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On time-interval transformations in special relativity

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Abstract

We revisit the problem of the Lorentz transformation of time-separations between events in the Minkowski spacetime to show that there exist a whole class of "**time-stretching formulas**" which "look" exactly like the well known **time-dilation formula** (TDF) in special relativity. We highlight the essential differences between the TDF and the similar looking time-stretching formulas in view of the fact that occasionally a time-stretching formula has been mistaken for the TDF in the literature. As a by-product of our discussion, we are able to present some new gedanken experiments in which from among the three formulas for time-dilation, length contraction and velocity addition, one can assume any two and derive the third. The novel feature of these gedanken experiments is that they use material particles instead of light rays.

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I. NOTATION AND CONVENTION

$\mathbb{M}$ denotes the Minkowski spacetime. We work in signature $+---$. Events in $\mathbb{M}$ are denoted by Euler-Script characters such as $\mathcal{P}$ and $\mathcal{Q}$. The 4-vector joining the event $\mathcal{P}$ to the event $\mathcal{Q}$ is denoted by $\overset{\rightsquigarrow}{\mathcal{PQ}}$. Latin suffixes are used for the spacetime range 0,1,2,3 and Greek suffixes for the space range 1,2,3. $S : \{ct, x, y, z\}$ and $S' : \{ct', x', y', z'\}$ are two inertial coordinate systems in $\mathbb{M}$. The standard symbols β and γ denote v/c and $1/\sqrt{1 - v^2/c^2}$.

II. INTRODUCTION

This paper takes a fresh look at the Lorentz transformation of time in special relativity. This exercise has been carried out to identify such features of the time-dilation formula (TDF) over the other **similar-looking formulas** that exist in special relativity which we may call **the time-stretching formulas**. This identification appears to be of some importance because at least on one occasion, one of the time-stretching formulas has been mistakenly identified as the TDF in the literature [Griffiths, Ref. 1, pp. 485-486]. To motivate our discussion, we first analyze two typical examples, both taken from the book by Griffiths [Ref. 1] and then pass on to a general discussion of the class of Lorentz time-transformation formulas.

First, we give a brief description of the Griffiths' method to "obtain" the TDF and the **Lorentz length-contraction formula**. Our description follows faithfully the method of Griffith although we do differ in some minor (unimportant) details.

A. Griffiths' gedanken experiment 1

In an inertial reference frame (IRF), say $S : OXYZ$ (Figure 1), a light ray leaves the spatial point $\vec{r}_1 : (x_1, y_1, 0)$ at time t_1 and arrives at the spatial point $\vec{r}_2 : (x_2 = x_1, y_2 = 0, 0)$ at time t_2 thus defining the two spacetime events $\mathcal{P}_1 : (ct_1, \vec{r}_1)$ and $\mathcal{P}_2 : (ct_2, \vec{r}_2)$. Then, $\Delta t_{_{\mathcal{P}_1\mathcal{P}_2}} \equiv (t_2 - t_1)$ is the time-separation between the events $\mathcal{P}_1$ and $\mathcal{P}_2$ in the IRF S and we calculate the corresponding time-separation $\Delta t'_{_{\mathcal{P}_1\mathcal{P}_2}} \equiv (t'_2 - t'_1)$ in the IRF $S' : O'X'Y'Z'$ which is related to IRF S by the standard x-boost

$$ct' = \gamma(ct - \beta x), \;\; x' = \gamma(x - \beta ct), \;\; y' = y, \;\; z' = z. \tag{1}$$

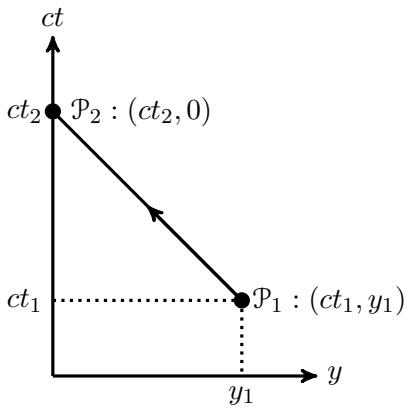

FIG. 1. The world-line of the light-ray in Griffiths' gedanken experiment 1 for obtaining the TDF. The events $\mathcal{P}_1$ and $\mathcal{P}_2$ lie in the $ct - y$ plane.

