Modeling of trapped fields by stacked (RE)BCO tape using angular transversal field dependency

Algirdas Baskys, Anup Patel, Simon C. Hopkins and Bartek A. Glowacki

***Abstract*—Stacks of superconducting (RE)BCO tape are gaining popularity as a potential alternative for superconducting bulks for trapped field applications. This is partly due to versatility and uniformity of the starting material, allowing for more deterministic prediction of field profile and magnitude. However, most FEM models of trapped field magnets do not incorporate such parameters as critical current and *n*-value dependency on the angle of applied magnetic field, leading to only qualitative modeling results. More quantitative results can be obtained from incorporating more data for superconductivity and thermal properties of the material. Such models can be used as a starting point for most geometries and both trapped field and current transport modeling problems. An FEM model of a stack of tapes was constructed using the *H* formulation, incorporating goniometric critical current and *n*-value measurements. The modeling results were compared to field cooling experiments for stacks of different heights. The experiment and modeling show good agreement.**

***Index Terms*— Field cooling, trapped field magnets, stack of HTS tapes, modeling of superconductors.**

I. INTRODUCTION

STACKS OF SUPERCONDUCTING TAPE layers can be used as magnetic field sources in the same way as (RE)BCO bulks by circulating a persistent current within the sample, as illustrated in Fig.1. Previous work by Patel *et al*. has shown that fields up to 7 T can be trapped [1] via field cooling in samples as small as 12 mm square and 13.8 mm high, compared to conventional ferromagnets limited to magnetic fields of about 1.5 T regardless of their size. Moreover, just like (RE)BCO bulks, stacks of (RE)BCO tape can be used for passive magnetic levitation in high speed contactless bearings [2]. Size, shape and achievable field can be tuned by adjusting the number of layers in the stack or their aspect ratio [3], [4].

With increasing (RE)BCO tape production, both the performance and uniformity of the superconducting properties are improving, which makes the behavior of these stacks or "composite bulks" very predictable. Hence various arrangements of stacked tape that suit the particular application can be modeled by only characterizing a single layer of the stack. This can be done by in-field goniometric measurements of critical current. This work shows results of an FEM model that incorporates data from such goniometric tests and predicts trapped fields in stacks of various heights. The modeling results were then compared to corresponding experimental data and another FEM model that does not incorporate the angular field dependency.

II. METHODS AND TECHNIQUES

A. Superconducting Tape

Superconducting tape used for this work was produced by SuperPower Inc. to specification SP12050 AP, i.e. $(Y,Gd)_{1+x}Ba_2Cu_3O_{7-\delta}$ with 7.5 % of Zr added. The stated $I_{c,min}$ was 240 A, at 77 K and self-field, over the whole length of the tape. However, segments tested showed much higher I_c, as shown in section III.A. The tape was 12 mm wide with a 50 µm thick Hastelloy substrate and a 2 µm thickness silver overlayer. The overall thickness of the tape was ~55 µm.

The tape was cut to 12x12 mm square pieces to make stacks for field-cooling tests and 40 mm long segments for goniometric critical current measurements.

B. Goniometric Critical Current Measurements

Goniometric measurements were performed at 77 K in liquid nitrogen and a magnetic flux density range of 0 to 0.5 T. The critical current goniometer was developed in house [5] (see Fig 2). All orientations of the magnetic field with respect

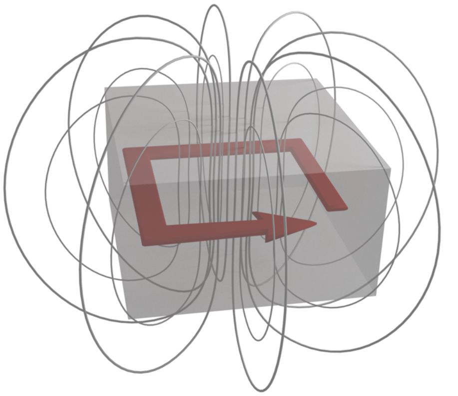

Fig. 1. An illustration of a persistent current within a sample generating a magnetic field.

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Additional data related to this publication are available at the University of Cambridge data repository: https://www.repository.cam.ac.uk/handle/1810/253585.

