

## Mathematical Modeling for Charlotte-Mecklenburg School Zones

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**ABSTRACT.** The Charlotte-Mecklenburg school district (CMS) is the 18<sup>th</sup> largest district in the United States and Charlotte is one of the fastest growing cities in the United States, leading to several severely overcrowded schools. Therefore, CMS is in desperate need of an effective mathematical model to create school attendance zone plans that maximizes efficiency and equity. This paper presents Voronoi mathematical models to fairly partition the CMS district. The models created balance school socioeconomic demographics, minimizes school overcrowdedness, and reduces inconvenient commutes. These factors are aligned with the CMS school board goals for attendance zone plans and are considered for their impact on student success. This paper provides a unique contribution to the pursuit of equity in the CMS district by constructing mathematical models which have not been seriously considered as a helpful tool in the fight against this complex issue.

### 1. Introduction

Gerrymandering is a practice intended to establish an unfair political advantage for a favored population by manipulating district boundaries (Svec et al., 2007). North Carolina politics has been riddled with gerrymandering since the late-1800s. To this day, gerrymandering is used to manipulate voting power and weaken the voices of voting citizens in North Carolina and other states around the country. For example, in February 2016, the U.S. Supreme Court ruled that North Carolina's Congressional districts were unconstitutional due to racial gerrymandering. After redrawing the districts, the lines were, once again, deemed unconstitutional due to partisan gerrymandering in August 2018 (Brosseau, 2018). The construction of districts that are discontinuous or that lack compactness tend to be in question when it comes to the issue of gerrymandering; however, there are many districts in question that do not possess such obvious physical traits when it comes to gerrymandering.

Unfair partitioning of areas occurs in other institutions as well. In each county across the United States, school board members and other leaders partition the county to designate public school districts (Richards, 2014). Approximately 87% of children attend their assigned school, yet about 60% of school districts across the US are potentially “gerrymandered” (Richards, 2014, 2017). In this context, gerrymandering means purposely excluding certain populations from school zones in favor of others (Richards, 2014). Since the *Brown v. Board of Education* Supreme Court case, some districts have used school boundaries to cripple integration efforts (Richards, 2014). While one-third of districts use school attendance zone plans as an instrument of integration, most school attendance zone plans continue to perpetuate racial and economic inequality by preserving or exacerbating residential segregation (Richards, 2014, 2017).

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*Key words and phrases.* School Attendance Zones; Voronoi Diagrams.

The Charlotte-Mecklenburg School District (CMS) has long been at the focal point of the national school desegregation debate. The Supreme Court ruling of *Swann v. Charlotte-Mecklenburg Board of Education* in 1971, which upheld desegregation busing, made CMS a role model in desegregation efforts (Smith, 2016). Between this ruling and 1990, CMS largely succeeded in overcoming high levels of white-black residential segregation to produce relatively integrated public schools thanks to school busing policies and multiracial attendance zones (Boger and Orfield, 2009). However, in 1999, the Fourth Circuit Court of Appeals ruled that race could not longer be used in school assignment decisions. This terminated the integration busing program in 2001 (Smith, 2016). With the adoption of the new Family Choice Student Assignment Plan in 2001, CMS shifted to a neighborhood-based school assignment system with controlled choice magnet schools (Boger and Orfield, 2009). Since most neighborhoods are economically and racially homogeneous due to residential segregation, the Family Choice Plan has dramatically increased school segregation (Frankenberg, 2013; Smith, 2016; Boger and Orfield, 2009). Nelson et al. (2015) argue that the level of segregation has almost returned to what it was before 1971. In fact, a study done in 2018 by the N.C. Justice Center's Education and Law Project found that more than 55% of CMS students would have to be reassigned in order to reach true racial diversity (Henderson, 2018). Therefore, CMS is in desperate need of an effective mathematical model to create school attendance plans which maximize efficiency and equity, and can be applied both now and as the district grows.