Then, as $(x_2 - x_1) = 0$, Eq.(1) gives,

$$(t'_2 - t'_1) = \gamma[(t_2 - t_1) - \beta(x_2 - x_1)] = \gamma(t_2 - t_1),$$

so that

$$\Delta t'_{_{\mathcal{P}_1\mathcal{P}_2}} = \gamma\, \Delta t_{_{\mathcal{P}_1\mathcal{P}_2}}. \tag{2}$$

Griffiths [Ref. 1, p.486] "identifies" the above relation Eq.(2) as the TDF.

B. Griffiths' gedanken experiment 2

Next, we describe how Griffiths derives the length-contraction formula using the TDF. We consider **a rigid rod at rest on the X'-axis** of the lRF S' with a point-lamp fixed at one end and a mirror at the other. A light ray leaves the point-lamp at the end $\vec{r}\,'_1 = (x'_1, 0, 0)$ of the rod at the time t'_1, reaches the mirror at the other end $\vec{r}\,'_2 = (x'_2 > x'_1, 0, 0)$ at time t'_2, gets reflected there and returns to $\vec{r}\,'_1$ at the time t'_3. The lRF S' is assumed to be related to another lRF S by the x-boost Eq.(1). Evidently $L' \equiv |\vec{r}\,'_2 - \vec{r}\,'_1| = x'_2 - x'_1$ is the **proper length of the rod**. Let $\mathcal{P}_1$, $\mathcal{P}_2$ and $\mathcal{P}_3$, respectively, be the events associated with the light ray leaving the lamp, arriving at the mirror and returning to the the lamp after reflection at the mirror.

Note that the rigid rod is assumed to move with the velocity $\hat{\imath}\beta/c = \hat{\imath}v$ relative to S. Therefore, if $\Delta t_{_{\mathcal{P}_1\mathcal{P}_2}} = (t_2 - t_1)$ is the time-separation between the events $\mathcal{P}_1$ and $\mathcal{P}_2$ in S,

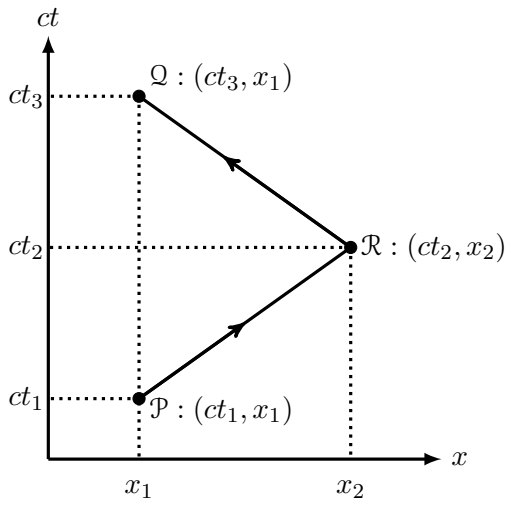

FIG. 2. World-line of the light-ray in Griffiths' gedanken experiment 2 for obtaining the length-contraction formula. The event-pairs $\mathcal{P}, \mathcal{R}$ and $\mathcal{R}, \mathcal{Q}$ are separated by null-intervals, but the event-pair $\mathcal{P}, \mathcal{Q}$ is separated by a time-like interval.