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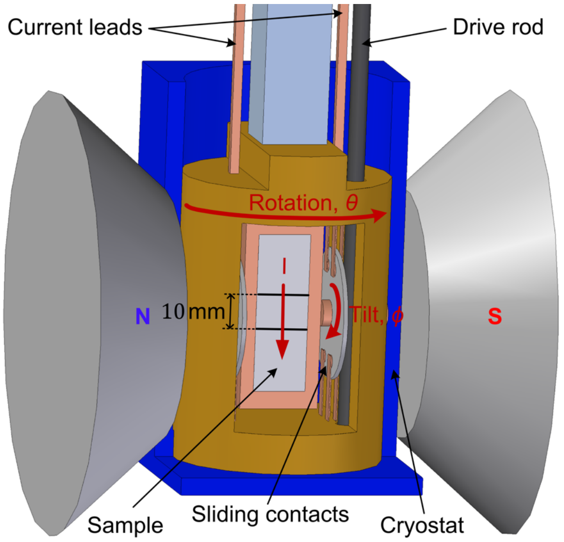

Fig. 2. Schematic representation of the experimental setup and the geometry of the applied field with respect to rotation and tilt angles.

to the sample can be explored via changing the rotational θ and tilt angles ϕ. For the purpose of this study, the rotational angle θ was varied between -180° and +180° while the tilt angle ϕ was kept at 0°. Orientation $\theta = 0°$ and $\phi = 0°$ corresponds to tape normal being parallel to the applied field.

The sample voltage was measured across 1 cm and an electric field criterion of 1 µV/cm was used to determine the critical current, although values of up to 5 µV/cm were measured to make power law fits to the I-V curves in order to determine the n-value.

Full width samples were tested at 0.5 T and 0.4 T, however, in order to keep I_c below 200 A due to limitations of the experimental setup, further tests were made on samples with width reduced to 6 mm. The measured I_c values were then extrapolated to those expected for full width samples by direct comparison of values obtained for both half-width and full width samples at 0.5 T, 0.4 T and self field. The half-width sample consistently showed 2.85 times lower I_c, likely due to properties of the tape being slightly worse at the edges of the tape compared to the center. Narrower width also makes the tape's performance more susceptible to localized defects. A total of three samples were measured, each showing I_c values within 5 A of each other.

It is worth noting that the effect of self-field in current transport measurements is a concern only for small applied fields, when the self field is comparable to the applied field. Simulations show that the self field on the tape (averaged across the cross-section of the tape) did not exceed 20 mT during the experimental tests. However, this could add to the discrepancy in FEM modeling when the stack contains only a few tape layers.

C. Field Cooling of Stacks of Tape

A number of field cooling tests were performed with a varying number of tape layers. The orientation of each layer in the stack was kept identical: both the up-down and longitudinal-transverse orientation. The top surface of the stack corresponds to the (RE)BCO facing side of the tape. Field cooling was performed at 77 K in liquid nitrogen. The magnetic field was applied via an electromagnet. The magnetic field was ramped down from 1 T at a rate of 0.1 T/s to ensure that the sample is fully magnetized. After field cooling the sample was removed from the electromagnet whilst keeping it in liquid nitrogen to ensure that the measurement is not affected by the proximity of ferromagnetic electromagnet poles. The field was recorded 2 minutes after the end of field cooling to ensure flux creep, which is most rapid just after magnetization, does not affect the results significantly. The field was measured 1 mm above the surface of the stack, centered in the middle.

D. Modeling Framework

Modeling was done in COMSOL Multiphysics 5.1 using the $\boldsymbol{H}$ formulation, which is widely used for modeling problems that involve superconductivity [6]. The E-J power law was modified to include magnetic field and field angle dependent superconductivity properties

$$E = E_0 \left(\frac{J}{J_c(B,\theta)}\right)^{n(B,\theta)} \tag{1}$$

where $J_c(B,\theta)$ and $n(B,\theta)$ are the critical current and n-value dependencies on the magnetic field magnitude B and orientation θ. The critical current density was obtained from the ratio of critical current (from goniometric measurements) and total cross-sectional area of the superconducting tape (see section II.A.). No functional form was fitted to $J_c(B,\theta)$ or

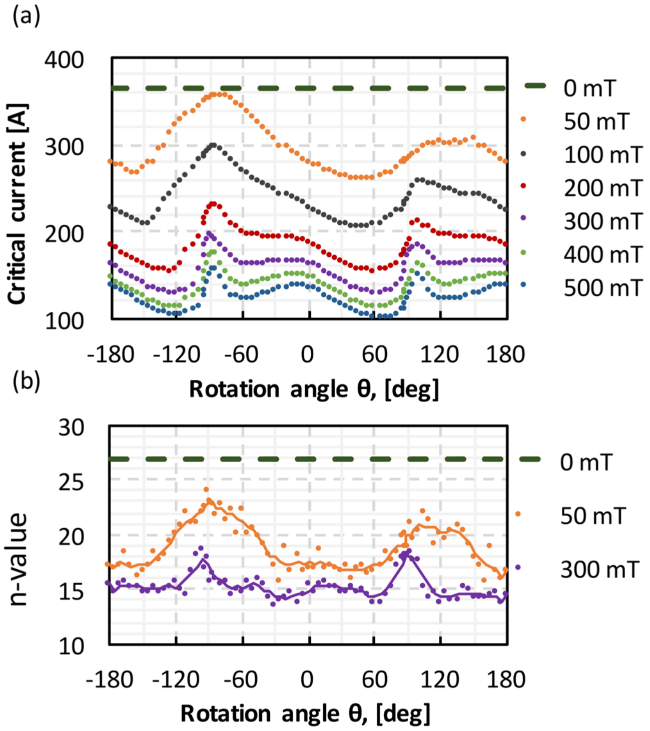

Fig. 3. Critical current for magnetic flux density up to 0.5 T across the whole rotational angle range (a). The peak near $\theta = 0°$ at higher fields corresponds to c-axis aligned pinning due to Zr additions. The n-values for selected field values (for clarity) are shown in (b). Note that the (b) also has an angular dependency.

