

# USPOREDBI NA PERFORMANSE UMJETNE GRANICE P3 I P4 STACEY OVISNO O INTERVALU

## COMPARISON OF THE PERFORMANCE OF THE ARTIFICIAL BOUNDARIES P3 AND P4 STACEY DEPENDING ON INTERVAL

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**Sadržaj:** *Ovaj rad predstavlja istraživanje obavljanje umjetne granice P3 i P4 od Stacey ovisno o intervalu od visine računalne mreže. Razmak između mreže točaka se mijenja stalno i tablice prikazani su u nastavku. Uvod, model, rezultati, zaključak i reference radu prikazani su u nastavku. Analitičke metode omogućuju nam da modelirati puno procesa u istraživanje zemlje i da uživati u značajnom uspjehu u studiju o trajnim deformacijama kao i za propagiranje valova za dugo vremena. U stvarnosti geološki mediji su isprekidani velikog razmjera, u kombinaciji s diskretnim česticama, razmotriti obje mikroskopske i makroskopske dijelove. Razvoj numeričkih shema pružaju nam proučavanje valova prenošenje kroz složene medijima. Ove metode su se pokazale kao vrijedan način za provedbu pravih podataka prekrivene seizmičkih istraživanja sa sintetičkim podacima iz numeričkih modela. Oni također imaju omogućiti uvid u procese poput raspršenja seizmičkih valova kao oni prolaze kroz koru. Naravno, tu su i druge metode kao metodom konačnih razlika, metoda graničnih elemenata, diskretne čestica sheme koji se ne temelje na valna jednačnja, ali je i fizike valova koji pokazuje na skali atoma. Ovih dana numerička seizmologija može pružiti nam računalne simulacije seizmičkih valova [4], koja predstavlja neprocjenjiv alat za razumijevanje fenomena valova, njihov stvaraju i njihove posljedice.*

*Rubni uvjeti uvijek će biti potrebno da se jamči jedinstvenu dobro formirane rješenje diferencijalne jednačnje.*

**Abstract:** *This paper presents an research of the performance of the artificial boundaries P3 and P4 of Stacey depending on the interval of the height of the computational grid. The interval between mesh points is changed constantly and the tables are presented below. Introduction, structure, results, conclusion and references of the paper are shown below. Analytical methods allow us to model a lot of processes in the exploring of the earth and to enjoy in a significant success in studies about permanent deformations as also for propagating of waves for a long time. In reality geological mediums are discontinuous of a large scale rang, combined from discrete particles, consider both microscopic and macroscopic parts. Development of numerical schemes provide us to study wave propagations through complex media. These methods have proven as valuable as a way for implementing of real data covered with seismic explorations with synthetic data from numerical models. They also have allow access to the processes like scattering of seismic waves as they pass through the crust. Of course there are other methods as finite difference method, boundary method of elements, discrete particle scheme who are not based on the wave equation, but to the physics of the wave propagation that shows on the atom scale. These days numerical seismology can provide us computer simulations of seismic wave propagation [4], that represent invaluable tool for understanding of the wave phenomena, their generating and their consequences.*

*Boundary conditions will always be required to guarantee unique well formed solution to the differential equation.*

### 1. INTRODUCTION

The numerical methods are shown very useful in cases of moderate and strong scatterings in complex mediums were multiple scattering of the waves are case of the investigating. Many researchers have used numerical methods for scattering of the waves, conversion of the waves through soil and making more precise artificial boundaries. Filtering of the incident waves also becomes very important part in the improving of the artificial boundaries. Tests in this paper

represent the performance of the artificial boundaries of Stacey P3 and P4 depending of the interval of height of the grid. The ratio between compressional and tangential waves is constant. As the crust becomes more known for the researchers, the artificial boundaries are more and more accurate, which is of essential meaning. The goal of all computations and tests that are made is to get more valuable predictions on that how the crust will answer if high density waves are spreading through it. That kind of waves for

example may be produced from seismological activity deep in the crust.