in the time-interval $\Delta t_{_{\mathcal{P}_1\mathcal{P}_2}}$, the mirror-end of the rod moves through the distance $v\,\Delta t_{_{\mathcal{P}_1\mathcal{P}_2}}$ while the light ray would travel the distance $c\,\Delta t_{_{\mathcal{P}_1\mathcal{P}_2}}$. Thus, $c\,\Delta t_{_{\mathcal{P}_1\mathcal{P}_2}} = L + v\,\Delta t_{_{\mathcal{P}_1\mathcal{P}_2}}$ where L is the **length of the (moving) rod** in the frame S and we get $\Delta t_{_{\mathcal{P}_1\mathcal{P}_2}} = L/(c-v)$. Similarly, by noting that the reflected ray travels in a direction opposite to the direction of motion of the rod in S, with the same speed c, we find that the time-separation between the events $\mathcal{P}_2$ and $\mathcal{P}_3$ is $\Delta t_{_{\mathcal{P}_2\mathcal{P}_3}} = L/(c+v)$. Adding the two trip times, we get $\Delta t_{_{\mathcal{P}_1\mathcal{P}_2}} + \Delta t_{_{\mathcal{P}_2\mathcal{P}_3}} = \Delta t_{_{\mathcal{P}_1\mathcal{P}_3}} = L/(c+v) + L/(c-v) = 2\gamma^2 L/c$ which is the time-separation between the events $\mathcal{P}_1$ and $\mathcal{P}_3$ in S. Similarly, since the rod is at rest and has a length L' in the lRF S, the events $\mathcal{P}_1$ and $\mathcal{P}_3$ are evidently separated in time in S by $\Delta t'_{_{\mathcal{P}_1\mathcal{P}_3}} = 2L'/c$. Now, following Griffith, we use the TDF and write $\Delta t_{_{\mathcal{P}_1\mathcal{P}_3}} = \gamma\,\Delta t'_{_{\mathcal{P}_1\mathcal{P}_3}}$. This gives $\gamma L = L'$ which is the length-contraction formula relating the proper-length L' of a rod to its relative length L.

C. Gedanken experiment 3

Prompted by the gedanken experiment 2 of Griffiths, we consider the following modified experiment. Looking at Figure 2, we wish to derive the length-contraction formula using only the world-line joining the events $\mathcal{P}$ and $\mathcal{R}$. Observing that S' is the rest-frame of the

rigid-rod, we use the TDF to relate $\Delta t_{_{\mathcal{PR}}}$ with $\Delta t'_{_{\mathcal{PR}}}$, and obtain $L/(c-v) = \gamma L'/c$ which may be rearranged as

$$L = \sqrt{(1-\beta)/(1+\beta)}\, L'. \tag{3}$$

But, this is not the length-contraction formula! This indicates that the TDF is, perhaps, not the correct relation between $\Delta t_{_{\mathcal{PR}}}$ and $\Delta t'_{_{\mathcal{PR}}} = L'/c$. If the TDF does not relate $\Delta t_{_{\mathcal{PR}}}$ and $\Delta t'_{_{\mathcal{PR}}} = L'/c$, it would mean that, while the **round-trip travel-times** $\Delta t_{_{\mathcal{PQ}}}$ and $\Delta t'_{_{\mathcal{PQ}}}$ of the light ray appear to be related by the TDF, as evidenced by the fact that we get the length-contraction formula by using this relation (gedanken experiment 2), the **one-way travel-times** $\Delta t_{_{\mathcal{PR}}}$ and $\Delta t'_{_{\mathcal{PR}}}$ are (perhaps) **not** related by it (TDF). In the following section, we get back to the basics, use the Lorentz transformation of time to check whether this conclusion is right.

III. LORENTZ TRANSFORMATION OF TIME-SEPARATIONS

We recall that two events $\mathcal{P}$ and $\mathcal{Q}$ in $\mathbb{M}$ are said to be **timelike-separated (TLS)**, **null-separated (NS)**, or **spacelike-separated (SLS)** according as

$$\Delta s^2_{_{\mathcal{PQ}}} \equiv c^2\, \Delta t^2_{_{\mathcal{PQ}}} - |\Delta \vec{r}_{_{\mathcal{PQ}}}|^2 = c^2\, \Delta t^2_{_{\mathcal{PQ}}} - \Delta x^2_{_{\mathcal{PQ}}} - \Delta y^2_{_{\mathcal{PQ}}} - \Delta z^2_{_{\mathcal{PQ}}} \gtreqless 0.$$

Further, we recall the following easily proved well known results concerning pairs of events of $\mathbb{M}$:

Lemma 1 A pair of TLS events is **contiguous** (i.e., they occur at the same spatial point) in an appropriate canonical inertial frame called the **proper frame of the TLS event-pair**.

