

THE PROBLEM OF CONDITIONALS

A Thesis

Presented to

The Faculty of the Department of Philosophy

University of Houston

In Partial Fulfillment

Of The Requirement for the Degree

Masters of Arts

by

Rayme Edward Engel

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This paper is devoted to the logical problem of conditionals. This is interpreted as the problem of formulating, in terms of simple non-modal logical notions, the general conditions for the truth of statements of the 'if...then---' form. It is argued in the first part of the paper that the compass of this analysis properly includes not only those conditionals expressed counterfactually with verbs in the subjunctive, but also those containing the verbs of the indicative. This view is defended through a series of arguments condensing in the conclusion that the appropriate mood for the verbs in a conditional is not determined by the kind of implication the writer or speaker claims between antecedent and consequent, but rather by opinion as to the probable truth or falsity of the antecedent. The second part of the paper is then given over to the search for a general definition for the truth of conditionals. An elaborate set of rules is finally formulated and claimed to provide an adequate analysis of the conditional. Difficulties shared by the problem of conditionals and that of explanation are mentioned, and it is suggested that counterexamples to earlier versions of the criterion for conditionals can be imitated to establish counterexamples to deductive-nomological models, and further that the necessary adjustments can be discovered in the strategies used to correct the conditional criterion.

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## INTRODUCTION

Conditionals in English are those ordered sentence compounds signaled by the connective constructions: "if... then--," "...only if--", "had...then--", and "...unless--". Conditional expressions are common in our everyday conversations, in the most lofty theoretical discourse, and in the most impassioned personal deliberations. But despite their importance and their pervasiveness, we stand without an adequate answer to the basic question: When is a conditional true? The Stoics and Megarians of ancient Greece vigorously debated the correctness of various answers; the dispute became so well-known that Callimachus found it fair game for an epigram: 'even the crows on the roof tops caw about the nature of conditionals.'<sup>1</sup>

Philo of Megara argued for the truth-functional interpretation: 'If P, then Q' if and only if ' $\neg(P \& \neg Q)$ .' Others insisted that a conditional was true only if it was not possible for its antecedent to be true and its consequent false. Today these two opposing views have found a comfortable compromise in the current wisdom which accepts Philo's answer for conditionals with indicative verbs while admitting the modal formulation as a vague, but concise, statement of the distinctive quality that characterizes conditionals with verbs in the subjunctive.

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<sup>1</sup>W & M Kneale, The Development of Logic, Oxford University Press, 1962, p 128.

Indeed, it is generally recognized that the proper analysis of conditionals with subjunctive verbs is not the truth-functional one, and that such conditionals call for a much more elaborate analysis. However, in this paper it will be argued: that this more elaborate analysis is equally the one called for by conditionals with indicative verbs; that the considerations usually put forward to underscore the inadequacy of a truth-functional interpretation for conditionals with verbs in the subjunctive testify as well against a truth-functional analysis for conditionals in the indicative; and further that the mood of a conditional is merely a device for expressing opinion as to the probable truth or falsity of the antecedent clause, and thus that it is not a device for signaling the kind of implication claimed between antecedent and consequent.

The utility of the truth-functional interpretation for assessing validity is not to be denied. But when our interest is in the truth of a conditional, and not in the validity of an argument, then the truth-table is as useless for determining the truth of an indicative conditional as it is for determining the truth of a conditional expressed in the subjunctive. Both the indicative and the subjunctive demand the more elaborate analysis.

The second part of this paper will be devoted to the problem of providing the requisite analysis. This is the problem usually referred to as the problem of subjunctive

or counterfactual conditionals, though the first part of this paper will argue that this is better described more simply as the problem of conditionals.

This problem is usually thought to have as a major component the task of defining natural or non-logical law. However, responsibility for this definition is appealed here on the following grounds: (1) that the task of defining non-logical law is a more general problem, one playing a major part in a number of other important projects in epistemology and the philosophy of science, and hence not one unique to the problem of conditionals; and (2) that the relations between the problem of (counterfactual) conditionals and the problem of law are not so intimate as the rumors suggest; more specifically that counterfactuals do not provide a criterion of lawlikeness.

The point of (1) can be illustrated through an analogy with someone attempting an analysis of knowledge through definition of the two-place predicate 's knows that p.' Suppose for the purpose of the illustration that the first three clauses in his proposal are: (a) 'p' is true; (b) s believes that p; (c) s has undefeatable evidence that p. Its author could reasonably protest that the problem of defining the nature of truth and that of defining the nature of belief are so general that the burden of definition should not be his. But, on the other hand, he would have

to admit that the problem of defining the general character of undefeatable evidence is sufficiently narrow to support the denial of a similar plea for relief from the labor of its exact definition. To draw out the analogy, it is thus claimed that the problem of defining 'natural law' is on a par with that of 'truth' and 'belief', and further that the task of specifying the relevant conditions presumed, but unstated, in the assertion of a conditional, is analogous to that of defining 'undefeatable evidence' for example in attempting an adequate interpretation of 's knows that p'.

The second charge, that the connection between the problem of conditionals and that of law is not so intimate as it is generally reputed to be, can be supported by a simple counterexample to the claim that counterfactual conditionals provide a criterion for lawlikeness, that is to Hempel's claim that "a law can, whereas an accidental generalization cannot, serve to support counterfactual conditionals."<sup>2</sup> Against this note that the counterfactual:

"If a rock had been taken out of this box, then it would have been one containing iron."

is supported by the accidental generalization;

"All rocks in this box contain iron."

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<sup>2</sup>Carl Hempel, Philosophy of Natural Science, Prentice Hall, Inc. p 56



The assumption that accidental generalizations will not support counterfactuals probably persists because it is so easy to find counterfactuals that are not supported by accidental generalizations. Counterfactuals that cannot be supported by the often cited accidental generalization "All men in this room understand English" are indeed plentiful, e.g. "If an Eskimo, a baby, Plato, or Mr. Khrushchev were in this room, then he would understand English." However, supportable counterfactuals are also abundant, e.g. "If an actor, Hume, Peter Pan, or Nixon had read an English translation of The Misanthrope to the men in this room, they would have understood it."

Dispensing with the burden of defining the nature of non-logical law does not clear away all the serious problems, as it will soon be seen.

## CHAPTER I

Conditionals are sentences. They are usually compound sentences of the form: "if...then---." Sentences for the purposes of this investigation can be thought of as universals where each sentence is viewed as a repeatable pattern for physical inscriptions, or as a repeatedly approximable norm.<sup>1</sup> With this interpretation, it cannot be said of every conditional, any more than it can be said of every declarative sentence, that it is definitely true or false. Only the dated individual inscriptions can be said to be true or false absolutely, and then only with respect to their authors when the sentences include the indexicals of personal reference. Those conditionals in which the individual components are eternal sentences (sentences whose truth values stay fixed through time and from speaker to speaker) will of course be true or false absolutely. But when the component sentences are not eternal, only the dated individual inscriptions ascribed to a certain author will unswervingly abide by the principle of the excluded middle. Individual inscriptions, however, can be eternalized by including specific references implicit in context.

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<sup>1</sup>This position is nearly that of Quine in Word and Object (Cambridge: The M.I.T. Press, 1960). See pages 191-195 where he begins his 'Flight from Intension,' for a fuller discussion of sentences as universal repeatable patterns of utterance.

By convention, the first component of a conditional cast in the form of the familiar "if...then--" is usually referred to as its antecedent, and the second component as its consequent. We can speak of the truth or falsity of the antecedent and consequent by imagining an eternalization of the sentences in question. There is, however, a difficulty in this when the verbs in the conditional are subjunctive and each component is an ungrammatical affair like the collocation "Dotson were happy" that appears in the perfectly well formed conditional:

If Dotson were in Boston, he would be happy.

We can bend the general convention to accomodate such expressions by taking the antecedent and consequent of each subjunctive conditional to be the first and second indicative sentences named in equivalent expressions patterned after paraphrases of the form:

If 'Dotson is in Boston' were true, then 'Dotson is happy' would be true.

Of course, the quoted sentences must first be eternalized before quotation, but otherwise the conversion we imagine proceeds rather smoothly. The names and definite descriptions in the first component are simply repeated in place of their pronominal representatives in the second component. The subjunctive verbs in the ungrammatical

sequents then give way to indicative correlates while whatever point they added to the sense of the original expression is picked up again when they reappear in the predicates affirming the truth of the sentences in quotation.

The process wrinkles somewhat, however, when the conditional in question is what Goodman calls, a countercomparative.<sup>2</sup> Consider for example:

- (i) If I had arrived one minute later, then I would have missed the train.

The paraphrases we imagine for the countercomparatives should not result in the self-contradictory components produced by the naive use of the conversion formula, e.g.

- (ii) If 'I arrived one minute later than I arrived' were true, then 'I missed the train' would have been true.

Goodman suggests a translation procedure which he claims appropriate for the troublesome countercomparatives. He notes that the self-contradictory component disappears when we translate (i) as the quantified whole:

- (iii)  $(\exists t)$  (t is a time. I arrived at t. If 'I arrive one minute later than t' were true, then 'I missed the train' would be true.)

An obvious objection to this as a paraphrase is that it

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<sup>2</sup>Nelson Goodman, Fact, Fiction, and Forecast, (Indianapolis, Bobbs-Merrill, 1965) p.6-7

represents the speaker as countenancing an ontology that includes temporal instances. A more formidable objection takes note of the fact that sentence quotation produces an opaque<sup>3</sup> construction.<sup>3</sup> No variable inside an an opaque construction can be bound by a quantifier outside it. The trouble becomes clear when we look at an immediate alphabetic variant of (iii) thus:

- (iv)  $(\exists w)$  ( $w$  is time. I arrived at  $w$ . If 'I arrive one minute later than  $t$ ' were true, then 'I miss the train' would have been true.)

Without the cross-reference that is blocked by the sentence quotation, the point of the paraphrase is irrevocably lost.

A solution to the problem, which undoubtedly has already been recognized, is found in the eternalization of the component sentences. The indexical 'I' and the variable 't' can be replaced with a name of the speaker and a definite description of the time in question; thus, the impossibility of binding a variable occurring in an opaque context never poses a problem. For example, there is no self-contradictory component in the possible expansion of (i) as follows:

- (v) If 'Fitchmeyer arrived at the Creekwood station at 10:07 P.M. on V.E. Day' were true, then 'Fitchmeyer missed the ten o'six from Springfield on V.E. Day' would have been true.

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<sup>3</sup>I owe this point completely to M.G. Yoes, though its manner and the remedy I suggest are not ones for which he should be held responsible.

If one finds need of a paraphrase that succeeds without demanding additional information from the context of utterance, a reparsing along the lines of the following should be appropriate:

(vi)  $(\exists t)$  ( $t$  is time. I arrived at  $t$ . If I arrived one minute after  $t$ , then I missed the train.)

With this tack cross-reference is secure, no opaque contexts are generated, and no additional information beyond that implicit in the original utterance is required for the conversion. We abandon the subjunctive verbs, but not their force; intimation of the antecedent's falsity is retained through the emergence of the second conjunct: 'I arrived at  $t$ '.

Nevertheless, some may argue that the paraphrase in (vi) fails in that the connection affirmed between the antecedent and consequent in the original subjunctive formulation (i) is much stronger than that affirmed by its indicative replacement. This objection holds only if the preceding chapters fail to establish the claim that the mood of a conditional is only designed to indicate something about its author's opinion as to the probable truth or falsity of the antecedent, and hence that the mood is not a signal as to whether the connection between antecedent and consequent is affirmed as lawful or as truth-functional.

