PERGAMON

Computers & Operations Research 29 (2002) 1115–1127

computers & operations research

www.elsevier.com/locate/dsw

A myopic policy for the gradual obsolescence problem with price-dependent demand

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Received 1 March 1999; received in revised form 1 September 1999

Abstract

The purpose of this paper is to develop a retailer's profit-maximizing myopic inventory policy for an item recognized as subject to gradual obsolescence. Demand is assumed to be a decreasing function of both the retailer's sale price and of time, up to a certain stochastic time point when obsolescence occurs and, as a result, the demand suddenly drops to zero. For each ordering cycle, the decision variables are the retailer's selling price and the order size. A stop-ordering rule is developed on the basis of finding the time point beyond which it is profitable to stop ordering, even if there is still some demand for the item. In addition, the sudden obsolescence problem is shown to be a limiting and non-trivial case of its gradual counterpart. The numerical example illustrates the main features of the model, including the importance of the vendor dropping the price charged to retailers, so as to provide the needed incentives for the retailers to drop the price charged to their own customers and thereby palliate as much as possible to negative effects of obsolescence.

Scope and purpose

This paper develops a myopic policy to evaluate a retailer's decision process, when an item is recognized as subject to gradual obsolescence. The model considers demand to be a decreasing function of both the retailer's sale price and of time, up to a certain stochastic time point when obsolescence occurs and, as a result, the demand suddenly drops to zero. The retailer's profit-maximizing policy consists of an optimal selling price and an order size for each ordering cycle, as well as the time point beyond which it is profitable to stop ordering, even if there is still some demand for the item. This is in contrast to alternate formulations, where the stopping rule is based upon minimizing the cost of obsolescence, rather than evaluating the profitability of the item in question. Finally, the numerical example illustrates the need for vendor/retailer collaboration in the development of the pricing policies. Otherwise, the retailer has no incentive to keep

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PII: S0305-0548(00)00093-9

prices low and thus counteract the normal decreases in demand that occur as time passes by and the probability of obsolescence increases.

Keywords: Gradual obsolescence; Price-dependent demand; Profit optimization; Myopic inventory policies

1. Introduction

This paper evaluates a retailer's inventory policy for an item subject to gradual obsolescence. In contrast to perishability or deterioration, which are a function of product quality and involve items with a fixed or stochastic lifetime, respectively, obsolescence is related to cases where changes in technology, fashion, style, environment and the like render the item useless for its intended use at some point in time and hence, its utility and therefore its demand drops to zero at that point. This discontinuity in demand is what differentiates obsolescence from deterioration/perishability. Such loss of utility may be relatively swift, which characterizes items subject to sudden obsolescence, where demand for the item collapses overnight [1–7]. Alternatively, the utility loss may occur gradually, through a period of declining demand, before the sudden collapse takes place [7]. Modelling the later case is the subject of this paper.

The gradual obsolescence problem exhibits several features, which are germane to the model of this paper. First, for a random period of time, it features a declining-demand market. Hence, the demand for the item may be modelled as a decreasing function of time. Barbosa and Friedman [8] and Chakravarty and Martin [9] are prototypes of declining-demand models. Second, the gradual obsolescence problem differs from its declining-demand counterpart in that the demand for the item does not reach the zero point gradually, according to the tenets of the demand function, usually exponential in nature. Rather, as a typical obsolescence problem, the demand for the item stops suddenly at a random point in time. Hence, the declining demand model must be truncated at some stochastic point, at which demand is suddenly zero. Such situations occur normally as a result of the introduction of a superior substitute, as it often happens with, for example, the release of the new version of a competitor's software. In fact, the rising rate of technological progress is the main reason advanced in the literature cited earlier for the saliency of the obsolescence problem. In addition, Hill et al. [10] present an excellent example of a DSS for handling gradual obsolescence, within a spare-parts inventory system.

