Analysis of Requirements and the Feasible Limit for Error Reduction in Two-Parameter Magnetic Determination of Steels' Hardness

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Abstract

All measurements of mechanical properties of materials in the magnetic structural analysis are indirect and relationships between the measured parameters are correlated. An important physical parameter of steel is hardness. An increase in the correlation coefficient R and a reduction in the standard deviation (SD) are achieved when controlling the hardness of steels with two-parameter magnetic methods compared to methods that use a single measured parameter. However, the specific conditions and requirements for application of the two-parameter methods remain unclear. The purpose of this article was to analyze conditions and the achievable error reduction limit for two-parameter indirect determination of steels hardness and to compare those with one-parameter methods.

In particular, we considered the mean Square Deviation (SD), σ_F, of indirect calculation of the physical quantity F using two measured parameters x_1 and x_2 that are correlated with F. It was found that reduction of σ_F is most pronounced when x_1 and x_2 are inversely correlated with the maximum modulus $|R|$ of the correlation coefficient R between them. The most significant reduction in σ_F occurs at similar values of the SDs σ_1 and σ_2 between the true value of F and the values calculated based on the results of indirect measurements of F using each of the parameters x_1 and x_2. The Results of the analysis are confirmed by an example of reduction in SD when determining the hardness of carbon steels by measuring their remanent magnetization and coercive force compared to use any one of these parameters.

This result can be applied to measurements in non-destructive testing and in related fields of physics and technology. The Results of the analysis allow us to compare different parameters for indirect two-parameter determination of a physical quantity, to select the optimal parameters, and to evaluate the minimum achievable measurement error of a physical quantity by a two-parameter method before performing the measurements.

Keywords: indirect physical measurements, correlation coefficient, hardness, remanent magnetization, coercive force.

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53.088 : 620.179.14

Анализ условий и достижимого предела снижения погрешности двухпараметрового магнитного определения твёрдости сталей

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Все измерения физико-механических свойств материалов в магнитном структурном анализе являются косвенными, а связи между параметрами имеют корреляционный характер. Важным физическим параметром стали является твёрдость. Исследователи добились повышения коэффициента R корреляции и снижения среднего квадратичного отклонения при контроле твёрдости сталей двухпараметровым магнитным методом по сравнению с однопараметровым. Но оптимальные условия применения двухпараметрового метода остаются не установленными. Целью статьи являлся анализ условий и достижимого предела снижения погрешности двухпараметрового косвенного определения твёрдости сталей по сравнению с однопараметровым.

Исследовано среднее квадратичное отклонение σ_F косвенного определения физической величины F с использованием двух параметров x_1 и x_2, корреляционно связанных с F. Получено, что эффект снижения σ_F сильнее всего проявляется при обратной корреляционной связи между x_1 и x_2 с максимальным модулем $|R|$ коэффициента R корреляции между ними. Наиболее существенное снижение σ_F имеет место при близких величинах средних квадратичных отклонений σ_1 и σ_2 между истинными значениями F и значениями, рассчитанными по результатам косвенных измерений F с использованием каждого из параметров x_1 и x_2. Результаты анализа подтверждены примером снижения среднего квадратичного отклонения определения твёрдости углеродистых сталей по результатам измерения их остаточной намагниченности и коэрцитивной силы по сравнению с использованием любого из этих параметров.

Область применения результата – измерения в неразрушающем контроле и смежных областях физики и техники. Результаты анализа позволят выбрать оптимальные параметры для косвенного двухпараметрового определения твёрдости сталей, оценить достижимую погрешность определения твёрдости.

Ключевые слова: косвенные физические измерения, коэффициент корреляции, твёрдость, остаточная намагниченность, коэрцитивная сила.

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Приборы и методы измерений
2020. – Т. 11, № 3. – С. 236–244
S.G. Sandomirski

Devices and Methods of Measurements
2020, vol. 11, no. 3, pp. 236–244
S.G. Sandomirski

Introduction

An important physical parameter of steel is hardness – property of a material to resist plastic deformation upon contact in the surface layer. HRC hardness according to Rockwell is most often used to characterize the hardness of heat-treated steel [1]. The importance of this parameter is also determined by the fact that the hardness of the metal is closely related to its mechanical properties: the conditional yield strength $\sigma_{0,2}$; the tensile strength σ_B; and the relative restriction ψ [1]. The results of HRC hardness measurements can be used to determine (or at least to provide a very accurate estimate) for the $\sigma_{0,2}$, σ_B, and ψ of steels [2]. This means that a non-destructive method for determining the hardness of steels allows one to control an entire set of its mechanical properties.