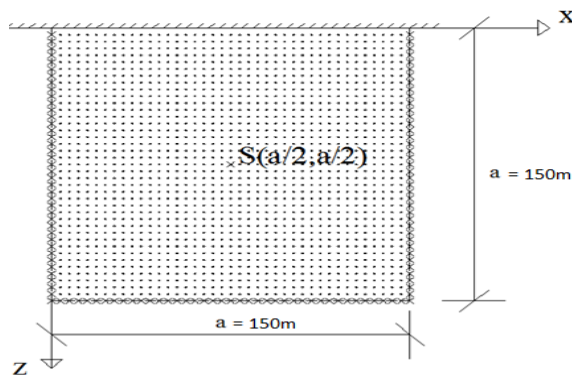
We use a concrete model were these waves are with artificial boundaries limited and making multiple tests, numerical simulations and variations with changing the interval of the height as shown in this paper. Problems that are of seismological character and earthquake engineering are solving also with stochastic methods [2]. Some research are added to this numerical methods to explore the seismic propagation of waves in velocity models that vary different in the space.

**2. THE MODEL**

Performance of the boundary conditions P3 and P4 have been explored on a mathematical 2D model, combined of grid with dimensions 150 x 150 [8]. In every direction there are 75 intervals on equal distance, in case when the index  $h=1$ . In this study, that distance will be changed from 0.6 to 2 meters. So, this part of medium should represent the shortened part from the whole medium. The medium is homogeneous and isotropic with propagating velocities of the compressional and tangential waves appropriate. At point (75, 75) explosive source is applied (picture 1).

**2.1. MOTION EQUATION**

The model is presented with two axis, horizontal – x and vertical – y axis. Values at x - axis are incrementing from left to right, and values at y – axis are incrementing from up to down so the upper left corner is (0, 0) point. With this settings two partial differential equations can be used for describing the motion of P and SV waves.



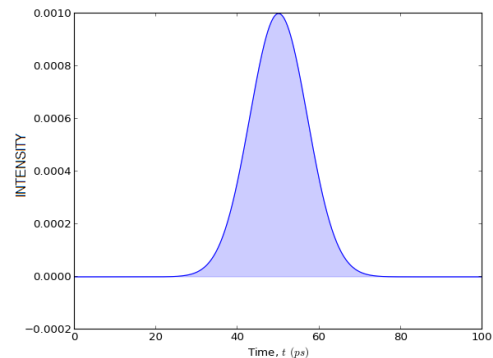
Picture 1. Test model

**2.2. INITIAL CONDITIONS**

At time  $t=0$  it is supposed there are no movements in the crust and the dislocations are  $u=w=0$ . While the numerical model is explicit two time steps are required two compute the next. The infinite media is made finite and from here computational with representing the artificial boundaries. To be more accurate the whole model, for the upper boundary

(free surface) also boundary conditions are used. In order to be all this functional a source is needed.

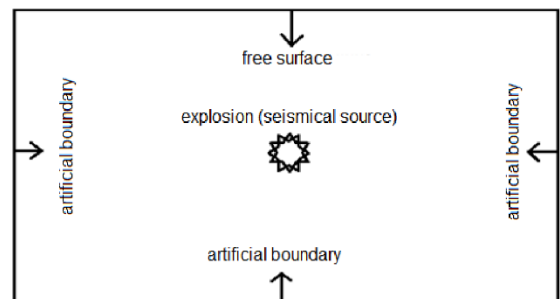
Explosive source will be used and it going to be Gaussian pulse (picture 2). Absorbing boundaries based on a combination of the analysis and the approaches to the modification from the wave field are developed for seismic simulations with hyperbolic systems [6]. One – dimensional absorbing boundary conditions are combined with approaches to other methods, as are paraxial approximations of free surface, anisotropic filtering, and a lot of other techniques. These methods absorbs effective all types of incident waves without exposing the local conditions.



Picture 2: Displacement function  $f(t)$  in time

Kelly et. al. [3] developed numerical model for equation of motion. Same numerical approximation is used in the presented model. All of the calculations have been made under workspace Eclipse, an algorithm with initial conditions and motion equations was developed and also graphical output is provided. There are two matrix outputs that represents the input and output energy of the model in matter of time  $t$  and height  $h$ .

It is important to know that all the calculations are been made with ratio between the velocities of the compressional and tangential waves of 2.0. In [6] is proven that the system is most stable for ratio between 1.5 and 3.0 and variant P3 is more stable than P4.