n(B,θ). Instead, linear interpolation was used between data points shown in Fig. 3. For *n(B,θ)*, the data was first smoothed by considering two nearest neighbors of each data point before using the data in the model (the smoothed curve can be seen as a solid line in Fig. 3(b)). For B > 0.5 T linear extrapolation was used, although fields of more than 0.5 T in the interior of the stack were achieved only for stacks larger than 40 layers. To match the experimental setup, the field was ramped from 1 T at the rate of 0.1 T/s and the trapped field was recorded after 120 s. The trapped field value was taken 1 mm above the stack, corresponding to the top side of Fig. 5.

In order to reduce computational time needed, a 2D-axissymetric model was used, although the stacks themselves are rectangular. However, experimental tests have shown that fields trapped in square and round stacks differ by less than 4 %, making this a reasonable simplification. Moreover, individual layers were not modeled, instead the stack was homogenized, as in [7], maintaining the same engineering critical current density as in the real stack. All tests were isothermal, hence the thermal physics were not considered, thus the magnetic field ramp rate is not critical for the simulation. On the other hand, the flux creep just after the magnetization procedure is significant, and it is crucial that the trapped field measurement is done at the same time after the magnetization for both the experiment and simulation.

It is worth noting that an axisymmetric model is more suitable for HTS tape showing symmetric $J_c(B,\theta)$ and $n(B,\theta)$ with respect to $\theta = 0^\circ$. This is often the case in tapes with no artificial pinning centers, or spherical pinning centers only, as produced by e.g. SuperOx [8] and Fujikura. The superconducting tape used in this study contains columnar pinning centers [9] giving rise to more intricate $J_c(B,\theta)$ curves.

In this instance the model fails to capture differing J_c values, for example, for two opposite radial directions, where the magnetic field orientations have an opposite sign. Given that for the tape used $J_c(B,\theta) \neq J_c(B,-\theta)$, especially at low fields (when the stack contains a low number of tape layers), this may introduce an error in the modeling results.

III. Results and Discussion

A. Goniometric Characterization

The results from goniometric tests are shown in Fig 3. The critical current shows marked asymmetry with respect to $\theta = 0^\circ$, which is chosen as the direction of tape normal, the *ab* plane peak is also shifted slightly away from $\theta = 90^\circ$ due to the *ab* plane not being parallel to the tape surface (vicinal) as described in [10]. At magnetic fields larger than 100 mT, another peak near $\theta = 0^\circ$ emerges due to *c*-axis aligned pinning.

Fig 3(b) also shows the *n*-value variation with magnetic field and its angle. The data is noisy, due to fact that data was only gathered until the electric field reached 5 μV/cm in order to avoid heating the sample. This resulted in relatively few data points for each fit. Nevertheless, it can be clearly seen that the *n* value is slightly larger near the *ab* plane peaks in the $I_c(\theta)$ curve.

B. Modeling Results and Comparison with Experiment

The experimental data shows that the trapped field tends to saturate rather quickly as the number of layers is increased. As can be seen from Fig. 4, the model that incorporates J_c dependence on the magnetic field direction (full $J_c(B,\theta)$ and $n(B,\theta)$ dependence) agrees well with the experimental data, whist the model that only considers the magnetic field magnitude (reduced $J_c(B, \theta = 0^\circ)$ and $n(B, \theta = 0^\circ)$ dependence) overestimates the trapped field for larger stacks. This highlights the importance of J_c anisotropy in predicting trapped fields in stacks of tape.

Small discrepancies are observed between experimental and modeling data for low numbers of layers, this is most likely due to the fact that small variations in the individual layer performance are most significant when there are only a few layers in the stack and limitations of the model described in section II.D. A small percentage of the layers may also suffer from damage during cutting [11].

Interestingly, the non-symmetric $J_c(B,\theta)$ gives rise to non-symmetric current density with respect to the midplane of the sample (Fig. 5), and trapped field below the sample is expected to be up to 5 % larger (in the 100 layer case). This still needs to be verified experimentally as the trapped field was only measured on one side of the stack (corresponding to the top side of Fig. 5. However, if the tape is stacked in random orientations, this effect should be diminished.

