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SOCIAL STRUCTURE, MARKETS AND INEQUALITY

Julien Gagnon (University of Cambridge)
Sanjeev Goyal (University of Cambridge)

ABSTRACT

The interaction between social structure and markets remains a central theme in the social sciences. In some instances, markets can build on and enhance social networks' economic role; in other contexts, markets appear to be in direct competition with social networks. The impact of markets on inequality and welfare is also varying: while markets can sometimes offer valuable outside options to marginalised individuals, in other situations only well connected and better off individuals can benefit from them.

In this paper, our goal is to understand the economic mechanisms that can explain these different empirical patterns.

We develop a simple model that combines social networks and a mix of network-exchange and market-exchange activities. The key to understanding the empirical patterns and phenomena lies in the relation between the two activities i.e., whether they are (strategic) complements or substitutes. Social connectedness facilitates the adoption of the market action if the two activities are complements; the converse is true in case of substitutes. Inequality in a social structure is typically reinforced by the market in case the two actions are complements; the converse holds true if they are substitutes.

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Julien Gagnon* Sanjeev Goyal†

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*Faculty of Economics and Trinity College, Cambridge

†Faculty of Economics and Christ's College, Cambridge. Email: sg472@cam.ac.uk. We thank Gabriel Arsenault, Francis Bloch, Yann Bramoullé, Nicolas Carayol, William Carpenter, Vasco Carvalho, Vessela Daskalova, Matthew Elliott, Edoardo Gallo, Alex Harris, Jean-William Laliberté, Kaivan Munshi, Anja Prummer, Fernando Vega-Redondo and seminar participants at Bordeaux, Cambridge, Brussels and Marseille for helpful comments. Julien Gagnon thanks the Gates Fellowship for financial support. Sanjeev Goyal is grateful to a Keynes Foundation and the Cambridge-INET Institute for financial support.

1 Introduction

"Commerce is a pacific system, operating to cordialise mankind, by rendering... individuals useful to each other... The invention of commerce... is the greatest approach towards universal civilisation that has yet been made." (Thomas Paine, 1792: 215)

"[The] legacy [of social morality] has diminished with time and with the corrosive contact of the active capitalist values - and more generally with the greater anonymity... of industrial society... As individual behavior has been increasingly directed to individual advantage, habits and instincts based on communal attitudes and objectives have lost out." (Fred Hirsch, 1976: 117-18)

The relationship between market institutions - based on anonymous transactions and ruled by price mechanisms - and exchange systems founded on social networks remains a central theme in the social sciences. In some contexts, markets are associated with the erosion of traditional institutions and indigenous cultures: for instance, market liberalization is found to erode caste networks in India by providing women with outside economic opportunities (Munshi and Rosenzweig, 2006). In other contexts, markets can be key to the revival of traditions and cultures: for example, when social networks are dense, emerging tourism markets can contribute to the revitalization of endangered languages (Kroshus Medina, 2006). The growth of new information technologies illustrates this tension also. Mobile telephony markets, for example, can build on and enhance social networks' information-sharing role. Jensen (2007) and Srinivasan and Burrell (2013) show that the large-scale adoption of mobile phones enabled fishermen in Kerala (India) to share information about local fish markets and fishing sites within their network of friends, relatives and business partners. On the other hand, the rise of online social networks such as Facebook and Twitter is intimately associated with the decline of the market for newspapers (Newman, 2009; Currah, 2009).

In this paper, our goal is to understand the economic mechanisms that can account for these different empirical outcomes.

Our model combines social structure and a mix of network and market exchange activities. Agents can partake in network exchange (action x) and in market exchange (action y), respectively at price $p_x > 0$ and $p_y > 0$. A player's payoffs to action x are increasing in the number of neighbours in the network who adopt the same action: this feature captures the personalized nature of network-based exchange, with one's payoffs depending on one's possible exchange partners in one's network. This feature can also be viewed as a reduced form specification for sustained reciprocal exchange between players. In contrast, market exchange is anonymous and short-term, and agents are price-takers: payoffs to action y thus depend solely on its price. The final ingredient is the relationship between the returns to the network and market actions: we allow for both a *complements* and a *substitutes* relation. This framework allows us to study who adopts the network and market action, respectively, and how this choice affects aggregate welfare and inequality.

We begin by examining the trade-offs an individual faces. She can adopt the network action x at price p_x, and the market action y at price p_y. To fix ideas, assume that the two activities are perfect *substitutes*. Letting the (gross) returns to the market action be 1, she compares a payoff of $1 - p_y$ with the payoff from the network action. This latter payoff depends on the number of her neighbors and their choices. The choice of her neighbors in turn depend on how many neighbors they have and how many of them adopt x. Given the prices and the returns, suppose it takes q neighbors adopting action x to justify her choice of action x. We are then led naturally to the notion of a set of individuals who each have q or more neighbors, whose neighbors in turn each have q or more neighbors, and so forth. We say a *q-connected club* is the maximal set of players having strictly more than q links with other players belonging to the club. Theorem 1 shows that equilibrium behavior in our model is fully characterized by choices in the q-connected club. The result also develops a relationship between prices and the complements *vs* substitutes relation and the threshold q.

We use this characterization to study the relation between social structure and market participation in detail. We begin with a simple question: who partake in market exchange? Theorem 1 states that if market exchange and network exchange are substitutes, then the members of the q-connected club adopt network action, while those outside choose the market action (if $1 - p_y > 0$). By contrast, in the case of complements, participation in market exchange goes hand in hand with social connections: the members of the q-connected club adopt both actions, while those outside may adopt neither action. We are also able to study the effect of different social structures on behavior: an increase in social connectedness expands the q-connected club and therefore diminishes market participation when the two actions are substitutes; the converse is true when they are complements (Proposition 1).

We then turn to the issue of welfare. Interestingly, we show that an increase in payoffs from the market action always raises welfare when the actions are complements, but that it may reduce welfare when they are substitutes (Proposition 2 and Corollary 1). The intuition behind this result is that when individual j switches from the network action x to the market action y when the latter's payoffs increase, she imposes a network externality on her neighbors who stay with x. This network externality may be larger than the benefits j achieves by opting for y. Conversely, in the complements case, the availability of market exchange always raises the returns from network exchange and has thereby the potential of a positive multiplier effect on welfare.

Finally, we examine inequality. We find that an increase in the payoffs from the market action y typically raises inequality when the market is a complement to the network action, while the converse holds true when they are substitutes (Propositions 3 and 4). In the complements case, the market action typically favors the players who are already better-off i.e., players in the q-connected club. In the substitutes case, the market action offers an outside option to players who benefit the least from the network, and as such clearly has the potential to reduce inequality.[1]

The principal contribution of our paper is a model that offers a parsimonious explanation for a range of empirical phenomena pertaining to the interaction between social structure and markets. We illustrate the scope of our model with a range of important applications. In some applications, markets are direct substitutes to networks. This is the case for market liberalization and caste networks in India: here markets crowd out networks by offering an outside option to poorly connected individuals. So doing, they reduce inequality, but also reduce welfare for those who keep exchanging in the network. In the case of online social networks and traditional media, it is the market that is crowded out by the raising payoffs to network exchange. Online social networks (e.g. Facebook) indeed provide individuals with opportunities to share information cheaply with their connections, thus making costly traditional sources of information (e.g. newspapers) redundant. On the other hand, in other applications (mobile telephony and fishermen in Kerala, India; tourism markets and the preservation of local language and culture), networks and markets complement each other. Better connected individuals are more able to take advantage of markets, which ultimately raise welfare but also lead to greater inequality. Section 6 below elaborates on these points. There we map these different contexts into our framework and then illustrate how our theoretical predictions are consistent with the empirical outcomes.

Our paper contributes to a long-standing and distinguished literature concerned with the social impact of markets and the interaction between market and non-market exchange. There is, on the one hand, the classical *doux-commerce* argument, going back to the eighteenth century, which purports that markets reinforce durable and peaceful social relations (e.g. Montesquieu, 1748; Paine, 1792; Condorcet, 1795). This argument relies on the view that markets open opportunities for exchange, which reinforce individuals' incentive to cooperate with each other to cease these opportunities. On the other hand, some scholars have argued that the expansion of markets, accompanied by wide ranging changes in attitudes and institutions, can crowd out social ties and deplete welfare (e.g. Polanyi, 1944; Thompson, 1963; Scott, 1977; Gudeman, 2008; for a popular recent statement close to this view, see Sandel, 2012).[2] This argument relies on the view that community-based economies, or *moral economies*, rest on norms of mutual support and reciprocity that "outside options" like markets undermine. Our

[1] In the basic model, individuals are identical except for differences in network connections. In an extension we consider the case of heterogeneity on other dimensions, such as human capital. Our main results on the relation between markets, networks and inequality continue to hold if this individual characteristic is positively correlated to network connections.

[2] For eloquent accounts of this debate, see Hirschman (1977; 1982) and Besley (2012).

framework accounts for these two opposed arguments in a single model: while markets can increase individuals' incentives to engage in network exchange when they are complements, the opposite occurs when they are substitutes. While welfare always increases in the former case, it may decrease in the latter.

Our paper also draws on and contributes to the literature on the limits of markets and the need for alternative forms of organization. Economists have explored these limits along a number of dimensions and have highlighted the important role of institutions, in particular the firm, to mitigate transaction costs (Coase, 1937; North and Thomas, 1973; Arrow, 1974; Williamson, 1975). In a similar vein, sociologists have emphasised social networks' role in information dissemination, which is essential for the proper functioning of markets (e.g .Granovetter, 1985). This idea, which suggests a *complements* relation between information, networks and markets, has been formalised and applied in a number of economic situations (e.g. Montgomery, 1991; Casella and Rauch, 2003; Calvo-Armengol and Jackson, 2004; Galeotti, 2010, Galeotti and Merlino, 2014). There is also a small but important body of work that views markets and social networks as substitutes (e.g. Kranton, 1996; Munshi and Rosenzweig, 2006). Our paper provides a unifying framework for this body of work by incorporating the complements/substitutes aspect of market vs network activity within a model of social networks.[3]

Finally, we contribute to the theoretical study of economic behavior in social networks. In particular, the study of games on networks is currently very active; see e.g., Ballester, Calvo-Armengol and Zenou (2006), Bramoulle and Kranton (2007), Galeotti et al., (2010), and Goyal and Moraga (2001). Jackson and Zenou (2014) provide a survey of this work. Interest has centered on games with a single action and payoffs that depend on own and neighbors' actions. The analysis of these games has highlighted the usefulness of the distinction between strategic substitutes and complements in understanding the effects of networks on behavior. Our paper extends this literature by adding an anonymous market action, in addition to the network action. This allows us to address substantive questions on the relation between networks and markets and the implications for inequality, that lie outside the scope of the existing literature.

The rest of this paper is organized as follows. Section 2 presents the model, while section 3 provides a characterization of equilibrium. Section 4 presents the study of market participation, welfare and inequality. Section 5 studies heterogeneity with respect to human capital. Section 6 discusses evidence from a number of empirical contexts to illustrate the scope of our model. Section 7 concludes.

[3]In section 6, we illustrate how the model in Munshi and Rosenzweig (2006) may be obtained as a special case of our framework.

2 The Model

Consider a group of players $N = \{1, 2, ...n\}$ with $n \geq 3$. A link between two players takes on binary values: $g_{ij} = 1$ signifies the existence of a link, while $g_{ij} = 0$ indicates the absence of a link. Denote by $\mathbf{g}$ the graph of links. We assume $g_{ii} = 0$ by convention. We define $N_i(\mathbf{g}) = \{j \in N : g_{ij} = 1\}$ as player i's *neighborhood* and denote her degree by $k_i = |N_i(\mathbf{g})|$.

Every player i chooses two actions, x_i and y_i, where $x_i \in X \equiv \{0, 1\}$ and $y_i \in Y \equiv \{0, 1\}$. Players' action set is denoted $\mathcal{A} = X \times Y$ and the set of all action profiles is denoted by $\mathcal{A}^n$. The action x is the "network exchange action", with price $p_x > 0$. The action y is the "market exchange action", with price $p_y > 0$. Let $\mathbf{p} = (p_x, p_y)$. Player i's payoffs function is written as:

$$\Pi_i(x, y|\mathbf{p}, \mathbf{g}) = \sum_{j \in N_i(\mathbf{g})} x_j x_i + y_i + \theta y_i \sum_{j \in N_i(\mathbf{g})} x_j x_i - p_x x_i - p_y y_i \tag{1}$$

where θ captures the substitutability or complementarity between x and y.

We now discuss the key elements of the payoff function (1).

First, note that the network action x displays local complementarity. This specification captures an important characteristic of network-based exchange systems: one's payoffs depends on the number of one's potential exchange partners in one's network. One may also think of this specification as a reduced form for sustained reciprocal exchange between players: such exchange takes place if and only if two players partake in it, and there is no free-riding in equilibrium (e.g. Kranton, 1996). Note also that action x has fixed cost p_x. One may however think of situations wherein p_x rises or falls with the number of neighbours partaking in x. For example, the cost of reciprocal exchange may be fixed for each reciprocal relationship, in which case p_x would rise in the number of neighbours engaging in x. Conversely, if x is "learning a communal language", for instance, then p_x may fall in opportunities to practice that language, and hence in the number of neighbours who partake in it. Our results are robust to such modifications

Second, the payoffs to the market action y are exogenous, for all players. Unlike network exchange, a player's payoffs from market exchange do not depend on her network. This specification applies for large markets where market thickness is not markedly influenced by players' decisions.[4]

[4]Market thickness, however, could easily be accounted for by assuming that players' payoffs to y display *global* complementarity. This would give rise to the possibility of coordination failure in market exchange.

Third, the relation between actions x and y is captured simply by the parameter θ. In particular, when $-1 < \theta < 0$, x and y are imperfect substitutes, while they are perfect substitutes when $\theta = -1$. When $\theta > 0$, x and y are complements (and the more so the larger θ). For simplicity we focus on and contrast the two cases $\theta = -1$ and $\theta = 1$. Appendix B provides a full characterization of equilibria for any $\theta \geq -1$.

We now describe the solution concept of the game. For a graph $\mathbf{g}$, a strategy profile $(\mathbf{x}^*, \mathbf{y}^*)$ is an *equilibrium* if for every $i \in N$, (x_i^*, y_i^*) maximizes $\Pi_i\left((x_i, y_i), \mathbf{x}_{-i}^*, \mathbf{y}_{-i}^*|\mathbf{g}\right)$. Local complementarity in x creates the potential for coordination failure; it is easy to see that $x_i = 0$ for all $i \in N$ is an equilibrium even in cases when all players would prefer to coordinate on $x = 1$. As our interest is in the relation between networks and markets, we wish to avoid these coordination issues. We say that a strategy profile $(\mathbf{x}, \mathbf{y})$ Pareto-dominates another profile $(\mathbf{x}', \mathbf{y}')$ if $\Pi_i(\mathbf{x}, \mathbf{y}|\mathbf{g}) \geq \Pi_i\left(\mathbf{x}', \mathbf{y}'|\mathbf{g}\right)$ for all $i \in N$, with an inequality strict for at least one $j \in N$. An equilibrium $(\mathbf{x}^*, \mathbf{y}^*)$ is said to be maximal if there does not exist another equilibrium $(\mathbf{x}', \mathbf{y}') \in \mathcal{A}^n$ that Pareto-dominates it. We focus solely on cases of maximal equilibrium (ME).

Furthermore, we are interested in understanding the impact of markets on social welfare. Given a network $\mathbf{g}$ and a price vector $\mathbf{p}$, aggregate welfare from a strategy profile $(\mathbf{x}, \mathbf{y})$ is given by:

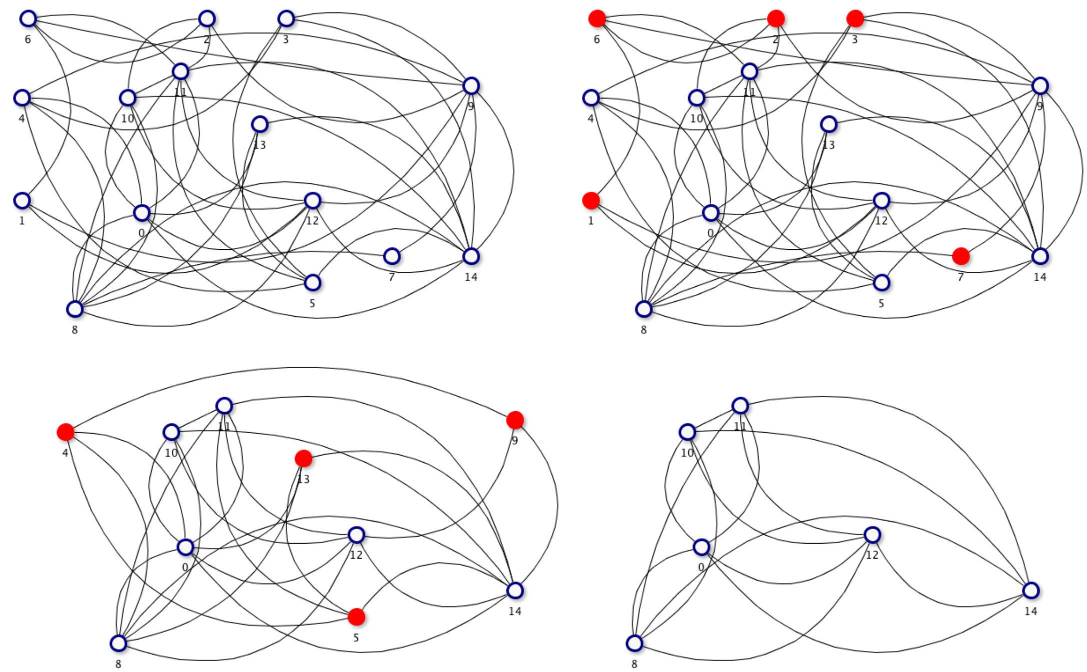
$$W(\mathbf{x}, \mathbf{y}|\mathbf{p}, \mathbf{g}) = \sum_{i \in N} \Pi_i(\mathbf{x}, \mathbf{y}|\mathbf{p}, \mathbf{g}). \tag{2}$$

3 Networks, prices and behavior

This section proves existence and uniqueness and then provides a characterization of the ME. It is useful to start the analysis with the decision problem of an individual who has a pure choice between $x = 1$ and $x = 0$. The payoff to $x_i = 1$ will be the number of her neighbors who choose $x = 1$ less the price p_x. Thus, she will choose $x = 1$ if and only if the number of her neighbors choosing $x = 1$ is higher than p_x. Similarly, her neighbors will choose $x = 1$ if a sufficient number of their own neighbors choose $x = 1$. This motivates the idea of a *q-connected club*, which we formally develop here.