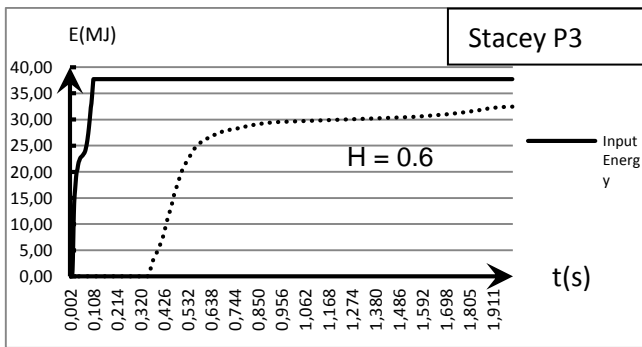
Picture 3: Soil part that includes source of explosion, free surface and three artificial boundaries

Many geological problems including the modeling of the earth crust and the seismic research for gas and oil are problems in wave propagating, seeking for solution of the two – dimensional (2-D) wave equation. With these model

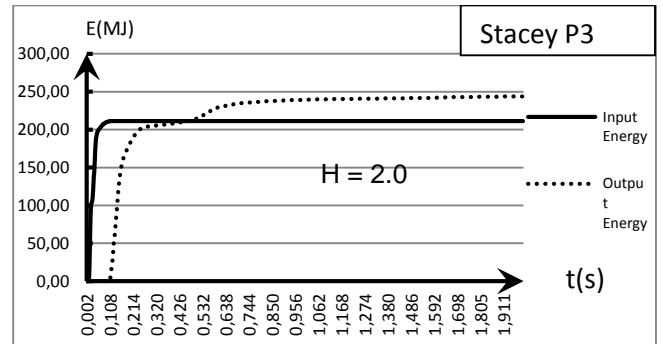
(picture 3) we define our area of computation with three artificial boundaries that are in the crust and the one we stand on the free surface. After the explosion the waves starting to move to the boundaries. As they reach the boundaries becomes a reflection of the boundaries “reflected waves” are produced. These waves are harmful for the system and we try to decrease their energy in the system. With better artificial boundaries we get better stability of the model.

Clayton and Engquis [1] used paraxial boundary conditions which are perfect absorbing for rays with normal impact. Also were constructed boundary conditions by direct working with differential approximations in order to minimize the amplitude of the reflection. Similar the coefficient of the reflection were minimized with factoring of the differential equation. Paraxial approximations are based on approximation of the dispersion of the one – dimensional wave equation. For polynomials these approximation over Taylor order.

### 3. RESULTS

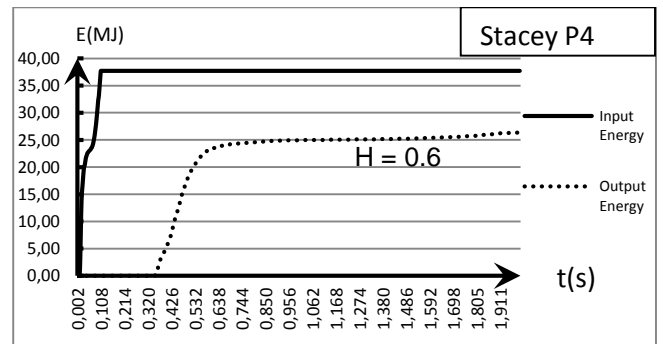


Picture 4: Input (full line) and output (dotted line) energy at  $h = 0.6$ , P3 variant

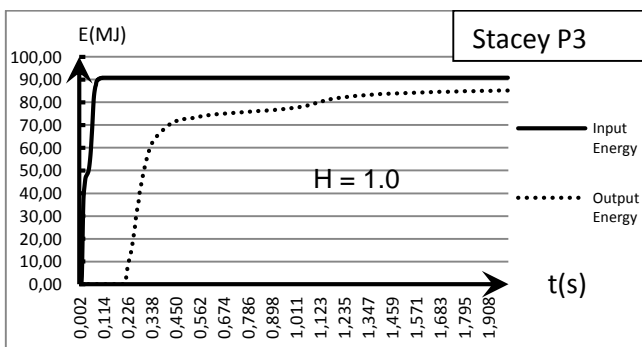


Picture 6: Input (full line) and output (dotted line) energy at  $h = 2.0$ , P3 variant

