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Applying of the Pseudoinverse Matrix in the Removal of Blur in Images

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***Abstract* – A method for digital image restoration, based on the pseudoinverse matrix, has many practical applications. We apply the method to remove blur in an image caused by uniform linear motion. This method assumes that linear motion corresponds to an integral number of pixels. Compared to other classical methods, this method attains higher values of the Improvement in Signal to Noise Ration (ISNR) parameter and of the Peak Signal-to-Noise Ratio (PSNR). We give an implementation in the MATLAB programming package.**

***Keywords:* deblurring, image restoration, matrix equation, pseudoinverse.**

I. INTRODUCTION

Recording and presenting helpful information is the purpose of producing images. Yet, the recorded image is a degraded form of the initial scene as a result of flaws in the imaging and capturing process. Images are rather unclear in numerous applications such as satellite imaging, medical imaging, astronomical imaging or poor-quality family portraits. It is vital to many of the subsequent image processing tasks to neutralize these flaws. One should consider an extensive variety of different degradations for example blur, noise, geometrical degradations, illumination and color imperfections [1-3].

Blurring is a form of bandwidth reduction of an ideal image owing to the imperfect image formation process. It can be caused by relative motion between the camera and the original scene, or by an optical system that is out of focus. When aerial photographs are produced for remote sensing purposes, blurs are introduced by atmospheric turbulence, aberrations in the optical system, and relative motion between the camera and the ground.

The field of image restoration is concerned with the reconstruction or estimation of the uncorrupted image from a blurred one. Essentially, it tries to perform an operation on the image that is the inverse of the imperfections in the image formation system. In the use of image restoration methods, the characteristics of the degrading system are assumed to be known a priori.

The method, based on pseudoinverse matrix, is apply for the removal of blur in an image caused by uniform linear motion. This method assumes that linear motion corresponds to an integral number of pixels. For comparison, we used two commonly used filters from the collection of least-squares filters, namely Wiener filter and the constrained least-squares filter [2]. Also we used in comparison the iterative nonlinear restoration based on the Lucy-Richardson algorithm [3].

This paper is organized as follows. In the second section we present process of image formation and problem formulation. In Section III we describe a method for the restoration of the blurred image. We observe certain enhancement in the parameters: *ISNR* and *PSNR*, compared with other standard methods for image restoration, which is confirmed by the numerical examples reported in the last section.

II. MODELING OF THE PROCESS OF THE IMAGE FORMATION

We assume that the blurring function acts as a convolution kernel or point-spread function $h(n_1, n_2)$ and the image restoration methods that are described here fall under the class of linear spatially invariant restoration filters. It is also assumed that the statistical properties (mean and correlation function) of the image do not change spatially. Under these conditions the restoration process can be carried out by means of a linear filter of which the point-spread function (PSF) is spatially invariant. These modeling assumptions can be mathematically formulated as follows. If we denote by $f(n_1, n_2)$ the desired ideal spatially discrete image that does not contain any blur or noise, then the recorded image $g(n_1, n_2)$ is modeled as [2]:

$$\begin{aligned} g(n_1, n_2) &= h(n_1, n_2) * f(n_1, n_2) \\ &= \sum_{k_1=0}^{N-1} \sum_{k_2=0}^{M-1} h(k_1, k_2) f(n_1 - k_1, n_2 - k_2). \end{aligned} \quad (1)$$

The objective of the image restoration is to make an estimate $f(n_1, n_2)$ of the ideal image, under the assumption that only the degraded image $g(n_1, n_2)$ and the blurring function $h(n_1, n_2)$ are given.

The problem can be summarized as follows: let H be a $m \times n$ real matrix. Equations of the form:

$$g = Hf, g \in \Re^m; f \in \Re^n; H \in \Re^{m \times n} \tag{2}$$

describe an underdetermined system of m simultaneous equations (one for each element of vector g) and $n = m + l - 1$ unknowns (one for each element of vector f). Where the index l indicates horizontal linear motion blur in pixels.

The problem of restoring an image that has been blurred by uniform linear motion, usually results of camera panning or fast object motion can be expressed as, consists of solving the underdetermined system (2). A blurred image can be expressed as:

$$\begin{bmatrix} g_1 \\ g_2 \\ g_3 \\ \vdots \\ g_n \end{bmatrix} = \begin{bmatrix} h_1 & \cdots & h_l & 0 & 0 & 0 & 0 \\ 0 & h_1 & \cdots & h_l & 0 & 0 & 0 \\ 0 & 0 & h_1 & \cdots & h_l & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & h_1 & \cdots & h_l \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ \vdots \\ f_m \end{bmatrix} \tag{3}$$

