Studies on consumers’ benefits from transformation of electricity markets

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Ryo Hase

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Chapter 1

Introduction

This chapter provides the introduction of the studies presented in this thesis. Firstly, backgrounds regarding the recent transformation of electricity markets will be explained. Secondly, research objectives of the studies will be presented. Finally, problems addressed in this thesis are presented after introducing research fields related to the studies.

1.1 Backgrounds

For a long time, many countries have vertically integrated electricity markets for ensuring a stable supply of electricity. In the vertically integrated electricity markets, a limited number of power companies have conducted generation, transmission, and retail of electricity. Nevertheless, those electricity markets with centralized structure mainly have issues on electricity charges and services offered by the suppliers [1, 2]. First, consumers cannot recognize whether current electricity charges are appropriate or not since they do not have any suppliers for the comparison. Second, electricity suppliers do not seem to actively improve their services since no competition occurs among a limited number of suppliers.

Structural transformation in electricity markets has emerged in many countries to ameliorate the services in electricity markets. As the first step of the transformation, traditionally centralized electricity markets have been deregulated and divided into several sectors. The deregulation aims at the participation of various electricity suppliers in the markets and enhancing competition among the suppliers. Electricity retail markets have been gradually liberalized to provide consumers with choices among electricity suppliers. As described above, power companies provide electricity to consumers at relatively higher charges due to no competition among the companies. Recently, many countries have deregulated their electricity markets to offer lower charges to consumers. For instance, the Japanese government has gradually deregulated their electricity markets. The deregulation has allowed various companies to enter electricity retail for all consumers in Japan since 2016.

As modernization of power grids, *smart grids* have been considered to increase reliability, resiliency, sustainability, and energy efficiency by using advanced metering and communication technologies [3]. The anticipated smart grid benefits are improving power reliability and quality, accommodating distributed power sources, improving resilience to disruption by natural disasters, and so on. Bidirectional power flow realized in smart grids might transforms the structure of electricity markets than ever. The expected benefits of smart grids in terms of consumers are (i) to provide consumers with actionable and timely information about consumers' energy usage and (ii) to increase consumers' choice and enable new products, services, and markets [3].

From the consumers' viewpoint, the roles of consumers in electricity markets have been greatly changed with the transformation. In traditionally centralized electricity markets, consumers do not have any alternatives to their electricity suppliers. After the deregulation of centralized electricity markets, on the other hand, consumers will have alternatives for their electricity suppliers. Moreover, consumers will possibly have chances to provide surplus electricity to its neighbors in future smart grids. Thus, consumers' choices are not only purchasing electricity from suppliers but also providing surplus electricity to others in the future.

1.2 Research objective

Although new types of trading will be available in the transformation of electricity markets, liberalization might pose issues regarding consumers' viewpoints. Consumers do not necessarily act in liberalized electricity markets as expected. For instance, many consumers have not switched their suppliers in many countries though consumers can choose electricity suppliers after the deregulation of electricity retail [4]. This trend might not lead to reduce electricity charges even though relatively lower charges is one of the expected outcomes of the deregulation. Besides, if consumers will not actively participate in electricity sharing markets in smart grids similar to the cases in electricity retail, mechanisms for electricity sharing might also fail since active participation of end-users is an inevitable factor for the success of resource sharing [5, 6]. Characteristics regarding the deregulated markets should be examined carefully to avoid causing large impacts to society such as California electricity crisis in 2001 [7].

The objective of the studies presented in this thesis is to analyze consumers' benefits in electricity markets to consider successful market mechanisms. These studies provide opportunities to understand consumers' benefits and provide insights for the mechanism design of electricity markets. These studies clarify the benefits of consumers about their decision in electricity trading. An example of insights is information about the decision making of consumers in electricity markets. One of the applications of these studies is a tool for the "proof of concept" of operations regarding electricity markets. For electricity suppliers, the tool can be used to check the effects on the settings of their electricity services. Besides, for policymakers such as governments or public offices, the tool gives insights to understand the characteristics of consumers such as their benefits, behaviors, etc.

1.3 Related works

Mechanisms for electricity markets require consideration in many factors such as engineering, economics, social and political aspects. Since electricity markets have transformed into a more complex style than ever, many kinds of problems must be addressed to provide desirable benefits for consumers. This section introduces related studies on the problems as an overview of research fields.

Managing production and demand of electricity

Matching production and demand for electricity in a particular time interval is inevitable to avoid power failure. These kinds of problems must be solved to continuously provide electricity to consumers as infrastructure for their daily life. Thus, managing production and demand is the fundamental constraints on electricity provided in electricity markets.

To determine electricity distribution according to constraints on production and demand, mathematical modeling techniques such as linear programming are utilized. For instance, Georgiou presents a mixed integer linear programming model for the long-term energy planning of power systems in [8]. Regarding deregulated electricity markets, Sen *et al.* introduce algorithms for multi-objective optimization about transmission line congestion, line loss, and price volatility [9].

Integrating energy storage to realize stable supply of electricity

Production from renewable energy cannot be controlled since the amounts of electricity production depend on uncertain conditions such as wind, solar, wave, and so on. Energy storage might facilitate this problem since energy storage can offset the gap between production and demand. Although a number of energy storage technologies have already been available, they need more advancements than ever since energy storage for electricity markets requires large-scale capacity [10].

Researchers propose models to assess the way to integrate energy storage systems into electricity networks. Babrowski *et al.* introduce a model to analyze the allocation and amount of energy storage systems required in Germany considering transmission from renewable energy supply to the storage systems [11]. Velik *et al.* present an architecture for managing prosumers' PV systems and battery storage in microgrids [12].

Designing mechanisms for electricity markets

Electricity markets require mechanisms to match suppliers and consumers by economic perspectives in addition to methods for distribution of electricity. Hence, mechanism design becomes an important process to realize successful electricity markets. In addition to consideration of the characteristics of electricity described above, economic aspects are involved in the mechanism design.

There are a variety of mechanisms for each type of electricity market. A typical example is auction mechanisms to determine electricity trading by adjusting electricity prices based on rules of the auction. Karaca *et al.* introduce a game theoretic approach to examine auction mechanisms for electricity markets to realize truthful bids and prevent strategic manipulations by participants [13]. Another example is the mechanism of Demand Response (DR), which mainly aims at reducing peak demand. In [14], Reka *et al.* propose a DR scheme for smart grids using game theory.

Forecasting production and demand of electricity

Demand forecasting will become more complex after the deregulation of electricity markets. For instance, DR might increase difficulty in forecasting demand [15]. Furthermore, forecasting is inevitable to utilize renewable energy since production from renewable energy is considered to be uncontrollable as noted above. Hence, forecasting technologies become crucial for the successful utilization of demand participation and renewable energy. These technologies are also related to the mechanism design of markets.

Regarding renewable energy, forecasting models are examined for each type of energy source. In [16], Bacher *et al.* present a method for online forecasting of production from PV systems based on data collected from PV systems on rooftops in Denmark. Wang *et al.* present a review of forecasting models of wind power in [17]. Reikard *et al.* present test results about the accuracy of forecasting models for ocean wave energy [18].

Analyzing characteristics of participants in electricity markets

The characteristics of participants are one of the crucial factors to design mechanisms for electricity markets. Regarding supply side, technologies for forecasting production are used for a stable supply of electricity. Similarly, mechanism design must address the characteristics of consumers at demand side. The insights of characteristics of consumers will improve the quality of mechanisms to induce desirable actions from demand side.

To examine consumers' behaviors, many studies consider modeling techniques about bounded rationality, which explains behaviors that do not aim at maximizing benefits [19]. For electricity consumers, Ruiz *et al.* [20] present a game theoretical model to investigate the effects of consumers' switching costs on competition of suppliers. Besides, Biglaiser *et al.* analyze the benefits of suppliers in a model where consumers have heterogeneous switching costs [21].

1.4 Problem statement

This thesis focuses on problems in modeling techniques for analyzing consumers' benefits in liberalized electricity markets. Behavior and decision of demand-side are equally or even more important than supply-side to deploy new technology even though many related studies have mainly focused on technology and cost-effectiveness of supply-side [22]. If proposed methods in this thesis can obtain insights about consumers' benefits, the insights will contribute to the effective mechanism design of electricity markets since consumers demonstrate vital roles in liberalized electricity markets. Moreover, methods to analyze the characteristics of consumers can be applied for both improving current situations and examining future novel mechanisms of electricity markets. This thesis deals with the following four problems about the analysis of consumers' benefits.

Problem 1. Constructing a modeling framework for electricity trading

Structure of electricity markets has became complex after liberalization. Especially, the networked structure will become important aspect since deregulation increases the number of participants in electricity networks. For extensive analysis of benefits for consumers in electricity trading, this thesis proposes a mathematical modeling framework. This approach is applied to electricity market models for Problem 2, Problem 3, and Problem 4.

Problem 2. Representing electricity trading in deregulated electricity markets

This problem aims at modeling participants in deregulated electricity markets to examine the benefits of them. Since there are multiple types of participants in the deregulated electricity markets, combinations of electricity trading among participants increase than ever. This thesis focuses on representing the deregulation of electricity retail since this kind of deregulation is closely related to electricity consumers. This problem is the first application of the model considered in Problem 1 for modeling benefits, prices, and trading of market participants.

Problem 3. Examining switching behavior of consumers in electricity retail

As the third problem, this thesis proposes a market model focusing on switching costs to analyze the behavior of consumers in electricity retail markets. Enhanced understanding of the switching behavior is crucial to improve the switching rate of consumers in electricity retail. Electricity retail markets in many countries see inactive switching behavior, which means the action of consumers to change their supplier. This topic aims at providing insights for promoting the switching behavior of consumers by modeling consumers' decision making. This approach is different from that in Problem 2 since the model in Problem 2 does not focus on the behavior of each participant.

Problem 4. Describing fairness among prosumers in electricity sharing

The fourth problem defines fairness measures for time-varying electricity sharing to decrease envy among prosumers about their allocation of electricity. This study examines electricity trading among prosumers, which has roles varying between suppliers or consumers from time to time. Electricity consumers do not necessarily act to maximize their benefits as considered in Problem 3. This problem focuses on envy-freeness, which is one of the fairness measures for resource allocation. This measure is selected since it is considered to be important even for irrational consumers. Since this problem deals with the time-varying roles of prosumers, this problem is considered to be a successor for Problem 2.

1.5 Thesis structure

The remaining chapters of this thesis are structured as follows. Chapter 2 presents mathematical modeling techniques for Problem 1 to represent electricity markets using graph theory. Chapter 3 proposes a market model to describe the benefits of participants in deregulated electricity markets for Problem 2. Chapter 4 introduces a model about Problem 3, which is to analyze the switching behavior of consumers. Chapter 5 introduces a model to examine envy-free allocation in electricity sharing among consumers for Problem 4. Chapter 6 concludes this thesis and future works regarding this study.

Chapter 2

Constructing a modeling framework for electricity trading

This chapter introduces a mathematical modeling framework about trading in electricity markets for Problem 1. Firstly, related works about mathematical modeling are introduced. Secondly, the proposed mathematical modeling framework for electricity markets is explained.

2.1 Related works

2.1.1 Overview of mathematical modeling

Mathematical modeling is a fundamental concept to describe a real-world problem as a mathematical problem and to acquire a solution for the real-world problem [23]. In [24], Meerschaert explains the process of mathematical modeling can be described as *5 step method*. The 5 step method includes the following steps.

Step 1. Focus on a real-world problem
Step 2. Define a mathematical model from the real-world problem
Step 3. Formulate an optimization problem on the mathematical model
Step 4. Solve the optimization problem
Step 5. Consider the relation between the results and the real-world problem

In this process, Step 5 is the most important step since the result of the formulated mathematical problem cannot be considered for the real-world problem without Step 5.

One of the key objectives of mathematical modeling is to consider the characteristics of the real-world problem. However, because the real-world problem is complex in many cases, all characteristics cannot be included in a solvable mathematical problem. Hence, proposed mathematical models should focus on the limited number of characteristics of the real-world problem.

2.1.2 Research fields related to mathematical modeling

Electricity market models presented in this thesis are composed of several approaches used in Operations Research (OR). As a research field to support appropriate decision making, OR deals with mathematical modeling techniques for decision making in many industries. Mathematical modeling in OR utilizes approaches proposed in various study areas: combinatorial optimization, graph theory, game theory, economics, etc. The rest of this section introduces an overview of the main research fields related to mathematical models proposed in this thesis.

Combinatorial optimization

Many real-world problems can be formulated as abstract combinatorial optimization problems [25]. The combinatorial optimization problem is described as a problem to select optimal elements in a set. The feasibility of computation for the problems is dependent on their search space. If a problem has a large search space, relatively longer computation time is required to obtain an optimal solution. Hence, the time complexity is important concepts to consider the computational efficiency of algorithms for combinatorial optimization [26].

Graph theory

Graph theory is one of the mathematical modeling techniques to solve problems. In graph theory, a graph is denoted by a set of vertices and edges. Many combinatorial optimization problems can be modeled and solved by graph theoretic approaches such as network flows, matching, and so on [27]. The complex structure and characteristics of electrical power networks have been represented and analyzed by using graph theory [28, 29]. In addition to the applications for conventional electricity grids, graph theory has been applied to methods for controlling future smart grids [30].

Recent applications on graph theory have focused on dynamical changes of the graph structure even though graph theory has originally dealt with the static structure of graphs. Temporal network theory appeared as an application of graph theory. The temporal network theory integrates time series of graph structure into classical graph theory [31]. *Temporal network* is also known as Time-Varying Graph (TVG) [32]. In studies about social networks, connections of vertices in a graph are focused on examining the characteristics of interactions among vertices (e.g. propagation of information, rumor, epidemic, and so on [33].

Regarding algorithms to solve problems about graphs, machine learning techniques have been adopted to improve the performance of the algorithms. In [34], Dai *et al.* introduce reinforcement learning is utilized to design algorithms to solve graph problems. He *et al.* propose a learning mechanism to improve branch-and-bound for mixed integer programs [35]. Although these studies improve the performance of algorithms for graph problems, machine learning techniques in these studies do not describe how real-world problems can be represented by graph problems. This thesis mainly focuses on the description of real-world problems using graph theory.

Game theory

Game theory analyzes the strategy of individuals in a situation where multiple players interact with each other. In [36], the concept of game theory is explained as follows; "game theory studies situations of competition and cooperation between several involved parties by using mathematical methods." A popular example of game theory is called prisoners' dilemma. This game can be utilized to consider the possible outcomes of each prisoner's strategy (either cooperation or defect) by examining each prisoner's benefit determined by the combination of the strategies. Besides, *evolutionary game theory* is considered as an extension of game theory [36]. An evolutionary game mainly focuses on the player's population in dynamical systems. For example, the transition of the share of strategy in the overall population is investigated by the techniques of evolutionary game theory.

Agent-based modeling

In *agent-based modeling* (ABM), a model of the real world is described by the set of autonomous agents and interaction between them [37, 38, 39, 40]. By conducting simulations with ABM, the behavior of a modeled system can be observed. One of the advantages of ABM simulations is the observation for emergent behavior in a model [41]. As a consequence of micro interactions between agents in a model, some macro features of the model are revealed by ABM simulations. Hence, the ABM simulations are used as a complement for analytical methods traditionally used to analyze real-world problems. If temporal networks and evolutionary games are mainly utilized to model interactions of individuals, they might be considered as ABM.

Reinforcement learning in multi-agent simulation is expected to discover adaptive solutions for problems that have uncertainty. However, although an agent learns appropriate actions against the environment, the environment is dependent on other agents' actions [42]. Hence, agents are more difficult to achieve better actions by reinforcement learning techniques through multi-agent environment compared to reinforcement learning with a single agent.

Social and economic network

Social and economics network is a relatively new research field to investigate economic behavior in a network representing a market [43]. This theory is also called *network market* [44]. Compared to microeconomics, network market has one simple advantage; a typical market network is assumed in a problem of network market. Ordinary microeconomics assumes an anonymous network as a market. However, this assumption cannot match with the actual environment of markets. Hence, network market is better in terms of the utilization of the market structure compared to the anonymous network. This type of research field is also known as *Agent-based Computational Economics* (ACE) [45].

Matching and buyer-seller network

In 1962, Gale and Shapley discussed *stable marriage problem* and *college admissions problem* in [46]. The stable marriage problem is a classic problem of matching, and this type of matching is classified into *one-to-one matching*; moreover, when the number of elements is the same on both sides of matching, the matching is called *perfect matching*. An objective of the college admissions problem is the construction of the matching between colleges and students. Colleges can accommodate some students, and, on the other hand, students must be linked to only one college; thus, this type of matching is called *many-to-one matching* [47]. Thus, matching has variation according to modeling targets.

Matching market is a method for constructing the matching between sellers and buyers in a networked market with a pricing mechanism [44, 48]. By changing the prices offered by sellers in a market model, demand of buyers in the model is changed, and the optimal distribution of sellers' items is realized if the prices of sellers are appropriate for buyers' demand for the items. The network structure used in matching market can be described as a *buyer-seller network* that is discussed in [49, 50]. The utilization of buyer-seller network provides two advantages. First, a buyer can choose a seller independently based on its evaluation to the seller, this is practical for the use in real markets because the markets do not ever have the central coordination of the choice of buyers. Second, the efficiency of the market can be examined by social welfare, which is the total of utilities of all market participants [51].

2.2 Mathematical modeling framework based on graph theory

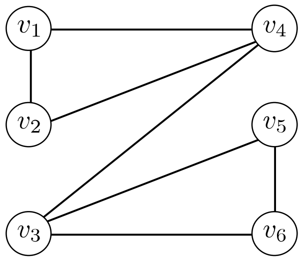
This section explains concepts for mathematical modeling of electricity markets. After explaining the basic notation of graph theory, this section proposes a modeling framework for electricity markets. Though this section proposes concepts of the framework, detailed notation specialized for each problem of this thesis is defined in subsequent chapters.

2.2.1 Fundamental concepts of graph theory

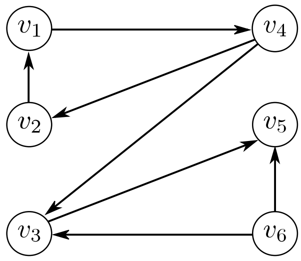
Undirected and directed graph

An undirected graph G consists of two sets: *node* set N and *edge* set E. G can be represented by $G = (N, E)$. Nodes are also called vertices. Every edge in E exists between two nodes in N. For instance, when E contains an edge between two nodes v and w, this edge can be denoted by (v, w) or (w, v). Both (v, w) and (w, v) represent the same edge because every edge in E has no direction. Fig. 2.1a shows an example of undirected graph G. In this Fig., an edge between v_1 and v_4 is denoted by (v_1, v_4) or (v_4, v_1).

A directed graph H is composed of a node set N and an *arc* set A. H is represented by $H = (N, A)$. An arc in A can be considered as a directed edge in H. An arc from v to w is represented as (v, w). Adversely, (w, v) denotes an arc with the direction opposite to (v, w). Fig. 2.1b is an example of directed graph H. In Fig. 2.1b, an arrow between v_4 and v_2 represents an arc (v_4, v_2).

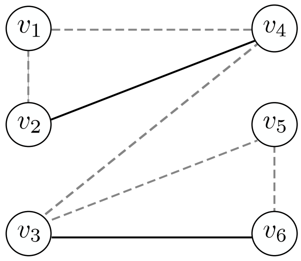


(a) Undirected graph.

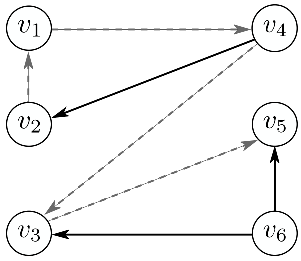


(b) Directed graph.

Fig. 2.1: Example of undirected graph and directed graph.



(a) Undirected path (v_2-v_6 path).



(b) Directed path (v_2-v_5 path).

Fig. 2.2: An undirected path and a directed path.

Path

In an undirected graph, a *path* is a sequence of edges. A path represents a route from the start node to the terminal node. In the sequence of edges of a path, the same node must appear only once. If a path has the start node s and the terminal node t, the path is called *s-t path*. In Fig. 2.2a, v_2-v_6 path ($v_2 - v_1 - v_4 - v_3 - v_5 - v_6$) is denoted by the dotted line. Of course, the number of v_2-v_6 paths in the graph shown in Fig. 2.2a is not necessarily only one. For example, a path $v_2 - v_4 - v_3 - v_6$ is also v_2-v_6 path. On the other hand, there is no v_2-v_6 path in the directed graph in Fig. 2.2b since no path can reach v_6 from v_2.

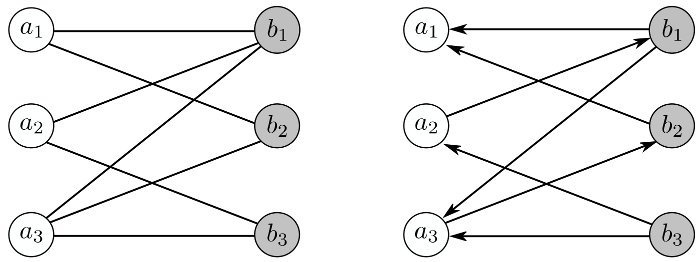
Degree

In graph theory, *degree* indicates the number of edges connected to a node. For undirected graphs, degree of node v is simply denoted by $deg(v)$. About a directed graph, two types of degree can be considered: *in-degree* and *out-degree*. For example, $deg_{in}(v)$ indicates in-degree of node v. Besides, $deg_{out}(v)$ denotes out-degree of node v.

Multipartite graph

A multipartite graph is a graph that consists of n partite sets of nodes [52]. For instance, for node set N of multipartite graph G, there are node sets N_i ($i = 1, 2, 3, ..., n$) that satisfy $N = N_1 \cup N_2 \cup N_3 \cup \ldots \cup N_n$. There is no edge between two nodes which belong to the same partite set of nodes. Hence, v and w must belong to the different type of node set each other about an arc (v, w) in a multipartite graph.

A bipartite graph has two types of node sets since a bipartite graph is a multipartite graph in which



(a) Undirected bipartite graph.
(b) Directed bipartite graph.

Fig. 2.3: An undirected bipartite graph and a directed bipartite graph.

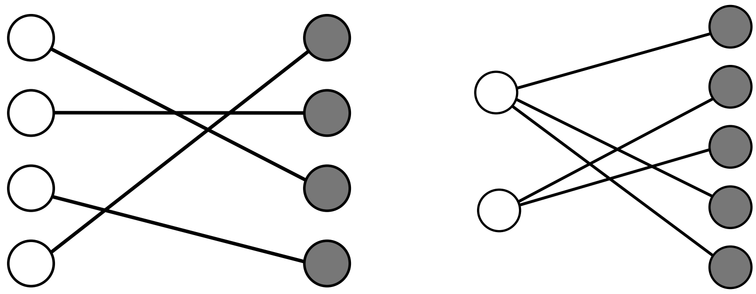
$n = 2$. For example, let N_a and N_b be two node sets of a bipartite graph. The bipartite graph has nodes $a_i \in N_a$ $(i = 1, 2, ..., |N_a|)$ and $b_j \in N_b$ $(j = 1, 2, ..., |N_b|)$. Thus, an undirected bipartite graph has edges represented by (a_i, b_j) or (b_j, a_i). Examples of undirected bipartite graph and directed bipartite graph are shown in Fig. 2.3.

2.2.2 Electricity market model represented by graphs

This thesis focuses on the following three aspects representing characteristics of trading in proposed models of electricity markets.

1. Representing network structure among market participants:
   As described in Section 1, the networked structure evolving after the deregulation of electricity markets has become an important aspect. Using graph theoretical concepts, the market models denote relationships of trading among participants in electricity markets.
2. Satisfying constraints on production and demand of electricity:
   Production and demand of electricity must be balanced in electricity trading to realize a stable supply of electricity to consumers. This thesis focuses on the constraints on the production and demand of market participants.
3. Describing benefits for consumers in markets:
   In the networked markets, consumers will face different benefits according to determined electricity trading since choices of consumers about suppliers increase than ever. Proposed market models define the benefit of consumers from electricity trading in the networked structure without violating the constraints on the production and demand of electricity.

The rest of this section presents mathematical modeling concepts for each aspect as the modeling framework.



(a) One-to-one matching. (b) Many-to-one matching.

Fig. 2.4: Examples of matching.

1. Matching in buyer-seller networks

The concept of matching is used for the first aspect of electricity markets. A bipartite graph in Fig. 2.4a is a one-to-one matching but not a perfect matching because one node in the black nodes is not assigned to any node in the white nodes; oppositely, a graph in Fig. 2.4b is a perfect one-to-one matching. By using the bipartite graph models of matchings, some researchers consider the matching problems based on the methods of graph theory.

Even though both matching market and general economics deal with markets, matching market does not use anonymous networks but given specified networks in which there are some buyers, sellers, and links between them. Therefore, matching market shows how market participants affect each other in the network that has the specified structure, and this attempt cannot be realized by general economics using the anonymous networks. An ordinary method of matching market assumes that sellers and buyers in a market deal with a single item, and an algorithm of matching market constructs only perfect one-to-one matching.

2. Network flows

As the second aspect of electricity markets, *network flow* [53] can be utilized to determine resource allocation in matching satisfying constraints on supply and demand of electricity. A directed graph (V, A) is used in a network flow problem. Arc $(v, w) \in A$ is a directed edge from v to w $(v, w \in V)$. $x(v, w)$ represents a nonnegative value of *flow* on $(v, w) \in A$. $c(v, w)$ denotes weight on arc (v, w). The cost of flow on arc (v, w) is calculated by $x(v, w) \cdot c(v, w)$. Fig. 2.5 shows the notation for the value of flow $x(v, w)$ between v and w.

A directed path is a set of arcs, which represents a route from the source to the sink in a graph. The route of a path must not pass through the same nodes that are already visited in the path. *o-t path*

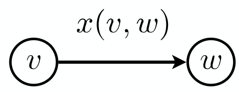


Fig. 2.5: Example of flow model on arc (v, w).

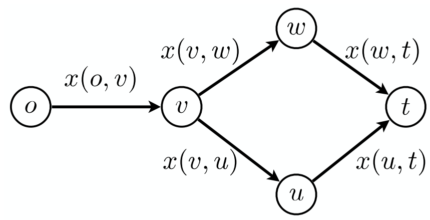


Fig. 2.6: A sample model with two o-t paths.

means a path which has the source o and the sink t. For instance, Fig. 2.6 shows a directed graph that has two o-t paths.

The main goal of a network flow problem is to find flows on o-t paths to optimize objective functions (e.g. maximizing amounts of flow, minimizing costs of flow, and so on). In a network flow problem, flow $x(v, w)$ must satisfy two constraints. One of the constraints is the *capacity constraint* on every arc in A, and the other is the *mass-balance constraint* on every node in V. For the capacity constraint, every arc $(v, w) \in A$ has capacity of flow $x(v, w)$. Let $ub(v, w)$ and $lb(v, w)$ be upper bound and lower bound of $x(v, w)$, respectively. As the capacity constraint, $x(v, w)$ must satisfy

$$lb(v, w) \leq x(v, w) \leq ub(v, w). \tag{2.1}$$

Besides, difference function $d : V \longrightarrow \mathbb{R}$ is utilized to define the mass-balance constraint. $d(v)$ describes difference in flow of input and output for v. $d(v)$ is calculated by

$$d(v) = \sum_{\{w:(v,w)\in A\}} x(v, w) - \sum_{\{w:(w,v)\in A\}} x(w, v). \tag{2.2}$$

In general, mass-balance constraint means $d(v) = 0$ except source and sink since structure of source and sink are different from the other nodes. let o and t represent source and sink, respectively. In terms of a source, $d(o)$ must satisfy

$$d(o) = \sum_{\{w:(o,w)\in A\}} x(o, w) - 0 \geq 0. \tag{2.3}$$

Table 2.1: Properties of a binary relation R.