Denote by $S_q \subseteq N$ the set of players who have strictly more than $q \in R_+$ links in network $\mathbf{g}$. Define $\hat{S}_q \subseteq S_q$ as a set of players who all have strictly more than q links with players who belong to $\hat{S}_q$. Denote by $\mathcal{S}_q$ the largest such set. We hereafter refer to $\mathcal{S}_q$ as the *q-connected club*. It is immediate that for any network $\mathbf{g}$ and for any $q \in R_+$, there is a unique q-connected

Figure 1: The 4-connected club



Top left: initial graph. **Top right**: delete all nodes with $k \leq 4$. **Bottom left**: among the nodes remaining, delete those with $k \leq 4$. **Bottom right**: the 4-connected club obtains when no further iteration is possible.

club.[5] We now provide an algorithm to obtain the q-connected club in any network.

Algorithm *Consider any network* $\mathbf{g}$. *To find its q-connected club (for* $q \in R_+$*), first delete all the nodes (and their links) in* $\mathbf{g}$ *for which* $k \leq q$. *Label the residual graph* $\mathbf{g_1}$. *In step 2, delete all the nodes (and their links) in* $\mathbf{g_1}$ *for which* $k \leq q$. *Iterate until no node with* $k \leq q$ *remains, which happens when* $\mathbf{g_t} = \mathbf{g_{t+1}}$. *The residual graph in this last step is the q-connected club.*

By way of illustration, consider the network on Figure 2. Suppose that we want to find the 4-connected club. First, find all the nodes with $k \leq 4$, and delete them and their links. In step 2, delete the nodes with 4 or less links in the residual network from step 1. Proceed likewise unless no node with $k \leq 4$ remains. The remaining nodes form the 4-connected club.

We now examine the incentives of individuals to choose different actions. Consider first

[5] In the complete network, $\mathcal{S}_1 = \mathcal{S}_2 ... = \mathcal{S}_{n-2} = N$, while $\mathcal{S}_{n-1} = \{\emptyset\}$. In a star network, $\mathcal{S}_0 = N$, while $\mathcal{S}_k = \{\emptyset\}$ for all $k > 0$.

the case where x and y are substitutes. In this case, a player will always choose either $(x_i = 1, y_i = 0)$, $(x_i = 0, y_i = 1)$ or $(x_i = 0, y_i = 0)$. Simple calculations reveal that for player i to prefer $x_i = 1$ to any other profile, she must have more than q_0 neighbours who play $x = 1$, where:

$$q_0 = \begin{cases} 1 - p_y + p_x & if\, p_y < 1 \\ p_x & if\, p_y \geq 1 \end{cases} \tag{3}$$

Second, consider the case of complements. This case is a little more involved as both actions can be chosen together. Denote by q_1 the number of links to neighbors playing $x = 1$ required for player i to be indifferent between $x_i = y_i = 1$ and any other action profile. Formally:

$$q_1 = \begin{cases} \frac{p_x}{2} & if\, p_y < 1 \\ \max\left\{p_y - 1,\, \frac{p_x + p_y - 1}{2}\right\} & if\, p_y \geq 1 \end{cases} \tag{4}$$

We are now ready to state our first main result.

Theorem 1 *An equilibrium exists and is generically unique. Let* $(\mathbf{x}^*, \mathbf{y}^*)$ *be the maximal equilibrium.*

- ***Substitutes*** $\theta = -1$*: If* $p_y < 1$*, then* $(x_i^*, y_i^*) = (1, 0)$ *for* $i \in \mathcal{S}_{q_0}$ *and* $(x_i^*, y_i^*) = (0, 1)$ *for* $i \notin \mathcal{S}_{q_0}$*. If* $p_y \geq 1$*, then* $(x_i^*, y_i^*) = (1, 0)$*, for* $i \in \mathcal{S}_{q_0}$ *and* $(x_i^*, y_i^*) = (0, 0)$ *for* $i \notin \mathcal{S}_{q_0}$*.*
- ***Complements*** $\theta = 1$*: If* $p_y < 1$*, then* $(x_i^*, y_i^*) = (1, 1)$ *for* $i \in \mathcal{S}_{q_1}$ *and* $(x_i^*, y_i^*) = (0, 1)$ *for* $i \in \mathcal{S}_{q_1}$*. If* $p_y \geq 1$*, then* $(x_i^*, y_i^*) = (1, 1)$ *for* $i \in \mathcal{S}_{q_1}$*,* $(x_i^*, y_i^*) = (1, 0)$ *for* $i \in \mathcal{S}_{p_x} \backslash \mathcal{S}_{q_1}$ *and* $(x_i^*, y_i^*) = (0, 0)$ *for* $i \in N \backslash \mathcal{S}_{q_1} \cup \mathcal{S}_{p_x}$*.*

Proof. *All proofs in Appendix A.*

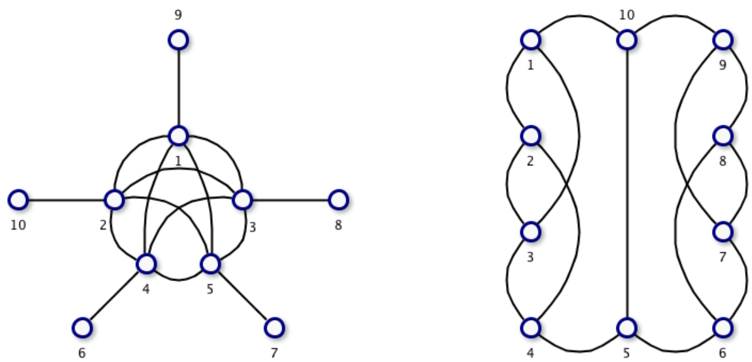
To illustrate the implications of Theorem 1, we examine the equilibrium in the regular network and the core-periphery, as presented in Figure 2.[6] Denote by $\mathcal{S}_q^{CP}$ and $\mathcal{S}_q^{R}$ the q-connected clubs in the CP and the regular networks, respectively. Observe that the relevant level of threshold connectivity q is determined by both $\mathbf{p}$ and θ.

Example 1 *Suppose that prices are given by* $(p_x, p_y) = (3.2, 0.5)$*.*

- ***Substitutes****: Then,* $q_0 = 1 - p_y + p_x = 3.7$*. Since* $\mathcal{S}_{3.7}^{CP} = N_c$ *and* $\mathcal{S}_{3.7}^{R} = \{\emptyset\}$*, then at the the maximal equilibrium* $\{x_i, y_i\}_{i \in N_P} = (0, 1)$ *and* $\{x_i, y_i\}_{i \in N_C} = (1, 0)$ *in the CP network, and* $\{x_i, y_i\}_{i \in N} = (0, 1)$ *in the regular network.*
- ***Complements****: Then,* $q_1 = \frac{p_x}{2} = 1.6$*. Since* $\mathcal{S}_{1.6}^{CP} = N_c$ *and* $\mathcal{S}_{1.6}^{R} = N$*, then at the maximal equilibrium* $\{x_i, y_i\}_{i \in N_P} = (0, 1)$ *and* $\{x_i, y_i\}_{i \in N_C} = (1, 1)$ *in the core-periphery network, and* $\{x_i, y_i\}_{i \in N} = (1, 1)$ *in the regular network.*

[6] In a CP network, core players (with population n_c) form a clique and have degree $k_c = n_c - 1 + n_p/n_c$, while periphery players (with population n_p) are only connected to a single core player (thus $k_p = 1$). In a regular network, $k_i = k \geq 1$ for all $i \in N$.

Figure 2: Core-periphery and regular networks



To conclude this section, we briefly relate our concept of q-connected club to q-cohesiveness discussed in Morris (2000). Morris (2000) defines a subset of players S as *q-cohesive* if all players in S have at least a fraction q of their neighbors in S. The notion of q-connected club is related to cohesiveness in the sense that it requires recursive connectivity, but there is one important difference. The concept of a q-connected club relies on an absolute number of links, while the cohesive set is defined in terms of proportion of links. This difference has a substantive content in our context as q-connected clubs will refer to well connected individuals. There is no such presumption in a q-cohesive set.

4 Market participation, welfare and inequality

4.1 Market participation

A central theme in the social sciences is how social structure and markets interact. We examine the receptivity of social structures to market activity: are sparse or dense social structures more receptive to markets? Within a society, are highly connected and central players or poorly connected and marginalized individuals more receptive to markets?

We know from Theorem 1 that, given a network $\mathbf{g}$, there generically exists a unique maximal equilibrium $(\mathbf{x}^*, \mathbf{y}^*)$. We define market participation, $\mathcal{Y}(\mathbf{p},\mathbf{g})$, as the fraction of players who choose $y = 1$ in this equilibrium.

$$\mathcal{Y}(\mathbf{p},\mathbf{g}) \equiv \frac{\sum_{i \in N} y_i^* (\mathbf{p}, \mathbf{g})}{N} \tag{5}$$

The following result summarizes our study of market participation.

Proposition 1 *For $\theta = -1$ and any two pairs $(\mathbf{p}, \mathbf{g})$ and $(\mathbf{p}', \mathbf{g}')$, $\mathcal{Y}(\mathbf{p}, \mathbf{g}) > \mathcal{Y}(\mathbf{p}', \mathbf{g}')$ if and only if $|\mathcal{S}_{q_0}(\mathbf{p}, \mathbf{g})| < |\mathcal{S}_{q_0}(\mathbf{p}', \mathbf{g}')|$. For $\theta = 1$, and any two pairs $(\mathbf{p}, \mathbf{g})$ and $(\mathbf{p}', \mathbf{g}')$, $\mathcal{Y}(\mathbf{p}, \mathbf{g}) > \mathcal{Y}(\mathbf{p}', \mathbf{g}')$ if and only if $p_y < 1$ and $|\mathcal{S}_{q_1}(\mathbf{p}, \mathbf{g})| > |\mathcal{S}_{q_1}(\mathbf{p}', \mathbf{g}')|$. In particular:*

- **Networks**. *$\mathcal{Y}(\mathbf{p}, \mathbf{g})$ (weakly) decreases with the addition of a link in the network when $\theta = -1$; the converse holds for $\theta = 1$.*

- **Prices**. *If $\theta = -1$, then $\mathcal{Y}(\mathbf{p}, \mathbf{g})$ is (weakly) decreasing in p_y and (weakly) increasing in p_x. If $\theta = 1$, then $\mathcal{Y}(\mathbf{p}, \mathbf{g})$ is (weakly) decreasing in both p_y and p_x.*

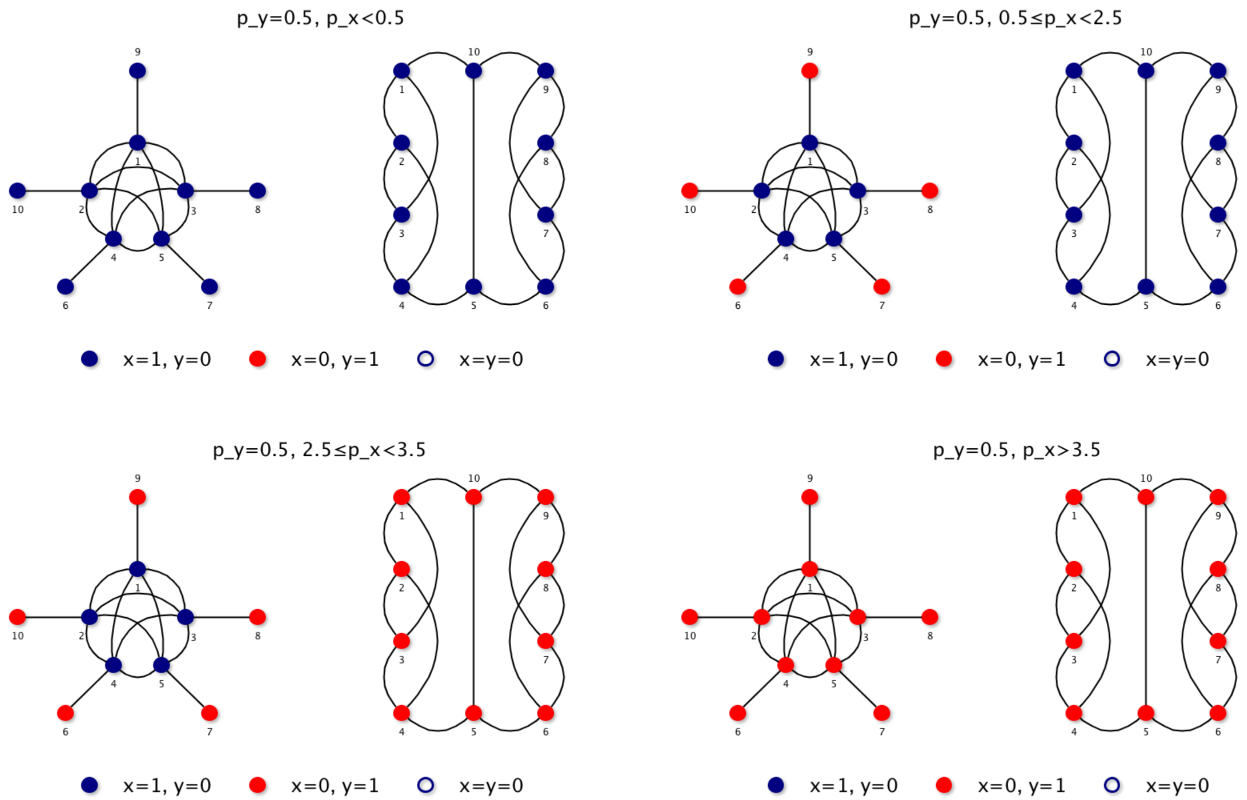
The first part of Proposition 1 is a corollary of Theorem 1: whether a price profile and social structure facilitates or hinders market participation depends on the q-connected club. In case of substitutes, the smaller this club, the deeper market participation; the converse holds true for complements. A first observation is thus that the effect of adding links on market participation will depend on whether x and y are complements or substitutes. Indeed, since adding links has the direct effect of weakly expanding the size of the q-connected club for any q, it will make y less attractive if $\theta = -1$, but will foster its adoption if $\theta = 1$.

We now examine more closely the relation between prices and networks, on the one hand, and the size of the q-connected club, on the other hand. It is useful to first observe the ways in which network architecture and location within a network affects market participation. To get an impression of the issues involved, consider the core-periphery (CP) and regular networks introduced in Figure 2. Denote by $\mathcal{Y}\left(\mathbf{p}, \mathbf{g}^{CP}\right)$ and $\mathcal{Y}\left(\mathbf{p}, \mathbf{g}^{R}\right)$ the market participation at the maximal equilibrium in the CP network and the regular network, respectively. We now compare market penetration in these two networks: to fix ideas, let us consider the case of substitutes, and fix $p_y = 0.5$. Note that adopting the action y brings $1 - p_y = 0.5$ to any player. When $p_x < 0.5$, then $q_0 = 0.5 + p_x < 1$, and so all players in both the regular and the CP networks strictly prefer x to y. Hence, $\mathcal{Y}\left(\mathbf{p}, \mathbf{g}^{R}\right) = \mathcal{Y}\left(\mathbf{p}, \mathbf{g}^{CP}\right) = 0$. If p_x lies in the range $0.5 \leq p_x < 2.5$, $1 < q_0 < 3$, and so periphery players prefer to switch to y while core players and players in the regular networks strictly prefer to stick to x. As a result, $\mathcal{Y}\left(\mathbf{p}, \mathbf{g}^{R}\right) = 0 < \mathcal{Y}\left(\mathbf{p}, \mathbf{g}^{CP}\right) = \frac{n_p}{n}$. If p_x lies in the range $2.5 \leq p_x < 3.5$, then $3 < q_0 < 4$, and all players in the regular network now strictly prefer to switch to $y = 1$, while only the core players prefer to stick to x. Consequently, market participation in the regular network is then higher than in the CP network, with $\mathcal{Y}\left(\mathbf{p}, \mathbf{g}^{R}\right) = 1 > v = \frac{n_p}{n}$. Finally, when $3.5 \leq p_x$, then all players strictly prefer y in both networks, and so $\mathcal{Y}\left(\mathbf{p}, \mathbf{g}^{R}\right) = \mathcal{Y}\left(\mathbf{p}, \mathbf{g}^{CP}\right) = 1$. Figure 3 summarizes these results.

Proposition 1 analyses market penetration in networks where there is no coordination failure on x. The next remark summarize the effects on $\mathcal{Y}(\mathbf{p},\mathbf{g})$ of a failure to coordinate on the ME.

