(24)}} \} ds_{(24)} \end{aligned}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$|(G_{27})^{(1)} - (G_{27})^{(2)}| e^{-(\bar{M}_{24})^{(4)}t} \leq$$

$$\frac{1}{(\widehat{M}_{24})^{(4)}} \left((a_{24})^{(4)} + (a'_{24})^{(4)} + (\widehat{A}_{24})^{(4)} + (\widehat{P}_{24})^{(4)} (\widehat{K}_{24})^{(4)} \right) d \left(((G_{27})^{(1)}, (T_{27})^{(1)}); (G_{27})^{(2)}, (T_{27})^{(2)} \right)$$

And analogous inequalities for G_i and T_i . Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a'_{24})^{(4)}$ and $(b'_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis, in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)}t}$ and $(\widehat{Q}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)}t}$ respectively of \mathbb{R}_+ .

If instead of proving the existence of the solution on \mathbb{R}_+ , we have to prove it only on a compact then it suffices to consider that $(a'_i)^{(4)}$ and $(b'_i)^{(4)}$, $i = 24, 25, 26$ depend only on T_{25} and respectively on (G_{27}) (and not on t) and hypothesis can be replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$

From 19 to 24 it results

$$G_i(t) \geq G_i^0 e^{-\int_0^t ((a'_i)^{(4)} - (a'_i)^{(4)}(T_{25}(s_{(24)}), s_{(24)})) ds_{(24)}} \geq 0$$

$$T_i(t) \geq T_i^0 e^{-(b'_i)^{(4)}t} > 0 \text{ for } t > 0$$

Definition of $((\widehat{M}_{24})^{(4)})_1, ((\widehat{M}_{24})^{(4)})_2$ and $((\widehat{M}_{24})^{(4)})_3$:

Remark 3: if G_{24} is bounded, the same property have also G_{25} and G_{26} . indeed if

$$G_{24} < (\widehat{M}_{24})^{(4)} \text{ it follows } \frac{dG_{25}}{dt} \leq ((\widehat{M}_{24})^{(4)})_1 - (a'_{25})^{(4)} G_{25} \text{ and by integrating}$$

$$G_{25} \leq ((\widehat{M}_{24})^{(4)})_2 = G_{25}^0 + 2(a_{25})^{(4)} ((\widehat{M}_{24})^{(4)})_1 / (a'_{25})^{(4)}$$

In the same way, one can obtain

$$G_{26} \leq ((\widehat{M}_{24})^{(4)})_3 = G_{26}^0 + 2(a_{26})^{(4)} ((\widehat{M}_{24})^{(4)})_2 / (a'_{26})^{(4)}$$

If G_{25} or G_{26} is bounded, the same property follows for G_{24} , G_{26} and G_{24} , G_{25} respectively.

Remark 4: If G_{24} is bounded, from below, the same property holds for G_{25} and G_{26} . The proof is analogous with the preceding one. An analogous property is true if G_{25} is bounded from below.

Remark 5: If T_{24} is bounded from below and $\lim_{t \rightarrow \infty} ((b'_i)^{(4)}((G_{27})(t), t)) = (b'_{25})^{(4)}$ then $T_{25} \rightarrow \infty$.

Definition of $(m)^{(4)}$ and ε_4 :

Indeed let t_4 be so that for $t > t_4$

$$(b_{25})^{(4)} - (b'_i)^{(4)}((G_{27})(t), t) < \varepsilon_4, T_{24}(t) > (m)^{(4)}$$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to

$$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4} \right) (1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t} \text{ If we take } t \text{ such that } e^{-\varepsilon_4 t} = \frac{1}{2} \text{ it results}$$

$$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2} \right), \quad t = \log \frac{2}{\varepsilon_4} \text{ By taking now } \varepsilon_4 \text{ sufficiently small one sees that } T_{25} \text{ is unbounded.}$$

The same property holds for T_{26} if $\lim_{t \rightarrow \infty} (b'_{26})^{(4)}((G_{27})(t), t) = (b'_{26})^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions ANALOGOUS

inequalities hold also for $G_{29}, G_{30}, T_{28}, T_{29}, T_{30}$

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}, \frac{(b_i)^{(5)}}{(\widehat{M}_{28})^{(5)}} < 1$ and to choose

$(\widehat{P}_{28})^{(5)}$ and $(\widehat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\widehat{M}_{28})^{(5)}} \left[(\widehat{P}_{28})^{(5)} + ((\widehat{P}_{28})^{(5)} + G_j^0) e^{-\left(\frac{(\widehat{P}_{28})^{(5)} + G_j^0}{G_j^0}\right)} \right] \leq (\widehat{P}_{28})^{(5)}$$

$$\frac{(b_i)^{(5)}}{(\widehat{M}_{28})^{(5)}} \left[((\widehat{Q}_{28})^{(5)} + T_j^0) e^{-\left(\frac{(\widehat{Q}_{28})^{(5)} + T_j^0}{T_j^0}\right)} + (\widehat{Q}_{28})^{(5)} \right] \leq (\widehat{Q}_{28})^{(5)}$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions G_i, T_i into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric

$$d\left(((G_{31})^{(1)}, (T_{31})^{(1)}), ((G_{31})^{(2)}, (T_{31})^{(2)}) \right) =$$

$$\sup_i \left\{ \max_{t \in \mathbb{R}_+} |G_i^{(1)}(t) - G_i^{(2)}(t)| e^{-(\widehat{M}_{28})^{(5)}t}, \max_{t \in \mathbb{R}_+} |T_i^{(1)}(t) - T_i^{(2)}(t)| e^{-(\widehat{M}_{28})^{(5)}t} \right\}$$

Indeed if we denote

Definition of $(\widetilde{G}_{31}, \widetilde{T}_{31}) : ((\widetilde{G}_{31}), (\widetilde{T}_{31})) = \mathcal{A}^{(5)}((G_{31}), (T_{31}))$

It results

$$|\widetilde{G}_{28}^{(1)} - \widetilde{G}_i^{(2)}| \leq \int_0^t (a_{28})^{(5)} |G_{29}^{(1)} - G_{29}^{(2)}| e^{-(\widehat{M}_{28})^{(5)}s_{(28)}} e^{(\widehat{M}_{28})^{(5)}s_{(28)}} ds_{(28)} +$$

$$\int_0^t \{ (a'_{28})^{(5)} |G_{28}^{(1)} - G_{28}^{(2)}| e^{-(\widehat{M}_{28})^{(5)}s_{(28)}} e^{-(\widehat{M}_{28})^{(5)}s_{(28)}} +$$

$$(a''_{28})^{(5)} (T_{29}^{(1)}, s_{(28)}) |G_{28}^{(1)} - G_{28}^{(2)}| e^{-(\widehat{M}_{28})^{(5)}s_{(28)}} e^{(\widehat{M}_{28})^{(5)}s_{(28)}} +$$

$$G_{28}^{(2)} | (a''_{28})^{(5)} (T_{29}^{(1)}, s_{(28)}) - (a''_{28})^{(5)} (T_{29}^{(2)}, s_{(28)}) | e^{-(\widehat{M}_{28})^{(5)}s_{(28)}} e^{(\widehat{M}_{28})^{(5)}s_{(28)}} \} ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$|(G_{31})^{(1)} - (G_{31})^{(2)}| e^{-(\widehat{M}_{28})^{(5)}t} \leq$$

$$\frac{1}{(\widehat{M}_{28})^{(5)}} \left((a_{28})^{(5)} + (a'_{28})^{(5)} + (\widehat{A}_{28})^{(5)} + (\widehat{P}_{28})^{(5)} (\widehat{k}_{28})^{(5)} \right) d\left(((G_{31})^{(1)}, (T_{31})^{(1)}); ((G_{31})^{(2)}, (T_{31})^{(2)}) \right)$$

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And analogous inequalities for G_i and T_i . Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis, in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)}t}$ and $(\widehat{Q}_{28})^{(5)} e^{(\widehat{M}_{28})^{(5)}t}$ respectively of \mathbb{R}_+ .

If instead of proving the existence of the solution on \mathbb{R}_+ , we have to prove it only on a compact then it suffices to consider that $(a'_i)^{(5)}$ and $(b'_i)^{(5)}$, $i = 28, 29, 30$ depend only on T_{29} and respectively on (G_{31}) (and not on t) and hypothesis can be replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$

From 19 to 28 it results

$$G_i(t) \geq G_i^0 e^{-\int_0^t \{ (a'_i)^{(5)} - (a''_i)^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \} ds_{(28)}} \geq 0$$

$$T_i(t) \geq T_i^0 e^{-(b'_i)^{(5)}t} > 0 \text{ for } t > 0$$

Definition of $((\widehat{M}_{28})^{(5)})_1, ((\widehat{M}_{28})^{(5)})_2$ and $((\widehat{M}_{28})^{(5)})_3 :$

Remark 3: if G_{28} is bounded, the same property have also G_{29} and G_{30} . indeed if

$G_{28} < (\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq ((\widehat{M}_{28})^{(5)})_1 - (a'_{29})^{(5)}G_{29}$ and by integrating

$$G_{29} \leq ((\widehat{M}_{28})^{(5)})_2 = G_{29}^0 + 2(a_{29})^{(5)}((\widehat{M}_{28})^{(5)})_1 / (a'_{29})^{(5)}$$

In the same way, one can obtain

$$G_{30} \leq ((\widehat{M}_{28})^{(5)})_3 = G_{30}^0 + 2(a_{30})^{(5)}((\widehat{M}_{28})^{(5)})_2 / (a'_{30})^{(5)}$$

If G_{29} or G_{30} is bounded, the same property follows for G_{28} , G_{30} and G_{28} , G_{29} respectively.

Remark 4: If G_{28} is bounded, from below, the same property holds for G_{29} and G_{30} . The proof is analogous with the preceding one. An analogous property is true if G_{29} is bounded from below.

Remark 5: If T_{28} is bounded from below and $\lim_{t \rightarrow \infty} ((b''_i)^{(5)}((G_{31})(t), t)) = (b'_{29})^{(5)}$ then $T_{29} \rightarrow \infty$.

Definition of $(m)^{(5)}$ and ε_5 :

Indeed let t_5 be so that for $t > t_5$

$$(b_{29})^{(5)} - (b''_i)^{(5)}((G_{31})(t), t) < \varepsilon_5, T_{28}(t) > (m)^{(5)}$$

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to

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$$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5} \right) (1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t} \text{ If we take } t \text{ such that } e^{-\varepsilon_5 t} = \frac{1}{2} \text{ it results}$$

$$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2} \right), \quad t = \log \frac{2}{\varepsilon_5} \text{ By taking now } \varepsilon_5 \text{ sufficiently small one sees that } T_{29} \text{ is unbounded.}$$

The same property holds for T_{30} if $\lim_{t \rightarrow \infty} (b''_{30})^{(5)}((G_{31})(t), t) = (b'_{30})^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions ANALOGOUS

inequalities hold also for $G_{33}, G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}} < 1$ and to choose

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$(\widehat{P}_{32})^{(6)}$ and $(\widehat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}} \left[(\widehat{P}_{32})^{(6)} + ((\widehat{P}_{32})^{(6)} + G_j^0) e^{-\left(\frac{(\widehat{P}_{32})^{(6)} + G_j^0}{G_j^0} \right)} \right] \leq (\widehat{P}_{32})^{(6)}$$

$$\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}} \left[((\widehat{Q}_{32})^{(6)} + T_j^0) e^{-\left(\frac{(\widehat{Q}_{32})^{(6)} + T_j^0}{T_j^0} \right)} + (\widehat{Q}_{32})^{(6)} \right] \leq (\widehat{Q}_{32})^{(6)}$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions G_i, T_i into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric

$$d \left(((G_{35})^{(1)}, (T_{35})^{(1)}), ((G_{35})^{(2)}, (T_{35})^{(2)}) \right) =$$

$$\sup_i \left\{ \max_{t \in \mathbb{R}_+} |G_i^{(1)}(t) - G_i^{(2)}(t)| e^{-(\widehat{M}_{32})^{(6)}t}, \max_{t \in \mathbb{R}_+} |T_i^{(1)}(t) - T_i^{(2)}(t)| e^{-(\widehat{M}_{32})^{(6)}t} \right\}$$

Indeed if we denote

Definition of $(\widetilde{G}_{35}), (\widetilde{T}_{35})$: $((\widetilde{G}_{35}), (\widetilde{T}_{35})) = \mathcal{A}^{(6)}((G_{35}), (T_{35}))$

It results

$$|\widetilde{G}_{32}^{(1)} - \widetilde{G}_{32}^{(2)}| \leq \int_0^t (a_{32})^{(6)} |G_{33}^{(1)} - G_{33}^{(2)}| e^{-(\widehat{M}_{32})^{(6)}s_{(32)}} e^{(\widehat{M}_{32})^{(6)}s_{(32)}} ds_{(32)} + \int_0^t ((a'_{32})^{(6)} |G_{32}^{(1)} - G_{32}^{(2)}| e^{-(\widehat{M}_{32})^{(6)}s_{(32)}} e^{-(\widehat{M}_{32})^{(6)}s_{(32)}} +$$

$$(a''_{32})^{(6)}(T_{33}^{(1)}, s_{(32)}) |G_{32}^{(1)} - G_{32}^{(2)}| e^{-(\widehat{M}_{32})^{(6)}s_{(32)}} e^{(\widehat{M}_{32})^{(6)}s_{(32)}} + G_{32}^{(2)} | (a''_{32})^{(6)}(T_{33}^{(1)}, s_{(32)}) - (a''_{32})^{(6)}(T_{33}^{(2)}, s_{(32)}) | e^{-(\widehat{M}_{32})^{(6)}s_{(32)}} \square (\widehat{M}_{32})^{(6)}s_{(32)} \} ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$|(G_{35})^{(1)} - (G_{35})^{(2)}| e^{-(\widehat{M}_{32})^{(6)}t} 37 \leq \frac{1}{(\widehat{M}_{32})^{(6)}} ((a_{32})^{(6)} + (a'_{32})^{(6)} + (\widehat{A}_{32})^{(6)} + (\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}) d(((G_{35})^{(1)}, (T_{35})^{(1)}); (G_{35})^{(2)}, (T_{35})^{(2)}))$$

And analogous inequalities for G_i and T_i . Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis, in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)}t}$ and $(\widehat{Q}_{32})^{(6)} e^{(\widehat{M}_{32})^{(6)}t}$ respectively of \mathbb{R}_+ .