Property	Description
reflexivity	$(s_k, s_k) \in R$
irreflexivity	$(s_k, s_k) \notin R$
symmety	$s_k R s_l \Rightarrow s_l R s_k$
asymmetry	$s_k R s_l \Rightarrow s_l R^c s_k$
transitivity	$s_k R s_l$ and $s_l R s_m \Rightarrow s_k R s_m$
intransitivity	$s_k R s_l$ and $s_l R s_m \Rightarrow s_k R^c s_m$
completeness	$(s_k, s_l) \in R$ or $(s_l, s_k) \in R$

Moreover, $d(t)$ must satisfy

$$d(t) = 0 - \sum_{\{w:(w,t)\in A\}} x(w,t) \leq 0. \tag{2.4}$$

For example, in the directed graph shown in Fig. 2.6, (2.2) must be satisfied in $d(v)$, $d(w)$, and $d(u)$. Likewise, (2.3) and (2.4) must be satisfied in $d(o)$ and $d(t)$, respectively.

3. Preference relation and utility function

The third aspect of electricity markets, which is to describe benefits for consumers, is represented by using concepts of preference and utility. *Preference relation* describes preference of a market participant over its alternatives to chose. For instance, let buyer b_j choose seller s_i based on preference over its alternatives of sellers. b_j is likely to choose an alternative that is more preferred than the current seller. The preference of b_j over every pair of alternatives $s_k, s_l \in X_j$ is expressed as preference relation, which is based on binary relation. Binary relation R is a subset of Cartesian product $X_j \times X_j$. For $s_k, s_l \in X_j$, let $(s_k, s_l) \in X_j \times X_j$ be a pair of alternatives. R can be denoted as $R = \{(s_k, s_l) \mid s_k, s_l \in X_j\}$. Let $s_k R s_l$ be $(s_k, s_l) \in R$, and $s_k R^c s_l$ denote $(s_k, s_l) \notin R$. R has properties shown in Table 2.1 based on (s_k, s_l) contained in R. In Table 2.1, s_k, s_l, s_m are alternatives arbitrarily chosen from X_j.

Utility function is used as a metric to represent the benefit of a participant in a model. For instance, utility of buyer b_j to choose alternative s_i is calculated by a utility function $\mu_j : X_j \rightarrow \mathbb{R}$. Utility function and preference relation are related each other. Considering P and I over the same set X_j, preference relation is rational if $P \cup I$ meets completeness and transitivity. Furthermore, if $P \cup I$ satisfies transitivity, each of P and I is also transitive. When preference relation is rational, preference relation can be defined by using utility function μ_j as follows.

$$s_k P s_l \iff \mu_j(s_k) > \mu_j(s_l).$$

Although $\mu_j(s_i)$ might contain many types of factors to determine utility obtained from each alternative, this thesis focuses on electricity charges as the factor of utility. Electricity is an almost homogeneous product since the function of electricity does not differ with suppliers from consumers' point of view [54, 55]. In general, the main factor for a comparison of homogeneous products is the charge of products. The definition of utility function is also based on the charge of product in game theory in some cases [56].

In addition to electricity charges, the other factors of consumers' utility can also be considered such as the environmental effects of energy sources, reliability of electricity supply, and so on.

- Rowlands *et al.* investigated several factors of consumers' decision making about choosing electricity suppliers [57]. Their results indicate consumers recognized electricity charges and environmental effects as important factors. Some electricity consumers prefer renewable energy due to its smaller environmental effects. Although consumers who prefer renewable energy are possibly willing to pay higher costs than conventional plants, the consumers' willingness to pay for renewable energy differs across countries [58].

- Some consumers might feel anxious about the reliability of new retailers in the markets. The consumers are willing to pay costs to avoid electric outages [59]. However, the reliability is not different across retailers if divisions for managing stable electricity supply have not been deregulated even after the liberalization of electricity retail (for example in Japan) [60].

Hence, this thesis assumes electricity charges are considered to be the common characteristics of consumers' utility compared to environmental effects and reliability described above. In other words, this thesis mainly focuses on economic benefits in the definition of utility functions in proposed electricity market models.

Utility function $\mu_j(s_i)$ can be used to represent the concept of *consumer surplus* widely used in microeconomics. Let v_j represent reservation price, which is willingness to pay of b_j for purchasing electricity from s_i. Besides, let p_i be an electricity price offered by s_i for one unit of electricity. If b_j

purchases d_j units of electricity, $\mu_j(s_i)$ is determined as follows in our modeling framework.

$$\mu_j(s_i) = \left(v_j - p_i\right) d_j. \tag{2.5}$$

Using network flow, d_j can be determined by the amount of flow between b_j and s_j in electricity trading. Since the amount of network flow must satisfy constraints on production and demand of electricity, network flow can be utilized to determine d_j without violating the constraints on electricity trading. Although utility functions defined in subsequent chapters are specialized for each problem, the values given by the utility functions commonly represent the benefit of consumers in matching in proposed market models.

2.3 Summary of this chapter

This chapter introduces a mathematical modeling framework to represent trading in electricity markets as an extension of the concept of matching market. By using concepts of graph theory, relationships about trading among market participants (e.g. suppliers, consumers) can be described. Network flow can denote the quantity of supply and demand of electricity in trading. Furthermore, as a method considered in social and economic networks, utility function can describe the benefits of each participant in electricity trading. This chapter explains the concepts of the modeling framework, and solutions for Problem 2, Problem 3, and Problem 4 are considered in the subsequent chapters based on the proposed modeling framework.

Chapter 3

Representing electricity trading in deregulated electricity markets

This chapter introduces a mathematical modeling technique regarding Problem 2. In this chapter, the mathematical modeling framework proposed will be utilized for describing consumers' benefits in deregulated electricity markets.

3.1 Introduction

3.1.1 Literature review

Numerous studies have been conducted to consider modeling techniques for deregulated electricity markets [61, 62]. Grine *et al.* present a multi-layer model to consider electricity prices with another energy commodity [63]. Triki *et al.* consider an optimal capacity allocation problem to maximize profits of electricity sellers [64]. Hussein *et al.* formulate an optimization problem for forecasting prices in day-ahead electricity markets [65]. Corchero *et al.* present a stochastic programming model for the Spanish electricity market [66].

Moreover, there are various studies on *efficiency*, which means the optimal allocation of the electricity with appropriate prices [67]. Efficiency can be measured by *social welfare* that is the sum of payoffs of all market participants [68]. Stern *et al.* consider the relation between market clearing price mechanisms and the maximization of social welfare in deregulated electricity markets [69]. Mechanism designs to maximize social welfare in double-sided electricity markets are presented in [70, 71]. Nicolaisen *et al.* propose a price setting problem in a double-price auction for wholesale markets by implementing a reinforcement learning algorithm [72]. Swami considers social welfare maximization with considering congestion of transmission lines [51]. As one of the research topics on market efficiency, *network market* based on graph theory is proposed in [49]. This model is considered to be more realistic than anonymous networks because actual trades occur between participants that can interact with each other. Since electricity trades also take place between participants connected by transmission lines, we proposed an algorithm to find optimal matchings in an electricity market model based on network market [73].

However, the previous works have not focused on electricity retailers, because the previous market models contain only suppliers and consumers, not a retailer. In [74], Babic notes advantages of an agent-based modeling technique for electricity retail markets; however, no characteristics regarding retailers are demonstrated in the thesis. To model activities of retailers in network markets, Blume *et al.* considers optimal price setting on a tripartite graph by utilizing a game theoretical approach [75]. However, the method cannot deal with a multi-unit commodity such as electricity because it is assumed that participants trade only a single-unit commodity. Besides, Nava introduces the competition model utilizing network flows in oligopolistic markets [76]. Although the model of Nava can cope with participants including retailers dealing with multi-unit commodities, the model cannot be applied to electricity markets because the roles of participants in electricity markets are determined before equilibrium prices are discovered.

3.1.2 Contribution

This thesis proposes a sequential solution method to determine prices and efficient trades in an electricity market model with electricity retailers. In this thesis, we formulated a determination problem for efficient electricity trades on a model with electricity as a multi-unit commodity, not a single-unit commodity. To solve the problem, we constructed the sequential solution method to choose electricity trades in the market model. In our solution method, a price setting algorithm extended from a price setting mechanism proposed in [75]. Moreover, to determine electricity trades on the model, a determination problem is formulated by utilizing integer programming and unsplittable flow [77]. Simulation results demonstrate the characteristics in deregulated electricity markets about efficiency in terms of social welfare and payoff allocation of each market participant. Although the simulation results about efficiency are similar to the result presented in [78], the parameter conditions about capacity of electricity sellers are different in this thesis. Moreover, the results regarding payoff allocation indicate that important factors for the payoff allocation are the market structure and the period of time having elapsed since the deregulation.

3.2 Electricity market model with agents

This section introduces our electricity market model based on graph theory. The model denotes three types of agents and electricity trades conducted between them.

3.2.1 Tripartite network representing electricity markets

Our electricity market model is represented by tripartite network $G = (S \cup B \cup T, A)$. Fig. 3.1 shows an example of G. G is composed of three types of agents: *buyer* $b_j \in B$, *seller* $s_i \in S$, and *trader*

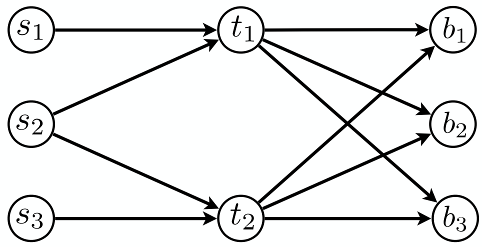


Fig. 3.1: Tripartite network G composed of agents.

$t_k \in T$. Each arc indicates that agents at endpoints of the arc can conduct electricity trades between them. Arc set A contains arcs (s_i, t_k) or (t_k, b_j) due to following three constraints.

1. Each arc connects two agents not belonging to the same type of agents.
2. b_j must be provided electricity from t_k.
3. t_k must purchase electricity from s_i.

3.2.2 Notation of electricity flow on market model

In the model, each seller has a *capacity* of electricity, and each buyer has a *demand* of electricity. The capacity of s_i and the demand of b_j are denoted by c_i^s and d_j^b respectively. Let $c_{\min}$ be the minimum capacity of all c_i^s, and let $d_{\min}$ be the minimum demand of all b_j. Since a seller in our model can supply electricity to at least one buyer via a trader, $c_{\min}$ must satisfy $c_{\min} \geq d_{\min}$.

To denote electricity trades in the model, the notation of network flow is utilized. Integer $x(b, a)$ is the quantity of electricity flow on arc (b, a). Lower bound and upper bound of $x(b, a)$ are represented

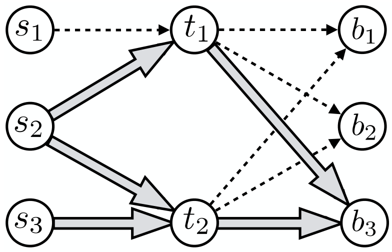


Fig. 3.2: Example of splittable flow.

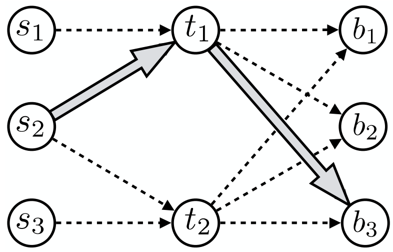


Fig. 3.3: Example of unsplittable flow.

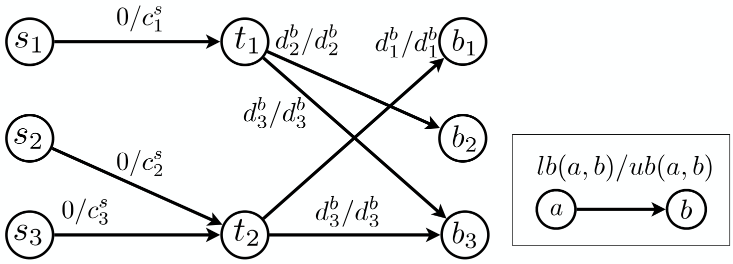


Fig. 3.4: Flow constraints on arcs in G.

by $lb(b, a)$ and $ub(b, a)$, respectively. If $x(b, a) > 0$, electricity currents on (b, a); otherwise, there is no electricity flow on (b, a).

In addition, *unsplittable flow* is utilized to avoid determining complicated electricity trades satisfying the demand of a buyer. In Fig. 3.2 and 3.3, solid arcs represent electricity flow to satisfy d_3^b. Electricity flow in Fig. 3.2 is considered as *splittable flow*. The type of flow in Fig. 3.3 is called unsplittable flow. Only one s_2-b_3 path is selected as unsplittable flow, and splittable flow adversely needs a larger number of arcs than unsplittable flow. Hence, our model uses unsplittable flow to determine electricity trades with a simple structure.

Unsplittable flow is realized by flow constraints on the model. s_i can supply electricity flow up to c_i^s. There is no electricity flow if s_i does not trade any electricity. Hence, $lb(s_i, t_k) = 0$, and $ub(s_i, t_k) = c_i^s$. Besides, b_j purchases d_j units of electricity, and flow constraints on (t_k, b_j) are denoted by $lb(t_k, b_j) = d_j^b$ and $ub(t_k, b_j) = d_j^b$. Fig. 3.4 indicates these capacity constraints.

3.3 Price setting game on market model

Agents in the model have property called valuation and payoff. The property is utilized in mechanisms for determining efficient electricity trades.

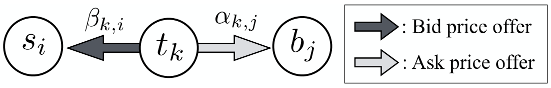


Fig. 3.5: Ask and bid prices.

3.3.1 Valuation and trade value

Electricity prices offered by traders are determined by a price setting game. Each seller and buyer in the model has *valuation*, which describes the utility for trading one unit of electricity. v_j^b indicates the valuation of b_j for purchasing one unit of electricity. v_i^s denotes the valuation of s_i for supplying one unit of electricity. The sets of each valuation are defined by

$$\mathbf{v_s} = \left\{v_i^s \mid s_i \in S, v_i^s > 0\right\}, \mathbf{v_b} = \left\{v_j^b \mid b_j \in B, v_j^b > 0\right\}.$$

Electricity trades occur between seller s_i and buyer b_j via trader t_k. About the electricity trades among them, each agent obtains payoff. The total of the payoff regarding the trade is called *trade value*. Since we assume that costs for supplying electricity through arcs between s_i and b_j are zero regardless of t_k, trade value for d_j^b units of electricity is described by

$$w_{i,j} = \left(v_j^b - v_i^s\right) d_j^b. \tag{3.1}$$

When t_k conducts a trade between b_j and s_i, t_k has its own strategy denoted by $(\alpha_{k,j}, \beta_{k,i})$. This strategy consists of two types of prices called *ask price* $\alpha_{k,j}$ and *bid price* $\beta_{k,i}$. t_k offers $\alpha_{k,j}$ to b_j adjacent to t_k. Besides, $\beta_{k,i}$ is offered to s_i adjacent to t_k. In Fig. 3.5, t_k offers $\alpha_{k,j}$ to b_j and $\beta_{k,i}$ to s_i. Since there will be traders who lost money if $\beta_{k,i} > \alpha_{k,j}$, the strategy of t_k must be a *no-crossing strategy* [79] represented by $\beta_{k,i} \leq \alpha_{k,j}$.

3.3.2 Payoff of each participant

b_j must purchase d_j^b units of electricity from one of the traders to satisfy its demand. The payoff of b_j for trading with t_k can be denoted by the following utility function $\mu_j(t_k)$.

$$\mu_j(t_k) = \left(v_j^b - \alpha_{k,j}\right) d_j^b. \tag{3.2}$$

Since this chapter assumes each buyer purchases electricity from only one trader, the total payoff

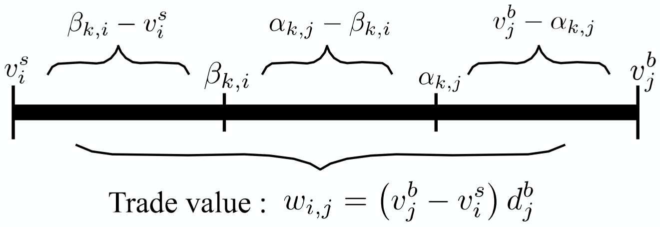


Fig. 3.6: Trade value and payoffs of agents.

of b_j for purchasing electricity from t_k is represented by

$$P\left(b_j\right) = \mu_j(t_k). \tag{3.3}$$

About sellers, s_i is offered $\beta_{k,i}$ by t_k. To provide electricity, s_i will choose t_k offering $\beta_{k,i}$ that maximizes payoff of s_i. Payoff of s_i for supplying d_j^b units of electricity to b_j through t_k is denoted by

$$\mu\big(s_i, (k, j)\big) = \left(\beta_{k,i} - v_i^s\right) d_j^b. \tag{3.4}$$

s_i can provide electricity to one or more buyers if the total of demands do not exceed c_i^s. Therefore, s_i can obtain the total payoff represented by

$$P\left(s_i\right) = \sum_{(t_k, b_j) \in pair(s_i)} \mu\big(s_i, (k, j)\big), \tag{3.5}$$

where $pair(s_i)$ denotes the set of pairs of b_j and t_k provided electricity from s_i.

t_k obtains payoff for trading d_j^b units of electricity between s_i and b_j, which is denoted by

$$\mu\big(t_k, (i, j)\big) = \left(\alpha_{k,j} - \beta_{k,i}\right) d_j^b. \tag{3.6}$$

Let $S(t_k)$ be the set of s_i adjacent to t_k, and let $B(t_k)$ be the set of b_j adjacent to t_k. The total payoff of t_k is represented by

$$P\left(t_k\right) = \sum_{s_i \in S(t_k), b_j \in B(t_k)} \mu\big(t_k, (i, j)\big)\, x_{k,i,j}. \tag{3.7}$$

Fig. 3.6 shows the relation between the trade value and payoffs of s_i, t_k, and b_j.

Algorithm 1 Price setting (G, $\mathbf{v_s}$, $\mathbf{v_b}$, λ)

```
for j ← 1 to |B| do
    if |adj(b_j)| = 1 then
        q(b_j) = 0.
    else
        Find ŝ_i with the minimum valuation v̂_i^s.
        q(b_j) = (v_j^b − v̂_i^s)λ.
        while q(b_j) = (v_j^b − v́_i^s)λ (ś_i ≠ ŝ_i, v́_i^s ≥ v̂_i^s) do
            Decrease q(b_j).
        end while
    end if
end for
for i ← 1 to |S| do
    if |adj(s_i)| = 1 then
        q(s_i) = 0.
    else
        Find b̂_j with the maximum valuation v̂_j^b.
        q(s_i) = (v̂_j^b − v_i^s)λ.
        while q(s_i) = (v́_j^b − v_i^s)λ (b́_j ≠ b̂_j, v́_j^b ≤ v̂_j^b) do
            Increase q(s_i).
        end while
    end if
end for
for k ← 1 to |T| do
    return α_{k,j} = v_j^b − q(b_j) (b_j ∈ B(t_k)).
    return β_{k,i} = v_i^s + q(s_i) (s_i ∈ S(t_k)).
end for
```

3.4 Procedure to determine electricity trades

To determine electricity trades on the model, we propose a sequential solution method. The method firstly calculates equilibrium electricity prices, and then the method determines electricity trades by using the prices.

3.4.1 Price setting algorithm

Algorithm 1 shows a price setting algorithm that calculates equilibrium prices. In this algorithm, each seller and buyer discovers its maximum payoff for trading electricity by considering the valuation of the other agents. This algorithm is based on price setting mechanism explained in [75]. We extend the algorithm to adjust payoffs of participants for ensuring no participant exclusively obtains larger payoff than other agents.

The process of Algorithm 1 is as follows. First, each buyer $b_j \in B$ calculates $q(b_j)$, which is the payoff of b_j for trading one unit of electricity. $q(b_j)$ is used to obtain ask prices $\alpha_{k,j}$. For all $a \in S \cup B$, let $adj(a)$ be the set of t_k connected to a. If $|adj(b_j) = 1|$, there is only one trader t_k adjacent to b_j.

There is no competition between b_j and t_k in this case, and thus $q(b_j) = 0$. If $|adj(b_j) > 1|$, two or more traders t_k are connected to b_j. Let $\hat{s_i}$ be a seller adjacent to $t_k \in adj(b_j)$. $\hat{s_i}$ has the valuation $\hat{v}_i^s$ that is the minimum in the valuation of sellers adjacent to $t_k \in adj(b_j)$. In our price setting algorithm, a real number λ ($0 < \lambda \leq 0.5$) is incorporated into the algorithm as a parameter that is used to adjust payoffs of sellers and buyers. The range of λ is set to realize the no-crossing strategy, explained in Section 3.3.1. With this notation, $q(b_j)$ is firstly set as $q(b_j) = (v_j^b - \hat{v}_i^s)\lambda$. Then, $q(b_j)$ is decreased until $q(b_j)$ becomes equal to $(v_j^b - \acute{v}_i^s)\lambda$, where $\acute{s_i} \neq \hat{s_i}$ is one of the sellers adjacent to $t_k \in adj(b_j)$ and has valuation $\acute{v}_i^s \geq \hat{v}_i^s$.

Second, each seller $s_i \in S$ determines $q(s_i)$, which is the payoff of s_i for trading one unit of electricity. By setting $q(s_i)$, bid prices $\beta_{k,i}$ can be determined. The process to set $q(s_i)$ is similar to the process to calculate $q(b_j)$. If $|adj(s_i) = 1|$, there is no competition between s_i and t_k, and $q(s_i) = 0$. If $|adj(s_i) > 1|$, let $\hat{b_j}$ be a seller adjacent to $t_k \in adj(s_i)$, and $\hat{b_j}$ has the valuation $\hat{v}_j^b$ that is the maximum in the valuation of buyers adjacent to $t_k \in adj(s_i)$. $q(s_i)$ is firstly set as $q(s_i) = (\hat{v}_j^b - v_i^s)\lambda$. Then, $q(s_i)$ is increased until $q(s_i)$ becomes equal to $(\acute{v}_j^b - v_i^s)\lambda$, where $\acute{b_j} \neq \hat{b_j}$ is one of the buyers adjacent to $t_k \in adj(s_i)$ and has valuation $\acute{v}_j^b \leq \hat{v}_j^b$.

Finally, ask price $\alpha_{k,j}$ and bid price $\beta_{k,i}$ of each trader t_k are set based on $q(b_j)$ and $q(s_i)$ for all b_j and s_i. These ask and bid prices finally determined are equilibrium prices on the market model.

3.4.2 Optimization problem for trade determination

Maximization of payoff for each trader

To determine electricity trades, each trader solves a maximization problem of payoff by using prices calculated by Algorithm 1. In this problem, trades are greedily chosen by each trader to maximize its payoff. This problem is formulated as an integer program similar to the generalized assignment problem [80].

The maximization problem is formulated as follows. Let $x_{k,i,j} \in [0, 1]$ denote the electricity trades on s_i-b_j path via t_k. $x_{k,i,j} = 1$ means electricity trades are conducted on the s_i-b_j path. Adversely, trades on the s_i-b_j path is not conducted if $x_{k,i,j} = 0$. The condition of network flow on the model determines $x_{k,i,j}$, and the condition is represented by

$$x_{k,i,j} = \begin{cases} 1 & \big(x(s_i, t_k) > 0 \cap x(t_k, b_j) > 0\big), \\ 0 & \big(x(s_i, t_k) = 0 \cup x(t_k, b_j) = 0\big). \end{cases}$$

By using $x_{k,i,j}$ and $\mu\big(t_k, (i, j)\big)$, the maximization problem of t_k is described as the following integer program.

$$\max P(t_k) = \sum_{s_i \in S(t_k), b_j \in B(t_k)} \mu\big(t_k, (i, j)\big)\ x_{k,i,j}. \tag{3.8}$$

$$0 \leq x_{k,i,j} \leq 1,$$

$$\mu\big(t_k, (i, j)\big) \geq 0,$$

$$\textstyle\sum_{b_j \in B(t_k)} x_{k,i,j}\ d^b_j \leq c^s_i.$$

In (3.8), $x_{k,i,j} = 1$ means trades on s_i-b_j path are selected to maximize payoff of t_k.

Trades satisfying all capacity and demand

Even though every trader determines all trades by (3.8), demands for some sellers might exceed their own capacity. This situation means some consumers cannot purchase electricity from a seller who does not have enough capacity to satisfy all demands. Hence, electricity trades satisfying all capacity and demand should be independently chosen from the electricity trades which each trader selected by solving (3.8). To choose the electricity trades considering capacity, a maximization problem of social welfare is utilized in this thesis.

To describe the maximization problem, a maximum unsplittable flow problem is utilized. In the problem, bipartite network $G_{\mathrm{bi}} = (S \cup B, A_{\mathrm{bi}})$ is constructed. Arc set A_{bi} corresponds to the set of possible trades x_t. Thus, for all $t_k \in T$, A_{bi} contains (s_i, b_j) if $x_{k,i,j} = 1$ in (3.8). The capacity of flow on (s_i, b_j) is denoted by $0 \leq x_{i,j} \leq 1$. s_i can supply flow up to c^s_i, and demand of flow of b_j is d^b_j. Finally, the following integer program gives $W(x_t)$ that is the maximum social welfare on G_{bi}.

$$\max\ W(x_t) = \sum_{(s_i, b_j) \in A_{\mathrm{bi}}} x_{i,j} w_{i,j}. \tag{3.9}$$

$$x_{i,j} \geq 0,$$

$$\textstyle\sum_{s_i \in adj(b_j)} x_{i,j} \leq 1,$$

$$\textstyle\sum_{b_j \in adj(s_i)} x_{i,j}\ d^b_j \leq c^s_i.$$

3.4.3 Overall procedure for trade determination

The overall procedure of our solution method to determine efficient trades is described in Procedure 1. First, equilibrium prices are calculated by Algorithm 1. Then, every trader discovers trades maximizing payoff of the trader. After that, efficient trades x_t will be determined in all trades that the traders want to conduct. Finally, social welfare $W(x_t)$ can be calculated.

Procedure 1 trade determination

Input: $G, \mathbf{v_s}, \mathbf{v_b}, \lambda$.

Output: $W(x_t), x_t,$.

Four steps of the procedure:

1. Price setting $(G, \mathbf{v_s}, \mathbf{v_b}, \lambda)$.
2. For all t_k, determine $x_{k,i,j}(s_i \in S(t_k), b_j \in B(t_k))$ by (3.8).
3. Construct G_{bi} by using $x_{k,i,j}(t_k \in T, s_i \in S, b_j \in B)$.
4. Obtain $W(x_t)$ and x_t by solving (3.9) with G_{bi}.

3.5 Market participants assigned to model

By setting conditions of the network and agents in the model, four types of market participants can be considered. Those participants can be utilized to reveal characteristics of deregulated electricity markets.

3.5.1 Participants in deregulated electricity markets

In this thesis, a day-ahead electricity market is focused on as deregulated electricity markets. Prices in the day-ahead markets are determined hourly or half hourly [81]. Participants supposed in this thesis are classified into four types: the public utility, independent power producers, retailers, and consumers. The participants are described by agents explained in Section 3.2.1.

1. *Public utility* (PU) conducts electricity generation and supply. The PU is a large firm that has conducted generation and supply since the market was regulated. In the deregulated market, the

PU can purchase electricity from another generator. The PU is denoted by a pair of seller and trader.

2. *Retailer* (R) conducts electricity trades with consumers. A retailer is described by one pair of a trader and a seller if the retailer has its own generator. Otherwise, one trader denotes a retailer.
3. *Independent power producer* (IPP) has its own generator to sell its electricity to customers. An IPP is assigned to one of the sellers, and it can supply electricity to PU and retailers.
4. *Consumer* (C) is an end-user of electricity. A consumer purchases electricity from one of the best suppliers connected to the consumer. The consumer is assigned to one of the buyers.