Lemma 2 A pair of SLS events is **simultaneous** (i.e., they occur at the same time) in an appropriate canonical inertial frame.

Lemma 3 A pair of NS events has space and time separations which are related by $c\,\Delta t = |\Delta \vec{r}|$ in **every inertial frame** S.

Next, we recall that a given (invariant) the space-time displacement between two events $\mathcal{P}$ and $\mathcal{Q}$ is split relative to an inertial frame uniquely into a time-separation Δt and a space-separation 3-vector $\Delta \vec{r}$ (Figure 3). To proceed further, we need to use the rule of

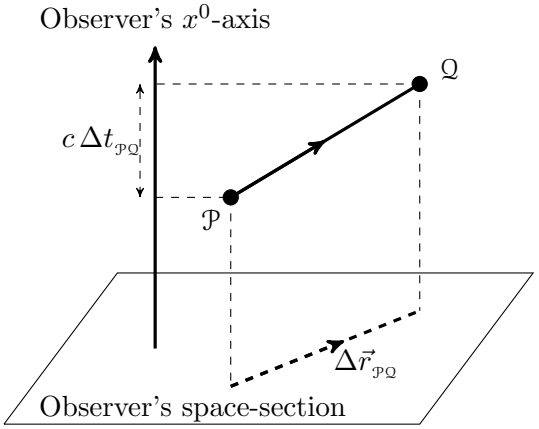

FIG. 3. The splitting of a spacetime displacement $\overset{\rightsquigarrow}{\mathcal{PQ}}$ into space and time displacements relative to an IRF.

transforming the time-separation between an (arbitrary) event-pair in one IRF $S : \{x^i\}$ to that in another IRF say, $S' : \{x'^i\}$. Since we do not want to restrict to any particular configuration between the frames S and S', we consider the frames to be connected by the **general Lorentz boost** [see for example, Weinberg, Ref 2]

$$x'^i = L^i{}_j x^j, \tag{4}$$

where the Lorentz-matrix L has the elements

$$L^0{}_0 = \gamma, \;\; L^0{}_\mu = L^\mu{}_0 = -\gamma\beta_\mu, \;\; L^\mu{}_\nu = \delta_{\mu\nu} + (\gamma - 1)\beta_\mu\beta_\nu/\beta^2, \tag{5}$$

in which $c\vec{\beta} = c(\beta_1\hat{\imath} + \beta_2\hat{\jmath} + \beta_3\hat{k})$ is the constant 3-velocity of the Cartesian frame S' relative to S, $\vec{\beta} = \beta\,\hat{\beta}$ and $\gamma = (1 - \beta^2)^{-1/2}$. Then, the zeroth component of Eq.(3) is the required time-transformation rule between a given pair of events $\mathcal{P}$ and $\mathcal{Q}$:

$$\Delta t' = \gamma\left[\Delta t - (\vec{\beta}/c)\,\cdot\,\Delta\vec{r}\right], \tag{6}$$

Here (Figure 3), we may recall that the spacetime-displacement (4-vector) $\overset{\rightsquigarrow}{\mathcal{PQ}}$ joining $\mathcal{P}$ and $\mathcal{Q}$ has components $(c\,\Delta t, \Delta\vec{r})$ in S and $(c\,\Delta t', \Delta\vec{r}')$ in S'. **Equation** (6) **is our key formula**. We note that it involves the chosen pair of events (as specified by the three parameters $\Delta\vec{r}$) as well as the Lorentz transformation used (which is specified by the three parameters $\vec{\beta}$).