If instead of proving the existence of the solution on \mathbb{R}_+ , we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(6)}$ and $(b''_i)^{(6)}$, $i = 32, 33, 34$ depend only on T_{33} and respectively on (G_{35}) (and not on t) and hypothesis can be replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$

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From 69 to 32 it results

$$G_i(t) \geq G_i^0 e^{-\int_0^t ((a'_i)^{(6)} - (a''_i)^{(6)}(T_{33}(s_{(32)}), s_{(32)})) ds_{(32)}} \geq 0$$

$$T_i(t) \geq T_i^0 e^{-(b'_i)^{(6)}t} > 0 \text{ for } t > 0$$

Definition of $((\widehat{M}_{32})^{(6)})_1, ((\widehat{M}_{32})^{(6)})_2$ and $((\widehat{M}_{32})^{(6)})_3$:

Remark 3: if G_{32} is bounded, the same property have also G_{33} and G_{34} . indeed if

$$G_{32} < (\widehat{M}_{32})^{(6)} \text{ it follows } \frac{dG_{33}}{dt} \leq ((\widehat{M}_{32})^{(6)})_1 - (a'_{33})^{(6)}G_{33} \text{ and by integrating}$$

$$G_{33} \leq ((\widehat{M}_{32})^{(6)})_2 = G_{33}^0 + 2(a_{33})^{(6)}((\widehat{M}_{32})^{(6)})_1 / (a'_{33})^{(6)}$$

In the same way, one can obtain

$$G_{34} \leq ((\widehat{M}_{32})^{(6)})_3 = G_{34}^0 + 2(a_{34})^{(6)}((\widehat{M}_{32})^{(6)})_2 / (a'_{34})^{(6)}$$

If G_{33} or G_{34} is bounded, the same property follows for G_{32} , G_{34} and G_{32} , G_{33} respectively.

Remark 4: If G_{32} is bounded, from below, the same property holds for G_{33} and G_{34} . The proof is analogous with the preceding one. An analogous property is true if G_{33} is bounded from below.

Remark 5: If T_{32} is bounded from below and $\lim_{t \rightarrow \infty} ((b''_i)^{(6)}((G_{35})(t), t)) = (b'_{33})^{(6)}$ then $T_{33} \rightarrow \infty$.

Definition of $(m)^{(6)}$ and ε_6 :

Indeed let t_6 be so that for $t > t_6$

$$(b_{33})^{(6)} - (b''_i)^{(6)}((G_{35})(t), t) < \varepsilon_6, T_{32}(t) > (m)^{(6)}$$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to

$$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6} \right) (1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t} \text{ If we take } t \text{ such that } e^{-\varepsilon_6 t} = \frac{1}{2} \text{ it results}$$

$$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2} \right), t = \log \frac{2}{\varepsilon_6} \text{ By taking now } \varepsilon_6 \text{ sufficiently small one sees that } T_{33} \text{ is unbounded.}$$

The same property holds for T_{34} if $\lim_{t \rightarrow \infty} (b''_{34})^{(6)}((G_{35})(t), t(t), t) = (b'_{34})^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions

Behavior of the solutions

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Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$:

$\sigma_1^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying

$$-(\sigma_2)^{(1)} \leq -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14}, t) + (a''_{14})^{(1)}(T_{14}, t) \leq -(\sigma_1)^{(1)}$$

$$-(\tau_2)^{(1)} \leq -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G, t) - (b''_{14})^{(1)}(G, t) \leq -(\tau_1)^{(1)}$$

Definition of $(v_1)^{(1)}, (v_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, v^{(1)}, u^{(1)}$:

By $(v_1)^{(1)} > 0, (v_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0, (u_2)^{(1)} < 0$ the roots of the equations

$$(a_{14})^{(1)}(v^{(1)})^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)} = 0 \text{ and } (b_{14})^{(1)}(u^{(1)})^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$$

Definition of $(\bar{v}_1)^{(1)}, (\bar{v}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$:

By $(\bar{v}_1)^{(1)} > 0, (\bar{v}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0, (\bar{u}_2)^{(1)} < 0$ the roots of the equations

$$(a_{14})^{(1)}(v^{(1)})^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)} = 0 \text{ and } (b_{14})^{(1)}(u^{(1)})^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$$

Definition of $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (v_0)^{(1)}$:-

If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by

$$(m_2)^{(1)} = (v_0)^{(1)}, (m_1)^{(1)} = (v_1)^{(1)}, \text{ if } (v_0)^{(1)} < (v_1)^{(1)}$$

$$(m_2)^{(1)} = (v_1)^{(1)}, (m_1)^{(1)} = (\bar{v}_1)^{(1)}, \text{ if } (v_1)^{(1)} < (v_0)^{(1)} < (\bar{v}_1)^{(1)},$$

$$\text{and } (v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}$$

$$(m_2)^{(1)} = (v_1)^{(1)}, (m_1)^{(1)} = (v_0)^{(1)}, \text{ if } (\bar{v}_1)^{(1)} < (v_0)^{(1)}$$

and analogously

$$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)}, \text{ if } (u_0)^{(1)} < (u_1)^{(1)}$$

$$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, \text{ if } (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)},$$

$$\text{and } (u_0)^{(1)} = \frac{T_{13}^0}{T_{14}^0}$$

$$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, \text{ if } (\bar{u}_1)^{(1)} < (u_0)^{(1)} \text{ where } (u_1)^{(1)}, (\bar{u}_1)^{(1)}$$

are defined by 59 and 61 respectively

Then the solution satisfies the inequalities

$$G_{13}^0 e^{((s_1)^{(1)} - (p_{13})^{(1)})t} \leq G_{13}(t) \leq G_{13}^0 e^{(s_1)^{(1)}t}$$

where $(p_i)^{(1)}$ is defined by equation 25

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((s_1)^{(1)} - (p_{13})^{(1)})t} \leq G_{14}(t) \leq \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(s_1)^{(1)}t}$$

$$\left(\frac{(a_{15})^{(1)} G_{13}^0}{(m_1)^{(1)} ((s_1)^{(1)} - (p_{13})^{(1)} - (s_2)^{(1)})} \left[e^{((s_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(s_2)^{(1)}t} \right] + G_{15}^0 e^{-(s_2)^{(1)}t} \right) \leq G_{15}(t) \leq$$

$$\frac{(a_{15})^{(1)} G_{13}^0}{(m_2)^{(1)} ((s_1)^{(1)} - (a'_{15})^{(1)})} \left[e^{(s_1)^{(1)}t} - e^{-(a'_{15})^{(1)}t} \right] + G_{15}^0 e^{-(a'_{15})^{(1)}t}$$

$$\boxed{T_{13}^0 e^{(R_1)^{(1)}t} \leq T_{13}(t) \leq T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t}}$$

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)}t} \leq T_{13}(t) \leq \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)}+(r_{13})^{(1)})t}$$

$$\frac{(b_{15})^{(1)} T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)}-(b'_{15})^{(1)})} \left[e^{(R_1)^{(1)}t} - e^{-(b'_{15})^{(1)}t} \right] + T_{15}^0 e^{-(b'_{15})^{(1)}t} \leq T_{15}(t) \leq$$

$$\frac{(a_{15})^{(1)} T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)}+(r_{13})^{(1)}+(R_2)^{(1)})} \left[e^{((R_1)^{(1)}+(r_{13})^{(1)})t} - e^{-(R_2)^{(1)}t} \right] + T_{15}^0 e^{-(R_2)^{(1)}t}$$

Definition of $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:-

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$

$$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$$

$$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$$

$$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$$

Behavior of the solutions

If we denote and define

Definition of $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$:

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$(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying

$$-(\sigma_2)^{(2)} \leq -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(T_{17}, t) + (a'_{17})^{(2)}(T_{17}, t) \leq -(\sigma_1)^{(2)}$$

$$-(\tau_2)^{(2)} \leq -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}((G_{19}), t) - (b'_{17})^{(2)}((G_{19}), t) \leq -(\tau_1)^{(2)}$$

Definition of $(v_1)^{(2)}, (v_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$:

By $(v_1)^{(2)} > 0, (v_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0, (u_2)^{(2)} < 0$ the roots

(e) of the equations $(a_{17})^{(2)}(v^{(2)})^2 + (\sigma_1)^{(2)}v^{(2)} - (a_{16})^{(2)} = 0$

and $(b_{14})^{(2)}(u^{(2)})^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and

Definition of $(\bar{v}_1)^{(2)}, (\bar{v}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$:

By $(\bar{v}_1)^{(2)} > 0, (\bar{v}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0, (\bar{u}_2)^{(2)} < 0$ the

roots of the equations $(a_{17})^{(2)}(v^{(2)})^2 + (\sigma_2)^{(2)}v^{(2)} - (a_{16})^{(2)} = 0$

and $(b_{17})^{(2)}(u^{(2)})^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$

Definition of $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$:-

(f) If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by

$$(m_2)^{(2)} = (v_0)^{(2)}, (m_1)^{(2)} = (v_1)^{(2)}, \text{ if } (v_0)^{(2)} < (v_1)^{(2)}$$

$$(m_2)^{(2)} = (v_1)^{(2)}, (m_1)^{(2)} = (\bar{v}_1)^{(2)}, \text{ if } (v_1)^{(2)} < (v_0)^{(2)} < (\bar{v}_1)^{(2)},$$

and $(v_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$

$$(m_2)^{(2)} = (v_1)^{(2)}, (m_1)^{(2)} = (v_0)^{(2)}, \text{ if } (\bar{v}_1)^{(2)} < (v_0)^{(2)}$$

and analogously

$$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}, \text{ if } (u_0)^{(2)} < (u_1)^{(2)}$$

$$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}, \text{ if } (u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)},$$

and $(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}$

$$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)}, \text{ if } (\bar{u}_1)^{(2)} < (u_0)^{(2)}$$

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \leq G_{16}(t) \leq G_{16}^0 e^{(S_1)^{(2)}t}$$

$(p_i)^{(2)}$ is defined by equation IN THE FOREGOING

$$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \leq G_{17}(t) \leq \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)}t}$$

$$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})} \left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)}t} \right] + G_{18}^0 e^{-(S_2)^{(2)}t} \right) \leq G_{18}(t) \leq$$

$$\frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a'_{18})^{(2)})} \left[e^{(S_1)^{(2)}t} - e^{-(a'_{18})^{(2)}t} \right] + G_{18}^0 e^{-(a'_{18})^{(2)}t}$$

$$\boxed{T_{16}^0 e^{(R_1)^{(2)}t} \leq T_{16}(t) \leq T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}}$$

$$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)}t} \leq T_{16}(t) \leq \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}$$

$$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b'_{18})^{(2)})} \left[e^{(R_1)^{(2)}t} - e^{-(b'_{18})^{(2)}t} \right] + T_{18}^0 e^{-(b'_{18})^{(2)}t} \leq T_{18}(t) \leq$$

$$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})} \left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)}t} \right] + T_{18}^0 e^{-(R_2)^{(2)}t}$$

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:-

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$

$$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$$

$$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)}$$

$$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$$

Behavior of the solutions

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Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$:

$\sigma_1^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \leq -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \leq -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \leq -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G, t) - (b''_{21})^{(3)}((G_{23}), t) \leq -(\tau_1)^{(3)}$$

Definition of $(v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$:

By $(v_1)^{(3)} > 0, (v_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0, (u_2)^{(3)} < 0$ the roots of the equations

$$(a_{21})^{(3)}(v^{(3)})^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$$

$$\text{and } (b_{21})^{(3)}(u^{(3)})^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0 \text{ and}$$

By $(\bar{v}_1)^{(3)} > 0, (\bar{v}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0, (\bar{u}_2)^{(3)} < 0$ the

$$\text{roots of the equations } (a_{21})^{(3)}(v^{(3)})^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$$

$$\text{and } (b_{21})^{(3)}(u^{(3)})^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$$

Definition of $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$:-

If we define $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$$(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}, \text{ if } (v_0)^{(3)} < (v_1)^{(3)}$$

$$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}, \text{ if } (v_1)^{(3)} < (v_0)^{(3)} < (\bar{v}_1)^{(3)},$$

and
$$(v_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}$$

$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}, \text{ if } (v_1)^{(3)} < (v_0)^{(3)}$

and analogously

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}, \text{ if } (u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}, \text{ if } (u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}, \text{ and } (u_0)^{(3)} = \frac{T_{20}^0}{T_{21}^0}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}, \text{ if } (\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \leq G_{20}(t) \leq G_{20}^0 e^{(S_1)^{(3)}t}$