This paper focuses on applying Voronoi-type models to the school attendance zone problem for CMS, a district saturated with racial segregation, income segregation, and overcrowdedness. The models presented, and the current CMS attendance zone plan, are compared based on the percentage of students in poverty, average traveling distance to school, and the number of schools over capacity. This research provides a unique contribution to the pursuit of equity in the district by constructing mathematical models that have not been seriously considered as a helpful tool in the fight against the complex issue of equity in school assignments.

## 2. The Data

The Mecklenburg County government provides public, geospatial data for Mecklenburg County, including the high schools' latitude and longitude measurements and the current attendance zone plan shapefile (Government, 2020). CMS population and poverty data is needed to construct the weighted Voronoi diagrams. The National Historical Geographic Information System (NHGIS) provides summary tables of population and economic data, along with GIS-compatible boundary files, for all levels of US census geography (including states, counties, tracts, and blocks) for the years from 1790 to 2020 (Manson et al., 2019). Charlotte's Open Data Portal is another source of poverty data (Portal, 2019). The 2010 US census provides the most detailed information of Charlotte-Mecklenburg population for each census block broken down by narrow age ranges. A population growth model is calculated using the school populations in 2000, 2010, and 2017 in order to project the number of high school-aged students in each census block in 2019.

The effectiveness of a model is dependent on how well it incorporates average traveling distance, the amount of overcrowdedness, and the percentage of students in poverty. These factors are calculated for all 18 zoned public high schools for every new model and the current model. The

average traveling distance is calculated using the travel distance Mathematica function command between the centroid of each census block/tract and the school. The travel distance command calculates the shortest driving path from the school to the centroid. The traveling distances from each census block/tract inside the school's cell to the school are averaged.

The amount of overcrowdedness or undercrowdedness of each school is calculated as the difference of the student population minus the student capacity. The student population of a high school in each Voronoi model is found by first identifying which census blocks fall inside its Voronoi cell. Since a census block could belong to more than one cell, the location of the centroid of the census block is used to assign the block to one cell in the Voronoi diagram. Then, the student population of a high school is the sum of high school student populations of all the blocks inside the high school's Voronoi cell. To determine the student capacity of each high school, the number of classrooms in the high school is multiplied by 20 (Helms, 2018). The student population data for the current model is from the CMS enrollment data at the beginning of the 2019-2020 school year (Charlotte-Mecklenburg Schools, 2019). The calculated difference of the student population minus the student capacity yields positive value if the school is over capacity and negative value if the school is under capacity.

Poverty data is not collected at the census block level, so this research uses the poverty data from 2010 US census tracts. The same population growth model is used to project the 2019 population in each tract and population in poverty in each tract. According to (The Annie E. Casey Foundation, 2020), 20% of school-age students in North Carolina live in poverty where the poverty rate across all age groups is closer to 11.4% (The United States Census Bureau, 2018). Using this information, the poverty rate in each tract is calculated by dividing the number of people in the tract below the poverty line by the total tract population. Since the poverty rate of children is 1.75 times the general poverty rate in the US, the poverty rate of each tract is multiplied by 1.75.

### **3. Tackling Travel Distance**

Poorly drawn attendance zone lines fail to minimize travel time and costs. Proximity to students' residences considerably affects cost of travel for both the families and the school (Ahmadi, 2006). The extra expense for travel may negatively impact the parents' involvement in their child's education (Williams and Sánchez, 2013). Along with the extra costs, any extra travel time reduces time for homework or other extracurricular activities (Ahmadi, 2006). For these reasons, children's distance from their school is more than just an inconvenience—it impacts children's academic success.

Voronoi diagrams have been used in many fields such as anthropology, crystallography, ecology, and economy, and now we will apply Voronoi diagrams to the Charlotte-Mecklenburg school districting problem (Austin, 2006). The first model created is a Euclidean Voronoi diagram, Figure 3.1. By definition, a Euclidean Voronoi diagram is the "partition of the plane with respect to nodes such that points are in the same region with a node if they are closer to that node than to any other node" (Svec et al., 2007). In this case, the nodes are the schools in the district.