A. Time-transformation formula in the transverse configuration

First, we consider the special case of Eq.(6) when the lRF S' is in what we may call the **transverse configuration** relative to S. This means that the frame S' moves in a direction **perpendicular** to the space-separation 3-vector $\Delta\vec{r}$ of the event-pair $\{\mathcal{P}, \mathcal{Q}\}$ in the frame S. In the transverse configuration, for an arbitrary (i.e., TLS, NS or SLS) pair of events $\{\mathcal{P}, \mathcal{Q}\}$ which have a space-separation 3-vector $\Delta\vec{r} \neq 0$ satisfying $\Delta\vec{r} \cdot \vec{\beta} = 0$ in S, Eq.(6) reduces to

$$\Delta t' = \gamma \Delta t. \tag{7}$$

This formula looks exactly like the TDF (8) that we discuss separately in the following subsection III-B. In the case $\mathcal{P}$ and $\mathcal{Q}$ are TLS, neither Δt in S because of the condition $\Delta\vec{r} \neq 0$, nor $\Delta t' = \gamma\, \Delta t$ in S' which is greater than Δt, and hence is not the minimal time-separation between the events, can be the proper-time separation between the events. On the other hand, when the event-pair $\{\mathcal{P}, \mathcal{Q}\}$ is NS or SLS, by definition, no lRF exists in which $\mathcal{P}$ and $\mathcal{Q}$ are separated by a pure (and hence proper) time-separation. **Thus, in all the three cases TLS/NS/SLS, both the time-separations $\Delta t'$ and Δt in Eq.**(7) **are *non-proper time intervals* unlike in the TDF** (8)**.**

B. Time-separation between TLS events

If neither of the frames S and S' is the proper frame of the TLS events, i.e., if neither of the time-intervals $\Delta t'$ and Δt is a proper-time interval, then one has to use the general formula Eq.(6) for the time-separation transformation transformation. However, if one of the time-intervals, say Δt, is proper, which requires $\Delta\vec{r} = 0$ in S, Eq.(6) reduces to

$$\Delta t' = \gamma \Delta \tau, \tag{8}$$

where we have denoted the **proper time-separation** Δt between $\mathcal{P}$ and $\mathcal{Q}$ by $\Delta\tau$. This is the well known **time-dilation formula (TDF)** [Refs.2-5]. We have discussed the other interesting special case of Eq.(6), namely, $\vec{\beta} \cdot \Delta\vec{r} = 0$ when $\Delta\vec{r} \neq 0$, in the subsection III-A.

C. Time-separation between NS events

For a pair of null-separated events $\{\mathcal{P}, \mathcal{Q}\}$ for which $|\Delta\vec{r}| = c\,\Delta t$ in S, Eq.(6) may be rewritten as

$$\Delta t' = \gamma\,\Delta t\,(1 - \beta\cos\theta)\,, \tag{9}$$

where θ is the angle between the 3-vectors $\Delta\vec{r}$ and $\vec{\beta}$ in S. **The special case $\theta = \pi/2$ is incidentally the Eq.(2) which Griffiths identifies as the TDF in his book [Ref.1]** which we have already discussed in the subsection III-A.

In particular, if we take the time-interval $\Delta t \equiv T$ as the **period** of a monochromatic light wave of frequency $\nu = 1/T$ emitted by a light-source at rest in the IRF S, then the time-interval $\Delta t' \equiv T'$ given by Eq.(9) would be the period of the monochromatic light wave IRF S' in which the light-source has a uniform velocity $c\vec{\beta}$. Thus, Eq.(9) gives

$$\frac{\nu'}{\nu} = \frac{\sqrt{1 - v^2/c^2}}{(1 - \beta\cos\theta)}, \tag{10}$$

