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Embracing the Vision: Our Work with Teachers Implementing GPS

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Abstract: In 2005, three Kennesaw State University mathematics education faculty members began a series of workshops titled "Implementing the Georgia Performance Standards [GPS]: Embracing the Vision." This workshop series has been underwritten by Georgia's Teacher Quality Higher Education Program. The first series of workshops began with 6th grade teachers the first year the GPS was implemented and the project has been funded each subsequent year since its inception. Currently, we are working with Math III teachers as they implement the course for the first time. The initial focus for the project was on conceptual understanding versus procedural understanding, writing tasks that facilitate students' conceptual understanding, assessment, and resources. While we have maintained these foci, we have found it necessary to emphasize mathematical content development and the process standards due to the needs of the participants. In this paper, we discuss the structure of our workshop series, focusing on process standards and mathematical content. We provide a specific example of a task we used with participants in our first Math III workshop to illuminate the features of our work.

Embracing the Vision: Our Work with Teachers Implementing GPS

Overview

In 2005, three Kennesaw State University mathematics education faculty members began a series of workshops titled "Implementing the Georgia Performance Standards [GPS]: Embracing the Vision." This workshop series has been underwritten by Georgia's Teacher Quality Higher Education Program. The first series of workshops began with 6th grade teachers the first year the GPS was implemented and the project has been funded each subsequent year since its inception. Currently, we are working with Math III teachers as they implement the course for the first time.

The series for a given year includes nine workshops: three full summer days and six workshops throughout the year (two full days and one evening in the fall, and two full days and one evening in the spring). This allows 50 seat hours so that we may award the participants 5 professional learning units. For each year funded, participating school districts have agreed to a cost-share obligation to pay for substitute teachers for the four full day academic-year workshops. We have received positive feedback from our participants via formal assessments and informal comments. Through sustained contact, we are able to address a variety of pedagogical issues and mathematical content.

The initial focus for the project was on conceptual understanding versus procedural understanding, writing tasks that facilitate students' conceptual understanding, assessment, and resources. While we have maintained these foci, we have found it necessary to emphasize mathematical content development and the process standards due to the needs of the participants. We found ourselves surprised when our participants were not able to name the five process standards after participating in our entire workshop series. This year, we have intensified our focus on the process standards during each session, with the intention of emphasizing their importance in the curriculum and helping participants use the process standards more to guide their own teaching and assessment practices.

Focus on the Process Standards

We introduced our participants to the Task Analysis Framework (Stein et al., 2009), and used it to analyze the cognitive demand level of tasks that are available in the GPS frameworks and others that we provide. We discussed the factors that can change the cognitive demand level of a task during the implementation phase and how to use the process standards as a guide to writing open-ended questions. For example, we began with the traditional question: Expand: $(x + 3)^2$. We discussed how to use the process standards to open the question and raise its cognitive demand. A first step in a Mathematics III classroom could be to simply ask students to explain how they determined their solution, which would incorporate the *communication* standard. We could take the question a step further and capitalize on a common student error and ask them to explain why $(x + 3)^2 \neq x^2 + 9$. Now the students would be using communication as well as *reasoning and proof.* Expanding further, we could ask students to provide two different explanations as to why $(x + 3)^2 \neq x^2 + 9$. By asking for multiple explanations, we can force students into using different *representations*. Some students might use a numerical approach and substitute a value for x to show that $(x + 3)^2 \neq x^2 + 9$. Other students could use a graphical approach, wherein they graphed $y_1 = (x + 3)^2$ and $y_2 = x^2 + 9$ and showed that the two graphs were different. Still others could use a geometric approach, showing algebra tile models of $(x + 3)^2$ and $x^2 + 9$ and the different areas represented by each expression. And, of course, there is the algebraic approach that students could use (traditionally called FOIL). Using this variety of representations allows students to make *connections* between algebraic, numeric, and geometric ideas. Because students are not provided a step-by-step procedure to answer this question, we consider the act of answering the question as *problem solving*.

As part of our workshops, we have participants actually solve this problem and present their solutions. Participants make comments such as, "I never would have thought of that in a million years" when they see the variety of ways to counteract this common student misconception. We point out the fact that making these relatively small changes to a traditional question raises the cognitive demand of the question considerably. We believe that engaging participants in these kinds of discussions broadens their own understanding of mathematics. After answering some open-ended questions themselves, participants work in groups to write open-ended questions for the GPS units which are the focus of that particular session. Each group presents their questions to the whole group, and the questions are modified based on group feedback.

