A Superconducting Magnetic Gear.

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Abstract

A comparison is made between a magnetic gear using permanent magnets and superconductors. The objective is to see if there are any fundamental reasons why superconducting magnets should not provide higher power densities than permanent magnets. The gear is based on the variable permeability design of Attilah and Howe [1] in which a ring of permanent magnets surrounding a ring of permeable pole pieces with a different spacing gives an internal field component at the beat frequency. Superconductors can provide much larger fields and forces but will saturate the pole pieces. However the gear mechanism still operates, but in a different way. The magnetisation of the pole pieces is now constant but rotates with angle at the beat frequency. The result is a cylindrical Halbach array which produces an internal field with the same symmetry as in the linear regime, but has an analytic solution. In this paper a typical gear system is analysed with finite elements using FlexPDE. It is shown that the gear can work well into the saturation regime and that the Halbach array gives a good approximation to the results. Replacing the permanent magnets with superconducting tapes can give large increases in torque density, and for something like a wind turbine a combined gear and generator is possible. However there are major practical problems. Perhaps the most fundamental is the large high frequency field which is inevitably present and which will cause AC losses. Also large magnetic fields are required, with all the practical problems of high field superconducting magnets in rotating machines. Nevertheless there are ways of mitigating these difficulties and it seems worthwhile to explore the possibilities of this technology further.

1.Introduction.

Magnetic gears are a very attractive concept as they need no lubrication, do not wear, and there is no contact between the moving parts. The simple idea of using poles on the surface of a magnetised cylinder is an old one but is not very practical. Even apart from the low coercive force of permanent magnets until the advent of NdFeB, only a small number of the magnets are in use at any one time so the torque density is inevitably low. However in 2001 Attilah and Howe described a most ingenious device which, when combined with Neodymium magnets, is competitive with mechanical gears [1] . Many developments have been described since and they are commercially available from a company called Magnomatics [2]. All the applications use NdFeB permanent magnets. The object of this paper is to explore the possibility of using the high current densities in superconductors to increase the torque density of a magnetic gear

2 Principle of Operation

The principle can be illustrated with the simplified linear version in fig.1. This shows part of a ring of alternating permanent magnets with a layer of soft iron on one side and a series of pole pieces on the other.

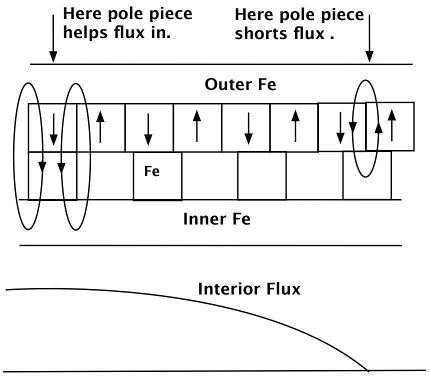

Fig.1. A simple linear model. The 'interior flux' shows the magnitude of the flux penetrating to the ends of the pole pieces.

The pole pieces have a slightly different spacing from the magnets so that at some points they coincide with the magnets, at others they are out of phase. If a pole piece coincides with a magnet it helps the flux to cross to the lower side where there is another iron layer, as in the leftmost magnet of fig.1 where the interior flux is a maximum. If the pole piece is across the joint between magnets, as is the rightmost, it shorts the flux and none crosses the layer to emerge from the pole pieces. The flux below the pole pieces therefore goes from a maximum to a minimum as the phase of the magnets and pole pieces changes. The period of the resulting flux variation is the difference of the periods of the magnets and pole pieces. It is essentially the beat frequency, or Moiré fringe pattern, of the two arrays. If we put N magnets round a cylinder and $N \pm p$ pole pieces inside we get an interior field with p poles. If we insert a rotor with p poles it will rotate at a rate $\pm N/p$. We have a magnetic gear, which can be a reversing gear if there are more pole pieces than magnet pairs. We can get other ratios by keeping the magnets stationary and rotating the poles, just as in a bicycle three speed planetary gear.

2.1 Analytic Solution, Linear regime.

If the magnets are in a thin layer on the surface of a cylinder we can get analytic expressions by modelling the magnets with a sinusoidal current sheet backed by infinite permeability on the outside, a sinusoidal variation of permeability μ inside it and a permeable core of infinite permeability. The total air-gap is g and the radius R. We replace the permanent outer magnets with a sinusoidal rotating current sheet j_o with n magnet pairs and angular velocity ω. The sheet current density is ;-

$$j = j_o \cos\left[n(\theta - \omega t)\right] \quad (1.1)$$

We replace the m discrete pole pieces with a material whose relative permeability varies sinusoidally from 1 to μ_{fe}+1.

$$\mu_r = 1 + 0.5\mu_{fe}\left[1 + \cos(m\theta)\right] \quad (1.2)$$

From Ampères theorem, integrating H across the gap,

$$g\delta H = jR\delta\theta = j_o R\delta\theta \cos\left[n(\theta - \omega t)\right] \quad (1.3)$$

$$H = \frac{j_o R}{ng}\sin\left[n(\theta - \omega t)\right] \quad (1.4)$$

$$B = \mu_r \mu_o H = \frac{\mu_o j_o R}{ng}\left[1 + 0.5\mu_r(1 + \cos(m\theta))\right]\sin\left[n(\theta - \omega t)\right] \quad (1.5)$$

Hence

$$B=\frac{\mu_o j_o R}{ng}\left[\sin\left[n(\theta-\omega t)\right]+0.5\mu_r\sin\left[n(\theta-\omega t)\right]+0.5\mu_r\cos(m\theta)\sin\left[n(\theta-\omega t)\right]\right]\quad(1.6)$$

The first term is small, the second term is at a high frequency and decays rapidly with distance. The last term can be written

$$B=\frac{\mu_r\mu_o j_o R}{4ng}\left[\sin\left[(n+m)\theta-n\omega t\right]+\sin\left[(n-m)\theta-n\omega t\right]\right]\quad(1.7)$$

Again the first term is high frequency but the last term is at the beat frequency of the two arrays and this is the one of interest.

The last term has m-n pole pairs and it rotates at an angular frequency of $n\omega/(m-n)$. If we insert a rotor with m-n pole pairs it will rotate with the outer ring but the speed is increased by a factor $n/(m-n)$ i.e. the gear ratio is the ratio of the number of poles on the outer ring to that on the central one. If n is greater than m the rotor rotates in the opposite direction. As in mechanical epicyclic gears, other ratios can be obtained by keeping other rings stationary.

One feature of the system brought out by this simple treatment is that there are inevitable large high frequency components to the field on top of the one we want. These average out in the torque, but may cause unacceptable AC losses in a superconductor.