which is the **relativistic Doppler formula** [see for example Landau and Lifshitz, Ref. 3, pp.116-17]. Here, in Eq.(10), θ is the angle between the direction of propagation (the wave vector) of the plane electromagnetic wave and the direction of motion ($\vec{\beta}$) of its source. When $\theta = \pi/2$, Eq.(10) gives the **transverse Doppler effect**. The transverse Doppler effect given by Eq.(10) with $\theta = \pi/2$ has been described sometimes as simply a manifestation of time-dilation. In this context, we quote a relevant remark from Weinberg's book [Ref. 2, p.30.] : "...time-dilation [given by the TDF] is not to be confused with the apparent time-dilation or contraction known as the Doppler effect [given by Eq.(10)] ". (The paranthetic remarks here are our own.)

D. Time-separation between SLS events

One has to use Eq.(6) in the general case when neither of the two frames is canonical for the SLS events. Apart from the special case $\vec{\beta}\,\cdot\,\Delta\vec{r} = 0$ with $\Delta\vec{r} \neq 0$ already discussed in subsection III-A, we have one other case in which the formula (6) takes on a reduced form: If one of the frames, say S, is the canonical frame of the two SLS events so that $\Delta t = 0$ in S, Eq.(6) becomes

$$\Delta t' = -\gamma(\vec{\beta}\,\cdot\,\Delta\vec{r})/c = -(\gamma\beta\,\Delta L_0/c)\cos\theta, \tag{11}$$

where $\Delta L_0 = |\Delta\vec{r}|$ is the **proper distance (length)** between the SLS events $\mathcal{P}$ and $\mathcal{Q}$ and θ is the angle betwen $\Delta\vec{r}$ and $\vec{\beta}$ in S.

E. The gedanken experiment 3 also gives length-contraction

The time-transformation formula Eq.(6) solves the riddle posed while discussing the gedanken experiment 3: In fact, in that experiment, we incorrectly used the TDF and arrived at the (erroneous) Eq.(3). Now, we know, from Eq.(6), that the correct formula to be used in experiment 3 is Eq.(9) with $\theta = 0$. Using Eq.(9), we get

$$\Delta t'_{\mathcal{PR}} = \gamma(1-\beta)\,\Delta t_{\mathcal{PR}}. \tag{12}$$

Then, if we substitute (see gedanken experiiment 2) $\Delta t_{\mathcal{PR}} = \Delta L/(c-v)$ and $\Delta t'_{\mathcal{PR}} = \Delta L_0/c$ in Eq.(12), we get $\Delta L_0/c = \gamma(1-\beta)\,\Delta L/(c-v)$ so that $\Delta L_0 = \gamma\,\Delta L$ which is precisely the desired length-contraction formula.

F. A different gedanken experiment

This experiment is a modification of the gedanken experiment 3. It is designed to derive the length-contraction formula specifically using a material particle (such as a bullet shot from a gun), instead of a light ray as in experiment 3, in order to demonstrate to the student that it is not always necessary to use light rays in such gedanken experiments. However, now, our calculations become a little clumsy in view of the fact that the speed of a material particle, unlike c, changes from frame to frame.

In its rest-frame $S' : O'X'Y'Z'$, let the two ends of the rod be $(x'_1, 0, 0)$ and $(x'_2, 0, 0)$. Then, $L' = x'_2 - x'_1$ is the proper-length of the rod. Let a bullet shot from a gun at $(x'_1, 0, 0)$, at time t'_1, travel with the uniform velocity $\hat{\imath}'\,u'$ and reach $(x'_2, 0, 0)$ at time t'_2. This trip of the bullet defines the two events $\mathcal{A}$ and $\mathcal{B}$, which have coordinates $(ct'_1, x'_1, 0, 0)$ and $(ct'_2, x'_2, 0, 0)$ in the IRF S'. In the IRF $S : OXYZ$ which is related to S' by the standard x-boost (obtained by setting $\vec{\beta} = \hat{\imath}'\,\beta$, $\beta > 0$ in Eq.(4) and Eq.(6)), let these events have the coordinates $\mathcal{A} : (ct_1, x_1, 0, 0)$ and $\mathcal{B} : (ct_2, x_2, 0, 0)$. Then, using the inverse of the time transformation Eq.(6), we get