Magnetic structural analysis consists in the non-destructive determination of the mechanical properties of materials by measuring their magnetic parameters [3]. Measurements of the mechanical properties of materials in magnetic structural analysis are indirect. The physical basis for the presence of correlations between the mechanical and magnetic properties of steels is that these properties (each in its own way) are determined by the structure of the metal (stresses, distribution of dispersed particles in the alloy matrix, defects in the crystal lattice, grain size) formed during heat treatment [3, 4]. These dependencies are influenced by random factors. Therefore, the relationships between the mechanical and magnetic parameters of steels are not functional, but correlative. The coefficient R of correlation between the results of direct and indirect measurements of the mechanical properties of steels is also influenced by the inevitable errors in the measurement of mechanical [1] and magnetic parameters [5].

To increase the reliability of magnetic structural analysis, a combined use of several magnetic parameters was proposed. Analysis of the theoretical foundations of such methods, experiments, and modeling showed a strong influence of errors in measuring the parameters used in multiparameter regression equations on the reliability of control [6–10]. Nevertheless, multi-parameter models are used to calculate the hardness of HRC steels [11, 12]. The results of measuring the coercive force H_c of steels, their relaxation magnetization M_{Hr}, relaxation magnetic susceptibility χ_r, quadratic terms H_c^2, χ_r^2 and the product $\chi_r M_{Hr}$ are summed with different weight coefficients. This made it possible to obtain ultra-high correlation coefficients (up to $R \approx 0.9999$ [11, Table 1]) and small mean square deviations (SD) between the results of measuring hardness and its determination using the developed indirect calculations. Similar results were obtained in studies [13–15] and other works. However, it was established in [5] that the numerical values of H_c, M_{Hr} and χ_r given in [4] are substituted into the multiparameter equations for calculating the HRC of steels in [11, 12]. Ultra-high correlation coefficients R between the results of calculation and measurement of HRC of steels and small SD between them in [11, 12] were obtained by choosing the weight coefficients of the terms. The inevitable errors in measuring magnetic parameters and calculating algebraic combinations were not taken into account. A set of statistical control data under the influence of interfering factors and correlation analysis were not carried out.

In [16], an analysis of the technique [11, 12] was performed. In the analysis, "for the sake of simplification", it was assumed that in the methodology [11, 12] "the initial independent variables had an error of 1 %, and those obtained by calculation from the initial ones had a 2 % error". It was found that the error in calculating the HRC of steels according to the method [11, 12] "can be tens of percent or more". However, it was shown in [17] that the measurement error for each of the relaxation magnetic parameters used in [11, 12] is not 1 %, but itself can be tens of percent. And algebraic operations inevitably increase the resulting calculation error [18]. Therefore, the practical application of multiparameter control of the structure of steels according to the method [11, 12] and similar ones has not been reported so far.

Meanwhile, the authors of [19] have achieved an increase in the correlation coefficient and a decrease in the root-mean-square error of control of the hardness of steel pipes in industrial conditions by the two-parameter method (from H_c and remanent induction B_r) in comparison with the one-parameter (from H_c) method. The optimal conditions for the application of the two-parameter method have not been established, however. The formula for the indirect measurement error for the case of a function of two variables is given in [20]. This formula does not take into account the correlations observed in practice between the measured variables. This hinders the effective use of multiparametric magnetic structural analysis methods in practice.

Приборы и методы измерений
2020. – Т. 11, № 3. – С. 236–244
S.G. Sandomirski

Devices and Methods of Measurements
2020, vol. 11, no. 3, pp. 236–244
S.G. Sandomirski

The purpose of the article was to analyze the conditions and the achievable limit for reducing the error of two-parameter indirect measurement of steel hardness in comparison with one-parameter methods.