Remark 1 *Suppose that players coordinate on an equilibrium $(\hat{\mathbf{x}}, \hat{\mathbf{y}})$ which is not the ME. Denote by $\hat{\mathcal{Y}}(\mathbf{p},\mathbf{g})$ the market participation at that equilibrium. Then, if $\theta = -1$, $\hat{\mathcal{Y}}(\mathbf{p},\mathbf{g}) \geq \mathcal{Y}(\mathbf{p},\mathbf{g})$; conversely, if $\theta = 1$, $\hat{\mathcal{Y}}(\mathbf{p},\mathbf{g}) \leq \mathcal{Y}(\mathbf{p},\mathbf{g})$.*

Figure 3: Market participation in the CP and regular networks
The case of substitutes



Remark 1 states that coordination failure will foster (hamper) market participation when x and y are substitutes (complements). When x and y are substitutes, a failure to coordinate on x (weakly) reduces the value of action x to players. This makes action y all the more attractive as an outside option. Conversely, when x and y are complements, coordination failures will make action x less profitable, which will reduce players' demand for y.

4.2 Aggregate welfare

This section studies the impact of markets on aggregate welfare. To do so, we compare welfare in a society prior to and after the introduction of the market action, y. Let $\mathcal{W}(\mathbf{p}, \mathbf{g})$ denote the aggregate welfare in the maximal equilibrium $(\mathbf{x}^*, \mathbf{y}^*)$, with:

$$\mathcal{W}(\mathbf{p}, \mathbf{g}) = W(\mathbf{x}^*, \mathbf{y}^* | \mathbf{p}, \mathbf{g}). \tag{6}$$

Given $(\mathbf{p}, \mathbf{g})$, we say that an outcome (x, y) is efficient if $W(\mathbf{x}, \mathbf{y}|\mathbf{p}, \mathbf{g}) \geq W(\mathbf{x}', \mathbf{y}'|\mathbf{p}, \mathbf{g})$, for all $(x', y') \in \mathcal{A}^n$.

We begin our study with the following result on aggregate welfare.

Proposition 2 *Aggregate welfare* $\mathcal{W}(\mathbf{p}, \mathbf{g})$ *(weakly) increases with the size of the* q*-connected club, for any* q*. In particular:*

- **Networks**. *In a regular network, the unique maximal equilibrium is efficient; in non-regular networks, it may be inefficient. Moreover, adding a link to a network (weakly) increases* $\mathcal{W}(\mathbf{p}, \mathbf{g})$.
- **Prices**. *When* $\theta = -1$, $\mathcal{W}(\mathbf{p}, \mathbf{g})$ *is (weakly) decreasing in* p_x *and non-monotonic in* p_y. *When* $\theta = 1$, $\mathcal{W}(\mathbf{p}, \mathbf{g})$ *is (weakly) decreasing in both* p_x *and* p_y.

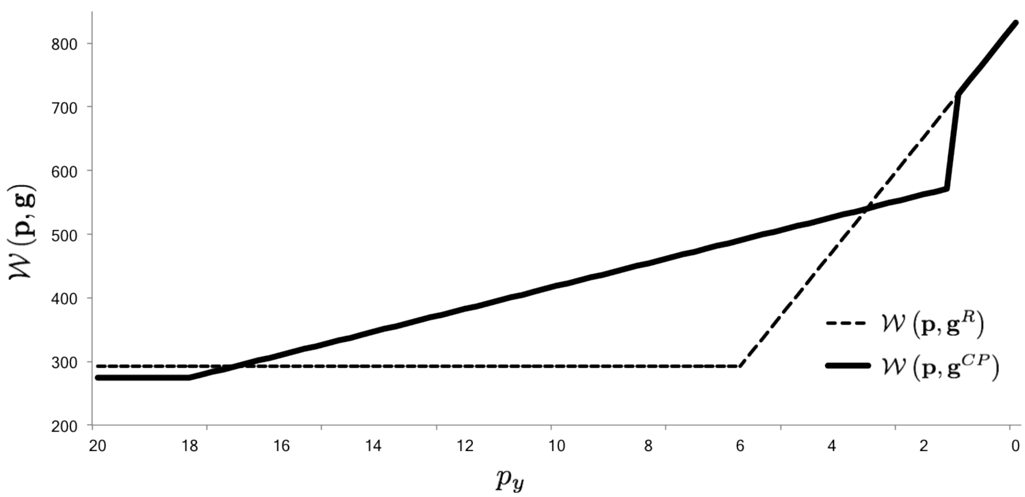
The first statement in the Proposition 2 follows from the observation that a bigger q-connected club always offers a larger potential for social earnings; therefore, it weakly raises aggregate welfare.

Since the network action offers local complementarities, players do not fully internalize the positive (social) payoffs of playing $x = 1$. There is thus a risk of under-provision of x compared to the social optimum. In regular networks, however, under-provision is avoided. Indeed, suppose that $x_i = 1$ for all $i \in N$ is the efficient outcome. Consider player j with k neighbours who are all playing $x = 1$. If j decides to play $x = 0$, it is necessarily because x is not profitable to her *even if she fully enjoys the local benefits of her neighbours playing* $x = 1$. Since the network is regular, what is true for j is true for all other players, which means that the actions x and y are undertaken in regular networks if and only if they maximize the welfare of all players. This reasoning clearly does not hold in non-regular networks. For instance, periphery players in CP networks may under-provide x relative to the efficient outcome.

We move now to the effects of additional network links on aggregate welfare. Proposition 2 establishes that aggregate payoffs are monotonically increasing in links. The intuition behind this result is straightforward. In the case of complements, an additional link weakly raises the marginal returns from action x and expands the q-connected club, for any q. In turn, greater adoption of action x weakly raises the marginal returns from action y. Thus, individual payoffs must weakly increase with additional links; this also holds true at the maximal equilibrium. In the case of substitutes, the result follows from the above reasoning and from noting that the payoffs to action y are independent of others' choices.

We now briefly study the role of network heterogeneity on aggregate welfare. Our analysis once again suggests that network heterogeneities have complex effects on aggregate payoffs: this notably stems from the complex patterns of market participation. To illustrate these effects, we compare again a regular network with a CP network. Denote by $\mathcal{W}\left(\mathbf{p}, \mathbf{g}^R\right)$ and $\mathcal{W}\left(\mathbf{p}, \mathbf{g}^{CP}\right)$ the aggregate welfare at the maximal equilibrium in the regular network and in

Figure 4: Aggregate welfare in a regular network and a CP network
$n = 10$, $l = 225$ and $p_x = 1.5$



the CP network, respectively. Suppose that x and y are complements and fix $p_x = 1.5$. Figure 4 plots the aggregate welfare attained in these networks as a function of p_y.

If $p_y \geq 6$, then y is too costly to be adopted in either network. All players in the regular network choose $x = 1$, while only core players choose $x = 1$ in the CP network. Even though the latter derive individually more payoffs than any player in the regular network, they are few, and so $\mathcal{W}(\mathbf{p}, \mathbf{g}^R) > \mathcal{W}(\mathbf{p}, \mathbf{g}^{CP})$. If p_y is raised to $4 \leq p_y < 5$, y becomes cheap enough for core players in the CP network, but remains too costly for all other players. For any $4 \leq p_y < 4.5$, higher market participation in the CP network entails $\mathcal{W}(\mathbf{p}, \mathbf{g}^R) < \mathcal{W}(\mathbf{p}, \mathbf{g}^{CP})$. If $1.5 \leq p_y < 4$, then y becomes cheap enough for players in the regular network. This entails full market participation in the regular network, leading to a welfare reversal again with $\mathcal{W}(\mathbf{p}, \mathbf{g}^R) > \mathcal{W}(\mathbf{p}, \mathbf{g}^{CP})$ for any $1.5 \leq p_y < 3.5$. Finally, if $p_y < 1.5$, then all players in both networks choose $x = y = 1$ and $\mathcal{W}(\mathbf{p}, \mathbf{g}^R) = \mathcal{W}(\mathbf{p}, \mathbf{g}^{CP})$.

The last part of Proposition 2 describes the welfare impact of p_x and p_y. Note first that players' utility is always weakly increasing in the number of their neighbors who choose $x = 1$. Therefore, a falling p_x always increases their utility either directly (since x becomes cheaper) or indirectly (if more of their neighbors decide to engage in x). The effect of p_y on welfare is however more intricate. Clearly, when $\theta = 1$, welfare increases when p_y falls. However, when $\theta = -1$, two effects may oppose each other. On the one hand, a falling p_y entails a direct increase in the utility of players who play $y = 1$. On the other hand, a falling p_y may push certain players to switch from x to y, entailing a decrease in the payoffs of their neighbors who play $x = 1$. The net effect of a decrease in p_y on welfare may thus be either negative or positive.

A long-lasting and important concern in the social sciences has been the potentially deleterious welfare effects of markets on welfare. Our framework allows an explicit examination of the circumstances under which the introduction of markets is welfare-enhancing. The following result summarizes our analysis.

Corollary 1 *In the case of complements, the introduction of the market action always (weakly) increases aggregate welfare. In the case of substitutes, the introduction of the market action may lower aggregate welfare.*

Observe first that in the case of complements, the introduction of y can indeed at best foster players' adoption of the network action x or, at worse, leave it unchanged. In that case, the introduction of y implies (weakly) larger individual payoffs, and hence also a larger aggregate welfare. However, if x and y are substitutes, the introduction of y is not generally welfare-enhancing due to network externalities. We provide an example to illustrate this possibility.

Consider the CP network on Figure 1, and suppose that $p_x < 1$. Prior to the introduction of y, all players necessarily choose $x = 1$. Suppose now that the market action y becomes available. If $p_y > p_x$, clearly no player desires to switch from $x = 1$ to $y = 1$. However, when $0 < p_y < p_x < 1$, then all periphery players switch to $y = 1$, while core players stick to $x = 1$. On the one hand, periphery players increase their payoffs by $p_x - p_y < 1$ following their switch. On the other hand, a periphery player's switch entails a decrease in the benefits of the core player she is connected to of exactly 1. The net effect is thus always strictly negative. Hence, in any CP network, the introduction of the market action y lowers aggregate welfare whenever $0 < p_y < p_x < 1$.

The above example motivates a closer examination of the nature of networks where markets lower welfare. Are there networks for which markets always raise welfare even when markets are a substitute for network actions? Proposition 2 provides us with a first response to this question. In particular, our analysis suggests that it is the heterogeneity in the social structure that may cause the aggregate welfare to fall with the introduction of markets (as the equilibrium in regular networks is always efficient and weakly increasing with the introduction of markets).

4.3 Inequality

We now turn to the impact of markets on inequality. The measurement of inequality is a vast subject and the literature has developed a wide and sophisticated set of measures over time (see e.g. Sen, 1997). In order to appreciate the key factors at work, we begin our analysis with a simple measure that captures the inequalities between the extremes of a payoff distribution; we then move to a much-used measure that is sensitive to the whole payoff distribution (Gini coefficient).

First, we examine the ratio of the highest payoffs to the lowest payoffs in the maximal equilibrium. Given network $\mathbf{g}$ and prices $\mathbf{p}$, this ratio is denoted by $\mathcal{R}(\mathbf{g}, \mathbf{p})$ and is defined as follows:

$$\mathcal{R}(\mathbf{p}, \mathbf{g}) \equiv \frac{1 + \max \{\pi_i(\mathbf{x}, \mathbf{y})\}_{i \in N}}{1 + \min \{\pi_i(\mathbf{x}, \mathbf{y})\}_{i \in N}} \tag{7}$$

Note that a rising $\mathcal{R}(\mathbf{p}, \mathbf{g})$ implies increasing inequality. Let $\mathcal{R}^0(\mathbf{p}, \mathbf{g})$ denote the inequality prior to the introduction of y, and $\mathcal{R}^1(\mathbf{p}, \mathbf{g})$ its level after.

The ratio $\mathcal{R}(\mathbf{p}, \mathbf{g})$ captures relative changes in the payoffs of the "wealthiest" players compared to those of the "poorest". It is close in spirit to other traditional metrics of inequality, including the *range*, the *20:20 ratio* or the *Palma ratio.* The range consists in the difference between the payoffs of the wealthiest and the poorest individuals of a population. The *20:20 ratio* and the *Palma ratio* consist respectively in the income ratio of the wealthiest 20% to the poorest 20% and in the income ratio of the wealthiest 10% to the poorest 40%. While $\mathcal{R}(\mathbf{p}, \mathbf{g})$ has the same structure as these two measures, it requires less information about the income distribution and the network structure.

Proposition 3 *When $\theta = -1$, the introduction of the market action y (weakly)* decreases $\mathcal{R}(\mathbf{p}, \mathbf{g})$. *When $\theta = 1$, it (weakly)* increases $\mathcal{R}(\mathbf{p}, \mathbf{g})$ *except when $\mathcal{S}_{q_1} = N$; in this case its effect is ambiguous.*

In the case of substitutes, individuals choose either x or y. It is players who are most disadvantaged in the traditional social structure who find action y to be an especially attractive outside option. Thus, the introduction of y has the potential of raising the payoffs of the players at the bottom of the income distribution. The payoffs of players at the top of the distribution, on the other hand, can only decline as some of their neighbors may switch from x to y. Putting together these two forces yields the result that markets unambiguously lower inequality in the case where the market and the network actions are substitutes.

When x and y are complements, the effect on inequality depends on the social structure and the prices of the two actions. This is because unlike the substitutes case, the lowest and highest payoffs may both increase with the introduction of markets. Therefore, the effect on inequality will depend on the relative magnitude of these increases. In spite of this complication, we show that so long as not everyone adopts both x and y, markets unambiguously increase inequality.

We sketch an outline of how inequality can be worsened by introducing a market for the case where $p_y \leq 1$. In this case, all players adopt y in the post-market equilibrium. Suppose however that no one adopts x: in that case, as x and y are complements, it must be the case that no one adopts x in the pre-market situation. For all individuals, payoffs equal 0 prior to and $1 - p_y$ after the introduction of the market action. Inequality thus remains unchanged. Suppose now that in the post-market equilibrium, some (but not all) players do adopt x. While the payoffs of x-adopters increase of *at least* $1 - p_y + q_1/2$, the payoffs of non-adopters increase of only $1 - p_y$. Hence, inequality increases. Similar arguments can be used to show that inequality worsens in case $p_y > 1$, so long as not everyone adopts both x and y. When *all* players adopt both x and y, however, matters are more complicated and the effect on inequality may go in either direction. The intuition is that the worst-off players may benefit relatively more or less than the best-off players from the newly available y, depending on prices. The following example illustrates this possibility.

Example 2 *Consider the example of the core-periphery network in Figure 1; suppose* $p_x = 1.1$. *Note that* $\mathcal{R}^0(\mathbf{p}, \mathbf{g}) = 3.9$. *Suppose y is introduced at* $p_y = 0.05$. *Then,* $\mathcal{R}^1(\mathbf{p}, \mathbf{g}) = 3.81$, *which indicates a* falling $\mathcal{R}(\mathbf{p}, \mathbf{g})$. *Conversely, suppose y becomes available at* $p_y = 0.5$. *then* $\mathcal{R}^1(\mathbf{p}, \mathbf{g}) = 5.21$. *This indicates an* increasing $\mathcal{R}(\mathbf{p}, \mathbf{g})$.

Proposition 3 provides a clear-cut prediction with regard to the impact of markets. While the result is sharp, it neglects all but the extreme payoffs of the distribution. To address this concern, we examine the effects of markets on inequality as measured by the *Gini coefficient*. The Gini coefficient, in addition to taking into consideration the poorest and the wealthiest individuals, fully accounts for those in between. Given $(\mathbf{p}, \mathbf{g})$, we denote the Gini-coefficient in the maximal equilibrium by $\mathcal{G}(\mathbf{g}, \mathbf{p})$. The following result summarizes the impact of markets on inequality, as measured by the Gini-Coefficient.

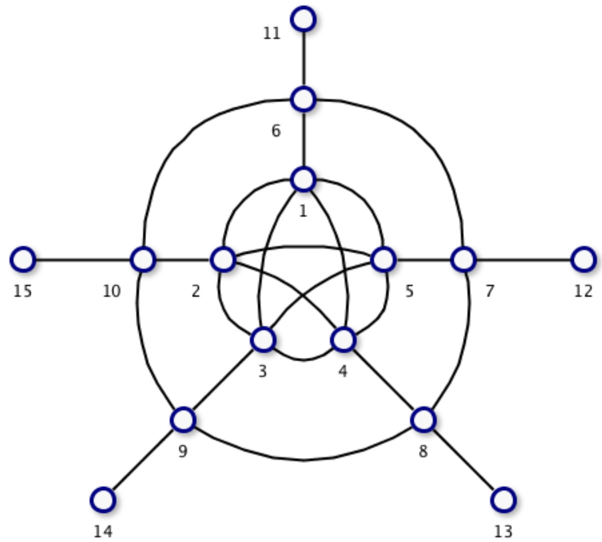
Proposition 4 *When* $\theta = -1$, *the introduction of the market action y (weakly)* decreases $\mathcal{G}(\mathbf{p}, \mathbf{g})$, *except if it decreases* $\mathcal{W}(\mathbf{p}, \mathbf{g})$ *and* $\mathcal{Y}(\mathbf{p}, \mathbf{g}) < 1$. *In that case, its effect on* $\mathcal{G}(\mathbf{p}, \mathbf{g})$ *is ambiguous. When* $\theta = 1$, *the introduction of y (weakly) increases* $\mathcal{G}(\mathbf{p}, \mathbf{g})$, *except if: (i)* $p_y < 1$; *or (ii)* $p_y \geq 1$, $S_{p_x} \setminus S_{q_1} = \{\emptyset\}$ *and* $S_{q_1} \neq \{\emptyset\}$. *In these two cases, the effect of y on* $\mathcal{G}(\mathbf{p}, \mathbf{g})$ *is ambiguous.*

The first thing to note about Proposition 4 is its similarity to Proposition 3. Indeed, a quick comparison between Propositions 3 and 4 shows that the effect of the introduction of markets on $\mathcal{G}(\mathbf{p}, \mathbf{g})$ and $\mathcal{R}(\mathbf{p}, \mathbf{g})$ typically goes in the same direction in most cases.