The elements of matrix H are defined as: $h_i = 1/l$ for i=1, 2,..., l. The objective is to estimate an original row per row f (contained in the vector f^T), given each row of a blurred g (contained in the vector g^T) and a priori knowledge of the degradation phenomenon H. We define the matrix F as the deterministic original image, its picture elements are F_{ij} for i=1,…, r and for j=1,…, n, the matrix G as the simulated blurred can be calculated as follows:

$$G_{ij} = \frac{1}{l} \sum_{k=0}^{l-1} F_{i, j+k}, i = 1, \cdots, r, j = 1, \cdots, m \tag{4}$$

with $n = m + l - 1$, where l is the linear motion blur in pixels. Equation (4) can be written in matrix form of the process of *horizontal* blurring as:

$$G = \left(HF^T\right)^T = FH^T \tag{5}$$

Since there is an infinite number of exact solutions for f or F in the sense that satisfy the equation $g = Hf$ or $G = FH^T$, an additional criterion that find a sharp restored matrix is required.

The process of blurring with vertical motion is with the form:

$$g = Hf, g \in \Re^m; f \in \Re^r; H \in \Re^{m \times r} \tag{6}$$

where $r = m + l - 1$, and l is linear vertical motion blur in pixels. The matrix H is Toeplitz matrix as the matrix given in (3), but with other dimensions. The matrix form of the process of *vertical* blurring of the images is:

$$G = HF, G \in \Re^{m \times n}; H \in \Re^{m \times r}; F \in \Re^{r \times n} \tag{7}$$

III. METHOD FOR THE REMOVAL OF BLUR IN THE IMAGES

The notion of pseudoinverse matrix of square or rectangular pattern is introduced by H. Moore in 1920 and again from R. Penrose in 1955, who was not aware of the work of Moore. Let T is real matrix with dimension $m \times n$ and $\Re(T)$ is the range of T. The relation of the form:

$$Tx = b, T \in R^{m \times n}, b \in R^m \tag{8}$$

are obtained in the analysis and modeling of many practical problems. It is known that when T is a singular matrix, its unique pseudoinverse matrix is defined.

In case when T is real matrix with dimension $m \times n$, Moore and Penrose proved that pseudoinverse matrix $T^\dagger$ is a unique matrix that satisfies the following four relations:

- $TT^\dagger T = T$
- $T^\dagger TT^\dagger = T^\dagger$
- $(TT^\dagger)^T = TT^\dagger$ (9)
- $(T^\dagger T)^T = T^\dagger T$

We will use the following proposition from [6]:

Let $T \in R^{m \times n}, b \in R^m, b \notin \Re(T)$ and we have a relationship $Tx = b$, then we have $T^\dagger b = u$, where u is the minimal norm solution and $T^\dagger$ is the pseudoinverse matrix of T.

Since relation (2) has infinitely many exact solutions for f, we need an additional criterion for finding the necessary vector for restoration. The criterion that we use for the restoration of blurred image is the minimum distance between the measured data:

$$\min(\left\|\hat{f} - g\right\|) \tag{10}$$

where $\hat{f}$ are the first m elements of the unknown image f, which is necessary to restore, with the following constraint:

$$\left\|Hf - g\right\| = 0. \tag{11}$$

Following the above proposal, only one solution of the relation $g = Hf$ minimizes the norm $\|Hf - g\|$. If this solution is marked by $\hat{f}$, then for it is true:

$$\hat{f} = H^{\dagger} g \tag{12}$$

Taking into account the relations of horizontal blurring (2) and (5), and relation (12) solution for the restored image is:

$$\hat{F} = G(H^T)^{\dagger} = G(H^{\dagger})^T \tag{13}$$

In the case of process of *vertical blurring* solution for the restored image, taking into account equations (6), (7) and (12), is:

$$\hat{F} = H^{\dagger} G \tag{14}$$

IV. EXPERIMENTAL RESULTS

In this section we have tested the method based on pseudoinverse matrix (PIM method) of images and present numerical results and compare with two standard methods for image restoration called least-squares filters: Wiener filter and constrained least-squares filter and the iterative method called Lucy-Richardson algorithm.

The experiments have been performed using Matlab programming language on an Intel(R) Core(TM) i5 CPU M430 @ 2.27 GHz 64/32-bit system with 4 GB of RAM memory running on the Windows 7 Ultimate Operating System.

In image restoration the improvement in quality of the restored image over the recorded blurred one is measured by the signal-to-noise ratio (*SNR*) improvement is defined as follows in decibels [7]:

$$\begin{aligned} ISNR &= SNR_{\hat{f}} - SNR_g \\ &= 10\log_{10}\left(\frac{\text{Variance of } g(n_1,n_2) - f(n_1,n_2)}{\text{Variance of } \hat{f}(n_1,n_2) - f(n_1,n_2)}\right) \end{aligned} \tag{15}$$

The improvement in *SNR* is basically a measure that expresses the reduction of disagreement with the ideal image when comparing the distorted and restored image. Note that all of the above signal-to-noise measures can only be computed in case the ideal image is available, i.e., in an experimental setup or in a design phase of the restoration algorithm.