Regardless of whether the omission of the subjunctive verbs in (vi) is or is not a legitimate move, trouble remains if adequate paraphrases for the countercomparatives are held to be only those unaided by context. We needn't go far to find countercomparatives that resist conversion through paraphrases such as (vi). Consider, for instance, the common refrain:

"If only I had more money, then I would...."

Given some information as to the standard unit of currency we can of course imitate the tact in (vi); but if allowed the advantage of context, we might as well exploit eternalizations of the component sentences to achieve paraphrases after the manner of (v). Whether or not recourse to context is legitimate in formulating paraphrases for countercomparatives is not a question that must be decided here. For we need not have the final paraphrase procedure in hand, it is enough that such a procedure may eventually be realized. This much is sufficient to anchor the convention adopted here, viz. conceiving of the antecedent and consequent of each subjunctive conditional as the first and second quoted sentences arising in paraphrases of the form: "If '...' were true, then '---' would be true." Under this convention the antecedent and consequent are always syntactically well-formed indicative sentences subject to the labels, 'true' and 'false'.

## CHAPTER II

The problem of conditionals is usually thought of as the problem of counterfactual conditionals, for it is generally assumed that conditionals with indicative verbs are more or less susceptible to a truth-functional analysis, and thus that only those with subjunctive verbs and a hint of counterfactual implication require a more elaborate analysis.

Before confronting this assumption directly, it would perhaps be worthwhile to pause for a moment to consider the intended application of the expression "counterfactual conditional." The term is almost invariably introduced into discussions by way of an illustration rather than by a definition. This practice would not be so troubling if a definition were at least attempted somewhere later in the proceedings. This unfortunately is rarely, if ever, the case. Usage would at least suggest that "counterfactual" is coextensive with "subjunctive" when applied to conditionals. The use of the subjunctive verbs however may only reflect the fact that the antecedents of these conditionals are presumed false.

Goodman's introduction to his landmark article on 'The Problem of Counterfactual Conditionals' may shed some light on this issue:

What, then, is the problem about counterfactual conditionals? Let us confine ourselves to those in which



antecedent and consequent are inalterably false--as, for example, when I say of a piece of butter that was eaten yesterday, and that had never been heated

If that piece of butter had been heated to 150° F., it would have melted.

Considered as truth-functional compounds, all counterfactuals are of course true, since their antecedents are false. Hence

If that piece of butter had been heated to 150° F., it would not have melted

would also hold. Obviously something different is intended, and the problem is to define the circumstances under which a given counterfactual holds while the opposing conditional with the contradictory consequent fails to hold. <sup>4</sup>

Goodman's explanation seems to suggest that it is the falsity of the antecedent rather than the subjunctive mood that defines the counterfactual, since:

If that piece of butter was heated to 150° F., it melted.

and:

If that piece of butter was heated to 150° F., it did not melt.

would also be true if considered as truth-functional compounds. And here too "obviously something different is intended," for surely no one who claims the latter intends agreement with someone asserting the former, or vice versa. But perhaps this is not so obvious, as it runs counter to the prevalent view protested in the next section concerning

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<sup>4</sup>Nelson Goodman, Fact, Fiction, and Forecast, (Indianapolis, Bobbs-Merrill, 1965) p. 4

the unique strength of the implication asserted by conditionals with verbs in the subjunctive.

In addition to the question of whether the subjunctive mood is an essential feature of those conditionals intended by the term 'counterfactual,' there is also some question as to the significance of the falsity or lack of falsity in the consequent. Is it, for example, a necessary condition for the truth of a counterfactual, or for an expression's being a counterfactual at all, that its consequent, like its antecedent, be false? If it is a necessary condition for the truth of a counterfactual, then half of the solution to the problem of counterfactual conditionals is at hand, given that "the problem is," as Goodman explains, "to define the circumstances under which a given counterfactual holds while the opposing conditional with the contradictory consequent fails to hold."<sup>2</sup> For the circumstances under which the opposing counterfactual with the contradictory consequent, i.e.

If that piece of butter had been heated to 150°F., it would not have melted.

fails to hold, are simply those in which its consequent is true, that is, when the piece of butter in question did not melt. Surely the problem of counterfactual conditionals

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<sup>2</sup>Nelson Goodman, Fact, Fiction, and Forecast, (Indianapolis, Bobbs-Merrill, 1965) p. 4.

is not so tame. This much seems certain, and might be taken as a reductio ad absurdum for the supposition that the falsity of the consequent is a necessary condition for the truth of a counterfactual. Thus, we might suppose that it is rather a necessary condition for an 'if-then' expression's being a counterfactual. Either way, it seems we would have to give up Goodman's partial description of the problem as that of defining the circumstances in which one of two opposing counterfactuals fails to hold. This consequence in itself is not difficult to accept, since a very serious problem still remains, i.e., that of defining the circumstances under which a given counterfactual holds. However, it is perhaps worth mentioning that Goodman's method throughout his analysis of counterfactuals is to argue against successive proposals by calling attention to pairs of opposing counterfactuals which would appear to hold under the proposal in question. This approach will not do if it is either a necessary condition for the truth of a counterfactual or for an expression's being a counterfactual that its consequent, like its antecedent, be false.

## CHAPTER III

Is a stronger and less tractable form of implication introduced into a conditional when the included verbs are subjunctive? Does the subjunctive conditional thus call for a special analysis basically different from the one required for the indicative conditional? We have already seen that the subjunctive verbs necessitate a twist in the usual convention for marking the antecedent and consequent of a conditional; but the question here is whether the contrast is such that the indicative conditional can be identified more or less with a simple truth-function while only those conditionals with subjunctive verbs actually demand the more elaborate analysis.

In this chapter it will be argued that the difference in mood does not reflect a major difference in the requisite analysis. The opposing opinion is well represented by W.V. Quine. Quine agrees that usage of the subjunctive conditional is at variance with the truth-functional interpretation represented by the truth table for material implication, but he claims that "there is no clear conflict between the [truth] table and the indicative conditional of ordinary usage".<sup>1</sup> He holds that "the indicative conditional can

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<sup>1</sup>W.V. Quine, Mathematical Logic, (New York, 1940) pp. 16, 17.

always be construed truth-functionally--even though its affirmation will ordinarily be motivated by considerations of causal connection."<sup>2</sup> Therefore, Quine claims that the subjunctive or "contrafactual conditional is best dissociated from the ordinary conditional in the indicative mood."<sup>3</sup>

He argues:

Whatever the proper analysis of the contrafactual conditional may be, we may be sure in advance that it cannot be truth-functional; for, obviously ordinary usage demands that some contrafactual conditionals with false antecedents and false consequences be true and that other contrafactual conditionals with false antecedents and false consequents be false.<sup>4</sup>

But if we are to be consistently sensitive to the "demands of ordinary usage", we must also say that the indicative conditional cannot be identified with a truth-function. For ordinary usage also seems to demand that some indicative conditionals with false antecedents and consequents be true and that some be false, for example (when Jones is in Virginia) that:

"If Jones is in Carolina but not in South Carolina, then he is in North Carolina"

be true, and that:

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<sup>2</sup>Ibid, p 14  
<sup>3</sup>Ibid  
<sup>4</sup>Ibid

"Jones is in Carolina only if he is in South Carolina"

be false. My purpose here will not be to champion ordinary usage, but simply equal treatment, i.e., to point out that the considerations of ordinary usage that underscore the inadequacy of the truth-functional analysis for the subjunctive conditional count as well against a truth-functional interpretation of the indicative conditional.

We say that a method of statement composition is truth-functional if and only if replacement of a component statement by another statement of like truth-value does not alter the truth-value of the compound. For example, we hold that conjunction is a truth-functional mode of statement composition since the truth of the whole is indifferent to any substitution of like valued statements. Some statement compounds are unquestionably non-truth-functional, for instance, the sentence:

a) Tom is sad because Mary left him.

By replacement of a component statement with another statement of like truth-value we can alter the truth-value of the whole:

b) Tom is sad because he won the derby.

Tom's sadness may have mellowed with victory, but it is unlikely that his victory was the cause of his sorrow.

'Because' we must agree is simply not a truth-functional connective.

Similar observations with conditionals show that 'if'--like 'because'--is not a truth-functional connective. Consider, for example, the indicative conditional:

- c) Tom will have a wife two years his senior if he marries his cousin.

If Tom was born two years before his cousin, then (c) must be granted as true, regardless of whether Tom marries or does not marry his cousin. But if he does not marry her, and the false statement, 'Tom marries his cousin', is supplanted by some other false statement such as 'Tom marries his younger sister', we obtain a patent falsehood:

- d) Tom will have a wife two years his senior if he marries his younger sister.

Indeed, there is no sure preservation of truth-value upon the substitution of a like valued component in an indicative conditional such as (d). Thus, it would appear that the indicative conditional cannot be readily identified with a truth-function.

This of course is a rather uncontroversial conclusion. Few would argue that the indicative conditional can be accurately identified with a simple truth-function. But nevertheless, if just this much is admitted, then it must be granted that the subjunctive conditional does not differ

from the indicative in the failure of the truth-functional analysis. Other crucial lines of divergence may of course be found.

Quine claims that the subjunctive conditional differs from the indicative conditional in that the latter suffers a truth-value gap whenever its antecedent is false.<sup>5</sup> He proposes closing truth-value gaps to smooth over the awkward humps in communication that arise when the antecedent of a conditional with indicative verbs is false. However, it will be maintained here that this artificial treatment is not only unreasonable but actually unnecessary. Indeed, there is something awkward about entertaining a conditional with indicative verbs when we are cognizant of the falsity of its antecedent; but it is grammatical in kind, and can be eliminated quite simply by changing the mood of the verbs. When the falsity of the antecedent in a conditional with indicative verbs, e.g.

"If Tom rides Kentucky Prince, he will win the derby"

produces a hump in communication, the difficulty is more naturally resolved if we reissue the point in the subjunctive than if we were to artificially rule that the conditional be labeled true no matter what the verdict is for its

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<sup>5</sup>W.V. Quine, Word and Object, (The M.I.T. Press, 1960) p 223-226



subjunctive edition:

If Tom were riding Kentucky Prince, he would win the derby,

This kind of move is not at all foreign to the common course of conversation. In general, discovery of the antecedent's falsity prompts reiteration of the point at issue through a parallel compound cast in the subjunctive when the speaker's interest extends beyond the contingencies of the moment.

The view that the indicative conditional suffers a truth-value gap whenever its antecedent is false has some fairly paradoxical results. For instance, one would expect its adherents to accept as a possible formulation of their position (and hence as true):

- e) If every indicative conditional has a false antecedent, then every indicative conditional has a truth-value gap.

But, of course, since this is an indicative conditional with a false antecedent one would also expect them to believe that it lacks a truth-value; thus, it seems they must accept both the statement that (e) is true (that it has a truth-value) and the statement that (e) does not have a truth-value. They might say that they were simply in error if the conditional was not diagnosed as suffering a truth-value gap when it was first introduced. But what then

could they say about its logically equivalent contrapositive:

- e') If some indicative conditional has a truth-value, then some indicative conditional does not have a false antecedent.

which has a true antecedent, and therefore must somehow be spared the pain of a truth-value gap? The discord cannot be attributed to a contrived and improbable example; for any indicative conditional marked with the symptoms of a truth-value gap (false antecedent and consequent) has as its contrapositive an equivalent, but curiously robust, expression with true antecedent and consequent.