To illustrate the effect of markets on inequality as measured by Gini-coefficient, consider the network presented in Figure 4. Figure 5 plots the changes to the Lorenz curve brough by the introduction of y, for both the cases where $\theta = 1$ and $\theta = -1$ and for specific $\mathbf{p}$. The Lorenz curve represents the cumulative distribution of payoffs.[7] Let $\mathcal{L}_0(i|\mathbf{p}, \mathbf{g})$ denote the Lorenz curve prior to the introduction of y, and $\mathcal{L}_1(i|\mathbf{p}, \mathbf{g})$ the Lorenz curve after. Note that the first

[7]Note that the Gini-Coefficient is simply the ratio of the area between the Lorenz curve and the 45-degree line to the total area below the 45-degree line.

Figure 5: A three-layer society



third of the population on the x axis comprises the poorest players of the network on Figure 4 (always players 11 to 15). The second third and the last third always comprise, respectively, players 6 to 10 and 1 to 5.

The effect of markets on $\mathcal{G}(\mathbf{p}, \mathbf{g})$ is explained as follows. Suppose $p_x = 2.2$. At the maximal equilibrium, before the introduction of y, players 1 to 10 play $x = 1$ while players 11 to 15 play $x = 0$. Players 1 to 5, 6 to 10 and 11 to 15, respectively, thus have payoffs of 2.8, 0.8 and 0, for a total aggregate welfare of $\mathcal{W}_0(\mathbf{p}, \mathbf{g}) = 18$. First consider the case of substitutes ($\theta = -1$) and suppose y is introduced at $p_y = 0.4$. While players 1 to 10 do not change their strategy, players 1 to 5 now play $y = 1$, which brings them payoffs of 0.6. The new aggregate welfare is $\mathcal{W}_1(\mathbf{p}, \mathbf{g}) = 21$. Since the payoffs of players 1 to 10 have not changed but total welfare has increased, the share of aggregate welfare going to players 1 to 10 falls. On the other hand, the payoffs of players 11 to 15 rises from 0 to 0.6, and so their share of aggregate welfare increases. As a result, the Lorenz curve after the introduction of y, $\mathcal{L}_1(i|\mathbf{p}, \mathbf{g})$, lies *above* $\mathcal{L}_0(i|\mathbf{p}, \mathbf{g})$, as shown on the left graph of Figure 5. This means that the Gini-coefficient falls after the introduction of y.

Next consider the case of complements ($\theta = 1$) and suppose that y is introduced at $p_y = 4.1$. While players 6 to 15 do not change their strategy, players 1 to 5 adopt y as they enjoy sufficiently large benefits from complementarity. The payoffs to players 6 to 15 do not change, but those of players 1 to 5 rise to 4.7. The aggregate welfare rises to $\mathcal{W}_1(\mathbf{p}, \mathbf{g}) = 27.5$. As a result, the share of aggregate welfare going to players 6 to 15 falls, while that of players 1 to 5 rises. The Lorenz curve after the introduction of y, $\mathcal{L}_1(i|\mathbf{p},\mathbf{g})$, is completely *below* $\mathcal{L}_0(i|\mathbf{p},\mathbf{g})$, as shown on the right graph of Figure 5. Therefore, $\mathcal{G}(\mathbf{p},\mathbf{g})$ must have increased after the introduction of y.

We finally turn to the case where the effects of markets on inequality differ depending on the measure of inequality we pick. The first difference occurs when $\theta = -1$: when $\mathcal{W}(\mathbf{p},\mathbf{g})$ decreases and $\mathcal{Y}(\mathbf{p},\mathbf{g}) \in (0,1)$, the effect of the introduction of y on $\mathcal{G}(\mathbf{p},\mathbf{g})$ can go either way, while it always weakly decreases $\mathcal{R}(\mathbf{p},\mathbf{g})$. The reason is that while the worst-off players always see their payoffs increase comparatively to those of the best-off players when markets are introduced, the latter may nevertheless see their "share of the pie" increase if $\mathcal{W}(\mathbf{p},\mathbf{g})$ decreases sufficiently. This may happen whenever the payoffs of players in between fall sharply. The following example illustrates this argument. Consider again the graph on Figure 5, and fix $p_x = 0.75$. Before the introduction of y, the individual payoffs of players 1 to 5, 6 to 10 and 11 to 15, respectively, amount to 4.25, 3.25 and 0.25, entailing $\mathcal{W}_0(\mathbf{p},\mathbf{g}) = 38.75$ and $\mathcal{G}_0(\mathbf{p},\mathbf{g}) = 0.3441$. Now suppose that y is introduced at a price $p_y = 0.7$. The individual payoffs of players 1 to 5, 6 to 10 and 11 to 15, respectively, then amount to 4.25, 2.25 and 0.3. While the payoffs of players 11 to 15 increase in comparison to those of players 1 to 5 (entailing a falling $\mathcal{R}(\mathbf{p},\mathbf{g})$), the payoffs of the latter clearly increase in proportion of $\mathcal{W}(\mathbf{p},\mathbf{g})$ due to the important fall in the payoffs of players 6 to 10. As a result, $\mathcal{W}_1(\mathbf{p},\mathbf{g}) = 34$ and $\mathcal{G}_1(\mathbf{p},\mathbf{g}) = 0.3873$, indicating a *rising* $\mathcal{G}(\mathbf{p},\mathbf{g})$. Now suppose that $p_y = 0.2$. Then, $\mathcal{W}_1(\mathbf{p},\mathbf{g}) = 36.5$ and $\mathcal{G}_1(\mathbf{p},\mathbf{g}) = 0.3150$, indicating a *falling* $\mathcal{G}(\mathbf{p},\mathbf{g})$ compared the situation without y.

The second difference occurs when $\theta = 1$. The effect of the introduction of y on $\mathcal{R}(\mathbf{p},\mathbf{g})$ is always weakly positive, except when players are all playing $x = y = 1$ after the introduction of y, in which case it is ambiguous. The effect on $\mathcal{G}(\mathbf{p},\mathbf{g})$ will naturally be ambiguous too in this case, but will be ambiguous whenever some (but not all) players who were playing $x = 0$ before the introduction of y play $x = y = 1$ after its introduction. The intuition is that while the payoffs of the best-off players obviously increase compared to those of the worst-off players, what the final distribution of payoffs looks like is not clear. For example, if the worst-off players (playing $x = y = 0$, for instance) are few but the introduction of y enables a large number of players to change their strategy from $x = 0$ to $x = y = 1$ after the introduction of y, then the final distribution may improve in comparison to the initial one. Conversely, if most players who player $x = y = 1$ after the introduction of y were also playing $x = 1$ before its introduction, then clearly y is likely to aggravate inequality.

5 Extension: Heterogeneous agents

While the framework developed so far highlights the consequences of network heterogeneity, individuals may be heterogeneous in other dimensions that affect the extent to which they can benefit from markets. For example, individuals may differ in their levels of human capital or initial wealth. Individuals with more human capital may be able to benefit more from market opportunities than others. Likewise wealthy individuals may find the opportunity cost of the market action y smaller than poorer peers. This motivates a study of the combined effect of network heterogeneity with other types of heterogeneity.

In the following analysis, we assume that players are also heterogeneous with respect to their *human capital*. Player i's human capital is captured by the parameter $h_i \in [0,1]$. *Ceteris paribus*, a higher h_i enables player i to derive greater benefits from the market action y. Taking into account this new parameter, player i's payoffs function is rewritten as:

$$\Pi_i\left(x,\, y|\mathbf{p},\mathbf{g}\right) = \sum_{j\in N_i(\mathbf{g})} x_j x_i + h_i y_i + \theta y_i \sum_{j\in N_i(\mathbf{g})} x_j x_i - p_x x_i - p_y y_i \tag{8}$$

We next redefine the threshold quantities that are used in our characterization of maximal equilibrium. Let $q_{0,i}$ be the number of links to neighbours playing $x = 1$ required for player i to be indifferent between $x_i = 1$ and $x_i = 0$ when x and y are *substitutes*, with:

$$q_{0,i} = \begin{cases} h_i - p_y + p_x & if\, p_y < h_i \\ p_x & if\, p_y \geq h_i \end{cases} \tag{9}$$

When x and y are *complements*, denote by $q_{1,i}$ the number of links to neighbours playing $x = 1$ required for player i to be indifferent between $x_i = y_i = 1$ and any other action profile. Formally:

$$q_{1,i} = \begin{cases} \frac{p_x}{2} & if\, p_y < h_i \\ \max\left\{p_y - h_i,\, \frac{p_x+p_y-h_i}{2}\right\} & if\, p_y \geq h_i \end{cases} \tag{10}$$

We next adapt the definition of the q-connected club introduced earlier. Consider a vector $\mathbf{q} = \{q_i\}_{i\in N}$ ascribing value q_i to each player in N. Denote by S_{q_i} the set of players who have a degree strictly larger than their ascribed value. Hence, $i \in S_{q_i}$ if and only if $k_i > q_i$, for all $i \in N$. Define $\hat{S}_{q_i} \subseteq S_{q_i}$ as a set of players who each have strictly more than links than their ascribed value q_i with players who belong to $\hat{S}_{q_i}$. Denote by $\mathcal{S}_{q_i}$ the largest such set, which is unique. We hereafter refer to $\mathcal{S}_{q_i}$ as the q_i-*connected club.*

Example 3 *Consider the arbitrary graph presented on Figure 7. Suppose now that players are either "high" or "low". Ascribe the value* 2 *to high types, and* 5 *to low types; hence,* $q_i = q_H = 2$ *iff* $i = H$ *and* $q_i = q_L = 5$ *iff* $i = L$*. To find the* q_i*-connected club, find all nodes with* $k_i \leq q_i$ *and delete them and their links. Then, repeat the previous step for the nodes in the remaining sub-graph. Repeat until no node with* $k_i \leq q_i$ *remains. The remaining nodes form the* q_i*-connected club.*

Theorem 2 *A maximal equilibrium exists and is generically unique. Let* $(\mathbf{x}^*, \mathbf{y}^*)$ *be the maximal equilibrium.*

- ***Substitutes*** $(\theta = -1)$*: If* $p_y < h_i$*, then* $(x_i^*, y_i^*) = (1, 0)$ *for* $i \in \mathcal{S}_{q_{0,i}}$ *and* $(x_i^*, y_i^*) = (0, 1)$ *for* $i \notin \mathcal{S}_{q_{0,i}}$*. If* $p_y \geq h_i$*, then* $(x_i^*, y_i^*) = (1, 0)$*, for* $i \in \mathcal{S}_{q_{0,i}}$ *and* $(x_i^*, y_i^*) = (0, 0)$ *for* $i \notin \mathcal{S}_{q_{0,i}}$*.*
- ***Complements*** $(\theta = 1)$*: If* $p_y < h_i$*, then* $(x_i^*, y_i^*) = (1, 1)$ *for* $i \in \mathcal{S}_{q_{1,i}}$ *and* $(x_i^*, y_i^*) = (0, 1)$ *for* $i \in \mathcal{S}_{q_{1,i}}$*. If* $p_y \geq h_i$*, then* $(x_i^*, y_i^*) = (1, 1)$ *for* $i \in \mathcal{S}_{q_{1,i}}$*,* $(x_i^*, y_i^*) = (1, 0)$ *for* $i \in \mathcal{S}_{p_x} \backslash \mathcal{S}_{q_{1,i}}$ *and* $(x_i^*, y_i^*) = (0, 0)$ *for* $i \in N \backslash \mathcal{S}_{q_{1,i}} \cup \mathcal{S}_{p_x}$*.*

Note that the results presented in Propositions 1, 2 and 3 are all robust to heterogeneity in human capital (with the exception of efficiency in regular networks: the maximal equilibrium is no longer necessarily efficient). The results on inequality, however, may change substantially if human capital is *negatively* related to membership in the q-connected club. To see why, consider the CP network presented of Figure 2. Suppose that core and periphery players have human capital $h_c = 1$ and $h_p = 10$, respectively. Suppose first that $\theta = -1$ and that $p_x = 3$. Before the introduction of y, only core players play $x = 1$, which brings them payoffs of 1 each. As a result, $\mathcal{W}_0(\mathbf{p}, \mathbf{g}) = 5$ and $\mathcal{R}_0(\mathbf{p}, \mathbf{g}) = 2$. Now suppose that the market action y is introduced at $p_y = 2$. While core players stick to $x = 1$, periphery players now play $y = 1$, which brings them payoffs of 8. Consequently, $\mathcal{W}_1(\mathbf{p}, \mathbf{g}) = 45$ and $\mathcal{R}_0(\mathbf{p}, \mathbf{g}) = 8$. This example thus shows that when $\theta = -1$, $\mathcal{R}(\mathbf{p}, \mathbf{g})$ may in fact *increase* following the introduction of y.

6 Applications

This section illustrates the scope of our framework through a discussion of number of empirical phenomena. We show that variations in θ and social structure account for the disparate outcomes on market adoption, welfare and inequality in these situations.

6.1 Language and Local Cultures

In this sub-section, we discuss the relation between markets, culture and languages. The empirical record is mixed: extensive studies show that markets and globalization are associated with both cultural change and persistence (Inglehart and Welzel, 2005; Inglehart and Baker, 2000). We illustrate how social structure and the strategic relation between markets and local cultures help explaining the different outcomes.

6.1.1 Caste Networks, Globalization and English Language Schooling

Munshi and Rosenzweig (2006) (henceforth MR) explore the impacts of market forces on traditional institutions. The economic liberalization of the Indian economy in the 1990s entailed a shift toward the corporate and finance sectors, which increased the returns to white-collar jobs for which knowledge of English was necessary. MR estimate that in the city of Mumbai, the liberalization of the economy increased the premium to English education (comparative to education in Marathi) by roughly 25% over the 1990s. However, they show that boys of working-class and heavily networked sub-castes (*jatis*) took much less advantage of the opportunities of the new economy than their female counterparts. As a result, while the gap in English education between girls of high and low castes disappeared, the gap for boys remained (roughly) intact. These educational differences have important implications for occupational choices: indeed, MR show that education in English prepares children for white-collar jobs,while education in Marathi channels them into blue-collar jobs.

We first map this empirical setting onto our model. Parents choose to send their child to an English school or to a Marathi school. The returns to English education are simply a function of the child's ability and the (exogenously given) premium to education in English. Let "English education" be the market action y, with exogenous benefits and cost p_y. The returns to education in Marathi, in contrast, depend on how many in the sub-caste also choose it: MR report in some parts of Bombay, 68% of the men in working-class jobs found employment through a relative or a member of the community.[8] There is thus a positive externality associated with participation in the network, and hence with the traditional occupational choice in the *jati*. Let "Marathi education" be the network action x, which exhibits local complementarity and has cost p_x. Observe that the choices of English or Marathi education are mutually exclusive; this is consistent with our assumption that $\theta = -1$. Finally, following the discussion in MR, we assume that men are connected to other men within a sub-caste, while girls have very few job connections. Finally, following the discussion in MR, we assume that while men are connected to other men within a sub-caste, girls cannot make significant use of these networks and have thus very few connections.

Given this mapping, Proposition 1 predicts that the adoption of y should *ceteris paribus* be higher for girls than for boys in working-class sub-castes. This prediction is clearly consistent with MR's findings.[9] Proposition 2 predicts that while the payoffs of girls should increase, the payoffs of boys who choose education in Marathi should (weakly) decrease due to boys "leaving" the network. While no direct evidence is provided on this, MR recognize this possibility and suggest that caste networks "might place tacit restrictions on the occupational

[8]In particular, they note that in a household survey conducted between 1982 and 2001, 62% of parents who chose education in Marathi reported "closer community ties" as a factor (while "career opportunities" was the most important factor for parents' decision to send their child to schools in English).

[9]In our basic model, in a regular network, all players should choose the same action. However, the authors argue that the boys of sub-castes networks who adopted English education are those who were more talented. This heterogeneity in outcomes is easily accounted for if we extend our model to human capital. With human capital (or "ability") the payoffs to English education can be written as $h_i - p_y$. Note that in this extended model, the adoption of y by more talented (higher h_i) individuals should leave the players who retain action x worse-off. This is exactly what is reported in MR.

mobility of their members to preserve the integrity of the network" and enhance welfare (MR: 1230). Lastly, Proposition 3 predicts that income inequality between girls and boys in working-class sub-castes should decrease. This is consistent with MR's finding that a previously disadvantaged group (girls) surpasses boys in educational attainment in the most heavily networked sub-castes. This is exactly what MR report: "while it is generally believed that the benefits of market liberalisation have accrued disproportionately to the elites in developing countries... we find, instead, that a previously disadvantaged group (girls) might surpass boys in educational attainment" (MR, 2006: 1250-51).

6.1.2 Tourism and the Preservation of Identity

The revival and preservation of endangered local cultures and languages is a major theme in social anthropology. Kroshus Medina's (2003) ethnographic work in the village of Succotz, situated next to the Mayan ruins of Xunantunich in Belize, illustrates how tourism markets may be instrumental in revitalizing and preserving local cultures and languages. She argues that since archaeological work made the site of Xunantunich available, tourism has presented

> ... new possibilities for Succotzenos to claim or reclaim Maya identity and culture... Tourism to Xunantunich has had a broader effect on local ethnic hierarchies: as tourists demonstrate interest in ancient Maya culture by generating demand for goods that reflect that culture, positive value attaches to the Maya label. Villagers are very cognizant of this fact. (Kroshus Medina, 2003: 361)

Tourism, she explains, has enhanced the value of traditional Maya knowledge – language and handicrafts – that most young Succotzenos lacked. De Azeredo Grunewald's (2002) anthropological study of the Pataxo Indians in Porto Seguro, Brazil, offers a very similar account. Tourist demands, he claims, "have sponsored a cultural revival process". In addition to handicraft, this cultural revitalization is particularly visible in the use of indigenous languages, which had disappeared: "With tourism, indigenous names began to be used to reinforce craft work sale identity... The introduction of "words in the indigenous language" is another strategy for crafts sale, or for interacting with tourists, or authorities. Therefore, the language undergoes a continuous process of revival" (De Azeredo Grunwald, 2002: 1013-14).