$(p_i)^{(3)}$ is defined by equation IN THE FOREGOING

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \leq G_{21}(t) \leq \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)}t}$$

$$\left(\frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})} \left[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)}t} \right] + G_{22}^0 e^{-(S_2)^{(3)}t} \right) \leq G_{22}(t) \leq$$

$$\frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})} \left[e^{(S_1)^{(3)}t} - e^{-(a'_{22})^{(3)}t} \right] + G_{22}^0 e^{-(a'_{22})^{(3)}t}$$

$$T_{20}^0 e^{(R_1)^{(3)}t} \leq T_{20}(t) \leq T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}$$

$$\frac{1}{(\mu_1)^{(3)}} T_{20}^0 e^{(R_1)^{(3)}t} \leq T_{20}(t) \leq \frac{1}{(\mu_2)^{(3)}} T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}$$

$$\frac{(b_{22})^{(3)} T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)} - (b'_{22})^{(3)})} \left[e^{(R_1)^{(3)}t} - e^{-(b'_{22})^{(3)}t} \right] + T_{22}^0 e^{-(b'_{22})^{(3)}t} \leq T_{22}(t) \leq$$

$$\frac{(a_{22})^{(3)} T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)} + (r_{20})^{(3)} + (R_2)^{(3)})} \left[e^{((R_1)^{(3)} + (r_{20})^{(3)})t} - e^{-(R_2)^{(3)}t} \right] + T_{22}^0 e^{-(R_2)^{(3)}t}$$

Definition of $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:-

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$

$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$

Behavior of the solutions

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$:

$(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$-(\sigma_2)^{(4)} \leq -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \leq -(\sigma_1)^{(4)}$

$-(\tau_2)^{(4)} \leq -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}(G_{27}, t) - (b''_{25})^{(4)}(G_{27}, t) \leq -(\tau_1)^{(4)}$

Definition of $(v_1)^{(4)}, (v_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, v^{(4)}, u^{(4)}$:

By $(v_1)^{(4)} > 0, (v_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0, (u_2)^{(4)} < 0$ the roots of the equations

$(a_{25})^{(4)}(v^{(4)})^2 + (\sigma_1)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$

and $(b_{25})^{(4)}(u^{(4)})^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

Definition of $(\bar{v}_1)^{(4)}, (\bar{v}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$:

By $(\bar{v}_1)^{(4)} > 0, (\bar{v}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0, (\bar{u}_2)^{(4)} < 0$ the

roots of the equations $(a_{25})^{(4)}(v^{(4)})^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$

and $(b_{25})^{(4)}(u^{(4)})^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

Definition of $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}, (v_0)^{(4)}$:-

(f) If we define $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (v_0)^{(4)}, (m_1)^{(4)} = (v_1)^{(4)}$, **if** $(v_0)^{(4)} < (v_1)^{(4)}$

$(m_2)^{(4)} = (v_1)^{(4)}, (m_1)^{(4)} = (\bar{v}_1)^{(4)}$, **if** $(v_4)^{(4)} < (v_0)^{(4)} < (\bar{v}_1)^{(4)}$,

and $(v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}$

$(m_2)^{(4)} = (v_4)^{(4)}, (m_1)^{(4)} = (v_0)^{(4)}$, **if** $(\bar{v}_4)^{(4)} < (v_0)^{(4)}$

and analogously

$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)}$, **if** $(u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}$, **if** $(u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)}$,

and $(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}$, **if** $(\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$

are defined respectively

Then the solution satisfies the inequalities

$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \leq G_{24}(t) \leq G_{24}^0 e^{(S_1)^{(4)}t}$$

where $(p_i)^{(4)}$ is defined by equation IN THE FOREGOING:

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \leq G_{25}(t) \leq \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)}t}$$

$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)} ((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})} \left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)}t} \right] + G_{26}^0 e^{-(S_2)^{(4)}t} \right) \leq G_{26}(t) \leq$$

$$\frac{(a_{26})^{(4)} G_{24}^0}{(m_2)^{(4)} ((S_1)^{(4)} - (a'_{26})^{(4)})} \left[e^{(S_1)^{(4)}t} - e^{-(a'_{26})^{(4)}t} \right] + G_{26}^0 e^{-(a'_{26})^{(4)}t}$$

$$\boxed{T_{24}^0 e^{(R_1)^{(4)}t} \leq T_{24}(t) \leq T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}$$

$$\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{(R_1)^{(4)}t} \leq T_{24}(t) \leq \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}$$

$$\frac{(b_{26})^{(4)} T_{24}^0}{(\mu_1)^{(4)} ((R_1)^{(4)} - (b'_{26})^{(4)})} \left[e^{(R_1)^{(4)}t} - e^{-(b'_{26})^{(4)}t} \right] + T_{26}^0 e^{-(b'_{26})^{(4)}t} \leq T_{26}(t) \leq$$

$$\frac{(a_{26})^{(4)} T_{24}^0}{(\mu_2)^{(4)} ((R_1)^{(4)} + (r_{24})^{(4)} + (R_2)^{(4)})} \left[e^{((R_1)^{(4)} + (r_{24})^{(4)})t} - e^{-(R_2)^{(4)}t} \right] + T_{26}^0 e^{-(R_2)^{(4)}t}$$

Definition of $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:-

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$

Behavior of the solutions

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$:

- (g) $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying
 $-(\sigma_2)^{(5)} \leq -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \leq -(\sigma_1)^{(5)}$
 $-(\tau_2)^{(5)} \leq -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}((G_{31}), t) - (b''_{29})^{(5)}((G_{31}), t) \leq -(\tau_1)^{(5)}$

Definition of $(v_1)^{(5)}, (v_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, v^{(5)}, u^{(5)}$:

- (h) By $(v_1)^{(5)} > 0, (v_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0, (u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}(v^{(5)})^2 + (\sigma_1)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$ and $(b_{29})^{(5)}(u^{(5)})^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

Definition of $(\bar{v}_1)^{(5)}, (\bar{v}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$:

By $(\bar{v}_1)^{(5)} > 0, (\bar{v}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0, (\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}(v^{(5)})^2 + (\sigma_2)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$ and $(b_{29})^{(5)}(u^{(5)})^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

Definition of $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (v_0)^{(5)}$:-

- (i) If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by
 $(m_2)^{(5)} = (v_0)^{(5)}, (m_1)^{(5)} = (v_1)^{(5)}$, **if** $(v_0)^{(5)} < (v_1)^{(5)}$
 $(m_2)^{(5)} = (v_1)^{(5)}, (m_1)^{(5)} = (\bar{v}_1)^{(5)}$, **if** $(v_1)^{(5)} < (v_0)^{(5)} < (\bar{v}_1)^{(5)}$, and $(v_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}$
 $(m_2)^{(5)} = (v_1)^{(5)}, (m_1)^{(5)} = (v_0)^{(5)}$, **if** $(\bar{v}_1)^{(5)} < (v_0)^{(5)}$
 and analogously
 $(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)}$, **if** $(u_0)^{(5)} < (u_1)^{(5)}$
 $(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}$, **if** $(u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)}$, and $(u_0)^{(5)} = \frac{T_{28}^0}{T_{29}^0}$
 $(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}$, **if** $(\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$ are defined by respectively

Then the solution satisfies the inequalities

$$G_{28}^0 e^{((s_1)^{(5)} - (p_{28})^{(5)})t} \leq G_{28}(t) \leq G_{28}^0 e^{(s_1)^{(5)}t}$$

where $(p_i)^{(5)}$ is defined by equation IN THE FOREGOING

$$\frac{1}{(m_5)^{(5)}} G_{28}^0 e^{((s_1)^{(5)} - (p_{28})^{(5)})t} \leq G_{29}(t) \leq \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(s_1)^{(5)}t}$$

$$\left(\frac{(a_{30})^{(5)} G_{28}^0}{(m_1)^{(5)} ((s_1)^{(5)} - (p_{28})^{(5)}) - (s_2)^{(5)}} \left[e^{((s_1)^{(5)} - (p_{28})^{(5)})t} - e^{-(s_2)^{(5)}t} \right] + G_{30}^0 e^{-(s_2)^{(5)}t} \leq G_{30}(t) \leq \frac{(a_{30})^{(5)} G_{28}^0}{(m_2)^{(5)} ((s_1)^{(5)} - (a'_{30})^{(5)})} \left[e^{(s_1)^{(5)}t} - e^{-(a'_{30})^{(5)}t} \right] + G_{30}^0 e^{-(a'_{30})^{(5)}t} \right)$$

$$\boxed{T_{28}^0 e^{(R_1)^{(5)}t} \leq T_{28}(t) \leq T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}$$

$$\frac{1}{(\mu_1)^{(5)}} T_{28}^0 e^{(R_1)^{(5)}t} \leq T_{28}(t) \leq \frac{1}{(\mu_2)^{(5)}} T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}$$

$$\frac{(b_{30})^{(5)} T_{28}^0}{(\mu_1)^{(5)} ((R_1)^{(5)} - (b'_{30})^{(5)})} \left[e^{(R_1)^{(5)}t} - e^{-(b'_{30})^{(5)}t} \right] + T_{30}^0 e^{-(b'_{30})^{(5)}t} \leq T_{30}(t) \leq$$

$$\frac{(a_{30})^{(5)} T_{28}^0}{(\mu_2)^{(5)} ((R_1)^{(5)} + (r_{28})^{(5)} + (R_2)^{(5)})} \left[e^{((R_1)^{(5)} + (r_{28})^{(5)})t} - e^{-(R_2)^{(5)}t} \right] + T_{30}^0 e^{-(R_2)^{(5)}t}$$

Definition of $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:-

Where $(S_1)^{(5)} = (a_{28})^{(5)}(m_2)^{(5)} - (a'_{28})^{(5)}$

$$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$$

$$(R_1)^{(5)} = (b_{28})^{(5)}(\mu_2)^{(5)} - (b'_{28})^{(5)}$$

$$(R_2)^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)}$$

Behavior of the solutions

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Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$:

(j) $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)} \leq -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \leq -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)} \leq -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}((G_{35}), t) - (b''_{33})^{(6)}((G_{35}), t) \leq -(\tau_1)^{(6)}$$

Definition of $(v_1)^{(6)}, (v_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, v^{(6)}, u^{(6)}$:

(k) By $(v_1)^{(6)} > 0, (v_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0, (u_2)^{(6)} < 0$ the roots of the

equations $(a_{33})^{(6)}(v^{(6)})^2 + (\sigma_1)^{(6)}v^{(6)} - (a_{32})^{(6)} = 0$

and $(b_{33})^{(6)}(u^{(6)})^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$ and

Definition of $(\bar{v}_1)^{(6)}, (\bar{v}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}$:

By $(\bar{v}_1)^{(6)} > 0, (\bar{v}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0, (\bar{u}_2)^{(6)} < 0$ the

roots of the equations $(a_{33})^{(6)}(v^{(6)})^2 + (\sigma_2)^{(6)}v^{(6)} - (a_{32})^{(6)} = 0$

and $(b_{33})^{(6)}(u^{(6)})^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

Definition of $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}, (v_0)^{(6)}$:-

(l) If we define $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$$(m_2)^{(6)} = (v_0)^{(6)}, (m_1)^{(6)} = (v_1)^{(6)}, \text{ if } (v_0)^{(6)} < (v_1)^{(6)}$$

$$(m_2)^{(6)} = (v_1)^{(6)}, (m_1)^{(6)} = (\bar{v}_6)^{(6)}, \text{ if } (v_1)^{(6)} < (v_0)^{(6)} < (\bar{v}_1)^{(6)}, \text{ and } (v_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}$$

$$(m_2)^{(6)} = (v_1)^{(6)}, (m_1)^{(6)} = (v_0)^{(6)}, \text{ if } (\bar{v}_1)^{(6)} < (v_0)^{(6)}$$

and analogously

$$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}, \text{ if } (u_0)^{(6)} < (u_1)^{(6)}$$

$$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}, \text{ if } (u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)}, \text{ and } (u_0)^{(6)} = \frac{T_{32}^0}{T_{33}^0}$$

$$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}, \text{ if } (\bar{u}_1)^{(6)} < (u_0)^{(6)} \text{ where } (u_1)^{(6)}, (\bar{u}_1)^{(6)} \text{ are defined}$$

respectively

Then the solution satisfies the inequalities

$$G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \leq G_{32}(t) \leq G_{32}^0 e^{(S_1)^{(6)}t}$$

where $(p_i)^{(6)}$ is defined by equation IN THE FOREGOING

$$\frac{1}{(m_1)^{(6)}} G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \leq G_{33}(t) \leq \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{(S_1)^{(6)}t}$$

$$\left(\frac{(a_{34})^{(6)} G_{32}^0}{(m_1)^{(6)} ((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})} \right) \left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)}t} \right] + G_{34}^0 e^{-(S_2)^{(6)}t} \leq G_{34}(t) \leq$$

$$\frac{(a_{34})^{(6)}G_{32}^0}{(m_2)^{(6)}((S_1)^{(6)}-(a'_{34})^{(6)})} \left[e^{(S_1)^{(6)}t} - e^{-(a'_{34})^{(6)}t} \right] + G_{34}^0 e^{-(a'_{34})^{(6)}t}$$

$$\boxed{T_{32}^0 e^{(R_1)^{(6)}t} \leq T_{32}(t) \leq T_{32}^0 e^{((R_1)^{(6)}+(r_{32})^{(6)})t}}$$

$$\frac{1}{(\mu_1)^{(6)}} T_{32}^0 e^{(R_1)^{(6)}t} \leq T_{32}(t) \leq \frac{1}{(\mu_2)^{(6)}} T_{32}^0 e^{((R_1)^{(6)}+(r_{32})^{(6)})t}$$

$$\frac{(b_{34})^{(6)}T_{32}^0}{(\mu_1)^{(6)}((R_1)^{(6)}-(b'_{34})^{(6)})} \left[e^{(R_1)^{(6)}t} - e^{-(b'_{34})^{(6)}t} \right] + T_{34}^0 e^{-(b'_{34})^{(6)}t} \leq T_{34}(t) \leq$$

$$\frac{(a_{34})^{(6)}T_{32}^0}{(\mu_2)^{(6)}((R_1)^{(6)}+(r_{32})^{(6)}+(R_2)^{(6)})} \left[e^{((R_1)^{(6)}+(r_{32})^{(6)})t} - e^{-(R_2)^{(6)}t} \right] + T_{34}^0 e^{-(R_2)^{(6)}t}$$