A third feature of the obsolescence problem is the usual retailer's practice of altering demand (and hence the order quantities) through the price mechanism, before obsolescence renders the inventoried items useless. For this purpose, the model considers a profit-maximizing rather than a cost minimizing retailer, using the selling price as a decision variable to alter demand. This is intended to palliate to the extent possible the negative effects of obsolescence. However, as shall be discussed latter on, these palliative efforts may not necessarily lead to price reductions in order to stimulate demand. Rather, the paper shows that it may be rational for a profit-maximizing retailer to increase the price of the item and thereby decrease its demand, if the vendor's price for the item is not sufficiently reduced, in light of the increasing probability of obsolescence. Instances of this practice often occurs in the case of spare parts for obsolete and/or discontinued products (e.g. ink cartridges for discontinued printers). This third characteristic represents the main contribution of this paper vis-à-vis the existing literature on gradual obsolescence [10,11,7], where demand is only a function of time, but not of price. At issue here is the treatment of the items subject to gradual obsolescence as assets and thus subject to evaluation for their profit-making potential [12]. Two important but mostly neglected manifestations of this behaviour includes (i) using the retailer's ability to influence demand through adroit manipulation of the selling price in order to develop profit-maximizing rather than cost-minimizing ordering policies; and (ii) allowing the retailer the choice of setting an ordering stopping rule on the basis of the profitability or lack thereof of the next order, rather than on the basis of demand shortage due to complete obsolescence.

To that effect, the paper is organized as follows. The next section develops the basic methodology for the gradual obsolescence problem subject of the current study. Modelling occurs at the point where occurrence of future obsolescence is finally recognized. A key issue is the realization that a full theoretical development requires the use of dynamic programming models, along the lines of the well-known Wagner and Whitin formulation [13]. These are difficult if not impossible at times to solve and sometimes impose rather taxing data requirements on the user. As a result, this section follows in the spirit of Silver and Meal [14] and develops a myopic profit-maximizing policy for the retailer, with the selling price and the order quantity being determined endogenously at the start of every inventory cycle. The last part of this section demonstrates the generality of the gradual obsolescence model by characterizing its sudden obsolescence counterpart as a limiting case. The analysis shows that, even though the profit difference between the two obsolescence cases decreases along with the declining demand rate, there exists a point of discontinuity when the rate is zero. Hence, the problem of finding the sudden obsolescence optimal policy is not trivial. Section 3 presents a numerical example designed to explore the nature of certain observations arising out of empirical regularities derived from our extensive computational experience, but not proven analytically due to the complexity of the algebraic expressions. A conclusions section completes the paper.

2. Modelling the gradual obsolescence problem

This section starts by describing the main features of the model, followed by the derivation of the objective function and of the optimality conditions. Included in the latter are the determination of the optimal retailer's policy and the optimal determination of when ordering should stop.

2.1. Elements of the model

For each cycle, i, let t_i be the length of time to deplete order i. Then, the retailer's decision model may be characterized by several features, which reflect the declining nature of the demand and its obsolescence features. With respect to the obsolescence features of the model, the following elements need to be identified:

(i) As in Arcelus et al. [1], a random variable, X, measuring the duration of time between the point of evaluation and the point of obsolescence, with $g(.)$ and $G(.)$ denoting its density and its cumulative probability distribution functions, respectively.

(ii) The time, T_i, at which the order i is depleted and the probability, p_i, that the obsolescence point occurs while the ith order is depleted, i.e.

$$T_i = \sum_{j=1}^{i} t_j \quad \text{with } T_0 = 0,$$

$$p_i = P(X \leqslant T_i | X > T_{i-1}) = [G(T_i) - G(T_{i-1})]/[1 - G(T_{i-1})] = \int_0^{t_i} f_i(y)\,\mathrm{d}y \tag{1}$$

where

$$f_i(y) = [g(x + T_{i-1})]/[1 - G(T_{i-1})].$$

With respect to demand considerations, the following elements are of importance.

(iii) A yearly pre-obsolescence demand rate, R_0, and a yearly demand rate during the obsolescence period, R_i, which decreases with time, y, at a constant rate of β and with the retailer's selling price, P_i, at a rate of α:

$$R_0 = r_0 P_0^{-\alpha} \quad \text{with } r_0 > 0 \text{ and } \alpha > 0$$

$$R_i(y, P_i) = R_0(P_0/P_i)^{\alpha} \exp(-\beta T_{i-1}) \exp(-\beta y) \quad \text{for } y \geqslant 0,\ R_0 > 0 \text{ and } \beta > 0. \tag{2}$$