Table 1

The results of measuring the magnetic parameters and HRC hardness of carbon steels in [4] and the results of calculating their HRC using different formulas

Steel	T_t, °C	Measurement results in [4]			HRC calculation results using the formulas:			
		H_c, kA/m	M_r, MA/m	HRC	(11)	(12)	(3), (11) and (12)	(13), (11) and (12)
30	20	2.3	0.865	46	52	49.8	50.9	50.9
	150	2.25	0.870	45	51.5	49.7	50.6	50.6
	200	2.1	0.876	44.5	49.9	49.7	49.8	49.8
	250	1.43	0.970	43.5	41.1	47.7	44.4	44.3
	300	1.22	1.007	44	37.4	46.3	41.85	41.6
	350	1.13	1.070	41	35.7	43.3	39.5	39.3
	400	0.995	1.145	38	32.8	38.5	35.65	35.5
	450	0.873	1.248	34	29.8	29.9	29.85	29.8
	500	0.876	1.265	32	29.8	28.2	29	29
	550	0.866	1.277	26	29.6	27	28.3	28.3
	600	0.834	1.280	23	28.7	26.7	27.7	27.7
	650	0.730	1.235	19	25.6	31.1	28.35	28.2
45	20	3.0	0.910	60	58.1	49.2	53.65	53.5
	150	2.7	0.919	55	55.7	49	52.35	52.2
	200	2.46	0.932	53	53.5	48.7	51.1	51
	250	1.55	1.027	50	42.9	45.5	44.2	44.2
	300	1.34	1.018	45	39.6	45.9	42.75	42.6
	350	1.26	1.021	45	38.2	45.8	42	41.8
	400	1.12	1.136	42	35.5	39.2	37.35	37.3
	450	1.02	1.271	37	33.3	27.6	30.45	30.3
	500	1.03	1.276	34	33.6	27.1	30.35	30.2
	550	1.05	1.280	29	34	26.7	30.35	30.1
	600	1.04	1.248	26	33.8	29.9	31.85	31.8
	650	0.880	1.250	20	29.9	29.7	29.8	29.8
Correlation coefficient *R* of the results of calculation and measurement of HRC					0.868	0.860	0.901	0.899
SD between calculation and measurement of HRC, HRC units					4.81	4.58	4.17	4.22

Physical model for analysis

Let us establish that the physical quantity *F* (in the particular case, the HRC hardness of a steel) can be determined by measuring the correlated parameters x_1 and x_2 related to it based on the dependencies:

$$F(x_1) = \phi_1(x_1) \tag{1}$$

and

$$F(x_2) = \phi_2(x_2). \tag{2}$$

Приборы и методы измерений
2020. – Т. 11, № 3. – С. 236–244
S.G. Sandomirski

Devices and Methods of Measurements
2020, vol. 11, no. 3, pp. 236–244
S.G. Sandomirski

Moreover, we denote the root-mean-square deviations (SD) of the results of determining the quantity F according to equations (1) and (2) from its true values as σ_1 and σ_2, respectively, and assume that the parameters x_1 and x_2 are correlated with each other with the correlation coefficient R ($-1 \le R \le 1$). We use the results of measurements of both parameters x_1 and x_2 to determine the physical quantity F. The value of F is calculated as the arithmetic mean of its definition by formulas (1) and (2):

$$F(x_1, x_2) = 0.5[\phi_1(x_1) + \phi_2(x_2)]. \qquad (3)$$

Analysis and its results

Let us analyze how the SD values, σ_1 and σ_2 and the value of the correlation coefficient R between the parameters x_1 and x_2 affect the SD σ_F of the indirect determination of the physical quantity F by equation (3). It is known from the theory of probability that the variance $D(x_1 + x_2)$ of the sum of two random variables x_1 and x_2 in the general case is equal to [21]:

$$D(x_1 + x_2) = D(x_1) + D(x_2) + 2K_{x_1 x_2}, \qquad (4)$$

where $D(x_1)$ is the variance of x_1; $D(x_2)$ – variance x_2; σ_1 and σ_2 – SD values x_1 and x_2; R is the correlation coefficient between them; $K_{x_1 x_2} = R\sigma_1\sigma_2$ – their correlation moment.