Focus on Mathematical Content

Another major focus of our workshops is mathematical content development. Many of our participants had been teaching the same courses for years prior to the GPS implementation. Due to the nature of the prior curriculum (the Quality Core Curriculum), it was possible for teachers to identify themselves as "geometry" or "algebra" teachers. Very few teachers taught statistics. In moving to the integrated curriculum, some teachers were required to teach content such as data analysis for the first time. Many of our participants reported being nervous about teaching all strands of the curriculum in each course.

At the beginning of each new workshop series, we conduct a needs assessment by giving our participants a "problem set" to complete in one hour. There are approximately 15 short answer and essay style questions that cover major topics in the curriculum. The statistics/data analysis content was the area in which the participants struggled most; however, we found that the participants needed support on all of the mathematical content to some degree. Though we recognized that data analysis/statistics was the weakest content area, this is the last unit to be addressed according to the Mathematics III Curriculum Map. Therefore, we began this year's series of workshops by focusing on the content in the algebra units, specifically those involving matrices, polynomial functions and conics. In each workshop, we investigate content in more depth using discovery tasks and activities found in the GPS frameworks and other places while emphasizing the process standards.

An Example

The following task is an example that we discussed in our first Math III workshop during the summer of 2010. A related task can be found in the frameworks involving finding a quadratic equation given three points of a parabola. This particular task was chosen to illustrate how one task can integrate content from multiple units: polynomial functions, conics, and solving equations. As we approached each task, we made a conscious effort to explicitly point out the process standards.

We began our investigation of a parabola with a geometric *representation*. We asked participants to use patty paper and paper folding to create an envelope of lines tangent to a parabola. Before using this task, we created a GSP file and, using the "fit to page" function in the print preview mode, we printed the sketch on a single sheet of paper. Gridlines and axes were included on the printout. We prepared the patty paper by laying each piece of patty paper on the printout and then tracing the focus and the directrix on each participant's patty paper as shown in Figure 1. We then asked the participants to fold the patty paper up so that the line (the directrix) went through the point (the focus). In other words, for a variety of points A on the directrix (see Figure 2), the participants folded the patty paper so that point A landed on top of the focus. The envelope of lines formed by the various creases in the paper is tangent to a parabola.

Next, we asked participants to consider the paper-folding activity and the mathematics behind it to produce a sketch using Geometer's Sketchpad that would generate a locus of points forming a parabola. Participants then tried to prove why the locus of points formed a parabola (see Figure 3).

The participants had to use some *problem solving* to come up with this sketch, and as they worked on their proof, they were, of course, using the *reasoning and proof* standard. We next asked the participants to put their patty paper aside and to work another problem. They worked in groups to find the equation of a parabola that contained the three points B, C, and D. By posing this task, we had turned the participants' attention to an algebraic *representation* of a parabola.

B: (-8

C: (-6

D: (1.