(iv) A purchasing quantity, q_i, of which $s_i(y)$ are sold at P_i per unit, before obsolescence occurs at y and $I_i(y)$ must be discarded as obsolete. Before y, the decrease in inventory over time equals the demand rate, since the sale function represents the only rationale for the change in the number of units on hand. Hence, the differential equation representing the dynamics of the inventory and the resulting functional form for the inventory level may be expressed as follows:

$$\partial I_i(y)/\partial y = -R_i = -A_P \exp(-\beta y) \quad \text{with } I_i(0) = q_i \quad \text{and}$$

$$A_P = R_0(P_0/P_i)^{\alpha} \exp(-\beta T_{i-1}) \quad \text{from (2)}$$

Hence,

$$I_i(y) = q_i - s_i(y) = q_i - A_P[1 - \exp(-\beta y)]/\beta \quad \text{if } 0 \leqslant y \leqslant t_i \tag{3}$$

and

$$I_i(t_i) = 0 \rightarrow q_i = A_P[1 - \exp(-\beta t_i)]/\beta = s_i(t_i).$$

(v) Using (3), a cumulative inventory held up to time y, $S_i(y)$, of

$$S_i(y) = \int_0^{y} I_i(x)\,\mathrm{d}x = q_i y - A_P y/\beta + s_i(y)/\beta \quad \text{for } 0 \leqslant y \leqslant t_i. \tag{4}$$

2.2. The retailer's decision

Let C, F, K and d be respectively the retailer's purchase price per item, the holding cost per dollar per year, the fixed cost component of each order placed and the salvage value, if any, of the obsolete unit. Further, define the following terms:

$$\theta_i = \int_0^{t_i} y f_i(y)\,\mathrm{d}y,$$

$$\xi_{1i} = q_i/A_P = [1 - \exp(-\beta t_i)]/\beta,$$

$$\xi_{2i} = \int_0^{t_i} (1/A_P) s_i(y_i) f_i(y)\,\mathrm{d}y = \int_0^{t_i} (1/\beta)[1 - \exp(-\beta y)] f_i(y)\,\mathrm{d}y \tag{5}$$

and

$$\xi_{3i} = (1/\beta)\int_0^{t_i} \{y - [1 - \exp(-\beta y)]/\beta\} f_i(y)\,\mathrm{d}y.$$

Then, using (1)–(5), the expected profit, $\phi_i(P_i, t_i)$, per unit time over the ith cycle may be expressed as follows:

$$\phi_i(P_i, t_i) = \psi_i(P_i, t_i)/t_i$$

where

$$\begin{aligned}
\psi_i(P_i, t_i) &= (1 - p_i)[(P_i - C)q_i - CFS_i(t_i) - K] \\
&\quad + \int_0^{t_i} [P_i s_i(y) + dI_i(y) - Cq_i - K - CFS_i(y)] f_i(y)\,\mathrm{d}y \\
&= (1 - p_i)[(P_i - C)q_i - CFS_i(t_i) - K] + (d - C)\int_0^{t_i} q_i f_i(y)\,\mathrm{d}y \\
&\quad + (P_i - d)\int_0^{t_i} s_i(y) f_i(y)\,\mathrm{d}y - \int_0^{t_i} K f_i(y)\,\mathrm{d}y - CF(q_i\theta_i - A_P\xi_{3i}) \\
&= (1 - p_i)[(P_i - C)q_i - CFS_i(t_i)] - K + (d - C)q_i p_i \\
&\quad + (P_i - d)A_P\xi_{2i} - CF(q_i\theta_i - A_P\xi_{3i}).
\end{aligned} \tag{6}$$

Note that the first expression for the expected profit over the ith cycle, $\psi_i(P_i, t_i)$ in (6), consists of two parts. The first gives the profit over the cycle (term in brackets), weighted by the probability, $(1 - p_i)$, that obsolescence does not occur during the period. The second considers the expected profit up to the point of obsolescence which may occur with probability p_i within the cycle. This includes the revenue from the regular units sold plus the salvage value of those left unsold, minus the purchasing (variable and fixed) and holding costs of the order. Further, observe that both

q_i and t_i appear in (6). For analytical convenience, t_i rather than q_i is used in this paper as decision variable. The relationship between the two is given in (3). Then the retailer's myopic policy consists of two parts: (i) find the sales price, P_i^*, and the cycle length, t_i^*, that maximize the expected profit per unit time per cycle; and (ii) determine when ordering should stop.