3.5.2 Constraints on network structure

To describe market participants, network G is constructed with following constraints. G has only one PU, and PU is connected to all consumers in G since PU existed in an electricity market before deregulation. The market model contains some IPP and retailers that have newly joined the market after deregulation. For simplicity, the model contains the same number of IPP and retailers. Each IPP is connected to all retailers and PU.

After the deregulation, a consumer cannot choose a retailer if the consumer does not know the retailer. To describe this situation, R is connected to C with the probability represented by $prob(R, C) \in (0, 1]$. If $prob(R, C) = 1$, all arcs between retailers and a consumer are constructed. As the time has elapsed since the deregulation, each consumer will increase the number of retailers which the consumer know. Hence, long time has passed since the deregulation if $prob(R, C)$ is high. Fig. 3.7 and Fig. 3.8 show example models with $prob(R, C) = 0.2$ and $prob(R, C) = 0.8$, respectively. In these figures, R' and R denote seller s_i and trader t_k of a retailer respectively. Besides, PU' and PU represent s_i and t_k of PU respectively in the figures.

3.5.3 Constraints on parameters of participants

The capacity of newly joining participants, such as IPP and retailers, is relatively lower than the capacity of PU in deregulated electricity markets. The capacity of PU is enough to supply electricity to all consumers since PU was responsible for electricity supply before deregulation. In this thesis, capacity of sellers is set according to the following constraint.

$$c_R \le c_{IPP} \le c_{PU}. \tag{3.10}$$

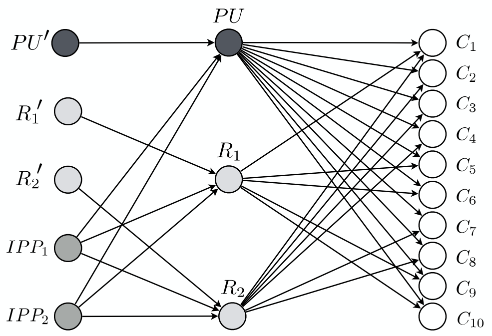


Fig. 3.7: Sample model with $prob(R, C) = 0.2$.

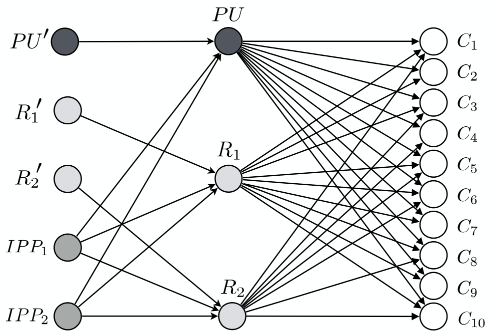


Fig. 3.8: Sample model with $prob(R, C) = 0.8$.

With regard to valuation of sellers, newly joining participants have lower valuation than PU in this thesis. This condition means newly joining participants can offer low-cost electricity than PU. Besides, valuation of all buyers is fixed to the same value for simplicity. The valuation of buyers is enough to purchase electricity from any sellers. Following constraint indicates the conditions described above.

$$v_R < v_{IPP} < v_{PU} < v_C. \tag{3.11}$$

3.5.4 Metrics indicating characteristics of markets

To evaluate trades determined by our method, two kinds of the metrics are considered in this thesis.

Efficiency rate

Although $W(x_t)$ is the maximum social welfare on G_{bi}, $W(x_t)$ does not necessarily correspond to $W(x)$ that is the upper bound of social welfare on G. $W(x)$ can be obtained by constructing bipartite network $G'_{\text{bi}} = (S \cup B, A'_{\text{bi}})$ from G. A'_{bi} is the set of possible trades x on G'_{bi}. If s_i can trade with b_j through at least one trader in G, arc set A'_{bi} contains an arc (s_i, b_j). With G'_{bi}, the following integer program gives $W(x)$.

$$\max \ W(x) = \sum_{(s_i,b_j)\in A'_{\text{bi}}} x_{i,j} w_{i,j}. \tag{3.12}$$

$$0 \le x_{i,j} \le 1,$$

$$w_{i,j} \ge 0,$$

$$\textstyle\sum_{b_j \in B} x_{i,j}\ d^b_j \le c^s_i.$$

Table 3.1: Payoff of market participants and agents.

Participant	Agent	Payoff of participant
PU	s_i and t_k	$P(PU) = P(s_i) + P(t_k)$.
R	s_i and t_k	$P(R) = P(s_i) + P(t_k)$.
IPP	s_i	$P(IPP) = P(s_i)$.
C	b_j	$P(C) = P(b_j)$.

$W(x_t)$ is not necessarily the same as $W(x)$ since the calculation of $W(x)$ does not consider electricity price setting. However, the difference between $W(x_t)$ and $W(x)$ should be small to keep high efficiency. Hence, the performance regarding efficiency of our sequential method can be evaluated by comparing $W(x_t)$ with $W(x)$. The comparison between $W(x_t)$ and $W(x)$ can be conducted by examining *Efficiency Rate* (ER), such that

$$ER(x_t, x) = \left\{W(x_t)/W(x)\right\} \times 100 \text{ [\%]}. \tag{3.13}$$

Payoff rate

In regulated electricity markets, PU is responsible for providing electricity to consumers. Hence, PU exclusively obtains payoff for supplying electricity. On the other hand, payoff for providing electricity is also allocated to newly joining market participants in deregulated electricity markets. Therefore, it is important to analyze who can acquire how much payoff to trade electricity in deregulated electricity markets.

In this thesis, *Payoff Rate* (PR) is utilized to indicate the rate of payoff of each participant in social welfare. The payoff of each participant can be described by using the payoff of agents. Table 3.1 shows the relation between the payoff of each participant and agents. In Table 3.1, $P(a)$ denotes payoff of market participant $a \in (PU \cup R \cup IPP \cup C)$. PR for a is denoted by

$$PR(a) = \left\{P(a)/W(x_t)\right\} \times 100 \text{ [\%]}. \tag{3.14}$$

3.6 Experimental results

This section demonstrates simulation results of our sequential solution method. After simulation conditions are introduced, results regarding efficiency rate and payoff rate are presented.

Table 3.2: Conditions of parameters in simulations.

Parameter	Assigned value
# of agents	$\|S\|$=5 or 7, $\|T\|$=3 or 5, $\|B\|$=10,15, or 30.
# of participants ($\|S\|$=5)	PU: 1, R: 2, IPP: 2, C : 10, 15, or 30.
# of participants ($\|S\|$=7)	PU: 1, R: 4, IPP: 4, C : 10, 15, or 30.
v_j^b	20 for all b_j.
v_i^s ($\|S\| = 5$)	$v_1^s = 10, v_2^s = 4, v_3^s = 3, v_4^s = 9, v_5^s = 8$.
v_i^s ($\|S\| = 7$)	$v_1^s = 10, v_2^s = 4, v_3^s = 3, v_4^s = 2, v_5^s = 9, v_6^s = 8, v_7^s = 7$.
d_j^b	$d_j^b = j$.
λ	0.25
$prob(R, C)$	0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0.
Iteration	100

3.6.1 Conditions

For conducting simulations of trade determination, a simulation software for our model was developed with Java and lp_solve, which is an integer programming solver. Simulation experiments were conducted with the condition described in Table 3.2. In the simulations, we assumed that each consumer has the unique demand of electricity. Hence, the index of each consumer is set as the fixed demand of the consumer since the index is unique to each consumer. Since the model structure depends on $prob(R, C)$, 100 times of iterations for every $prob(R, C)$ and CP were conducted.

Table 3.3: Capacity pattern ($|B| = 10, 15, 30$).

$\|B\|$	10				
CP	1	2	3	4	5
PU	55	55	55	55	55
IPP	3	6	13	27	31
R	1	2	6	18	69

$\|B\|$	15				
CP	1	2	3	4	5
PU	120	120	120	120	120
IPP	7	15	30	60	84
R	1	4	13	40	69

$\|B\|$	30				
CP	1	2	3	4	5
PU	465	465	465	465	465
IPP	29	58	116	232	328
R	5	17	51	155	268

In deregulated electricity markets, newly joining participants expands their capacity for electricity supply as the time elapses. Hence, different types of capacity of newly joining participants were set in the simulations. Table 3.3 shows Capacity Patterns (CP), which are the conditions of capacity of each seller. CP 1 indicates newly joining participants do not have large capacity because not a long period has elapsed since the start of deregulation. In CP 2, more periods of time have passed after the deregulation than CP 1, and the difference of capacity between market participants became smaller than CP 1. In CP 3, 4, and 5, newly joining participants got more capacity as the index of CP increases. For all CP, capacity of PU is is equal to the total of all demands, and all consumer can purchase electricity from PU as the worst choice.

Table 3.4: Ask and bid prices obtained in a simulation.

(s_i, t_k)	v_i^s	$\beta_{k,i}$
(PU', PU)	10.00	10.00
(R_1', R_1)	4.00	4.00
(R_2', R_2)	3.00	3.00
(IPP_1, PU)	9.00	12.67
(IPP_1, R_1)	9.00	12.67
(IPP_1, R_2)	9.00	12.67
(IPP_2, PU)	8.00	12.00
(IPP_2, R_1)	8.00	12.00
(IPP_2, R_2)	8.00	12.00

(t_k, b_j)	v_j^b	$\alpha_{k,j}$
(PU, C_1)	20.00	14.67
(PU, C_2)	20.00	16.00
(PU, C_3)	20.00	16.00
(PU, C_4)	20.00	16.00
(PU, C_5)	20.00	16.00
(PU, C_6)	20.00	16.00
(PU, C_7)	20.00	14.67
(PU, C_8)	20.00	16.00
(PU, C_9)	20.00	16.00
(PU, C_{10})	20.00	16.00

(t_k, b_j)	v_j^b	$\alpha_{k,j}$
(R_1, C_1)	20.00	14.67
(R_1, C_2)	20.00	16.00
(R_1, C_4)	20.00	16.00
(R_1, C_7)	20.00	14.67
(R_1, C_9)	20.00	16.00
(R_1, C_{10})	20.00	16.00

(t_k, b_j)	v_j^b	$\alpha_{k,j}$
(R_2, C_1)	20.00	14.67
(R_2, C_3)	20.00	16.00
(R_2, C_5)	20.00	16.00
(R_2, C_6)	20.00	16.00
(R_2, C_7)	20.00	14.67
(R_2, C_8)	20.00	16.00

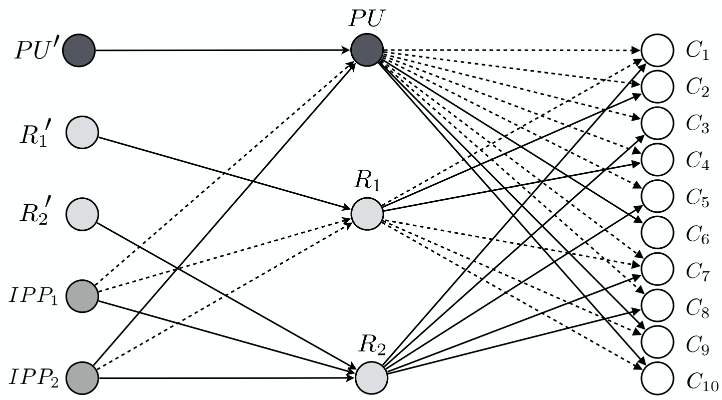


Fig. 3.9: Determined trades on G.

3.6.2 Results and discussion

Determined electricity trades

First, the result of one of the iterations is focused on to examine determined electricity trades. In this result, $prob(R, C) = 0.3$, and CP 3 was selected. Table 3.4 represents ask and bid prices determined by Algorithm 1. All bid price $\beta_{k,i}$ were more than valuation v_i^s. Moreover, all ask price $\alpha_{k,j}$ were less than valuation v_j^b. Based on the prices shown in Table 3.4, Procedure 1 determined electricity trades x_t. In Fig. 3.9, solid arcs denote trades x_t, and dotted arcs show no electricity trades are conducted on the arcs.

Table 3.5: $\mu\big(t_k, (i, j)\big)$ and $w_{i,j}$ of each determined trade.

Determined trade	$\mu\big(t_k, (i, j)\big)$	$w_{i,j}$	d_j^b	$w_{i,j}\ d_j^b$
$R_2' - R_2 - C_1$	11.67	17	1	17
$R_1' - R_1 - C_2$	12.00	16	2	32
$R_2' - R_2 - C_3$	13.00	17	3	51
$R_1' - R_1 - C_4$	12.00	16	4	64
$IPP_1 - R_2 - C_5$	3.33	11	5	55
$IPP_2 - PU - C_6$	4.0	12	6	72
$IPP_2 - R_2 - C_7$	2.67	12	7	84
$IPP_1 - R_2 - C_8$	3.33	11	8	88
$PU' - PU - C_9$	6.00	10	9	90
$PU' - PU - C_{10}$	6.00	10	10	100
$W(x_t)$				653

Table 3.6: Payoff rate of each participant.

Participant	PR [%]
C_1	0.82
C_2	1.23
C_3	1.84
C_4	2.45
C_5	3.06

Participant	PR [%]
C_6	3.68
C_7	5.72
C_8	4.90
C_9	5.51
C_{10}	6.13

Participant	PR [%]
PU	21.13
R_1	11.03
R_2	17.26
IPP_1	7.30
IPP_2	7.96

Table 3.5 shows $\mu\big(t_k, (i, j)\big)$ and trade value $w_{i,j}$ for each determined trade. The first column at Table 3.5 represents determined trades described by the notation such as $s_i - t_k - b_j$. Since $\mu\big(t_k, (i, j)\big) \geq 0$ for all t_k, no-crossing trades were conducted in this simulation. In terms of social welfare, $W(x_t) = 653.0$, and $W(x) = 667.0$. Therefore, $ER(x_t, x) = 97.9$ % in this example. More detailed analysis on $ER(x_t, x)$ is presented in Section 3.6.2.

Table 3.6 shows PR of each participant in the simulation. As shown in Table 3.6, no participant obtains zero payoffs and acquires a lot of payoffs exclusively. Allocation of PR of PU, IPP, and retailers varied in each iteration since the number of trades that each participant involves in each iteration is not the same as other iterations. In Section 3.6.2, PR is examined in more detail with other simulation conditions.

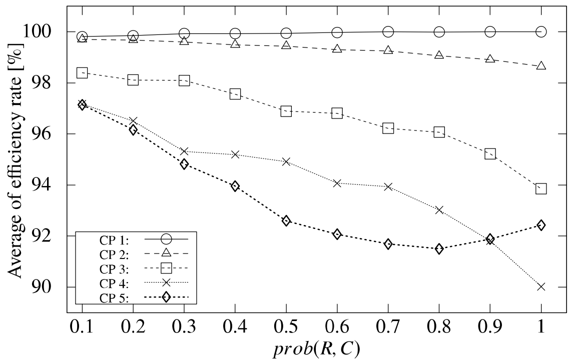


Fig. 3.10: Average of efficiency rate ($|S| = 5, |B| = 10$).

Analysis regarding efficiency rate

Our sequential solution method demonstrated high ER in the simulation. Fig. 3.10 shows the average of $ER(x_t, x)$ in the simulation with the model in which $|S| = 5$ and $|B| = 10$. In Fig. 3.10, the horizontal axis denotes $prob(R, C)$, and the vertical axis corresponds to the average of $ER(x_t, x)$ in the simulation. The average of $ER(x_t, x)$ was larger than 90 % for all $prob(R, C)$ and CP. This result shows our solution method can be used to determine efficient trades of a multi-unit commodity such as electricity.

For all CP, the average of ER became small as $prob(R, C)$ increased. The reason for this decline might relate to the number of possible trades on G. If $prob(R, C)$ is high, each trader has a larger number of possible trades on G. In this condition, however, demands of traders concentrate on participants providing inexpensive electricity. Hence, our method determines trades satisfying all capacity and demand, and this determined trades demonstrates relatively lower ER.

Regarding CP, high ER was obtained when the capacity of IPP and retailers is relatively lower than PU. Adversely, ER became worse when the difference of capacity of participants became smaller. Newly joining participants also have large capacity similar to PU with CP 5. Hence, many consumers could purchase cheaper electricity from newly joining participants in the condition.

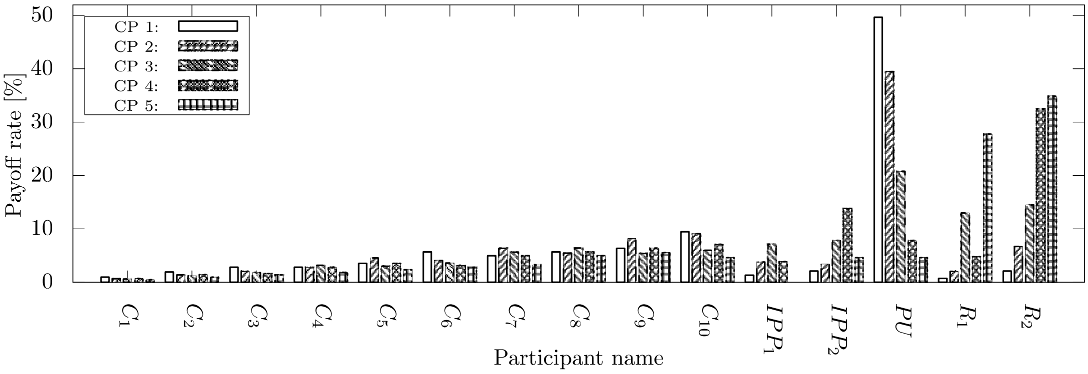
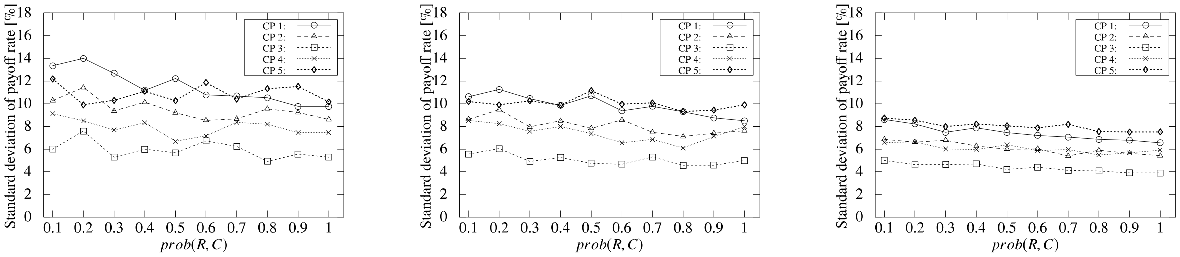


Fig. 3.11: Payoff rate of each participants with each CP.

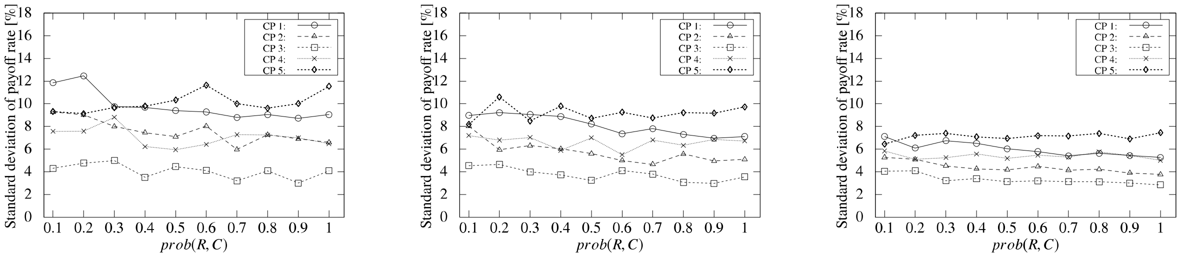
Analysis regarding payoff rate

The relation between payoff allocation and structure of market network were examined by investigating PR of market participants. First, Fig. 3.11 shows simulation result about PR of each market participant for each CP. In Fig. 3.11, the horizontal axis corresponds to the name of each participant, and the vertical axis indicates PR of each participant. The parameters of G in this simulation were set to $|S| = 5, |B| = 10$, and $prob(R, C) = 0.5$. Large part of PR was exclusively allocated to PU for CP 1. For CP 2, PR of the PU decreased, and PR of newly joining participants increased. Furthermore, for CP 3, PR was fairly allocated to market participants compared to other CP. However, large part of PR was adversely allocated to retailers for CP 4 and 5. Hence, if the valuation of market participants has been fixed to the same value, large part of PR is exclusively allocated to retailers obtaining enough capacity to supply electricity.



(a) $|B| = 10$. (b) $|B| = 15$. (c) $|B| = 30$.

Fig. 3.12: Standard deviation of payoff rate ($|S| = 5$).



(a) $|B| = 10$. (b) $|B| = 15$. (c) $|B| = 30$.

Fig. 3.13: Standard deviation of payoff rate ($|S| = 7$).

For examining the characteristics of PR with various structures of G, the standard deviation of PR was analyzed. The standard deviation of PR indicates whether payoff allocation is fair or not. For instance, large standard deviation means payoff is not allocated to participants evenly. Fig. 3.12 indicates the standard deviation of PR with $|S| = 5$. Each of Fig. 3.12 (a), (b), and (c) show the result with the different setting of $|B|$. The horizontal axis denotes $prob(R, C)$, and the vertical axis shows the average of the standard deviation of PR in the 100 iterations. In terms of CP, Fig. 3.12 indicates similar characteristics found in Fig. 3.11. The smallest standard deviation was obtained with CP 3, and the standard deviation became high with CP 1 and CP 5.

With regard to the axis denoting $prob(R, C)$, the number of buyers $|B|$ affected variation of the standard deviation. In Fig. 3.12 (a), the standard deviation varies widely as $prob(R, C)$ increases. The variation of the standard deviation in Fig. 3.12 (b) is smaller than that of Fig. 3.12 (a). Besides, the standard deviation of Fig. 3.12 (c) almost remains stable. Hence, the dispersion of the standard deviation of PR will become stable if the number of buyers increases.

Fig. 3.13 also demonstrated the characteristics shown in Fig. 3.12. Since $|S| = 7$ in Fig. 3.13, the number of sellers is larger than that of Fig. 3.12. The standard deviation in Fig. 3.13 is smaller than that in Fig. 3.12. These results indicate the variation of PR will decrease if there is a large number of sellers in the model.

3.7 Summary of this chapter

Regarding deregulated electricity markets, this chapter proposed a market model to describe the benefits of market participants and a sequential solution method to determine efficient electricity trades in the model. Social welfare and payoff allocation on the electricity market model were investigated by conducting simulation experiments. About the consumers' choice over suppliers in simulation, however, demonstrated the difference compared to general situations of electricity retail markets since simulation results indicated many consumers chose a supplier that is not the PU. Chapter 4 refines the electricity market model for the analysis of the consumers' choice of suppliers. Furthermore, Chapter 5 will integrate the concepts of the dynamical transition of production and consumption of electricity into the market model since the model explained in this chapter only represents a static situation of trading.

Chapter 4

Examining switching behavior of consumers in electricity retail

This chapter introduces a mathematical modeling technique regarding Problem 3. As a supplement for this chapter, Appendix A explains the proof about the transition of the strategy of consumers in algorithms presented in this chapter.

4.1 Introduction

4.1.1 Literature review

Many studies deal with analytical market models involving price competitions among suppliers with consumers' switching costs. Ruiz *et al.* [20] present a game theoretical model to investigate the effects of consumers' switching costs on competitions of suppliers. Ruiz's model does not represent dynamic interaction between market participants since the model has two suppliers and assumes two-stage games. Biglaiser *et al.* analyze benefits of suppliers in a model where consumers have heterogeneous switching costs [21]. Though Biglaiser's model related to a model for deregulated markets since Biglaiser's model assumes incumbents and entrants, this model also assumes only two-stage games. Rhodes proposes a dynamic model to examine the influence of switching costs on price competitions and discuss switching costs can be beneficial for forward-looking consumers [82]. Consumers in real electricity retail markets, however, are not assumed to be forward-looking because they do not tend to switch their seller while lower charges are available.

As studies considering switching costs of consumers focusing on electricity markets, Giuletti *et al.* propose a model to explain the difference in electricity prices observed in British electricity markets

[55]. However, [55] considers only searching cost and does not focus on other kinds of switching costs such as burden on consumers for switching their supplier. Ruiz *et al.* present a static model and a dynamical model to determine the equilibrium of electricity prices to enhance consumers' behavior to switch electricity retailer [54]. Though Ruiz's model considers switching behavior of consumers by using network structure, the model does not assume interactions between consumers.

Related studies described above mainly focus on interactions among suppliers such as price competition. Moreover, the related studies aim at promoting switching behavior of consumers by changing charges. Nevertheless, if consumers are not active in terms of switching sellers, suppliers do not seem to actively change their charges since the expected outcome by updating charges is obscure. For promoting the switching behavior of consumers, this chapter focuses on interactions between consumers under the situation where suppliers do not change their charges.

As studies dealing with the interaction between agents, *evolutionary game theory* has been used for promoting cooperative strategies among agents. Notably, network structure where agents placed is important aspects for investigating the transition of the strategies. For instance, *scale-free networks* have degree distribution following a power law, which can be observed in real networks such as the Internet, the network of acquaintance, and so on [83]. Durán *et al.* examine the evolution of cooperation in evolutionary prisoner's dilemma between agents on random graphs [84]. Lieberman *et al.* introduce evolutionary graph theory, which deals with evolutionary dynamics of agents on random graphs including scale-free networks [85]. Wu *et al.* explain that cooperation in a public goods game was promoted by giving rewards or punishments to agents [86]. These studies cannot be simply applied to express interaction among consumers in electricity markets since the characteristics of cooperators and defectors are not defined for electricity markets.

4.1.2 Contribution

This section lists contributions of proposals regarding each topic presented in Chapter 1 as follows. First, main contributions regarding Problem 3 are itemized below.

- To express the irrational behavior of consumers, this chapter focuses on two types of switching costs: (i) effort at switching suppliers and (ii) searching costs. When a consumer switches suppliers, an effort is needed to cancel the contract with the previous supplier, create a new contract with a new supplier, and so on. Moreover, costs to search for information on alternatives such as their charges are also required.

- The preference of consumers over suppliers involving an effort at switching is modeled as preference relation with *interval order* [87]. The preference relation describes a situation where consumers do not switch suppliers unless the benefit of switching is more significant than the effort. Electricity charges offered by suppliers are also modeled based on Japanese electricity retail markets to determine the preference relation of consumers.
- To describe interaction among consumers with searching costs, we propose an evolutionary game on a network among consumers. The game expresses the dynamics of the share of two strategies of consumers: searching alternatives actively (cooperators) and waiting for suggestions from the other consumers (defectors).
- For promoting cooperation, this chapter focuses on giving rewards to consumers as compensation for searching costs by referring to [86]. Although [86] shows that punishments are more effective than rewards to promote cooperation, this thesis does not deal with punishments since it's highly unlikely that suppliers punish consumers. Simulation results of the evolutionary game show the influence of rewards on cooperation and switching behavior.

4.2 Model representation

4.2.1 Bipartite graph denoting market participants

This section firstly defines consumer's rational preference over suppliers as a simpler model than irrational preference. The relationship between sellers and buyers in electricity market is modeled by bipartite graph $N\ (S \cup B, E)$. Seller $s_i \in S\ (|S| \geq 2, i = 1, 2, \ldots, |S|)$ denotes an electricity supplier, and buyer $b_j \in B\ (j = 1, 2, \ldots, |B|)$ is a consumer. An edge between s_i and b_j is denoted by $e(s_i, b_j)$. b_j can purchase electricity from s_i if $e(s_i, b_j)$ exists. N is a complete bipartite graph; therefore, b_j is connected to all seller $s_i \in S$. s_i can be divided into two types: the incumbent and entrants. The incumbent s_1 is a supplier that has provided electricity to buyers for a long time. Besides, the entrants $s_i \in S\ (i \neq 1)$ are suppliers newly entered into a market after the deregulation.