Definition of $(S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}$:-

Where $(S_1)^{(6)} = (a_{32})^{(6)}(m_2)^{(6)} - (a'_{32})^{(6)}$

$$(S_2)^{(6)} = (a_{34})^{(6)} - (p_{34})^{(6)}$$

$$(R_1)^{(6)} = (b_{32})^{(6)}(\mu_2)^{(6)} - (b'_{32})^{(6)}$$

$$(R_2)^{(6)} = (b'_{34})^{(6)} - (r_{34})^{(6)}$$

Proof : From GLOBAL EQUATIONS we obtain

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$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14}, t) \right) - (a''_{14})^{(1)}(T_{14}, t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

Definition of $v^{(1)}$:- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$- \left((a_{14})^{(1)}(v^{(1)})^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)} \right) \leq \frac{dv^{(1)}}{dt} \leq - \left((a_{14})^{(1)}(v^{(1)})^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)} \right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$:-

(a) For $0 < \boxed{(v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \geq \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)} e^{-(a_{14})^{(1)}((v_1)^{(1)} - (v_0)^{(1)})t}}{1 + (C)^{(1)} e^{-(a_{14})^{(1)}((v_1)^{(1)} - (v_0)^{(1)})t}}, \quad \boxed{(C)^{(1)} = \frac{(v_1)^{(1)} - (v_0)^{(1)}}{(v_0)^{(1)} - (v_2)^{(1)}}$$

$$\text{it follows } (v_0)^{(1)} \leq v^{(1)}(t) \leq (v_1)^{(1)}$$

In the same manner , we get

$$v^{(1)}(t) \leq \frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)} e^{-(a_{14})^{(1)}((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)})t}}{1 + (\bar{C})^{(1)} e^{-(a_{14})^{(1)}((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)})t}}, \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{v}_1)^{(1)} - (v_0)^{(1)}}{(v_0)^{(1)} - (\bar{v}_2)^{(1)}}$$

From which we deduce $(v_0)^{(1)} \leq v^{(1)}(t) \leq (\bar{v}_1)^{(1)}$

(b) If $0 < (v_1)^{(1)} < (v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0} < (\bar{v}_1)^{(1)}$ we find like in the previous case,

$$(v_1)^{(1)} \leq \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)} e^{-(a_{14})^{(1)}((v_1)^{(1)} - (v_2)^{(1)})t}}{1 + (C)^{(1)} e^{-(a_{14})^{(1)}((v_1)^{(1)} - (v_2)^{(1)})t}} \leq v^{(1)}(t) \leq$$

$$\frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)} e^{-(a_{14})^{(1)}((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)})t}}{1 + (\bar{C})^{(1)} e^{-(a_{14})^{(1)}((\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)})t}} \leq (\bar{v}_1)^{(1)}$$

(c) If $0 < (v_1)^{(1)} \leq (\bar{v}_1)^{(1)} \leq \boxed{(v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$, we obtain

$$(\nu_1)^{(1)} \leq \nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)} + (\bar{C})^{(1)}(\bar{\nu}_2)^{(1)} e^{[-(a_{14})^{(1)}((\bar{\nu}_1)^{(1)} - (\bar{\nu}_2)^{(1)})t]}}{1 + (\bar{C})^{(1)} e^{[-(a_{14})^{(1)}((\bar{\nu}_1)^{(1)} - (\bar{\nu}_2)^{(1)})t]}} \leq (\nu_0)^{(1)}$$

And so with the notation of the first part of condition (c), we have

Definition of $\nu^{(1)}(t)$:-

$$(m_2)^{(1)} \leq \nu^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{\nu^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

Particular case : If $(a''_{13})^{(1)} = (a''_{14})^{(1)}$, then $(\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)}G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}$, then $(\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$ if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)}T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

Proof : From GLOBAL EQUATIONS we obtain

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$$\frac{dv^{(2)}}{dt} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(T_{17}, t) \right) - (a''_{17})^{(2)}(T_{17}, t)v^{(2)} - (a_{17})^{(2)}v^{(2)}$$

Definition of $\nu^{(2)}$:- $\boxed{\nu^{(2)} = \frac{G_{16}}{G_{17}}}$

It follows

$$- \left((a_{17})^{(2)}(v^{(2)})^2 + (\sigma_2)^{(2)}v^{(2)} - (a_{16})^{(2)} \right) \leq \frac{dv^{(2)}}{dt} \leq - \left((a_{17})^{(2)}(v^{(2)})^2 + (\sigma_1)^{(2)}v^{(2)} - (a_{16})^{(2)} \right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$:-

(d) For $0 < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \geq \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)} e^{[-(a_{17})^{(2)}((\nu_1)^{(2)} - (\nu_0)^{(2)})t]}}{1 + (C)^{(2)} e^{[-(a_{17})^{(2)}((\nu_1)^{(2)} - (\nu_0)^{(2)})t]}} , \quad \boxed{(C)^{(2)} = \frac{(\nu_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\nu_2)^{(2)}}$$

it follows $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\nu_1)^{(2)}$

In the same manner, we get

$$\nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)} + (\bar{C})^{(2)}(\bar{\nu}_2)^{(2)} e^{[-(a_{17})^{(2)}((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)})t]}}{1 + (\bar{C})^{(2)} e^{[-(a_{17})^{(2)}((\bar{\nu}_1)^{(2)} - (\bar{\nu}_2)^{(2)})t]}} , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)} - (\nu_0)^{(2)}}{(\nu_0)^{(2)} - (\bar{\nu}_2)^{(2)}}$$

From which we deduce $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\bar{\nu}_1)^{(2)}$

(e) If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case,

$$(\nu_1)^{(2)} \leq \frac{(\nu_1)^{(2)} + (C)^{(2)}(\nu_2)^{(2)} e^{[-(a_{17})^{(2)}((\nu_1)^{(2)} - (\nu_2)^{(2)})t]}}{1 + (C)^{(2)} e^{[-(a_{17})^{(2)}((\nu_1)^{(2)} - (\nu_2)^{(2)})t]}} \leq \nu^{(2)}(t) \leq$$

$$\frac{(\bar{v}_1)^{(2)} + (\bar{C})^{(2)} (\bar{v}_2)^{(2)} e^{[-(a_{17})^{(2)} (\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)}] t}}{1 + (\bar{C})^{(2)} e^{[-(a_{17})^{(2)} (\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)}] t}} \leq (\bar{v}_1)^{(2)}$$

(f) If $0 < (v_1)^{(2)} \leq (\bar{v}_1)^{(2)} \leq (v_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$, we obtain

$$(v_1)^{(2)} \leq v^{(2)}(t) \leq \frac{(\bar{v}_1)^{(2)} + (\bar{C})^{(2)} (\bar{v}_2)^{(2)} e^{[-(a_{17})^{(2)} (\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)}] t}}{1 + (\bar{C})^{(2)} e^{[-(a_{17})^{(2)} (\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)}] t}} \leq (v_0)^{(2)}$$

And so with the notation of the first part of condition (c), we have

Definition of $v^{(2)}(t)$:-

$$(m_2)^{(2)} \leq v^{(2)}(t) \leq (m_1)^{(2)}, \quad \boxed{v^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(2)}(t)$:-

$$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}, \quad \boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$$

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

Particular case :

If $(a''_{16})^{(2)} = (a''_{17})^{(2)}$, then $(\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(v_1)^{(2)} = (\bar{v}_1)^{(2)}$ if in addition $(v_0)^{(2)} = (v_1)^{(2)}$ then $v^{(2)}(t) = (v_0)^{(2)}$ and as a consequence $G_{16}(t) = (v_0)^{(2)} G_{17}(t)$

Analogously if $(b''_{16})^{(2)} = (b''_{17})^{(2)}$, then $(\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$ if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)} T_{17}(t)$ This is an important consequence of the relation between $(v_1)^{(2)}$ and $(\bar{v}_1)^{(2)}$

Proof : From GLOBAL EQUATIONS we obtain

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$$\frac{dv^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)} (T_{21}, t) \right) - (a''_{21})^{(3)} (T_{21}, t) v^{(3)} - (a_{21})^{(3)} v^{(3)}$$

Definition of $v^{(3)}$:- $\boxed{v^{(3)} = \frac{G_{20}}{G_{21}}}$

It follows

$$- \left((a_{21})^{(3)} (v^{(3)})^2 + (\sigma_2)^{(3)} v^{(3)} - (a_{20})^{(3)} \right) \leq \frac{dv^{(3)}}{dt} \leq - \left((a_{21})^{(3)} (v^{(3)})^2 + (\sigma_1)^{(3)} v^{(3)} - (a_{20})^{(3)} \right)$$

From which one obtains

For $0 < (v_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (v_1)^{(3)} < (\bar{v}_1)^{(3)}$

$$v^{(3)}(t) \geq \frac{(v_1)^{(3)} + (C)^{(3)} (v_2)^{(3)} e^{[-(a_{21})^{(3)} ((v_1)^{(3)} - (v_0)^{(3)}) t]}}{1 + (C)^{(3)} e^{[-(a_{21})^{(3)} ((v_1)^{(3)} - (v_0)^{(3)}) t]}} , \quad \boxed{(C)^{(3)} = \frac{(v_1)^{(3)} - (v_0)^{(3)}}{(v_0)^{(3)} - (v_2)^{(3)}}$$

it follows $(v_0)^{(3)} \leq v^{(3)}(t) \leq (v_1)^{(3)}$

In the same manner, we get

$$v^{(3)}(t) \leq \frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)} (\bar{v}_2)^{(3)} e^{[-(a_{21})^{(3)} ((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}) t]}}{1 + (\bar{C})^{(3)} e^{[-(a_{21})^{(3)} ((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}) t]}} , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{v}_1)^{(3)} - (v_0)^{(3)}}{(v_0)^{(3)} - (\bar{v}_2)^{(3)}}$$

Definition of $(\bar{v}_1)^{(3)}$:-

From which we deduce $(v_0)^{(3)} \leq v^{(3)}(t) \leq (\bar{v}_1)^{(3)}$

If $0 < (v_1)^{(3)} < (v_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\bar{v}_1)^{(3)}$ we find like in the previous case,

$$(v_1)^{(3)} \leq \frac{(v_1)^{(3)} + (C)^{(3)}(v_2)^{(3)} e^{[-(a_{21})^{(3)}((v_1)^{(3)} - (v_2)^{(3)})t]}}{1 + (C)^{(3)} e^{[-(a_{21})^{(3)}((v_1)^{(3)} - (v_2)^{(3)})t]}} \leq v^{(3)}(t) \leq \frac{(\bar{v}_1)^{(3)} + (C)^{(3)}(\bar{v}_2)^{(3)} e^{[-(a_{21})^{(3)}((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)})t]}}{1 + (C)^{(3)} e^{[-(a_{21})^{(3)}((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)})t]}} \leq (\bar{v}_1)^{(3)}$$

If $0 < (v_1)^{(3)} \leq (\bar{v}_1)^{(3)} \leq (v_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}$, we obtain

$$(v_1)^{(3)} \leq (t) \leq \frac{(\bar{v}_1)^{(3)} + (C)^{(3)}(\bar{v}_2)^{(3)} e^{[-(a_{21})^{(3)}((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)})t]}}{1 + (C)^{(3)} e^{[-(a_{21})^{(3)}((\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)})t]}} \leq (v_0)^{(3)}$$

And so with the notation of the first part of condition (c), we have

Definition of $v^{(3)}(t)$:-

$$(m_2)^{(3)} \leq v^{(3)}(t) \leq (m_1)^{(3)}, \quad \boxed{v^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(3)}(t)$:-

$$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}, \quad \boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$$

Now, using this result and replacing it in GLOBAL EQUATIONMS we get easily the result stated in the theorem.