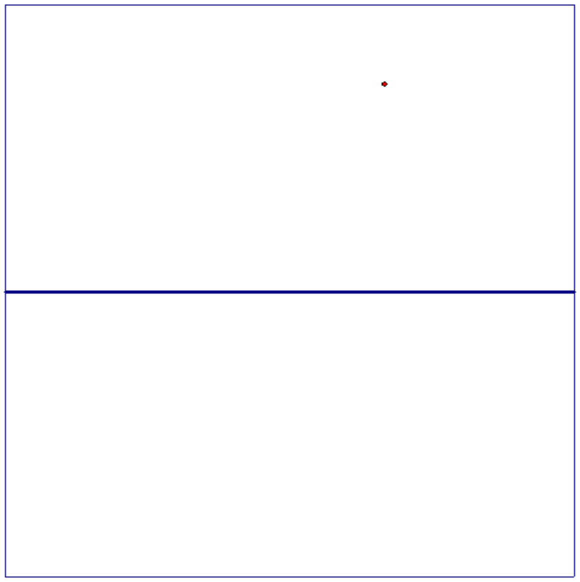

Figure 1. Patty paper with focus & directrix

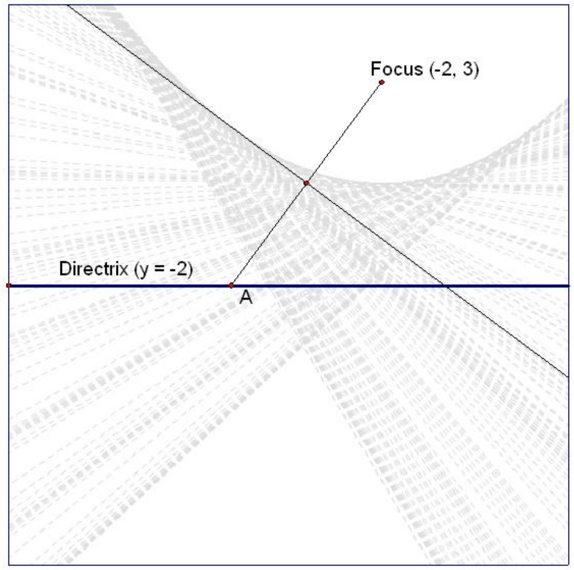

Figure 2. Paper folding parabola

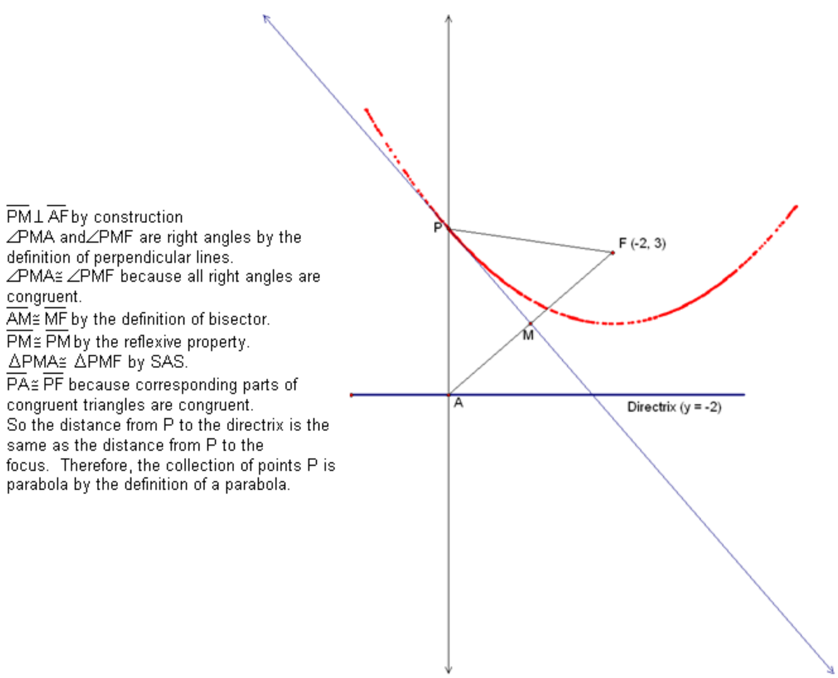

Figure 3. Informal proof of a parabola construction using The Geometer's Sketchpad.

In order to find the equation, most participants set up a system of equations and solved it using matrices. During this part of our investigation, our participants were drawing on *connections* between conic sections and matrices: two different units in the GPS.

$$4.61 = a\,(-8.41)^2 + b\,(-8.41) + c$$
$$2.56 = a\,(-6.54)^2 + b\,(-6.54) + c$$
$$2.05 = a\,(1.93)^2 + b\,(1.93) + c$$

Rewriting this system of equations, we get

$$70.73a - 8.41b + c = 4.61$$
$$42.77a - 6.54b + c = 2.56$$
$$3.72\,a + 1.93b + c = 2.05$$

If $A = \begin{bmatrix} 70.73 & -8.41 & 1 \\ 42.77 & -6.54 & 1 \\ 3.72 & 1.93 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 4.61 \\ 2.56 \\ 2.05 \end{bmatrix}$, then $A^{-1}B = \begin{bmatrix} .1 \\ .4 \\ .9 \end{bmatrix}$ (see Figure 4).

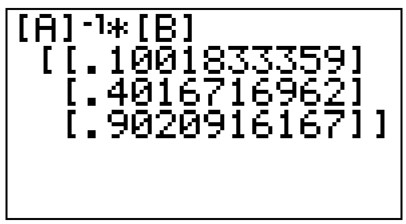

Figure 4. Matrix Solution.

Their resulting equation was $y = .1x^2 + .4x + .9$. Some participants even used a quadratic regression to find the equation, which made *connections* with the data analysis and probability strand of the curriculum.