As to the first part, P_i^* and t_i^* may be obtained from the first-order conditions as given in the following lemma.

Lemma 1. *Optimal values of the decision variables*:

$$\phi(t_i^*, P_i^*) = \partial\psi_i/\partial t_i|_{t_1=t_i^*} = R_i^*\{(P_i^* - C) - p_i^*(P_i^* - d) - CF[(1 - p_i^*)t_i^* + \theta_i^*]\},$$

$$P_i^* = H\{C\xi_{1i} - d(p_i\xi_{1i} - \xi_{2i}) + CF[(1 - p_i)(\xi_{1i}t_i + \xi_{1i}/\beta - t_i/\beta) + \xi_{1i}\theta_i - \xi_{3i}]\}, \tag{7}$$

where

$$H = \alpha/\{(\alpha - 1)[(1 - p_i)\xi_{1i} + \xi_{2i}]\}.$$

θ_i and p_i are defined in (5) and (1), respectively. The first-order conditions in (7) exhibit a very intuitively appealing economic interpretation. The first expression indicates that the optimal cycle length is that for which the expected profit per unit time equals its marginal total profit. The second sets the price at the point where the revenue of an extra unit equals its marginal cost. Further, θ_i denotes the average time before obsolescence occurs, if at all, in the ith cycle, conditional on having survived the previous $i - 1$ cycles. For computational purposes, the system of equations in (7) can be reduced to a sequential solution procedure since, in the second expression, P_i^* appears as a direct function of t_i^*. More details will be given in the next section.

We know examine when the retailer should stop ordering. From (7), it is clear that ordering should be done in the ith cycle, if its optimal expected profit is positive, i.e. if

$$\phi(t_i^*, P_i^*) > 0 \rightarrow (P_i^* - C) - p_i^*(P_i^* - d) > CF[(1 - p_i^*)t_i^* + \theta_i^*] \tag{8}$$

or, as shown by the second inequality of (8), if the expected unit profit exceeds the additional expected unit costs. Let m be the maximum number of orders for which the inequalities in (8) hold. Then the retailer should place m orders, that will be depleted by T_m, after which the item in question is no longer sold. This is one of the key contributions of this paper. Stopping on the basis of (8) is done for profitability reasons, rather than as a cost trade-off, even if there may still be some demand for the item.

The preceding discussion assumes that m is finite and thus the ordering will stop in a finite number of steps. The veracity of this assertion is parameter-specific, since it depends upon the probability distribution being used. Nevertheless, the following property gives a sufficient condition for m to be finite. This result is likely to hold for the increasing-failure-rate (IFR) distributions applicable to this type of problems (i.e. gamma, Weibull and the like).

Property 1. *A sufficient condition for m to be finite is that*

$$p_m > (P_m^* - C)/(P_m^* - d). \tag{9}$$

The expression in (9) is obtained from (8), on the simplifying assumption of zero holding costs. Thus, when $F = 0$, (8) results in (9). Its economic interpretation is clear. Ordering should stop, when the expected benefits of not buying an extra unit exceeds the actual per-unit profit if the purchase takes place. However, it is not clear whether, as the number of orders increases, the ratio of the RHS of (9) will decrease far enough for the inequality to hold. The resolution of such an issue is parameter specific. That this ratio will decrease can be seen by observing that both the declining demand and the threat of obsolescence encourage the retailer to continue lowering the selling price to counteract to the largest extent possible the negative effect of these two factors on the demand and hence on profits. This together with the fact that $C > d$ renders the RHS of (9) a decreasing function of time. Further, the conditional probability of obsolescence, p_i, also increases with time, since IFR distributions are applicable for the gradual obsolescence problem.