The simplest and most widely used full-reference quality metric is the mean squared error (*MSE*) [8], computed by averaging the squared intensity differences of restored and reference image pixels, along with the related quantity of peak signal-to-noise ratio (*PSNR*). These are appealing because they are simple to calculate, have clear physical meanings, and are mathematically convenient in the context of optimization.

The advantages of *MSE* and *PSNR* are that they are very fast and easy to implement. However, they simply and objectively quantify the error signal. With *PSNR* greater values indicate greater image similarity, while with *MSE* greater values indicate lower image similarity. Below *MSE*, *PSNR* are defined:

$$MSE = \frac{1}{rm}\sum_{i=1}^{r}\sum_{j=1}^{m}\left|f_{i,j} - \hat{f}_{i,j}\right|^2 \tag{16}$$

$$PSNR = 20\log_{10}\left(\frac{MAX}{\sqrt{MSE}}\right)(\text{dB}), \tag{17}$$

where *MAX* is the maximum pixel value.

A. Horizontal motion

The X-ray image making provides a crucial method of diagnostic by using the image analysis. Fig. 1, Original Image, shows such a deterministic original X-ray image. Fig. 1, Degraded Image, presents the degraded X-ray image for l=35.

Finally, from Fig. 1, PIM Restored Image, Wiener Restored Image, Constrained LS Restored Image and Lucy-Richardson Restored Image, it is clearly seen that the details of the original image have been recovered.

These figures demonstrate four different methods of restoration, method based on pseudoinverse matrix, Wiener filter, Constrained least-squares (LS) filter, and Lucy-Richardson algorithm, respectively.

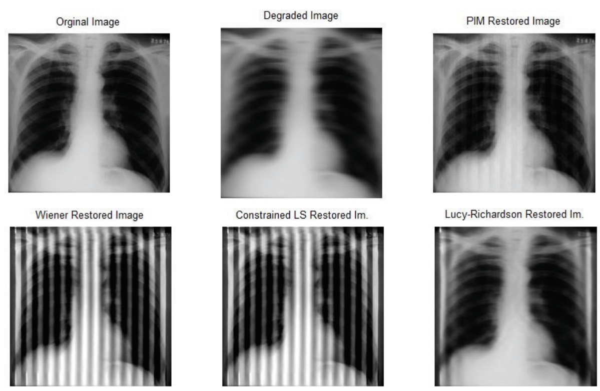

Fig. 1 Restoration in simulated degraded X-ray image for length of the horizontal blurring process, l=35

The difference in quality of restored images can hardly be seen by human eye. For this reason, the *ISNR* and *PSNR* have been chosen in order to compare the restored images. Fig. 2 and Fig. 3 shows the corresponding *ISNR* and *PSNR* values.

The figures illustrate that the quality of the restoration is as satisfactory as the classical methods or better from them (l<100 pixels). Realistically speaking, large motions do not occur frequently in radiography.

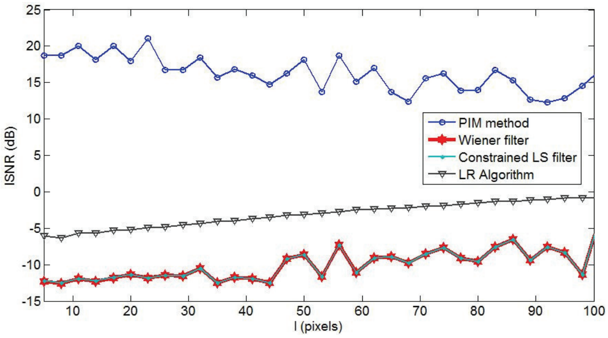

Fig. 2 Improvement in signal-to-noise-ratio vs. length of the blurring process in pixels

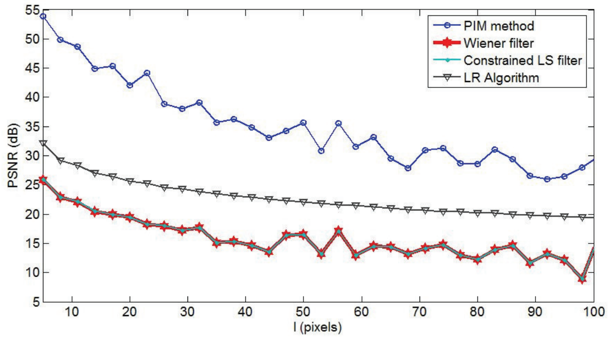

Fig. 3 Peak signal-to-noise-ratio vs. length of the blurring process in pixels

B. Vertical Motion

Obviously the method is not restricted to restoration of X-ray images. We can consider another practical example with images of ANPR (Automatic Number Plate Recognition) system. The results present in Fig. 4 – 6 refer when we have vertical blurring process.