From (4), the SD σ_Σ of the sum of the quantities x_1 and x_2 is [21]:

$$\sigma_\Sigma = \sqrt{\sigma_1^2 + 2R\sigma_1\sigma_2 + \sigma_2^2}. \qquad (5)$$

Taking into account (5) and (3), we obtain the SD σ_F of the indirect determination of the physical quantity F according to equation (3) an analytical expression:

$$\sigma_F = 0.5\sqrt{\sigma_1^2 + 2R\sigma_1\sigma_2 + \sigma_2^2}. \qquad (6)$$

To generalize the analysis, we introduce the relative values:

$$\bar{\sigma}_2 = \sigma_2 / \sigma_1 \text{ and } \bar{\sigma}_F = \sigma_F / \sigma_1. \qquad (7)$$

The value $\bar{\sigma}_F$ shows how the use of the second parameter x_2 to determine the value of F decreases (if $\bar{\sigma}_F < 1$) or increases (if $\bar{\sigma}_F > 1$) the SD of the indirect measurement according to equation (3) compared to using only the parameter x_1. In the notation (7), from (6) we obtain:

$$\bar{\sigma}_F = 0.5\sqrt{1 + 2R\bar{\sigma}_2 + \bar{\sigma}_2^2}. \qquad (8)$$

Figure 1 shows the results of calculating the dependences $\bar{\sigma}_F = \bar{\sigma}_F(\bar{\sigma}_2)$ by formula (8) in the range $0 \le \bar{\sigma}_2 \le 3$ for different coefficients R of the correlation between the parameters x_1 and x_2 in the possible range $-1 \le R \le 1$ of its change. Figure 2 shows the results of calculating the dependences by $\bar{\sigma}_F = \bar{\sigma}_F(R)$ formula (8) at different values in the range of $-1 \le R \le 1$. Taking into account the symmetric influence of the parameters x_1 and x_2 on the result of calculating the value of $F(x_1, x_2)$ by formula (3), for analysis in the case of $0 \le \bar{\sigma}_2 \le 1$, the parameters x_1 and x_2 can be swapped and the case $\bar{\sigma}_2 \ge 1$ can be considered.

The analysis of the dependences shown in Figures 1 and 2 shows that values $\bar{\sigma}_F < 1$ cannot be achieved at any values of R if $\bar{\sigma}_2 \ge 3$. In these cases, the use of the two-parameter method cannot provide a decrease in SD for indirect measurement of F in comparison with the one-parameter method. At $1 \le \bar{\sigma}_2 < 3$, the value decreases as R decreases and approaches the value -1. From (8) it follows that the condition $\bar{\sigma}_F \le y$ is satisfied for values of R satisfying the equation:

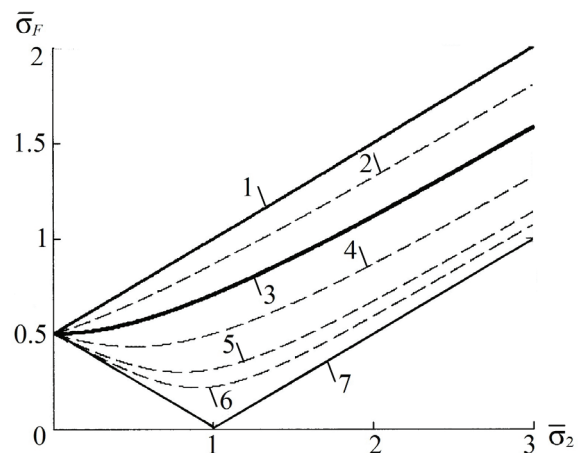$$R \le \left(4y - 1 - \bar{\sigma}_2^2\right) / (2\bar{\sigma}_2). \qquad (9)$$

Figure 1 – Dependence of the relative standard deviation $\bar{\sigma}_F$ of the results of determining the physical quantity F according to the formula (3) on the relative standard deviation $\bar{\sigma}_2$ of the second parameter: $1 – 7$ – respectively at $R = 1$; 0.5; 0; -0.5; -0.8; -0.9; -1. Calculation according to the formula (8)

