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# Hybrid Method for Inverse Electromagnetic Coil Optimization Using Multi-transition and Hopfield Neural Networks

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**Abstract.** In this paper, a hybrid method for inverse optimization of electromagnetic coils utilizing the multi-transition neural network and the Hopfield neural network is proposed. Due to the discrete character of the neural network, an optimization problem is transformed into a discrete problem through the division of the entire coil area into elemental coils with constant current density. The minimization of the objective function is performed by the multi-transition neural network and the Hopfield neural network in turns. Subdivision of the elemental coils is performed in order to achieved better accuracy of the results which are verified using 2-D finite element analysis. The application of the proposed method for inverse optimization of MRI device is also presented.

## 1. Introduction

Design and optimization of shapes and parameters of various electromagnetic devices can be performed by two general methods: direct methods which are usually very time consuming and require treatment of one variable as a parameter while other variables are changeable, and inverse methods which are faster than direct methods but usually very case sensitive.

For solving inverse optimization problems, various procedures such as simulated annealing, neural networks or genetic algorithms have been proposed. [1, 2] Recently, the authors proposed a new inverse method for optimization of coil position using the Hopfield neural network [3]. However, several problems were experienced, such as long computation time and achievement of a local minimum of the objective function instead of the desired global minimum.

In this paper, the authors propose an improved hybrid method for optimal position design of electromagnetic coils by using the multi-transition neural network (MTNN) [4] and the Hopfield neural network (HNN). Due to the discrete character of the neural network, the discretization of the entire available coil area into a set of elemental coils is performed. Initially, the MTNN, which is less sensitive to the local minimum of the objective function than the HNN [4], is employed for global minimization of the objective function. However, the MTNN minimization is discrete and strongly influenced by neuron number — in our case the discretization pattern of the entire available coil area. To improve the approaching characteristic of the minimization process, the HNN was used in conjunction with the MTNN. While the Dirack Step-function is utilized as an input-output function of the MTNN, for the HNN the ordinary sigmoid function is employed. This results in a continuous spectrum of output values between 0 and 1, rather than two discrete binary values 0 and 1, as previously proposed [3]. The elemental coils with neuron output values between 0 and 1 can be subdivided into smaller sub-elemental coils, resulting in greater accuracy of the optimization process. The entire algorithm can be repeated until the desired accuracy is achieved.

In this paper, the algorithm of the proposed hybrid method is described. Then, the proposed method is successfully applied for optimization of the electromagnetic coils in an electromagnetic MRI device.

## 2. Outline of Proposed Method

As mentioned above, due to the discrete character of the neural network where the neuron status is restricted to binary values 1 or 0, the optimization problem of the coil position, which is rather a continuous problem of determining the terminal coordinates of coils, is transformed into a discrete problem. The entire available coil area is discretized into a set of elemental coils $n$ (see Fig. 1). In the neural network each elemental coil is represented by a separate neuron with status $p_n$. After the minimization process is completed, each neuron has either binary value 1 or 0 in case of the MTNN or any other value between 0 and 1 in case of the HNN. In the real physical model, it means that the elemental coil with value $p_i = 1$ is carrying current with constant current density, while the elemental coil with value $p_i = 0$ is not carrying current.

Using the following objective function

$$O = \sum_{k=1}^{m} \left\{ \sum_{j=x,y,z,r\cdots} [B_{0kj} - B_{kj}(\mathbf{p})]^2 \right\} \quad \rightarrow \quad min\,, \tag{1}$$

we are able to control not only the intensity of magnetic flux density $\mathbf{B}$, which is the monitoring parameter in the analysis, but also its components $j$ and direction in the space. In (1), $m$ is the number of observation points with prescribed value of magnetic flux density component $j$, and $\mathbf{p}$ is a vector define as $\mathbf{p}^T = [p_1, p_2, \cdots, p_i, \cdots, p_n]$, where $n$ is the number of elemental coils (see Fig. 1).

### 2.1. Multi-transition Neural Network

A new multi-transition neural network (MTNN) [4] is employed for initial minimization of the objective function. In minimization of the objective function using of the HNN, the minimum is very likely to be local, which points to the main disadvantage of the HNN. On the other hand, using the MTNN, we perform multi-transition on all or a group of neurons, changing their status so that now the energy of the network is smaller than the energy of the network at its local minimum (see Fig. 4). Since we use a discrete Dirack Step-function for the input-output function of the MTNN, we are likely to jump at each multi-transition step in the direction of the global minimum. After only a few minimization steps of the MTNN, we can approach the global minimum of the objective function. Further minimization is executed using the HNN, mainly for reasons related to its greater sensitivity than that of the MTNN.