We now map this evidence onto our mode. Let y stand for "tourism activity" (e.g. selling handicraft) and x stand for "cultural activity" (e.g. learning indigenous language). The returns to x depend on the number of neighbours who adopt it: for instance, the returns to learning a local language depend on the number of people in one's network one can speak the local language with. In contrast, the returns to y depend on (exogenous) market opportunities. As discussed above, x and y may reinforce each other: cultural activities expand tourism activities (due to tourists' cultural demand), and tourism activities increase the benefits of engaging in cultural activities (e.g. through increased status of local cultures, increased business opportunities). This corresponds in our model to the situation where $\theta = 1$. Proposition

1 predicts that an increase in market opportunities raises the returns and hence the viability of local culture and language. This is consistent with the evidence.

A common finding in sociolinguistics points to the importance of social structure in the preservation and revitalization of local cultures and especially languages (e.g. Milroy and Milroy, 1985, 1999; Fishman, 1990, 1991; Milroy, 2002). In particular, close-knit social networks are necessary for language revitalization while the loosening of social networks may well be a factor in language erosion (Sallabank, 2010). The reason is that speakers of a local language require other speakers to interact with, who themselves require local language speakers to interact with, and so forth. Close-knit social networks thus provide the necessary opportunities to interact with other speakers and learn (Sallabank, 2010; Hulsen et al., 2002). This finding is consistent with the key role of the q-connected club in sustaining action X, in our model (Theorem 1 and Proposition 1).

6.2 Information Technology

The development and spread of modern information and communication technologies has had large economic effects. We now illustrate how our model helps to explain important empirical phenomena associated with these technologies.

6.2.1 Online Social Networks and The Decline of Traditional Media

The explosive growth of online social networks is a the defining features of the last decade. Some of these social networks (e.g. Facebook) have, overtime, become prominent platforms for news sharing. The Reuters Institute for the Study of Journalism (RISJ) reports that more than half of the population of many countries (e.g. Brazil, Spain, Italy and Finland) use Facebook for news purposes, and roughly 60% of Facebook users find, share or discuss news every week (RISJ, 2014). The use of online social networks is strongly related to age: in the countries it surveyed, the RISJ reports that roughly 40% of $18-24$ year-olds find news via online social networks, as opposed to only 17% for people aged over 55. Online sources (including social networks) are already, by far, the most important sources of news for younger individuals. This rise of online news exchange in the last few years has been parallel to a sharp decline of traditional media such as print newspapers (Newman, 2009; Currah, 2009).

These sweeping changes in the media can be analysed through the lens of our model. Suppose that to access the news, individuals can either exchange information among neighbours in their network (e.g. join Facebook and share news: action x) or exchange in the market (e.g. buy print newspapers or a subscription to TV channels: action y). The returns to x for a player i depend on how many of i's friends exchange and discuss news in the network, while the returns to y are exogenous. We denote by p_x the time cost of exchanging information with friends (e.g. sharing articles or "posting" information on Facebook), while p_y represents the price of marketed media, such as print newspapers. Since the same news can be accessed from both sources, traditional media and social networks are substitutes ($\theta = -1$). This relationship is even clearer in the case of online social networks: indeed, within all age cohorts, online news consumption and traditional media consumption (i.e. print and TV) are strongly negatively correlated (RISJ, 2014: 45).

The key point here is the dramatic fall in price of exchanging information within networks brought about by online social networks such as Facebook. Theorem 1 and Proposition 1 predict that with a fall in p_x, 'better connected' individuals (i.e. individuals in the q_0-connected club) will switch from the market action y to the network action. Wrzus et al. (2013) and others have shown that individuals' network size falls with age: *ceteris paribus* our model thus predicts that exchanging in networks should be particularly popular among young people. This is strongly consistent with the empirical evidence (see e.g. RISJ, 2014).

Propositions 2 and 3 predict that a rise in aggregate welfare and inequality should accompany a decrease in p_x. Here the available evidence is suggestive: online social networks users have access to a much wider range of news and information sources than individuals using only traditional media (Currah, 2009; RIJS, 2014). Given the size of the online community, aggregate welfare (measured by access to information and news) has probably gone up, but the disparities in participation on online media also suggest that inequality in information access has increased.

6.2.2 The Digital Provide: Networks and Mobile Phones

The widespread adoption of mobile telephones in developing countries has been extensively studied (see e.g. Aker and Mbiti, 2010). Jensen (2007) studies their economic effects on fishermen in Kerala, India. Prior to the introduction of cellphones in 1997, fishermen fished and sold their catch almost exclusively within their local catchment zone, which led to high levels of waste and price discrepancies between different markets on the coast. The introduction of cellphones, however, changed this state of affairs. With phones, fishermen can exchange information with buyers directly while at sea, therefore obtaining precious information about the demand in different markets.[10] In addition to buyers, fishermen also use their phone to share information about prices, demand and supply with auctioneers they are connected to,[11] as well as information about fishing sites with friends and relatives (Srinivasan and Burrell, 2013). By 2001, more than 65% of all fishing boats in Kerala owned a cellphone. Jensen

[10] Jensen notes in that respect that fishermen with cellphones "often carry lists with the numbers of dozens or even hundreds of potential buyers" (p.891) whom they know personally.

[11] For an extensive discussion of this point, see Srinivasan and Burrell (2013). In their ethnographic study of fishermen of Kerala, the authors note indeed that private investors, necessary to fund most boats, would appoint an anctioneer to auction fishermen's catch on the beach market. Many fishermen would thus choose investors located in different markets so as to have an auctioneer at their disposal in different markets. With the arrival of mobile phones, boat owners could directly"call their auctioneers at different landing sites to ascertain prices... as soon as they get within range" (Srinivasan and Burrell, 2013: 6).

(2007) also finds that mobile phones significantly increased daily profits of all fishermen but also increased economic inequality between them.

We map this empirical setting onto our model. Suppose that players are located on a bipartite network, with buyers on one side and sellers on the other: for simplicity, we focus on fishermen (sellers). Fishermen' payoffs are given by the value of their (expected) sales in fish markets. *Ceteris paribus*, better information about prices and fishing sites increases fishermen' expected payoffs as it allows them to sell their catch where the demand is higher. Let the network action x stand for "information sharing" (e.g. information about prices, the local demand or fishing sites). The payoffs to x for a fisherman i depends positively on the number of i's neighbours (e.g. buyers, auctioneers and friends) who also exchange information. Let the market action y stand for "owning a cellphone". Without a cellphone, sharing information (x) yields very low returns as it is impossible for fishermen to learn precisely where to fish or where to sell their catch while at sea or before going at sea, without a mobile phone.[12] Owning a cellphone, however, increases the returns to x, and vice-versa: "sharing information" and "owning a cellphone" are complements, i.e., $\theta = 1$.[13]

Given this mapping, Proposition 1 predicts that for p_y not too large, x may become profitable when combined with *y for better connected fishermen.* This prediction is in direct line with Jensen's (2007) finding that bigger and better-connected boats adopted mobile phones the most.[14] Further, Proposition 2 predicts that for $\theta = 1$, markets lead to an unambiguous increase in aggregate welfare. This is in line with Jensen's (2007: 913) finding of "net welfare gains [for both buyers and sellers], due to more efficient allocation of fish". Finally, Proposition 3 suggests that the introduction of cellphones should lead to an increase in inequality between owners and non-owners of cellphones. This is in line with Jensen's finding that owners of cellphones increased their payoffs substantially more than non-owners, which led to an increase in inequality.

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[12]Note that our model captures the network externalities entailed by mobile phones. Indeed, observe that players play $x = 1$ if and only if in combination with $y = 1$, since sharing information without mobile phones is not possible. Player i's decision to play $x_i = y_i = 1$ will thus depend on how many of i's neighbours also choose $x = y = 1$; hence, i's decision to buy a mobile phone is intrinsically linked to i's neighbours decision to buy a mobile phone.

[13]Note however that the payoffs to y are not all contingent to the network. Mobile banking is an example of mobile phone use that does not necessarily depend on one's connections.

[14]Srinivasan and Burrell (2013) explain well why bigger boats are also better connected. They explain that bigger boats (*ring-seine*), due to their cost, typically require multiple investors. Since each investor comes with an appointed auctioneer, *ring-seine* owners end up with multiple auctioneers (in many different markets on the coast, strategically chosen) that they can call to obtain price- and demand-related information. In contrast, smaller boats (*gillnet*) would have one or few investors. Note finally that bigger boats, independently of their connections, might derive more benefits from mobile phones compared to smaller boats (e.g. their typically much larger catch make them more sensitive to price differences): this heterogeneity is captured in our extended model and does not change the predictions of the basic model in terms of welfare and inequality.

7 Concluding remarks

The interaction between social structure and markets remains a central theme in the social sciences. Individuals' personal relationships (or *network*) provide opportunities for interaction and economic exchange. Individuals also obtain goods and services from other sources that lie outside these relations: these include the state and markets. Some social scientists have argued that the expansion of markets, accompanied by wide ranging changes in attitudes and institutions, can crowd out social ties and aggravate inequality. There is also a distinguished school of thought, going back to the eighteenth century, that asserts that markets enhance welfare and reinforce reciprocity. These conflicting views find an echo in the varieties of empirical experience. This paper develops a theoretical framework in an attempt to account for the empirical evidence.

We develop a model where individuals located in a social network choose a network action and a market action. We show that the key to understanding the diverse empirical patterns lies in the relation between the network action and the market action, i.e., whether they are (strategic) complements or substitutes.

Social connectedness facilitates adoption of market action if the two activities are complements; the converse is true in case of substitutes. Inequality in a social structure is typically reinforced by the market in case the two actions are complements; the converse holds true if they are substitutes.

Appendix A

Proof to Theorem 1 Existence of equilibrium follows from standard considerations: if $p_y < 1$, then $x_i = 0$ and $y_i = 1$ for all $i \in N$ is always an equilibrium. If $p_y \geq 1$ then $x_i = y_i = 0$ for all $i \in N$ is always equilibrium. Existence of maximal equilibrium now follows from noting that the set of strategies and hence the set of equilibria is finite.

We turn next to the issue of uniqueness. Suppose that there exist two distinct profiles $(\mathbf{x}, \mathbf{y})$ and $(\mathbf{x}', \mathbf{y}')$ that are both maximal equilibria. Observe that for generic values of parameters p_x and p_y, there must exist players i and j such that i does strictly better under $(\mathbf{x}, \mathbf{y})$, while j fares strictly better under $(\mathbf{x}', \mathbf{y}')$. Consider first the case of complements ($\theta = 1$). Construct a profile $(\hat{\mathbf{x}}, \hat{\mathbf{y}})$, where $\hat{x}_i = \max\{x_i, x_i', x_i^*\}$ and $\hat{y}_i = \max\{y_i, y_i', y_i^*\}$ for all i, where x_i^* and y_i^* are i's best-response in x and y, respectively, to the strategy profile $(\hat{\mathbf{x}}_{-i}, \hat{\mathbf{y}}_{-i})$. It is easily verified that $(\hat{\mathbf{x}}, \hat{\mathbf{y}})$ constitutes an equilibrium. As $\theta = 1$, it also follows that in the equilibrium $(\hat{\mathbf{x}}, \hat{\mathbf{y}})$, payoffs of all individuals are weakly higher than their payoff in either equilibrium $(\mathbf{x}, \mathbf{y})$ or $(\mathbf{x}', \mathbf{y}')$. As there is a strict inequality for at least a pair of agents, it follows that $(\hat{\mathbf{x}}, \hat{\mathbf{y}})$ Pareto-dominates $(\mathbf{x}, \mathbf{y})$ and $(\mathbf{x}', \mathbf{y}')$. This contradicts the hypothesis that $(\mathbf{x}, \mathbf{y})$ and $(\mathbf{x}', \mathbf{y}')$ are maximal equilibria.

Next consider the case of substitutes ($\theta = -1$). Construct a profile $(\hat{\mathbf{x}}, \hat{\mathbf{y}})$, where $\hat{x}_i = \max\{x_i, x_i', x_i^*\}$ and $y_i = \min\{y_i, y_i', y_i^*\}$ for all i. It is easily verified that $(\hat{\mathbf{x}}, \hat{\mathbf{y}})$ constitutes an equilibrium. As the number of players choosing $x = 1$ has weakly grown from both $(\mathbf{x}, \mathbf{y})$ and $(\mathbf{x}', \mathbf{y}')$ to $(\hat{\mathbf{x}}, \hat{\mathbf{y}})$, the payoffs of every individual choosing $x = 1$ under $(\hat{\mathbf{x}}, \hat{\mathbf{y}})$ must be weakly larger. Moreover, an individual k switches from $y_k = 1$ or $y_k' = 1$ to $\hat{y}_k = 0$ only if $\min\{y_k, y_k', y_k^*\} = 0$. As payoffs from y are independent of others' choices, this must entail a weak increase in player k's payoffs. As there is a strict inequality for at least a pair of agents, i and j, we have shown that $(\hat{\mathbf{x}}, \hat{\mathbf{y}})$ Pareto-dominates $(\mathbf{x}, \mathbf{y})$ and $(\mathbf{x}', \mathbf{y}')$. This again contradicts the hypothesis that $(\mathbf{x}, \mathbf{y})$ and $(\mathbf{x}', \mathbf{y}')$ are maximal equilibria. The parameter conditions for different types of maximal equilibrium are straightforward, given the definitions of q_0 and q_1.
■

Proof to Proposition 1 The first part of Proposition 1 along with the part on ***Networks*** are corollaries to Theorem 1; the proof is thus omitted.

Prices. Consider first the case where $\theta = -1$. We know that $\mathcal{Y}(\mathbf{p}, \mathbf{g}) = 0$ whenever $p_y \geq 1$. When $p_y < 1$, then $\mathcal{Y}(\mathbf{p}, \mathbf{g})$ depends *negatively* on the size of the q_0-connected club. Hence, whenever q_0 increases, then the size of $\mathcal{S}_{q_0}$ weakly decreases, entailing a weakly larger $\mathcal{Y}(\mathbf{p}, \mathbf{g})$. Since q_0 weakly decreases with p_y and weakly increases with p_x, then it follows straightforwardly that $\mathcal{Y}(\mathbf{p}, \mathbf{g})$ weakly decreases with p_y and weakly increases with p_x. Consider second the case where $\theta = 1$. We know that $\mathcal{Y}(\mathbf{p}, \mathbf{g}) = 1$ whenever $p_y < 1$. When $p_y \geq 1$, then $\mathcal{Y}(\mathbf{p}, \mathbf{g})$ depends *positively* on the size of the q_1-connected club. Hence, whenever q_1 increases, then the size of $\mathcal{S}_{q_1}$ weakly decreases, entailing a weakly smaller $\mathcal{Y}(\mathbf{p}, \mathbf{g})$. Since q_1 weakly decreases with p_y and p_x, then it follows straightforwardly that $\mathcal{Y}(\mathbf{p}, \mathbf{g})$ weakly decreases with p_y and

p_x. ■

Proof to Proposition 2 *For any q, $\mathcal{W}(\mathbf{p},\mathbf{g})$ weakly increases with the size of the q-connected club. Adding a link to a network always (weakly) increases $\mathcal{W}(\mathbf{p},\mathbf{g})$.*

Observe first that the payoffs of players outside the q-connected club, for the threshold values q_0, q_1, and p_x, are always independent of the size of the q-connected club. Thus, expanding the size of the latter leaves the payoffs of those players unchanged. However, observe that the payoffs of players inside the q-connected club are always weakly increasing in its size. Hence, increasing the size of the q-connected club always weakly enhances aggregate welfare.

In a regular network, the maximal equilibrium is efficient. We first show that the maximal equilibrium in any regular network is always efficient. First observe that in a regular network, from Theorem 1, all players always adopt the same strategy at the maximal equilibrium. Consider the following Lemma.

Lemma 1 *In a regular network, the efficient outcome is generically symmetric.*

Proof. Suppose a contrario that at the efficient outcome $(\mathbf{x}^*,\mathbf{y}^*)$, there is (at least) one player j and one player i such that $(x_i^*,y_i^*) \neq \left(x_j^*,y_j^*\right)$. For generic $\mathbf{p}$, either i or j must be strictly better off; assume without loss of generality that player i is actually the best-off player in N and j the worst-off. This implies notably that $\prod_i(\mathbf{x}^*,\mathbf{y}^*) > \prod_j(\mathbf{x}^*,\mathbf{y}^*)$.