Particular case :

If $(a''_{20})^{(3)} = (a''_{21})^{(3)}$, then $(\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(v_1)^{(3)} = (\bar{v}_1)^{(3)}$ if in addition $(v_0)^{(3)} = (v_1)^{(3)}$ then $v^{(3)}(t) = (v_0)^{(3)}$ and as a consequence $G_{20}(t) = (v_0)^{(3)}G_{21}(t)$

Analogously if $(b''_{20})^{(3)} = (b''_{21})^{(3)}$, then $(\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$ if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)}T_{21}(t)$ This is an important consequence of the relation between $(v_1)^{(3)}$ and $(\bar{v}_1)^{(3)}$

Proof : From GLOBAL EQUATIONS we obtain

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$$\frac{dv^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25}, t) \right) - (a''_{25})^{(4)}(T_{25}, t)v^{(4)} - (a_{25})^{(4)}v^{(4)}$$

Definition of $v^{(4)}$:- $\boxed{v^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$- \left((a_{25})^{(4)}(v^{(4)})^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)} \right) \leq \frac{dv^{(4)}}{dt} \leq - \left((a_{25})^{(4)}(v^{(4)})^2 + (\sigma_1)^{(4)}v^{(4)} - (a_{24})^{(4)} \right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(4)}, (v_0)^{(4)}$:-

(d) For $0 < \boxed{(v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}} < (v_1)^{(4)} < (\bar{v}_1)^{(4)}$

$$v^{(4)}(t) \geq \frac{(v_1)^{(4)} + (C)^{(4)}(v_2)^{(4)} e^{[-(a_{25})^{(4)}((v_1)^{(4)} - (v_0)^{(4)})t]}}{4 + (C)^{(4)} e^{[-(a_{25})^{(4)}((v_1)^{(4)} - (v_0)^{(4)})t]}} , \quad \boxed{(C)^{(4)} = \frac{(v_1)^{(4)} - (v_0)^{(4)}}{(v_0)^{(4)} - (v_2)^{(4)}}$$

it follows $(v_0)^{(4)} \leq v^{(4)}(t) \leq (v_1)^{(4)}$

In the same manner, we get

$$v^{(4)}(t) \leq \frac{(\bar{v}_1)^{(4)} + (\bar{c})^{(4)} (\bar{v}_2)^{(4)} e^{[-(a_{25})^{(4)} (\bar{v}_1)^{(4)} - (\bar{v}_2)^{(4)}] t}}{4 + (\bar{c})^{(4)} e^{[-(a_{25})^{(4)} (\bar{v}_1)^{(4)} - (\bar{v}_2)^{(4)}] t}}, \quad \boxed{(\bar{c})^{(4)} = \frac{(\bar{v}_1)^{(4)} - (v_0)^{(4)}}{(v_0)^{(4)} - (\bar{v}_2)^{(4)}}$$

From which we deduce $(v_0)^{(4)} \leq v^{(4)}(t) \leq (\bar{v}_1)^{(4)}$

(e) If $0 < (v_1)^{(4)} < (v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{v}_1)^{(4)}$ we find like in the previous case,

$$(v_1)^{(4)} \leq \frac{(\bar{v}_1)^{(4)} + (\bar{c})^{(4)} (v_2)^{(4)} e^{[-(a_{25})^{(4)} ((v_1)^{(4)} - (v_2)^{(4)}) t]}}{1 + (\bar{c})^{(4)} e^{[-(a_{25})^{(4)} ((v_1)^{(4)} - (v_2)^{(4)}) t]}} \leq v^{(4)}(t) \leq \frac{(\bar{v}_1)^{(4)} + (\bar{c})^{(4)} (\bar{v}_2)^{(4)} e^{[-(a_{25})^{(4)} ((\bar{v}_1)^{(4)} - (\bar{v}_2)^{(4)}) t]}}{1 + (\bar{c})^{(4)} e^{[-(a_{25})^{(4)} ((\bar{v}_1)^{(4)} - (\bar{v}_2)^{(4)}) t]}} \leq (\bar{v}_1)^{(4)}$$

(f) If $0 < (v_1)^{(4)} \leq (\bar{v}_1)^{(4)} \leq \boxed{(v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$, we obtain

$$(v_1)^{(4)} \leq v^{(4)}(t) \leq \frac{(\bar{v}_1)^{(4)} + (\bar{c})^{(4)} (\bar{v}_2)^{(4)} e^{[-(a_{25})^{(4)} ((\bar{v}_1)^{(4)} - (\bar{v}_2)^{(4)}) t]}}{1 + (\bar{c})^{(4)} e^{[-(a_{25})^{(4)} ((\bar{v}_1)^{(4)} - (\bar{v}_2)^{(4)}) t]}} \leq (v_0)^{(4)}$$

And so with the notation of the first part of condition (c), we have

Definition of $v^{(4)}(t)$:-

$$(m_2)^{(4)} \leq v^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{v^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(4)}(t)$:-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

Particular case :

If $(a''_{24})^{(4)} = (a''_{25})^{(4)}$, then $(\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(v_1)^{(4)} = (\bar{v}_1)^{(4)}$ if in addition $(v_0)^{(4)} = (v_1)^{(4)}$ then $v^{(4)}(t) = (v_0)^{(4)}$ and as a consequence $G_{24}(t) = (v_0)^{(4)} G_{25}(t)$ **this also defines** $(v_0)^{(4)}$ **for the special case.**

Analogously if $(b''_{24})^{(4)} = (b''_{25})^{(4)}$, then $(\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then

$(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)} T_{25}(t)$ This is an important consequence of the relation between $(v_1)^{(4)}$ and $(\bar{v}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

Proof : From GLOBAL EQUATIONS we obtain

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$$\frac{dv^{(5)}}{dt} = (a_{28})^{(5)} - \left((a'_{28})^{(5)} - (a'_{29})^{(5)} + (a''_{28})^{(5)} (T_{29}, t) \right) - (a''_{29})^{(5)} (T_{29}, t) v^{(5)} - (a_{29})^{(5)} v^{(5)}$$

$$\text{Definition of } v^{(5)} \text{ :- } \boxed{v^{(5)} = \frac{G_{28}}{G_{29}}}$$

It follows

$$- \left((a_{29})^{(5)} (v^{(5)})^2 + (\sigma_2)^{(5)} v^{(5)} - (a_{28})^{(5)} \right) \leq \frac{dv^{(5)}}{dt} \leq - \left((a_{29})^{(5)} (v^{(5)})^2 + (\sigma_1)^{(5)} v^{(5)} - (a_{28})^{(5)} \right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(5)}, (v_0)^{(5)}$:-

(g) For $0 < \boxed{(v_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}} < (v_1)^{(5)} < (\bar{v}_1)^{(5)}$

$$v^{(5)}(t) \geq \frac{(v_1)^{(5)} + (C)^{(5)}(v_2)^{(5)} e^{[-(a_{29})^{(5)}((v_1)^{(5)} - (v_0)^{(5)})t]}}{5 + (C)^{(5)} e^{[-(a_{29})^{(5)}((v_1)^{(5)} - (v_0)^{(5)})t]}} , \quad \boxed{(C)^{(5)} = \frac{(v_1)^{(5)} - (v_0)^{(5)}}{(v_0)^{(5)} - (v_2)^{(5)}}$$

it follows $(v_0)^{(5)} \leq v^{(5)}(t) \leq (v_1)^{(5)}$

In the same manner, we get

$$v^{(5)}(t) \leq \frac{(\bar{v}_1)^{(5)} + (\bar{C})^{(5)}(\bar{v}_2)^{(5)} e^{[-(a_{29})^{(5)}((\bar{v}_1)^{(5)} - (\bar{v}_2)^{(5)})t]}}{5 + (\bar{C})^{(5)} e^{[-(a_{29})^{(5)}((\bar{v}_1)^{(5)} - (\bar{v}_2)^{(5)})t]}} , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{v}_1)^{(5)} - (v_0)^{(5)}}{(v_0)^{(5)} - (\bar{v}_2)^{(5)}}$$

From which we deduce $(v_0)^{(5)} \leq v^{(5)}(t) \leq (\bar{v}_5)^{(5)}$

(h) If $0 < (v_1)^{(5)} < (v_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0} < (\bar{v}_1)^{(5)}$ we find like in the previous case,

$$(v_1)^{(5)} \leq \frac{(v_1)^{(5)} + (C)^{(5)}(v_2)^{(5)} e^{[-(a_{29})^{(5)}((v_1)^{(5)} - (v_2)^{(5)})t]}}{1 + (C)^{(5)} e^{[-(a_{29})^{(5)}((v_1)^{(5)} - (v_2)^{(5)})t]}} \leq v^{(5)}(t) \leq \frac{(\bar{v}_1)^{(5)} + (\bar{C})^{(5)}(\bar{v}_2)^{(5)} e^{[-(a_{29})^{(5)}((\bar{v}_1)^{(5)} - (\bar{v}_2)^{(5)})t]}}{1 + (\bar{C})^{(5)} e^{[-(a_{29})^{(5)}((\bar{v}_1)^{(5)} - (\bar{v}_2)^{(5)})t]}} \leq (\bar{v}_1)^{(5)}$$

(i) If $0 < (v_1)^{(5)} \leq (\bar{v}_1)^{(5)} \leq \boxed{(v_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain

$$(v_1)^{(5)} \leq v^{(5)}(t) \leq \frac{(\bar{v}_1)^{(5)} + (\bar{C})^{(5)}(\bar{v}_2)^{(5)} e^{[-(a_{29})^{(5)}((\bar{v}_1)^{(5)} - (\bar{v}_2)^{(5)})t]}}{1 + (\bar{C})^{(5)} e^{[-(a_{29})^{(5)}((\bar{v}_1)^{(5)} - (\bar{v}_2)^{(5)})t]}} \leq (v_0)^{(5)}$$

And so with the notation of the first part of condition (c), we have

Definition of $v^{(5)}(t)$:-

$$(m_2)^{(5)} \leq v^{(5)}(t) \leq (m_1)^{(5)}, \quad \boxed{v^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(5)}(t)$:-

$$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad \boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$$

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

Particular case :

If $(a''_{28})^{(5)} = (a''_{29})^{(5)}$, then $(\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(v_1)^{(5)} = (\bar{v}_1)^{(5)}$ if in addition $(v_0)^{(5)} = (v_5)^{(5)}$ then $v^{(5)}(t) = (v_0)^{(5)}$ and as a consequence $G_{28}(t) = (v_0)^{(5)}G_{29}(t)$ **this also defines $(v_0)^{(5)}$ for the special case.**

Analogously if $(b''_{28})^{(5)} = (b''_{29})^{(5)}$, then $(\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then

$(u_1)^{(5)} = (\bar{u}_1)^{(5)}$ if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)}T_{29}(t)$ This is an important consequence of the relation between $(v_1)^{(5)}$ and $(\bar{v}_1)^{(5)}$, **and definition of $(u_0)^{(5)}$.**

Proof : From GLOBAL EQUATIONS we obtain

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$$\frac{dv^{(6)}}{dt} = (a_{32})^{(6)} - \left((a'_{32})^{(6)} - (a'_{33})^{(6)} + (a''_{32})^{(6)}(T_{33}, t) \right) - (a''_{33})^{(6)}(T_{33}, t)v^{(6)} - (a_{33})^{(6)}v^{(6)}$$

Definition of $v^{(6)}$:- $\boxed{v^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}(v^{(6)})^2 + (\sigma_2)^{(6)}v^{(6)} - (a_{32})^{(6)}\right) \leq \frac{dv^{(6)}}{dt} \leq -\left((a_{33})^{(6)}(v^{(6)})^2 + (\sigma_1)^{(6)}v^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(6)}, (v_0)^{(6)}$:-

(j) For $0 < \boxed{(v_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}} < (v_1)^{(6)} < (\bar{v}_1)^{(6)}$

$$v^{(6)}(t) \geq \frac{(v_1)^{(6)} + (C)^{(6)}(v_2)^{(6)} e^{[-(a_{33})^{(6)}(v_1)^{(6)} - (v_0)^{(6)}]t}}{1 + (C)^{(6)} e^{[-(a_{33})^{(6)}(v_1)^{(6)} - (v_0)^{(6)}]t}}, \quad \boxed{(C)^{(6)} = \frac{(v_1)^{(6)} - (v_0)^{(6)}}{(v_0)^{(6)} - (v_2)^{(6)}}$$

it follows $(v_0)^{(6)} \leq v^{(6)}(t) \leq (v_1)^{(6)}$

In the same manner , we get

$$v^{(6)}(t) \leq \frac{(\bar{v}_1)^{(6)} + (\bar{C})^{(6)}(\bar{v}_2)^{(6)} e^{[-(a_{33})^{(6)}(\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)}]t}}{1 + (\bar{C})^{(6)} e^{[-(a_{33})^{(6)}(\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)}]t}}, \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{v}_1)^{(6)} - (v_0)^{(6)}}{(v_0)^{(6)} - (\bar{v}_2)^{(6)}}$$

From which we deduce $(v_0)^{(6)} \leq v^{(6)}(t) \leq (\bar{v}_1)^{(6)}$

(k) If $0 < (v_1)^{(6)} < (v_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0} < (\bar{v}_1)^{(6)}$ we find like in the previous case,

$$(v_1)^{(6)} \leq \frac{(v_1)^{(6)} + (C)^{(6)}(v_2)^{(6)} e^{[-(a_{33})^{(6)}(v_1)^{(6)} - (v_2)^{(6)}]t}}{1 + (C)^{(6)} e^{[-(a_{33})^{(6)}(v_1)^{(6)} - (v_2)^{(6)}]t}} \leq v^{(6)}(t) \leq \frac{(\bar{v}_1)^{(6)} + (\bar{C})^{(6)}(\bar{v}_2)^{(6)} e^{[-(a_{33})^{(6)}(\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)}]t}}{1 + (\bar{C})^{(6)} e^{[-(a_{33})^{(6)}(\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)}]t}} \leq (\bar{v}_1)^{(6)}$$

(l) If $0 < (v_1)^{(6)} \leq (\bar{v}_1)^{(6)} \leq \boxed{(v_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$, we obtain

$$(v_1)^{(6)} \leq v^{(6)}(t) \leq \frac{(\bar{v}_1)^{(6)} + (\bar{C})^{(6)}(\bar{v}_2)^{(6)} e^{[-(a_{33})^{(6)}(\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)}]t}}{1 + (\bar{C})^{(6)} e^{[-(a_{33})^{(6)}(\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)}]t}} \leq (v_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(6)}(t)$:-

$$(m_2)^{(6)} \leq v^{(6)}(t) \leq (m_1)^{(6)}, \quad \boxed{v^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(6)}(t)$:-

$$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}, \quad \boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$$

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

Particular case :

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}$, then $(\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(v_1)^{(6)} = (\bar{v}_1)^{(6)}$ if in addition $(v_0)^{(6)} = (v_1)^{(6)}$ then $v^{(6)}(t) = (v_0)^{(6)}$ and as a consequence $G_{32}(t) = (v_0)^{(6)}G_{33}(t)$ **this also defines $(v_0)^{(6)}$ for the special case.**

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}$, then $(\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then

$(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)}T_{33}(t)$ This is an important consequence of the relation between $(v_1)^{(6)}$ and $(\bar{v}_1)^{(6)}$, **and definition of $(u_0)^{(6)}$.**

We can prove the following

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Theorem 3: If $(a_i'')^{(1)}$ and $(b_i'')^{(1)}$ are independent on t , and the conditions