Next, we asked participants to find the equation of a parabola given that the focus was (-2, 3) and the directrix was y = -2. This is where the real fun began! Some participants used the formula $(x - h)^2 = 4p(y - k)$, where p is the focal distance (the distance from the focus to the vertex) to find the equation. We mentioned that there were other ways to determine the answer. When the participants presented their solutions, the group that used the formula showed their work on the board. We said their answer was correct, and then asked them why the formula worked. The room grew very quiet. And then one participant laughed and said, "'cause that's what the book says". Laughter erupted. The participant realized that she had been teaching this formula without making connections to meaning (Stein, et al., 2009). We asked the participants to try to figure out why the formula worked. One participant came to the board and began to work on the problem. The participants worked together and derived the formula from the definition of a parabola. The excitement in the room grew. The participant at the board reflected on the experience and said,

> As experienced educators, we sometimes forget that the true beauty in math is in the way everything develops from basic algorithms and procedures. I was recently allowed the opportunity to rejuvenate that feeling of mathematical euphoria in a training class. As we were discussing the various forms of the equation of a parabola, my mind began to wander to the theoretical side of the process of developing these equations and I remembered how invigorating it is to allow the beauty of the mathematics to support the things we take for granted. The instructor…began by asking probing questions that stimulated the portion of my mind that makes me feel the need to take risks. I began to ask a great deal of "what if" questions and writing things down in hopes that the pieces would come together and produce something useful. I was allowed

> to present my findings to the class and reveled in development of the proof of the standard form of the equation of a parabola!! I am not sure if the others around me immediately supported my math mania, but as I communicated my ideas, others began to get sucked in. There were still others that I am not sure ever came to the level of involvement in the problem that I was, but they certainly appreciated the excitement I was exhibiting and shared with me in the joy of my accomplishments. Not only did I leave the session that day excited about what had taken place, but I continued to work on the proof I had done and rewrote it so that everyone could, hopefully, take the time to look at how it worked. That day in class has challenged me to be more like this with my students and realize that when they take ownership in the development of processes there is a remote chance of them having that "euphoric" feeling.

The next day, this participant wrote up the derivation of the formula to share with the group (see Figure 5).

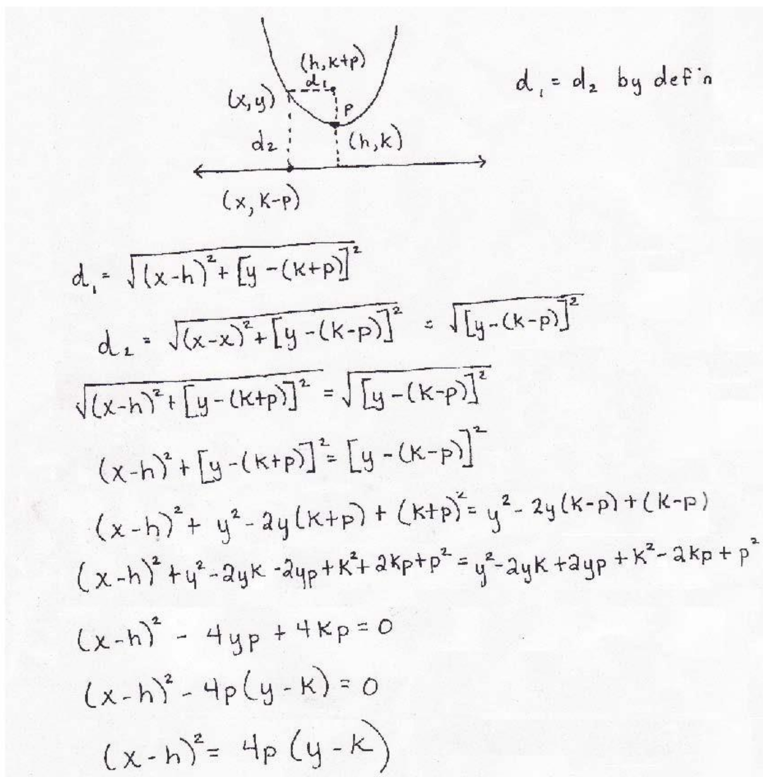

Figure 5. Participant's derivation of equation of parabola given focus and directrix.