Once the value of m has been determined, the expression for the total profit for the m cycles, Φ_m, can be derived as follows:

$$\Phi_m(P_i^*, t_i^*) = \sum_{i=1}^{m} \left\{ B_1(P_i^*, t_i^*)[1 - G(T_i)] + \int_{T_{i-1}}^{T_i} B_2(P_i^*, t_i^*, x) g(x)\, \mathrm{d}x \right\}, \tag{10}$$

where $B_1(*)$ represents the profit per cycle if obsolescence does not occur in cycle i, and $B_2(*)$ represents the profit per cycle if obsolescence does occur in cycle i. The values for $B_1(*)$ and $B_2(*)$ are given by the two terms in brackets on the first RHS of (6). Note that in (10) ordering occurs as long as is profitable to do so, in accordance with the condition in (8). This implies that $B_1(*)$ does not necessarily have to be zero for the mth order.

2.3. The sudden obsolescence problem as a limiting case

To illustrate the generality of the formulation in (6), the myopic policy's profit function (Eq. (17)) of Arcelus et al. [1] for the sudden obsolescence problem is derived as a limiting case of that in (6). For simplicity, the same notation is used for both models (except for the use of A_P instead of R_i to denote the price-induced demand).

Theorem 1.

$$\begin{aligned} \lim_{\beta \to 0} \phi(P_i, t_i) = {} & (P_i - C)A_P - (P_i - d)A_P(p_i - \theta_i/t_i) - K/t_i \\ & - CFA_P[(1 - p_i)t_i/2 + \theta_i - \mu_i/(2t_i)] \end{aligned} \tag{11}$$

where $\mu_i = \int_0^{t_i} y^2 f_i(y)\,\mathrm{d}y$ *and* θ_i *is defined in* (5).

Proof. Using L'Hospital's rule, it can be readily shown that the following limits hold as $\beta \to 0$:

$$\lim_{\beta \to 0} s_i(y) = \lim_{\beta \to 0} A_P[1 - \exp(-\beta y)]/\beta = A_p y \quad \text{for } 0 < y \leqslant t_i,$$

$$\lim_{\beta \to 0} \xi_{1,i} = \lim_{\beta \to 0} s_i(t_i)/A_P = t_i,$$

$$\lim_{\beta \to 0} q_i = \lim_{\beta \to 0} s_i(t_i) = A_P t_i,$$

$$\lim_{\beta \to 0} \xi_{2i} = \lim_{\beta \to 0} \int_0^{t_i} (1/A_P) s_i(y_i) f_i(y)\, \mathrm{d}y = \theta_i,$$

$$\lim_{\beta \to 0} \xi_{3i} = \lim_{\beta \to 0} \int_0^{t_i} \{\beta y - [1 - \exp(-\beta y)]/\beta^2\} f_i(y)\, \mathrm{d}y$$

$$= \lim_{\beta \to 0} \int_0^{t_i} \{y[1 - \exp(-\beta y)]/2\beta\} f_i(y)\, \mathrm{d}y$$

$$= \int_0^{t_i} (y^2/2) f_i(y)\, \mathrm{d}y = \mu_i/2. \tag{12}$$

Observe that only the ξ_{3i} case requires more than one use of L'Hospital's rule to reach the desired limit. In any case, combining (6) and (12) yields (11). □

2.4. Modelling the pre-obsolescence case

The analysis of the next section requires numerical comparisons between the optimal policies developed in Lemma 1 and those related to the case when future obsolescence is not yet recognized. For the later and following Arcelus and Srinivasan (1987), the retailer's profit and the resulting optimality conditions may be written as

$$\operatorname{Max} \Phi(P_0, t_0) = (P_0 - C)R_0 - K/t_0 - CFR_0 t_0/2$$

with R_0 defined in (2) and the optimality conditions given by

$$P_0^* = \alpha K/[R_0^* t_0^*(\alpha - 1)] \tag{13}$$

and

$$t_0^* = [2K/(CFR_0^*)]^{1/2}.$$

The objective function in (13) reflects the standard expression for the yearly profit for the deterministic case, where profits are defined as the yearly revenues minus the costs of purchasing, of placing the order and of holding inventory. However, it should be noted from its definition in (2) that the demand rate, R_0, used in (13), is neither constant nor subject to obsolescence, but a function of price only.