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$$

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0,$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$$

with $(p_{13})^{(1)}, (r_{14})^{(1)}$ as defined by equation IN THE FOREGOING are satisfied, then the system

If $(a_i'')^{(2)}$ and $(b_i'')^{(2)}$ are independent on t , and the conditions (SECOND MODULE)

$$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$$

$$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$$

$$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0,$$

$$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$$

with $(p_{16})^{(2)}, (r_{17})^{(2)}$ as defined by equation IN THE FOREGOING are satisfied, then the

system (THIRD MODULE)

Theorem 3: If $(a_i'')^{(3)}$ and $(b_i'')^{(3)}$ are independent on t , and the conditions

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$$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$$

$$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$$

$$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0,$$

$$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$$

with $(p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation IN THE FOREGOING are satisfied, then the system

We can prove the following (FOURTH MODULE CONSEQUENCES)

Theorem 3: If $(a_i'')^{(4)}$ and $(b_i'')^{(4)}$ are independent on t , and the conditions

$$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$$

$$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$$

$$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0,$$

$$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$$

with $(p_{24})^{(4)}, (r_{25})^{(4)}$ as defined by equation IN THE FOREGOING are satisfied, then the system

Theorem 3: If $(a_i'')^{(5)}$ and $(b_i'')^{(5)}$ are independent on t , and the conditions (FIFTH MODULE

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CONSEQUENCES)

$$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$$

$$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$$

$$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0,$$

$$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$$

with $(p_{28})^{(5)}, (r_{29})^{(5)}$ as defined by equation IN THE EQUATION STATED IN THE FOREGOING are

satisfied, then the system

Theorem 3: If $(a_i'')^{(6)}$ and $(b_i'')^{(6)}$ are independent on t , and the conditions

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$$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$$

$$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$$

$$(a'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0 ,$$

$$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$$

with $(p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation IN THE FOREGOING are satisfied , then the system

$$(a_{13})^{(1)}G_{14} - [(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})]G_{13} = 0 \quad 241$$

$$(a_{14})^{(1)}G_{13} - [(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})]G_{14} = 0 \quad 242$$

$$(a_{15})^{(1)}G_{14} - [(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})]G_{15} = 0 \quad 243$$

$$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)]T_{13} = 0 \quad 244$$

$$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)]T_{14} = 0 \quad 245$$

$$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)]T_{15} = 0 \quad 246$$

has a unique positive solution , which is an equilibrium solution for the system

$$(a_{16})^{(2)}G_{17} - [(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})]G_{16} = 0 \quad 247$$

$$(a_{17})^{(2)}G_{16} - [(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})]G_{17} = 0 \quad 248$$

$$(a_{18})^{(2)}G_{17} - [(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})]G_{18} = 0 \quad 249$$

$$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})]T_{16} = 0 \quad 250$$

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})]T_{17} = 0 \quad 251$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})]T_{18} = 0 \quad 252$$

has a unique positive solution , which is an equilibrium solution

$$(a_{20})^{(3)}G_{21} - [(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})]G_{20} = 0 \quad 253$$

$$(a_{21})^{(3)}G_{20} - [(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})]G_{21} = 0 \quad 254$$

$$(a_{22})^{(3)}G_{21} - [(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})]G_{22} = 0 \quad 255$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})]T_{20} = 0 \quad 256$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})]T_{21} = 0 \quad 257$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})]T_{22} = 0 \quad 258$$

has a unique positive solution , which is an equilibrium solution

$$(a_{24})^{(4)}G_{25} - [(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})]G_{24} = 0 \quad 259$$

$$(a_{25})^{(4)}G_{24} - [(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})]G_{25} = 0 \quad 260$$

$$(a_{26})^{(4)}G_{25} - [(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})]G_{26} = 0 \quad 261$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27}))]T_{24} = 0 \quad 262$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}((G_{27}))]T_{25} = 0 \quad 263$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27}))]T_{26} = 0 \quad 264$$

has a unique positive solution , which is an equilibrium solution

$$(a_{28})^{(5)}G_{29} - [(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})]G_{28} = 0 \quad 265$$

$$(a_{29})^{(5)}G_{28} - [(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})]G_{29} = 0 \quad 266$$

$$(a_{30})^{(5)}G_{29} - [(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})]G_{30} = 0 \quad 267$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})]T_{28} = 0 \quad 268$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})]T_{29} = 0 \quad 269$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})]T_{30} = 0 \quad 270$$

has a unique positive solution , which is an equilibrium solution

$$(a_{32})^{(6)}G_{33} - [(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})]G_{32} = 0 \quad 271$$

$$(a_{33})^{(6)}G_{32} - [(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})]G_{33} = 0 \quad 272$$

$$(a_{34})^{(6)}G_{33} - [(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})]G_{34} = 0 \quad 273$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})]T_{32} = 0 \quad 274$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})]T_{33} = 0 \quad 275$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})]T_{34} = 0 \quad 276$$

has a unique positive solution , which is an equilibrium solution

Proof: 277

(a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if

$$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$$

Proof: 278

(a) Indeed the first two equations have a nontrivial solution G_{16}, G_{17} if

$$F(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a''_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$$

(a) Indeed the first two equations have a nontrivial solution G_{20}, G_{21} if 279

$$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a''_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$$

(a) Indeed the first two equations have a nontrivial solution G_{24}, G_{25} if 280

$$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$$

(a) Indeed the first two equations have a nontrivial solution G_{28}, G_{29} if 281

$$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$$

Proof: 282

(a) Indeed the first two equations have a nontrivial solution G_{32}, G_{33} if

$$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$$

Definition and uniqueness of T_{14}^* :- 283

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a'_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T_{14}^* for which $f(T_{14}^*) = 0$. With this value , we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T_{14}^*)]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T_{14}^*)]}$$

Definition and uniqueness of T_{17}^* :-

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(2)}(T_{17})$ being increasing, it follows that there exists a unique T_{17}^* for which $f(T_{17}^*) = 0$. With this value, we obtain from the three first equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T_{17}^*)]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T_{17}^*)]}$$

Definition and uniqueness of T_{21}^* :-

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(1)}(T_{21})$ being increasing, it follows that there exists a unique T_{21}^* for which $f(T_{21}^*) = 0$. With this value, we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T_{21}^*)]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T_{21}^*)]}$$

Definition and uniqueness of T_{25}^* :-

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(4)}(T_{25})$ being increasing, it follows that there exists a unique T_{25}^* for which $f(T_{25}^*) = 0$. With this value, we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T_{25}^*)]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T_{25}^*)]}$$

Definition and uniqueness of T_{29}^* :-

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(5)}(T_{29})$ being increasing, it follows that there exists a unique T_{29}^* for which $f(T_{29}^*) = 0$. With this value, we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T_{29}^*)]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T_{29}^*)]}$$

Definition and uniqueness of T_{33}^* :-

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(6)}(T_{33})$ being increasing, it follows that there exists a unique T_{33}^* for which $f(T_{33}^*) = 0$. With this value, we obtain from the three first equations

$$G_{32} = \frac{(a_{32})^{(6)}G_{33}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^*)]} \quad , \quad G_{34} = \frac{(a_{34})^{(6)}G_{33}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^*)]}$$

(e) By the same argument, THE SOLUTIONAL EQUATIONS OF THE GLOBAL EQUATIONS ADMIT solutions G_{13}, G_{14} if

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - [(b'_{13})^{(1)}(b'_{14})^{(1)}(G) + (b'_{14})^{(1)}(b'_{13})^{(1)}(G)] + (b'_{13})^{(1)}(G)(b'_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0, \varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi(G^*) = 0$

(f) By the same argument, the GLOBAL EQUATIONS admit solutions G_{16}, G_{17} if

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})] + (b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{17} taking into account the hypothesis $\varphi(0) > 0, \varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi((G_{19})^*) = 0$

(g) By the same argument, SOLUTIONAL EQUATIONS admit solutions G_{20}, G_{21} if

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - [(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})] + (b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{21} taking into account the hypothesis $\varphi(0) > 0, \varphi(\infty) < 0$ it follows that there exists a unique G_{21}^* such that $\varphi((G_{23})^*) = 0$

(h) By the same argument, the GLOBAL EQUATIONS admit solutions G_{24}, G_{25} if

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - [(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})] + (b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{25} taking into account the hypothesis $\varphi(0) > 0, \varphi(\infty) < 0$ it follows that there exists a unique G_{25}^* such that $\varphi((G_{27})^*) = 0$

(i) By the same argument, the GLOBAL EQUATIONS AND CONCOMITANT DERIVED EQUATIONS admit solutions G_{28}, G_{29} if 285

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - [(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})] + (b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{29} taking into account the hypothesis $\varphi(0) > 0, \varphi(\infty) < 0$ it follows that there exists a unique G_{29}^* such that $\varphi((G_{31})^*) = 0$

(j) By the same argument, the GLOBAL EQUATIONS admit solutions G_{32}, G_{33} if 286

$$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - [(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})] + (b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values It is easy to see that φ is a decreasing function in G_{33} taking into account the hypothesis $\varphi(0) > 0, \varphi(\infty) < 0$ it follows that there exists a unique G_{33}^* such that $\varphi(G^*) = 0$

Finally we obtain the unique solution

$$G_{14}^* \text{ given by } \varphi(G^*) = 0, T_{14}^* \text{ given by } f(T_{14}^*) = 0 \text{ and}$$

$$G_{13}^* = \frac{(a_{13})^{(1)}G_{14}^*}{[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}^*)]}, \quad G_{15}^* = \frac{(a_{15})^{(1)}G_{14}^*}{[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}^*)]}$$

$$T_{13}^* = \frac{(b_{13})^{(1)}T_{14}^*}{[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G^*)]}, \quad T_{15}^* = \frac{(b_{15})^{(1)}T_{14}^*}{[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G^*)]}$$

Obviously, these values represent an equilibrium solution THE SYSTEM

Finally we obtain the unique solution of THE SYSTEM

G_{17}^* given by $\varphi((G_{19})^*) = 0, T_{17}^*$ given by $f(T_{17}^*) = 0$ and

$$G_{16}^* = \frac{(a_{16})^{(2)}G_{17}^*}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T_{17}^*)]} , G_{18}^* = \frac{(a_{18})^{(2)}G_{17}^*}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T_{17}^*)]}$$

$$T_{16}^* = \frac{(b_{16})^{(2)}T_{17}^*}{[(b'_{16})^{(2)}-(b''_{16})^{(2)}((G_{19})^*)]} , T_{18}^* = \frac{(b_{18})^{(2)}T_{17}^*}{[(b'_{18})^{(2)}-(b''_{18})^{(2)}((G_{19})^*)]}$$

Obviously, these values represent an equilibrium solution of THE GLOBAL EQUATIONS

Finally we obtain the unique solution of THE GLOBAL EQUATIONS 287

G_{21}^* given by $\varphi((G_{23})^*) = 0$, T_{21}^* given by $f(T_{21}^*) = 0$ and

$$G_{20}^* = \frac{(a_{20})^{(3)}G_{21}^*}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T_{21}^*)]} , G_{22}^* = \frac{(a_{22})^{(3)}G_{21}^*}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T_{21}^*)]}$$

$$T_{20}^* = \frac{(b_{20})^{(3)}T_{21}^*}{[(b'_{20})^{(3)}-(b''_{20})^{(3)}(G_{23}^*)]} , T_{22}^* = \frac{(b_{22})^{(3)}T_{21}^*}{[(b'_{22})^{(3)}-(b''_{22})^{(3)}(G_{23}^*)]}$$

Obviously, these values represent an equilibrium solution of GLOBAL SYSTEM

Finally we obtain the unique solution of THE SYSTEM 288

G_{25}^* given by $\varphi(G_{27}) = 0$, T_{25}^* given by $f(T_{25}^*) = 0$ and

$$G_{24}^* = \frac{(a_{24})^{(4)}G_{25}^*}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T_{25}^*)]} , G_{26}^* = \frac{(a_{26})^{(4)}G_{25}^*}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T_{25}^*)]}$$

$$T_{24}^* = \frac{(b_{24})^{(4)}T_{25}^*}{[(b'_{24})^{(4)}-(b''_{24})^{(4)}((G_{27})^*)]} , T_{26}^* = \frac{(b_{26})^{(4)}T_{25}^*}{[(b'_{26})^{(4)}-(b''_{26})^{(4)}((G_{27})^*)]} \quad 289$$

Obviously, these values represent an equilibrium solution of THE GLOBAL SYSTEM

Finally we obtain the unique solution of THE GLOBAL SYSTEM 290

G_{29}^* given by $\varphi((G_{31})^*) = 0$, T_{29}^* given by $f(T_{29}^*) = 0$ and

$$G_{28}^* = \frac{(a_{28})^{(5)}G_{29}^*}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T_{29}^*)]} , G_{30}^* = \frac{(a_{30})^{(5)}G_{29}^*}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T_{29}^*)]}$$

$$T_{28}^* = \frac{(b_{28})^{(5)}T_{29}^*}{[(b'_{28})^{(5)}-(b''_{28})^{(5)}((G_{31})^*)]} , T_{30}^* = \frac{(b_{30})^{(5)}T_{29}^*}{[(b'_{30})^{(5)}-(b''_{30})^{(5)}((G_{31})^*)]} \quad 291$$

Obviously, these values represent an equilibrium solution of THE SYSTEM

Finally we obtain the unique solution of THE DERIVED EQUATIONS OF THE GLOBAL EQUATIONS