Our model represents the situation in which only one supplier s_1 provides electricity for all consumers at the beginning. This is considered to be one of the features of electricity retail markets. For a comparison, let us consider cellular phone services, which have similar characteristics compared with electricity retail since the services have monthly charges and several alternatives that consumers can choose. The statistics regarding the share of careers in Japan indicate that the maximum share was about 60% when cellular phones started to be used widely [88]. Thus, users of cellular phone services had multiple choices from the beginning, and it is different from the feature of electricity retail markets.

The combination of electricity trades on N can be denoted as *matching* M, which is a subset of E based on graph theory. b_j purchases electricity from only one seller s_i since b_j is assumed to avoid complicated contracts with multiple s_i. Conversely, s_i can supply electricity to multiple b_j and satisfy all demand of b_j. Hence, M can be called *many-to-one matching* that is similar to the model for a college admission problem [89]. Let $\delta_M(b_j) = \{s_i \mid s_i \in S, e(b_j, s_i) \in E\}$ be the neighbors of b_j on M. Besides, let $\delta_M(s_i) = \{b_j \mid b_j \in B, e(s_i, b_j) \in E\}$ denote neighbors of s_i on M. Thus, $\delta_M(s_i)$ and $\delta_M(b_j)$ must satisfy $0 \leq |\delta_M(s_i)| \leq |B|$ and $|\delta_M(b_j)| = 1$.

4.2.2 Preference relation

Buyer b_j must choose only one alternative of seller $s_i \in S$ to purchase electricity as a decision problem of b_j. This section defines rational preference relation that is simpler than irrational preference relation presented in Section 4.3. Let $X_j \subseteq S$ be a set of alternatives for b_j. X_j contains s_i if $e(s_i, b_j)$ exists; hence, X_j is denoted by $X_j = \{s_i \mid e(s_i, b_j) \in E\}$. Adversely, X_j does not contain s_i if $e(s_i, b_j) \notin E$. M is determined after all $b_j \in B$ obtain the solution for their decision problem. Let $a_j \in X_j$ be an alternative s_i chosen by b_j. M is represented by $M = \{e(a_j, b_j) \mid a_j \in X_j, b_j \in B\}$. The preference of b_j over every pair of alternatives $s_k, s_l \in X_j$ is expressed as preference relation explained in Chapter 2.

This chapter deals with two types of preference relation: strong preference relation and indifference relation. Strong preference relation P contains (s_k, s_l) if b_j prefers s_k to s_l. In other words, s_k is recognized as a preferred seller compared with s_l for b_j. $s_k P s_l$ denotes P contains (s_k, s_l). P satisfies irreflexivity and asymmetry. Indifference relation I contains (s_k, s_l) if b_j is indifferent to s_k and s_l. $s_k I s_l$ means $I \in (s_k, s_l)$. If $s_k I s_l$, both $s_k P s_l$ and $s_l P s_k$ do not hold. I is an equivalence relation and satisfies reflexivity and symmetry.

4.2.3 Utility function

Utility is a metric to represent the benefit of b_j for choosing an alternative. Utility of b_j to choose alternative s_i is calculated by a utility function $\mu_j : X_j \to \mathbb{R}$. Utility function and preference relation are related each other. Considering P and I over the same set X_j, preference relation is rational if

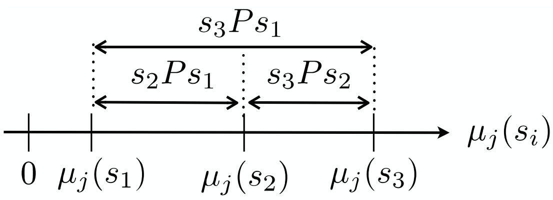


Fig. 4.1: Example of $\mu_j(s_i)$ for alternative s_1, s_2, and s_3.

$P \cup I$ meets completeness and transitivity. Furthermore, if $P \cup I$ satisfies transitivity, each of P and I is also transitive. When preference relation is rational, preference relation can be defined by using utility function μ_j as follows.

$$s_k P s_l \iff \mu_j(s_k) > \mu_j(s_l). \tag{4.1}$$

Fig. 4.1 depicts an example of the relationship between a utility function and preference relation. $\mu_j(s_i)$ gives different values over s_i, and P is set based on $\mu_j(s_i)$.

As described in Chapter 2, this thesis only considers electricity charge as the factor of utility function. Let $c_i \in \mathbb{R}$ be the charge offered by seller s_i. Moreover, let $v_j \in \mathbb{R}$ be *valuation* for purchasing electricity by b_j. v_j is considered to be the budget of b_j for purchasing electricity. $\mu_j(s_i)(s_i \in X_j)$ is defined as $\mu_j(s_i) = v_j - c_i$. We assumed b_j switches its present seller s_l to alternative s_i if $s_i P s_l$. If $s_i I s_l$, b_j does not change its seller from s_l to s_i.

4.2.4 Electricity charge

Electricity charge is one of the factors to determine preference relation over s_i by b_j. Though many types of service plans exist in Japan, this chapter considers the charges based on *Meter-rate lighting B*, which is a common service plan offered by a retailer that formerly supplied electricity as a PU in Japan. Let H denote the set of months; for instance, we focus on the months from September 2017 to August 2018 as $H = \{\text{Sep.2017}, \text{Oct.2017}, ..., \text{Aug.2018}\}$. Let $c_i^m : H \rightarrow \mathbb{R}$ be the monthly charge offered by s_i at month $h \in H$. $c_i^m(h)$ is mainly composed of two factors: basic charge β and electricity amount charge α. Hence, $c_i^m(h) = \beta + \alpha$. $\beta \in \mathbb{R}$ is a fixed charge determined by the amperes contracted by buyers. Besides, $\alpha \in \mathbb{R}$ varies based on electricity demand of each buyer. Let $d_j : H \rightarrow \mathbb{R}$ denote the demand of b_j at h. α have three tiered rate α_1, α_2, and $\alpha_3 \in \mathbb{R}$ that are applied according to $d_j(h)$

Table 4.1: β and α offered by sellers.

s_i	β	α_1	α_2	α_3
s_1	842.4	19.52	26	30.02
s_2	972	18.24	24.87	28.75
s_3	842.4	23.24	23.45	25.93
s_4	842.4	21.43	22.63	25.24
s_5	842.4	20.76	22.62	25.31
s_6	842.4	19.52	24.95	25.92
s_7	842.4	19.52	24.09	25.75
s_8	842.4	19.4	25.8	25.9
s_9	842.4	19.43	24.81	25.99
s_{10}	842.4	19.33	25.74	29.72
s_{11}	800.28	18.54	24.7	28.51
s_{12}	800.28	18.46	24.62	28.44
s_{13}	780	18.07	24.07	27.79
s_{14}	754.37	19.31	24.33	27.21
s_{15}	421.2	24.03	24.03	24.03
s_{16}	0	26	26	26

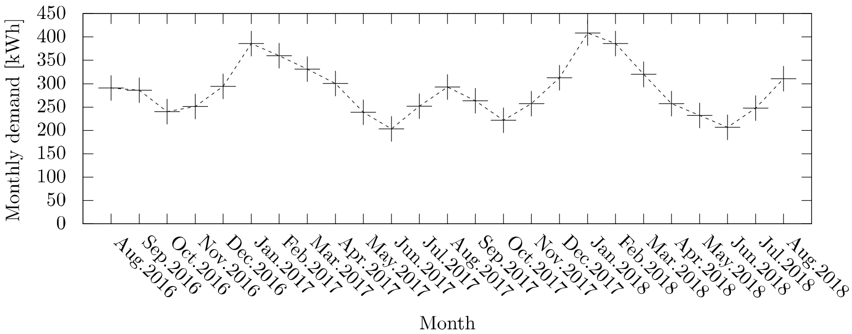


Fig. 4.2: Monthly electricity demand of consumers.

as follows.

$$\alpha = \begin{cases} \alpha_1 d_j(h) & (d_j(h) \leq 120), \\ 120\alpha_1 + \alpha_2 d_j(h) & (120 < d_j(h) \leq 300), \\ 120\alpha_1 + 180\alpha_2 + \alpha_3 d_j(h) & (300 < d_j(h)). \end{cases}$$

To consider a model of the charges, we picked up Japanese electricity retailers offering service plans that are similar to Meter-rate lighting B. Table 4.1 shows the examples of β and α. Contracted amperes are set to 30A to determine β since 30A is the most common contracted amperes in Japan. As shown in Table 4.1, each seller offers different patterns of β and α. Since demand in the future cannot be determined in advance, b_j calculates $c_i^m(h)$ by using $d_j(h)$ in the past. Fig. 4.2 shows the average electricity demand of buyers in Japan from a report [90]. Fig. 4.2 indicates $d_j(h)$ varies month to month, and $c_i^m(h)$ is also assumed to become different by the variation of $d_j(h)$.

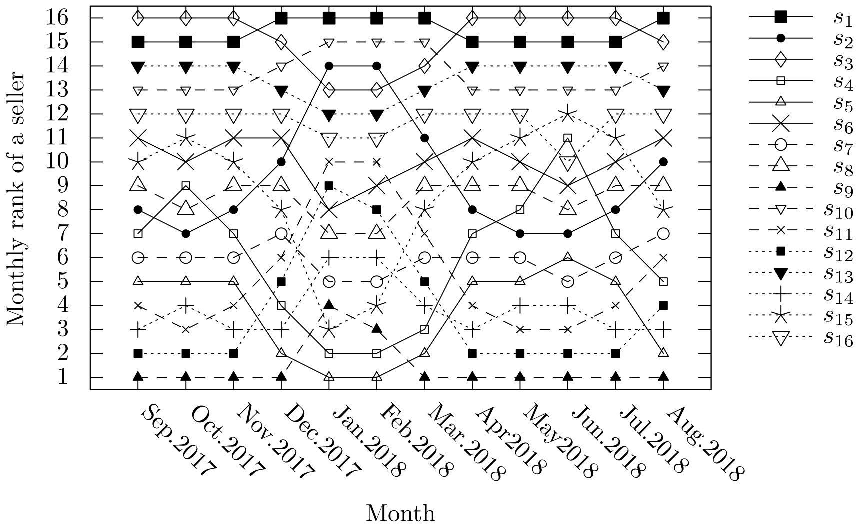


Fig. 4.3: Monthly ranking of sellers based on monthly demand.

4.2.5 Buyer's preference over sellers

Preference relation over s_i is determined by the magnitude of c_i as indicated by (4.1). To express typical examples of preference relation, Fig. 4.3 shows the ranking of monthly charge $c_i^m(h)$ $(h \in H)$ calculated by using $d_j(h)$ shown in Fig. 4.2. The highest rank is given to s_i offering the highest charge. The ranking widely differs since $d_j(h)$ also differs month to month. Hence, preference relation is difficult to decide based on the ranking of $c_i^m(h)$.

To determine preference relation, we focused on the pattern of transitions of electricity demand. Fig. 4.2 shows that the demand changing on an annual basis. The difference over months is assumed to decrease by considering the total charge through a year. Let $c_i^a : H \to \mathbb{R}$ be the annual total charge at month $k \in H$. Furthermore, let $H_a(k)$ represent twelve months by k. For instance, when k = Aug. 2018, H_a(Aug. 2018) is defined as H_a(Aug. 2018) $= \{h \mid$ h is a month from Sep. 2017 to Aug. 2018$\}$. $c_i^a(k)$ is defined by

$$c_i^a(k) = \sum_{h \in H_a(k)} c_i^m(h).$$

Fig. 4.4 shows the ranking of sellers based on the total of charges through a year. The horizontal axis shows month h, and the vertical axis indicates the ranking of sellers based on $c_i^a(k)$. The difference in ranking becomes not so large compared to Fig. 4.3. Let c_i denote a charge offered by s_i. We assume b_j uses the average of c_i^a(Sep.2017), c_i^a(Oct.2017), ..., and c_i^a(Aug.2018) as c_i to consider preference relation over s_i and $\mu_j(s_i)$.

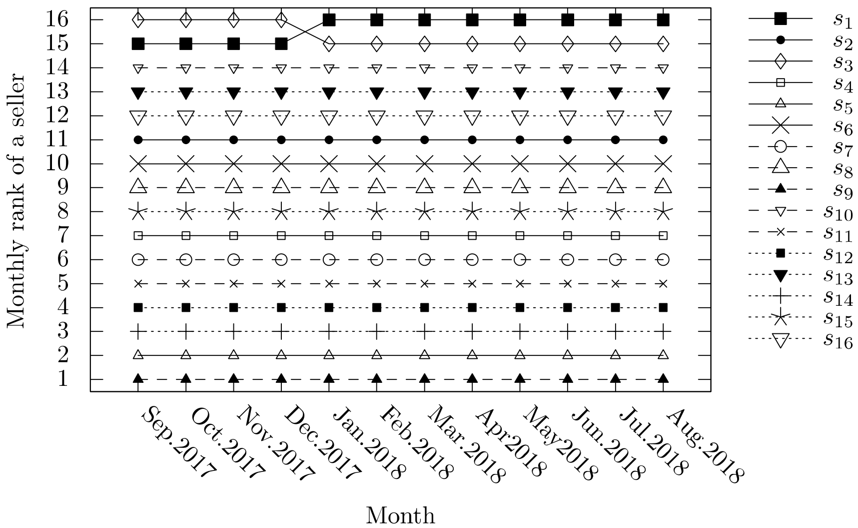


Fig. 4.4: Monthly ranking of sellers based on annual demand.

The feature of electricity charges described above is not common compared with other homogeneous products such as gasoline, which is also an energy product [91]. Although charges for gasoline are calculated based on demand, the calculation is not based on tiered rates in general.

4.3 Switching costs of buyers

4.3.1 Preference and utility considering switching costs

This section proposes a model to express irrational preference relation of consumers considering switching costs. Considering the buyer's effort at switching, b_j might not change its current seller s_l even if b_j has another alternative that offers a lower charge than s_l. In this situation, for an alternative $s_k \in X_j$, $s_k I s_l$ holds even if $c_k < c_l$ and $\mu_j(s_k) > \mu_j(s_l)$. Nevertheless, $s_k P s_l$ holds if the utility obtained by switching is assumed to be larger than the effort. Regarding preference relation denoted by (4.1), $s_k I s_l$ holds only if $c_k = c_l$ and $\mu_j(s_k) = \mu_j(s_l)$. Thus, (4.1) cannot represent preference relation considering the effort at switching.

To investigate the preference of buyers under switching costs, effort at switching should be examined into preference relation described by (4.1). This modification enables preference relation to describe the situation in which transitivity of an indifference relation does not hold. For instance, for $s_k, s_l, s_m \in X_j$, let b_j currently purchase electricity from s_k, and each seller offers different charges. If $s_k I s_l$, $s_l I s_m$, and $s_k P s_m$ are satisfied, transitivity of I does not hold since $s_k P s_m$ is satisfied instead of $s_k I s_m$.

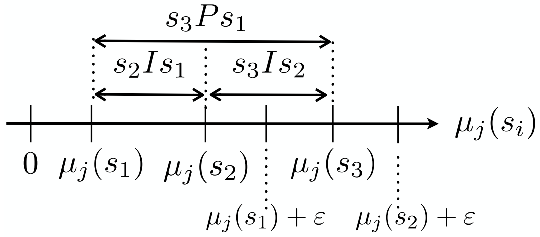


Fig. 4.5: Preference relation with $\mu_j(s_i)$ and ε.

One of the methods to represent an indifference relation with intransitivity is an interval order [87]. An interval order is an irreflexive binary relation and satisfies the following condition for $s_k, s_l, s_m, s_n \in X_j$.

$$s_k P s_l \text{ and } s_m P s_n \Rightarrow s_k P s_n \text{ or } s_m P s_l. \tag{4.2}$$

In an interval order, a threshold to switch from s_i by b_j is integrated into the relationship between a preference relation and utility function. Let function $\epsilon_j : X_j \Rightarrow \mathbb{R}$ denote the threshold. $\epsilon_j(s_i)$ indicates switching costs regarding effort at switching from s_i. In this chapter, $\epsilon_j(s_i)$ is assumed to satisfy $\epsilon_j(s_i) = \varepsilon$ ($\varepsilon \geq 0, \varepsilon \in \mathbb{R}$) for all $b_j \in B$ and $s_i \in X_j$. Thus, we use ε instead of $\epsilon_j(s_i)$ in the following discussion. The relationship between a preference relation and utility function in an interval order is denoted as

$$s_k P s_l \iff \mu_j(s_k) - \mu_j(s_l) > \varepsilon. \tag{4.3}$$

Let b_j currently purchase electricity from s_l. Based on (4.3), b_j switches its seller to s_k if the difference of utility between s_k and s_l is larger than ε. Adversely, b_j does not switch its seller if $s_k I s_l$ is satisfied since $\mu_j(s_k) - \mu_j(s_l) \leq \varepsilon$. Fig. 4.5 shows the relationship between a preference relation and utility function in an interval order. In Fig. 4.5, preference relation has intransitivity of an indifference relation since s_2Is_1, s_3Is_2, and s_3Ps_1 are satisfied. Thus, preference relation based on utility functions with the effort at switching can be defined by an interval order.

4.3.2 Cooperators and defectors among buyers

b_j is assumed to switch its seller if (4.3) is satisfied as described in Section 4.3.1. However, b_j cannot switch its seller to any alternative if b_j is not able to discover an alternative satisfying (4.3). Especially,

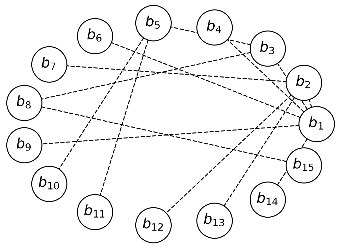


Fig. 4.6: Example of graph G ($|B| = 15, m = 1$).

b_j needs searching cost, which are costs to search for information on alternatives and charges. b_j might not search any sellers if searching costs are considered to be higher for b_j, and b_j cannot find s_i satisfying (4.3) contained in X_j. In this situation, b_j possibly expects the other buyers who had already switched their seller to share the information about sellers. We define two strategies among buyers as follows.

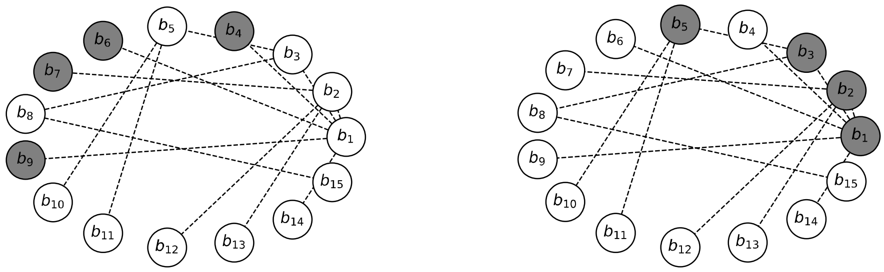
C**:** Exploring alternatives with searching cost

Strategy C (Cooperator) demonstrates cooperative activities. In addition to search for information about alternatives, b_j will suggest the alternative chosen by b_j to the other buyers. b_j with this strategy will search for alternatives and decide whether b_j should switch its seller or not based on condition (4.3). This chapter analyzes the effect of rewards given by entrants to buyers for enhancing cooperation even though the rewards are costs for entrants. Hence, cooperators who induced switching behavior of buyers can gain rewards.

D**:** Avoiding searching cost without exploration

This strategy is called D (Defector) since b_j might obtain beneficial information on alternatives as a free-rider. Though b_j with strategy D does not search for alternatives in person, b_j can obtain knowledge from other buyer b_k with strategy C. If an alternative suggested by b_k meets (4.3), b_j will switch its seller without searching cost.

The interactions are conducted on graph $G = (B, E')$ where each edge in E' represents a pair of neighbors of buyers that can share information each other. Thus, after sharing information on graph G, b_j finally determines its seller on graph N. Since each buyer is assumed to interact with a limited number of buyers, Barabasi-Albert (BA) model is used for G because BA model is utilized to represent social networks in many studies [92]. In an algorithm to create G based on BA model, every newly added vertex will be connected to the other m vertices already existed in G. BA model is a scale-free



(a) Ascending pattern. (b) Descending pattern.

Fig. 4.7: Example of the initial assignment of strategy.

network in which each vertex has a different degree. Fig. 4.6 shows an example of G with $|B| = 15$ and $m = 1$. Dotted lines in Fig. 4.6 denote edges in E'.

4.4 Evolutionary game among buyers

4.4.1 Preliminaries

This section explains an evolutionary game using graph N and G to investigate transitions of strategies of buyers occurring from time to time. Algorithm 2 shows the overall process of the game based on an agent-based approach. Due to searching costs, b_j chooses one of the two strategies C and D. Let $\sigma_j^t \in \{C, D\}$ be the strategy chosen by b_j at iteration t in a game. Based on σ_j^t, B can be divided into two subsets: $B_C^t = \{b_j \mid b_j \in B, \sigma_j^t = C\}$ and $B_D^t = \{b_j \mid b_j \in B, \sigma_j^t = D\}$. $\Sigma^t = \{\sigma_1^t, \sigma_2^t, ..., \sigma_{|B|}^t\}$ is a set of strategies of $b_j \in B$ at t.

The initial condition of Σ^t is denoted by Σ^0. Σ^0 is determined by the initial assignment of the strategy of buyers. Let the share of $b_j \in B_C^t$ over B at iteration t be denoted by $\theta_t = |B_C^t|/|B|$. θ_0 denotes the initial share of strategy C. θ_0 is initialized according to either of two patterns: ascending pattern and descending pattern. In ascending pattern, buyers are sorted by their degree with ascending order. Adversely, buyers are sorted by their degree in descending order with descending pattern. C will be set to the first θ_0% of buyers based on sorted order in each pattern. Fig. 4.7a and Fig. 4.7b depict Σ_0 in each pattern. Shadowed vertices denote $b_j \in B_C^t$, and the other vertices represent $b_j \in B_D^t$.

4.4.2 Process of a game

At each iteration t, b_j will decide one of the alternatives $a_j^t \in X_j$. $A^t = \{a_1^t, a_2^t, ..., a_{|B|}^t\}$ is a set of alternatives chosen by b_j at t. A^0 is the initial condition of and A^t. Let $T \in \mathbb{N}$ be the final iteration in a game. A^T is the set of sellers finally chosen by buyers, and M is determined by A^T.

$b_j \in B_C^t$ searches for the other alternatives by Algorithm 3. s_l is an alternative chosen by b_j. Let $X_j' \subset X_j$ denote a set of alternatives already searched by b_j. At iteration $t = 0$, $X_j' = \{s_l\}$. Since b_j is assumed not to have information about seller $s_i \notin X_j'$ in advance, a new alternative $s_k \notin X_j$ is randomly chosen and added to X_j' at line 3 and 8 of Algorithm 3. Then, b_j will compare $\mu_j(s_k)$ with $\mu_j(s_l)$. To search for alternatives, b_j needs a searching cost $\gamma_j^t \in \mathbb{R}$. All b_j have the same constant γ $(\gamma \geq 0, \gamma \in \mathbb{R})$ as γ_j^t. According to [93], searching activity for prices is assumed to continue if the benefit from searching is larger than searching cost. Hence, b_j is assumed to search for another alternative if condition (4.3) holds between s_k and s_l. Otherwise, b_j chooses an alternative that has the maximum utility in X_j'.

Besides, based on Algorithm 4, the interaction between each pair of buyers on $e(b_j, b_k) \in E'$ $(b_j, b_k \in B_C^t \cup B_D^t)$ will be conducted to share the information of alternatives with other neighbors. The neighbors of b_j on G is denoted by $\delta_G(b_j) = \{b_k \mid b_k \in B, e(b_j, b_k) \in E'\}$. Algorithm 4 determines the sets of suggestions made for b_j. The set of suggestions for b_j is denoted by $\Psi_j^t = \{a_k^t \mid a_k^t \in X_j, \mu_j(a_k^t) - \mu_j(a_j^t) > \varepsilon\}$. b_j determines whether b_j switches its seller from a_j^t to one of the sellers in Ψ_j^t or not. If b_j switching its seller according to the suggestion given by another buyer $b_k \in B_C^t$, reward r $(r \geq 0, r \in \mathbb{R})$ will be given to b_k who suggested the seller. Let $S_r \subseteq S \setminus \{s_1\}$ be the set of sellers who gives rewards. Let λ $(0 \leq \lambda \leq 1)$ be the rate of $s_i \in S_r$ over S. Hence, $|S_r| = \lambda\ |S \setminus \{s_1\}|$. $s_i \in S_r$ is randomly selected at $t = 0$ in a game. r_j^t indicates rewards obtained by b_j at iteration t. As a constraint, b_k can get at most r as a reward at each iteration. Therefore, r_j^t will become 0 or r.

At the end of t, b_j will update σ_j^t based on payoff, which represents the benefit obtained by b_j considering $\mu_j(a_j^t)$, γ_j^t, and r_j^t. b_j determines σ_j^t by comparing own payoff with payoff of its neighbors. $\rho : B \to \mathbb{R}$ is a function denoting payoff of b_j. Let b_m be the buyer who has the maximum payoff among $\delta_G(b_j) \cup b_j$. Besides, let $\rho_{\max}$ denote $\rho(b_m)$. Thus, b_j chooses its strategy σ_j^t considering $\rho_{\max}$.

Algorithm 2 Process of an evolutionary game

Input: $N, G, \varepsilon, \gamma, r, \Sigma^0, A^0, T, X_j = S, X'_j = \{s_1\}(b_j \in B)$
Output: Σ^T, A^T

$t = 1$
while $t \leq T$ **do**
  for each $b_j \in B$ **do**
    $\sigma_j^t \Leftarrow \sigma_j^{t-1}$
    $r_j^t \Leftarrow 0$
    $\Psi_j^t \Leftarrow \varnothing$
    $v_j \Leftarrow c_i$ of a_j^{t-1}
    if $\sigma_j^t = C$ **then**
      Determines alternative a_j^t by Algorithm 3 using X_j and X'_j.
      $\gamma_j^t \Leftarrow \gamma$
    else
      $a_j^t \Leftarrow a_j^{t-1}$
      $\gamma_j^t \Leftarrow 0$
    end if
  end for
  Determines Ψ_j^t for all b_j by Algorithm 4 for all $e(b_i, b_j) \in E'$.
  for each $b_j \in B$ **do**
    if $\Psi_j^t \neq \varnothing$ **then**
      $a_j^t \Leftarrow a_k^t$ offering the lowest c_i in all sellers in $\Psi_j^t \cup \{a_j^t\}$.
      if $r_k^t = 0$ **then**
        $r_k^t \Leftarrow r$.
      end if
    end if
  end for
  for each $b_j \in B$ **do**
    $\rho(b_j) \Leftarrow \mu_j(a_j^t) - \gamma_j^t + r_j^t$.
  end for
  for each $b_j \in B$ **do**
    if $\rho_{\max} < 0$ **then**
      if $\rho(b_j) < 0$ **then**
        $\sigma_j^t \Leftarrow D$
      end if
      $X'_j \Leftarrow X'_j \cup \Psi_j^t$
    else
      if $\rho_{\max} > \rho(b_j)$ **then**
        if $\sigma_m^{t-1} \neq \sigma_j^{t-1}$ **then**
          $\sigma_j^t \Leftarrow \sigma_m^{t-1}$
        end if
      end if
    end if
  end for
  $t \Leftarrow t + 1$
end while

Algorithm 3 Process for searching for alternatives by b_j

Input: X_j, X'_j

Output: $a^t_j = s_i \ (s_i \in X_j), X'_j$

```
s_l ⇐ s_j^{t-1}
if X'_j ≠ X_j then
    s_k ⇐ s_k ∈ X_j (s_k ∉ X'_j)
    if μ_j(s_k) − μ_j(s_l) > ε then
        s_l ⇐ s_k
        X'_j ⇐ X'_j + {s_k}
        while X'_j ≠ X_j do
            s_k ⇐ s_k ∈ X_j (s_k ∉ X'_j)
            if μ_j(s_k) − μ_j(s_l) > ε then
                s_l ⇐ s_k
                X'_j ⇐ X'_j + {s_k}
            else
                if μ_j(s_k) > μ_j(s_l) then
                    s_l ⇐ s_k
                    break
                else
                    break
                end if
            end if
        end while
    end if
end if
a_j^t ⇐ s_l
```

Algorithm 4 Interaction between buyers

Input: $e(b_i, b_j), \Psi_i^t, \Psi_j^t$
Output: Ψ_i^t, Ψ_j^t

if $\sigma_i^{t-1} = C$ **then**
    if $\sigma_j^{t-1} = C$ **then**
        if $\mu_j(a_i^t) - \mu_j(a_j^t) > \varepsilon$ **then**
            Add a_i^t to Ψ_j^t
        else if $\mu_i(a_j^t) - \mu_i(a_i^t) > \varepsilon$ **then**
            Add a_j^t to Ψ_i^t
        end if
    else if $\sigma_j^{t-1} = D$ **then**
        if $\mu_j(a_i^t) - \mu_j(a_j^t) > \varepsilon$ **then**
            Add a_i^t to Ψ_j^t
        end if
    end if
else if $\sigma_i^{t-1} = D$ **then**
    if $\sigma_j^{t-1} = C$ **then**
        if $\mu_i(a_j^t) - \mu_i(a_i^t) > \varepsilon$ **then**
            Add a_j^t to Ψ_i^t
        end if
    end if
end if

Table 4.2: The values used as parameters in simulation.