3. A numerical example

This section is designed to illustrate some economic regularities of the model that are not amenable to rigorous theoretical analysis, due to the complexity of the profit function. To that effect, it summarizes the results of one of the numerical examples studied for this purpose. The starting point is a base-case example, using a two-parameter (a, b) Weibull distribution as the prototype of IFR distributions that can be used to model X, the duration of the gradual obsolescence. Then, the density function for X, with a mean, θ, shape parameter a and scale parameter, b, may be written as follows:

$$g(x) = ba\, x^{a-1}\exp(-bx^a) \quad \text{for } x > 0;\ a > 0 \text{ and } b > 0$$

with

$$\theta = [\Gamma(1/a + 1)]/b^{1/a},$$
$$EX^2 = [\Gamma(2/a + 1)]/b^{2/a}, \tag{14}$$

and

$$E(NX) = \theta \sum_{n=0}^{\infty} \int_{b(nT)^a}^{\infty} [b^{1/a+1} x^{1/a} \exp(-bx)/\Gamma(1/a + 1)]\, \mathrm{d}x.$$

The last two expressions are used when deriving the optimal policies and are included here for expository convenience, even though proofs are omitted. The base-case parameter values, which remain constant throughout the simulation, unless otherwise stated explicitly, are

$$[C, F, K, d, r_0, \theta, a, \alpha, \beta] = [20, 0.1, 350, 1, 312500, 0.5, 5, 1.75, 1.5]. \tag{15}$$

Note that for comparability purposes, the values of θ and of the scale parameter, a, are fixed at 0.5 and at 5, respectively, with the shape parameter, b, obtained from the second expression in (14). All computations were performed through the use of Matlab's [15] Optimization Toolbox.

Table 1 summarizes the first and most important economic regularity of this section. It arises out of studying the effect on the retailer's ordering and pricing policies of changes in C, the retailer's unit purchasing cost, ranging from the base case of 20 (100% of C) of down to \$4 or 20% of the original value. For each value of C and with the values for the other parameters as given in (15), the table provides (i) the orders to be placed, all of which with a non-negative profit, as given in (8); (ii) for each order, the optimal values of the decision variables, as given in (7) of Lemma 1, along with the demand from (2) and the order quantity as the product of the order's demand and depletion time; (iii) the combined values for each of the variables described in (ii), computed as described in footnote (a) of Table 1; and (iv) the optimal annual policy for the situation before future obsolescence is suspected, computed from (13).

The data from Table 1 suggests the following findings. First, for each value of C, it is clear that as time passes by (i.e. as the value of i increases) and the probability of obsolescence increases, the retailer (i) purchases fewer units from the vendor (lower values for Q_i); (ii) places orders more frequently as the inventory depletion rate decreases (lower values for t_i); (iii) charges higher prices to its own customers (higher values for P_i); and consequently (iv) services a lower demand base (lower values for R_i). Hence, the profit-maximizing retailer finds the strategy of encouraging a lowering of the demand for the product to avoid being left with valueless merchandise on hand more profitable than the strategy of attempting to continuously increase the demand through price decreases.

Table 1
Effect of changes in the retailer's unit purchasing price, C

Order i	100% of C					80% of C					60% of C				
	ϕ_i	t_i	P_i	Q_i	R_i	ϕ_i	t_i	P_i	Q_i	R_i	ϕ_i	t_i	P_i	Q_i	R_i
1	5614	0.165	47.1	48	288	7039	0.152	37.63	66	435	9318	0.136	28.19	101	739
2	3255	0.141	49.4	30	215	4568	0.134	38.93	45	335	6676	0.125	28.8	74	590
3	599	0.116	58.17	16	135	1742	0.11	43.77	25	232	3642	0.103	31.13	46	441
4											780	0.091	36.31	27	294
Total[a]	1602	0.422	50.25	94	223	2002	0.396	39.19	136	343	2759	0.455	29.8	248	545
Before[b]	9292	1.009	49.02	347	344	11051	0.924	39.06	473	512	13808	0.827	29.16	706	854
	40% of C					20% of C									
1	1362	0.116	18.77	180	1551	25236	0.09	9.37	481	54541					
2	1066	0.113	18.97	145	1286	21485	0.09	9.4	428	47409					
3	733	0.1	19.81	99	1032	17560	0.08	9.53	341	40839					
4	403	0.08	21.53	65	789	13507	0.07	9.81	253	34733					
5	109	0.08	24.62	42	558	9651	0.06	10.27	186	29146					
6						6158	0.06	10.92	138	24032					
7						3054	0.05	11.85	103	19224					
8						324	0.05	13.34	75	14462					
Total[a]	4144	0.482	19.82	531	1102	8059	0.562	9.93	2005	3568					
Before[b]	18878	0.706	19.33	1239	1754	32132	0.541	9.59	3236	5984					

[a]The *Total* values for t_i and Q_i are the sum of the corresponding values for each order. The total profit is Φ_m from (10). Total R_i is the average yearly demand, computed as the ratio of the total quantity to the total inventory depletion length. Total price is the average price per order, weighted by the various order quantities.
[b]The data in the *Before* row corresponds to that in (13) for the period before obsolescence.