G_{33}^* given by $\varphi((G_{35})^*) = 0$, T_{33}^* given by $f(T_{33}^*) = 0$ and

$$G_{32}^* = \frac{(a_{32})^{(6)}G_{33}^*}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^*)]} , G_{34}^* = \frac{(a_{34})^{(6)}G_{33}^*}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^*)]}$$

$$T_{32}^* = \frac{(b_{32})^{(6)}T_{33}^*}{[(b'_{32})^{(6)}-(b''_{32})^{(6)}((G_{35})^*)]} , T_{34}^* = \frac{(b_{34})^{(6)}T_{33}^*}{[(b'_{34})^{(6)}-(b''_{34})^{(6)}((G_{35})^*)]}$$

Obviously, these values represent an equilibrium solution of THE SYSTEM

Asymptotic Stability Analysis Of The System Space –Time –Mass –Energy- Quantum Gravity- Perception-Strong Nuclear Force-Weak Nuclear Force-Gravity-Electromagnetism-Vacuum Energy and Quantum Field 292

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a'_i)^{(1)}$ and $(b'_i)^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: _Denote

Definition of G_i, T_i :-

$$G_i = G_i^* + \mathbb{G}_i , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a'_{14})^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)}, \quad \frac{\partial(b''_i)^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account DERIVED EQUATIONS OF THE GLOBAL EQUATIONS neglecting the terms of power 2, we obtain

$$\frac{dG_{13}}{dt} = -((a'_{13})^{(1)} + (p_{13})^{(1)})G_{13} + (a_{13})^{(1)}G_{14} - (q_{13})^{(1)}G_{13}^*T_{14} \quad 293$$

$$\frac{dG_{14}}{dt} = -((a'_{14})^{(1)} + (p_{14})^{(1)})G_{14} + (a_{14})^{(1)}G_{13} - (q_{14})^{(1)}G_{14}^*T_{14} \quad 294$$

$$\frac{dG_{15}}{dt} = -((a'_{15})^{(1)} + (p_{15})^{(1)})G_{15} + (a_{15})^{(1)}G_{14} - (q_{15})^{(1)}G_{15}^*T_{14} \quad 295$$

$$\frac{dT_{13}}{dt} = -((b'_{13})^{(1)} - (r_{13})^{(1)})T_{13} + (b_{13})^{(1)}T_{14} + \sum_{j=13}^{15}(s_{(13)(j)})T_{13}^*G_j \quad 296$$

$$\frac{dT_{14}}{dt} = -((b'_{14})^{(1)} - (r_{14})^{(1)})T_{14} + (b_{14})^{(1)}T_{13} + \sum_{j=13}^{15}(s_{(14)(j)})T_{14}^*G_j \quad 297$$

$$\frac{dT_{15}}{dt} = -((b'_{15})^{(1)} - (r_{15})^{(1)})T_{15} + (b_{15})^{(1)}T_{14} + \sum_{j=13}^{15}(s_{(15)(j)})T_{15}^*G_j \quad 298$$

If the conditions of the previous theorem are satisfied and if the functions $(a'_i)^{(2)}$ and $(b'_i)^{(2)}$ belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Proof: Denote

Definition of G_i, T_i :-

$$G_i = G_i^* + G_i, \quad T_i = T_i^* + T_i$$

$$\frac{\partial(a'_{17})^{(2)}}{\partial T_{17}}(T_{17}^*) = (q_{17})^{(2)}, \quad \frac{\partial(b''_i)^{(2)}}{\partial G_j}(G_{19}^*) = s_{ij}$$

taking into account equations DERIVED EQUATIONS OF THE GLOBAL EQUATIONS and neglecting the terms of power 2, we obtain

$$\frac{dG_{16}}{dt} = -((a'_{16})^{(2)} + (p_{16})^{(2)})G_{16} + (a_{16})^{(2)}G_{17} - (q_{16})^{(2)}G_{16}^*T_{17} \quad 299$$

$$\frac{dG_{17}}{dt} = -((a'_{17})^{(2)} + (p_{17})^{(2)})G_{17} + (a_{17})^{(2)}G_{16} - (q_{17})^{(2)}G_{17}^*T_{17}$$

$$\frac{dG_{18}}{dt} = -((a'_{18})^{(2)} + (p_{18})^{(2)})G_{18} + (a_{18})^{(2)}G_{17} - (q_{18})^{(2)}G_{18}^*T_{17}$$

$$\frac{dT_{16}}{dt} = -((b'_{16})^{(2)} - (r_{16})^{(2)})T_{16} + (b_{16})^{(2)}T_{17} + \sum_{j=16}^{18}(s_{(16)(j)})T_{16}^*G_j$$

$$\frac{dT_{17}}{dt} = -((b'_{17})^{(2)} - (r_{17})^{(2)})T_{17} + (b_{17})^{(2)}T_{16} + \sum_{j=16}^{18}(s_{(17)(j)})T_{17}^*G_j$$

$$\frac{dT_{18}}{dt} = -((b'_{18})^{(2)} - (r_{18})^{(2)})T_{18} + (b_{18})^{(2)}T_{17} + \sum_{j=16}^{18}(s_{(18)(j)})T_{18}^*G_j$$

If the conditions of the previous theorem are satisfied and if the functions $(a'_i)^{(3)}$ and $(b'_i)^{(3)}$ belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of G_i, T_i :-

$$G_i = G_i^* + G_i, \quad T_i = T_i^* + T_i$$

$$\frac{\partial(a'_{21})^{(3)}}{\partial T_{21}}(T_{21}^*) = (q_{21})^{(3)}, \quad \frac{\partial(b''_i)^{(3)}}{\partial G_j}(G_{23}^*) = s_{ij}$$

Then taking into account equations DERIVED FROM THE GLOBAL EQUATIONS and neglecting the terms of power 2, we obtain

$$\frac{dG_{20}}{dt} = -((a'_{20})^{(3)} + (p_{20})^{(3)})G_{20} + (a_{20})^{(3)}G_{21} - (q_{20})^{(3)}G_{20}^*T_{21} \quad 300$$

$$\begin{aligned} \frac{dG_{21}}{dt} &= -((a'_{21})^{(3)} + (p_{21})^{(3)})G_{21} + (a_{21})^{(3)}G_{20} - (q_{21})^{(3)}G_{21}^* T_{21} \\ \frac{dG_{22}}{dt} &= -((a'_{22})^{(3)} + (p_{22})^{(3)})G_{22} + (a_{22})^{(3)}G_{21} - (q_{22})^{(3)}G_{22}^* T_{21} \\ \frac{dT_{20}}{dt} &= -((b'_{20})^{(3)} - (r_{20})^{(3)})T_{20} + (b_{20})^{(3)}T_{21} + \sum_{j=20}^{22} (s_{(20)(j)})T_{20}^* G_j \\ \frac{dT_{21}}{dt} &= -((b'_{21})^{(3)} - (r_{21})^{(3)})T_{21} + (b_{21})^{(3)}T_{20} + \sum_{j=20}^{22} (s_{(21)(j)})T_{21}^* G_j \\ \frac{dT_{22}}{dt} &= -((b'_{22})^{(3)} - (r_{22})^{(3)})T_{22} + (b_{22})^{(3)}T_{21} + \sum_{j=20}^{22} (s_{(22)(j)})T_{22}^* G_j \end{aligned}$$

If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}$ and $(b_i'')^{(4)}$ belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable. (FOURTH MODULE)

Proof: Denote

Definition of G_i, T_i :-

$$\begin{aligned} G_i &= G_i^* + \mathbb{G}_i, \quad T_i = T_i^* + \mathbb{T}_i \\ \frac{\partial (a_{25}'')^{(4)}}{\partial T_{25}} (T_{25}^*) &= (q_{25})^{(4)}, \quad \frac{\partial (b_i'')^{(4)}}{\partial G_j} ((G_{27})^*) = s_{ij} \end{aligned}$$

Then taking into account equations DERIVED EQUATIONS OF THE GLOBAL EQUATIONS MENTIONED HEREINBEFORE and neglecting the terms of power 2, we obtain

$$\begin{aligned} \frac{dG_{24}}{dt} &= -((a'_{24})^{(4)} + (p_{24})^{(4)})G_{24} + (a_{24})^{(4)}G_{25} - (q_{24})^{(4)}G_{24}^* T_{25} \\ \frac{dG_{25}}{dt} &= -((a'_{25})^{(4)} + (p_{25})^{(4)})G_{25} + (a_{25})^{(4)}G_{24} - (q_{25})^{(4)}G_{25}^* T_{25} \\ \frac{dG_{26}}{dt} &= -((a'_{26})^{(4)} + (p_{26})^{(4)})G_{26} + (a_{26})^{(4)}G_{25} - (q_{26})^{(4)}G_{26}^* T_{25} \\ \frac{dT_{24}}{dt} &= -((b'_{24})^{(4)} - (r_{24})^{(4)})T_{24} + (b_{24})^{(4)}T_{25} + \sum_{j=24}^{26} (s_{(24)(j)})T_{24}^* G_j \\ \frac{dT_{25}}{dt} &= -((b'_{25})^{(4)} - (r_{25})^{(4)})T_{25} + (b_{25})^{(4)}T_{24} + \sum_{j=24}^{26} (s_{(25)(j)})T_{25}^* G_j \\ \frac{dT_{26}}{dt} &= -((b'_{26})^{(4)} - (r_{26})^{(4)})T_{26} + (b_{26})^{(4)}T_{25} + \sum_{j=24}^{26} (s_{(26)(j)})T_{26}^* G_j \end{aligned}$$

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ASYMPTOTIC STABILITY ANALYSIS(FIFTH MODULE)

If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}$ and $(b_i'')^{(5)}$ belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Denote

Definition of G_i, T_i :-

$$\begin{aligned} G_i &= G_i^* + \mathbb{G}_i, \quad T_i = T_i^* + \mathbb{T}_i \\ \frac{\partial (a_{29}'')^{(5)}}{\partial T_{29}} (T_{29}^*) &= (q_{29})^{(5)}, \quad \frac{\partial (b_i'')^{(5)}}{\partial G_j} ((G_{31})^*) = s_{ij} \end{aligned}$$

Then taking into account equations DERIVED EQUATIONS OF THE GLOBAL EQUATIONS and neglecting the terms of power 2, we obtain

$$\begin{aligned} \frac{dG_{28}}{dt} &= -((a'_{28})^{(5)} + (p_{28})^{(5)})G_{28} + (a_{28})^{(5)}G_{29} - (q_{28})^{(5)}G_{28}^* T_{29} \\ \frac{dG_{29}}{dt} &= -((a'_{29})^{(5)} + (p_{29})^{(5)})G_{29} + (a_{29})^{(5)}G_{28} - (q_{29})^{(5)}G_{29}^* T_{29} \\ \frac{dG_{30}}{dt} &= -((a'_{30})^{(5)} + (p_{30})^{(5)})G_{30} + (a_{30})^{(5)}G_{29} - (q_{30})^{(5)}G_{30}^* T_{29} \\ \frac{dT_{28}}{dt} &= -((b'_{28})^{(5)} - (r_{28})^{(5)})T_{28} + (b_{28})^{(5)}T_{29} + \sum_{j=28}^{30} (s_{(28)(j)})T_{28}^* G_j \end{aligned}$$

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$$\frac{dT_{29}}{dt} = -((b'_{29})^{(5)} - (r_{29})^{(5)})T_{29} + (b_{29})^{(5)}T_{28} + \sum_{j=28}^{30} (s_{(29)(j)} T_{29}^* G_j)$$

$$\frac{dT_{30}}{dt} = -((b'_{30})^{(5)} - (r_{30})^{(5)})T_{30} + (b_{30})^{(5)}T_{29} + \sum_{j=28}^{30} (s_{(30)(j)} T_{30}^* G_j)$$

**ASYMPTOTIC STABILITY ANALYSIS(SIXTH MODULE RAMIFICATIONS ON THE
 CONCATENATED GLOBAL EQUATIONS)**

If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}$ and $(b_i'')^{(6)}$ belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Denote

Definition of G_i, T_i :-

$$G_i = G_i^* + G_i \quad , \quad T_i = T_i^* + T_i$$

$$\frac{\partial (a_{33}'')^{(6)}}{\partial T_{33}} (T_{33}^*) = (q_{33})^{(6)} \quad , \quad \frac{\partial (b_i'')^{(6)}}{\partial G_j} ((G_{35})^*) = s_{ij}$$

Then taking into account equations DERIVED FROM THE CONCATENATED GLOBAL EQUATIONS and neglecting the terms of power 2, we obtain

$$\frac{dG_{32}}{dt} = -((a'_{32})^{(6)} + (p_{32})^{(6)})G_{32} + (a_{32})^{(6)}G_{33} - (q_{32})^{(6)}G_{32}^* T_{33} \tag{303}$$

$$\frac{dG_{33}}{dt} = -((a'_{33})^{(6)} + (p_{33})^{(6)})G_{33} + (a_{33})^{(6)}G_{32} - (q_{33})^{(6)}G_{33}^* T_{33}$$

$$\frac{dG_{34}}{dt} = -((a'_{34})^{(6)} + (p_{34})^{(6)})G_{34} + (a_{34})^{(6)}G_{33} - (q_{34})^{(6)}G_{34}^* T_{33}$$

$$\frac{dT_{32}}{dt} = -((b'_{32})^{(6)} - (r_{32})^{(6)})T_{32} + (b_{32})^{(6)}T_{33} + \sum_{j=32}^{34} (s_{(32)(j)} T_{32}^* G_j)$$

$$\frac{dT_{33}}{dt} = -((b'_{33})^{(6)} - (r_{33})^{(6)})T_{33} + (b_{33})^{(6)}T_{32} + \sum_{j=32}^{34} (s_{(33)(j)} T_{33}^* G_j)$$

$$\frac{dT_{34}}{dt} = -((b'_{34})^{(6)} - (r_{34})^{(6)})T_{34} + (b_{34})^{(6)}T_{33} + \sum_{j=32}^{34} (s_{(34)(j)} T_{34}^* G_j)$$