Figure 3 shows the isolines of the function $\bar{\sigma}_F = y$ at different y in coordinates $(\bar{\sigma}_2, R)$. The analysis of the dependences shown in Figure 3 shows that a necessary condition for a two-fold ($y \approx 0.5$) decrease in the SD of the measurement of the parameter F by the two-parameter method compared with the one-parameter method is the simultaneous fulfillment of

Приборы и методы измерений
2020. – Т. 11, № 3. – С. 236–244
S.G. Sandomirski

Devices and Methods of Measurements
2020, vol. 11, no. 3, pp. 236–244
S.G. Sandomirski

the conditions $R < -0.5$ and $1 \leq \overline{\sigma}_2 < 2$. In practice, on can note that an approximately four-fold ($y \approx 0.25$) reduction of the SD can be achieved for a two-parameter measurement of a physical quantity F in comparison with a one-parameter measurement: this requires the simultaneous fulfillment of the conditions $R < -0.9$ and $1 \leq \overline{\sigma}_2 < 1.5$ (area below curve 4 in Figure 3).
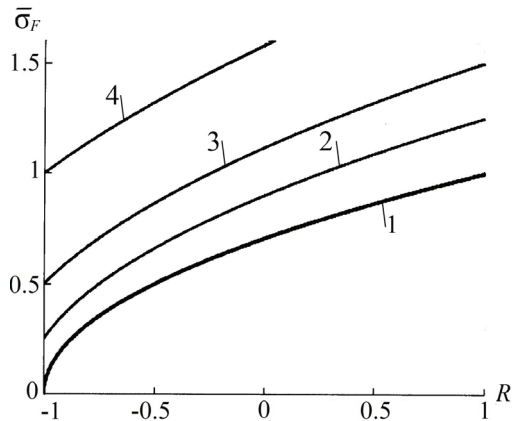

Figure 2 – Dependence of the relative root-mean-square deviation $\overline{\sigma}_F$ of the results of determining the physical quantity F according to the formula (3) on the coefficient R of the correlation between the parameters x_1 and x_2: 1 – 4 – with the relative standard deviation $\overline{\sigma}_2$ of the second parameter, respectively, equal to 1; 1.5; 2; 3. The calculation according to the formula (8)
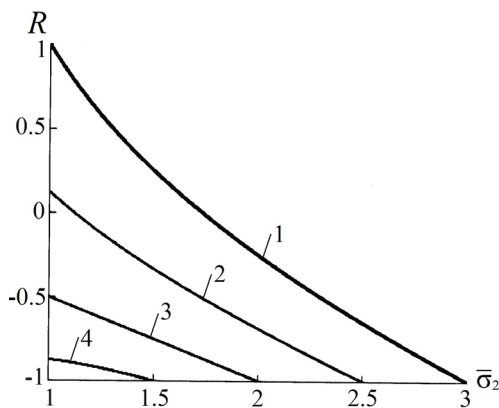

Figure 3 – Isolines of the function $\overline{\sigma}_F = y$ in coordinates $(\overline{\sigma}_2, R)$: 1 – 4 – when y is equal to: respectively, 1; 0.75; 0.5; 0.25. Calculation by the formula (9)

Solving inequality (9) with respect to the parameter $\overline{\sigma}_2$, we obtain:

$$\overline{\sigma}_2 \leq -R + \sqrt{R^2 + 4y^2 - 1}. \qquad (10)$$

Knowing the correlation coefficient R between the parameters x_1 and x_2, it is possible to calculate the ratio $\overline{\sigma}_2 = \sigma_2 / \sigma_1$ ($1 \leq \overline{\sigma}_2 < 3$) using equation (10), which, when determining the value of F by the

parameters x_1 and x_2, will provide the required decrease ($0 < y \leq 1$) in the SD $\overline{\sigma}_F$ for the indirect measurement of F by the two-parameter method compared to the one-parameter method.

As an example, the results of the analysis are confirmed by a decrease in the error in determining the hardness HRC of carbon steels after tempering at a temperature T_t, by measuring their coercive force H_c and remanent magnetization M_r, compared to using any one of these parameters (Table 1, Figures 4, 5). For analysis, we used the measurement results in [4, Tables 1.1 and 1.3] of H_c, M_r, and HRC of steels 30 and 45. The parameters H_c and M_r given in [4] were measured by standard methods GOST 8.377–80 "Magnetically soft materials. Measurement technique for determining static magnetic characteristics". The relative error of their measurement does not exceed $\pm 2\,\%$ and $\pm 3\,\%$, respectively. The correlation coefficient R between H_c and M_r according to Table 1 was -0.853.