Other facts that may influence the accuracy of the predictions is the fact that the stacks in reality are square and not 2D-axisymetric. Also, regions in the stack, near corners experience magnetic field that is not in the plane perpendicular to the current direction, i.e. $\phi \neq 0^\circ$. However, it is expected that in those regions, the critical current density is in fact larger as goniometric scans spanning a range of ϕ and θ values [10]

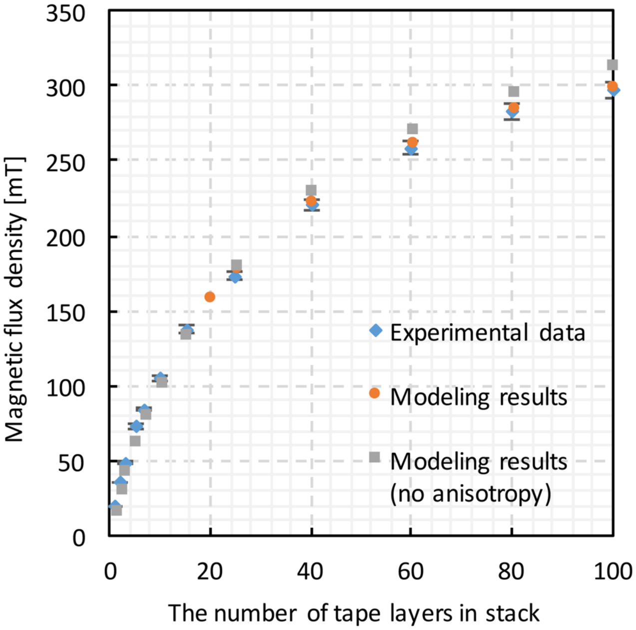

Fig. 4. Experimental data and modeling results of trapped field after field cooling a stack of (RE)BCO tape with varying number of layers. FEM model that includes anisotropic J_c agrees with experiment very well, while the model that did not include anisotropy shows overestimates for trapped field for large stacks.

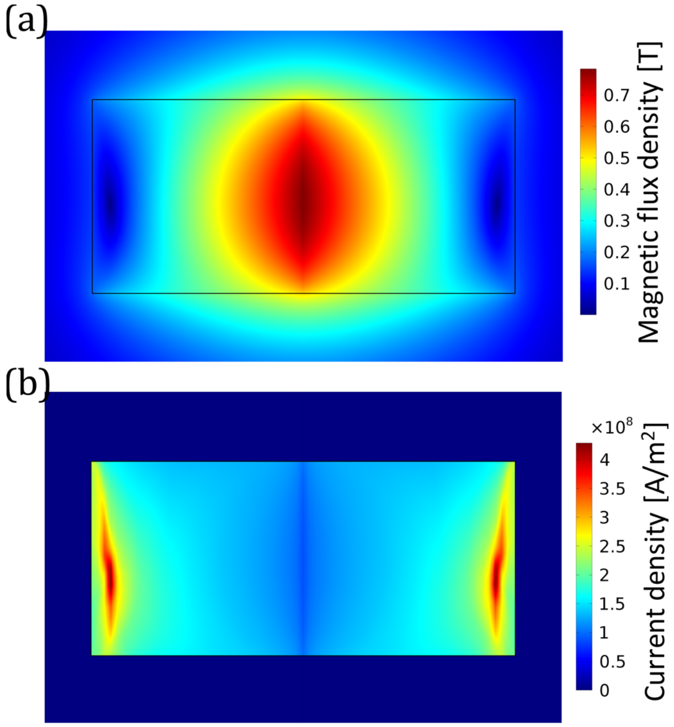

Fig. 5. Modeled magnetic flux density (a) and the current distribution (b). Interestingly, the figures are not symmetrical in the vertical direction as the $J_c(B,\theta)$ is not an even function.

show that the lowest critical current densities are recorded for $\phi = 0^\circ$. Hence, the current is limited by sections in the stack where ϕ is indeed 0° and J_c dependence on the tilt angle ϕ is not the limiting factor for trapped field. Nevertheless, these factors are secondary and the model predictions fit the data within experimental error.

IV. CONCLUSIONS AND FURTHER WORK

Goniometric critical current measurements were performed on a single layer of superconducting tape at 77 K and applied magnetic field of up to 0.5 T. The $J_c(B,\theta)$ and $n(B,\theta)$ data collected were used to make a FEM model to predict the trapped field in a stack of such tapes after field cooling. The model showed good agreement with experimental data.

Further work will investigate how the peak in $J_c(B,\theta)$ due to *c*-axis pinning influences the trapped field values especially if ferromagnetic layers are used in the stack, that can alter the magnetic field profile, and in particular the magnetic field direction within the stack.

In addition, similar experiments will be performed for pulsed field magnetization, which requires simulation of thermal physics as well.

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