The characteristic equation of this system is

$$\begin{aligned} & ((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)}) \{ (\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)} \} \\ & \left[\left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)} \right) (q_{14})^{(1)} G_{14}^* + (a_{14})^{(1)} (q_{13})^{(1)} G_{13}^* \right] \\ & \left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)} \right) s_{(14),(14)} T_{14}^* + (b_{14})^{(1)} s_{(13),(14)} T_{14}^* \\ & + \left((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)} \right) (q_{13})^{(1)} G_{13}^* + (a_{13})^{(1)} (q_{14})^{(1)} G_{14}^* \\ & \left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)} \right) s_{(14),(13)} T_{14}^* + (b_{14})^{(1)} s_{(13),(13)} T_{13}^* \\ & \left((\lambda)^{(1)} \right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)} \right) (\lambda)^{(1)} \\ & \left((\lambda)^{(1)} \right)^2 + \left((b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)} \right) (\lambda)^{(1)} \\ & + \left((\lambda)^{(1)} \right)^2 + \left((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)} \right) (\lambda)^{(1)} (q_{15})^{(1)} G_{15} \\ & + \left((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)} \right) \left((a_{15})^{(1)} (q_{14})^{(1)} G_{14}^* + (a_{14})^{(1)} (a_{15})^{(1)} (q_{13})^{(1)} G_{13}^* \right) \\ & \left((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)} \right) s_{(14),(15)} T_{14}^* + (b_{14})^{(1)} s_{(13),(15)} T_{13}^* \} = 0 \\ & + \\ & ((\lambda)^{(2)} + (b'_{18})^{(2)} - (r_{18})^{(2)}) \{ (\lambda)^{(2)} + (a'_{18})^{(2)} + (p_{18})^{(2)} \} \end{aligned}$$

$$\begin{aligned}
 & \left[\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)} \right) (q_{17})^{(2)} G_{17}^* + (a_{17})^{(2)} (q_{16})^{(2)} G_{16}^* \right] \\
 & \left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)} \right) s_{(17),(17)} T_{17}^* + (b_{17})^{(2)} s_{(16),(17)} T_{17}^* \\
 & + \left((\lambda)^{(2)} + (a'_{17})^{(2)} + (p_{17})^{(2)} \right) (q_{16})^{(2)} G_{16}^* + (a_{16})^{(2)} (q_{17})^{(2)} G_{17}^* \\
 & \left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)} \right) s_{(17),(16)} T_{17}^* + (b_{17})^{(2)} s_{(16),(16)} T_{16}^* \\
 & \left((\lambda)^{(2)} \right)^2 + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)} \right) (\lambda)^{(2)} \\
 & \left((\lambda)^{(2)} \right)^2 + \left((b'_{16})^{(2)} + (b'_{17})^{(2)} - (r_{16})^{(2)} + (r_{17})^{(2)} \right) (\lambda)^{(2)} \\
 & + \left((\lambda)^{(2)} \right)^2 + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)} \right) (\lambda)^{(2)} (q_{18})^{(2)} G_{18} \\
 & + \left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)} \right) \left((a_{18})^{(2)} (q_{17})^{(2)} G_{17}^* + (a_{17})^{(2)} (a_{18})^{(2)} (q_{16})^{(2)} G_{16}^* \right) \\
 & \left. \left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)} \right) s_{(17),(18)} T_{17}^* + (b_{17})^{(2)} s_{(16),(18)} T_{16}^* \right\} = 0 \\
 & + \\
 & \left((\lambda)^{(3)} + (b'_{22})^{(3)} - (r_{22})^{(3)} \right) \left\{ (\lambda)^{(3)} + (a'_{22})^{(3)} + (p_{22})^{(3)} \right\} \\
 & \left[\left((\lambda)^{(3)} + (a'_{20})^{(3)} + (p_{20})^{(3)} \right) (q_{21})^{(3)} G_{21}^* + (a_{21})^{(3)} (q_{20})^{(3)} G_{20}^* \right] \quad 690 \\
 & \left((\lambda)^{(3)} + (b'_{20})^{(3)} - (r_{20})^{(3)} \right) s_{(21),(21)} T_{21}^* + (b_{21})^{(3)} s_{(20),(21)} T_{21}^* \\
 & + \left((\lambda)^{(3)} + (a'_{21})^{(3)} + (p_{21})^{(3)} \right) (q_{20})^{(3)} G_{20}^* + (a_{20})^{(3)} (q_{21})^{(1)} G_{21}^* \\
 & \left((\lambda)^{(3)} + (b'_{20})^{(3)} - (r_{20})^{(3)} \right) s_{(21),(20)} T_{21}^* + (b_{21})^{(3)} s_{(20),(20)} T_{20}^* \\
 & \left((\lambda)^{(3)} \right)^2 + \left((a'_{20})^{(3)} + (a'_{21})^{(3)} + (p_{20})^{(3)} + (p_{21})^{(3)} \right) (\lambda)^{(3)} \\
 & \left((\lambda)^{(3)} \right)^2 + \left((b'_{20})^{(3)} + (b'_{21})^{(3)} - (r_{20})^{(3)} + (r_{21})^{(3)} \right) (\lambda)^{(3)} \\
 & + \left((\lambda)^{(3)} \right)^2 + \left((a'_{20})^{(3)} + (a'_{21})^{(3)} + (p_{20})^{(3)} + (p_{21})^{(3)} \right) (\lambda)^{(3)} (q_{22})^{(3)} G_{22} \\
 & + \left((\lambda)^{(3)} + (a'_{20})^{(3)} + (p_{20})^{(3)} \right) \left((a_{22})^{(3)} (q_{21})^{(3)} G_{21}^* + (a_{21})^{(3)} (a_{22})^{(3)} (q_{20})^{(3)} G_{20}^* \right) \\
 & \left. \left((\lambda)^{(3)} + (b'_{20})^{(3)} - (r_{20})^{(3)} \right) s_{(21),(22)} T_{21}^* + (b_{21})^{(3)} s_{(20),(22)} T_{20}^* \right\} = 0 \\
 & + \\
 & \left((\lambda)^{(4)} + (b'_{26})^{(4)} - (r_{26})^{(4)} \right) \left\{ (\lambda)^{(4)} + (a'_{26})^{(4)} + (p_{26})^{(4)} \right\} \\
 & \left[\left((\lambda)^{(4)} + (a'_{24})^{(4)} + (p_{24})^{(4)} \right) (q_{25})^{(4)} G_{25}^* + (a_{25})^{(4)} (q_{24})^{(4)} G_{24}^* \right] \\
 & \left((\lambda)^{(4)} + (b'_{24})^{(4)} - (r_{24})^{(4)} \right) s_{(25),(25)} T_{25}^* + (b_{25})^{(4)} s_{(24),(25)} T_{25}^* \\
 & + \left((\lambda)^{(4)} + (a'_{25})^{(4)} + (p_{25})^{(4)} \right) (q_{24})^{(4)} G_{24}^* + (a_{24})^{(4)} (q_{25})^{(4)} G_{25}^* \\
 & \left((\lambda)^{(4)} + (b'_{24})^{(4)} - (r_{24})^{(4)} \right) s_{(25),(24)} T_{25}^* + (b_{25})^{(4)} s_{(24),(24)} T_{24}^* \\
 & \left((\lambda)^{(4)} \right)^2 + \left((a'_{24})^{(4)} + (a'_{25})^{(4)} + (p_{24})^{(4)} + (p_{25})^{(4)} \right) (\lambda)^{(4)} \\
 & \left((\lambda)^{(4)} \right)^2 + \left((b'_{24})^{(4)} + (b'_{25})^{(4)} - (r_{24})^{(4)} + (r_{25})^{(4)} \right) (\lambda)^{(4)} \\
 & + \left((\lambda)^{(4)} \right)^2 + \left((a'_{24})^{(4)} + (a'_{25})^{(4)} + (p_{24})^{(4)} + (p_{25})^{(4)} \right) (\lambda)^{(4)} (q_{26})^{(4)} G_{26} \\
 & + \left((\lambda)^{(4)} + (a'_{24})^{(4)} + (p_{24})^{(4)} \right) \left((a_{26})^{(4)} (q_{25})^{(4)} G_{25}^* + (a_{25})^{(4)} (a_{26})^{(4)} (q_{24})^{(4)} G_{24}^* \right)
 \end{aligned}$$

$$\begin{aligned}
 & \left\{ ((\lambda)^{(4)} + (b'_{24})^{(4)} - (r_{24})^{(4)})s_{(25),(26)}T_{25}^* + (b_{25})^{(4)}s_{(24),(26)}T_{24}^* \right\} = 0 \\
 & + \\
 & ((\lambda)^{(5)} + (b'_{30})^{(5)} - (r_{30})^{(5)})\{(\lambda)^{(5)} + (a'_{30})^{(5)} + (p_{30})^{(5)}\} \\
 & \left[\left((\lambda)^{(5)} + (a'_{28})^{(5)} + (p_{28})^{(5)} \right) (q_{29})^{(5)} G_{29}^* + (a_{29})^{(5)} (q_{28})^{(5)} G_{28}^* \right] \\
 & \left((\lambda)^{(5)} + (b'_{28})^{(5)} - (r_{28})^{(5)} \right) s_{(29),(29)} T_{29}^* + (b_{29})^{(5)} s_{(28),(29)} T_{29}^* \\
 & + \left((\lambda)^{(5)} + (a'_{29})^{(5)} + (p_{29})^{(5)} \right) (q_{28})^{(5)} G_{28}^* + (a_{28})^{(5)} (q_{29})^{(5)} G_{29}^* \\
 & \left((\lambda)^{(5)} + (b'_{28})^{(5)} - (r_{28})^{(5)} \right) s_{(29),(28)} T_{29}^* + (b_{29})^{(5)} s_{(28),(28)} T_{28}^* \\
 & \left((\lambda)^{(5)} \right)^2 + \left((a'_{28})^{(5)} + (a'_{29})^{(5)} + (p_{28})^{(5)} + (p_{29})^{(5)} \right) (\lambda)^{(5)} \\
 & \left((\lambda)^{(5)} \right)^2 + \left((b'_{28})^{(5)} + (b'_{29})^{(5)} - (r_{28})^{(5)} + (r_{29})^{(5)} \right) (\lambda)^{(5)} \\
 & + \left((\lambda)^{(5)} \right)^2 + \left((a'_{28})^{(5)} + (a'_{29})^{(5)} + (p_{28})^{(5)} + (p_{29})^{(5)} \right) (\lambda)^{(5)} (q_{30})^{(5)} G_{30} \\
 & + \left((\lambda)^{(5)} + (a'_{28})^{(5)} + (p_{28})^{(5)} \right) \left((a_{30})^{(5)} (q_{29})^{(5)} G_{29}^* + (a_{29})^{(5)} (a_{30})^{(5)} (q_{28})^{(5)} G_{28}^* \right) \\
 & \left\{ ((\lambda)^{(5)} + (b'_{28})^{(5)} - (r_{28})^{(5)})s_{(29),(30)}T_{29}^* + (b_{29})^{(5)}s_{(28),(30)}T_{28}^* \right\} = 0 \\
 & + \\
 & ((\lambda)^{(6)} + (b'_{34})^{(6)} - (r_{34})^{(6)})\{(\lambda)^{(6)} + (a'_{34})^{(6)} + (p_{34})^{(6)}\} \\
 & \left[\left((\lambda)^{(6)} + (a'_{32})^{(6)} + (p_{32})^{(6)} \right) (q_{33})^{(6)} G_{33}^* + (a_{33})^{(6)} (q_{32})^{(6)} G_{32}^* \right] \\
 & \left((\lambda)^{(6)} + (b'_{32})^{(6)} - (r_{32})^{(6)} \right) s_{(33),(33)} T_{33}^* + (b_{33})^{(6)} s_{(32),(33)} T_{33}^* \\
 & + \left((\lambda)^{(6)} + (a'_{33})^{(6)} + (p_{33})^{(6)} \right) (q_{32})^{(6)} G_{32}^* + (a_{32})^{(6)} (q_{33})^{(6)} G_{33}^* \\
 & \left((\lambda)^{(6)} + (b'_{32})^{(6)} - (r_{32})^{(6)} \right) s_{(33),(32)} T_{33}^* + (b_{33})^{(6)} s_{(32),(32)} T_{32}^* \\
 & \left((\lambda)^{(6)} \right)^2 + \left((a'_{32})^{(6)} + (a'_{33})^{(6)} + (p_{32})^{(6)} + (p_{33})^{(6)} \right) (\lambda)^{(6)} \\
 & \left((\lambda)^{(6)} \right)^2 + \left((b'_{32})^{(6)} + (b'_{33})^{(6)} - (r_{32})^{(6)} + (r_{33})^{(6)} \right) (\lambda)^{(6)} \\
 & + \left((\lambda)^{(6)} \right)^2 + \left((a'_{32})^{(6)} + (a'_{33})^{(6)} + (p_{32})^{(6)} + (p_{33})^{(6)} \right) (\lambda)^{(6)} (q_{34})^{(6)} G_{34} \\
 & + \left((\lambda)^{(6)} + (a'_{32})^{(6)} + (p_{32})^{(6)} \right) \left((a_{34})^{(6)} (q_{33})^{(6)} G_{33}^* + (a_{33})^{(6)} (a_{34})^{(6)} (q_{32})^{(6)} G_{32}^* \right) \\
 & \left\{ ((\lambda)^{(6)} + (b'_{32})^{(6)} - (r_{32})^{(6)})s_{(33),(34)}T_{33}^* + (b_{33})^{(6)}s_{(32),(34)}T_{32}^* \right\} = 0
 \end{aligned}$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

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