Suppose first that $\theta = -1$, and $p_y \geq 1$. Since $\prod_i(\mathbf{x}^*,\mathbf{y}^*) > \prod_j(\mathbf{x}^*,\mathbf{y}^*)$, then necessarily $x_i^* = 1$, while $y_i^* = y_j^* = 0$. Construct a profile $(\hat{\mathbf{x}},\hat{\mathbf{y}})$, where $\hat{x}_l = 1$ and $\hat{y}_l = 0$ for all $l \in N$. Note first that player i must be weakly better off in $(\hat{\mathbf{x}},\hat{\mathbf{y}})$ than in $(\mathbf{x}^*,\mathbf{y}^*)$ as the number of her neighbours playing $x = 1$ has weakly grown; hence, $\prod_i(\hat{\mathbf{x}},\hat{\mathbf{y}}) \geq \prod_i(\mathbf{x}^*,\mathbf{y}^*)$. Note further that player j is necessarily strictly better off as since the network is regular, then $\prod_j(\hat{\mathbf{x}},\hat{\mathbf{y}}) = \prod_i(\hat{\mathbf{x}},\hat{\mathbf{y}}) \geq \prod_i(\mathbf{x}^*,\mathbf{y}^*) > \prod_j(\mathbf{x}^*,\mathbf{y}^*)$. All players in between i and j are also trivially weakly better-off. It follows necessarily that $W(\mathbf{x}^*,\mathbf{y}^*|\mathbf{p},\mathbf{g}) < W(\hat{\mathbf{x}},\hat{\mathbf{y}}|\mathbf{p},\mathbf{g})$. This contradicts the hypothesis that $(\mathbf{x}^*,\mathbf{y}^*)$ is the efficient outcome. The argument trivially extends for cases where $\theta = -1$ and $p_y < 1$, and so the proof for these cases is omitted.

Suppose second that $\theta = 1$ and $p_y < 1$. Since $\prod_i(\mathbf{x}^*,\mathbf{y}^*) > \prod_j(\mathbf{x}^*,\mathbf{y}^*)$, then necessarily $x_i^* = 1 > x_j^* = 0$, while $y_i^* = y_j^* = 1$. Construct a profile $(\hat{\mathbf{x}},\hat{\mathbf{y}})$, where $\hat{x}_l = 1$ and $\hat{y}_l = 1$ for all $l \in N$. Note first that player i must be weakly better off in $(\hat{\mathbf{x}},\hat{\mathbf{y}})$ than in $(\mathbf{x}^*,\mathbf{y}^*)$ as the number of her neighbours playing $x = 1$ has weakly grown. Note further that player j is necessarily strictly better off as $\prod_j(\hat{\mathbf{x}},\hat{\mathbf{y}}) = \prod_i(\hat{\mathbf{x}},\hat{\mathbf{y}}) \geq \prod_i(\mathbf{x}^*,\mathbf{y}^*) > \prod_j(\mathbf{x}^*,\mathbf{y}^*)$. Since this is true for all players, then necessarily $W(\mathbf{x}^*,\mathbf{y}^*|\mathbf{p},\mathbf{g}) < W(\hat{\mathbf{x}},\hat{\mathbf{y}}|\mathbf{p},\mathbf{g})$. This contradicts the hypothesis that $(\mathbf{x}^*,\mathbf{y}^*)$ is the efficient outcome. The argument trivially extends for cases where $\theta = 1$ and $p_y \geq 1$, and so the proof for these cases is omitted. This completes the proof to Lemma 1. ■

Lemma 2 *In a regular network, the efficient outcome is an equilibrium.*

Proof. We prove by contradiction that if the efficient outcome is not an equilibrium in a regular network, then it is not the efficient outcome. Suppose first that $\theta = -1$ and $p_y \geq 1$. Trivially, $y_i = 0$ for all $i \in N$, both at equilibrium and at the efficient outcome. Suppose that the efficient outcome $(\mathbf{x}^*, \mathbf{0})$ is not an equilibrium. Then, there must be some player j who wants to deviate from $\left(x_j^*, 0\right)$ to $\left(1 - x_j^*, 0\right)$. If $x_i^* = 0$ for all $i \in N$, then clearly j does not want to deviate to $x_j = 1$ as none of her neighbours plays $x = 1$ (entailing that playing $x = 1$ would only impose a cost p_x on j, without any benefit). If $x_i^* = 1$ for all $i \in N$, then j wants to deviate to $x_j = 0$ if and only if $0 > \prod_j (\mathbf{x}^*, \mathbf{0})$. However, if this is the case for j, then all players can strictly improve their payoffs by playing $x = y = 0$, which contradicts the hypothesis that $(\mathbf{x}^*, \mathbf{0})$ is the efficient outcome. The argument trivially extends for cases where $\theta = -1$ and $p_y < 1$, and so the proof for these cases is omitted.

Suppose second that $\theta = 1$ and $p_y < 1$. Trivially, $y_i = 1$ for all $i \in N$, both at equilibrium and at the efficient outcome. Suppose that the efficient outcome $(\mathbf{x}^*, \mathbf{1})$ is not an equilibrium. Then, there must be some player j who wants to deviate from $\left(x_j^*, 0\right)$ to $\left(1 - x_j^*, 0\right)$. If $x_i^* = 0$ for all $i \in N$, then clearly j does not want to deviate to $x_j = 1$ as none of her neighbours plays $x = 1$ (entailing that playing $x = 1$ would only impose a cost p_x on j, without any benefit). If $x_i^* = 1$ for all $i \in N$, then j wants to deviate to $x_j = 0$ if and only if $1 - p_y > \prod_j (\mathbf{x}^*, \mathbf{1})$. However, if this is the case for j, then all players can strictly improve their payoffs by playing $x = 0$ and $y = 1$, which contradicts the hypothesis that $(\mathbf{x}^*, \mathbf{1})$ is the efficient outcome. The argument trivially extends for cases where $\theta = 1$ and $p_y \geq 1$, and so the proof for these cases is omitted. ■

We conclude the proof by showing that the efficient outcome is the maximal equilibrium. We have already shown that the efficient outcome $(\mathbf{x}^*, \mathbf{y}^*)$ is always an equilibrium. Suppose by contradiction that it is not a maximal equilibrium. Then, there exists another profile $(\mathbf{x}', \mathbf{y}')$ such that $\prod_i (\mathbf{x}', \mathbf{y}') \geq \prod_i (\mathbf{x}^*, \mathbf{y}^*)$ for all $i \in N$, with inequality strict for at least one j. This in turn entails that $W(\mathbf{x}^*, \mathbf{y}^* | \mathbf{p}, \mathbf{g}) < W(\mathbf{x}', \mathbf{y}' | \mathbf{p}, \mathbf{g})$, which contradicts the hypothesis that $(\mathbf{x}^*, \mathbf{y}^*)$ is the efficient outcome. Hence, the efficient outcome is always the maximal equilibrium. Since both are (generically) unique, this also entails that the maximal equiibrium is efficient in regular networks. ■

In an irregular network, the maximal equilibrium may be inefficient. We now prove by construction that in irregular networks, the maximal equilibrium is not generically unique. Consider the CP network in Figure 1. Suppose first that $\theta = -1$, and suppose that $p_y < p_x < 1$. Then, at the maximal equilibrium, players in the periphery play $x = 0$ and $y = 1$, while players in the core play $x = 1$ and $y = 0$, entailing that $\mathcal{W}(\mathbf{p}, \mathbf{g}) = 25 - 5p_x - 5p_y$. Now construct a profile $(\hat{\mathbf{x}}, \hat{\mathbf{y}})$, where $\hat{x}_i = 1$ and $\hat{y}_i = 0$ for all $i \in N$. The resulting aggregate welfare amounts to $W(\hat{\mathbf{x}}, \hat{\mathbf{y}} | \mathbf{p}, \mathbf{g}) = 30 - 10p_x$. Since $W(\hat{\mathbf{x}}, \hat{\mathbf{y}} | \mathbf{p}, \mathbf{g}) - \mathcal{W}(\mathbf{p}, \mathbf{g}) = 5 - 5p_x + 5p_y > 0$, then the maximal equilibrium is clearly not efficient. It can finally be easily shown that if

$p_x < p_y < 1$, then $x_i = 1$ and $y_i = 0$ for all $i \in N$ at the maximal equilibrium, and the latter is efficient. ■

Adding a link to a network always (weakly) increases $\mathcal{W}(\mathbf{p}, \mathbf{g})$. Finally, since adding a link to the network always weakly expand the q-connected club for any q, then clearly adding a link always weakly enhances welfare.

Prices have two effects on welfare. The first (direct) effect is the *cost effect*: players have to pay more for x and y respectively when p_x and p_y increase. Trivially, increasing p_x and p_y have both a negative cost effect on welfare.

The second (indirect) effect is the *q-connected club effect*: recall that aggregate welfare always depends positively on the size of the q-connected club at the threshold values q_0, q_1, and p_x, and that these values depend on $\mathbf{p}$. Note first that all of these threshold values are increasing in p_x; hence, an increasing p_x will always weakly reduce the size of the q-connected club for any of these threshold values. Hence, since the *cost effect* and the *q- connected club effect* of an increasing p_x on $\mathcal{W}(\mathbf{p}, \mathbf{g})$ are both negative, then clearly $\mathcal{W}(\mathbf{p}, \mathbf{g})$ decreases in p_x. Second, when $\theta = 1$, then an increasing p_y always has a weakly negative *q-connected club effect* (since p_x is independent of p_y, while q_1 decreases in p_y). In that case, the *cost effect* and *the q-connected club effect* of an increasing p_y on $\mathcal{W}(\mathbf{p}, \mathbf{g})$ are both weakly negative, and so $\mathcal{W}(\mathbf{p}, \mathbf{g})$ decreases in p_y. However, when $\theta = -1$, then q_0 weakly increases with p_y. Hence, the *cost effect* and the *q-connected club effect* oppose each other. The net effect of an increasing p_y on $\mathcal{W}(\mathbf{p}, \mathbf{g})$ is thus ambiguous when $\theta = -1$.

Proof to Proposition 3: We first take up the case of substitutes. Observe that an individual never adopts both x or y. Moreover, $y_i = 0$, for all $i \in N$, if $p_y > 1$. So in this case choice and hence inequality is unaffected by markets. The interesting case therefore is $p_y < 1$. There are three possible outcomes with respect to action y: one, where no one adopts it, two, where some adopt it while others do not adopt it, and three, where everyone adopts it. Let us take up these cases in turn.

If no one adopts y then the market does not have any impact on choice; so, inequality is unaffected.

Next consider the case where, in a maximal equilibrium, some individuals adopt y while others do not adopt it. Since some individuals do not choose y, and y yields a positive payoff, they must choose x. Observe that everyone choosing x must earn weakly more than $1 - p_y$, as choosing y is always an option. Thus $\min_{i \in N} \{\pi_i^1(\mathbf{p}, \mathbf{g})\} = 1 - p_y$, while $\max_{i \in N} \{\pi_i^1(\mathbf{p}, \mathbf{g})\}$ is earned by someone who chooses x. Now observe that in a maximal equilibrium the neighbors of any individual choosing x must weakly decline, after the introduction of y. Hence maximum payoff of someone choosing x and hence the maximum payoff (weakly) falls with the introduction of the market. Finally, observe that minimum payoff must weakly increase as the market simply offers an outside option with a positive payoff. Thus, the inequality level $\mathcal{R}()$ must weakly fall.

Finally in case three, if everyone adopts y then the equilibrium inequality level $\mathcal{R}^1(\mathbf{p}, \mathbf{g}) = 1$; as this is the minimum level possible, inequality must (weakly) decrease, in comparison to the pre-market situation.

We next take up the case of complements. Let us start with the case where $p_y \leq 1$. Everyone must choose $y_i = 1$ in any equilibrium. There are three cases to consider: one, where no one chooses x, two, where some individuals choose x while others don't choose x, and three, where everyone chooses x. Let us taken them up in turn.

If no one chooses x, then given that x and y are complementary, and we are considering maximal equilibrium, it must be the case that no one must choose x in the pre-market maximal equilibrium. Thus in the pre-market equilibrium, there is no inequality, $\mathcal{R}^0(\mathbf{p}, \mathbf{g}) = 1$. Inequality must (weakly) increase with markets.

If some individuals choose x, while others don't then there are two possible scenarios in the pre-market maximal equilibrium. One, where no one adopts x and two, with partial adoption. If no one adopts x then inequality ratio in pre-market equilibrium is 1. Inequality can only rise in the post market world. In the latter case, assume without loss of generality that player i has in fact the highest payoffs before the introduction of y. Then, $\mathcal{R}_0\left(\mathbf{p}, \mathbf{g}\right)$ can be written at length as follows:

$$\mathcal{R}_0\left(\mathbf{p}, \mathbf{g}\right) = \frac{1 + m_i - p_x}{1 + \min_{i \in N}\left\{\pi_i^0\left(\mathbf{p}, \mathbf{g}\right)\right\}} = 1 + m_i - p_x \tag{11}$$

where m_i is the number of i's neighbours who play $x = 1$ before the introduction of y. Since the number of i's neighbours playing $x = 1$ weakly grows after the introduction of y, the following inequality must hold true:

$$\mathcal{R}_1\left(\mathbf{p}, \mathbf{g}\right) \geq \frac{2 + 2m_i - p_x - p_y}{1 + \min_{i \in N}\left\{\pi_i^1\left(\mathbf{p}, \mathbf{g}\right)\right\}} = \frac{2(1 + m_i) - p_x - p_y}{2 - p_y} \tag{12}$$

Combining equation (11) with inequality (12), and noting that the last expression in 12 is rising in p_y, we obtain the following expression:

$$\mathcal{R}_0\left(\mathbf{p}, \mathbf{g}\right) = 1 + m_i - p_x < \frac{2(1 + m_i) - p_x - p_y}{2 - p_y} \leq \mathcal{R}_1\left(\mathbf{p}, \mathbf{g}\right). \tag{13}$$

Thus inequality strictly increases with the introduction of a market in this case.

Finally, in case everyone chooses x, we know from the discussion in the main text that the effects of markets on inequality are ambiguous.

We turn next to the case where $p_y > 1$. If the maximal equilibrium entails $y_i = 0$, for all $i \in N$, then the market has no effect on choice and on inequality. Consider next the other limiting case, where $y_i = 1$, for all $i \in N$. Given that $p_y > 1$, this must mean that $x_i = 1$, for all $i \in N$. We have already taken up this case in the main text, via example. The impact of markets on inequality is ambiguous in this case.

We turn finally to the case of partial adoption of the market action, y, in a maximal equilibrium. In this equilibrium, everyone who chooses y must also choose x (as $p_y > 1$). From Theorem 1, when $p_y \geq 1$, players can be partitioned into three strategy groups, namely $\mathcal{S}_{q_1}$, $\mathcal{S}_{p_x} \setminus \mathcal{S}_{q_1}$ and $N \setminus \{\mathcal{S}_{q_1} \cup \mathcal{S}_{p_x}\}$. Label these groups A, B and C, respectively. Hence, for any $i \in A$, $x^1_{i,A} = 1$ and $y_{i,A} = 1$; for any $j \in B$, $x^1_{j,B} = 1$ and $y_{j,B} = 0$; while for any $l \in C$, $x^1_{l,C} = y_{l,C} = 0$.

We take up first the case where A, B and C are all non-empty. We will show that $\mathcal{R}(\mathbf{p}, \mathbf{g})$ strictly increases. If both A and B are non-empty, then $q_1 > p_x$. If this were not the case then everyone doing x would prefer to do x and y and B would be empty. Next, note that all players in $\mathcal{S}_{p_x}$ must find the adoption of x profitable before the introduction of y. This is because every player in B has strictly more than p_x links to other players in $\mathcal{S}_{p_x}$; she can therefore attain positive payoffs if all others in S_{p_x} also choose x. So in a maximal equilibrium, $x^0_{i,A} = x^0_{j,B} = 1$.

Furthermore, players in A have more links to other players in $\mathcal{S}_{p_x}$ than do players in B. If not then there is a player $j \in B$ with more links to other players in S_{p_x} than a player $i \in A$. But then j has more neighbors choosing x and expects a higher return from action y than do players in S_{q_0}. But then she should optimally choose a and y and must belong to A. A contradiction.

Since players in A clearly have strictly more links to other players in $\mathcal{S}_{p_x}$ than players in B, it follows immediately that $\pi^1_{i,A}(\mathbf{p}, \mathbf{g}) > \pi^1_{j,B}(\mathbf{p}, \mathbf{g})$ and $\pi^0_{i,A}(\mathbf{p}, \mathbf{g}) > \pi^0_{j,B}(\mathbf{p}, \mathbf{g})$. Hence, we obtain that $\max_{i \in N}\{\pi^1_i(\mathbf{p}, \mathbf{g})\} \in \{\pi^1_i(\mathbf{p}, \mathbf{g}) : i \in A\}$, and $\max_{i \in N}\{\pi^0_i(\mathbf{p}, \mathbf{g})\} \in \{\pi^0_i(\mathbf{p}, \mathbf{g}) : i \in A\}$. Since every player in A strictly increases her payoffs with the introduction of y, clearly $\max_{i \in N}\{\pi^1_i(\mathbf{p}, \mathbf{g})\} > \max_{i \in N}\{\pi^0_i(\mathbf{p}, \mathbf{g})\}$.

Lastly, note that since the introduction of y can only weakly foster the adoption of x, $x^1_{l,C} = 0$ implies that $x^0_{l,C} = 0$ too for all $l \in C$. Since $x^1_{l,C} = x^0_{l,C} = y_{l,C} = 0$ for all $l \in C$, it immediately follows that $\pi^1_{l,C}(\mathbf{p}, \mathbf{g}) = \pi^0_{l,C}(\mathbf{p}, \mathbf{g}) = 0$ for all $l \in C$. Hence, it follows that $\min_{i \in N}\{\pi^1_i(\mathbf{p}, \mathbf{g})\} = \min_{i \in N}\{\pi^0_i(\mathbf{p}, \mathbf{g})\} = 0$.

The proof that inequality strictly increases with the introduction of y when $p_y \geq 1$ and A, B and C are all non-empty immediately now follows since we have shown that maximum earnings increase while the minimum payoffs remain constant with the introduction of market.