Those who urge Quine's view could avoid the conflict either 1) by showing that the rule of contraposition does not hold apart from a formal interpretation in which all the truth-value gaps are first closed, or 2) by revising their position to allow that truth-value gaps occur equally in conditionals in which both antecedent and consequent are true. Let us consider the two ploys separately:

(1) Stalnaker, (though not a proponent of the truth-value gap interpretation), argues that contraposition, while valid for the truth-functional horseshoe, is invalid for the conditional:

For an example in support of this conclusion, we take another item from the political opinion survey:

'If the U.S. halts the bombing, then North Vietnam will not agree to negotiate.'

A person would believe this statement if he thought that the North Vietnamese were determined to press for a complete withdrawal of U.S. troops. But he would surely deny the contrapositive.

'If North Vietnam agrees to negotiate, then the U.S. will not have halted the bombing.'

He would believe that a halt on the bombing, and much more is required to bring the North Vietnamese to the negotiating table.<sup>6</sup>

Nevertheless, one wrinkle in the wide spread of conditionals for which contraposition goes through would not seem to warrant its immediate dismissal; moreover, the charge of invalidity, it will be maintained, is better directed at the opinion survey than at contraposition. The conditional in question would most likely be understood as the semi-factual:

f) Even if the U.S. halts the bombing, North Vietnam still won't agree to negotiate.

while its wording in the survey form:

g) If the U.S. halts the bombing, then North Vietnam will not agree to negotiate.

is better suited, for querrying opinion as to whether a U.S. bombing halt would cause North Vietnam to refuse negotiations. If someone did actually claim to believe that a U.S.

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<sup>6</sup>Robert C. Stalnaker, "A Theory of Conditionals", American Philosophical Quarterly, Oxford, 1968 p 107

bombing halt would somehow cause North Vietnam to refuse negotiation, then he should have registered the same response to the contrapositive of (g), i.e., to

- h) If North Vietnam agrees to negotiate, then the U.S. has not halted the bombing.

In fact, if the pollster was really employed to ascertain public opinion on (g) rather than on (f), then (h) should have been included along with, but not of course adjacent to (g) as one of the items on the survey questionnaire to allow a check on the internal consistency of the interviews. Those forms in which the response to (g) and (h) differed could then have been discarded as unreliable data for an accurate estimate of the public's present persuasion as to proper course to follow in ending the war.

If contraposition is not valid for conditionals it would seem that modus tollens would also be invalid, since contraposition can be derived simply from modus tollens and the deduction theorem. But Stalnaker claims that "although contraposition fails, modus tollens is valid for the conditional."<sup>7</sup> Thus according to his position one who believes (g) could consistently deny its contrapositive, (h), but all the same from:

- i) North Vietnam agrees to negotiate

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<sup>7</sup>Ibid.

be prepared to infer:

j) The U.S. has not halted the bombing.

This seems extremely odd, for a denial of (h) would surely appear to be the natural way of gainsaying any inference from (i) to (j). Since (h) is nothing more than the conditional 'If (i), then (j)', Stalnaker must also argue for the invalidity of conditional proof. But the wiser course would be to give up the original claim and admit that contraposition, like modus tollens, is valid for the conditional.

(2) If contraposition is vindicated, the advocates of Quine's position must admit that the indicative conditional with true antecedent and consequent suffers the same truth-value gap as its contrapositive with false antecedent and consequent. Allowing this would not seem to involve any special difficulty for their view. Nevertheless, it means that the second line, TF, is the only line in the truth table that the material conditional shares with the indicative conditional, and in this it is surely no different from the conditional in the subjunctive mood.

All of these consequences may be admitted, the important question remains to be asked. Why suppose truth-value gaps at all? Why for instance, should we represent Toby's claim:

If Hawkshaw saw me, the jig is up.

as suffering a truth-value gap if Hawkshaw is blind while readily deeming it false if Toby later chances to speak in the subjunctive:

If Hawkshaw had seen me, the jig would be up.

and Hawkshaw is actually a fellow collaborator, and not the police agent Toby mistook him for.

When an indicative conditional of no theoretical concern is put on exhibit without sufficient brief, one might well be tempted to speak of it as suffering a truth-value gap when its antecedent is false; however, the gap it suffers should not be attributed to the indicative verbs, but rather to the gap in the author's specification of reference, context, and relevant information, e.g. of 'Toby', the jig, and Hawkshaw's employ.

## CHAPTER IV

The notion that a stronger and less tractable form of implication is introduced into a conditional when the included verbs are subjunctive is not confirmed by the grammatical considerations that regulate the mood of such sentences. The mood of a conditional is chosen to accord with opinion as to the probable truth or falsity of its antecedent clause. When Toby opens with the indicative:

"If Helen is even somewhat attractive,..."

we can guess that Toby has not yet seen Helen, but when we later overhear his comment in the subjunctive:

"If Helen were even somewhat attractive,..."

we surmise that he has seen Helen, and that he is not now in her presence if he has any tact at all. If the mood of a conditional is decided by its author's opinion as to the truth or falsity of the antecedent (or by the one he wishes to affect), then the mood of a conditional cannot be supposed a function decided by the degree of connection its author wishes to affirm between antecedent and consequent. The two conventions would be at odds whenever one wishing to affirm the stronger connection also wishes to express his open-mindedness with respect to the probable truth of the antecedent. Thus, we either run the risk of piquing

Tom's sister with:

"If Tom were honest...."

or we fail to express the strength of our claim by using:

"If Tom is honest...."

Similarly, the conflict would arise whenever one wished to assert the weaker connection while evidencing his belief in the antecedent's falsity. The problem evaporates if we grant that the connective force and the mood of the verbs are fully independent. The connection affirmed by a soldier when he reports:

If the mine is stepped on, then it will explode

is the same connection as that affirmed after he learns that his trap was circumvented and maintains:

If the mine had been stepped on, it would have exploded.

The difference in the mood of the two sentences is not explained by our supposing in the soldier first a desire to assert some weak connection he could defend to his sergeant when his faulty preparation of the mine was discovered, and then later a surge of courage that prompted him to claim a very strong connection between the mine's being stepped on, and its exploding. The difference in mood is explained by a contrast in the soldier's beliefs before



and after the enemy's path was reported. Thus, when he believed it quite possible that enemy troops would march on the mine, his report was worded in the indicative. But later, when he learned that the line of march had changed, his replies perforce took on the verbs of the subjunctive.

Admittedly, there are exceptions to the rule requiring indicative verbs in a conditional uttered by one unprejudiced as to the truth of the antecedent clause. Indeed, one fully open-minded on the question of the antecedent's truth or falsity may use the subjunctive mood for reasons of politeness ("If you would), or for reasons of formality ("as it were"). Sometimes the subjunctive is also used to accommodate the views of a listener. For example, in Smith's argument with Jones, Suppose that Jones takes up a line of reasoning that begins:

"If the Democrats are right, then there will be..."

But suppose further that Smith is an ardent Republican and the assumption that the Democrats are right is so irritating and inconsistent with his present set of beliefs that he resists assuming even conditionally that they might be correct on this point. Hence, he firmly insists:

"But the Democrats are not right!"

Thus, in order to meet the resistance in Smith's mind to the assumption in the antecedent, Jones retreats to a

conditional with subjunctive verbs:

"But, look, if the Democrats were right, then there would be..."

With this tact, Jones is able to avoid debate on a question of little or no consequence to the shape of his argument by using the subjunctive verbs in a conditional with an antecedent which he need not believe false or contrary-to-fact.<sup>1</sup>

For the sake of arguments where our interest goes beyond the contingencies of the moment to some broader, possibly theoretical issue, we can usually carry on the discussion where a conversant balks at a conditional by employing the subjunctive. The choice of mood may then be determined by the resistance we anticipate in the mind of our listeners. But in the standard case we only have regard for the needs of avoiding resistance in our own minds. Although the hypothetical exchange between Jones and Smith does provide an exception to the general rule concerning the use of indicative verbs, it also again illustrates the fact that the mood of a conditional is not directed by the degree of connection the speaker claims between antecedent and consequent, but rather is gauged to comport with

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<sup>1</sup>A peculiar feature of the subjunctive mood in the English language is its attenuated store of distinctive verbs, a feature which made it necessary to fill in the auxillary verbs of the consequent clauses in the examples above. Out of context the verb in "If the Democrats

opinion over the probable truth of the antecedent clause: indicative if his attitude (affected, feigned, or real) is 'one of open-mindedness, but subjunctive if his belief (affected, feigned, or real) is that the antecedent is more probably false or contrary-to-fact than true.

Something of the point intended can be seen in saying that the same conditional can be expressed by two sentences differing only in the mood of the included verbs (used to fit the varying attitudes of the speaker), just as the same proposition may be expressed by two sentences differing only in the tense of the verbs (used to fit the varying temporal situation). Thus, the sentence 'There will be a recession in 1972' said before 1972, and 'We had a recession in 1972', said after 1972, might be said to express the same proposition. In similar metaphors the conditional:

"If the Democrats are right, then there will be a recession"

said by Jones who suspects that the Democrats may be right, and his retreat:

"Okay, if the Democrats were right, then there would be a recession"

(designed by Jones to accommodate the unquestionable views

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were" could be construed as indicative past, or subjunctive present.

of his listener) might be said to express the same proposition. It seems in fact that if one were to mark a difference in the strength of the two, it would be the one with the subjunctive verbs that he would deem the weaker. But of course the perceived weakness of the latter lies not in the connection Jones alleges but rather in his stand on the possibility of the Democrats being right.

## CHAPTER V

One might hold to the view that conditionals with subjunctive verbs assert a stronger form of implication on the grounds that some uses of the indicative conditional (unlike any of those for the subjunctive) can actually be identified with the truth-function we call material implication. Copi, for instance, claims that

Some conditional statements in English do assert merely material implications, as for example 'If Communist China is a peace-loving nation, then I'm a Dutchman' This sort of conditional is ordinarily intended as emphatic or humorous method of denying the truth of its antecedent for it always contains a notorious or ridiculously false statement as a consequent.<sup>1</sup>

But if the assertion here is merely the truth-functional claim of material implication, then it should not fade when the "notorious or ridiculously false statement" is replaced by a less ridiculous statement, e.g. "...then China is a U.N. member." Moreover, if Copi's example expresses merely a material implication, then its truth follows simply from the falsity of its antecedent, thus the truth-value of its consequent is quite irrelevant. Also, note that the force of this conditional as a rhetorical device actually depends, like modus tollens, on the one feature that is common to material implication and to all conditionals: the whole compound is false if its antecedent is true and its consequent false. When someone, affirms a conditional 'If p

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<sup>1</sup>Irving Copi, Symbolic Logic, Macmillan Co. 1965, 2nd ed.

then q' where it is assumed by all that 'q' is false, he is simply inviting a mental application of modus tollens to focus attention on the passion in his belief that 'p' is false; the rhetorical purpose of which is to deny his opponent the opportunity to return with a polite reply expressing agreement with the last remark while he continues disagreement on the point at issue, that is, to preclude a reply such as:

"Yes, that is probably true, but of course it doesn't rule out p."