2.2. The Model Magnetic System

This is certainly very beautiful, but in a cylindrical machine there is clearly a lot of attenuation of the field of the magnets before it reaches the rotor. Before considering a superconducting gear we analyse a typical permanent magnet gear for comparison. A finite element model using FlexPDE, and with 1.5T permanent magnets, shows an applied field at the rotor surface of 70 mT, which is comparable to what can be produced by a conventional copper stator, so useful torques are possible. The following are the dimensions and materials used and illustrated in figs 2 and 3.

Figure 1. shows the system. There are 30 iron pole pieces and 26 magnet pairs so we expect 4 magnet pairs in the rotor space. This can be seen in figs.2 and 3.

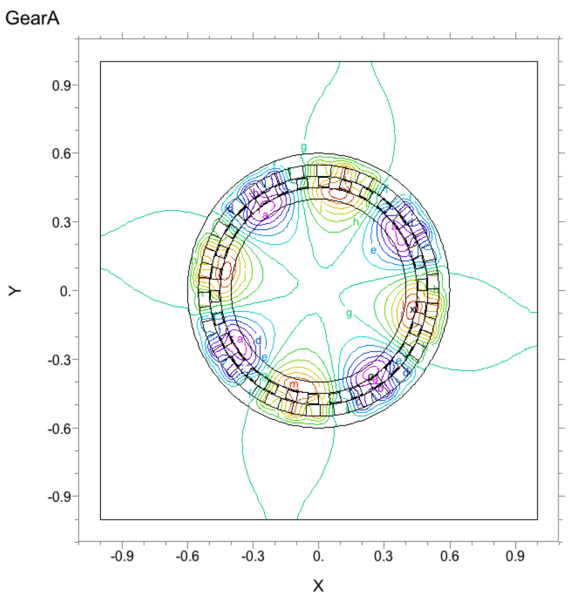

Fig 2. An FE Plot of the Flux Lines.
This shows concentric rings of, from the outside, a ferromagnetic shield, permanent magnets, iron pole-pieces, rotor magnets, and rotor iron

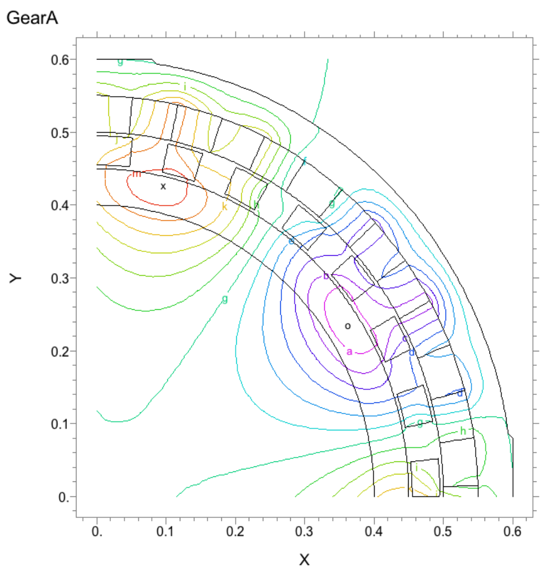

Fig.3. The first quadrant of fig.2.

Figures 2 and 3 show a finite element presentation of the flux lines of the system using FlexPDE. Figure 2 is the whole system, figure 3 the first quadrant. The four layers are, from the outside, a layer of iron, the outer permanent magnets, the permeable pole pieces, and the inner magnets on what will be called the rotor. Inside this is an iron cylinder. Solutions were obtained both with and without the outer and inner iron.

2.2.1. Geometry and Materials

Geometry

No of magnet pole-pairs	26
No of Pole pieces	30
Outer radius	0.6m
Outer iron thickness	5 cm
Outer permanent magnet thickness	5cm
Pole pieces thickness	4cm
Rotor magnet thickness	5cm
Air-gaps	3mm.

Materials

Magnets	NdFeB,	Magnetisation	1.5 Tesla
Iron	Hiperco-50	Saturation Magnetisation	2.5 Tesla.

We consider an air cored machine and also one with iron rotor and outer ring. As well as the torques on the outer ring and rotor we quote the shear stress in the outer air-gap. This is a common measure of the torque density [3]. It tends to be in the range 100-120kPa. and for geometrically similar systems is independent of the scale. (This is not true of superconducting systems. Superconductors cannot compete with permanent magnets in machines at small sizes

2.2.2. Results

Torques are divided by the volume of the whole system to give the torque per unit volume.

a) With Iron

Outer ring Torque	286 kPa/m^3
Rotor Torque	44.4 kPa/m^3.
Gap Stress	33.8 kPa
Ratio of Torques	6.44

b) Air Cored

Outer ring Torque	152 kPa/m^3
Rotor Torque	23.3 kPa/m^3.
Gap Stress	17.8 kPa
Ratio of Torques	6.52

The effect changing of μ in a linear system was also tried. For μ =1000 the stress was 33.8 kPa and for μ=30 it was 29.6kPa.

We can summarise these results as follows.
1) The field in the centre rotates at the expected speed, i.e. a ratio of 6.5, and as a further check of the validity of the solution the ratio of the torques on the outer and inner rings is also 6.5. This is necessary for conservation of energy, and was found to hold well for all subsequent calculations.
2) The gap stress is significantly lower than in [3], but in the right ball-park. However no attempt has been made to optimise the dimensions, of which there are many, the objective here is to compare permanent magnets and superconductors in similar systems.
3) The pole pieces have a large demagnetising factor and so are difficult to magnetise. The reluctance is dominated by the air-gaps, so the results are not sensitive to the permeability of the iron. The stress is not significantly different for permeabilities between 30 and infinity. Adding iron increases torques and fields by a factor of about two, which is in line with the result that putting iron inside, or outside, an air cored cylindrical coil halves the reluctance and so doubles the fields.

3. Saturation Regime.

If we are to make the most of superconductors we need to use high fields which will saturate the pole pieces. The linear treatment above suggests that the reduction in relative permeability will prevent the use of the gear in this regime, but this is not the case. Since a superconducting rotor introduces other complications this regime will be explored by modelling permanent magnets with an unrealistically high magnetisation.

3.1 Large Outer magnetisation

To determine the effects of saturation on the central field the magnetisation of the outer magnets was increased while the rotor magnetisation was set at a low level (0.5T) so that its field did not influence the pole pieces. Also for clarity the outer iron and rotor iron were removed.
As the pole pieces saturate, those immediately opposite a magnet magnetise radially in the direction of the magnetisation of the adjacent magnet (fig.4.). Those half way between magnets will still saturate, but the magnetisation is mainly azimuthal rather than radial.