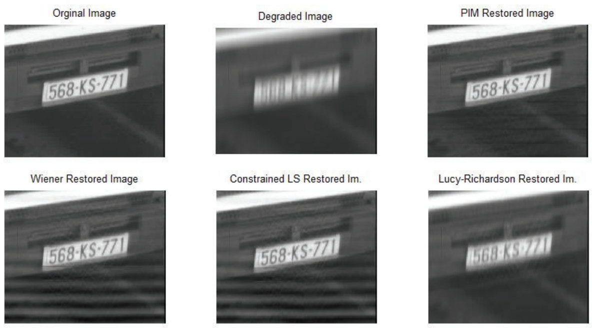

Fig. 4 Restoration in simulated vertical degraded image for length of the blurring process, l=30

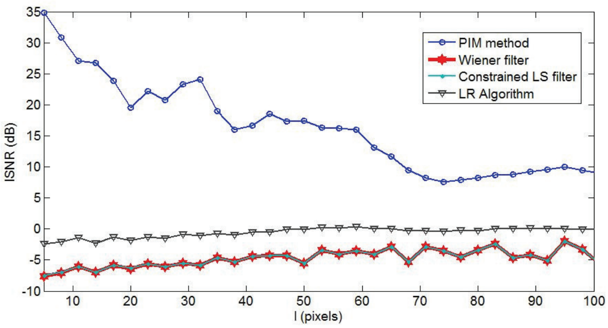

Fig. 5 Improvement in signal-to-noise-ratio vs. length of the blurring process in pixels

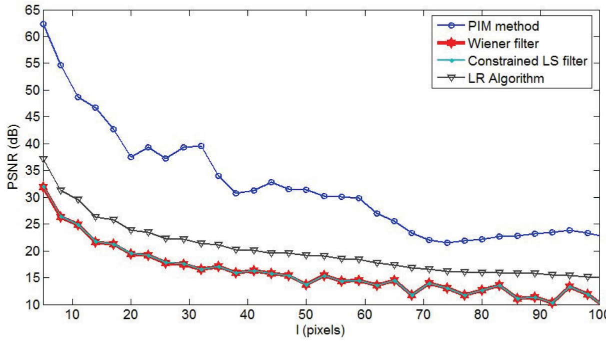

Fig. 6 Peak signal-to-noise-ratio vs. length of the blurring process in pixels

V. CONCLUSIONS

We introduce a computational method, based on the pseudoinverse matrix, to restore an image that has been blurred by uniform linear motion. We are motivated by the problem of restoring blurry images via well developed mathematical methods and techniques based on pseudoinverse matrix in order to obtain an approximation of the original image.

We present the results by comparing our method and that of the Wiener filter, Constrained least-squares filter and Lucy-Richardson algorithm, well established methods used for fast recovery and restoration of high resolution images.

In the method we studied, the resolution of the restored image remains at a very high level, yet the *ISNR* is considerably higher while the computational efficiency is improved in comparison to other methods and techniques.

REFERENCES

[1] J. Biemond, R. L. Lagendijk, and R. M. Mersereau, "*Iterative methods for image deblurring*", Proc. IEEE, 78(5):856–883, 1990.

[2] Al Bovik, *The essential guide to the image processing*, Academic Press, 2009.

[3] Rafael C. Gonzalez, Richard E. Woods, *Digital Image Processing*, 2nd Edition, Prentice-Hall, 2002.

[4] Rafael C. Gonzalez, Richard E. Woods, Steven L. Eddins, *Digital Image Processing Using MATLAB*, Prentice-Hall, 2003.

[5] Domingo Mery, Dieter Filbert, "*A Fast non-Iterative Algorithm for the Removal of Blur Caused by Uniform Linear motion in X-Ray Images*", 15th World Conference on Non-Destructive Testing, Computer Processing and Simulation , Rome, 15-21 October 2000.

[6] Spiros Chountasis, Vasilios N. Katsikis, and Dimitrios Pappas, "*Applications of the Moore-Penrose Inverse in Digital Image Restoration*", Mathematical Problems in Engineering Volume 2009 (2009).

[7] Ahmet M. Eskicioglu and Paul S. Fisher, "*Image Quality Measures and Their Performance*", IEEE Transactions on Communications, vol. 43, pp. 2959-2965, Dec. 1995.

[8] Zhou Wang and Alan C. Bovik, "*Mean Squared Error: Love It or Leave It? A New Look at Signal Fidelity Measures*", IEEE Signal Processing Magazine, vol. 26, no. 1, pp. 98-117, Jan. 2009.