The consensus that 'q' is false and acceptance of 'If p, then q' would indeed rule out p . To illustrate the point let us suppose that Smith, in a still cool argument, firmly asserts:

"If the Chinese are honest, then I'm a monkey's uncle"

Thus, confronted with this strong assertion, the champion of Asian sincerity, who naturally tries to accommodate rather than refute the more strongly held convictions of his opponent, is faced with the prospect of either pointing out the number of characteristically simian features in the progeny of Smith's brother, and thereby considerably raising the tone of the argument, or simply giving up the attempt to get across his point; for if he doesn't want to assume the task of refuting the conditional but

wants to maintain the truth of its antecedent, 'The Chinese are honest', then he has to uphold the claim in the consequent, viz. that Smith is a monkey's uncle. The dilemma forced by the rhetorical assertion of such conditionals thus depends upon the property which is shared by all conditionals, and not just by those asserting material implications, i.e. that the compound cannot be true if the antecedent is true and the consequent is false.

The claim that some indicative conditionals are properly construed as truth-functional compounds is also often supported by a consideration of conditionals used to make promises, bets, or predictions. The plausibility of construing an indicative conditional as a material implication is actually at its best when the conditional in question is being used for the expression of a promise, a bet, or a prediction, especially if there is no firm test for its truth when the antecedent clause is false. Thus, when the store manager promises:

If your washer breaks before 1970, we will fix it without cost.

the material construal may appear quite plausible; for there is no obvious way of proving him insincere if the antecedent is unfulfilled. In fact, if we let 'T' correspond to 'promise not broken,' or 'prediction does not fail,' and 'F' correspond to 'promise broken,' or 'prediction fails;'

then we actually obtain the standard truth-table for material implication. The promise can't be broken, nor can the prediction fail if the antecedent is false, i.e. if the washer does not break before 1970. And this may be all anybody really cares about in such cases. But, if one is concerned about the question of truth or falsity, 'T' must correspond to 'conditional true' and 'F' to 'conditional false.'

In bets or predictions where there is a firm independent way of determining the truth value of the conditional in question, e.g. when Sadie claims:

- a) If Tom rides the Clover Hill mare he will win the Derby

the simple falsity of the antecedent clause is not likely to draw assent to the conditional as a whole, but rather the investigation of some matter relevant to the content of the conditional, for example, the order of finish at the May Day Derby will likely decide the issue. It is important to note that if (a) were treated in the common conversational situation as a material implication, then we would interpret George's reply:

- b) Tom would never ride the Clover Hill mare.



as a show of agreement since it logically entails the truth of (a) materially construed. But (b), it must be admitted has the quality of an objection. Of course, it is not an objection to the claim in (a), but rather an objection to Sadie's ability to entertain the assumption that Tom might ride a horse owned by the philistines at Clover Hill, and thus indirectly to the mood of the conditional. George may agree with Sadie that the Clover Hill mare is the fastest entry in the May Day Derby, and thus also that Tom would win on her, but his belief in (b) does not commit him to Sadie's claim in (a), as it would if (a) were merely a simple truth-function like material implication.

It may well be difficult to say what the test of truth is when the antecedent clause is false in a conditional such as:

If your washer breaks, we will fix it without cost

but it is surely no different from that for:

If your washer had broken, we would have fixed it without cost.

Here the occasion of utterance is of course different, but not the conditional, the grounds for its truth, or its truth-value. Lacking an easy way of determining this truth-value does not justify our forthwith assuming one. We cannot bridge a gap in our knowledge merely by calling it a

truth-value gap.

The truth of a conditional, whether indicative or subjunctive, simply does not follow from the falsity of its antecedent clause. For an ordered sentence compound whose truth does follow from the falsity of its antecedent, we must look to a sentence like:

'The moon is green' materially implies 'The moon is not green.'

or to 'if-then' compounds uttered by someone who indicates that he wishes to be so understood.

Perhaps, the complexity of the conditional idiom is more readily grasped with those conditionals in which the verbs happen to occur in the subjunctive. But the shift from the indicative:

"If a is an F, then it is a G"

to the subjunctive:

"If a were an F, then it would be a G."

is more clearly prompted by a shift in opinion as to whether a is an F, rather than by a shift in opinion as to the adhesive quality of the bond between F's and G's.

## CHAPTER VI

How do we decide whether a conditional is true or false? We know that any conditional with true antecedent and false consequent is false. The conditional:

"If Willoby eats the fish, he will die"

is definitely false if Willoby ate the fish, but did not die, i.e., if the antecedent is true and the consequent false. None of the other three truth-value combinations (TT, FT, or FF) delivers a decisive verdict. For the case in which both antecedent and consequent are found to be true one might suppose that the question about the conditional as a whole could be decisively answered in the affirmative, as when Willoby eats the fish and dies a few hours later. But such a hasty confirmation would be in error; it overlooks the possibility that Willoby did not die because he ate the fish, but because of an automobile accident, a stray bullet, heart attack, or some other unfortunate event while the fish in question was actually quite edible.

If the case of the conditional with true antecedent and consequent had such an obvious solution, it would be just as obvious for the case of its contrapositive in which the antecedent and consequent are both false, e.g. for:

"If Willoby does not die, then he did not eat the fish"

Actually there are only two truth-value combinations that need to be considered for a set of necessary and sufficient conditions for the truth of a conditional. Every conditional with true antecedent and consequent will have an equivalent in its contrapositive with false antecedent and consequent, and the formal truth conditions for the one are the formal truth conditions for the other. The case of true antecedent and false consequent has already been decided. Thus, we can simplify our discussion, without loss of territory, by confining it to conditionals in which the antecedents are false.

It may be helpful to begin by considering how a person who asserts a conditional, 'If 'A' were true, then 'C' would be true', might go about justifying his assertion to someone who questions its truth. We might imagine that his defense proceeds as follows:

- 1(1) Suppose that A.
- 2(2) We've agreed that B.
- 3(3) Note that 'L' is a law or a general principle of which 'If A and B, then C' is an instance.
- 1,2,3(4) Thus, with our initial assumption we can infer C.
- 2,3(5) Therefore, we can say if A, then C.

This ploy is usually collapsed in everyday conversation, but it is still the informal analogue of the method used for conditional proof. The antecedent of the conditional

to be deduced is assumed tentatively as a premise to illustrate the consequences of this assumption. Then some statements we assert categorically are introduced as premises. Any consequence of the total set of premises, can then be taken as the consequent of a conditional using the original assumption as its antecedent. The resulting conditional thus depends only on the premises granted categorically in the first lines of the deduction. With this analogy in mind we might design a criterion for conditionals: A conditional "If 'A' were true, then 'C' would be true" (where 'A' is actually false or contrary-to-fact) is true if and only if there is a deduction of C depending upon the assumption A, some general principle or law L, and a set S of true sentences presumed, but unstated, in the assertion of the conditional.<sup>1</sup>

The problem is now to formulate the general specifications for membership in the set S. One might naturally ask why the simple condition that the sentences in S be true is not a sufficient condition, for it would seem to make no difference whether S includes every true sentence or just those necessary for the deduction of C from A and S.

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<sup>1</sup>So as not to obscure the debt I owe to his valuable discussion, and also to facilitate subsequent references to his arguments, I am adopting Goodmar's notation: " 'A' for the antecedent 'C' for the consequent and 'S' for the set of statements of the relevant conditions or, indifferently, for the conjunction of these statements." p 9, Fact, Fiction, and Forecast (Indianapolis, Bobbs-Merrill, 1965)

The answer, put picturesquely, is that the world of the false or contrary-to-fact antecedent is not described by the sentences that happen to be true for this world. Put more concretely, since  $\underline{A}$  is false,  $\neg \underline{A}$  will be true; and with  $\underline{A}$  and  $\neg \underline{A}$  among the assumption sentences, there will be no sentence  $\underline{C}$  that cannot be deduced from  $\underline{A} \cdot \underline{S}$ . Thus, every conditional would be deemed true with the simple rule proposed. Unfortunately, it is only slightly less naive to suppose that this consequence can be avoided by specifying that  $\neg \underline{A}$  not be a member of  $\underline{S}$ . For where  $\underline{Q}$  is any false sentence other than  $\underline{A}$ , both  $\neg \underline{Q}$  and  $\underline{Q} \vee \underline{A}$  will be true, and thus qualify as members of  $\underline{S}$ , though together they will logically entail  $\neg \underline{A}$ . Therefore, the criterion would still fail to mark a distinction between those conditionals which are true and those which are false; for every conditional with a false antecedent would satisfy the stipulations of our rule without a clause restricting  $\neg \underline{Q}$  and  $\underline{Q} \vee \underline{A}$  from membership in  $\underline{S}$ . We must, therefore, add an amendment requiring that  $\underline{A} \cdot \underline{S}$  be self-compatible, or that  $\underline{S}$  not entail  $\neg \underline{A}$ :

(I)  $\neg(\underline{S} \vdash \neg \underline{A})$

The rule so amended should accomplish the very minimum of drawing at least some distinctions.

Conditionals with false antecedent and true consequent present a special problem; for whenever the consequent  $\underline{C}$  is true, there will always be a trivial deduction of  $\underline{C}$

from  $A \cdot S$  since  $C$  as true qualifies as a member of  $S$ . Thus, we must also include a provision stipulating that  $S$  be compatible with  $\neg C$ , or simply that  $S$  not entail  $C$ :

(II)  $\neg(S \vdash C)$

The need for amendments (I) and (II) might have been guessed in the initial planning, however future revisions will be less predictable and more surprising to our preanalytical expectations. For instance, Goodman points out that another difficulty arises with a conditional beginning:

"If Jones were in Carolina...."

The antecedent of such a conditional will be entirely compatible with:

a) Jones is not in Carolina.

and with:

b) North Carolina plus South Carolina is identical with Carolina.

These taken together as  $S$  and conjoined with the antecedent allow a deduction of 'Jones is in South Carolina,' hence the tentative criterion accepts the conditional:

c) If Jones were in Carolina, he would be in South Carolina

And if we replace 'Jones is not in North Carolina' with:

d) Jones is not in South Carolina

then we can also establish the acceptance of the opposing conditional:

e) If Jones were in Carolina, he would be in North Carolina.

In response to this difficulty W. T. Parry proposed the addition of a requirement stipulating that  $\underline{S}$  not follow by law from  $\underline{A}$ .<sup>2</sup> This restriction would appear to proscribe 'Jones is not in South Carolina' and 'Jones is not in North Carolina' if they do, in fact, follow by law from 'Jones is not in Carolina' (as both Goodman and Parry seem to suppose). Nevertheless, Goodman shows that this amendment is not enough to insure the rejection of the unacceptable conditionals, (c) and (e).<sup>3</sup> He points out that if Jones is actually in South Dakota we will have as true, both:

f) Jones is in a state whose name contains the word 'south'

and:

g) Jones is in a state north of South Carolina.