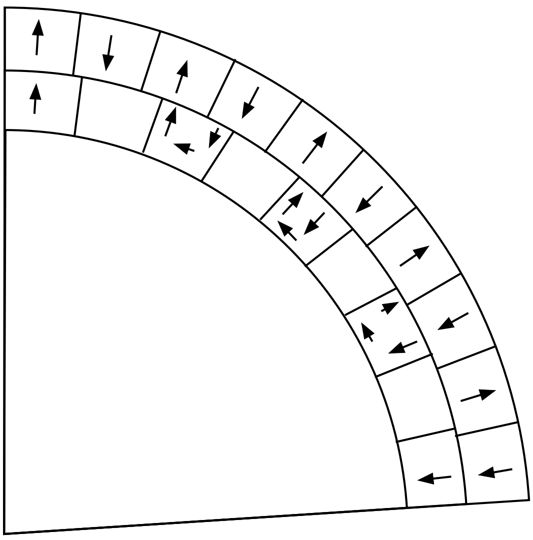

Fig.4. The pole magnetisation if the outer magnets saturate them.

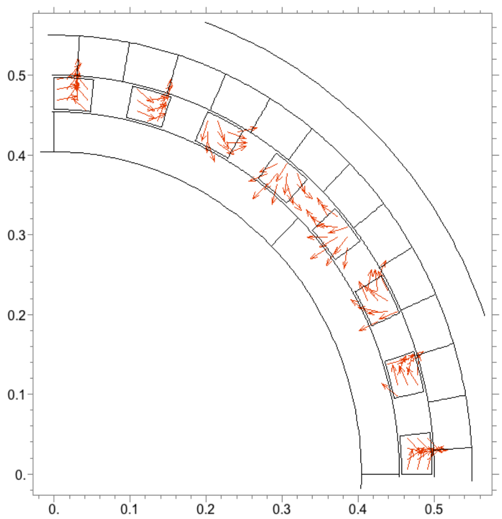

Fig.5. An FE vector plot of the pole magnetisation

This shown in fig.4. The FE solution is in fig 5 and shows the same pattern. In other words the magnetisation is constant, but rotates with the beat frequency. Apart from the gaps, this makes a cylindrical Halbach array which can be analysed analytically, and this is done below.
We expect that initially in the linear region the torque, or stress in the gap, will be proportional to the magnetisation of the pole pieces and hence to that of the outer magnets. When pole pieces saturate they form a cylindrical Halbach array and provide a constant field with the same symmetry as before. Therefore the stress should saturate. This is indeed what happens.

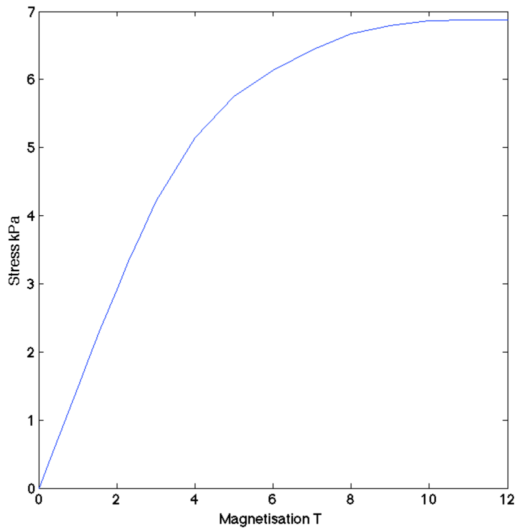

Fig.6. The stress as a function of outer magnetisation 0.5T rotor magnetisation.

Figure 6 shows the mean shear stress in the gap between the pole pieces and the outer magnets as the magnetisation of the outer magnets is increased to 12T. The pole pieces are made of Hyperco-50 with a saturation of 2.5T. However it needs an outer magnetisation of about 7T to saturate these in the geometry above.

3.2 Analytic Solution

A large number of cylindrical Halbach arrays have been analysed in the literature usually for specific purposes. See references [4] and [5] for examples and a review. For the purposes of this paper we take a simple 2D infinitely long array. We assume the magnetisation, M_s, rotates as $p\theta$. In free space the vector potential $A(r,\theta)$ is of the form $A(r,\theta)=r^{\pm p}\sin(p\theta)$ and in the pole piece layer we need to add a term $M_s\,[r/(p-1)]\sin(p\theta)$. Matching boundary conditions in the usual way gives the vector potential in the interior, and also shows that there is no field outside the gear. (In xy coordinates the angle of M varies as $(p+1)\theta$ and B as $(p-1)\theta$. $p=1$ gives a uniform field)
If the outer radius is b and the inner a then

(1.8)

$$A(r,\theta)=M_s r\left[\left(\frac{r}{a}\right)^{p-1}-\left(\frac{r}{b}\right)^{p-1}\right]\sin(p\theta) \qquad (1.9)$$

$$B_\theta = -M_s p\left[\left(\frac{r}{a}\right)^{p-1} - \left(\frac{r}{b}\right)^{p-1}\right]\cos(p\theta) \qquad (1.10)$$

To compare the vector potential with that of the gear we first need to divide the magnetisation by 2 since only half the Halbach magnets are present. Also a closer examination of the FE results shows that the pole piece magnetisation is not uniform in direction. The mean magnetisation is 1.8T rather than the 2.5T of the uniform material. In fig 7 equation (1.9) with these corrections is compared with the finite element solution. The agreement is satisfactory given the approximations involved. Therefore the Halbach array is a suitable model for approximate design purposes in the saturation regime.

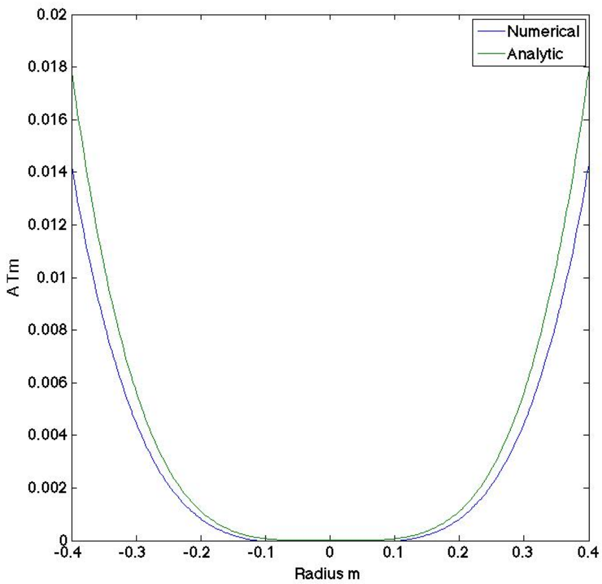

Fig.7. The central vector potential from FE and eq (1.9)