Second, if the vendor wishes to encourage higher sales, lowering the unit cost to the retailer appears to be a profitable strategy. As Table 1 indicates, lowering the value of C may not reverse the retailer's policy outlined in the previous paragraph, but its negative consequences are lessened. The average yearly demand for the product increases from 223 units, for the 100% of C case, to 3568 units for its 20% of C counterpart. Total profits increase from \$1602 to \$8059. Both are a consequence of lower prices per unit, larger number of orders and higher quantities per order.

Third, a direct consequence of this last result is the increasing percentage of yearly sales recuperated as the vendor's selling price is decreased, hence lessening the negative impact of obsolescence. Since the average depletion time has been fixed for this example at a constant $\theta = 0.5$, the ratio of the Total Profit to the Before Profit yields the desired percentage. For the example of Table 1, these values are (0.172, 0.181, 0.200, 0.220, 0.251) for (100% of C, 80% of C, 60% of C, 40% of C, 20% of C), respectively.

Fourth, the first two findings do not imply that price decreases do not form part of the retailer's profit maximising strategy. Retail prices always decrease, when obsolescence becomes a problem. This can be readily seen by comparing the retail prices of at least the first order to the retail price before obsolescence becomes an issue. For example, prices decrease from \$49.02 to \$47.10 for the

Table 2
Effect of changes in α, β and a on the number of orders (NO), on the total depletion time (T_m) and on the total profit (Φ_m)

Parameters			100% of C			80% of C			60% of C			40% of C			20% of C		
α	β	a	NO	T_m	Φ_m	NO	T_m	Φ_m	NO	T_m	Φ_m	NO	T_m	Φ_m	NO	T_m	Φ_m
2.25	3	3	0	0	0	0	0	0	0	0	0	1	0.177	151	3	0.362	971
2.25	3	7	0	0	0	0	0	0	1	0.281	3	1	0.208	194	3	0.39	1116
2.25	3	9	0	0	0	0	0	0	1	0.296	6	1	0.211	197	3	0.397	1142
1.75	4	5	2	0.266	533	2	0.237	710	3	0.336	1078	4	0.391	1764	5	0.365	3719
2	4	5	0	0	0	1	0.192	80	1	0.157	191	2	0.269	505	4	0.394	1704
2.25	4	5	0	0	0	0	0	0	0	0	0	1	0.19	84	2	0.237	699
2.25	2	5	0	0	0	0	0	0	0	0	0	1	0.212	327	2	0.269	1599
2.25	4	5	0	0	0	0	0	0	0	0	0	1	0.19	84	2	0.237	699
2.25	6	5	0	0	0	0	0	0	0	0	0	0	0	0	2	0.229	323

100% of C case and from, say, \$19.33 to \$18.77 for its 40% of C counterpart. However, if the vendor wishes to encourage a consistent policy of lower retail prices, a strategy of continuous decreases in C is required.

From the discussion about Table 1, it is clear that the tendency of the retailer is to operate at the lowest demand rate possible and thereby decreasing as much as possible the average inventory and thus the expected obsolescence costs. However, the impact of obsolescence on the retailer's ordering and pricing policies is also affected by fluctuations in the values of the other parameters of the model. The magnitude and direction of these marginal effects, with the values of all other parameters fixed at those given in (15), follow the tenets of microeconomic theory. Hence, a direct positive impact on profits is obtained from decreases in F and K or increases in d. Similarly, indirect profitability increases result from any parametric change which yields increases in the demand rate (higher values for r_0 or lower price, $-\alpha$, or time, β, demand elasticities) or decreases in the right skewness of the duration-of-the-gradual-obsolescence distribution, as measured by a, with θ fixed at 0.5. As an example, Table 2 summarizes the effects of three important parameters, namely a, α and β.