Thus, there will still be a deduction satisfying the conditions of the amended rule to establish the opposing,

<sup>2</sup> W. T. Parry, 'Reexamination of the Problem of Counterfactual Conditional,' Journal of Philosophy, Vol 54, (1957) p. 90

<sup>3</sup> Parry on Counterfactuals' Journal of Philosophy, Vol 54, (1957), pp442-5



unacceptable conditionals. For the conditional 'If Jones were in Carolina, he would be in North Carolina' there is a set  $\underline{S}$  including (g) and the true sentence:

- h) Either Jones is in North Carolina, or Jones is not both in Carolina and in the state north of South Carolina.

and such that  $\underline{A \cdot S}$  leads by law to  $\underline{C}$ , that is, to 'Jones is in North Carolina.' Therefore, Goodman concludes that we must amend the rule with a clause specifying that there be "no set  $\underline{S}$ ' compatible with  $\underline{C}$  and with  $-\underline{C}$ , and such that  $\underline{A \cdot S}$ ' is self-compatible and leads by law to  $-\underline{C}$ ."<sup>4</sup> But notice that there being an suitable  $\underline{S}$  such that  $\underline{A \cdot S}$  leads by law to  $\underline{C}$  for both:

- i) If Jones were in Carolina, he would be in South Carolina.

and:

- j) If Jones were in Carolina, he would be in North Carolina.

does not guarantee us a suitable  $\underline{S}$ ' for each of these, such that  $\underline{A \cdot S}$ ' leads by law to  $-\underline{C}$ . The consequents of the two conditionals unfortunately are not direct denials of one another. The appropriate sentence  $\underline{S}$ ' for (i), according to Goodman's proposal, must be compatible with  $\underline{C}$ ,

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<sup>4</sup>Nelson Goodman, Fact, Fiction, and Forecast, (Indianapolis, Bobbs-Merrill, 1965) p. 13.

'Jones is in South Carolina, and yet when conjoined with 'Jones is in Carolina' must lead by law to  $\neg C$ , 'Jones is not in South Carolina'. Such a sentence would seem to be hard to find. Nevertheless, consider the sentence:

k) Jones is west of South Carolina.

Strictly speaking, (k) is logically compatible with 'Jones is in South Carolina'. The incompatibility only arises if we add some principle about the two-place predicate 'is west of' such as:

1)  $(x)(y)$  (if  $x$  is west of  $y$ , then  $x$  is not in  $y$ )

With this as the appropriate non-logical law,  $A \cdot S$  leads to  $\neg C$ , that is to 'Jones is not in South Carolina.' Thus, (i) is rejected with the rule amended as Goodman suggests. Likewise (j) is rejected as demonstrated when  $S$  is 'Jones is west of North Carolina'. Note that we are assured of a suitable  $S$  for all similar conditionals about Jones or anyone who is not in Carolina. For if the person in question is not in Carolina, he will either be north, south, east, or west of the Carolina under mention, and there is a non-logical law identical with, or parallel to, (1) for each of these cases. One might wonder then whether the amendment is too strong, whether it causes the rejection of some acceptable conditionals along with its rejection of the unacceptable conditionals we have been focusing on

so narrowly. Consider for example a conditional such as:

- m) If Jones were in Carolina, but not in South Carolina he would be in North Carolina.

which would appear to have impeccable credentials. Indeed, we need only add to the antecedent the true sentence 'North Carolina plus South Carolina is identical with Carolina' to obtain a deduction of the consequent. But, as just noted, we are assured here of a suitable  $\underline{S}$ ' such that  $\underline{A \cdot S}$ ' leads by law to  $\underline{-C}$ . Hence without Goodman's amendment, the rule admits unwelcome conditionals like (i) and (j); but if it is instituted, then conditionals like (m), which would obviously be true, are nevertheless wrongly deemed false. With the amendment, our rule is too stern, without it, too lax: either way it is overly democratic in not favoring the true over the false.

Something has gone afoul, and the problem seems to stem from the fact that in each of these cases  $\underline{S}$ ' alone leads by law to  $\underline{-C}$ . The appropriate deduction would seem to be one in which  $\underline{A}$  must play a leading role. Thus, we might modify Goodman's amendment with the stipulation that  $\underline{A \cdot S}$ ', but not  $\underline{S}$ ' alone, lead by law to  $\underline{-C}$ .

The rule now allows for the proper acceptance of (k), but the desired rejection of (i) and (j) appears uncertain. To demonstrate the exclusion of (i), for example, we would need to find some true sentence  $\underline{S}$ ', such that the assumption

'Jones is in Carolina', (which would normally lend credence to 'Jones is in South Carolina') is essential to the inference of 'Jones is not in South Carolina'. It seems doubtful that there is such a sentence. Possibly Goodman finds the way out through the definition of some special non-logical law. But, as mentioned earlier, that investigation is beyond the scope of this paper. Perhaps there is no general solution for conditionals of this sort. At any rate, we can return to this question after considering a number of other difficulties, the solution of which may yield an answer.

## CHAPTER VII

Apart from the problem of defining the nature of non-logical law, the major problem concerning conditionals is usually taken to be the problem of cotenability. This is the problem of specifying without vicious circularity the true statements that will, and the true statements that will not, withstand the assumption that the false or contrary-to-fact antecedent is true. Goodman explains that the problem of cotenability arises in the familiar case where we would affirm:

(i) If match m had been scratched, it would have lighted,

but deny:

(ii) If match m had been scratched, it would not have been dry.

The rule should provide for the acceptance of (i) and the rejection of (ii) in the familiar case we are supposing, but, as Goodman points out, the latter passes the test of the tentative criterion. We may have as an element in S the true sentence "Match m did not light," and then as A·S:

Match m is scratched; it does not light; it is well made; oxygen enough is present...etc.

from which by means of a legitimate general law we can infer:

m was not dry.

Goodman suggests that the trouble is caused by including a true sentence in S ('It does not light') which would not be true if A were true, that is to say, a sentence not cotenable with A. He notes that this sentence is not excluded from S with the tentative rule, and that the unwanted counterfactual would thus be established as "there would seem to be no suitable set of sentences S' such that A·S' leads by law to the negate of the consequent."<sup>1</sup>

My objection to this is simply that there is a suitable set of sentences S' such that A·S' leads by law to -C. Take, for instance, the set S' which has as its sole member:

Match m lighted.

Clearly, S' is logically compatible with C and with -C, that is with:

Match m was not dry

and with:

Match m was dry.

Also, S' as such certainly does not follow from -A, ('Match m was not scratched'), and it is of course compatible with

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<sup>1</sup>Nelson Goodman, Fact, Fiction, and Forecast, (Indianapolis, Bobbs-Merrill, 1965) pp. 15-17

A. Nevertheless, with the conjunction of A and S', i.e., with:

Match m was scratched and m lighted

and an equally legitimate general law, we can infer -C, that match m was dry. For if it is known of a certain match that it was scratched and lighted, then it can be inferred that the match in question must have been dry. Therefore our finding a suitable set S', such that A.S' leads by law to -C, shows that the unwanted conditional is not, as Goodman supposes, accepted by his tentative rule.

There may be some discomfort with this rejection due to a suspicion that the sentence 'Match m lighted' would be false in the familiar case we are imagining for the sake of Goodman's argument. For no obvious reason, however, Goodman never requires that S' be true in the several formulations of his tentative rule. One suspects, nevertheless, that Goodman may have assumed this as a requirement especially since (i) would be found unacceptable on his rule if the false sentence 'm is wet' were suitable for S'. Let us thus discount the foregoing as a legitimate rejection of the unacceptable conditional, for there is no reason not to institute the standard of truth for the member sentences of S'.

Is there still a suitable set S' for (ii), such that A.S' leads by law to the negate of its consequent? If there

is no such set, then we must face the problem of finding a non-circular definition of cotenability. But the moment of confrontation is still well into the future, for there is a set  $\underline{S}'$  that meets all the conditions of the amended rule. Consider the set  $\underline{S}'$  containing the disjunction:

Some undry matches light when scratched, or  $\underline{m}$  was dry.

This must be true in the case we are supposing since the truth of the second disjunct is necessary for the truth of (i), the acceptable conditional. The set  $\underline{S}'$  selected for (ii) here also accords with all the provisions in Goodman's criterion: it is compatible with  $\underline{C}$  (' $\underline{m}$  was not dry') and with  $-\underline{C}$ ; it does not follow by law from  $-\underline{A}$  (' $\underline{m}$  is not scratched'); and it is compatible with  $\underline{A}$  nevertheless, from  $\underline{A} \cdot \underline{S}'$  and a legitimate general law claiming:

No undry match lights when scratched

we can again infer  $-\underline{C}$ , that match  $\underline{m}$  was dry. Hence the illegitimate conditional (ii) is still rejected by the tentative rule, even though it lacks a clause covering the cotenability of the sentences in  $\underline{S}$ .

However, we cannot now conclude that a problem over the cotenability of the sentences in  $\underline{S}$  never arises, for the same ploy can of course be used to demonstrate the exclusion of (i), the perfectly acceptable conditional, with the rule in its present form. In fact, the rule is



so strong that no conditional (in which the consequent is false) will satisfy the stipulation that there be no suitable set  $\underline{S}'$  such that  $\underline{A} \cdot \underline{S}'$  leads by law to  $\underline{-C}$ . For where  $\underline{L}$  is the legitimate general law, and  $\underline{C}$  is false, there will always be a sentence  $\underline{-Lv-C}$ , admissible in  $\underline{S}'$ , which will insure that  $\underline{A} \cdot \underline{S}'$  leads by law to  $\underline{-C}$  in a deduction of the form:

- (1) A
  - (2)  $\underline{-Lv-C}$
  - (3) L
- C

Moved by the superfluousness of  $\underline{A}$  in this sequence one would naturally think of amending the criterion still further by ruling that the deduction of  $\underline{-C}$  depend upon  $\underline{A}$ , or more precisely that  $\underline{A} \cdot \underline{S}'$  lead by law to  $\underline{-C}$ , where  $\underline{S}'$  alone does not lead by law to  $\underline{-C}$ . But this restriction is easily evaded. The untoward results are readily illustrated. Counterexamples can be generated systematically with the deductive formula:

- (1) A
  - (2)  $\underline{-Av(-Lv-C)}$
  - (3) L
- C

Here  $\underline{A}$  obviously plays a key role in the deduction of  $\underline{-C}$

since (2) and (3) do not alone entail  $\neg C$ . Moreover, this contrivance is not limited to those conditionals in which  $C$  is false; the disjunction of  $\neg A$ ,  $\neg L$ , and  $\neg C$  is true just in case one disjunct, e.g.  $\neg A$ , is true. Thus, there will be a set  $S'$  of true sentences for every conditional such that  $A \cdot S'$ , but not  $S'$  alone, leads by law to  $\neg C$ ; therefore, every conditional will be rejected by the rule. To avoid this unhappy situation, we can discard the amendment requiring the non-existence of the set  $S'$ . Goodman introduced this amendment to exclude the unacceptable conditionals speculating upon Jones' probable whereabouts in Carolina. But the provision failed in its intent unless the consequents:

...Jones is in South Carolina

and

...Jones is in North Carolina

can be counted as direct denials of one another. But they cannot. Of course the unacceptable conditionals about Jones would be excluded by the rule with Goodman's amendment, but the exclusion has nothing to do with the specific nature of these conditionals as unacceptable, but rather derives from the fact that they are conditionals, and all conditionals are unacceptable according to the criterion with Goodman's amendment.