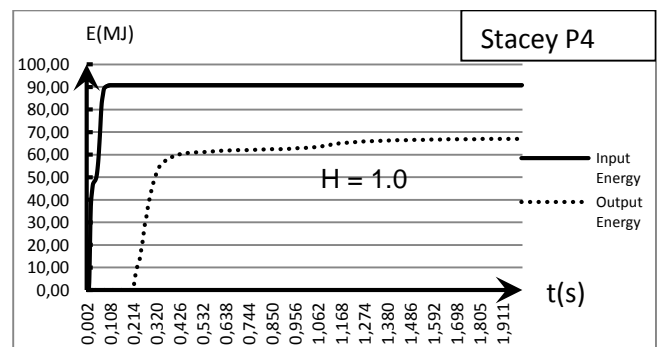
Now, if we closely take a look of this results, actually pictures 4 and 7, we can conclude that P3 is with much smaller error, and on other hand P4 is with high error. Continuing on situations at pictures 5 and 8, error at P3 is now present but small, while error at P4 variant is remaining enough big to fully declaring of an mode with big error. At pictures 6 and 9 the errors in both cases are now far too big to make further discussions. So we can make a conclusion that variant P3 at interval 1.0 is the most accurate situation.



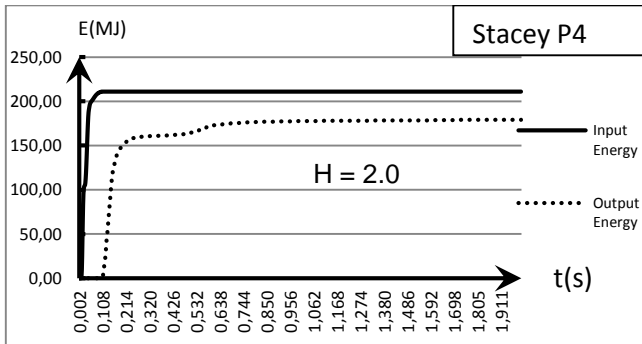
Picture 7: Input (full line) and output (dotted line) energy at  $h = 0.6$ , P4 variant



Picture 5: Input (full line) and output (dotted line) energy at  $h = 1.0$ , P3 variant

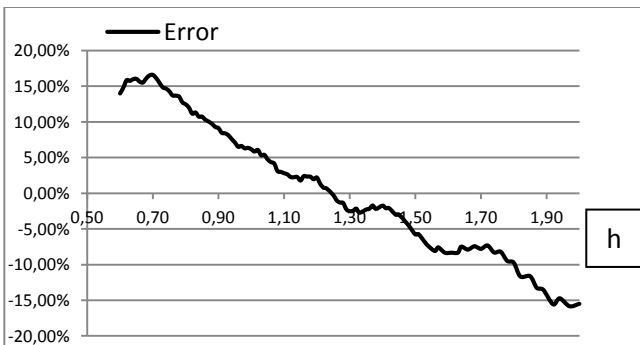


Picture 8: Input (full line) and output (dotted line) energy at  $h = 1.0$ , P4 variant



Picture 9: Input (full line) and output (dotted line) energy at  $h = 2.0$ , P4 variant

As the interval rises to values of 2.0 for P3, the error is big and that situation further in analysis should not be taken. Variant P4 is constantly with high error from values of 0.6 to 2.0. This mode in this concrete situation give us conclusion that is less accurate and with high error instead P3 mode. Interesting for mention is that exactly at  $h = 2.0$  P4 give less error than  $h = 1.0$ , but because the error is constantly present at P4 instead of P3 we still can say that P3 is better mode as you can see from picture 10.



Picture 10: Error at Stacey P3 in matter of  $h$

#### 4. CONCLUSION

In different cases every mode gives different results. To say which mode is better very large analysis should be done. The numerical solutions shown above support theoretical analyze of the boundary schemes. However the point is to prove the importance of these boundary conditions. Exploring of artificial boundaries is of great importance for seismology.

Since last century researchers were trying to better know the natural phenomena. If our nature is more known for us with more her characteristics, we can easily predict some natural activities like earthquakes we mentioned above. In seismology exploring is directed to the waves that appear at every reaction in the crust, and that's how artificial boundaries appear. With better artificial boundaries the changes in the earth will be more predictable. So from all of this we can conclude that this field of investigation has a lot more to give.

#### 5. REFERENCES

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