Statistical processing of correlation dependencies between H_c and HRC (Figure 4*a*), M_r and HRC (Figure 4*b*) and plotting the trend lines of these dependencies were performed in the Microsoft Excel program. The following equations were obtained for determining the hardness HRC of the investigated steels from the results of measuring their H_c and M_r (where $\tau_1 = 1$ m/kA and $\tau_2 = 1$ m/MA are dimensional factors):

$$\text{HRC} = \phi_1(H_c) = 22.965 \cdot \ln(\tau_1 H_c) + 32.874 \cdot \tau_1 H_c; \qquad (11)$$

$$\text{HRC} = \phi_2(M_r) = -115.76 \cdot (\tau_2 M_r)^2 + 192.61 \cdot \tau_2 M_r - 30.228. \qquad (12)$$

The data given in Table 1 show that the value of the SD between the results of calculating the hardness HRC of the investigated steels according to formula (3), using formulas (11) and (12), and the results of its measurement (Figure 5) amounted to 86 % and 91 %, respectively, of the SD values between the results of calculating the HRC hardness of these steels according to formulas (11) and (12) and its measurement.

The analysis results can be extended to other functional processing of the indirect measurements $F(x_1)$ and $F(x_2)$, that provide the same additional relative error of calculation as the algorithm (3) [18]. For example, the analysis can be applied to the geometric mean of the results of determining the physical quantity F according to formulas (1) and (2):

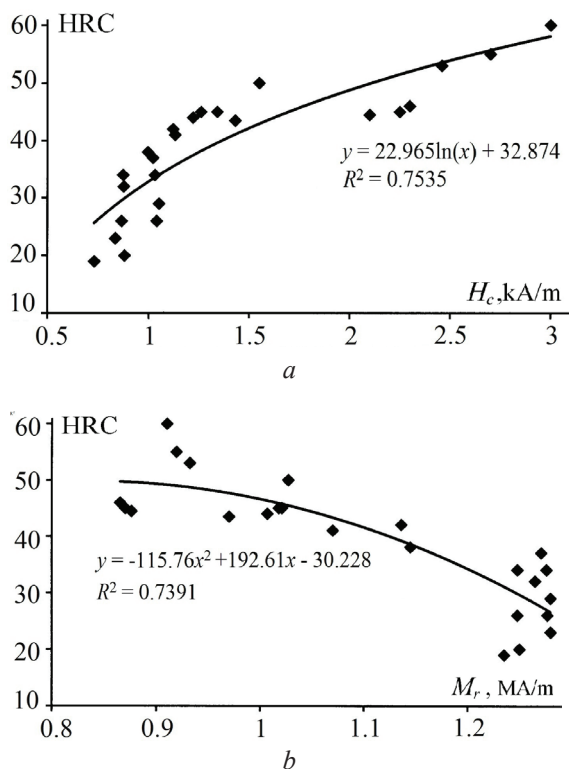$$F(x_1, x_2) = \sqrt{\phi_1(x_1) \cdot \phi_2(x_2)}. \qquad (13)$$

Приборы и методы измерений
2020. – Т. 11, № 3. – С. 236–244
S.G. Sandomirski

Devices and Methods of Measurements
2020, vol. 11, no. 3, pp. 236–244
S.G. Sandomirski

a

b

Figure 4 – Correlation fields of dependences between H_c and HRC (*a*), M_r and HRC (*b*) of steels 30 and 45, the results of measuring the parameters of which in [4] are given in Table 1, and the trend lines of these dependences
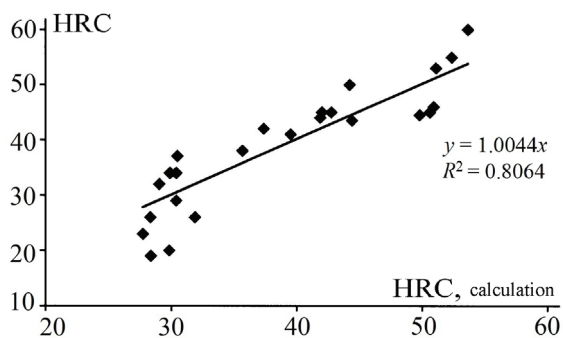