Unfortunately, dispensing with the non-existence clause has done little to improve our situation. The abbreviated rule simply errors now in the other direction; it accepts every conditional; for there is an analogous formula which will always select a set  $\underline{S}$  satisfying the requirements of our rule. No matter what the conditional,  $\underline{S}$  need only include  $\text{-AvC}$  to provide for a deduction of the consequent thus:

- (1) A  
 (2) -AvC  
 C

Goodman acknowledges this problem in the second edition of Fact, Fiction, and Forecast;<sup>2</sup> but he claims that the difficulty can be eliminated if "we add the requirement that neither  $\underline{S}$  nor  $\underline{S}'$  follow by law from  $\text{-A}$ ." The point, however, seems to deserve more attention; the difficulty cannot be cast aside so easily. Goodman's amendment is at once too weak and too strong. It is obviously too weak, since for any conditional,  $\underline{S}$  may still be the set containing  $\text{-AvC}$  and some other true sentence  $\underline{P}$  which does not follow by law from  $\text{-A}$ . The most natural proposal to avert this difficulty would be to add the stipulation that no member of the set  $\underline{S}$  follow by law from  $\text{-A}$ ; but, as it will soon be shown, this

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<sup>2</sup>Goodman, op. cit., p. 13

is hardly sufficient. That Goodman's rule is too strong, is not at once so apparent. The argument, however, is simple. Suppose it is found that a certain piece of metal c conducts electricity. Consider the hypothesis:

(iii) If c is not copper, then some pieces of metal which are not copper conduct electricity.<sup>3</sup>

The warrant for this conditional would usually be that its antecedent plus the true sentence S:

Piece of metal c conducts electricity

together entail the consequent, that some pieces of metal which are not copper conduct electricity. But if the criterion for the truth of conditionals includes the requirement that S does not follow by law from -A, then (iii) will be rejected, because the natural candidate for S, ('c conducts electricity'), follows by law ('All copper conducts electricity') from -A, that is from 'c is copper.'

It seems we must relax the regulation that S not follow by law from -A, most naturally perhaps by requiring only that -A not entail S. Hopefully nothing hinged on Goodman's more rigid formulation. There was a suggestion to exclude 'Jones is not in South Carolina' as a suitable S for the unacceptable conditional:

<sup>3</sup>This of course may be read "If c were copper,...." since differences in mood, as it was argued earlier, do not demand differences in the logical analysis, but rather indicate opinion as to the truth or falsity of the antecedent.

- (iv) If Jones were in Carolina, he would be in North Carolina

by requiring that  $\underline{S}$  not follow by law from  $\neg A$ . But it is quite doubtful that there is really a non-logical law by which 'Jones is not in South Carolina' can be inferred from 'Jones is not in Carolina'; and even if there were such a law, the restriction, as it may be recalled, was easily circumvented. Thus, there is no apparent reason why Goodman's amendment could not be revised to rule just that neither  $\underline{S}$  nor  $\underline{S}'$  be logically entailed by  $\neg A$ . The amendment as recast here would still be inadequate since, as noted before, for any conditional whatsoever, there is a suitable set  $\underline{S}$  not entailed by  $\neg A$  and yet compounded so that  $A \cdot \underline{S}$  leads by law to  $\underline{C}$  whenever  $\underline{S}$  embraces  $\neg A \vee \underline{C}$  and at least one true sentence  $\underline{P}$  not entailed by  $\neg A$ . But the loophole exploited by choosing  $\underline{S}$  as some multimembered set comprising  $\neg A \vee \underline{C}$  and true sentences, like  $\underline{P}$ ,  $\underline{Q}$ , and  $\underline{R}$  (not entailed by  $\neg A$ ), can be closed with a simple adjustment. Where  $\underline{S}$  is the set of true sentences  $(S_1, \dots, S_n)$ , there should be no sentence  $S_i$  such that  $\neg A$  entails  $S_i$ . This is a necessary stipulation, but it needs tightening. It takes the inclusive sense of 'or' into account, but not its exclusive sense. For where 'o' is taken to represent the exclusive sense of 'or', there will always be a set  $\underline{S}$ , viz. the unit set of  $\neg A \circ \underline{C}$ , which is not entailed by  $\neg A$ , but yet such that  $A \cdot \underline{S}$  leads by law to  $\underline{C}$ . Thus, we must elaborate the amendment

a bit further, to demand that no member  $S_i$  of  $\underline{S}$  be such that it is entailed by  $-A$  and  $-\underline{C}$ :

(III)  $-(\neg A, \neg C \vdash S_i)$  .

This regulation adequately handles any difficulty arising from the use of the exclusive 'or'. Conditionals with false antecedent and true consequent present another special problem that can be handled by a simple amendment. Where  $\underline{C}$  is true,  $-\underline{L}\vee\underline{C}$  can always be included in  $\underline{S}$  to obtain a set  $\underline{S}$  such that the conjunction of  $\underline{A}$ ,  $\underline{S}$ , and  $\underline{L}$  logically entails  $\underline{C}$ ; however, we can simply introduce a clause specifying that  $\underline{S}$  and  $\underline{L}$  alone not entail  $\underline{C}$ :

(IV)  $-(S, L \vdash C)$

to insure that  $-\underline{L}\vee\underline{C}$  will never be reckoned among the sentences in  $\underline{S}$ . With the addition of this amendment we can also drop (II), the stipulation that  $\underline{S}$  alone not entail  $\underline{C}$ .

In sum the tentative criterion should now read that a conditional "If 'A' were true, 'C' would be true" (where 'A' is false or contrary-to-fact) is true if and only if there is some general principle or law  $L$  and some set  $S$  of true sentences  $(S_1, \dots, S_n)$  such that:

(1)  $A, S, L \vdash C$

(2)  $-(S \vdash \neg A)$  (I)

(3)  $-(\neg A, \neg C \vdash S_i)$  (III)

(4)  $-(S, L \vdash C)$ 

(IV)

Nevertheless, there is still a simple formula for finding an acceptable set  $\underline{S}$  to secure for every conditional a simple deduction from  $\underline{A \cdot S}$  to  $\underline{C}$ ; for where 'P' stands for any true sentence not entailed by  $\underline{-A}$  or  $\underline{-C}$ , counterexamples can be generated systematically through the form:

(1)  $A$ (2)  $\underline{(-A \cdot P) \vee C}$  $C$ 

Clearly it will not help to add an amendment stipulating that  $S_i$  not follow by law from  $\underline{C}$ : this would be useless because another mischievous, but simple, evasion can be carried out by taking  $\underline{(-A \cdot P) \vee (C \cdot P)}$  as a unit in  $\underline{S}$ .

At this point, one might be tempted to introduce a piece of legislation to the effect that no member  $\underline{S}_i$  be such that it follows from  $\underline{-A}$  and some other true sentence  $\underline{P}$  unless  $\underline{S}_i$  follows from  $\underline{P}$  alone. Indeed, this would insure that an acceptable set  $\underline{S}$  could not contain a sentence like  $\underline{(-A \cdot P) \vee C}$ , but it would also insure that the only acceptable set for  $\underline{S}$  would be the null set. For where  $\underline{R}$  is any true sentence, then  $\underline{A \vee R}$  is a true sentence such that  $\underline{R}$  follows from  $\underline{-A}$  and  $\underline{A \vee R}$ , but not from  $\underline{A \vee R}$  alone; hence no sentence would qualify as a member of  $\underline{S}$ . A less demanding amendment might be found in the stipulation that  $\underline{A}$  and  $\underline{-C}$

not entail  $\underline{S}$ :

(V)  $-(A, -C \vdash -S)$

This would exclude sentences such as  $(\underline{-A \cdot P}) \vee C$ , since  $\underline{A \cdot -C}$  logically entails  $\underline{-((\underline{-A \cdot P}) \vee C)}$ . However, there is a class of conditionals which we would intuitively accept as true, but yet such that they would not meet the conditions of the criterion if this stipulation were added. These are conditionals that have an antecedent which alone will not entail the consequent, but will when conjoined with some true sentence which is not a natural or non-logical law. For example, when it is said of a particular ball-point pen  $\underline{b}$ :

(vi) If pen  $\underline{b}$  does not write, some ball-point pens do not write

This quite reputable conditional would not meet the overly rigorous standards of our criterion with the proposed amendment. Note that the obvious candidate for  $\underline{S}$  in this case:

$\underline{b}$  is a ball-point pen

is such that its denial,  $\underline{-S}$ , strictly follows from  $\underline{A \cdot -C}$ , i.e., from:

Pen  $\underline{b}$  does not write, but every ball-point pen writes.



The requirement that  $\underline{A \cdot C}$  not entail  $\underline{-S}$ , is obviously too strong for those conditionals in which  $\underline{C}$  follows by logic alone from  $\underline{A}$  and a sentence describing some relevant fact about the situation; (V) will only do for those conditionals in which the antecedent  $\underline{A}$ , coupled with the natural set of relevant true statements, is presumed to lead by a causal or non-logical law to the consequent  $\underline{C}$ . The trouble with deductions such as:

- (1)  $A$   
 (2)  $\underline{(-A \cdot P) \vee C}$   
 $C$

does not lie simply in the fact that  $\underline{A \cdot S}$  entails  $\underline{C}$  without reference to a causal or non-logical law; there are many wholly acceptable conditionals, like (vi), for which the appropriate set  $\underline{S}$  of true sentences is such that  $\underline{A \cdot S}$  leads by a logical law to  $\underline{C}$ . The problem lies elsewhere, and it is not obvious where it is or just how it can be solved.

The scope of the criterion might be confined to those conditionals in which a causal connection is affirmed between the antecedent and the consequent, and thus to those for which  $\underline{A \cdot S}$  is presumed to lead by a non-logical or causal law to  $\underline{C}$ . Within this scope, (V) would not be too stern a measure, and it would insure the exclusion of  $\underline{(-A \cdot P) \vee C}$  as a possible sentence in  $\underline{S}$ . But besides being a fairly undesirable move, there are other serious difficulties that

promptly appear when we turn our attention to conditionals in which the sequence from antecedent to consequent is the one from cause to effect.

## CHAPTER VIII

Consider the fairly absurd, but quite ordinary conditional (at least with respect to the causal ordering):

- (i) If Match  $m$  had been scratched, it would have giggled without shame, grace, or the proper reserve.

To sharpen focus on the logical features of the problem as it arises with such a conditional, we will introduce a set of symbols for the relevant open sentences thus:

'Sx' for 'Match x is scratched'

'Dx' for 'x is dry, well-made, and in oxygen'

'Lx' for 'x lights'

'Gx' for 'x giggles without shame, grace, or the proper reserve'

Now if we suppose ' $(x) (Sx \cdot Dx \rightarrow Lx)$ ' the name of the relevant non-logical law covering the scratching and lighting of matches, we can show that there is a set  $\underline{S}$  which insures the acceptance of the absurdity in (i) under our present criterion. For by taking  $\underline{S}$  to be the conjunction of 'Dm' and '-Lm,' we can produce a deduction of  $G_m$  as follows:

1(1)	$S_m$	<u>A</u>
2(2)	$D_m \cdot \neg L_m$	<u>S</u>
3(3)	$(x)(Sx \cdot Dx \rightarrow Lx)$	<u>L</u>
3(4)	$S_m \cdot D_m \rightarrow L_m$	3
1,2(5)	$S_m \cdot D_m$	1,2

1,2,3(6)	$L_m$	4,5
1,2,3(7)	$L_m \vee G_m$	6
1,2,3(8)	$G_m$	2,7

The problem with our criterion here is that it allows in  $\underline{S}$  a true sentence like ' $-L_m$ ' that is not cotenable with the antecedent, that is to say, a sentence which would not be true if the antecedent ' $S_m$ ' were true. This is the problem of cotenability. The solution, nevertheless, seems obvious; merely require that  $\underline{S}$  and  $\underline{L}$  not entail  $-A$ :

(VI)  $-(S, L \vdash -A)$

since the conjunction of ' $(D_m \cdot -L_m)$ ' and ' $(x)(S_x \cdot D_x \rightarrow L_x)$ ' logically entails ' $-S_m$ '. This restriction also allows us to discard the first amendment (I) requiring that  $\underline{S}$  not entail  $-A$ . However, the general problem remains; for the illegitimate member of  $\underline{S}$ , the sentence 'Match  $m$  did not light' (' $-L_m$ '), can now be replaced with ' $-L_m \vee G_m$ ' without violating the strictures of (VI); hence, there is still a convenient deduction of ' $G_m$ ' from  $\underline{A}$ ,  $\underline{S}$ , and  $\underline{L}$ , e.g.:

1(1)	$S_m$	$\underline{A}$
2(2)	$D_m \cdot (-L_m \vee G_m)$	$\underline{S}$
3(3)	$(x)(S_x \cdot D_x \rightarrow L_x)$	$\underline{L}$
1,2,3(4)	$L_m$	1,2,3
2(5)	$-L_m \vee G_m$	2
1,2,3(6)	$G_m$	4,5

Thus, the scandalous conditional, 'If match  $\underline{m}$  had been scratched, it would have giggled without shame, grace, or the proper reserve,' would be established as true if (VI) was adequate.