$$\Delta t_{_{\mathcal{AB}}} = \gamma[\Delta t'_{_{\mathcal{AB}}} + (\beta/c)\,\Delta x'_{_{\mathcal{AB}}}], \tag{13}$$

where $\Delta x'_{_{\mathcal{AB}}} = x'_2 - x'_1 = L'$ and $\Delta t'_{_{\mathcal{AB}}} = L'/u'$. Also, we note that $\Delta t_{_{\mathcal{AB}}} = L/(u-v)$. Thus,

$$\Delta t_{_{\mathcal{AB}}} = L/(u-v) = L'\gamma[1/u' + (v/c^2)]. \tag{14}$$

Now, using the Einstein velocity addition formula $u' = (u-v)/(1-vu/c^2)$, we may rewrite the above equation as $L/L'\gamma = [(1-vu/c^2)/(u-v) + (v/c^2)](u-v)$, which simplifies to $L/L'\gamma = 1 - vu/c^2 + (u-v)v/c^2 = 1 - v^2/c^2 = 1/\gamma^2$, so that $L = L'\gamma$ which is the length-contraction formula.

G. Other gedanken experiments

Two variants of the above gedanken experiment can be tried out for fun. In the first, we may use a material particle doing a round trip along the x-axis of the IRF S' instead of doing a one-way trip as in the above gedanken experiment. Alternatively, one may consider a material particle doing a one-way trip in the transverse configuration (for example, along the y-axis of the IRF S') along the x-axis of the IRF S'. We leave the details to the interested reader.

IV. ON THE TDF AND THE OTHER TIME-STRETCHING FORMULAS

The general time transformation equation (6) gives a large number of relations connecting the time-separation between the various possible event-pairs in two inertial frames. Let us call the special case of Eq.(6) corresponding to $\vec{\beta} \cdot \Delta\vec{r} = 0$ as a **time-stretching formula**. Note that the TDF is also a time-stretching formula. However, while the TDF satisfies the condition $\vec{\beta} \cdot \Delta\vec{r} = 0$ because $\Delta\vec{r} = 0$, all other time-stretching formulas satisfy $\vec{\beta} \cdot \Delta\vec{r} = 0$ with a non-zero $\Delta\vec{r}$ which is perpendicular to $\vec{\beta}$. Therefore, the TDF arises in a completely different situation when compared to the other time-stretching formulas. Hence, none of the time-stretching formulas, in particular the one in Eq.(2), qualifies to be called the TDF. In support of this conclusion, we may also recall some known features of the TDF which distinguish it from other time-stretching formulas. **The TDF** which is summarized by the statement that **a moving clock goes slow** [Refs. 1-5], **is a relation connecting the proper-time-separation of a TLS event-pair with its corresponding non-proper-time interval in some other lRF**. In general, a given pair of TLS events is separated by

different time-intervals in different lRF's. Of these, the time-interval measured in the proper-frame of the TLS events, called the **proper-time interval**, is the **minimal** time-separation between the two TLS events. As such, a **proper-time interval is always dilated** in any other inertial frame (and is never shortened). On the other hand, both the time-separations that occur on either side of the time-stretching formula Eq.(7) are non-proper separations as already observed towards the end of the subsection III-A. Although the non-proper time interval Δt in S is shorter than $\Delta t'$ in the transverse configuration for S', in some other appropriate non-transverse configuration for S', also given by Eq.(6), the **same** non-proper time interval Δt in S can become **greater** than $\Delta t'$ also. Thus, a non-proper time interval (specified in some inertial frame) can get **dilated** in some inertial frame and as well get **contracted** in some other (appropriate) inertial frame. Hence it is not proper to call its transformation rule as a "time-dilation formula".

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