When the participants had presented their various solutions, we used Geometer's Sketchpad to graph the equation of the parabola. We handed out a printout of the graph that we used to prepare the patty paper and asked the participants to lay their patty paper on top of the graph. We had graphed the equation of the function, and the equation was included on the printout. The envelope of lines they generated through paper-folding was perfectly tangent to the graph of the equation of the parabola that they determined algebraically. When they put the patty paper on the graph, there was a collective gasp in the room. Then there was a chorus of "cool", and "awesome", and "how about that?". One participant commented,

> What a great 3 days! Regarding the parabola problem.....It was great to take the parabola apart and put it back together. We looked at it from every angle....what an understanding to carry with me into the classroom. Thanks for encouraging me to make the students really think. I will put more thought into the questions I ask the kids.

We concluded our discussion of the parabola problem by considering the standards the problem addressed, particularly the process standards. The participants agreed that all five process standards were evident in the problem. They had to *problem solve* to derive the formula for the equation of the parabola. They *communicated* with their group members as they solved various parts of the problem, and the whole group *communicated* together to derive the equation. They made *connections* between the graph, the equation, and the definition of a parabola. They used algebraic and geometric *representations* of a parabola. They used *reasoning and proof* to derive the equation of the parabola and to prove that the locus of points we created on Geometer's Sketchpad formed a parabola. With respect to the content standards in Mathematics III, the parabola investigation cuts across several units (Matrices, Conic Sections, Graphs of Polynomial Functions). The investigation also relies on prior knowledge from previous courses (e.g., congruence proofs from Math I, quadratic regression from Math II).

Conclusions

In order to inform ourselves more about participants' mathematical content learning, we ask them during the final survey if their understanding of the content increased through their participation in this project and to provide specific examples. Comments from our Math II workshops included "increased confidence," "better understanding," and "gained a thorough understanding of content." Content specific comments included "statistics has always been a weak spot for me but this workshop helped me refresh my skills" and "[I am] much more comfortable with the expectation of data analysis which I have not done in a long time. Median/median lines I don't recall ever doing."

We used tasks and discovery activities for the purpose of developing our participants' mathematical content knowledge, and we found that our participants appreciated observing different teaching strategies including group collaborations (jigsaw, think-pair-share, etc.). Initially, some participants discussed having a negative view of using tasks in their classrooms, citing reasons such as too much reading, too much class time, too much recall of prior mathematics, difficulty in assessment, and too little skill practice. With discussion of how the tasks may be modified to better meet their teaching needs, our participants found at least pieces of tasks that they felt comfortable using in their classrooms. Participants commented that their exposure to multiple teaching strategies enabled them to envision how students could be guided to a more conceptual understanding of content.

These types of mathematical investigations have been the hallmark of the professional development we provide for teachers. Our emphasis on the process standards, using higher-order questioning, and learning new mathematical content through investigation, discovery, proof, and verification, have brought participants back to our workshops year after year. Nearby district personnel are quick to recommend our workshops to their participants. We believe we have developed a high-quality model for professional development to support teachers as they implement a new curriculum. After visiting our project, a representative from the Teacher Quality office emailed us and said,

> You have designed a high quality program for your Math III teachers—and they appreciate it. One participant told me, 'I'll take anything that has the name Sanchez, Ledford, Fox or Garner on it. I always know it's going to be worthwhile.'

One thing we believe keeps our participants coming back to work with us is the climate we create during the workshops. It is possible to be very successful in mathematics and even to become a successful mathematics teacher without ever developing a deep, conceptual and connected understanding of mathematics. When teachers are confronted with their lack of conceptual understanding, it can be uncomfortable. For example, when the participant was at the board showing her answer to part of the parabola problem and we asked her why the formula worked, she was visibly uncomfortable. The next day, she even teased and said, "see if I ever go to the board again!" She was, however, just as engaged and involved in subsequent workshops.

We have a "heart to heart" discussion with our participants early in the series wherein we explain that we know that there will topics for which their own understanding is superficial and procedural. We tell our own stories of transformation from our early years of teaching when we focused on procedures to where we are today. We explain that our workshops are a "safe space" for them to ask questions, to admit that they do not understand, and for us to investigate mathematics together. We encourage participants to ask "why" questions and reinforce them when they are asked. We will make comments such as, "I'm so glad you asked that question". This kind of atmosphere helps bond our group and makes participants comfortable to explore mathematics and re-learn it when necessary. We feel privileged to be working with such dedicated teachers to improve mathematics education in Georgia.

References

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