Goodman's amendment requiring that there be no similar set  $\underline{S}$ ' such that  $\underline{A \cdot S}$ ' leads by law to  $\underline{-C}$  would indeed provide for the rejection of (i), but its revival offers little advantage, since it would also assure the rejection of the acceptable conditional

(ii) 'If Match  $\underline{m}$  had been scratched it would have lighted'

To help illustrate this rejection, let 'Mx' correspond to 'x loses mass', and '(x)(Sx · Dx → Mx)' correspond to a non-logical law covering the loss of mass by dry, well-made matches scratched in sufficient oxygen, then there is a set  $\underline{S}$ ' containing ' $\underline{Dm} \cdot (-Mm \vee -Lm)$ ', which is such that  $\underline{A \cdot S}$ ' leads by law to  $\underline{-C}$ , that is, to ' $\underline{-Lm}$ ' ; thus:

1(1)	$S_m$	$\underline{A}$ ,
2(2)	$D_m \cdot (-M_m \vee -L_m)$	$\underline{S}$ '
3(3)	$(x)(S_x \cdot D_x \rightarrow M_x)$	
1,2,3(4)	$M_m$	1,2,3
2(5)	$-M_m \vee -L_m$	2
1,2,3(6)	$-L_m$	4,5

The resurrection of the non-existence clause thus offers as much loss as profit. The root of our trouble goes deeper

than the frivolity of (i) would suggest. While the absurdity of this conditional underscores the inadequacy of our formula, it misleadingly belittles the conceptual difficulty which might have been better appreciated with a less improbable example. The problem here stems from the fact that no single event has just one causal consequence. It is not only true of a particular match scratched under favorable conditions that it will light, but also that it will lose mass, possibly that someone will get burnt, and inevitably that the number of unused matches will decrease by at least one. The problem this fact presents for an analysis of conditional expressions can be viewed as an aspect of the problem of cotenability. For the trouble arises when sentences are permitted in  $\underline{S}$ , (e.g. ' $\neg L_m \vee M_m$ ') which would not be true if the antecedent (e.g. ' $S_m$ ') were true, in other words, when the sentences in  $\underline{S}$  are not 'jointly tenable' or cotenable with  $\underline{A}$ . Adding the requirement that  $\underline{S}$  be cotenable with  $\underline{A}$  will not solve the problem, for then the definition would be tightly circular. Cotenability is defined in terms of conditionals, and conditionals as such would then be defined in terms of cotenability. In order to determine whether 'If  $\underline{A}$  were true,  $\underline{C}$  would be true' is true, we must first decide whether the conditional 'If  $\underline{A}$  were true, then  $\underline{S}$  would not be true' is itself true. Thus, to establish any conditional, we would always have to determine the truth of another, and then another one before that one, and so on ad infinitum.

The problem seems serious, but a number of possible solutions suggest themselves. Take, for example, the rather unintuitive, but fairly simple ruling, that there be no set  $\underline{S}$ ' (of true sentences) such that  $\underline{A \cdot S' \cdot -C}$ , but not  $\underline{A \cdot -C}$  alone, leads by law to  $-\underline{S}$ . Unacceptable conditionals like (i) could then be easily rejected, since there is an appropriate set  $\underline{S}$ ', namely the unit set of ' $\underline{Dm}$ ', such that  $\underline{A \cdot S' \cdot -C}$ , but not  $\underline{A \cdot -C}$  alone, leads by law to  $-\underline{S}$ , i.e. to  $\neg(\underline{Dm} \cdot (-\underline{Lm} \vee \underline{Gm}))'$ ; to wit:

1(1)	$S_m$	$\underline{A}$
2(2)	$D_m$	$\underline{S}'$
3(3)	$-G_m$	$-\underline{C}$
4(4)	$(x)(Sx \cdot Dx \rightarrow Lx)$	
1,2,4(5)	$L_m$	1,2,4
1,2,3,4(6)	$L_m \cdot -G_m$	3,5
1,2,3,4(7)	$\neg(\neg L_m \vee G_m)$	6
1,2,3,4(8)	$\neg D_m \vee \neg(\neg L_m \vee G_m)$	7
1,2,3,4(9)	$\neg(D_m \cdot (\neg L_m \vee G_m))$	8

Note that this last condition is at once satisfied by any conditional in which  $\underline{A \cdot S}$  logically entails  $\underline{C}$ ; for in all such cases  $\underline{A \cdot -C}$  alone leads by law to  $-\underline{S}$ , thus, of course there can be no set  $\underline{S}'$  such that  $\underline{A \cdot S' \cdot -C}$ , but not  $\underline{A \cdot -C}$  alone, leads by law to  $-\underline{S}$ .

The real danger is not that the rule, so amended, is too strong, but rather that it is still too weak. The

untoward results are readily illustrated. Consider a variant on (i), equally scandalous, though a bit more verbose:

- (iii) If match  $\underline{m}$  were dry, well-made, in sufficient oxygen and scratched, it would have giggled without shame, grace, or the proper reserve.

The set  $\underline{S}$ , whose sole member is ' $\neg L_{\underline{m}}vG_{\underline{m}}$ ', can be conjoined with the antecedent ' $S_{\underline{m}} \cdot D_{\underline{m}}$ ', to obtain a satisfactory deduction of ' $G_{\underline{m}}$ ' thus:

1(1)	$S_{\underline{m}} \cdot D_{\underline{m}}$	<u>A</u>
2(2)	$\neg L_{\underline{m}}vG_{\underline{m}}$	<u>S</u>
3(3)	$(x)(Sx \cdot Dx \rightarrow Lx)$	
1,2,3(4)	$L_{\underline{m}}$	1,2,3
1,2,3(5)	$G_{\underline{m}}$	2,4

Moreover, there is no suitable set  $\underline{S}'$  such that  $\underline{A} \cdot \underline{S}' \cdot \neg C$ , but not  $\underline{A} \cdot \neg C$  alone, leads by law to  $\neg \underline{S}$ ; for  $\underline{A} \cdot \neg C$  by itself leads by law to  $\neg \underline{S}$  as follows:

1(1)	$S_{\underline{m}} \cdot D_{\underline{m}}$	<u>A</u>
2(2)	$\neg G_{\underline{m}}$	<u><math>\neg C</math></u>
3(3)	$(x)(Sx \cdot Dx \rightarrow Lx)$	
1,3(4)	$L_{\underline{m}}$	1,3
1,2,3(5)	$L_{\underline{m}} \cdot \neg G_{\underline{m}}$	2,4
1,2,3(6)	$\neg(\neg L_{\underline{m}}vG_{\underline{m}})$	5

It seems the criterion still requires further revision. Narrowing its scope, confining our consideration to causal



conditionals, did not rid us of serious difficulties; however, it may have served to make them more clear.

Note that in the model case where  $\underline{A} \cdot \underline{S}$  entails or leads by non-logical law to  $\underline{C}$ , in such cases  $\underline{S} \cdot \underline{-C}$  entails or leads by non-logical law to  $\underline{-A}$ . The minimal set suitable for  $\underline{S}$  will be such that  $\underline{-A}$  is the only significant consequence gained by conjoining  $\underline{S}$  and  $\underline{-C}$ . One might say that  $\underline{S}$  is overloaded (or perhaps "loaded") if  $\underline{S} \cdot \underline{-C}$  has a non-trivial consequence other than  $\underline{-A}$ . To put the point more precisely, let us introduce the term 'material consequent.'  $Z$  is a material consequence of  $X$  and  $Y$  if and only if:

- (1)  $X \cdot Y \vdash Z$
- (2)  $\neg(X \vdash Z)$  [ $X$  does not entail  $Z$ ]
- (3)  $\neg(Y \vdash Z)$
- (4)  $\neg(Z \vdash X \cdot Y)$
- (5) if  $X \vdash W$  but  $\neg(W \vdash X)$ , and  $Y \vdash U$ , but  $\neg(U \vdash Y)$ , then  $\neg(Z \vdash W \cdot U)$

For example, where  $X$  is  $P \cdot Q$ , and  $Y$  is  $R \cdot S$ , none of the following consequences of  $X$  and  $Y$  are material consequences of  $X$  and  $Y$ :

- |   |     |
|---|-----|
| $P, Q, P \cdot Q$                                 | (2) |
| $R, S, R \cdot S$                                 | (3) |
| $(P \cdot Q), (R \cdot S)$                        | (4) |
| $Q \cdot R, P \cdot S$                            | (5) |
| $((P \cdot Q) \vee M) \cdot ((R \cdot S) \vee N)$ | (5) |

With this definition, we can now say that the appropriate set  $\underline{S}$  of true sentences  $(S_1, \dots, S_n)$  is such that:

- (VII) Every material consequence of  $S_1$  and  $-C$  entails or is entailed by  $-A$ .

This amendment is sufficient for the rejection of (i) and (iii), the facetious, but seriously troublesome conditionals, concerned with the social propriety of  $\underline{m}$ 's response to scratching. The crucial sentence in  $\underline{S}$ , ' $-L_{\underline{m}} \vee G_{\underline{m}}$ ', for both (i) and (iii), was such that  $\underline{S} \cdot -C$ ,

$$(-L_{\underline{m}} \vee G_{\underline{m}}) \cdot -G_{\underline{m}}$$

had a material consequence, ' $-L_{\underline{m}}$ ', not entailing or entailed by  $-A$ , that is, by ' $-S_{\underline{m}}$ ', or by ' $-(S_{\underline{m}} \cdot D_{\underline{m}})$ '. Hence, the unacceptable conditionals are properly proscribed by the rule with this new regulation limiting the material consequences of  $\underline{S}$  and  $-C$ .

This amendment has another asset which strongly recommends it; this amendment can also be used to exclude from  $\underline{S}$  the troublesome sentence  $(-A \cdot P) \vee C$  that almost caused us to adopt (V) and abandon the search for a criterion that would cover conditionals not assuming causal laws. The sentence  $(-A \cdot P) \vee C$  is now restricted from  $\underline{S}$  without the use of (V) since  $\underline{S} \cdot -C$  as such would have  $\underline{P}$  as a material consequence. Thus, the amendment prompted by a difficulty arising from an idiosyncrasy of conditionals affirming

a causal connection, turns out to provide a solution for the most irksome problem uncovered in attempting to formulate a criterion adequate for conditionals not affirming a causal connection.