Parameter	Value
$\|B\|$	1,000
$\|S\|$	16
m	1
a_j^0	s_1
v_j at $t = 0$	c_1
ε	1000, 2000, 3000, 4000, 5000, or 6000
γ	ε, 1.5ε, or 2ε
r	γ
T	50

Table 4.3: Total monthly charge of each seller s_i in simulation.

s_i	c_i [JPY]	Rank	s_i	c_i [JPY]	Rank
$s1$	90,576	16	$s14$	87,262	8
$s8$	90,555	15	$s13$	86,352	7
$s16$	89,779	14	$s11$	86,292	6
$s10$	89,176	13	$s3$	86,040	5
$s6$	88,947	12	$s4$	85,768	4
$s2$	88,020	11	$s5$	85,668	3
$s9$	87,850	10	$s12$	85,384	2
$s7$	87,487	9	$s15$	83,852	1

4.5 Experimental results

4.5.1 Conditions

This section explains computational results about simulation experiment of the proposed evolutionary game. Table 4.2 shows values chosen for parameters in the simulation. Furthermore, Table 4.3 indicates the monthly charge c_i offered by sellers at every iteration in a game. As described in 4.2.5, c_i is the average of c_i^a(Sep.2017), c_i^a(Oct.2017), ..., and c_i^a(Aug.2018). We conducted 10,000 sets of the experiment for each set of parameters since the process of the game contains random factors such as the structure of G, the assignment of strategy to buyers, and so on. In the simulation, the following two metrics were investigated.

(I) The maximum share of buyers with strategy C:

This metric indicates how the share of cooperators changed from the initial share θ_0 through a game. Let Θ be the set of θ_t for all iteration t before σ_j^t finally becomes D for all $b_j \in B$. Hence, Θ is represented as $\Theta = \{\theta_t \mid t = 0, 1, 2, ..., T\}$. $\theta_{\max}$ denotes the metric that indicates the maximum share of $b_j \in B_C^t$ in B in a game. $\theta_{\max}$ is defined as $\theta_{\max} = \max \Theta$. If cooperation was not enhanced through overall iterations, $\theta_{\max} = \theta_0$.

(II) Switching rate:

Switching rate ϕ indicates the share of buyers who switched their sellers from the incumbent to entrants in M obtained at iteration T. ϕ will also converge as well as $\theta_{\max}$ since no switching behavior will be observed after iteration T.

To analyze the effect of rewards for buyers, fixed values are chosen as several parameters to focus on the different pattern of the other parameters. The simulation was conducted based on each of the following two types of conditions.

(A) $|S_r| = 0$:

This condition examines the cooperation and switching rate of buyers in the case that no sellers give reward to buyers since $|S_r| = 0$. The effort to switch ε and searching cost γ are assumed to affect these metrics. Since no sellers give reward to buyers, $|S_r| = 0$ and $\lambda = 0$. θ_0 is chosen from 0.1, 0.2, 0.3, 0.4, or 0.5.

(B) $|S_r| \geq 0$:

This condition investigates the situation in which sellers give reward to buyers for cooperation. In this condition, $\theta_0 = 0.1$ to simplify the results for focusing on the rewards. $|S_r|$ is set by using $\lambda = 0, 0.25, 0.5, 0.75$, or 1.

4.5.2 Results and discussion

(A) Examining behavior if $|S_r| = 0$

(I) Maximum share of C ($\theta_{\max}$):

Fig. 4.8a to Fig. 4.8f show $\theta_{\max}$ without rewards for buyers. Each figure shows different conditions based on ε and ascending/descending pattern. For descending pattern shown in Fig. 4.8a to Fig. 4.8c, $\theta_{\max}$ decreased as γ increased in almost every condition. If $\gamma = \varepsilon$, cooperation was well promoted compared with the situation in which $\gamma > \varepsilon$. The reason might be that $b_j \in B_C^t$ can obtain $\rho(b_j) \geq 0$ if $\gamma \leq \varepsilon$. Interestingly, Fig. 4.8a indicates cooperation under $\varepsilon > 5,000$ is promoted as much as $\varepsilon = 1,000$. Adversely, $b_j \in B_C^t$ cannot obtain positive payoff $\rho(b_j)$ though they conduct switching if $\gamma > \varepsilon$. Therefore, γ is considered to be an important factor to promote cooperation.

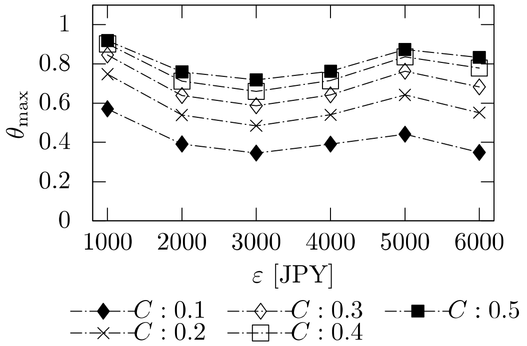
For ascending pattern indicated in Fig. 4.8d to Fig. 4.8f, $\theta_{\max}$ did not change even though ε and γ were different. Actually, $\theta_{\max}$ is the same value as θ_0. These results indicate that the number of neighbors of each buyer has a crucial role in promoting cooperation and switching behavior among buyers. Conversely, if each buyer does not have many neighbors, large $\theta_{\max}$ is important because cooperation among buyers is not expected to be enhanced.

In all sets of simulation with the condition $|S_r| = 0$, σ_j^t finally became D for all buyers $b_j \in B$. In other words, no buyers chose strategy C at iteration T.

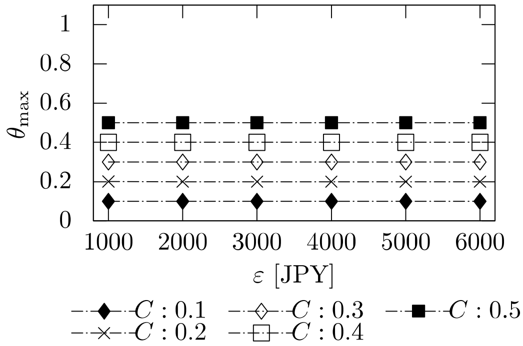
(II) Switching rate (ϕ):

The switching rate ϕ is shown in Fig. 4.9a to Fig. 4.9f. All figures indicate that the switching rate became smaller as ε increased. Especially, when $\gamma > \varepsilon$, ϕ became highly smaller with $\varepsilon \geq 5,000$. Considering cooperation, Fig. 4.8b and Fig. 4.8c shows the cooperation is not actively promoted if $\varepsilon \geq 5,000$. These results mean b_j experienced difficulty to discover any preferred seller s_i without cooperation if $\varepsilon \geq 5,000$. Thus, ϕ is also promoted by cooperation under descending pattern.

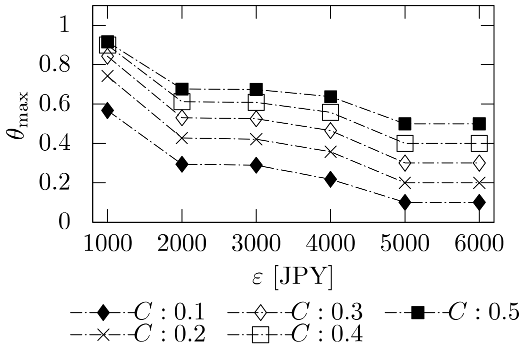
For ascending pattern depicted in Fig. 4.9d to Fig. 4.9f, ϕ was not larger than descending pattern and demonstrated almost the same value in every figure regardless of γ. On the other hand, the difference between ϕ under ascending pattern and descending pattern became smaller if $\varepsilon \geq 5,000$. This situation indicates that network structure does not affect ϕ when buyers have relatively large ε.



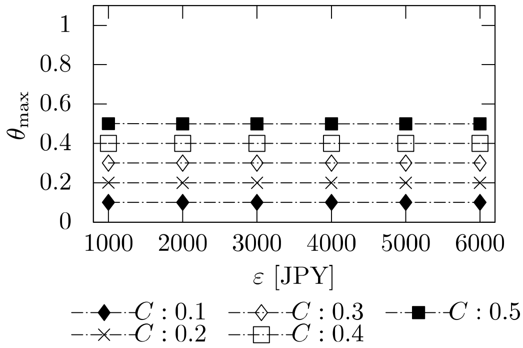
(a) $\gamma = \varepsilon$, descending pattern.



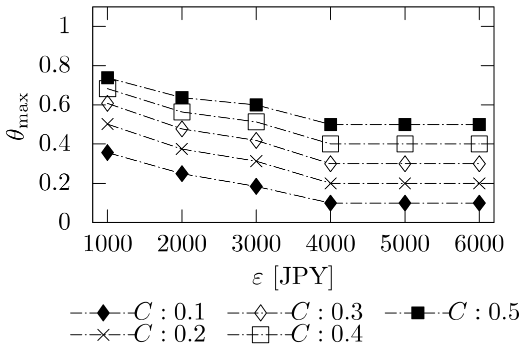
(d) $\gamma = \varepsilon$, ascending pattern.



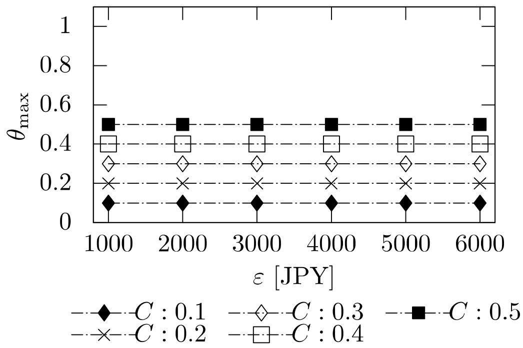
(b) $\gamma = 1.5\varepsilon$, descending pattern.



(e) $\gamma = 1.5\varepsilon$, ascending pattern.

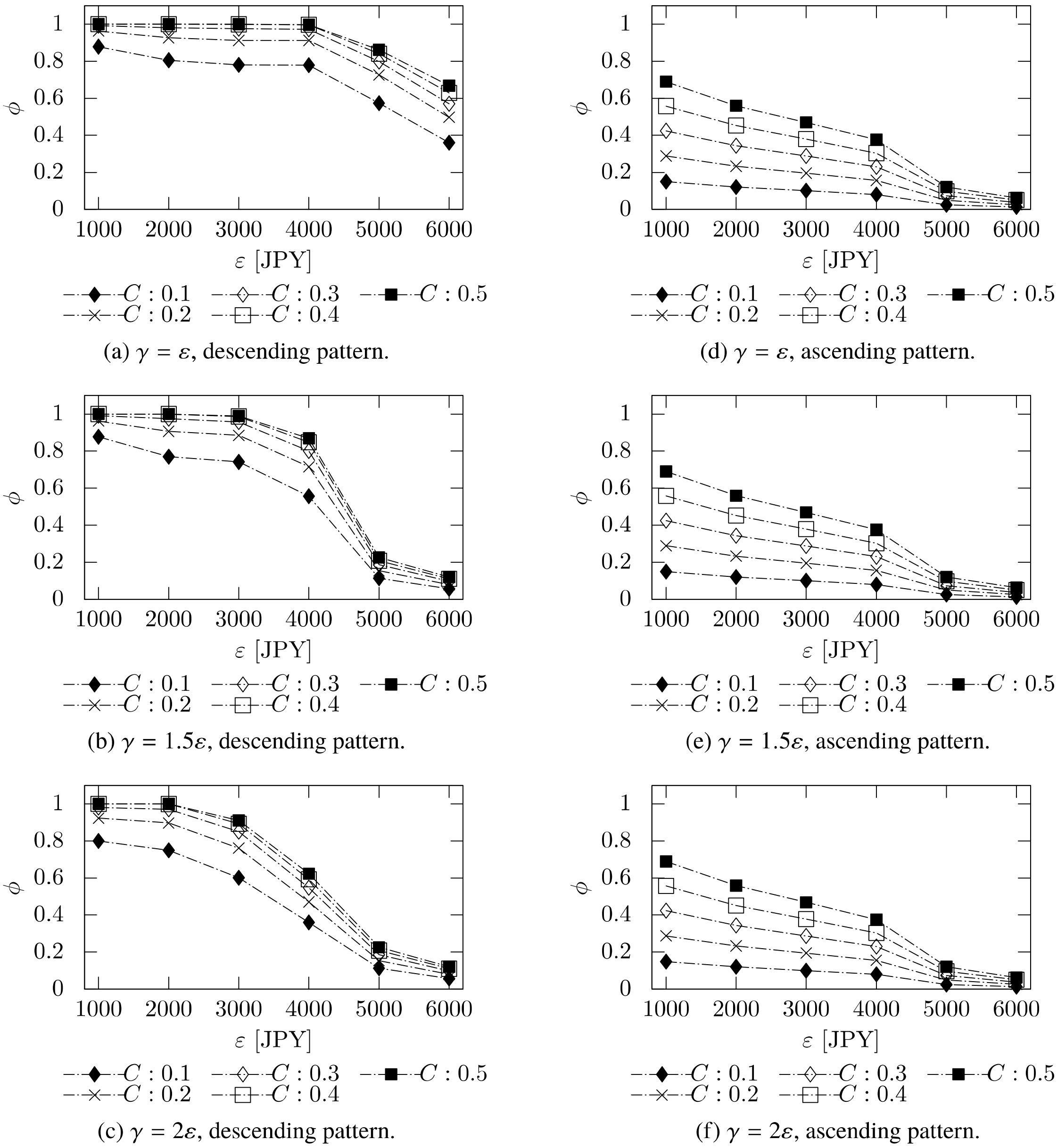


(c) $\gamma = 2\varepsilon$, descending pattern.



(f) $\gamma = 2\varepsilon$, ascending pattern.

Fig. 4.8: The difference of $\theta_{\max}$.



(a) $\gamma = \varepsilon$, descending pattern.

(d) $\gamma = \varepsilon$, ascending pattern.

(b) $\gamma = 1.5\varepsilon$, descending pattern.

(e) $\gamma = 1.5\varepsilon$, ascending pattern.

(c) $\gamma = 2\varepsilon$, descending pattern.

(f) $\gamma = 2\varepsilon$, ascending pattern.

Fig. 4.9: Switching rate of buyers.

(B) Examining behavior if $|S_r| \geq 0$

(I) Maximum share of C ($\theta_{\max}$):

Fig. 4.10a to Fig. 4.10f show $\theta_{\max}$ if $|S_r| \geq 0$. For descending patterns depicted in Fig. 4.10a to Fig. 4.10c, $\theta_{\max}$ basically increased as $|S_r|$ expanded except the following two situations. First, the effect of $|S_r|$ was not significant for buyers with $\varepsilon = 1,000$ since they actively switched sellers by their search, and rewards are not necessary for these buyers. Those buyers did not need the suggestion from other buyers, and the rewards did not affect $\theta_{\max}$. Besides, the effect of giving rewards was not apparent if $\varepsilon \geq 5,000$ especially with $\gamma > \varepsilon$. The reason is that buyers can rarely discover preferred sellers if $\varepsilon \geq 5,000$. Except for the two situations, $\theta_{\max}$ was improved by giving rewards to buyers.

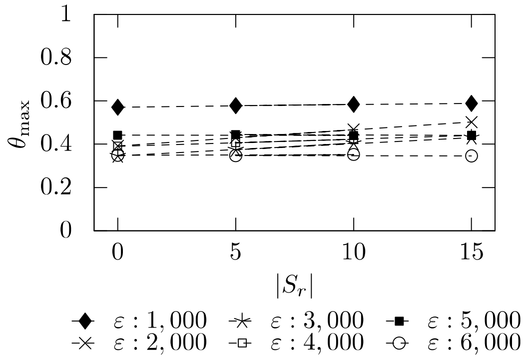
For ascending patterns depicted in Fig. 4.10d to Fig. 4.10f, $\theta_{\max}$ keeps almost the same value as θ_0. This result indicates that strategy C might not be propagated if buyers do not have many neighbors even if sellers give rewards.

σ_j^t also finally became D for all buyers $b_j \in B$ in all sets of simulation with $|S_r| \geq 0$. σ_j^t might eventually converge to D, and this perspective should be examined in the future.

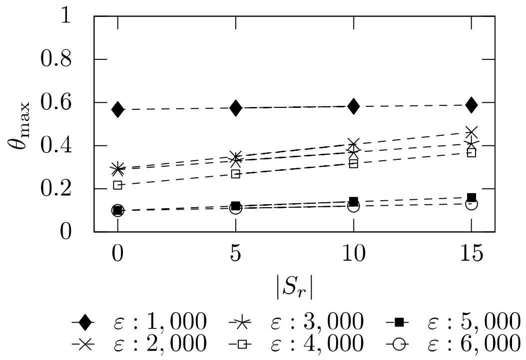
(II) Switching rate (ϕ):

In the results about descending pattern in Fig. 4.11a to Fig. 4.11c, ϕ basically increased according to $|S_r|$ as same as $\theta_{\max}$. If $\gamma > \varepsilon$, however, switching behavior was not much enhanced whereas $|S_r|$ increased. This result means that ϕ might not be improved with higher ε even if rewards offset γ. Since the rewards give little effects to ϕ with relatively large ε, it is more important to relieve ε of buyers.

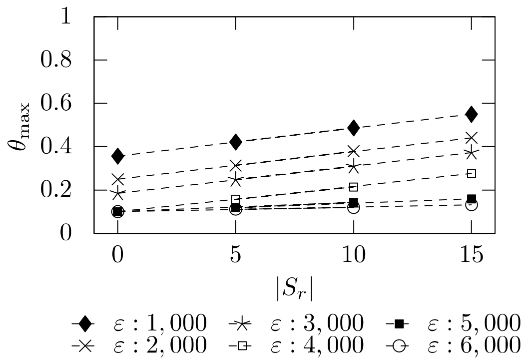
In Fig. 4.11d to Fig. 4.11f, the effect of rewards with ascending pattern cannot be observed. Since ascending pattern initially allocated strategy C to b_j with a relatively smaller degree, the effect of promoting switching behavior was also small. Though the rewards for $b_j \in B_C^t$ is considered to be a factor to increase ϕ, it is more important which buyers initially choose strategy C in a network among buyers. Furthermore, methods to increase connections among buyers is also beneficial to improve the switching rate.



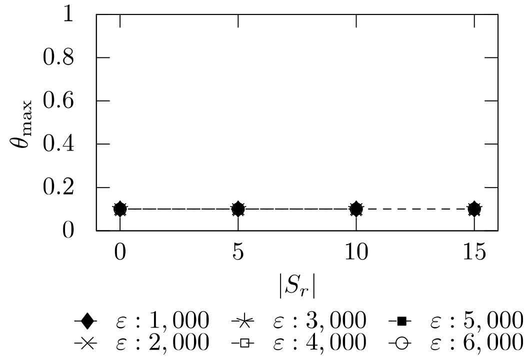
(a) $\gamma = \varepsilon$, descending pattern.



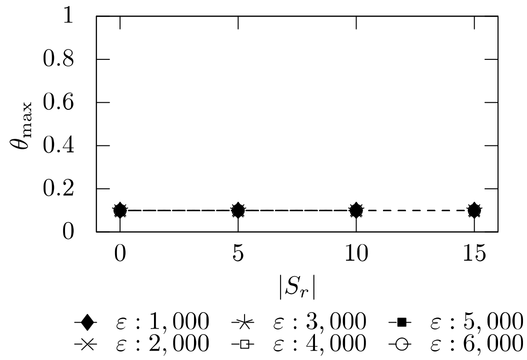
(b) $\gamma = 1.5\varepsilon$, descending pattern.



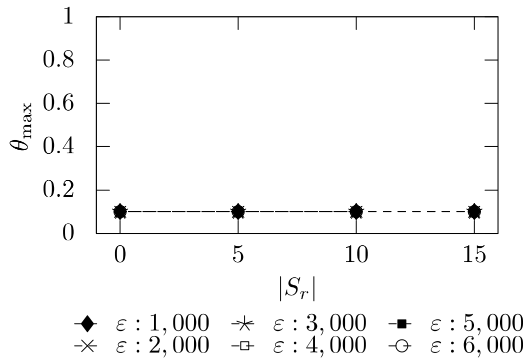
(c) $\gamma = 2\varepsilon$, descending pattern.



(d) $\gamma = \varepsilon$, ascending pattern.

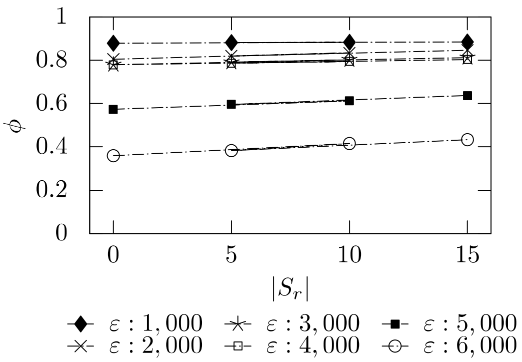


(e) $\gamma = 1.5\varepsilon$, ascending pattern.

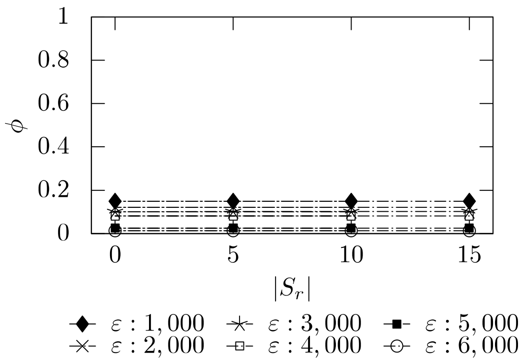


(f) $\gamma = 2\varepsilon$, ascending pattern.

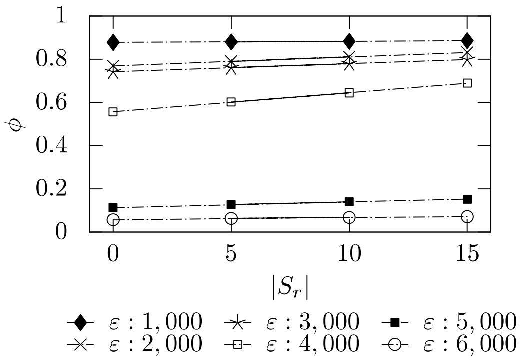
Fig. 4.10: The difference of $\theta_{\max}$ ($\theta_0 = 0.1$).



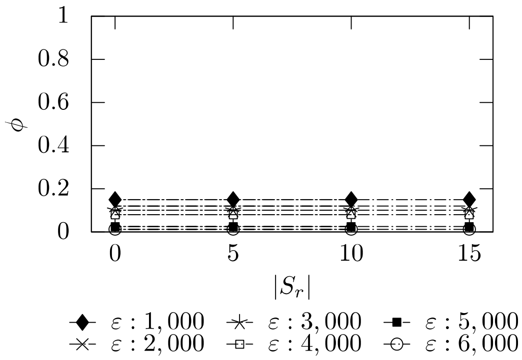
(a) $\gamma = \varepsilon$, descending pattern.



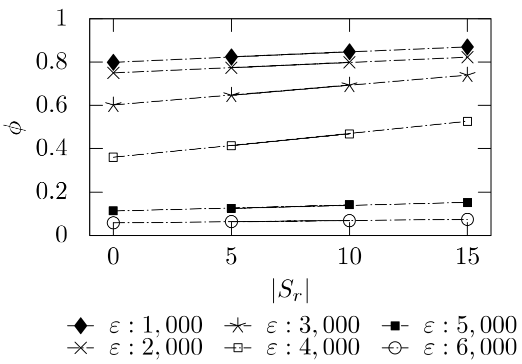
(d) $\gamma = \varepsilon$, ascending pattern.



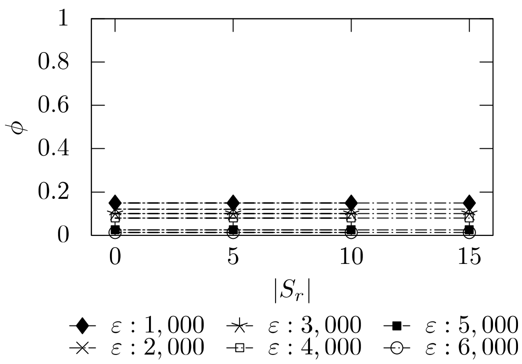
(b) $\gamma = 1.5\varepsilon$, descending pattern.



(e) $\gamma = 1.5\varepsilon$, ascending pattern.



(c) $\gamma = 2\varepsilon$, descending pattern.



(f) $\gamma = 2\varepsilon$, ascending pattern.

Fig. 4.11: Switching rate of buyers with sellers giving reward ($\theta_0 = 0.1$).

4.6 Summary of this chapter

This chapter proposed an evolutionary game on a graph to analyze the switching behavior of consumers. Preference relation of consumers involving switching costs was defined and expressed situations where consumers do not tend to switch their suppliers though lower charges are available. We examined the conditions to promote cooperation to search for alternatives and switching suppliers in the computational experiment of the evolutionary game. The results demonstrated that the share of cooperators and the switching rate were not improved by simply giving rewards for cooperators. These results suggest that the degree of cooperators in a network among consumers demonstrates a vital role to increase the share of cooperators and the switching rate.

Chapter 5

Describing fairness among prosumers in electricity sharing

This chapter presents a mathematical modeling technique for Problem 4.

5.1 Introduction

5.1.1 Literature review

Interaction between agents can be modeled as *matching* in graph theory. The concept of matching is firstly introduced by Gale and Shapley in [46]. In a college admission problem presented in [46], a matching model can be represented by a bipartite graph consists of two types of nodes: colleges and students. As one of the applications of matching, Easley and Kleinberg proposed matching market, which determines matching between sellers and buyers as partners to trade a single indivisible item [44]. A study in [17] proposes a matching problem to describe electricity retail markets to represent trading for electricity as multi-unit items. Above studies on matching, however, only deal with static aspects of markets and cannot describe interactions of trading varying with time.

A resource sharing model based on a multi-agent system is called El Farol Bar Problem (EFBP) [94]. In the EFBP, the capacity in a bar is considered as a resource shared among agents. Each agent decides whether the agent go to a bar or not every week. To avoid congestion and enjoy the bar, agents must achieve cooperative behavior without pursuing their selfish goal. A potluck problem is proposed as the generalized version of the EFBP [95]. In the potluck problem, each agent has time-varying supply and demand for dinner in every week. The goal of the potluck problem is to minimize the difference between the total demand and total supply among all agents. The potluck problem only focuses on minimizing the difference in total supply and demand as overall efficiency and does not consider envy of each agent.

As a metric for envy of each agent, *envy-freeness* is considered in many resource allocation problems. If the allocation is envy-free, every agent is satisfied with its allocation and does not envy the allocation for other agents [96]. Problems about envy-freeness is classified into two types: problems for indivisible goods and problems for divisible goods. As an example of studies for indivisible goods, Netzer *et al.* propose a distributed algorithm for minimizing envy among agents [97]. As a dynamic version of envy-free allocation problems, online cake-cutting problem is discussed in [98]. Though dynamic aspects of resource allocation problems are considered in [98], the problems deal with only fixed amounts of resources over time. This chapter focuses on divisible goods such as electricity.

Resource sharing among prosumers might be conducted with limited neighbors that are connected to each other. Hence, resources and envy can be partially observable from a prosumer's viewpoint. The partial observation is considered in a resource allocation problem proposed in [99]. Even though this problem deals with dynamic settings, it considers indivisible resource allocation. Besides, resources shared among agents are not changed dynamically in [99] though each agent cannot observe all resources. Beynier also introduces methods for envy-free allocation in social networks, which can describe a more realistic situation about partial observation of agents [100]. However, methods proposed in [100] are tailored for the allocation for indivisible resources.

To represent conditions of prosumers varying with time, our model for resource sharing among prosumers [101] utilizes Time-Varying Graph (TVG) [32]. TVG is also called temporal network. On the temporal networks, community structures change with time [31]. For example, [31] explains several events about communities such as growth, contraction, merge, split, and so on. Hence, it is required to consider the time-varying structural change about networks of prosumers for resource sharing.

Rossetti *et al.* proposed *stream graphs*, which is an extension of several concepts of graph theory to analyze the temporal characteristics of networks. Especially, [102] defines bipartite stream, which is an extension of bipartite graphs to temporal networks. Although the bipartite stream can model the appearance/disappearance of nodes and edges on a bipartite graph, every node belongs to the same group in the bipartite graphs through a time span. Hence, the bipartite streams cannot represent periodical changes in the role of nodes in bipartite networks.

5.1.2 Contribution

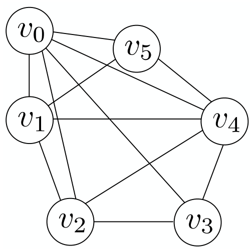
This chapter proposes a model for analyzing envy in resource sharing among prosumers in a temporal network. Main contributions of this chapter are itemized below.

- This chapter introduces a resource sharing model among prosumers in a social network. The model deals with time-varying supply and demand of prosumers based on the model proposed in our previous study [101]. As an extension of the model in [101], the relationship among prosumers for resource sharing is represented as social network. Thus, prosumers share their resources only with their neighbors in a model of social network. As models of social network, this chapter utilizes Barabasi-Albert (BA) model [92] and Watts-Strogatz (WS) model [103].

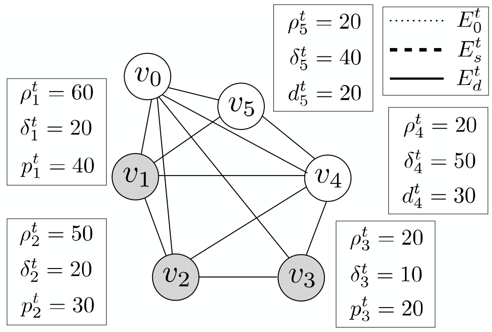
- The extended concepts of envy are introduced since resource sharing is conducted on a social network in the proposed model. The envy in resource sharing discussed in related works is defined in the situation where all agents share the same resources each other over time. In our assumption of resource sharing in a social network, however, the amount of resources to be shared is different in each prosumer and varies with time. Therefore, this chapter defines indices to evaluate envy-freeness among prosumers that focuses on a resource allocation problem in a social network.

- We formulate a minimum cost circulation problem to discover matching that maximizes utilization of resources of prosumers and satisfies the constraints of supply and demand of prosumers. Furthermore, to reduce envy among prosumers in the circulation problem, this chapter discusses the adjustment of weight assigned to arcs (directed edges) in a flow network for the circulation problem. Experimental results indicate that appropriate weight settings on arcs can decrease the envy among prosumers over time while maximizing the utilization of resources of prosumers.

This chapter is structured as follows. Section 5.2 explains fundamental definitions of our resource sharing model used in the following sections. The concept of envy of prosumers in a social network is also introduced in Section 5.2. Section 5.3 formulates a minimum cost circulation problem to discover matching in the resource sharing model. Section 5.4 demonstrates experimental results about the envy among prosumers in the matching. Section 6.1 concludes this chapter and explains future works.



(a) Example of N.



(b) Example of $\mathcal{G}$ at t.

Fig. 5.1: Example of graph N and $\mathcal{G}$.

5.2 Definitions on temporal network

5.2.1 Model representation

Agents share their resources on a social network represented by undirected graph N $(V \cup \{v_0\}, E)$. V is a set of node $v_i \in V$ $(i = 1, 2, ..., |V|)$, and each node v_i denotes an agent of a prosumer. v_0 is a node to cover imbalance of resource sharing among agent $v_i \in V$. v_0 does not denote a prosumer. E is a set of undirected edge (v_i, v_j) $(v_i, v_j \in V)$. Since N is undirected graph, both (v_i, v_j) and (v_j, v_i) are the same edge. Edge (v_i, v_j) represents the relationship where v_i is a neighbor of v_j. Agents are willing to share resources only with their neighbors. Adversely, v_i and v_j do not share their resources if (v_i, v_j) does not exist. E also contain edges (v_0, v_i) for any $v_i \in V$. Thus, arbitrary v_i share resources with v_0. Fig. 5.1a shows an example of N.

In resource sharing, the amounts of available resources for each agent will vary with time. To represent the variation of resources, we utilize a temporal network constructed based on N. Temporal network $\mathcal{G}$ $(V \cup \{v_0\}, E^t, \mathcal{T})$ is an undirected graph that has a set of agents V, which is the same as the agents in N. $\mathcal{T}$ is a time span for $\mathcal{G}$, and $t \in \mathcal{T}(t = 1, 2, 3..., T)$ is a round to describe each state of $\mathcal{G}$. E^t is a set of edges at t in $\mathcal{G}$. E^t does not contain (v_i, v_j) if $(v_i, v_j) \notin E$. The set of agents V does not change through $\mathcal{T}$.

5.2.2 Roles of agents based on their production and demand

Every agent $v_i \in V$ has its own demand and production of resources at t. The demand of resources of v_i is δ_i^t, and the production of resources of v_i is ρ_i^t. At each round t, v_i uses ρ_i^t to cover its δ_i^t. Based on the condition about δ_i^t and ρ_i^t, v_i will be assigned to one of the following three roles.

1. $\rho_i^t > \delta_i^t$

   Since surplus is more than deficit, v_i has surplus resource indicated by $p_i^t = \rho_i^t - \delta_i^t$. v_i will share the surplus as *seller* to the other agents. The set of sellers is $V_S^t \subseteq V$.

2. $\delta_i^t > \rho_i^t$

   v_i has the deficit of resource denoted by $d_i^t = \delta_i^t - \rho_i^t$ since its deficit is more than its surplus. v_i will try to cover its deficit as *buyer* by getting surplus from other agents. The set of buyers is $V_B^t \subseteq V$.

3. $\delta_i^t = \rho_i^t$

   v_i can cover its own demand δ_i^t by using its own production ρ_i^t at t. Regarding resource sharing, v_i has no surplus and deficit for sharing at t. Hence, v_i does not join the resource sharing at t, and role of v_i is called *none* at t.

Fig. 5.1b shows an example of production and demand of agents $v_i \in V$ of $\mathcal{G}$ at t. Regarding v_1, production $\rho_1^t = 60$, and demand $\delta_1^t = 20$. Thus, v_1 is a seller at t with its surplus $p_1^t = 40$. About v_4, production $\rho_4^t = 20$, and demand $\delta_4^t = 50$. Hence, v_4 becomes a buyer at t with its deficit $d_4^t = 30$.

As a constraint for resource sharing in our model, all surplus p_i^t and deficit d_i^t must be shared at t. However, the imbalance of surplus and deficit of resources will occur since every agent v_i individually determines demand δ_i^t and production ρ_i^t without considering the balance among other agents. Hence, surplus p_i^t and deficit d_i^t are not necessarily offset by resource sharing only among agents.

To offset the imbalance of the surplus and deficit, v_0 has both surplus and deficit to offset the all resources of v_i. Let d_0^t be the deficit of v_0 that is sufficient for all surplus of $v_i \in V_S^t$. P^t represents the total surplus of all $v_i \in V_S^t$. Thus, d_0^t is defined as $d_0^t = P^t$. P^t is denoted by

$$P^t = \sum_{v_i \in V_S^t} p_i^t. \tag{5.1}$$

Regarding surplus of v_0, let p_0^t be the surplus of v_0 that is enough for all deficit of $v_i \in V_B^t$. D^t denotes the total of deficit of all $v_i \in V_B^t$. Therefore, p_0^t is defined as $p_0^t = D^t$. D^t is represented as

$$D^t = \sum_{v_i \in V_B^t} d_i^t. \tag{5.2}$$

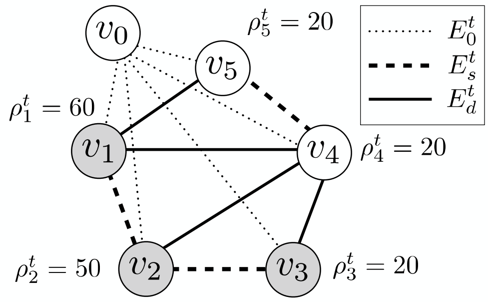


Fig. 5.2: Example of edges in $\mathcal{G}$ at t.

5.2.3 Neighbors of each agent according to agent's role

Agent $v_i \in V$ shares its resources with its neighbors. To explain the relationship to describe resource sharing with neighbors, we divide E^t into three edge sets: E_d^t, E_s^t, and E_0^t ($E^t = E_d^t \cup E_s^t \cup E_0^t$). Fig. 5.2 shows an example of these edge sets. The definitions of the edge sets are as follows.

- E_d^t contains edges with two agents which belong to different sets. Thus, $(v_i, v_j) \in E_d^t$ ($v_i \in V_S^t, v_j \in V_B^t$). Edges in E_d^t describe the opportunity for resource sharing between v_i and v_j.

- E_s^t contains edges between agents which belong to the same set. Hence, $(v_i, v_j) \in E_s^t$ ($v_i, v_j \in V_S^t$ or $v_i, v_j \in V_B^t$). No resource is shared on edges in E_s^t that are used to describe envy among agents. The definition of envy is explained in Section 5.2.6.

- E_0^t is a set of edges between $v_i \in V$ and v_0. Therefore, $(v_i, v_0) \in E_0^t$ ($v_i \in V_S^t \cup V_B^t$). Edges in E_0^t can be used to cover resources that are not offset by resource sharing only among v_i.

The set of neighbors of $v_i \in V$ over an edge set E^t is denoted by the following function.

$$adj(v_i, E^t) = \{v_j \in V \mid (v_i, v_j) \in E^t\}. \tag{5.3}$$

Similarly, the set of neighbors of $v_i \in V$ over an edge set E_d^t, E_s^t, and E_0^t are represented by $adj(v_i, E_d^t)$, $adj(v_i, E_s^t)$, and $adj(v_i, E_0^t)$, respectively. For example, regarding v_1 in Fig. 5.2, $adj(v_1, E_d^t) = \{v_4, v_5\}$ and $adj(v_1, E_s^t) = \{v_2\}$.

5.2.4 Utility function

Benefit of $v_i \in V$ for resource sharing is represented by utility function $\mu^t : V \rightarrow \mathbb{R}$. This chapter focuses on benefit of agents $v_i \in V$, and benefit of v_0 is not considered. In $\mathcal{G}$, v_0 might need more costs to offset the difference than agents v_i since v_0 must prepare for p_0^t and d_0^t according to surplus and deficit of v_i every round. Hence, it is assumed that resources of v_0 are more expensive than that of other agents. Because of this condition, v_i prefers resource sharing with $v_j \in V$ to resource sharing with v_0.

Utility function μ^t is defined by benefit from resources and the amounts of resources in sharing at round t. Let ϕ and ϕ_0 be the benefit for sharing one unit of resource with $v_j \in V$ and v_0, respectively. Since resources of v_0 is more expensive than v_j, ϕ and ϕ_0 are considered to satisfy at least $\phi > \phi_0$. Let y^t be the total amounts of resources obtained from all neighbors $adj(v_i, E_d^t)$ at round t. For instance, in Fig. 5.2, y^t of v_1 represents the amounts of resources shared with v_4 and v_5. Moreover, y_0^t represents the amounts of resources obtained from v_0. Thus, the utility of $v_i \in V_S^t \cup V_B^t$ at round t is calculated by

$$\mu^t(v_i) = \phi \cdot y^t + \phi_0 \cdot y_0^t. \tag{5.4}$$

For simplicity, we assume $\phi = 1$ and $\phi_0 = 0$. Therefore, equation (5.4) can be denoted as

$$\mu^t(v_i) = y^t. \tag{5.5}$$

Since all surplus must be shared at t, for sellers $v_i \in V_S^t$, $y^t + y_0^t = p_i^t$. Likewise, for buyers $v_i \in V_B^t$, $y^t + y_0^t = d_i^t$ because all deficit d_i^t must be shared.

To increase utility of agents, resource sharing among agents $v_i \in V$ should be conducted as much as possible since $\phi > \phi_0$. In other words, resources of v_0 should be utilized only if the amounts of resources of v_i are not sufficient to offset all surplus and deficit. The conditions for resource allocation described above can be discovered by maximizing the total of y^t over all agents. Methods to discover the resource allocation for the maximization will be presented in Section 5.3. *Available resources* of v_i means the amounts of resources of neighbors $v_j \in adj(v_i, E_d^t)$.

5.2.5 Utility rate

The size of utility is quite different in each agent since each agent has a different amount of available resources (surplus or deficit) of neighbors. Hence, utility $\mu^t(v_i)$ cannot be directly compared with other agents. To compare the utility over agents, it is required to define a criterion to describe a normalized measure of utility over agents. This chapter defines *utility rate* $\omega_i^t \in [0, 1]$ that indicates how much amount of agent's resource is shared among agents $v_i \in V$. Definitions to determine ω_i^t are presented later in this section.

The available resources for v_i must be defined to determine ω_i^t. Seller $v_i \in V_S^t$ share its resources with buyers $v_j \in V_B^t$. Buyer $v_i \in V_B^t$ share its resources with sellers $v_j \in V_S^t$. Thus, the available resources of v_i is defined using the sum of resources of neighbors $v_j \in adj(v_i, E_d^t)$. For instance, in Fig. 5.1a, seller v_1 recognizes the deficit of neighbors v_5 and v_4 as resources. For $v_i \in V_S^t$, the total of deficit d_j^t of neighbors $v_j \in adj(v_i, E_d^t)$ is denoted by

$$D_i^t = \sum_{v_j \in adj(v_i, E_d^t)} d_j^t. \tag{5.6}$$

Besides, as available resource for $v_i \in V_B^t$, the total surplus p_j^t of neighbors $v_j \in adj(v_i, E_d^t)$ is represented by

$$P_i^t = \sum_{v_j \in adj(v_i, E_d^t)} p_j^t. \tag{5.7}$$

Let α_i^t represent the maximum amounts of available resources for v_i at round t. α_i^t is determined as follows.

- For seller $v_i \in V_S^t$:
  α_i^t is obtained by either D_i^t or p_i^t. If $D_i^t > p_i^t$, $\alpha_i^t = p_i^t$ since v_i can share resources up to its surplus p_i^t. This situation means v_i can utilize only a part of D_i^t. Adversely, if $p_i^t > D_i^t$, $\alpha_i^t = D_i^t$ since v_i can only share resources up to D_i^t. Thus, surplus p_i^t can be covered up to only D_i^t if $p_i^t > D_i^t$. Besides, surplus p_i^t can be offset by D_i^t if $D_i^t = p_i^t$. According to the conditions described above, α_i^t can be defined as the minimum value of D_i^t or p_i^t.

- For buyer $v_i \in V_B^t$:
  α_i^t is determined by either P_i^t or d_i^t, and conditions to obtain α_i^t is similar to the conditions of sellers described above. If $P_i^t > d_i^t$, $\alpha_i^t = d_i^t$ since v_i can share resources up to its deficit d_i^t. Conversely,

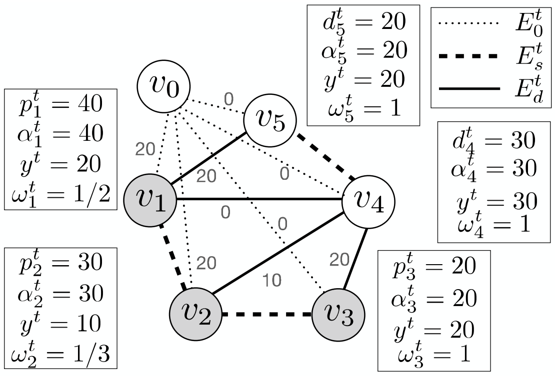


Fig. 5.3: Example of utility rate.

if $d_i^t > P_i^t$, $\alpha_i^t = P_i^t$ since v_i can only share resources up to P_i^t. If $P_i^t = d_i^t$, deficit d_i^t can be offset by P_i^t. Hence, α_i^t can be defined as the minimum value of P_i^t or d_i^t.

Therefore, α_i^t is calculated as follows.

$$\alpha_i^t = \begin{cases} \min\left(D_i^t, p_i^t\right) & (v_i \in V_S^t), \\ \min\left(P_i^t, d_i^t\right) & (v_i \in V_B^t). \end{cases} \tag{5.8}$$

All resources of α_i^t are not necessarily allocated to v_i because some resources in α_i^t might also be shared with the other agents and allocated to them. A part of α_i^t allocated to v_i can be denoted by $y^t = \mu^t(v_i)$ as equation (5.5). Thus, utility rate ω_i^t is obtained as follows.

$$\omega_i^t = \mu^t(v_i)/\alpha_i^t. \tag{5.9}$$

When $\alpha_i^t = 0$, v_i has no resource to share, and ω_i^t cannot be defined. Besides, α_i^t and ω_i^t cannot be defined if $|adj(v_i, E_d^t)| = 0$. If one of the conditions described above holds, ω_i^t is not defined at t. Hence, envy of v_i explained in Section 5.2.6 is not defined at t with the above conditions, either.

Fig. 5.3 shows an example of the amounts of available resources α_i^t and utility rate ω_i^t. The amounts of resources shared on edges are denoted on each edge. Regarding v_1, its surplus is represented by $p_1^t = 40$. Moreover, the total of available deficit of neighbors are obtained by $D_1^t = d_4^t + d_5^t = 50$. Thus, $\alpha_1^t = \min(D_1^t, p_1^t) = 40$. In this example, v_1 obtained 20 units of resources from v_5. Hence, $y^t = 20$, and $\mu^t(v_1) = 20$. Therefore, utility rate $\omega_1^t = 20/40 = 1/2$.

5.2.6 Envy among agents

At every round t, v_i possibly has envy to each of its neighbors about allocated resources. Envy exists if v_i and its neighbor v_j are in the same set V_S^t or V_B^t. If $v_i, v_j \in V_S^t$, both v_i and v_j are willing to share their surplus to $v_k \in V_B^t$. Conversely, both of $v_i, v_j \in V_B^t$ will desire resources of the same agent $v_k \in V_S^t$. Thus, v_i has envy over v_j if $v_j \in adj(v_i, E_s^t)$.

The definition of envy in this chapter is similar to [97]. However, this chapter defines envy by using utility rate even though [97] utilize utility function. Envy of v_i to v_j about utility rate is defined as *local envy* $\xi : V \times V \to [0, 1]$. $\xi(v_i, v_j)$ is defined as

$$\xi(v_i, v_j) = \begin{cases} \omega_j^t - \omega_i^t & (\omega_i^t < \omega_j^t), \\ 0 & (\omega_i^t \geq \omega_j^t). \end{cases} \tag{5.10}$$

where $v_j \in adj(v_i, E_s^t)$. The range of $\xi(v_i, v_j)$ is $[0, 1]$ because the minimum of ω_i^t is zero, and maximum value of ω_i^t is one. Because of this definition, $\xi(v_i, v_j)$ is not defined at round t if either of the following conditions is satisfied at t.

1. $|adj(v_i, E_s^t)| = 0$. (There is no neighbor to define envy.)

2. $|adj(v_i, E_d^t)| = 0$. (ω_i^t cannot be defined in this case.)

3. $\alpha_i^t = 0$. (ω_i^t cannot be defined in this case.)

Due to the above condition 1 and 2, $\xi(v_i, v_j)$ cannot be determined throughout $\mathcal{T}$ if $v_i \in V$ has zero or one neighbor in N. Thus, we assume $v_i \in V$ has two or more neighbors in N.

To compare envy among agents, envy of v_i at t must be denoted as one value. However, $\xi(v_i, v_j)$ can be defined for multiple neighbors $v_j \in adj(v_i, E_s^t)$. Hence, for the comparison of envy among agents, the maximum value of $\xi(v_i, v_j)$ is chosen as envy of v_i at t. Envy of v_i at t is represented by $\eta_i^t \in [0, 1]$. Ξ_i represents the set of $\xi(v_i, v_j)$ determined for v_i and neighbors $v_j \in adj(v_i, E_s^t)$. Since η_i^t is the maximum value in Ξ_i, η_i^t is represented by

$$\eta_i^t = \max\ \Xi_i. \tag{5.11}$$

If $|\Xi_i| = 0$ at the round t, η_i^t is not defined at t. $\mathcal{T}_e \subseteq \mathcal{T}$ is the set of rounds in which η_i^t is defined. Since η_i^t is not determined in some t as described above, $|\mathcal{T}_e| \leq |\mathcal{T}|$.

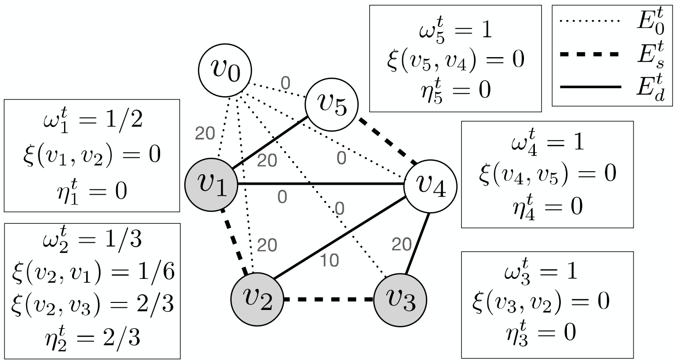


Fig. 5.4: Example of envy.

Fig. 5.4 shows examples of $\xi(v_i, v_j)$ and η_i^t in $\mathcal{G}$ at t. Utility rate ω_i^t is the same as Fig. 5.3. About seller v_2, $\xi(v_2, v_1) = 1/6$ and $\xi(v_2, v_3) = 2/3$. Thus, envy of v_2 at t is $\eta_2^t = 2/3$. Regarding sellers v_1 and v_3, $\eta_1^t = 0$ and $\eta_3^t = 0$ since both ω_1^t and ω_3^t are larger than ω_2^t. For buyers v_4 and v_5, $\eta_4^t = 0$ and $\eta_5^t = 0$ since ω_4^t and ω_5^t have the same value.

5.2.7 Index to denote envy of each agent

To describe envy of each agent through time span $\mathcal{T}$, this chapter defines two kinds of index as follows.

1. *Envy-free amount index* $\epsilon_i^a \in [0, 1]$:

   ϵ_i^a indicates the envy of v_i in terms of the amounts of resources that v_i shared among agents. ϵ_i^a is defined as

$$\epsilon_i^a = \left\{ \sum_{k \in \mathcal{T}_e} \eta_i^k \right\} / |\mathcal{T}_e|. \tag{5.12}$$

   High ϵ_i^a means v_i shared little amounts of resources among agents. Adversely, v_i does not have much envy if ϵ_i^a is low. Since ϵ_i^a is defined by the actual amount of resources which v_i obtained, ϵ_i^a might not be suitable for prosumers that do not consider the difference of amounts of resources in detail.

2. *Envy-free round index* $\epsilon_i^r \in [0, 1]$:

   Envy-free round index ϵ_i^r indicates the number of rounds in which v_i have envy to other agents. ϵ_i^r will be used to examine only whether v_i has envy or not at each round. To describe rounds at which v_i has envy, r_i^t is defined as

$$r_i^t = \begin{cases} 1 & (\eta_i^t > 0), \\ 0 & (\eta_i^t = 0). \end{cases} \tag{5.13}$$

v_i has envy at round t if $r_i^t = 1$. Hence, ϵ_i^r is defined as

$$\epsilon_i^r = \left\{ \sum_{k \in \mathcal{T}_e} r_i^k \right\} / |\mathcal{T}_e|. \tag{5.14}$$

Considering bounded rationality, prosumers are possibly not sensitive to the difference in the amounts of resources. ϵ_i^r is assumed to be utilized in the above situations.

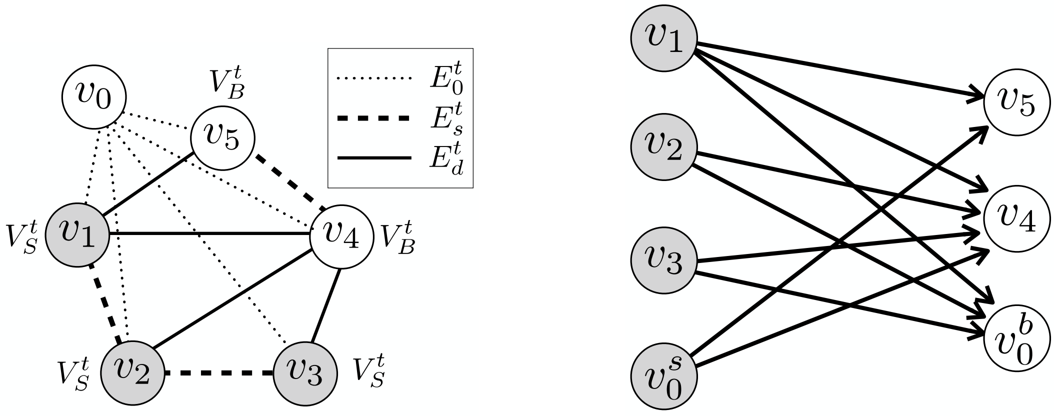
5.3 Problem formulation

5.3.1 Bipartite graph and matching for resource sharing

Resource allocation among agents at round t can be represented as matching M. This section explains the definition of M, and methods to discover M are presented in Section 5.3.2 and 5.3.3. M is defined on G $(V \cup \{v_0\}, E^t)$, which is an undirected graph at t in $\mathcal{G}$. M is denoted as many-to-many matching since each node in M has one or more edges. To determine M, G is converted to a directed bipartite graph G' $(S^t \cup B^t, A^t)$, which is a directed graph with two types of node sets denoted by S^t and B^t. For every arc $(v, w) \in A^t$, nodes v and w belong to S^t and B^t, respectively. The existence of arc $(v, w) \in A^t$ means v and w can share resources on the arc. Fig. 5.5 shows examples of G and G'.

Nodes in G are mapped into nodes in G'. S^t contains all $v_i \in V_S^t$. Similarly, B^t includes all $v_i \in V_B^t$. Since G' has two types of node sets S^t and B^t, node v_0 in G is divided into two nodes: v_0^s and v_0^b ($v_0^s \in S^t$, $v_0^b \in B^t$). v_0^s offsets the gap of all deficit of $v_j \in V_B^t$. Conversely, v_0^b covers the gap of all surplus of $v_i \in V_S^t$. Therefore, $S^t = V_S^t \cup \{v_0^s\}$ and $B^t = V_B^t \cup \{v_0^b\}$. Fig. 5.5b depicts a bipartite graph G' converted from G shown in Fig. 5.5a. In Fig. 5.5a, $V_S^t = \{v_1, v_2, v_3\}$ and $V_B^t = \{v_4, v_5\}$. Hence, $S^t = \{v_1, v_2, v_3, v_0^s\}$ and $B^t = \{v_4, v_5, v_0^b\}$ in Fig. 5.5b.

Regarding edges and arcs, edges (v_i, v_j) ($v_i \in V_S^t, v_j \in V_B^t$) in E_d^t are converted to arcs $(v_i, v_j) \in A^t$. For instance, G' in Fig. 5.5b has arcs converted from edges in G shown in Fig. 5.5a. Edges in E_0^t are also converted to arcs in A^t. v_0^s are connected to all $v_j \in V_B^t$ with arc (v_0^s, v_j). Similarly, A^t has (v_i, v_0^b)



(a) Graph G at round t.

(b) Bipartite graph G'.

Fig. 5.5: Example of G and G'.

($v_i \in V_S^t$). On the other hand, any edges in E_s^t are not converted to arcs in A^t because no resource sharing is conducted on edges in E_s^t.

By using resource allocation obtained as M, the value of utility function $\mu^t(v_i)$ can be calculated. To obtain $\mu^t(v_i)$, variables y^t and y_0^t are used as explained in Section 5.2.4. M is denoted by the set of arcs and the values of flow on the arcs in G'. The flow corresponds to y^t and y_0^t in equation (5.4). As defined in Section 5.2.4, y^t is the total amounts of resources obtained by v_i from all neighbors except v_0. Let $adj(v_i, A^t)$ denote neighbors of v_i in G' except v_0^b and v_0^s. $adj(v_i, A^t)$ is defined according to the set to which v_i belongs in G'. If $v_i \in S^t$, $adj(v_i, A^t) = \{v_j \in (B^t \setminus \{v_0^b\}) \mid (v_i, v_j) \in A^t\}$. Thus, when $v_i \in S^t$, y^t is defined by using flow as follows.

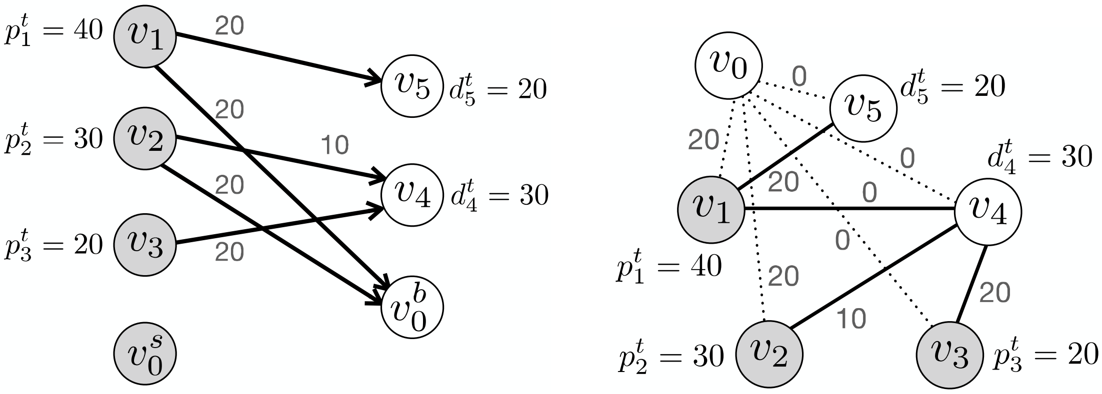
$$y^t = \sum_{v_j \in adj(v_i, A^t)} x(v_i, v_j). \tag{5.15}$$

If $v_i \in B^t$, $adj(v_i, A^t) = \{v_j \in (S^t \setminus \{v_0^s\}) \mid (v_j, v_i) \in A^t\}$. Hence, when $v_j \in B^t$, y^t is obtained by

$$y^t = \sum_{v_j \in adj(v_i, A^t)} x(v_j, v_i). \tag{5.16}$$

Besides, y_0^t is the total amounts of resources obtained from v_0. By using flow, y_0^t is defined as follows for $v_i \in V_S^t \cup V_B^t$.

$$y_0^t = \begin{cases} x(v_i, v_0^b) & (v_i \in V_S^t), \\ x(v_0^s, v_i) & (v_i \in V_B^t). \end{cases} \tag{5.17}$$



(a) Matching M in G'.

(b) Resources shared in G.

Fig. 5.6: Relationship between matching M and graph G.

Fig. 5.6a is an example of M determined in G' shown in Fig. 5.5b. The value of flow is denoted on each arc in Fig. 5.5b. As shown in Fig. 5.6a, M is represented by the set of arcs and the values of flow on the arcs. If the value of flow on an arc is not zero, M contains the arc. An arc with zero flow is not included in M. Fig. 5.6b shows G that corresponds to M in G'. In Fig. 5.6b, the value of resource shard on each edge is denoted on the edge. Thus, the allocation of resources in G can be determined by M in G'. The allocation of resources determines variables y^t and y_0^t to obtain utility rate and the indices about envy for each agent. For instance, in both Fig. 5.6a and Fig. 5.6b, y^t and y_0^t of v_1 are determined as $y^t = 20$ and $y_0^t = 20$ since $x(v_1, v_5) = 20$ and $x(v_1, v_0^b) = 20$, respectively.

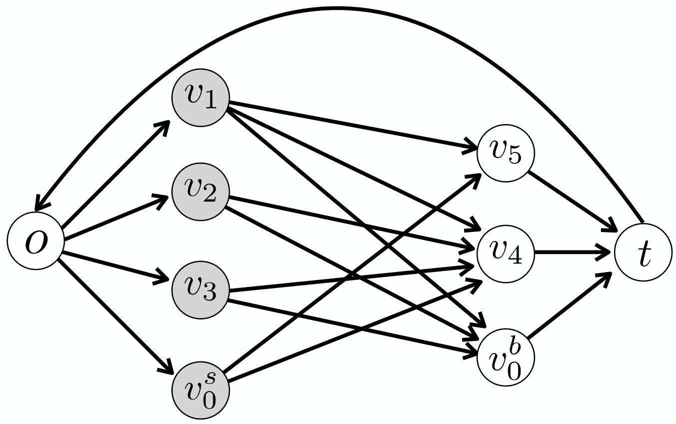


Fig. 5.7: A directed graph H for modeling circulation.

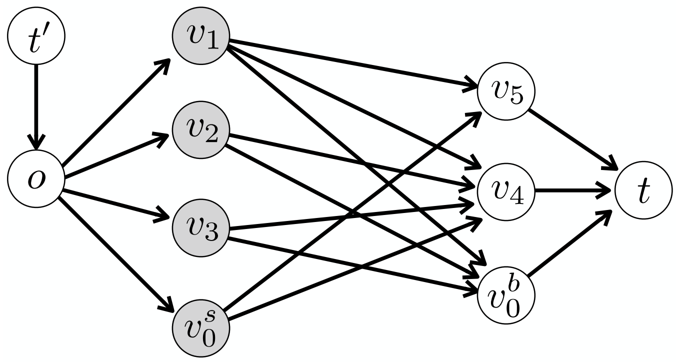


Fig. 5.8: A directed graph H' to solve circulation.

5.3.2 Circulation to discover matching

Resources of agents v_i should be efficiently shared in matching M. The most efficient resource allocation in M can be achieved by maximizing flow among v_i in G'. We consider a minimum cost circulation problem [27] to discover M that maximizes flow among all agents. Fig. 5.7 shows a directed graph H to define the circulation problem. In this circulation problem, the objective is to discover the minimum cost feasible circulation, which satisfies two constraints (2.1) and (2.2). The circulation problem is different from general network flow problems since the graph H does not have source and sink. Hence, constraint (2.2) must be satisfied in all nodes in the circulation problem. H can be constructed from G' such as the graph depicted in Fig. 5.5b. G' can be converted to H by adding node o and t and arcs explained in Table 5.1.

Matching M can be obtained as the minimum cost feasible circulation on H satisfying the capacity constraints shown in Table 5.1. Weights of arcs $c(v, w)$ shown in Table 5.1 are set to maximize flow among v_i. About flow from source o, the weight of arcs for v_i is set to $c(o, v_i) = 0$. The weight of an

Table 5.1: Arc (v, w) added to G' for constructing H.

v	w	$lb(v, w)$	$ub(v, w)$	$c(v, w)$
$v_i \in V_S^t$	$v_j \in V_B^t$	0	$\max(p_i^t, d_j^t)$	0
$v_i \in V_S^t$	v_0^b	0	p_i^t	0
v_0^s	$v_j \in V_B^t$	0	d_j^t	0
o	$v_i \in V_S^t$	p_i^t	p_i^t	0
o	v_0^s	0	D_j^t	$z > 0$
$v_j \in V_B^t$	t	d_j^t	d_j^t	0
v_0^b	t	0	P_i^t	$z > 0$
t	o	$\max(P^t, D^t)$	$\max(P^t, D^t)$	0

arc for v_0^s, on the other hand, is set to $c(o, v_0^s) = z$ $(z > 0)$. According to the minimum cost feasible circulation, resource sharing among v_i is selected in preference to resource sharing with v_0^s. Likewise, regarding flow toward sink t, $c(v_j, t) = 0$ and $c(v_0^b, t) = z$ $(z > 0)$. Therefore, flow from v_j to t is chosen in preference to flow from v_0^b in the minimum cost feasible circulation.

The circulation problem on H can be solved with general algorithms for a minimum cost maximum flow problem by changing arcs in H. We utilize H' which can be obtained by removing arc (t, o) from H and adding arc (t', o) to H. Fig. 5.8 shows an example of H' converted from H depicted in Fig. 5.7. Flow bounds of (t', o) are the same as the bounds of (t, o). Thus, $lb(t', o) = 0$ and $ub(t', o) = \max(P^t, D^t)$. Then, the minimum cost feasible circulation on H is obtained as the minimum cost maximum flow on t'-t path in H'.

Table 5.2: $c(v, w)$ considering envy weight.

v	w	$c(v, w)$
o	$v_i \in V_S^t$	$\psi_i^t \in [0, 1]$
o	v_0^s	$z > 1$
$v_j \in V_B^t$	t	$\psi_j^t \in [0, 1]$
v_0^b	t	$z > 1$

5.3.3 Circulation considering envy among participants

As described above, matching M maximizing resource sharing among v_i at round t can be obtained by solving the circulation problem. However, that circulation problem does not consider the aspects of envy-freeness among prosumers. Furthermore, since the circulation problem focuses on the flow only at round t, the circulation problem needs a modification to realize desirable resource sharing through time span in a temporal network.

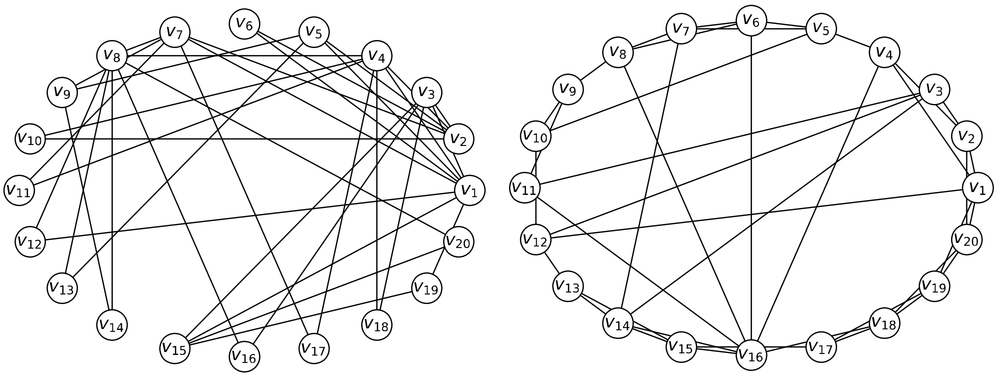
Envy weight is defined for the circulation problem to discover M with lower envy among prosumers while maximizing efficiency in resource allocation. The envy weight is the weight of arc that reflects envy of prosumers to the resource allocation in future rounds. Thus, prosumers who have more envy than others will be preferentially allocated resources by using envy weight in the circulation problem.

ψ_i^t denotes envy weight for v_i at t. As the initial condition at $t = 1$, $\psi_i^t = 0$. $\mathcal{T}' \subseteq \mathcal{T}$ is a time span before round t ($1 < t \leq T$). For instance, for $t = 4$, $\mathcal{T}' = \{1, 2, 3\}$. As well as envy-free amount index ϵ_i^a, η_i^t is used for ψ_i^t. $\mathcal{T}_e' \subseteq \mathcal{T}'$ denotes the set of rounds in which η_i^t is defined. If $|\mathcal{T}_e'| = 0$, $\psi_i^t = 0$ since this condition means envy does not exist before round t. By using the definition described above, ψ_i^t is defined as

$$\psi_i^t = 1 - \left\{ \left(\sum_{k \in \mathcal{T}_e'} \eta_i^k \right) / |\mathcal{T}_e'| \right\}. \tag{5.18}$$

Therefore, ψ_i^t is in the interval $[0, 1]$.

Table 5.2 shows weight of arcs $c(v, w)$ modified from the definition in Table 5.1 to the definition considering ψ_i^t. Low ψ_i^t makes the cost of flow of v_i high. Thus, in the minimum cost circulation problem, the priority of v_i in resource allocation will decrease compared to v_j, which has ψ_j^t higher than ψ_i^t. If ψ_i^t is used to calculate circulation, the value of z must be larger than ψ_i^t so that resource sharing among prosumers will be preferentially selected rather than v_0^s and v_0^b. Hence, z must satisfy $z > 1$ since $\psi_i^t \in [0, 1]$ when ψ_i^t is applied.



(a) BA model.

(b) WS model

Fig. 5.9: Examples of social network models.

Table 5.3: The values used as parameters in simulation.

Parameter	Value
T	100
$\|V\|$	20, 40, 60, 80, or 100
ρ_i^t	randomly chosen from $\{\rho_i^t \in \mathbb{N}_0 \mid \rho_i^t \in [0, 100)\}$
δ_i^t	randomly chosen from $\{\delta_i^t \in \mathbb{N}_0 \mid \delta_i^t \in [0, 100)\}$
m (BA model)	2
k (WS model)	4
p (WS model)	0.2 ($\|V\| = 20$), 0.3 ($\|V\| = 40$), 0.35 ($\|V\| = 60$), 0.4 ($\|V\| = 80, 100$)

5.4 Experimental results

5.4.1 Conditions

To analyze envy among agents in efficient resource allocation determined by the proposed circulation problem, simulation experiments were conducted. As common models to construct social networks, this chapter utilizes BA model and WS model in the simulation. Fig. 5.9a depicts an example of the BA model. A graph constructed as BA model is called scale-free network in which only a small part of nodes has larger degree compared to the other nodes. We focus on this characteristics because achieving lower envy might become difficult if there is larger deviation about degree of nodes in a graph. On the other hand, WS model does not have scale-free property and relatively smaller deviation of degree of nodes compared to BA model. Fig. 5.9b shows an example of WS model utilized for comparison.

Table 5.3 shows parameter conditions used in the simulation. T means the number of rounds in a time span $\mathcal{T}$ as one set of the experiments. $|V|$ was chosen from five values to analyze influences of the number of agents. Parameter m for BA model and parameter k for WS model were chosen so that the similar number of edges are generated in both types of graphs. Parameter p for WS model influences clustering coefficient of graph, which indicates the existence of clustering in the graph. In the simulation, p was set to make clustering coefficient of graphs of WS model closer to clustering coefficient of graphs of BA model.

The simulation has ten patterns of parameter conditions since five values of $|V|$ were used for the two types of graph structure: BA model and WS model. The simulation experiment is conducted 1,000 sets for each parameter condition since the parameters contains random factors such as graph structure, production ρ_i^t, and demand δ_i^t. Hence, 1,000 instances of $\mathcal{G}$ were generated for each parameter condition. To impartially compare the proposed problem with envy weight and without envy weight, matching M was determined by using the same instance of $\mathcal{G}$ for both conditions every round. According to the condition of degree of agent v_i of N in Section 5.2.6, every instance of graph N used in the simulation satisfies that every agent v_i in the instance has two or more neighbors.

A simulation software was developed with Python. In the simulation, a python library called NetworkX [104] was utilized for constructing graphs based on BA model and WS model. Moreover, the proposed circulation problem is solved by using a method to solve a minimum cost maximum flow algorithm defined in NetworkX.

As an evaluation metric, the average of ϵ_i^a and ϵ_i^r over agents is calculated to examine the magnitude of envy-freeness of overall agents. Moreover, the standard deviation of ϵ_i^a and ϵ_i^r for all agent $v_i \in V$ is obtained to investigate the size of variance in each index of envy-freeness over agents.

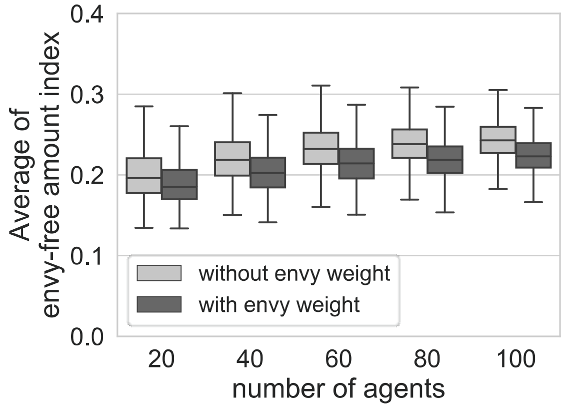
5.4.2 Results and discussion

Following results and discussion explain how envy weight influenced envy among prosumers in matching, which represents efficient resource utilization determined by the proposed circulation problem.

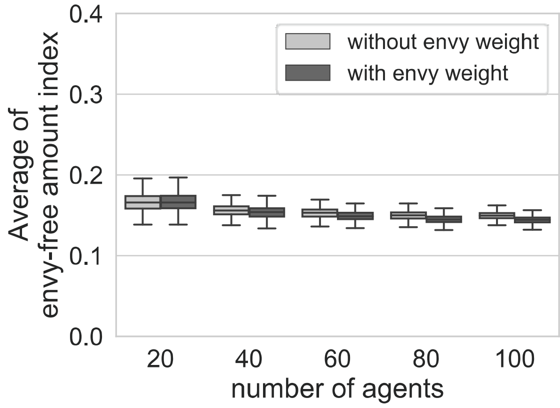
Average of each index over agents

The average of ϵ_i^a among agents is shown in Fig. 5.10. The charts in this section depict the results about 1,000 sets of experiments for each parameter condition. Fig. 5.10a shows the average of ϵ_i^a regarding BA model. These results indicate that the difference between the two weight conditions is not so significant. If the number of agents increased, the average of ϵ_i^a of matching with envy weight is slightly lower than matching without envy weight. Fig. 5.10b shows the average of ϵ_i^a about WS model. The average value of ϵ_i^a regarding WS model is lower compared to that of BA model. Unlike BA model, the average of ϵ_i^a in WS model slightly decreased as the number of agents increased.

The average of ϵ_i^r is shown in Fig. 5.11. Fig. 5.11a shows the average of ϵ_i^r regarding BA model. The characteristics of the difference of ϵ_i^r between two weight conditions are similar to that of ϵ_i^a. With almost every condition, the average value of ϵ_i^r is relatively larger than that of ϵ_i^a. Besides, Fig. 5.11b shows the average of ϵ_i^r about WS model. Similar to the results about ϵ_i^a, the average of ϵ_i^r in WS model was lower compared to ϵ_i^r in BA model. About WS model, the value of ϵ_i^r is also slightly larger than ϵ_i^a.

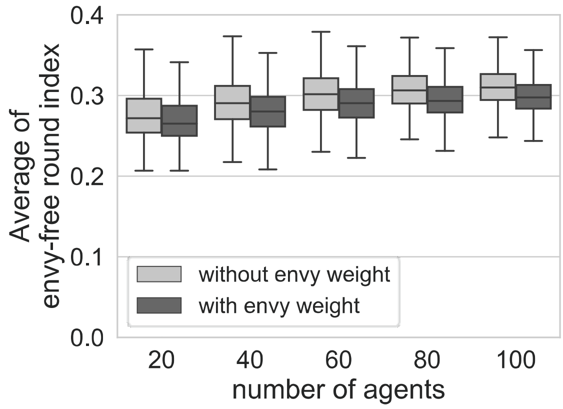


(a) Average of ϵ_i^a (BA model).

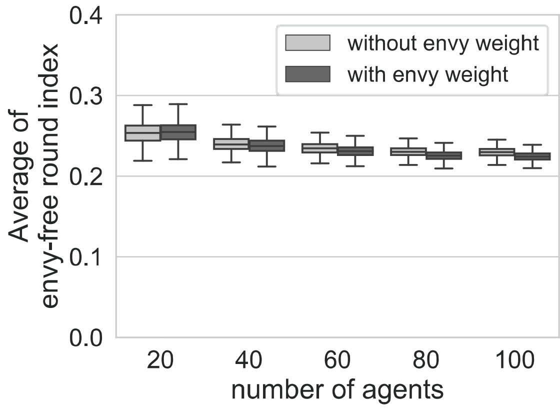


(b) Average of ϵ_i^a (WS model).

Fig. 5.10: Average of envy-free amount index ϵ_i^a.



(a) Average of ϵ_i^r (BA model).



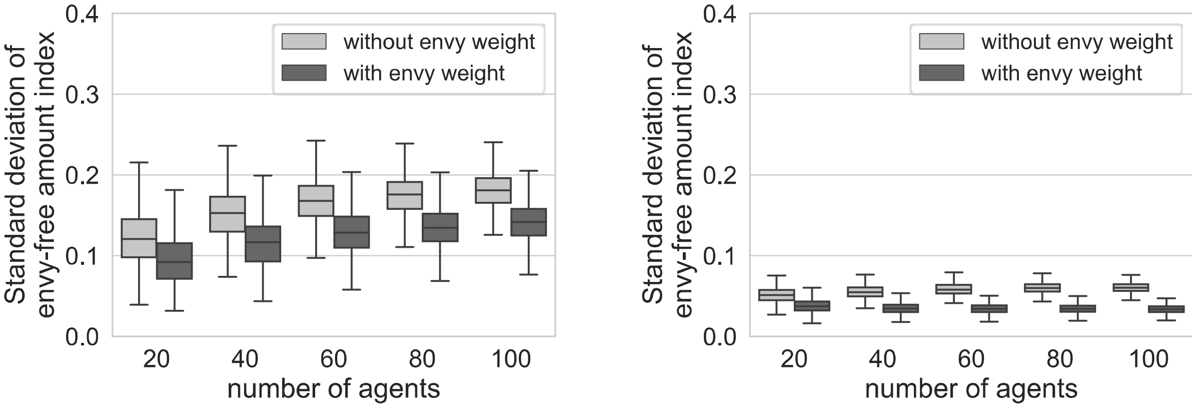
(b) Average of ϵ_i^r (WS model).

Fig. 5.11: Average of envy-free round index ϵ_i^r.

Standard deviation of each index over agents

The standard deviation of ϵ_i^a over agents is shown in Fig. 5.12. Fig. 5.12a shows the standard deviation of ϵ_i^a in BA model. The standard deviation of ϵ_i^a with envy weight was relatively lower than ϵ_i^a without envy weight. Fig. 5.12b shows the standard deviation of ϵ_i^a in WS model. By comparing Fig. 5.12b with Fig. 5.12a, the deviation of ϵ_i^a in WS model is clearly lower than that in BA model. The standard deviation of ϵ_i^a with envy weight was also lower than ϵ_i^a without envy weight in WS model. However, the difference of the standard deviation of ϵ_i^a of two conditions in WS model was smaller than that of BA model. Therefore, although the magnitude of improvement is different according to BA model and WS model, the matching with lower standard deviation of ϵ_i^a was discovered by using envy weight for both models.

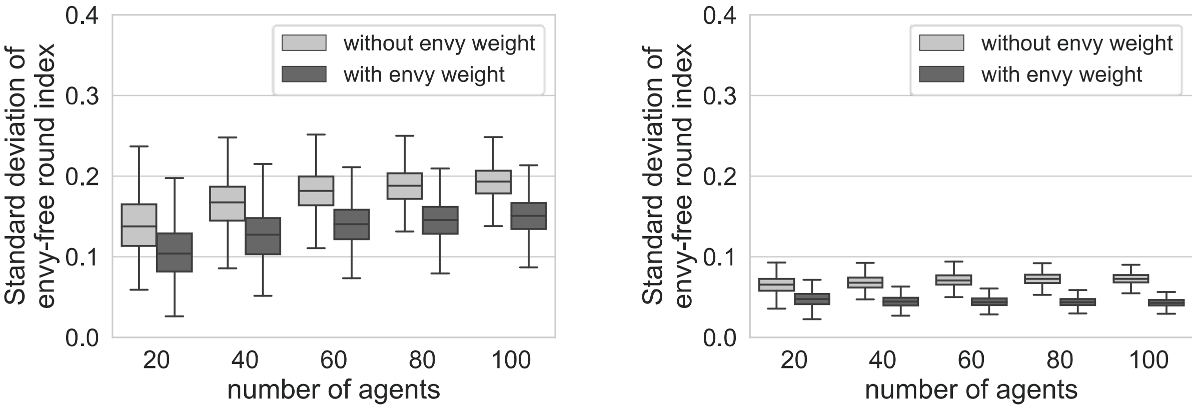
Fig. 5.13 shows the standard deviation of ϵ_i^r among agents. Fig. 5.13a shows the standard deviation of ϵ_i^r in BA model. As shown in Fig. 5.13a, ϵ_i^r in BA model demonstrated similar characteristics of ϵ_i^a shown in Fig. 5.12a. There is no large difference between the values of ϵ_i^a and ϵ_i^r. Fig. 5.13b shows the standard deviation of ϵ_i^r in WS model. As well as BA model, the characteristics of the standard deviation of ϵ_i^r shown in Fig. 5.13b is almost similar to that of ϵ_i^a demonstrated in Fig. 5.12b. Hence, as well as ϵ_i^a, the matching with lower standard deviation of ϵ_i^r was found by applying envy weight in the circulation problem.



(a) Standard deviation of ϵ_i^a (BA model).

(b) Standard deviation of ϵ_i^a (WS model).

Fig. 5.12: Standard deviation of envy-free amount index ϵ_i^a.



(a) Standard deviation of ϵ_i^r (BA model).

(b) Standard deviation of ϵ_i^r (WS model).

Fig. 5.13: Standard deviation of envy-free round index ϵ_i^r.

5.4.3 Discussions

Regarding weight conditions, envy weight contributed to improve both ϵ_i^a and ϵ_i^r mainly in terms of variance of the indices over agents for both BA model and WS model. By integrating envy weight into the minimum cost circulation problem, envy among agents can be reduced in resource allocation determined as matching compared to matching obtained without considering envy weight. However, the magnitude of difference in each index between two weight conditions is not so large especially regarding ϵ_i^a. Since the proposed circulation problem determines matching maximizing efficiency of resource utilization of agents, the circulation problem with envy weight demonstrates the existence of matching that represents resource allocation improving fairness while maximizing efficiency. Because our proposed approach to find matching is heuristics, considering theoretical bounds for the value of each index is an interesting future topic.

Comparing ϵ_i^a with ϵ_i^r, the average of each index demonstrates the difference in both BA model and WS model. Results indicate the average of ϵ_i^r is relatively larger than the average of ϵ_i^a. Thus, prosumers will feel larger envy with ϵ_i^r as a metric of equity compared to ϵ_i^a at least by using conditions in Table 5.3. Simulation experiments with additional parameter conditions will give us more insights about the difference in ϵ_i^a and ϵ_i^r.

The standard deviation of ϵ_i^a and ϵ_i^r, on the other hand, does not demonstrate a large difference between ϵ_i^a and ϵ_i^r for each of BA model and WS model. These results indicate that both of the matching determined with two weight conditions have almost the same performance about the variance of envy among agents. Regarding the results of BA model with those of WS model, the results indicate that the difference of envy over prosumers in BA model might be larger than that of WS model since the standard deviation of ϵ_i^a and ϵ_i^r in BA model is greater than WS model. Therefore, the structure of a social network might also affect the size of envy for each prosumer.

5.5 Summary of this chapter

This chapter proposed model deals with resource sharing among agents representing prosumers. To evaluate envy-freeness for resource sharing among agents in a social network, we proposed two indices: envy-free amount index and envy-free round index. Simulation results demonstrated the envy among agents is reduced by using the proposed minimum cost circulation problem and the condition of weight on arcs in a flow network for the problem. Additionally, the simulation results demonstrated that agents will face the different magnitude of unfairness in each index of envy. Although the concept of envy-freeness for resource sharing was proposed in this chapter, more realistic datasets of electricity should be used in simulation experiments to obtain practical insights about the benefits of prosumers.

Chapter 6

Concluding remarks

6.1 Conclusion

The results of studies on four problems contextualize the objective of this thesis, which is to analyze consumers' benefits in electricity markets to consider successful mechanisms. Regarding Problem 1, concepts of graph theory were utilized to propose market models for electricity markets, which represent both benefits of participants and constraints on supply and demand. A market model to represent deregulated electricity markets was presented for Problem 2, and this study explained the relationship between the benefits of participants and electricity prices in energy trading. About Problem 3, consumers' switching behavior was modeled as an evolutionary game. The results demonstrated that both the network structure and the existence of consumers active to switch suppliers become important factors to promote switching behavior. Furthermore, in the resource sharing model presented for Problem 4, envy-freeness was utilized as one of the metrics of electricity sharing among prosumers.

Considering the above examples of results, the market models presented in this thesis demonstrated several insights about consumers' benefits in liberalized electricity markets. Therefore, these models are expected to contribute to providing insights for considering and examining market mechanisms that will be developed in the transformation of electricity markets from now on. The advanced understanding of the benefits of consumers will contribute to decision making by both supply-side and demand-side of the markets.

6.2 Limitations and future works

The proposed modeling techniques have several limitations regarding the detailed characteristics of electricity markets that have not currently been integrated into the models. To enhance the applicability of the models for real-world problems and obtain sophisticated insights from the models, the following topics are left as future works.

1. **Using datasets about electricity production and consumption in experiments:**
   To construct more realistic electricity market models and validate the models effectively, experiments should use datasets of production and consumption of electricity. Many modeling teams have traditionally restricted access for data about electricity production and demand [105]. Fortunately, there has been the movement toward opening up energy related data for improving reproducibility and transparency of energy modeling research recently [106]. Open Power System Data is a platform of power data (including conventional and renewable plants) for electricity system researchers [107]. Besides, Renewable.ninja provides simulation data about the hourly output of worldwide solar and wind generation plants [108]. Regarding consumers' behavioral data, however, have been still treated as closed data since actual behavioral data about consumers can be considered as sensitive data [109]. Thus, it is important to consider what kinds of data are available to improve the insights obtained from the models.

2. **Considering detailed time intervals in time-varying electricity trading:**
   Regarding Problem 3 and 4, detailed time intervals in time-varying trading models should be defined to clarify the characteristics of electricity markets that the models represent. Focal points will be different according to the size of the time intervals represented by the models (e.g. hourly, daily, monthly, and yearly basis) [110]. For instance, short-term time intervals mainly focus on aspects regarding controls on supply and demand in every second, minute, hour, etc. Besides, long-term time intervals can be utilized for planning over a few months about generation combination, legislation, and so on.

3. **Integrating multiple factors of the utility of consumers:**
   Although the utility function in this thesis only considers electricity charges as the factor of consumers' utility, the other factors should be considered in the utility function according to future transitions of electricity markets. Especially, it is inevitable to address the environmental concerns regarding electricity generation and consumption worldwide for sustainable development. Moreover, since choices for consumers in electricity markets might increase and change, novel factors of the utility will be enabled by new market mechanisms in the future.

4. **Increasing the number of nodes in network models:**
   For all of the four problems presented in this thesis, expanding scales of the market models is important since the number of nodes used in the simulation experiments is considered to be smaller than that in the real world. For this purpose, algorithms to determine trading should be improved to solve larger sizes of problems effectively.

5. **Assuming heterogeneous switching costs of consumers:**
   Regarding the model for analyzing the switching behavior of consumers for Problem 3, adding heterogeneity of switching costs of consumers might provide additional insights about the behavior of consumers. This topic requires extensive numerical experiments since the number of patterns in parameter conditions will considerably increase if consumers' switching costs are heterogeneous.

6. **Adding dynamical transitions of electricity prices:**
   Adjusting prices in the market model for Problem 3 and Problem 4 will demonstrate more practical information than current models. Since electricity prices are assumed to be fixed in the proposed models in this thesis, market models should be extended to allow changes in electricity prices.

Appendix A

Convergence of consumers' strategies in algorithms for Problem 3

In the process of the evolutionary game for Problem 3 explained in Chapter 4, strategies of all buyers will converge to D. We show the convergence of the strategies as follows.

Lemma 1 *Each buyer b_j will eventually discover only indifferent sellers after a sufficient number of iterations if X_j and preference relation of b_j over s_i do not change.*

Proof: Suppose b_j can find a preferred seller at every iteration. For iteration t ($t = 1, 2, \ldots$), let $a_j^{t-1} \in X_j$ be the seller from which b_j purchases electricity at iteration $t-1$. b_j can divide X_j into two subsets: the set of preferred sellers $\mathcal{P}_j^t = \{s_k \mid \mu_j(s_k) - \mu_j(a_j^{t-1}) > \varepsilon\}$ and the set of indifferent sellers $\mathcal{I}_j^t = X_j \setminus \mathcal{P}_j^t$ ($\mathcal{P}_j^t \cap \mathcal{I}_j^t = \varnothing$). For only $b_j \in B_C^t$, b_j conducts the search for new alternatives at line 3 and 8 of Algorithm 3. For b_j with both C and D, b_j compares its current alternative with the other alternatives suggested by the other buyers at line 3, 5, 9, and 15 of Algorithm 4.

The relationship between $\mathcal{I}_j^t$, $\mathcal{P}_j^t$, and $\mathcal{I}_j^{t+1}$ can be explained as follows. In this proof, it is assumed that b_j can find preferred seller every iteration t. In this case, for a_j^{t-1} discovered at $t-1$ and a_j^t ($a_j^t \neq a_j^{t-1}$) found at t, the following condition must be satisfied.

$$\mu_j(a_j^t) - \mu_j(a_j^{t-1}) > \varepsilon. \tag{A1.1}$$

Let $\mathcal{P}_j^t = \{s_m \in X_j \mid \mu_j(s_m) - \mu_j(a_j^{t-1}) > \varepsilon\}$ and $\mathcal{I}_j^t = \{s_m \in X_j \mid \mu_j(s_m) - \mu_j(a_j^{t-1}) \leq \varepsilon\}$. The relationship between $\mathcal{P}_j^t$ and $\mathcal{I}_j^t$ can be represented as $\mathcal{P}_j^t \cup \mathcal{I}_j^t = X_j$ and $\mathcal{P}_j^t \cap \mathcal{I}_j^t = \varnothing$. Thus, a_j^t is contained in either $\mathcal{P}_j^t$ or $\mathcal{I}_j^t$. When condition (A1.1) is satisfied, $a_j^t \in \mathcal{P}_j^t$.

We show that $\mathcal{I}_j^t \subset \mathcal{I}_j^{t+1}$ if (A1.1) is satisfied. In other words, we explain $s_m \in \mathcal{I}_j^{t+1}$ if $s_m \in \mathcal{I}_j^t$.

Since $\mathcal{I}_j^t = \{s_m \in X_j \mid \mu_j(s_m) - \mu_j(a_j^{t-1}) \leq \varepsilon\}$, $\mu_j(s_m) - \mu_j(a_j^{t-1}) \leq \varepsilon$. Therefore,

$$\mu_j(s_m) \leq \mu_j(a_j^{t-1}) + \varepsilon. \tag{A1.2}$$

Moreover, because $\mathcal{I}_j^{t+1} = \{s_m \in X_j \mid \mu_j(s_m) - \mu_j(a_j^t) \leq \varepsilon\}$, $\mu_j(s_m) - \mu_j(a_j^t) \leq \varepsilon$. Thus,

$$\mu_j(s_m) \leq \mu_j(a_j^t) + \varepsilon. \tag{A1.3}$$

Furthermore, by (A1.1),

$$\mu_j(a_j^{t-1}) + \varepsilon < \mu_j(a_j^t). \tag{A1.4}$$

Since $\varepsilon \geq 0$, even by adding ε to right side of (A1.4), the following condition is satisfied.

$$\mu_j(a_j^{t-1}) + \varepsilon < \mu_j(a_j^t) + \varepsilon. \tag{A1.5}$$

By (A1.2) and (A1.5),

$$\mu_j(s_m) \leq \mu_j(a_j^{t-1}) + \varepsilon < \mu_j(a_j^t) + \varepsilon. \tag{A1.6}$$

Condition (A1.6) means $\mu_j(s_m)$ satisfies (A1.3) if $\mu_j(s_m)$ satisfies (A1.2). Thus, if $s_m \in \mathcal{I}_j^t$ based on (A1.2), $s_m \in \mathcal{I}_j^{t+1}$ is also satisfied based on (A1.3). Therefore, $\mathcal{I}_j^t \subset \mathcal{I}_j^{t+1}$.

Because $a_j^t \in \mathcal{P}_j^t$, $a_j^t \notin \mathcal{I}_j^t$. Furthermore, since $\mathcal{I}_j^t \subset \mathcal{I}_j^{t+1}$ and $a_j^t \notin \mathcal{I}_j^t$, $a_j^t \in (\mathcal{I}_j^{t+1} \setminus \mathcal{I}_j^t)$. Let $s_q \in (\mathcal{I}_j^{t+1} \setminus \mathcal{I}_j^t)$. Since $s_q \in \mathcal{I}_j^{t+1}$, $\mu_j(s_q) - \mu_j(a_j^t) \leq \varepsilon$. Moreover, since $s_q \notin \mathcal{I}_j^t$, $s_q \in \mathcal{P}_j^t$. Hence, $(\mathcal{I}_j^{t+1} \setminus \mathcal{I}_j^t)$ can be denoted as follows by using s_q.

$$(\mathcal{I}_j^{t+1} \setminus \mathcal{I}_j^t) = \{s_q \mid \mu_j(s_q) - \mu_j(a_j^t) \leq \varepsilon, s_q \in \mathcal{P}_j^t\}.$$

Therefore, we obtain the following relationship between $\mathcal{I}_j^{t+1}$ and $\mathcal{I}_j^t$.

$$\begin{aligned}\mathcal{I}_j^{t+1} &= \mathcal{I}_j^t \cup (\mathcal{I}_j^{t+1} \setminus \mathcal{I}_j^t), \\ &= \mathcal{I}_j^t \cup \{s_q \mid \mu_j(s_q) - \mu_j(a_j^t) \leq \varepsilon, s_q \in \mathcal{P}_j^t\}.\end{aligned}$$

In the comparison of sellers, b_j can find a preferred seller s_k only if compared seller s_k is contained in $\mathcal{P}_j^t$. For a_j^{t-1} discovered at $t-1$, if seller s_k ($s_k \in X_j$) chosen at t is a preferred seller, the following condition is satisfied based on condition (4.1) in Chapter 4.

$$\mu_j(s_k) - \mu_j(a_j^{t-1}) > \varepsilon.$$

The set of preferred seller s_k over a_j^{t-1} is denoted by $\mathcal{P}_j^t = \{s_k \in X_j \mid \mu_j(s_k) - \mu_j(a_j^{t-1}) > \varepsilon\}$.

a_j^t at the start of Algorithm 4 can be considered as follows to clarify the relationship between Algorithm 4 and $\mathcal{P}_j^t$.

- If $\sigma_j^{t-1} = D$:
  $a_j^t = a_j^{t-1}$ at the start of Algorithm 4 since a_j^t is initially set as $a_j^t = a_j^{t-1}$ at line 12, Algorithm 2.
- If $\sigma_j^{t-1} = C$:
  If $b_j \in B_C^t$, Algorithm 3 will be executed at line 9, Algorithm 2. At iteration t, a_j^t is initially set as $a_j^t = a_j^{t-1}$ at line 1, Algorithm 3. However, if $s_m(s_m \in X_j, s_m \neq a_j^{t-1})$ satisfying

  $$\mu_j(s_m) - \mu_j(a_j^t) > \varepsilon \ (a_j^t = a_j^{t-1}) \tag{A1.7}$$

  was discovered as a result of Algorithm 3, $a_j^t = s_m$. Therefore, $a_j^t = a_j^{t-1}$ or $a_j^t = s_m$ at the start of Algorithm 4.

As described above, $a_j^t = a_j^{t-1}$ or $a_j^t = s_m$ at the start of Algorithm 4. We will show preferred seller $s_k \in X_j$ over a_j^t is contained in $\mathcal{P}_j^t$ in each case of $a_j^t = a_j^{t-1}$ or $a_j^t = s_m$ as follows.

- If $a_j^t = a_j^{t-1}$:
  If $s_k \in X_j$ satisfies the following condition, $s_k \in \mathcal{P}_j^t$.

  $$\mu_j(s_k) - \mu_j(a_j^{t-1}) > \varepsilon. \tag{A1.8}$$

- If $a_j^t = s_m$:

  Let $s_k \in X_j$ be a seller satisfying the following condition.

$$\mu_j(s_k) - \mu_j(s_m) > \varepsilon.(s_m \in X_j, s_m \neq a_j^{t-1}) \tag{A1.9}$$

  Since $\mu_j(s_k) > \mu_j(s_m) + \varepsilon$ by (A1.9) and $\varepsilon \geq 0$, the following condition is satisfied.

$$\mu_j(s_k) > \mu_j(s_m). \tag{A1.10}$$

  By subtracting $\mu_j(a_j^{t-1})$ from both sides of (A1.10),

$$\mu_j(s_k) - \mu_j(a_j^{t-1}) > \mu_j(s_m) - \mu_j(a_j^{t-1}). \tag{A1.11}$$

  Condition (A1.7) indicates that

$$\mu_j(s_m) - \mu_j(a_j^{t-1}) > \varepsilon. \tag{A1.12}$$

  By (A1.11) and (A1.12),

$$\mu_j(s_k) - \mu_j(a_j^{t-1}) > \mu_j(s_m) - \mu_j(a_j^{t-1}) > \varepsilon. \tag{A1.13}$$

  Therefore, the relationship between $\mu_j(s_k)$ and $\mu_j(a_j^{t-1})$ can be denoted as

$$\mu_j(s_k) - \mu_j(a_j^{t-1}) > \varepsilon. \tag{A1.14}$$

  Because (A1.14) is the same as (A1.8), these conditions indicate $s_k \in \mathcal{P}_j^t$.

Based on the conditions described above, we will show that preferred sellers contained in $\mathcal{P}_j^t$ in line 3, 5, 9, and 15 of Algorithm 4.

- line 3, Algorithm 4 ($\sigma_j^{t-1} = C, \sigma_i^{t-1} = C$)
  b_j regards $a_i^t \in X_j$ as a preferred seller if a_i^t satisfies $\mu_j(a_i^t) - \mu_j(a_j^t) > \varepsilon$, which is the condition of line 3. Since $\sigma_j^{t-1} = C$, a_j^t satisfies $a_j^t = a_j^{t-1}$ or $a_j^t = s_m(s_m \neq a_j^{t-1})$. If $a_j^t = a_j^{t-1}$, since the condition of line 3 is identical to $\mu_j(a_i^t) - \mu_j(a_j^{t-1}) > \varepsilon$, it becomes the same as (A1.8). Besides, if $a_j^t = s_m$, the condition of line 3 can be described as $\mu_j(a_i^t) - \mu_j(s_m) > \varepsilon$. Let $a_i^t = s_k$, and the condition of line 3 is identical to (A1.10) in this case. Thus, $a_i^t \in \mathcal{P}_j^t$ if the condition of line 3 is satisfied.

- line 5, Algorithm 4 ($\sigma_j^{t-1} = C, \sigma_i^{t-1} = C$)
  b_i regards $a_j^t \in X_i$ as a preferred seller if a_j^t satisfies $\mu_i(a_j^t) - \mu_i(a_i^t) > \varepsilon$, which is the condition of line 5. Since $\sigma_i^{t-1} = C$, a_i^t satisfies $a_i^t = a_i^{t-1}$ or $a_i^t = s_m(s_m \neq a_i^{t-1})$. If $a_i^t = a_i^{t-1}$, since the condition of line 5 is identical to $\mu_i(a_j^t) - \mu_i(a_i^{t-1}) > \varepsilon$, it becomes the same as (A1.8). Besides, if $a_i^t = s_m$, the condition of line 5 can be described as $\mu_i(a_j^t) - \mu_i(s_m) > \varepsilon$. Let $a_j^t = s_k$, and the condition of line 5 is identical to (A1.10). Therefore, $a_j^t \in \mathcal{P}_i^t$ if the condition of line 5 is satisfied.

- line 9, Algorithm 4 ($\sigma_j^{t-1} = C, \sigma_i^{t-1} = D$)
  b_j regards $a_i^t \in X_j$ as a preferred seller if a_i^t satisfies $\mu_j(a_i^t) - \mu_j(a_j^t) > \varepsilon$, which is the condition of line 9. Since $\sigma_j^{t-1} = D$, $a_j^t = a_j^{t-1}$. Because the condition in line 9 is identical to $\mu_j(a_i^t) - \mu_j(a_j^{t-1}) > \varepsilon$, $a_i^t \in \mathcal{P}_j^t$ as the same as condition (A1.8).

- line 15, Algorithm 4 ($\sigma_j^{t-1} = D, \sigma_i^{t-1} = C$)
  b_i regards $a_j^t \in X_i$ as a preferred seller if a_j^t satisfies $\mu_i(a_j^t) - \mu_i(a_i^t) > \varepsilon$, which is the condition of line 15. Since $\sigma_i^{t-1} = D$, $a_i^t = a_i^{t-1}$. Because the condition in line 15 is identical to $\mu_i(a_j^t) - \mu_i(a_i^{t-1}) > \varepsilon$, $a_j^t \in \mathcal{P}_i^t$ as the same as condition (A1.8).

Otherwise, b_j discovers $s_k \in \mathcal{I}_j^t$, which is an indifferent seller. The following discussion is based on the iteration t at which the condition $|\mathcal{P}_j^t| > 0$ is satisfied. Let $a_j^t \in X_j$ be a seller chosen by b_j at t. For iteration $t+1$, $\mathcal{I}_j^{t+1}$ can be defined as $\mathcal{I}_j^{t+1} = \{s_m \mid \mu_j(s_m) - \mu_j(a_j^t) \leq \varepsilon\}$. For $\mathcal{I}_j^t$, $\mathcal{P}_j^t$, and $\mathcal{I}_j^{t+1}$, $\mathcal{I}_j^{t+1} = \mathcal{I}_j^t \cup \{s_q \mid \mu_j(s_q) - \mu_j(a_j^t) \leq \varepsilon, s_q \in \mathcal{P}_j^t\}$. Since a_j^t is at least contained in $\mathcal{I}_j^{t+1}$, the size of $\mathcal{I}_j^{t+1}$ will be denoted by $|\mathcal{I}_j^{t+1}| \geq |\mathcal{I}_j^t| + 1$. Adversely, $|\mathcal{P}_j^{t+1}| \leq |\mathcal{P}_j^t| - 1$ because $\mathcal{P}_j^{t+1} = \{s_k \mid S \setminus \mathcal{I}_j^{t+1}\}$. Thus, $|\mathcal{P}_j^t|$ decreases at every iteration. Since $|\mathcal{P}_j^t|$ is finite, there must be an iteration u that satisfies $|\mathcal{P}_j^u| = 0$ after the sufficiently large number of iterations. However, this contradicts the assumption since there must be $s_k \in \mathcal{P}_j^u$ in the search of b_j according to the assumption. $\square$

Lemma 2 *With fixed X_j and preference relation of b_j over s_i, $|B_C^t|$ becomes 0 if $\mathcal{I}_j^t$ steadily enlarges.*

Proof: Let us explain b_j must reach iteration t at which b_j can only find $a_k^t \in \mathcal{I}_j^t$. As explained above, $\mathcal{I}_j^t$ increases if a_j^t is updated due to the discovery of preferred seller in $\mathcal{P}_j^t$. From the assumption of Lemma 1, $|X_j|$ is fixed. Moreover, $\mathcal{P}_j^t \cup \mathcal{I}_j^t = X_j$, and $\mathcal{P}_j^t \cap \mathcal{I}_j^t = \varnothing$. Thus, $\mathcal{P}_j^t$ decreases if $\mathcal{I}_j^t$ increases. If $\mathcal{I}_j^t$ continues increasing, finally $\mathcal{I}_j^t$ becomes $\mathcal{I}_j^t = X_j$. In this case, b_j must discover only $a_j^t \in \mathcal{I}_j^t$ in Algorithm 3 and Algorithm 4.

However, even if $\mathcal{I}_j^t \neq X_j$, b_j can reach the situation in which b_j only find $a_j^t \in \mathcal{I}_j^t$. In this case, $\mathcal{I}_j^t$ does not increase. Thus, Algorithm 2 can terminate without satisfying $\mathcal{I}_j^t = X_j$. Besides, by the assumption of Lemma 1, c_i is fixed for all $s_i \in X_j$. Hence, $\mu_j(s_i)$ is also fixed. Thus, if $s_i \in \mathcal{I}_j^t$ at t, there is no iteration t' at which $s_i \in \mathcal{P}_j^{t'}$ after t. Therefore, b_j must reach the iteration t at which b_j can only find $a_k^t \in \mathcal{I}_j^t$.

We show that $\rho(b_j) < 0$ only if $\sigma_j^t = C$. As described in line 26 of Algorithm 2, the definition of $\rho(b_j)$ is as follows.

$$\rho(b_j) = \mu_j(a_j^t) - \gamma_j^t + r_j^t. \tag{A2.1}$$

Since $\mu_j(a_j^t) \geq 0, r_j^t \geq 0$, and $\gamma_j^t \geq 0$, (A2.1) becomes $\rho(b_j) < 0$ only if $\gamma_j^t > 0$.

- If $\sigma_j^t = C$:
  By line 10 of Algorithm 2, $\gamma_j^t > 0$. By (A2.1), $\rho(b_j)$ can be $\rho(b_j) < 0$.

- If $\sigma_j^t = D$:
  By line 13 of Algorithm 2, $\gamma_j^t = 0$. By (A2.1), if $\sigma_j^t = D$, $\rho(b_j)$ must satisfy $\rho(b_j) \geq 0$.

Therefore, $\rho(b_j) < 0$ is satisfied only if $\sigma_j^t = C$.

Next, we show that $\rho(b_j) < 0$ if b_j can only find $a_j^t \in \mathcal{I}_j^t$ at t. At iteration t, if b_j find only $a_j^t \in \mathcal{I}_j^t$, $a_j^t = a_j^{t-1}$ in Algorithm 3 and Algorithm 4. In this case, $\mu_j(a_j^t) = 0$. Moreover, because $\Psi_j^t = \emptyset$ in Algorithm 2 and Algorithm 4 in this case, the procedures of line 18-23 of Algorithm 2 will not be executed. Therefore, $r_j^t = 0$. Furthermore, since $\sigma_j^t = C$, $\gamma_j^t > 0$ according to line 10 of Algorithm 2. By (A2.1), $\rho(b_j) < 0$. Therefore, for b_j with $\sigma_j^t = C$, finally $\rho(b_j) < 0$.

Finally, let us consider the transition of σ_j^t in the situation where $\rho(b_j) < 0$ for all $b_j \in B_C^t$. σ_j^t is determined after the process from line 28 to line 41 of Algorithm 2.

- If $\rho_{\max} < 0$:
  Since $\rho_{\max} < 0$, for b_j and all buyer b_k adjacent to b_j, $\rho(b_j) < 0, \rho(b_k) < 0$. Thus, $\sigma_j^t = C$ and $\sigma_k^t = C$. σ_j^t will be changed to $\sigma_j^t = D$ by line 29-31, Algorithm 2.

- If $\rho_{\max} \geq 0$:
  σ_j^t will be determined to either C or D according to the condition of line 35-37 of Algorithm 2. Since $\rho(b_j) < 0$ for b_j with $\sigma_j^t = C$, $\sigma_m^t = D$ for b_m with $\rho_{\max} \geq 0$. Because $\rho(b_j) < 0$ and $\sigma_m^t = D$, the conditions of line 35, 36 of Algorithm 2 are satisfied. Thus, strategy of b_j becomes $\sigma_j^t = D$ in this case.

As described above, since all buyer $b_j \in B_C^t$ finally decide $\sigma_m^t = D$, finally $|B_C^t| = 0$. □

Theorem 1 *After sufficiently large number of iterations, all buyer $b_j \in B$ eventually choose $\sigma_j^t = D$ at an iteration t if X_j and preference relation of b_j over s_i do not change.*

Proof: Suppose $|B_C^t| > 0$ at every iteration t. As shown in Lemma 1, $b_j \in B$ can only find $s_i \in \mathcal{I}_j^t$ after a sufficient number of iterations. For $b_j \in B_C^t$, $\mu(b_j) = 0$ since $s_i \in \mathcal{P}_j^t$ cannot be found.

Suggestion conducted by b_j cannot be succeeded because any other $b_k \in B$ do not have $s_i \in \mathcal{P}_j^t$. As described above, $a_j^t \in \mathcal{P}_i^t$ holds if each of conditions in line 5 and 15 of Algorithm 4 is met. Then, a_j^t will be added to Ψ_i^t in line 6 and 16 in Algorithm 4. Similarly, $a_i^t \in \mathcal{P}_j^t$ holds if each of conditions in line 3 and 9 of Algorithm 4 is met. a_i^t will be added to Ψ_j^t in line 4 and 10 in Algorithm 4.

Conversely, conditions of line 5 and 15 in Algorithm 4 are not met if $a_j^t \in \mathcal{I}_i^t$. Since no seller will be added to Ψ_i^t in this case, $\Psi_i^t = \varnothing$. Likewise, conditions of line 3 and 9 in Algorithm 4 are not met if $a_j^t \in \mathcal{I}_j^t$. Because no seller will be added to Ψ_j^t in this case, $\Psi_j^t = \varnothing$.

If $\mathcal{P}_j^t = \varnothing$ for b_j, $\Psi_j^t = \varnothing$ as described above since $\mathcal{I}_j^t = X_j$ and $a_j^t \in \mathcal{I}_i^t$. The condition of line 18 in Algorithm 2 is not met in this condition. Therefore, line 19 of Algorithm 2 will not be reached in this case.

Hence, $r_j^t = 0$, and $\rho(b_j) = -\gamma_j^t$. Therefore, $\rho(b_j) < 0$. Considers the following two cases divided according to $|B_D^t|$.

Case 1. $|B_D^t| \neq 0$:

Both of strategies C and D remain at any iteration t in this case. For $b_k \in B_D^t$, $\rho(b_k) \geq 0$ since $r_k^t = 0$ due to $\sigma_k^t = D$. Thus, for $b_j \in B_C^t$ and $b_k \in B_D^t$, $\rho(b_k) > \rho(b_j)$. As described in line 29-31 in Algorithm 2, b_j change σ_j^t from C to D. Hence, $|B_C^t|$ finally becomes zero at an iteration t. This contradicts the assumption.

Case 2. $|B_D^t| = 0$:

In this case, all buyers choose strategy C at any iteration. Since $|B_C^t| = |B|$, $\rho(b_j) < 0$ and $\rho(b_k) < 0$ for any pair of buyers $b_j, b_k \in B_C^t$. As described in line 29-31 in Algorithm 2, $b_j \in B_C^t$ change σ_j^t from C to D if $\rho(b_j) < 0$. Thus, $|B_C^t|$ finally becomes zero at an iteration t. This contradicts the assumption. □

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