3.2.1 Analytic torque on the rotor.

If we put a rotor in the centre with a radial magnetisation $\boldsymbol{M_r}$ the local torque per unit volume is $\boldsymbol{M_r}\times\boldsymbol{B_q}$ where $\boldsymbol{B_a}$ is the applied field. If the rotor magnets have inner radius a_r and outer radius b_r and the corresponding pole piece radii are a and b, the total torque on the rotor is T where T is given by :-

$$T = \frac{4pM_rM_sb_r^2}{\mu_o(p^2-1)}\left\{\left[\left(\frac{b_r}{a}\right)^{p-1} - \left(\frac{b_r}{b}\right)^{p-1} - \left(\frac{a_r}{b}\right)^{p-1}\left(\frac{a_r}{b}\right)^2\right] + \left(\frac{a_r}{b_r}\right)^2\left(\frac{a_r}{a}\right)^{p-1}\right\} \qquad (1.11)$$

Note that as pointed out above, if divided by the volume , this depends only on the ratios of the dimensions and so is independent of scale.

3.3 Large Rotor and Outer Magnetisation.

If both rotor and outer magnetisation are increased together, the rotor magnetisation does affect the pole pieces, but the same rotation occurs so that the field applied to the rotor is the same, but the torque increases in proportion to the rotor magnetisation. Therefore in the linear regime the stress increases as the square of the magnetisation, but when the pole pieces saturate the stress increases linearly. This is shown in fig.8.

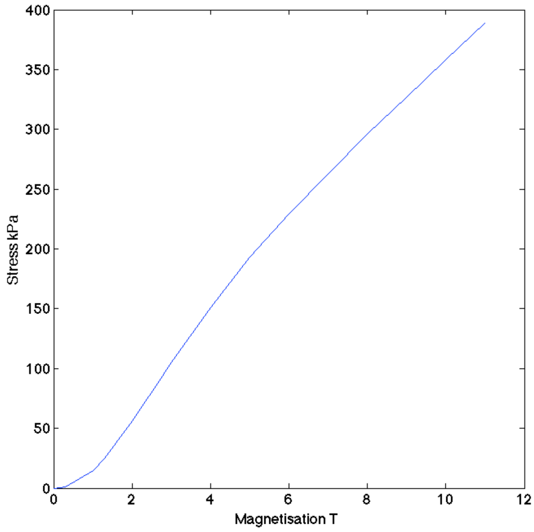

Fig.8. The stress with equal rotor and outer magnetisations.

3.4 Large rotor magnetisation.

Finally we consider what happens if the effect of the rotor magnetisation dominates the pole pieces. In this case the magnetisation just follows that of the rotor and we lose the rotating effect. The torque therefore begins to reduce. This is shown in fig.9. where the outer magnetisation is kept at 1.5T

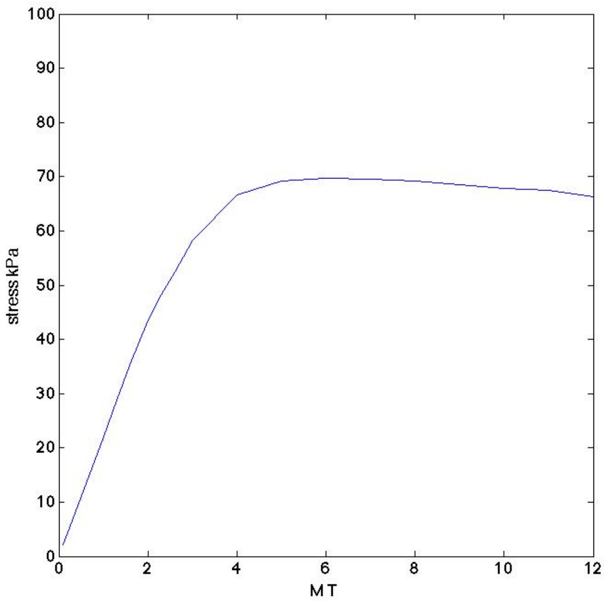

Fig 9. Stress as a function of rotor magnetisation.

4. Superconductors

Since superconductors can produce much higher fields than permanent magnets it should be possible to get much higher torques [6]. However one problem is immediately apparent from (1.7). There are large fluctuating fields of a similar size to the dipole field we want, which will cause significant losses. These might be mitigated by putting the rotor conductors or in slots or by a copper shield. Also, being high frequency, they decay rapidly as the air-gap is increased. Losses will not be considered in this paper. Also the response of a superconducting rotor to the external field of the outer layers is not as simple as merely multiplying the magnetisation by the external field.

The problem was treated in the following way. The permanent magnets in the rotor were replaced by superconducting tapes in the same positions. (The outer magnets are too small to be replaced by superconductors). The torques, and stresses in the outer gap were then calculated as a function of critical current density in the tapes, assumed the same in rotor and outer ring. The Lorentz force on the tapes due to the field of the outer rings was integrated over the rotor magnets to find the torque on the rotor. It is then necessary to take into account the lowering of J_c due to the large magnetic fields. The tapes were assumed to obey the Kim model for the field dependence to obtain the zero field J_c. The tape properties were taken from reference [7] , (Superpower tape), at 30K, so the values of J_c were multiplied by $(1+B/B_o)$ where B was the peak field in the rotor and B_o is 2T. This gives the zero field critical current density which is needed to provide the calculated torque. The maximum engineering current density considered was 2.2×10^{10} A/m^2,

The results are shown in fig.10. It can be seen that there is in principle the possibility of a very large increase in the power density of the magnetic gear by a factor of up to 10. However fig 11 shows the drawback. The peak field in the rotor is 15T so we are bringing in all the complications of a 15T magnet, such as support structures, which will reduce he mean current density by a large factor.

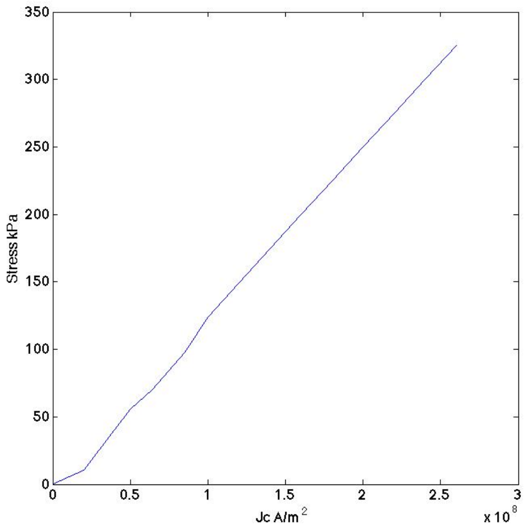

Figure 10. Stress as a function of zero field critical current density

However this is a preliminary comparison and there are many variables that could be optimised. AC losses can be reduced by putting conductors in slots, adding a copper shield and increasing the air-gaps. It is also possible to replace the rotor with the stationary armature of a generator thus avoiding the complication of a separate gear and generator. It therefore seems worthwhile to explore these possibilities further.