We next examine cases where one of A, B or C is empty. Suppose first that A is empty. Then, $y_i = 0$ for all $i \in N$, which entails that $\mathcal{R}(\mathbf{p}, \mathbf{g})$ is left unchanged. If B is empty, then N can be partitioned into 2 groups, namely A and C. The proof that $\mathcal{R}(\mathbf{p}, \mathbf{g})$ strictly increases in this case is analogous to the case where $p_y < 1$. Next suppose that only C is empty. We know that (i) $x_i = 1$ for all $i \in N$ and (ii) $\pi^1_{i,A}(\mathbf{p}, \mathbf{g}) > \pi^1_{j,B}(\mathbf{p}, \mathbf{g})$ and $\pi^0_{i,A}(\mathbf{p}, \mathbf{g}) > \pi^0_{j,B}(\mathbf{p}, \mathbf{g})$, for any $i \in A$ and $j \in B$. Since the payoffs of players in B are left unchanged by the introduction of y (since $y_{j,B} = 0$ for all $j \in B$), then it follows that $\min_{i \in N}\{\pi^1_i(\mathbf{p}, \mathbf{g})\} = \min_{i \in N}\{\pi^0_i(\mathbf{p}, \mathbf{g})\}$. However, as the payoffs of players in A strictly increase (since $y_{i,A} = 1$ for all $i \in A$), then we know that $\max_{i \in N}\{\pi^1_i(\mathbf{p}, \mathbf{g})\} > \max_{i \in N}\{\pi^0_i(\mathbf{p}, \mathbf{g})\}$, which means that $\mathcal{R}(\mathbf{p}, \mathbf{g})$ strictly increases.

We turn finally to the case where two out of A, B and C are empty. If $C = N$, then the introduction of y does not affect choice or inequality. A similar argument applies when $B = N$. Finally, we prove by example that if $A = N$, then $\mathcal{R}(\mathbf{p}, \mathbf{g})$ may increase or decrease following the introduction of y. Consider indeed the network on Figure 8.

Fix $p_x = 4.1$. In such case, the best-off players before the introduction of y are players 1 to 6 with payoffs of 0.9, while all other players have payoffs of 0. In such case, $\mathcal{R}_0(\mathbf{p}, \mathbf{g}) = 1.9$. Now suppose that y is introduced at a price $p_y = 1.05$, such that $A = N$. Then, the payoffs to players 1, 7 and 3 to 6 amount to 5.85, while those of player 2 and players 8 to 11, respectively, amount to 7.85 and 3.85. Consequently, $\mathcal{R}_1(\mathbf{p}, \mathbf{g}) = 1.825$, which indicates a falling $\mathcal{R}(\mathbf{p}, \mathbf{g})$. Now suppose that $p_y = 2$. Then, the payoffs to players 1, 7 and 3 to 6 amount to 4.9, while those of player 2 and players 8 to 11, respectively, amount to 6.9 and 2.9. Consequently, $\mathcal{R}_1(\mathbf{p}, \mathbf{g}) = 2.026$, which shows a rising $\mathcal{R}(\mathbf{p}, \mathbf{g})$. This completes the proof. ■

Proof to Proposition 4: Denote by $\mathcal{L}(i|\mathbf{p}, \mathbf{g})$ the *Lorenz curve* at the maximal equilibrium. Consider the sequence of players $(1, 2...i, ...n)$ such that $\prod_j(\mathbf{p}, \mathbf{g}) \leq \pi_i(\mathbf{p}, \mathbf{g})$ for all $j < i$.[15] Then, the Lorenz curve, at a given player i, takes the following value:

$$\mathcal{L}(i|\mathbf{p}, \mathbf{g}) = \frac{\sum_{j<i}\pi_j(\mathbf{p}, \mathbf{g}) + \pi_i(\mathbf{p}, \mathbf{g})}{\mathcal{W}(\mathbf{p}, \mathbf{g})} \tag{14}$$

Hence, $\mathcal{L}(0|\mathbf{p}, \mathbf{g}) = 0$ and $\mathcal{L}(n|\mathbf{p}, \mathbf{g}) = 1$, by definition. We denote by $\Delta\mathcal{L}(i|\mathbf{p}, \mathbf{g})$ the slope of the Lorenz curve at a player i, with

$$\Delta\mathcal{L}(i|\mathbf{p}, \mathbf{g}) = \frac{\pi_i(\mathbf{p}, \mathbf{g})}{\mathcal{W}(\mathbf{p}, \mathbf{g})} \tag{15}$$

Given the ordering of individuals from lowest to highest, the Lorenz curve is increasing and convex: this means that $\Delta\mathcal{L}(i|\mathbf{p}, \mathbf{g}) \leq \Delta\mathcal{L}(j|\mathbf{p}, \mathbf{g})$ if and only if $i < j$ in the support.

We say that a Lorenz curve A, $\mathcal{L}^A(i|\mathbf{p}, \mathbf{g})$, dominates a Lorenz curve B, $\mathcal{L}^B(i|\mathbf{p}, \mathbf{g})$, if $\mathcal{L}^A(i|\mathbf{p}, \mathbf{g}) \geq \mathcal{L}^B(i|\mathbf{p}, \mathbf{g})$ for all $i \in N$, with inequality strict for at least one i. Denote

[15] As multiple players can have the same payoffs, this sequence may not be unique.

by $\mathcal{L}_0(i|\mathbf{p},\mathbf{g})$ and by $\mathcal{L}_1(i|\mathbf{p},\mathbf{g})$ the Lorenz curve before and after the introduction of y, respectively, and define $\mathcal{G}_0(\mathbf{p},\mathbf{g})$ and $\mathcal{G}_1(\mathbf{p},\mathbf{g})$ analogously. Observe that it is sufficient to show $\mathcal{L}_0(i|\mathbf{p},\mathbf{g})$ dominates (is dominated by) $\mathcal{L}_1(i|\mathbf{p},\mathbf{g})$ to prove that $\mathcal{G}_0(\mathbf{p},\mathbf{g}) \leq \mathcal{G}_1(\mathbf{p},\mathbf{g})$ $(>)$.

Case A: Proof for $\theta = -1$:

From Theorem 1, note that players can be partitioned into two strategy groups, namely S_{q_0} and $N \setminus S_{q_0}$. Label the former group A and the latter group B. Denote by $\mathcal{L}^A(i|\mathbf{p},\mathbf{g})$ and $\mathcal{L}^B(i|\mathbf{p},\mathbf{g})$ the Lorenz curve over the support segments A and B, respectively. Recall from the proof of Proposition 3 that $\pi^1_{i,A}(\mathbf{p},\mathbf{g}) > \pi^1_{j,B}(\mathbf{p},\mathbf{g})$ for any $i \in A$ and $j \in B$. Hence, $\mathcal{L}^B_0(i|\mathbf{p},\mathbf{g}) > \mathcal{L}^A_0(j|\mathbf{p},\mathbf{g})$ and $\Delta\mathcal{L}^A_1(i|\mathbf{p},\mathbf{g}) > \Delta\mathcal{L}^B_1(j|\mathbf{p},\mathbf{g})$ for any $i \in A$ and $j \in B$. In other words, $\mathcal{L}^B_1(i|\mathbf{p},\mathbf{g})$ is the (left) portion of the Lorenz curve (with $\mathcal{L}^B(0|\mathbf{p},\mathbf{g}) = 0$), while $\mathcal{L}^A_1(i|\mathbf{p},\mathbf{g})$ is the right portion (with $\mathcal{L}^A(n|\mathbf{p},\mathbf{g}) = 1$). Note that by definition, $\mathcal{L}^B(n_B|\mathbf{p},\mathbf{g}) = \mathcal{L}^A(0|\mathbf{p},\mathbf{g})$, where $n_B = |B|$.

(i) $\mathcal{Y}(\mathbf{p},\mathbf{g}) \in \{0,1\}$ and/or $\mathcal{W}_1(\mathbf{p},\mathbf{g}) \geq \mathcal{W}_0(\mathbf{p},\mathbf{g})$ Observe that if $\mathcal{Y} = 0$ then clearly the introduction of y has no impact on inequality. Next consider the case $\mathcal{Y} = 1$. We show that $\mathcal{G}(\mathbf{p},\mathbf{g})$ must weakly decrease. If $\mathcal{Y}(\mathbf{p},\mathbf{g}) = 1$, then $\pi^1_i(\mathbf{p},\mathbf{g}) = \pi^1_j(\mathbf{p},\mathbf{g}) = 1 - p_y$ for any $i, j \in N$. Hence, $\mathcal{G}_1(\mathbf{p},\mathbf{g}) = 0$. The proof follows.

Now we turn to the situation where $\mathcal{Y} \in (0,1)$. We take up the case where y weakly increases $\mathcal{W}(\mathbf{p},\mathbf{g})$. We show that that either $\mathcal{L}_0(i|\mathbf{p},\mathbf{g})$ dominates $\mathcal{L}_1(i|\mathbf{p},\mathbf{g})$ or $\mathcal{L}_0(i|\mathbf{p},\mathbf{g}) = \mathcal{L}_1(i|\mathbf{p},\mathbf{g})$.[16] Recall from the proof to Proposition 3 that $\pi^1_{i,A}(\mathbf{p},\mathbf{g}) \leq \pi^0_{i,A}(\mathbf{p},\mathbf{g})$. Since $\mathcal{W}_1(\mathbf{p},\mathbf{g}) \geq \mathcal{W}_0(\mathbf{p},\mathbf{g})$, then (i) $\Delta\mathcal{L}^A_1(i|\mathbf{p},\mathbf{g}) \leq \Delta\mathcal{L}^A_0(i|\mathbf{p},\mathbf{g})$ for any $i \in A$ and (ii) $\mathcal{W}^A_1(\mathbf{p},\mathbf{g}) / \mathcal{W}_1(\mathbf{p},\mathbf{g}) < \mathcal{W}^A_0(\mathbf{p},\mathbf{g}) / \mathcal{W}_0(\mathbf{p},\mathbf{g})$, where $\mathcal{W}^A_1(\mathbf{p},\mathbf{g}) = \sum_{i \in A} \pi_i(\mathbf{p},\mathbf{g})$. The latter also implies that $\mathcal{W}^B_1(\mathbf{p},\mathbf{g}) / \mathcal{W}_1(\mathbf{p},\mathbf{g}) > \mathcal{W}^B_0(\mathbf{p},\mathbf{g}) / \mathcal{W}_0(\mathbf{p},\mathbf{g})$, and so $\mathcal{L}^B_1(n_B|\mathbf{p},\mathbf{g}) = \mathcal{L}^A_1(0|\mathbf{p},\mathbf{g}) > \mathcal{L}^B_0(n_B|\mathbf{p},\mathbf{g}) = \mathcal{L}^A_0(0|\mathbf{p},\mathbf{g})$.

It further follows from points (i) and (ii) above that $\mathcal{L}^A_1(i|\mathbf{p},\mathbf{g})$ dominates $\mathcal{L}^A_0(i|\mathbf{p},\mathbf{g})$. To see why, suppose *a contrario* that there exists a $j \in A$ such that $\mathcal{L}^A_0(j|\mathbf{p},\mathbf{g}) > \mathcal{L}^A_1(j|\mathbf{p},\mathbf{g})$. Remark that $\mathcal{L}^A_0(n|\mathbf{p},\mathbf{g}) = \mathcal{L}^A_1(n|\mathbf{p},\mathbf{g})$ by definition. Since both $\mathcal{L}^A_0(0|\mathbf{p},\mathbf{g})$ and $\mathcal{L}^A_1(0|\mathbf{p},\mathbf{g})$ are continuous and strictly increasing, $\mathcal{L}^A_0(j|\mathbf{p},\mathbf{g}) > \mathcal{L}^A_1(j|\mathbf{p},\mathbf{g})$ implies there there is at least one $l \in A$ such that $\Delta\mathcal{L}_1(l|\mathbf{p},\mathbf{g}) > \Delta\mathcal{L}_0(l|\mathbf{p},\mathbf{g})$. This entails that there is at least one $l \in A$ such that $\pi^1_l(\mathbf{p},\mathbf{g}) > \pi^0_l(\mathbf{p},\mathbf{g})$, which is a contradiction.

We now show that if the introduction of y weakly increases $\mathcal{W}(\mathbf{p},\mathbf{g})$, then $\mathcal{L}^B_1(i|\mathbf{p},\mathbf{g})$ dominates $\mathcal{L}^B_0(i|\mathbf{p},\mathbf{g})$ too. To see why, suppose a contrario that there exists a $j \in B$ such that $\mathcal{L}^A_0(B|\mathbf{p},\mathbf{g}) > \mathcal{L}^A_1(B|\mathbf{p},\mathbf{g})$. Recall first that $\mathcal{L}^B_0(i|\mathbf{p},\mathbf{g})$ is convex by definition. Further, since $\pi^1_j(\mathbf{p},\mathbf{g}) = 1 - p_y$ for all $j \in B$, then $\Delta\mathcal{L}_1(j|\mathbf{p},\mathbf{g})$ is the same for all $j \in B$. In other

[16]For this proof, suppose that the ordering of players does not change (hence, suppose that the support of $\mathcal{L}_0$ is the same as for $\mathcal{L}_1$). While the resulting $\mathcal{L}_1$ is not the "true" Lorenz curve as players are not necessarily ordered by payoffs, the resulting $\mathcal{G}_1$ is the right one, and hence this assumption is made without loss of generality. Furthermore, since the proof for $\mathcal{Y}(\mathbf{p},\mathbf{g}) = 0$ is immediate, we focus on the case that $\mathcal{Y}(\mathbf{p},\mathbf{g}) \in (0,1)$ for the remainder of the proof for Case A.

words, $\mathcal{L}_1^B(j|\mathbf{p},\mathbf{g})$ is a straight line from $(0,0)$ to $\left(n_B, \mathcal{L}_1^B(n_B|\mathbf{p},\mathbf{g})\right)$. Since $\mathcal{L}_1^B(j|\mathbf{p},\mathbf{g})$ is a straight line, $\mathcal{L}_0^B(j|\mathbf{p},\mathbf{g})$ is convex, $\mathcal{L}_1^B(0|\mathbf{p},\mathbf{g}) = \mathcal{L}_1^B(0|\mathbf{p},\mathbf{g})$ and there exists a j such that $\mathcal{L}_0^A(j|\mathbf{p},\mathbf{g}) > \mathcal{L}_1^A(j|\mathbf{p},\mathbf{g})$, then that $\mathcal{L}_0^B(n_B|\mathbf{p},\mathbf{g}) > \mathcal{L}_1^B(n_B|\mathbf{p},\mathbf{g})$. However, this contradicts the hypothesis that $\mathcal{L}_0^B(n_B|\mathbf{p},\mathbf{g}) < \mathcal{L}_1^B(n_B|\mathbf{p},\mathbf{g})$.

Hence, we have shown that $\mathcal{L}_1^A(i|\mathbf{p},\mathbf{g})$ dominates $\mathcal{L}_0^A(i|\mathbf{p},\mathbf{g})$ and that $\mathcal{L}_1^B(i|\mathbf{p},\mathbf{g})$ dominates $\mathcal{L}_0^B(i|\mathbf{p},\mathbf{g})$ when $\mathcal{W}_1(\mathbf{p},\mathbf{g}) \geq \mathcal{W}_0(\mathbf{p},\mathbf{g})$ and $\mathcal{Y}(\mathbf{p},\mathbf{g}) \in (0,1)$.
(ii). $\mathcal{Y}(\mathbf{p},\mathbf{g}) \notin \{0,1\}$ **and** $\mathcal{W}_1(\mathbf{p},\mathbf{g}) < \mathcal{W}_0(\mathbf{p},\mathbf{g})$
The example given in the text proves by construction this part of the proposition.

Case B: Proof for $\theta = 1$**:**
(i) $p_y < 1$**:** The proof proceeds through an example. $\mathcal{G}(\mathbf{p},\mathbf{g})$ can either increase or decrease following the introduction of y. Consider the graph on Figure 9, and assume that $\theta = 1$. Fix $p_y = 0.5$ in what follows.

1. Let $p_x = 7.1$: Before the introduction of y, all players' payoffs amount to 0 (as all players play $x = 0$), and so $\mathcal{W}_0(\mathbf{p},\mathbf{g}) = 0$ and $\mathcal{G}_0(\mathbf{p},\mathbf{g}) = 0$. After the introduction of y, the individual payoffs of players 9 to 16 rise to 7.3 (as they now play $x = y = 1$), while those of all other players rise to 0.5. As a result, $\mathcal{W}_1(\mathbf{p},\mathbf{g}) = 63.2$ and $\mathcal{G}_1(\mathbf{p},\mathbf{g}) = 0.4367$, indicating a *rising* $\mathcal{G}(\mathbf{p},\mathbf{g})$.

2. Let $p_x = 3.5$: Prior to introduction of y, players 9 to 16 play $x = 1$, while all others play $x = 0$. As a result, $\mathcal{W}_0(\mathbf{p},\mathbf{g}) = 28$ and $\mathcal{G}_1(\mathbf{p},\mathbf{g}) = 0.5$. After the introduction of y, payoffs of players 9 to 16, 1 to 4 and 5 to 8 are respectively 11, 3 and 0.5. As a result, $\mathcal{W}_1(\mathbf{p},\mathbf{g}) = 102$ and $\mathcal{G}_1(\mathbf{p},\mathbf{g}) = 0.3873$, indicating a *falling* $\mathcal{G}(\mathbf{p},\mathbf{g})$.

(ii): $p_y \geq 1$**,** $S_{p_x} \setminus S_{m_2} = \{\emptyset\}$ **and** $S_{m_2} \neq \{\emptyset\}$
From Theorem 1, when $p_y \geq 1$, players can be partitioned into three strategy groups, namely S_{m_2}, $S_{p_x} \setminus S_{m_2}$and $N \setminus \{S_{m_2} \cup S_{p_x}\}$. Label these groups A, B and C, respectively. Hence, for any $i \in A$, $x_{i,A}^1 = 1$ and $y_{i,A} = 1$; for any $j \in B$, $x_{j,B}^1 = 1$ and $y_{j,B} = 0$; while for any $l \in C$, $x_{l,C}^0 = y_{l,C} = 0$.