Figure 5 – Correlation field of the relationship between the results of calculating the HRC hardness of the studied steels according to formula (3) and the results (Table 1) of its measurement in [4] and the trend line of this dependence

Indeed, let the value of *F* be determined using the results of measuring the parameter x_1 by formula (1) with a relative error δ_1, and using the results of measuring the parameter x_2 using formula (2) – with a relative error δ_2. In this case, $F(x_1) \approx F(x_2)$. We use (Table 2) formulas [22] to calculate the relative measurement errors introduced by algebraic operations.

Table 2

Absolute and relative errors arising from the application of certain algebraic functions

Function type	Absolute error	Relative error
$A = a + b$	$\Delta A = \Delta a + \Delta b$	$\dfrac{\Delta A}{A} = \dfrac{\Delta a + \Delta b}{a + b}$
$A = ab$	$\Delta A = a\Delta b + b\Delta a$	$\dfrac{\Delta A}{A} = \dfrac{\Delta a}{a} + \dfrac{\Delta b}{b}$
$A = a^n$	$\Delta A = na^{n-1}\Delta a$	$\dfrac{\Delta A}{A} = n\dfrac{\Delta a}{a}$

We obtain for the relative errors δ_3 and δ_{13} the determination of the physical quantity *F*, respectively, by formulas (3) and (13):

$$\delta_3 \approx 0.5(\delta_1 + \delta_2), \qquad (14)$$

(taking into account the fact that $F(x_1) \approx F(x_2)$),

$$\delta_{13} = 0.5(\delta_1 + \delta_2). \qquad (15)$$

Close relative errors δ_3 and δ_{13} of determining the physical quantity *F* by formulas (3) and (13), in addition to formulas (14) and (15), are shown (Table 1) by close results of applying algorithms (3) and (13) to determine hardness HRC of carbon steels according to the results of their M_r and H_c measurements, given in Table 1.

The analysis of the influence of different factors on the mean square deviation σ_F of the indirect determination of the physical quantity *F* using two parameters x_1 and x_2 correlated with *F* allows us to draw the following conclusions.

Conclusion

The effect of reducing the mean square deviation σ_F for determining the steel hardness (physical quantity *F*) when using the second parameter x_2 is most pronounced when the inverse correlation between the parameters x_1 and x_2 with the maximum modulus $|R|$ the correlation coefficient *R* between them. In this case, the most significant decrease in σ_F occurs at close values of the SD σ_1 and σ_2 between the true values of *F* and the values calculated from the results of indirect measurements of *F* using each of the parameters x_1 and x_2.

The application of the two-parameter method will not provide a decrease in σ_F for determining

Приборы и методы измерений
2020. – Т. 11, № 3. – С. 236–244
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Devices and Methods of Measurements
2020, vol. 11, no. 3, pp. 236–244
S.G. Sandomirski

the hardness of steel (physical quantity F) compared with the one-parameter method if the SD σ_2 of measurements of F using the second parameter x_2 is more than 3 times greater than the SD σ_1 of calculating F using the first parameter x_1.

Knowing the correlation coefficient R between the parameters x_1 and x_2, it is possible to calculate the ratio σ_2 / σ_1 that, when determining the hardness of steel (physical quantity F) by the parameters x_1 and x_2, will provide a specified reduction in the error of its determination by the two-parameter method compared to the one-parameter method. Achievable in practice is an approximately four-fold decrease in σ_F when measuring the hardness of steels by the two-parameter method compared to the one-parameter method: this requires the simultaneous fulfillment of the conditions $R < -0.9$ and $\sigma_2 / \sigma_1 < 1.5$.

The field of application of the result is measurements in non-destructive testing and related fields of physics and technology. The results of the analysis will make it possible to select the optimal parameters for the indirect two-parameter determination of the hardness of steels and to estimate the achievable error in determining the hardness with their use.

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