### 2.2. Hopfield Neural Network

Before minimization of the objective function with the HNN begins, each neuron status is defined by the last output of the MTNN, and not by some random function like in [3]. Therefore, the status of each neuron $p_i$ (each neuron represents one elemental coil) can be: $p_i = 0$ resulting in no current flow in elemental coil $i$, or $p_i = 1$ resulting in constant current flow in elemental coil $i$. This status is more likely to be closer to the global minimum of the objective function than any other neuron status. Therefore, the minimization of the objective function can be executed faster using the HNN. However, due to the difference between input-output functions used for the MTNN and for the HNN, which for the latter is a continuous sigmoid function, the status of each neuron might change significantly. While the minimization process evolves with time, the status of each neuron $p_i$ changes. Finally, after a stopping criterion is achieved, the status of each neuron might have values in the entire region between 0 and 1. It is reasonable to conclude that the elemental coil with neuron status 1 is carrying a constant amount of source current, while the elemental coil with neuron status 0 is not carrying any current. The possibility is high, however, that some of the elemental coils will end with a neuron status somewhere between 0 and 1. For these elemental coils, the subdivision process is performed by further dividing each of these elemental coils into smaller sub-elemental coils, and the assignment of a new neuron to each new generated sub-elemental coil. This newly generated neural network is again minimized employing the MTNN. By the repetition of this procedure, it is possible to reach the global minimum of the network faster and easier. The simplified block-diagram of the proposed algorithm is presented in Fig. 2. To reduce the number of circles, and therefore to speed up the computation in general, a simple procedure that changes the status of some of the neurons is enabled. We used two threshold values: 0.9 and 0.1 as margins, and neurons with a status larger than 0.9 or lower than 0.1 were changed into a new status of 1 and 0, respectively. The threshold values can be defined

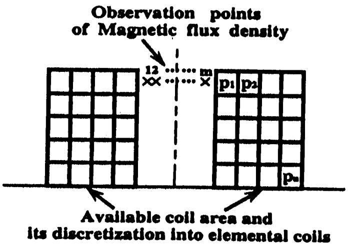


Fig. 1. Entire coil area and observation points.

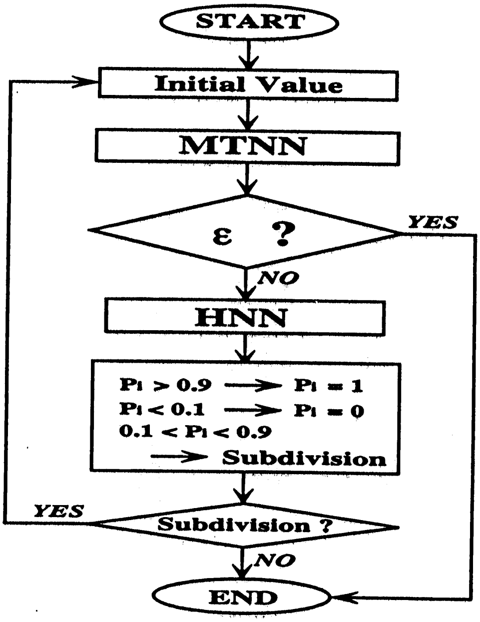


Fig. 2. Algorithm of proposed hybrid method.

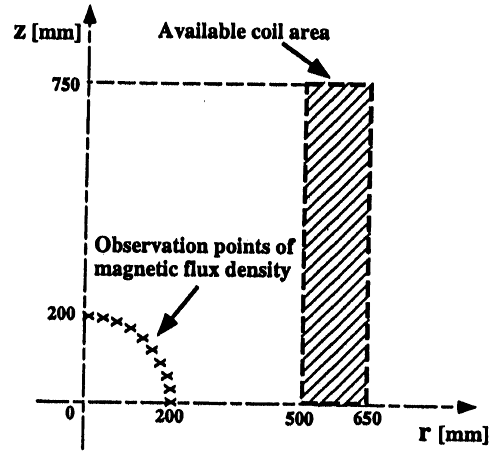


Fig. 3. Model of MRI device.

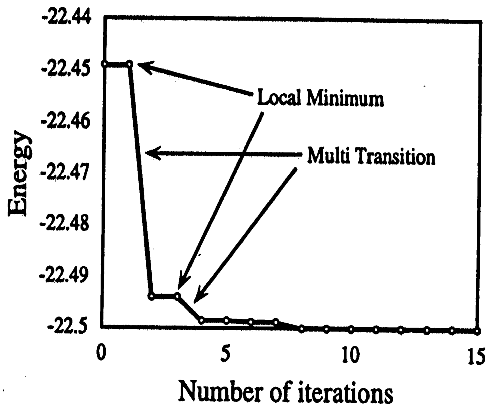


Fig. 4. Energy minimization by MTNN–step #1.

by the user. For more accurate results, narrow margins are advisable.