4. Some concluding comments

This paper has contributed to the literature in several significant ways. First, it has developed a myopic policy to evaluate a retailer's decision process when confronted with an item subject to gradual obsolescence, a problem largely left relatively unattended so far. Second, the modelling of the demand combines three streams of thought, by considering it to be a decreasing function of both the retailer's sale price and of time, up to a certain stochastic time point when obsolescence occurs and, as a result, the demand suddenly drops to zero. Third, the retailer's profit-maximizing policy consists of an optimal selling price and an order size for each ordering cycle, as well as the time point beyond which it is profitable to stop ordering, even if there is still some demand for the item. This is in contrast to alternate formulations, where the stopping rule is based upon minimizing the cost of obsolescence, rather than evaluating the profitability of the item in question. Fourth, consideration of the gradual obsolescence problem for any probability distribution brings up the important notion of a stopping rule, which does not arise in the exponential case [5], because the assumption of constant obsolescence rate leads to a constant profit per period . Finally, the sudden obsolescence problem is shown to be a limiting case of its gradual counterpart.

In addition, the numerical example has served to illustrate several features of economic significance. With the model allowing the retailer the opportunity to alter demand through the price mechanism, the ordering process ends earlier, since it stops whenever the next order is expected to be profitable, rather than having to wait until there is no longer any demand for the product. Further an important supply chain property is illustrated, namely the need for vendor/retailer collaboration in the development of the pricing policies. Otherwise, the retailer has no incentive to keep prices low and thus counteract the normal decreases in demand that occur as time passes by and the probability of obsolescence increases.

Acknowledgements

Financial assistance for the completion of this research from the Natural Sciences and Engineering Research Council of Canada is gratefully acknowledged, as is the computational assistance of Tamara Djokic.

References

[1] Arcelus FJ, Pakkala TPM, Srinivasan G. Operational policies for the sudden obsolescence problem, with price-dependent demand. Presented at the International Conference of OR for Better Tomorrow, Delhi, India, 1998.

[2] Cobbaert K, Van Oudheusden D. Inventory models for fast moving spare parts subject to 'sudden death' obsolescence. International Journal of Production Economics 1996;44:239–48.

[3] Dohi T, Osaki S. Optimal inventory policies under product obsolescent circumstance. Computer and Mathematical Applications 1995;29:23–30.

[4] Joglekar P, Lee P. An exact formulation of inventory costs and optimal lot size in face of sudden obsolescence. Operations Research Letters 1993;14:283–90.

[5] Joglekar P, Lee P. A profit-maximization model for a retailer's stocking decisions on products subject to sudden obsolescence. Production and Operations Management 1996;5:288–94.

[6] Masters JM. A note of the effect of sudden obsolescence on the optimal lot size. Decision Sciences 1991;22:1180–6.

[7] Van Delft Ch, Vial JP. Discounted costs, obsolescence and planned stockouts with the EOQ formula. International Journal of Production Economics 1996;44:255–65.

[8] Barbosa LC, Friedman M. Inventory lot size models with vanishing market. Journal of the Operational Research Society 1979;30:1129–32.

[9] Chakravarty AK, Martin GE. Discount pricing policies for inventories subject to declining demand. Naval Research Logistics 1989;36:89–102.

[10] Hill AV, Giard V, Mabert VA. A decision support system for determining optimal retention stocks for service parts inventories. IIE Transactions 1989;21:221–9.

[11] Jain K, Silver EA. Lot sizing for a product subject to obsolescence or perishability. European Journal of Operational Research 1994;75:287–95.
[12] Arcelus FJ, Srinivasan G. Inventory Policies under various optimizing criteria and variable markup rates. Management Science 1987;33:756–62.
[13] Wagner H, Whitin TM. Dynamic version of the economic lot size model. Management Science 1958;5:89–96.
[14] Silver EA, Meal HC. A heuristic for selecting lot size quantities for the case of a deterministic time-varying demand rate and discrete opportunities for replenishment. Production and Inventory Management 1973;14:64–74.
[15] Matlab. Optimization toolbox. Natick, MA: The Math Works, Inc., 1997.

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