Note that if $B = \{\emptyset\}$ and $A \neq \{\emptyset\}$, then players can be partitioned into two groups only, A and C. The introduction of y has an ambiguous effect on $\mathcal{G}(\mathbf{p},\mathbf{g})$. To see this, consider indeed Figure 9 again and fix $p_y = 1.5$.

1. $p_x = 7.1$: Before the introduction of y, all players' payoffs amount to 0 (as all players play $x = 0$), and so $\mathcal{W}_0(\mathbf{p},\mathbf{g}) = 0$ and $\mathcal{G}_0(\mathbf{p},\mathbf{g}) = 0$. After the introduction of y, only players 9 to 16 adopt both x and y, while all other players play $x = y = 0$. The individual payoffs of players 9 to 16 thus rise to 6.4, while those of all other players remain unchanged. As a result, $\mathcal{W}_1(\mathbf{p},\mathbf{g}) = 51.2$ and $\mathcal{G}_1(\mathbf{p},\mathbf{g}) = 0.5$, indicating a *rising* $\mathcal{G}(\mathbf{p},\mathbf{g})$.

2. $p_x = 3.5$: Before the introduction of y, players 9 to 16 play $x = 1$, while all others play $x = 0$. As a result, $\mathcal{W}_0(\mathbf{p},\mathbf{g}) = 28$ and $\mathcal{G}_1(\mathbf{p},\mathbf{g}) = 0.5$. After the introduction of y, players 1 to 4 and 9 to 16 all play $x = y = 1$, while players 5 to 8 stick to $x = y = 0$. The individual payoffs of players 9 to 16, 1 to 4 and 5 to 8 then are respectively 10, 2 and 0. As a result, $\mathcal{W}_1(\mathbf{p},\mathbf{g}) = 88$ and $\mathcal{G}_1(\mathbf{p},\mathbf{g}) = 0.4318$, indicating a *falling* $\mathcal{G}(\mathbf{p},\mathbf{g})$.

(iii): $p_y \geq 1$ **and (i)** $S_{m_2} = \{\emptyset\}$**; or (ii)** $S_{m_2} \neq \{\emptyset\}$**,** $S_{p_x} \setminus S_{m_2} \neq \{\emptyset\}$ **and** $N \setminus \{S_{m_2} \cup S_{p_x}\} \neq \{\emptyset\}$**.**
First, note that whenever $p_y \geq 1$ and $S_{m_2} = \{\emptyset\}$, then $y_i = 0$ for all $i \in N$. In this case, the Gini coefficient is left unchanged by the introduction of y.

Suppose now that $p_y \geq 1$ and that the three sets A, B and C are non-empty. Clearly, since $\pi^1_{l,C}(\mathbf{p},\mathbf{g}) = \pi^0_{l,C}(\mathbf{p},\mathbf{g}) = 0$ for all $l \in C$, $\mathcal{L}^C_0(l|\mathbf{p},\mathbf{g}) = \mathcal{L}^C_1(l|\mathbf{p},\mathbf{g}) = 0$ for all $l \in C$. Likewise, we know that $\pi^1_{j,B}(\mathbf{p},\mathbf{g}) = \pi^0_{j,B}(\mathbf{p},\mathbf{g})$ for all $j \in B$. This is true because the the set of individuals who adopt $x = 1$ remains unchanged after the introduction of y (because $q_1 > p_x$.) Finally, since we know that $\mathcal{W}_1(\mathbf{p},\mathbf{g}) > \mathcal{W}_0(\mathbf{p},\mathbf{g})$ from Proposition 2, then it follows immediately that $\mathcal{L}^B_0(j|\mathbf{p},\mathbf{g}) > \mathcal{L}^B_1(j|\mathbf{p},\mathbf{g})$ for all $j \in B$.

We lastly prove that $\mathcal{L}^A_0(i|\mathbf{p},\mathbf{g})$ dominates $\mathcal{L}^A_1(i|\mathbf{p},\mathbf{g})$. First note that since $\mathcal{L}^B_0(j|\mathbf{p},\mathbf{g}) > \mathcal{L}^B_1(j|\mathbf{p},\mathbf{g})$, then $\mathcal{L}^A_0(0|\mathbf{p},\mathbf{g}) > \mathcal{L}^A_1(0|\mathbf{p},\mathbf{g})$ by definition. Further, note that the support of $\mathcal{L}^A_0(i|\mathbf{p},\mathbf{g})$ is the same as for $\mathcal{L}^A_1(i|\mathbf{p},\mathbf{g})$ since the order of players by payoffs does not change. Indeed, for any $i \in A$ with m_i links to other players in $\mathcal{S}_{p_x}$, we can write the payoffs before and after the introduction of y respectively as $\pi^0_{i,A}(\mathbf{p},\mathbf{g}) = m_i - p_x$ and $\pi^1_{i,A}(\mathbf{p},\mathbf{g}) = 2m_i + 1 - p_y - p_x$. Hence,$\pi^0_{i,A}(\mathbf{p},\mathbf{g}) > \pi^0_{j,A}(\mathbf{p},\mathbf{g}) \Leftrightarrow \pi^1_{i,A}(\mathbf{p},\mathbf{g}) > \pi^1_{i,A}(\mathbf{p},\mathbf{g})$.

The next step is to show that $\Delta\mathcal{L}^A_1(i|\mathbf{p},\mathbf{g}) > \Delta\mathcal{L}^A_0(i|\mathbf{p},\mathbf{g})$ for one $i \in A$, then $\Delta\mathcal{L}^A_1(l|\mathbf{p},\mathbf{g}) > \Delta\mathcal{L}^A_0(l|\mathbf{p},\mathbf{g})$ for all $l > i$. Observe that $\Delta\mathcal{L}^A_1(i|\mathbf{p},\mathbf{g}) > \Delta\mathcal{L}^A_0(i|\mathbf{p},\mathbf{g})$ implies

$$\frac{2m_i + 1 - p_x - p_y}{\mathcal{W}_1(\mathbf{p},\mathbf{g})} > \frac{m_i - p_x}{\mathcal{W}_0(\mathbf{p},\mathbf{g})} \tag{16}$$

which in turns implies that

$$\frac{2m_i + 1 - p_x - p_y}{m_i - p_x} > \frac{\mathcal{W}_1(\mathbf{p},\mathbf{g})}{\mathcal{W}_0(\mathbf{p},\mathbf{g})} \tag{17}$$

Since $\frac{\mathcal{W}_1(\mathbf{p},\mathbf{g})}{\mathcal{W}_0(\mathbf{p},\mathbf{g})}$ is a constant and $\frac{2m_i+1-p_x-p_y}{m_i-p_x}$ is strictly increasing in m_i, this inequality must hold fo all $l > i$ since $m_l > m_i$.

Finally, suppose *a contrario* that $\mathcal{L}^A_0(i|\mathbf{p},\mathbf{g})$ does not dominate $\mathcal{L}^A_1(i|\mathbf{p},\mathbf{g})$, such that there is a i such that $\mathcal{L}^A_0(i|\mathbf{p},\mathbf{g}) < \mathcal{L}^A_1(i|\mathbf{p},\mathbf{g})$. Since $\mathcal{L}^A_0(0|\mathbf{p},\mathbf{g}) > \mathcal{L}^A_1(0|\mathbf{p},\mathbf{g})$ and $\mathcal{L}^A_0(n|\mathbf{p},\mathbf{g}) = \mathcal{L}^A_1(n|\mathbf{p},\mathbf{g})$ and both $\mathcal{L}^A_0(i|\mathbf{p},\mathbf{g})$ and $\mathcal{L}^A_1(i|\mathbf{p},\mathbf{g})$ are convex by definition, then there must exist one $l \in A$ and one $j \in A$ such that $j > l$ and $\Delta\mathcal{L}^A_1(l|\mathbf{p},\mathbf{g}) > \Delta\mathcal{L}^A_0(l|\mathbf{p},\mathbf{g})$ and $\Delta\mathcal{L}^A_1(j|\mathbf{p},\mathbf{g}) <$

$\Delta\mathcal{L}_0^A(j|\mathbf{p},\mathbf{g})$. This contradicts the above step. We have thus proved that $\mathcal{L}_0^A(i|\mathbf{p},\mathbf{g})$ dominates $\mathcal{L}_1^A(i|\mathbf{p},\mathbf{g})$. ■

Appendix B

We now provide a complete characterization of the unique maximal equilibrium for $\theta > -1$. We distinguish between the cases of (imperfect) substitutes ($\theta \in (-1, 0)$) and complements ($\theta \geq 0$).
Consider first the case of *substitutes*. Consider the following thresholds values:

$$\begin{aligned} s_1 &= \tfrac{1-p_y}{-\theta} \\ s_2 &= \tfrac{p_x}{1+\theta} \end{aligned} \tag{18}$$

The threshold value s_1 represents the minimum number of links to neighbours playing $x = 1$ for the adoption of y to incur a loss larger than the nominal gains. The threshold s_2 represents the minimum number of links to neighbours playing $x = 1$ for the adoption of x to be profitable even when $y = 1$.

Consider next the case of *complements*. As for Theorem 1, denote by c_1 the number of links to neighbours playing $x = 1$ required for player i to be indifferent between $x_i = y_i = 1$ and any other action profile, as follows:

$$c_1 = \begin{cases} \frac{p_x}{1+\theta} & if\, p_y < 1 \\ \max\left\{\frac{p_y-1}{\theta},\ \frac{p_x+p_y-1}{1+\theta}\right\} & if\, p_y \geq 1 \end{cases} \tag{19}$$

Theorem 3 *A maximal equilibrium exists and is generically unique. Let $(\mathbf{x}^*, \mathbf{y}^*)$ be the maximal equilibrium.*

- ***Substitutes** ($\theta \in (-1, 0)$): If $p_y < 1$, then $(x_i^*,\, y_i^*) = (1, 0)$ if $i \in \mathcal{S}_{\max\{p_x, s_1\}}$; $(x_i^*,\, y_i^*) = (1, 1)$ if $i \in \{\mathcal{S}_{s_2} \setminus \mathcal{S}_{s_1}\}$; $(x_i^*,\, y_i^*) = (0, 1)$ if $i \notin \mathcal{S}_{\max\{s_1, s_2\}}$; and $(x_i^*,\, y_i^*) = (0, 0)$ otherwise. If $p_y \geq 1$, then $(x_i^*,\, y_i^*) = (1, 0)$, for $i \in \mathcal{S}_{p_x}$ and $(x_i^*,\, y_i^*) = (0, 0)$ for $i \notin \mathcal{S}_{p_x}$.*
- ***Complements** ($\theta \geq 0$): If $p_y < 1$, then $(x_i^*,\, y_i^*) = (1, 1)$ for $i \in \mathcal{S}_{c_1}$ and $(x_i^*,\, y_i^*) = (0, 1)$ for $i \in \mathcal{S}_{c_1}$. If $p_y \geq 1$, then $(x_i^*,\, y_i^*) = (1, 1)$ for $i \in \mathcal{S}_{c_1}$, $(x_i^*,\, y_i^*) = (1,\, 0)$ for $i \in \mathcal{S}_{p_x} \setminus \mathcal{S}_{c_1}$ and $(x_i^*,\, y_i^*) = (0,\, 0)$ for $i \in N \setminus \mathcal{S}_{c_1} \cup \mathcal{S}_{p_x}$.*

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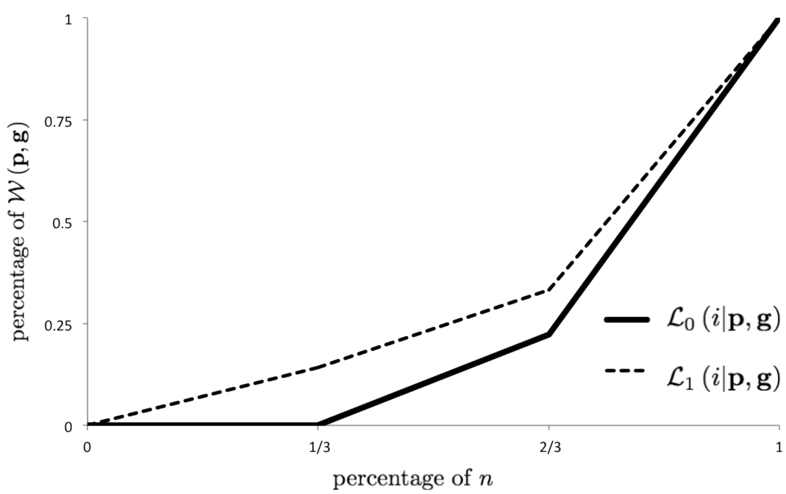
Figure 6: The Gini-Coefficient
(Left graph: $\theta = -1$, $p_x = 2.2$ and $p_y = 0.4$; Right graph: $\theta = 1$, $p_x = 2.2$ and $p_y = 4.1$)



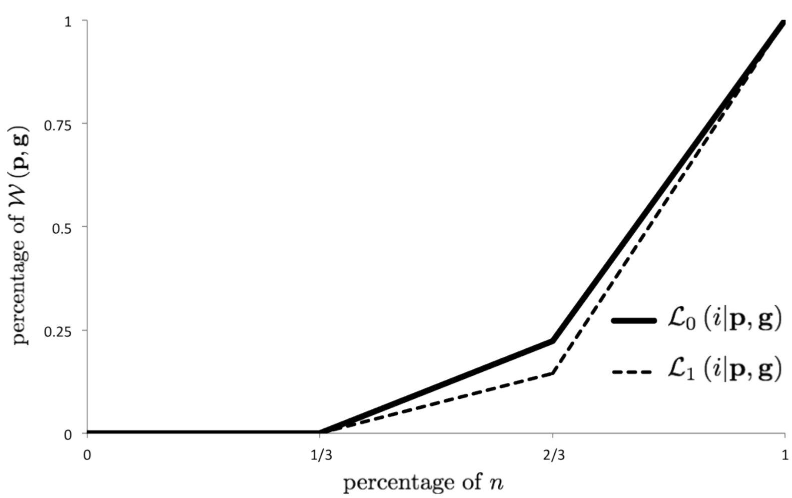
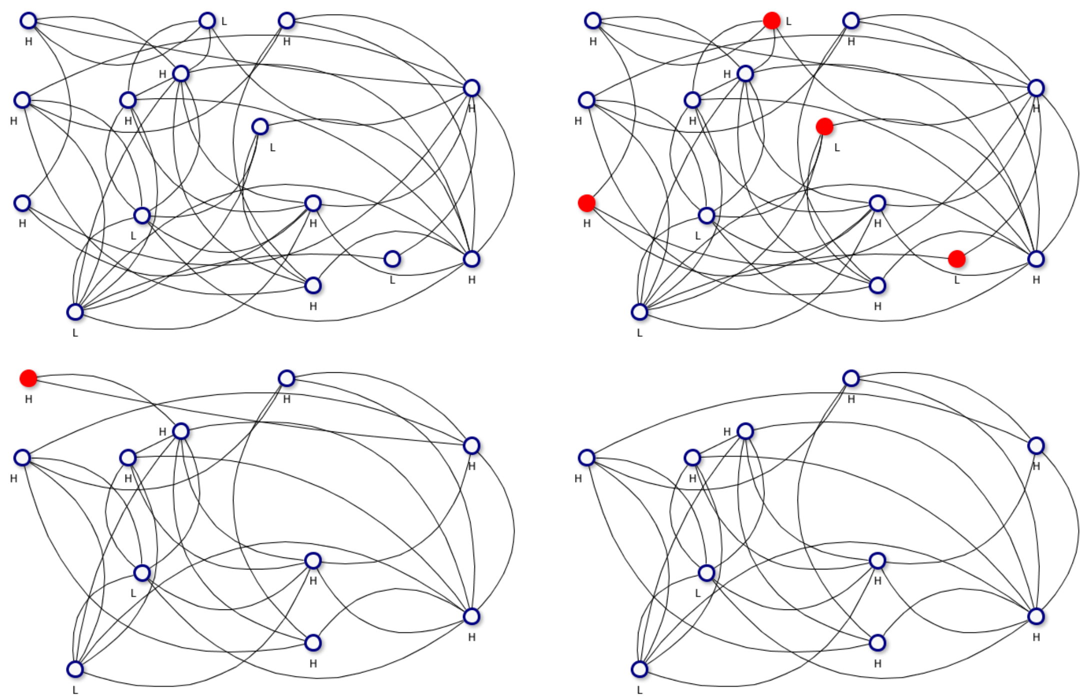


Figure 7: The q_i-connected club in an arbitrary network, with $q_H = 2$ and $q_L = 5$



Top left: initial graph. **Top right**: delete all L nodes with $k \leq 5$ and H nodes with $k \leq 2$. **Bottom left**: among the nodes remaining, delete all L nodes with $k \leq 5$ and H nodes with $k \leq 2$. **Bottom right**: the q_i-connected club obtains when no further iteration is possible.

Figure 8: The Ambiguous Effect of y on $\mathcal{R}(\mathbf{p}, \mathbf{g})$ when $\theta = 1$: An Example

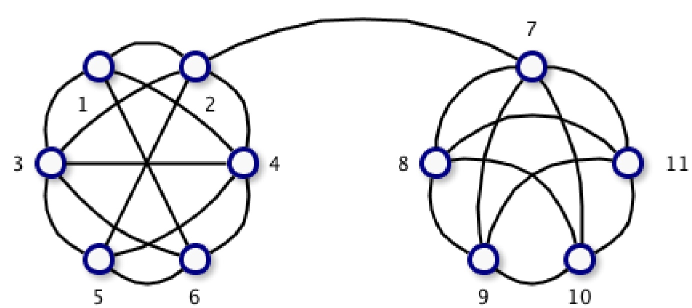


Figure 9: