ORIGINAL ARTICLE

Bounded and Continuous Linear Operators on Linear 2-Normed Space

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Abstract: In this article, at first basic definitions and properties of linear 2-normed space are presented. Then the author defines bounded operator as an introduction to defined norm of an operator. In addition, the definition of continuous operator on a linear 2-normed space is given. This article proved an operator Γ from a linear 2-normed space $(U, ||.||_U)$ into a linear 2-normed space $(V, ||.||_V)$ is bounded operator if and only if Γ is continuous operator.

Keywords: Linear 2-Normed Space; Linear Operator; Bounded Operator; Continuous Linear Operator

1. Introduction

The idea of linear 2-normed spaces method first has been introduced by S. Gahler[1]. Then many mathematicians studied this subject like Cho, Gupta, White, Freese and others[2-9], and they participated in growth this branch of mathematics. Recently, many authors came out with several results in 2-normed spaces, analogous with that in the ordinary normed spaces. Authors in[10-11] introduced some results of the concept of best approximation of bounded linear 2-functionals on real linear 2-normed spaces. They presented different characterizations of the best approximation elements related these spaces. In[12] Harikrishnan and Ravindran discussed some properties of an operator in linear 2-normed spaces. In addition, they focused on the concept of contraction mapping and fixed point of contraction mappings in linear 2-normed spaces.

The main goal of this article is to define bounded operator on a linear 2-normed spaces in order to defined linear 2-normed of an operator. Then main theorems of the bounded linear operator are proved. In pure and applied physics and mathematics, this method can be applied[13-92].

The structure of this article is as follows. In Section 2 some properties and basic definitions of linear 2-normed space are given. Section 3 is devoted to introduce some concepts such as bounded operator, continuous operator and discuss main results of these concepts.

2. The effect of wavelet and SPIHT in image analysis

In this section some definitions of linear 2-normed space and basic properties are given.

2.1 Definition[93]

A linear 2-normed space (L2N-space) is an ordered pair $(U, \|.,.\|)$, where $\|.,.\|$ is a mapping defined on $U \times U$ satisfying the following conditions:

For all a, b, c in U and $\gamma \epsilon K$,

(N1) $\|a, b\| = 0$ if and only if a and b are linearly dependent.

doi: 10.18686/esta.v7i3.129

(N2) $\|a,b\| = \|b,a\|$.

(N3) $\|\alpha a,b\| = |\gamma|\|a,b\|$

(N4) $\|a,b+c\| \le \|a,b\| + \|b,c\|$.

2.2 Example[94]

Let $U = \mathbb{R}^2$. For every $a,b \in U$ and $\alpha\epsilon(0,1]$, a linear 2-normed define by $\|a,b\| = |a_1 b_2 - a_2 b_1|$ where $a = (a_1,a_2)$ and $b = (b_1,b_2)$ is called standard linear 2-normed.

2.3 Definition[94]

Let $(U,\|.,.\|)$ be an L2N-space. We define open ball $O_e(a,r)$ with center $a \in U$ and radius $r >$ us $r > 0$ as follows:

$O_e(a,r) = \{b \in U: \|b - a, e\| < r\}$

2.4 Definition[94]

Let $(U,\|.,.\|)$ be an L2N-space. We define the closed ball $C_e[a,r]$ with center $a \in U$ and radius $r > 0$ as follows:

$C_e[a,r] = \{b \in U: \|b - a, e\| < r\}$

2.5 Definition[94]

In a L2N-space $(U,\|.,.\|)$, a sequence (a_n) is said to be convergent if $\lim_{n\to\infty}\|a_n - a, c\| = 0$ for each $c \in U$.

2.6 Definition[94]

In a L2N-space $(U,\|.,.\|)$, a sequence (a_n) is said to be cauchy if there exists two elements $b, c \in U$ and positive integer N such that b *and* c are linearly independent $\lim_{m,n\to\infty}\|a_m - a_n, b\| = 0$ and $\lim_{m,n\to\infty}\|a_m - a_n, c\| = 0$ for all $m,n > N$.

2.7 Definition[94]

A L2N-space $(U,\|.,.\|)$, is called a 2-Banach space if every cauchy sequence in U is convergent.

3. Bounded and continuous linear operators

In this section, the author presented the definition of bounded and continuous linear operator on linear 2-normed space and studied some properties of this operator.

3.1 Definition

Suppose each of $(U,\|.,.\|_U)$ and $(V, \|.,.\|_V)$ is a L2N-space. Then an operator $\Gamma: U \to V$ is called bounded operator (B-operator) if there is a real number p such that $\|\Gamma(a),\Gamma(b)\|_V \le p\,\|a,b\|_U$ for all, $b \in U$.

Now, the concept of continuous operator on L2N-space is introduced.

3.2 Definition

Suppose each of $(U,\|.,.\|_U)$ and $(V, \|.,.\|_V)$ is a L2N-space. Then an operator $\Gamma: U \to V$ is called continuous operator (C-operator) at $a \in U$ if for all $\varepsilon > 0$ there is $\delta > 0$ such that $\|\Gamma(b) - \Gamma(a), \Gamma(c)\|_V < \varepsilon$ satisfying $\|b - a, c\|_U < \delta$ for each, $c \in U$.

In the following, the definition of the set of all bounded linear operators is presented.

3.3 Definition

Let $(U,\|.,.\|_U)$ and $(V,\|.,.\|_V)$ be a L2N-space. We say that $\mathbb{B}(U,V) = \{\Gamma: U \to V, \Gamma \text{ is bounded operator}\}$ is the set of all bounded linear operators $\Gamma: U \to V$.

3.4 Theorem

Let $(U,\|.,.\|_U)$ and $(V,\|.,.\|_V)$ be a L2N-space. Then an operator $\Gamma: U \to V$ is (B-operator) if and only if $\Gamma(A)$ is bounded for each bounded set A of U.

Proof

Assume that operator Γ is bounded, then by def. (3.1) there is a real number p such that $\|\Gamma(a), \Gamma(b)\|_V \le p\,\|a,b\|_U$, $\forall\, a,b \in U$. Suppose $A \subseteq U$ be a bounded set, then by there is a real number c *with* $\|a,b\| \le c$, $\forall\ a \in U$. Set $s \ge pc$ where $s > 0$, then we get $\|\Gamma(a),\Gamma(b)\|_V \le p\, c \le s$ implies $\Gamma(A)$ is bounded.

Now for the converse, let $\Gamma(A)$ be bounded, $\exists$ real number $t \ni \|\Gamma(a),\Gamma(b)\|_V \le t, \forall\ a \in U$ then we can find $\in \mathbb{R}$, $\ni$ $t \le c\,\|a,b\|_U$.

Thus, $\|\Gamma(a),\Gamma(b)\|_V \le t \le c\,\|a,b\|_U, \forall a \in U$. Hence Γ is bounded.

3.5 Remark

Suppose that $(U,\|.,.\|_U)$ and $(V, \|.,.\|_V)$ be a L2N-space, furthermore $\Gamma: U \to V$ is a (B-operator). Then $\|\Gamma(a),\Gamma(b)\|_V \le \|\Gamma(a),\Gamma(b)\|\,\|a,b\|_U$.

3.6 Lemma

Let $(U,\|.,.\|_U)$ be a L2N-space. If A and B be a bounded set in U then $A + B$ is a bounded set.

Proof

For all $, \mathscr{b} \in U$, since A and B be a bounded set then $\exists \quad c_1 \in \mathbb{R} \quad \ni \|a, \mathscr{b}\|_U \leq c_1$ and $\exists \quad c_2 \in \mathbb{R} \ni \|\mathscr{b}, z\|_U \leq c_2$ For all a, $\mathscr{b}, z$in U. Now by condition (N4) in definition of L2N-space we get $\|a, \mathscr{b} + z\|_U \leq \|a, \mathscr{b}\|_U + \|\mathscr{b}, z\|_U$.

Thus $\|a, \mathscr{b} + z\|_U \leq c_1 + c_2$. Set $d = c_1 + c_2$ for some $d \in \mathbb{R}$. Hence $\|a, \mathscr{b} + z\|_U \leq d$ implies $A + B$ is bounded set .

3.7 Theorem

Let $(U, \|.,.\|_U)$ and $(V, ||.,.||_V)$ be a L2N-space and $\Gamma: U \to V$ be a (B-operator). We define $\|\Gamma(a), \Gamma(\mathscr{b})\| = \sup_{a \in U} ||\Gamma(a), \Gamma(\mathscr{b})||_V$ for each $\Gamma \in \mathbb{B}(U, V)$ then an ordered pair $(\mathbb{B}(U, V), ||.,.||)$ is a L2N-space.

Proof

Let a , $\mathscr{b}, c \in U$. Then $\|a, \mathscr{b} + c\| \leq \|a, \mathscr{b}\| + \|\mathscr{b}, c\|$.

(N1) $\|\Gamma(a), \Gamma(\mathscr{b})\| = 0 \leftrightarrow \sup_{a \in U} ||\Gamma(a), \Gamma(\mathscr{b})||_V = 0 \leftrightarrow ||\Gamma(a), \Gamma(\mathscr{b})||_V = 0 \leftrightarrow$

a and $\mathscr{b}$ are linearly dependent

(N2) $\|\Gamma(a), \Gamma(\mathscr{b})\|$=$\sup_{a \in U} ||\Gamma(a), \Gamma(\mathscr{b})||_V$= $\sup_{a \in U} ||\Gamma(\mathscr{b}), \Gamma(a)||_V$=$\|\Gamma(\mathscr{b}), \Gamma(a)\|$.

(N3)$\forall a \in U$, $\|\gamma\Gamma(a), \Gamma(\mathscr{b})\| = \sup_{a \in U} ||\gamma\Gamma(a), \Gamma(\mathscr{b})||_V = |\gamma| \sup_{a \in U} ||\Gamma(a), \Gamma(\mathscr{b})||_V = |\gamma| \|\Gamma(a), \Gamma(\mathscr{b})\|$.

(N4) $\|\Gamma(a), \Gamma(\mathscr{b} + c)\| = \sup_{a \in U} ||\Gamma(a), \Gamma(\mathscr{b} + c)||_V \leq \sup_{a \in U} ||\Gamma(a), \Gamma(\mathscr{b})||_V \sup_{a \in U} ||\Gamma(a), \Gamma(c)||_V$= $\|\Gamma(a), \Gamma(\mathscr{b})\|$+$\|\Gamma(a), \Gamma(c)\|$.

Thus $(\mathbb{B}(U, V), ||.,.||)$ is a L2N-space.

3.8 Theorem

Suppose each of $(U, \|.,.\|_U)$ and $(V, ||.,.||_V)$ be a L2N-space. Then an operator $\Gamma: U \to V$ is B-operator if and only if Γ is C-operator.

Proof

Assume that Γ is bounded and consider any $a \in U$ and $\epsilon > 0$. Since Γ is linear, then for every $a, \mathscr{b}, c \in U$ there is $\delta = \frac{\epsilon}{\|\Gamma(a), \Gamma(\mathscr{b})\|}$ such that $||\mathscr{b} - a, c||_U < \delta$. We obtain $||\Gamma(\mathscr{b} - a), \Gamma(c)||_V = ||\Gamma(\mathscr{b} - a, c)||_V \leq \|\Gamma(a), \Gamma(\mathscr{b})\| \; ||\mathscr{b} - a, c||_U < \|\Gamma(a), \Gamma(\mathscr{b})\| \, \delta = \epsilon$. Hence Γ is continuous.

Now for the converse, suppose that Γ is continuous at $a \in U$. Then by def. (3.2), for each $\varepsilon > 0$ there is $\delta > 0$ such that $||\Gamma(\mathscr{b}) - \Gamma(a), \Gamma(c)||_V < \varepsilon$ satisfying $\|\mathscr{b} - a, c\|_U < \delta$ for each $, c \in U$. Let $A \subseteq U$ be a bounded set and take any $s \in A$. Set $= (a + s)$, where. Then $||\Gamma(s), \Gamma(c)||_V = ||\Gamma(\mathscr{b} - a), \Gamma(c)||_V = ||\Gamma(\mathscr{b}) - \Gamma(a), \Gamma(c)||_V < \varepsilon$. Thus $\Gamma(A)$ is bounded and by Th. (3.4) Γ is bounded.

3.9 Theorem

Suppose each of $(U, \|.,.\|_U)$ and $(V, ||.,.||_V)$ be a L2N-space. Then an operator $\Gamma: U \to V$ is C-operator if Γ is a C-operator at single point $a \in U$.

Proof

Suppose that Γ is C-operator at a point $a \in U$, then by Th. (3.8) Γ is a C-operator.

4. Conclusion

The main goal in the present article is to define the linear 2-normed of a bounded linear operator $\Gamma: U \to V$. Main theorems of the bounded linear operator Γ are investigated such as Γ is bounded operator if and only if Γ is continuous.

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