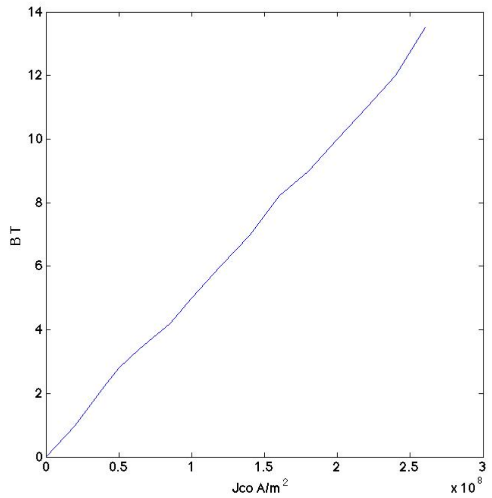

Fig.11. The peak field in the rotor gap as a function of zero field current density

Conclusions

Magnetic gears can work well into the saturation regime so it is possible that superconducting magnets could be used. The cylindrical Halbach array provides a useful model for analysing this system. Replacing the permanent magnets with superconductors can produce very large increases in torque density. In addition the rotor could be replace by the armature of a generator so that we do dot need a separate gear and generator. However the only parameter considered has been the critical current density of the superconductor to see if there are any fundamental reasons to exclude the possibility of using superconductors. No fundamental problem as found, but this limited analysis exposes problems which will need to be solved in a realistic design. A complete design of a superconducting gear and comparison with a conventional system will require an optimisation of dimensions, calculations of AC losses and inclusion of magnet reinforcements as the fields need to be large. This is a large project but appears worth doing.

References

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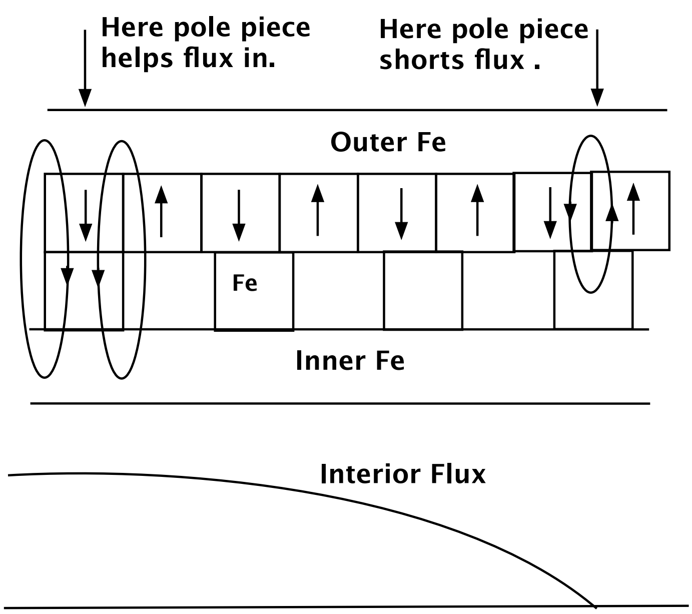
Figures



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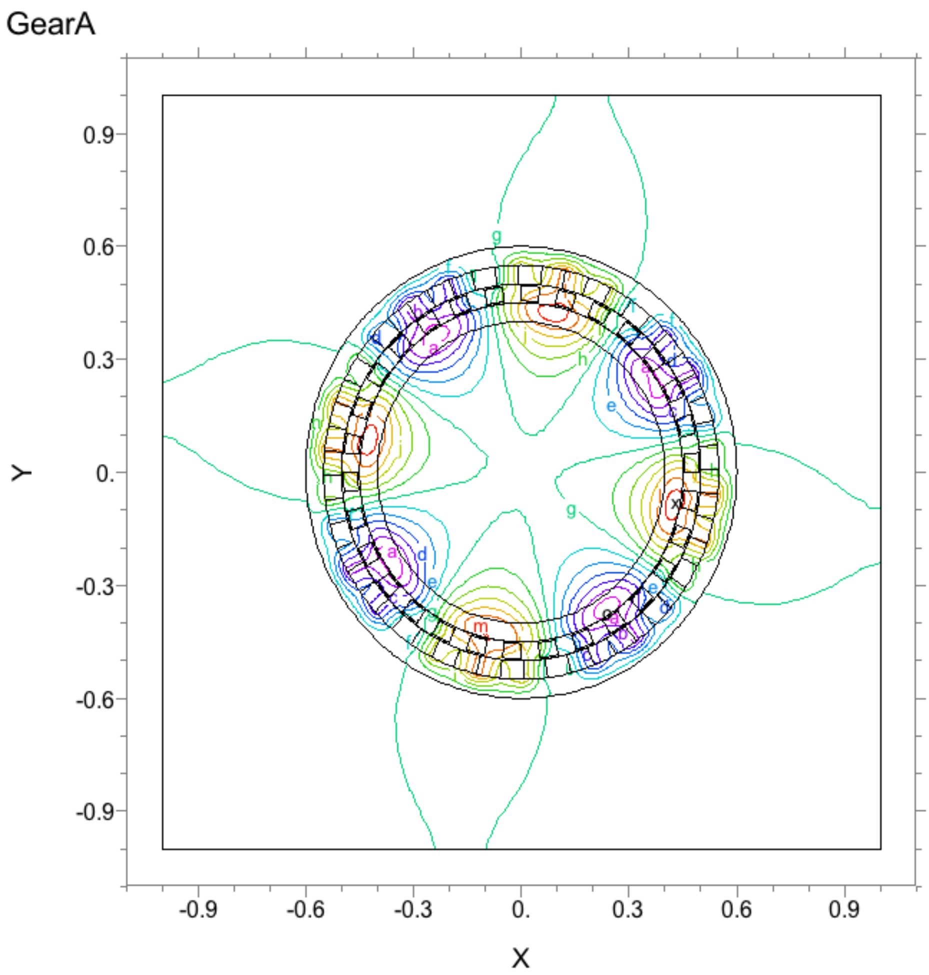

Fig 2. An FE Plot of the Flux Lines.