## CHAPTER IX

Having eliminated the need for amendments (I), (II), and (V), the proposed rule can be stated in sum as follows: a conditional "If 'A' were true, then 'C' would be true" (where 'A' is false or contrary-to-fact) is true if and only if there is some general principle or law L and a set S of true sentences  $(S_1, \dots, S_n)$  such that:

- |     |   |       |
|-----|---|-------|
| (1) | $A, S, L \vdash C$  |       |
| (2) | $-(S, L \vdash C)$  | (IV)  |
| (3) | $-(S, L \vdash -A)$   | (VI)  |
| (4) | $-(-A, -C \vdash S_i)$  | (III) |
| (5) | Every material consequence of $S_i$ and $-C$ entails or is entailed by $-A$ . | (VII) |

The conditions are somewhat involved and possibly redundant. This is most likely the result of employing an unintuitive ad hoc method of discovery. Perhaps a more stream-lined and cogent formulation could have been effected by indulging in extended reflection on the general traits of the real world, preserved and altered in our mental vision of the feigned world of the contrary-to-fact antecedent. But this method of discovery has severe limitations. Complicated difficulties not guessed in our preanalytical musings inevitably arise. Nevertheless, such reflection could still be used to argue for and against the reasonableness and desirability of instituting each of the various

amendments. But there are two important questions which must be answered first:

- a) Is the rule too strong? Does it reject some conditionals that would be perfectly acceptable on other grounds?
- b) Is the rule adequate? Are there still some unacceptable conditionals that are not rejected by the rule?

We have already given considerable attention to (b); however, it still deserves some discussion, especially with respect to Goodman's problem of cotenability. But first in reply to (a), let us consider some of the more likely sources of error in this direction. For example, acceptable conditionals suggesting where Jones would be if he were in Carolina, e.g.

- (i) If Jones is in Carolina, but not in South Carolina, he is in North Carolina.

The antecedent of (i) coupled with the true sentence for S:

- (ii) North Carolina plus South Carolina is identical with Carolina

leads by virtue of the non-logical general principle:

- (iii)  $(w)(x)(y)(z)(\text{if}(w \text{ plus } x)=y, \text{ then } z \text{ is in } y \text{ only if } z \text{ is in } w \text{ or } z \text{ is in } x)$

to its consequent, 'Jones is in North Carolina'. S as such now easily satisfies each of the conditions of the proposed criterion; hence, our rule is at last not so demanding

as to reject the more acceptable conditionals concerned with Jones' probable whereabouts in Carolina.

The unacceptable conditionals, speculating on where Jones would be if he were in Carolina, are adequately outlawed by the requirement that there be no sentence  $S_i$  in  $\underline{S}$  such that  $S_i$  is entailed by  $\underline{-A.-C}$ . For example, in the case of the conditional:

- (iv) If Jones were in Carolina, he would be in North Carolina

the crucial sentence for  $\underline{S}$  was:

- (v) Either Jones is in North Carolina, or Jones is not both in Carolina and in a state north of South Carolina

This is now excluded, as it is entailed by  $\underline{-A.-C}$ , i.e., by

- (vi) Jones is not in Carolina; and Jones is not in North Carolina,

Even if it is insisted that the exclusive sense of 'or' is the one in question here, the disqualification of (v) is maintained, since  $\underline{-A.-C}$ , entails the denial of the first disjunct of (v) and the affirmation of the second.

How, it should now be asked, do those conditionals for which  $\underline{A.S}$  is presumed to entail  $\underline{C}$  fare on the proposed rule? Consider for example one claiming of ball-point pen  $b$ :

- (vii) If  $\underline{b}$  does not write, some ball-point pen does not write.

The appropriate choice for S would naturally be the sentence 'b is a ball-point pen.' This would plainly satisfy condition that -A·-C not entail S since from -A·-C:

(viii) b writes. Every ball-point pen writes.

we would not infer S, i.e. that b is a ball-point pen except on pain of committing the fallacy of affirming the consequent. Of course the conjunction of S and -C:

(ix) b is a ball-point pen. Every ball-point pen writes.

has the material consequence that b writes, but this is entailed by -A, i.e., by 'Its not the case that b does not write.' Thus, (vi) satisfies the stipulation in (5). Conditions (1)-(3) are obviously met, hence (vii) is properly accepted by our rule.

For a stiff test, let us see how the proposed criterion handles the familiar case Goodman uses to illustrate the difficulty over cotenability, the one where for a given match m we would affirm:

(x) If match m had been scratched, it would have lighted.

but deny:

(xi) If match m had been scratched, it would not have been dry.

Our criterion must be flexible enough to accept (x) but

rigid enough to reject (xi). For convenience in testing the criterion in its treatment of these conditionals, let the symbols:

$S_x, L_x, D_x, W_x,$  and  $O_x$

correspond respectively to the relevant open sentences:

Match  $x$  is scratched,  $x$  lights,  $x$  is dry,  $x$  is well made, and  $x$  is in sufficient oxygen.

In the case of (x) the natural cast for  $\underline{S}$  will include ' $D_m$ ', ' $W_m$ ', and ' $O_m$ '. These sentences are presumed true in the familiar case we are supposing. No one of them is such that it is entailed by the conjunction of  $\underline{-A}$  and  $\underline{-C}$ , i.e. by ' $\underline{-S_m} \cdot \underline{-L_m}$ .' Note further that  $\underline{S} \cdot \underline{-C}$ , ' $(D_m \cdot W_m \cdot O_m) \cdot \underline{-L_m}$ ', does not even have  $\underline{-A}$ , ' $\underline{-S_m}$ ', as a material consequence. And of course  $\underline{A} \cdot \underline{S}$ , but not  $\underline{S}$  alone, leads by law to  $\underline{C}$ , to wit:

- |          |  |       |
|----------|--|-------|
| 1(1)     | $S_m$  |       |
| 2(2)     | $D_m \cdot W_m \cdot O_m$                                |       |
| 3(3)     | $(x)(S_x \cdot D_x \cdot W_x \cdot O_x \rightarrow L_x)$ |       |
| 3(4)     | $S_m \cdot D_m \cdot W_m \cdot O_m \rightarrow L_m$      | 3     |
| 1,2,3(5) | $L_m$  | 1,2,4 |

Thus, in the familiar case where we would affirm (x), the proposed criterion properly accepts the conditional as true.

The unwanted conditional (xi), however is not rejected.



The requisite set  $\underline{S}$ , consisting of the sentences,

$-L_m$ ,  $W_m$ , and  $O_m$

satisfies the stipulations in (3)-(5); and  $\underline{A \cdot S}$ , but not  $\underline{S}$  alone leads by law to  $\underline{C}$ , thus:

- 1(1)  $S_m$   
 2(2)  $-L_m \cdot W_m \cdot O_m$   
 3(3)  $(x)(S_x \cdot W_x \cdot O_x \cdot \neg L_x \rightarrow D_x)$   
 1,2,3(4)  $-D_m$  1,2,3

Unfortunately, while the criterion is adequate for the problem of cotenability as it arises in conditionals where a causal sequence is involved, e.g. in:

If match  $\underline{m}$  had been scratched, it would have giggled.

the criterion is obviously not sufficient as it stands for conditionals such as (xi). Conditionals like (xi), the ones Goodman uses to illustrate the problem of cotenability, e.g. the conditional in (xi) and:

- (xii) If the temperature of bolt  $\underline{b}$  had been 650 degrees at  $\underline{t}$ , then it would not have been iron at  $\underline{t}$ .

exhibit what we shall call an inferential sequence, as opposed to the causal sequence expressed by "If  $\underline{m}$  were scratched, it would light" and "If  $\underline{b}$  were heated, it would expand." We might say that the former affirm the validity

of an inference while the latter affirm the inevitability of a certain sequence of events.

While it is true that the proposed criterion is only designed to cover the problem of cotenability as it is found in conditionals affirming a causal connection, it is still adequate for conditionals of inference, like "If bolt b did not expand, then it was not heated" if they have a contrapositive that expresses a causal sequence, e.g. "If b was heated, then it expanded." Moreover, we can define cotenability for the remaining inferential conditionals in terms of those conditionals affirming a causal sequence. Note for example, how this would take care of the troublesome counterfactual:

(xi) If match m had been scratched, it would not have been dry.

that Goodman first finds to exhibit the problem of cotenability. That the true statement, 'm did not light' is not cotenable with the antecedent of this conditional, could be determined by the warrant under our criterion for the causal conditional:

If match m had been scratched, it would have lighted.

Thus, we can construct a general rule for the conditionals of inference by incorporating a clause excluding statements from S which are not cotenable with A. Circularity

can then be avoided by thus defining cotenability in terms of those conditionals asserting a causal connection since it was not necessary to institute an explicit restriction for  $\underline{S}$  on the statements not cotenable with the antecedents of conditionals in this form.

The normal run of conditionals will of course include only a) those in which  $\underline{C}$  follows from  $\underline{A \cdot S}$  by logic alone, b) those in which a causal connection is affirmed, and c) those having a contrapositive in which a causal connection is affirmed. For all these, the unamended criterion is adequate. Conditionals like (xi) are improbable in practice, but can, as we have seen, be handled by an amendment requiring the cotenability of  $\underline{A}$  and  $\underline{S}$ , where cotenability is defined in terms of the more usual sort of conditionals.

There is still the problem of defining the natural or non-logical laws which we invoke in support of conditionals affirming causal connections. But this is also a major problem in a number of other popular projects, e.g. in the attempt to construct an adequate model for deductive-nomological explanations.

There may be some worth in remarking that many of the deductive forms that prompted new amendments to our rule can be recast to smoke out similar problems in deductive-nomological models. For example, where 'L' stands for the general covering law, 'E' for the explanandum, and 'P' for any statement about a particular fact, we can see that

not much labor or material is required to construct "explanations" when we use as our blueprint the relettered version of a familiar formula:

(1) L

(2) (-L·P) v E

E

All such "explanations" fulfill the four conditions Hempel and Oppenheim list for a suitable explanation: the explanans must 1) be true, 2) have empirical content, 3) logically entail the explanandum, and 4) contain general laws required for the derivation of the explanandum.<sup>1</sup> To preclude "explanations" contrived under this formula, we might draw on our experience with conditionals to recommend a fifth condition reminiscent of one from our own rule, e.g. that no two premises  $C_i$ ,  $C_j$  of the explanans have a material consequence not entailing or entailed by  $C_i$ , or by  $C_j$ .

Thus, for this and other problems a map of the twisted road we have traveled might provide guidance for the formulation of an adequate definition of deductive-nomological explanations. Also our difficulty with conditionals of inference is paralleled by a problem with the laws covering such inferences when used in deductive-nomological explanations.

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<sup>1</sup>C. G. Hempel and P. Oppenheim, "Studies in the Logic of Explanation," Philosophy of Science, Vol. 15, 1948, pp 135-75.

As a more immediate profit from our venture with conditionals, we now have the means to formulate one of the necessary conditions for the truth of elliptical explanations of the form 'E because C', (where 'E' describes an event to be explained and 'C' refers to some antecedent or concomitant event or state of affairs), for example:

Willoby died because he ate the fish.

Locutions of this form may be thought of as deductive-nomological explanations stated elliptically, that is, omitting mention of relevant assumptions presupposed by the explanation, but taken for granted in the context of utterance. We can say that an elliptical explanation 'E because C' (where 'E' and 'C' are both fulfilled) is true only if the conditional 'If C, then E' is true by our definition. This obviously wouldn't do as a sufficient condition since we count all conditionals of the form 'If E, then E' as true. Perhaps there are counterexamples to this proposal due to peculiarities in the logic of 'because' and the variations in what will and what will not satisfy us as an explanation. But the appropriate revisions should not involve near the labor that the task would entail without the support of an analysis for 'if...then.'

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