## 3. Application

The proposed hybrid method was applied for inverse position optimization of electromagnetic coils inside a MRI device. The schematic layout of the application model is presented in Fig. 3. The available coil area, presented as a striped region in Fig. 3, was bounded as $500$ [mm] $\leq r \leq 650$ [mm] and $0$ [mm] $\leq z \leq 750$ [mm] — only 1/4 of the entire optimization area was analyzed. The entire coil area was divided into twenty elemental coils, initially developing a neural network of only twenty neurons. Ten observation points symmetrically positioned along a circle with a radius of 200 [mm] around the center of the model (see Fig. 3) were considered. The source current value was constant and equal to 20 [A/mm$^2$].

Figure 4 shows the minimization of the energy of the neural network by the MTNN. It can be seen that as the network evolves with time the minimization of its energy can be executed faster using the MTNN, and at the same time, avoiding any local minimum of the objective function. The status of each neuron was established initially by randomly generating output values of each neuron. In this stage only the discrete values 0 and 1 are allowed to exist as output values of each neuron (see Fig. 5*a*). Then, using the MTNN, the minimization of the energy due to the objective function (1) was performed. The final status of each neuron following the MTNN minimization process, is presented in Fig. 5*b*. The next step in the analysis is checking the error of the obtained solution by the following error criterion:

$$\varepsilon = \frac{|B_{zi} - B_{z0i}|}{|B_{z0i}|} \cdot 100 \ [\%] \,, \qquad (2)$$

where the $B_{z0i}$ and $B_{zi}$ are the desired and obtained intensity values of the $z$–component of magnetic flux density vector $\mathbf{B}$ at each observation point $i$, respectively.

If the maximum error obtained by means of (2) was smaller than the prescribed one, the computation was terminated. Otherwise, the neural network's energy was further minimized by the HNN, and the neuron status presented in Fig. 5*c* was obtained. Notice that now some of the neurons have an output value between discrete terminal values 0 and 1. To speed up the computation to each neuron with a status larger than 0.9, a new status value 1 was assigned, and to those with a status smaller than 0.1, value 0 (see Fig. 5*d*). The elemental coils represented in the neural network by neurons with a status between 0.1 and 0.9 are further subdivided into smaller sub-elemental coils, and each new neuron is assigned to each new sub-elemental coil. This procedure must be repeated until the desired error criterion is reached (see Figs. 2 and 5).

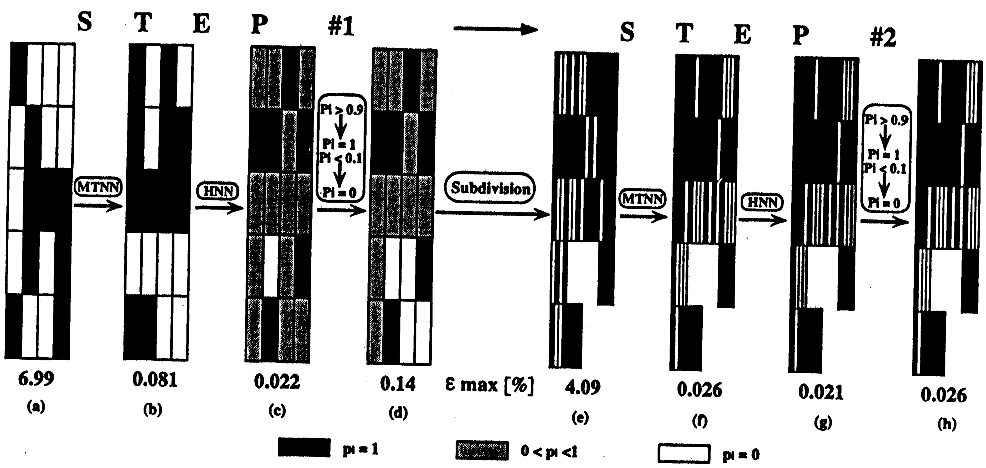


Fig. 5. Obtained results for MRI model.

## 4. Conclusions

A new hybrid method for inverse optimization of electromagnetic coils using the multi-transition and the Hopfield neural networks in combination was proposed. Avoiding local minimum points by using the MTNN and preserving the sensitivity of the network by using the HNN results in a new algorithm that provides highly accurate results with less computation effort. The accuracy of the results is also increased by subdivision of elemental coils into smaller sub-elemental coils, a procedure that can be performed using the proposed method. The successful application of the proposed method for inverse optimization of electromagnetic coils in MRI device is very promising.

# References

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