This shows concentric rings of, from the outside, a ferromagnetic shield, permanent magnets, iron pole-pieces, rotor magnets, and rotor iron

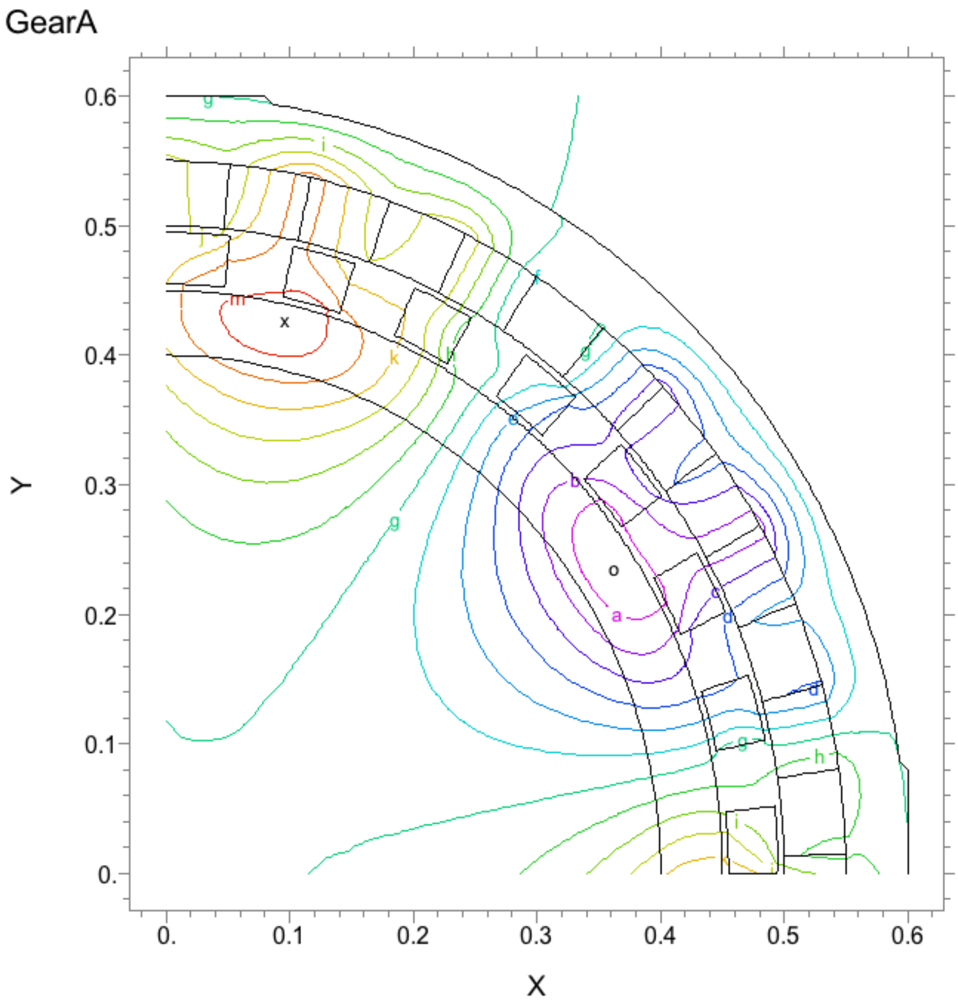

Fig.3. The first quadrant of fig.2

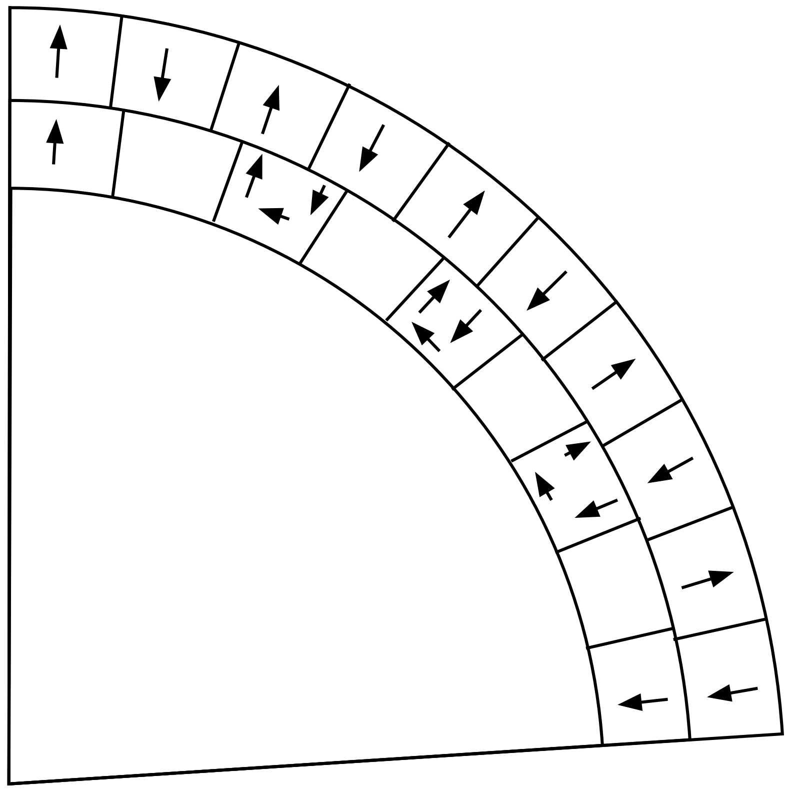

Fig.4. The pole magnetisation the outer magnets saturate them.

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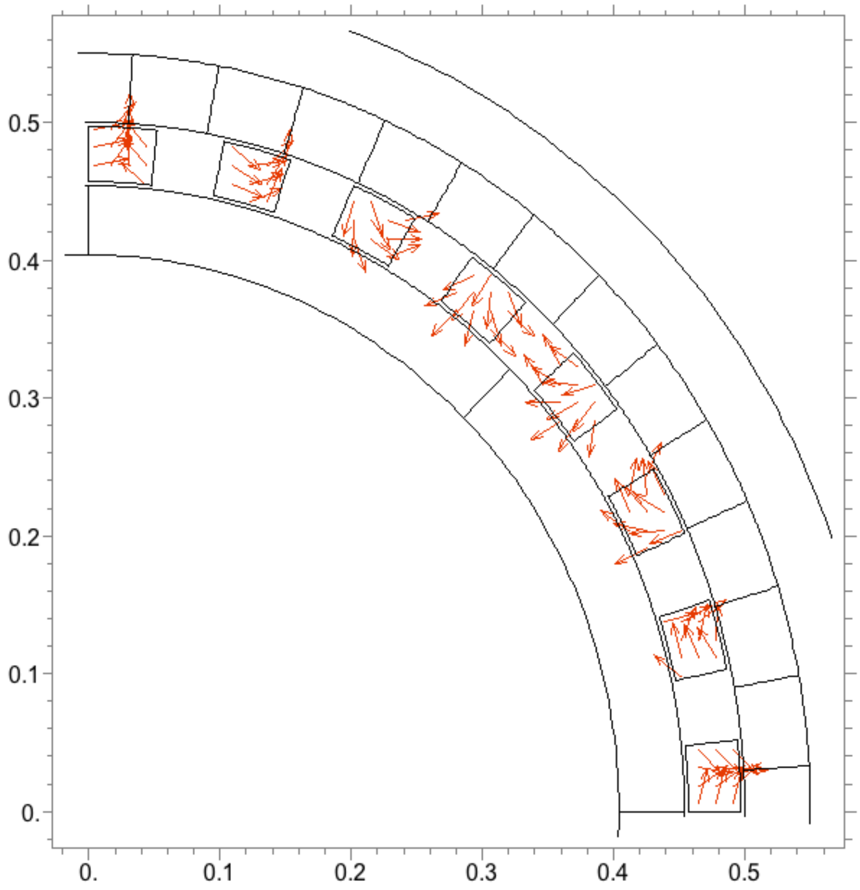


Fig.5. An FE vector plot of the pole magnetisation.

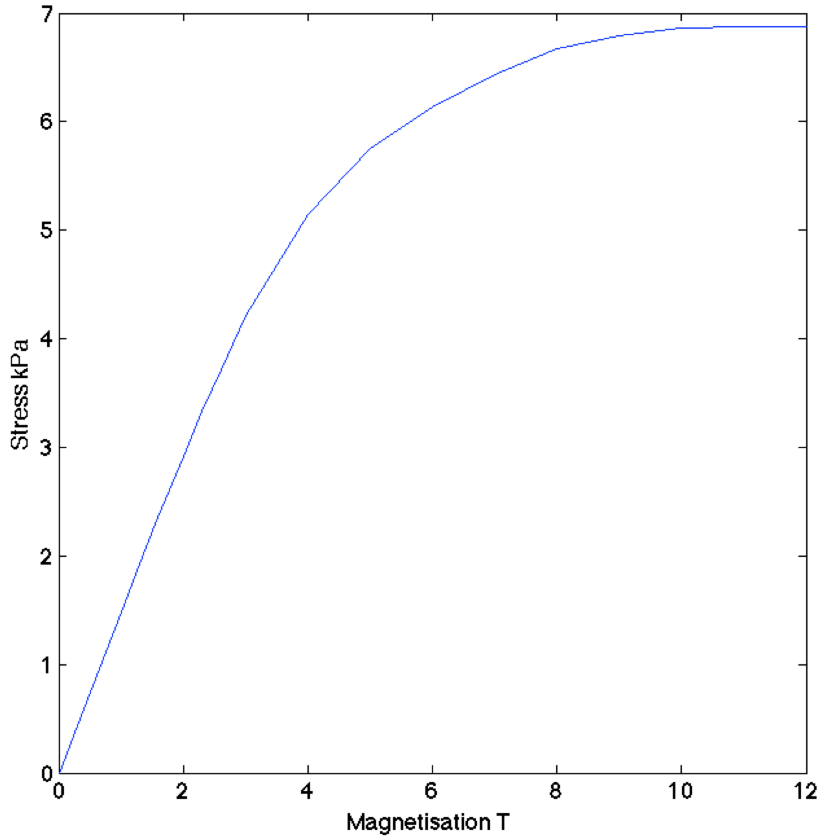
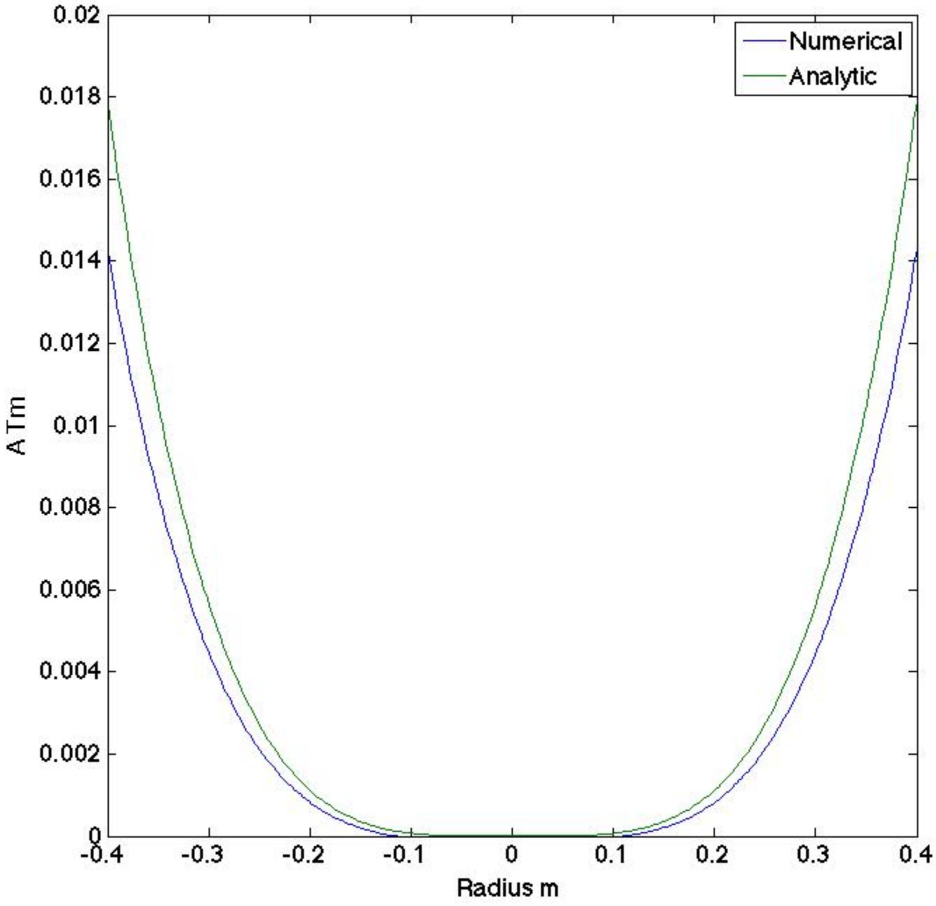


Fig.6. The stress as a function of outer magnetisation 0.5T rotor magnetisation.



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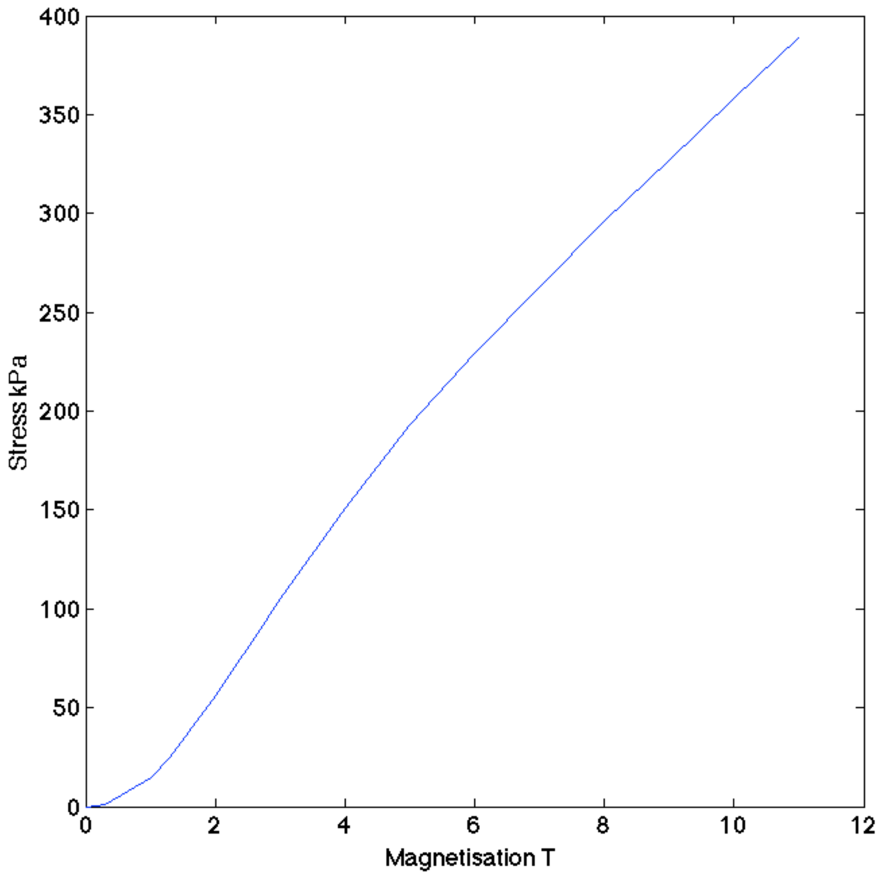

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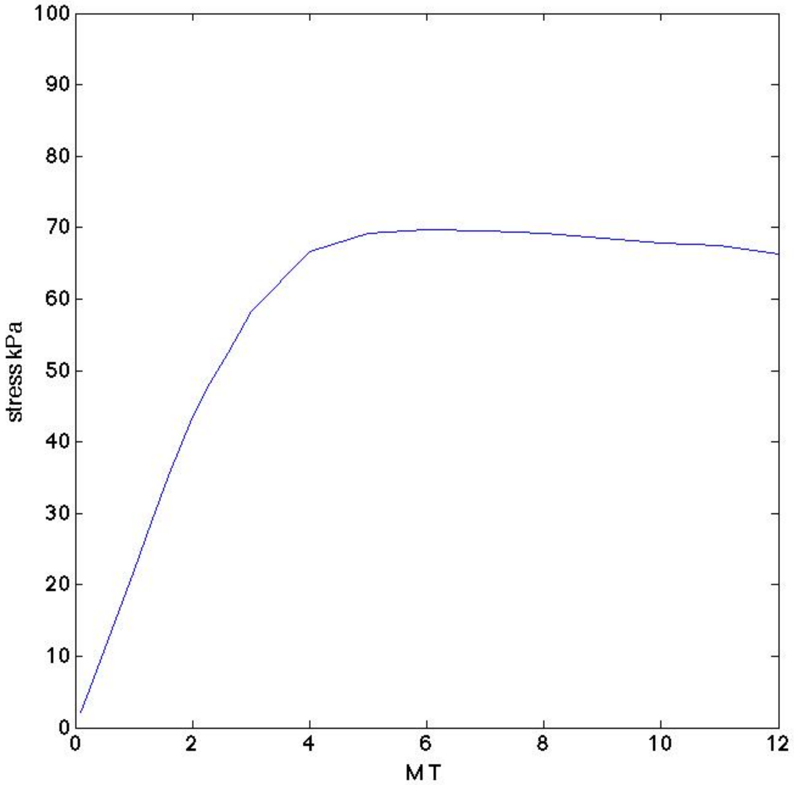

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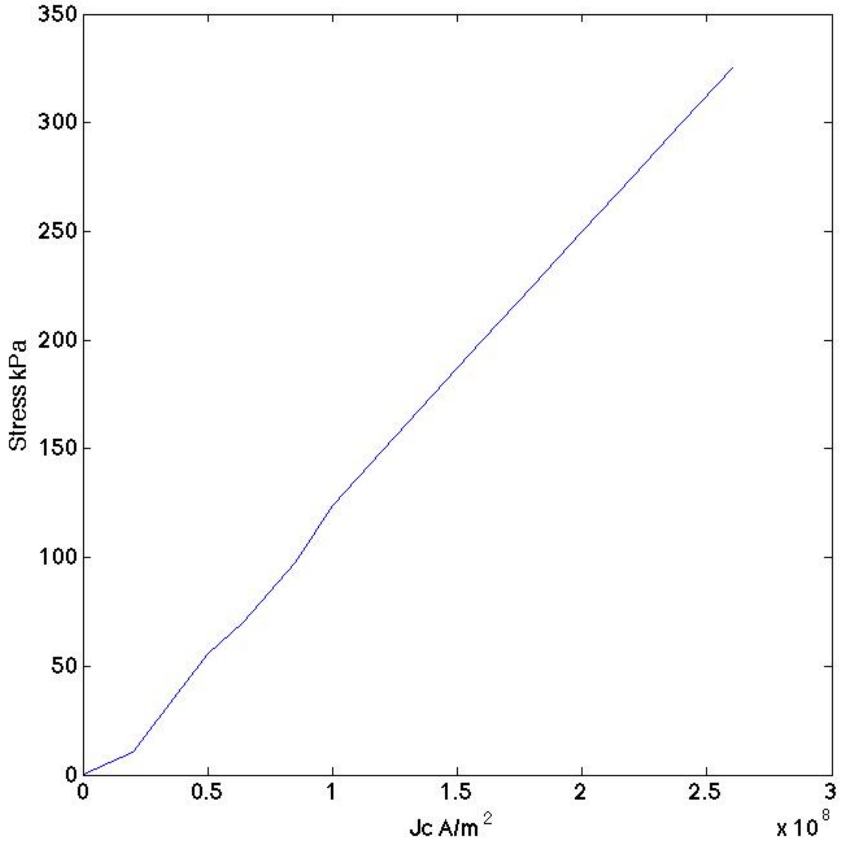

Figure 10. Stress as a function of zero field critical current density