More complex Voronoi diagram models can consider other factors or a combination of other factors such as travel time, school capacity, and socioeconomic balancing. In Wang et al. (2014), the authors created another multifaceted approach that addressed nature barriers and road networks, which construct a more accurate model of traveling time. Constructing a Voronoi model that accounts for natural barriers and road networks like Wang et al. (2014) would maximize transportation efficiency. Ahmadi (2006) used the multiplicatively weighted Voronoi diagram method to construct an attendance area system for public female junior high schools in Rasht, Iran. Unlike basic Voronoi diagrams, weighted Voronoi diagrams can divide up a region with respect to population density, or other factors, as well as distance. Hence, the Euclidean Voronoi model, Taxicab Voronoi model, a road network Voronoi model, and weighted Voronoi diagrams will be applied to this districting problem because they can account for multiple factors that are important to consider for school districting.

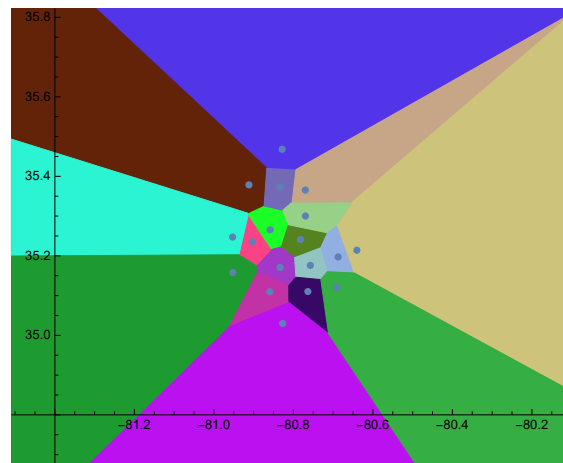


FIGURE 3.1. A Euclidean Voronoi diagram of CMS public high schools.

The second Voronoi model created is based on taxicab distance, Figure 3.2. Taxicab distance is measured differently than Euclidean distance, and the properties of perpendicular bisectors change as a result. Taxicab distance between two points is defined as the sum of the vertical and horizontal distances between the two points. In both taxicab and Euclidean geometry, a perpendicular bisector is defined as the set of points equidistant from two points, but in taxicab geometry a perpendicular bisector is not a straight line; the configuration of a perpendicular bisector is dependent on the slope between the two points.

#### 4. Tackling Overcrowding

Inadequate plans can also lead to overcrowding in some schools while leaving other schools underutilized, and overcrowding can be detrimental to student learning and teacher instruction. When an overcrowded school was compared to an underutilized school, Rivera-Batiz and Marti (1995) reported that the overcrowded school had lower passing rates for both reading exams and math exams (Earthman, 2002). For students and teachers alike, overcrowding raises absence rates and reduces time to explore material beyond the curriculum (Earthman, 2002; Rivera-Batiz and

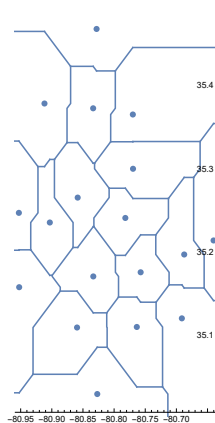


FIGURE 3.2. A Taxicab Voronoi diagram of CMS public high schools.

Marti, 1995).

In this case, a weighted module algorithm is constructed to adapt previous models including the Euclidean Voronoi model to incorporate weights. As described in the previous section, a boundary line between two schools in the Euclidean Voronoi model are perpendicular bisectors and by definition are equidistant between the two schools. In contrast, a weighted Voronoi model does not bisect the distance between the two schools, instead it splits the distance based on a fractional amount. To summarize the steps of the weighted module, consider two schools  $s_1$  and  $s_2$  with weights  $w_1$  and  $w_2$  respectively, where  $w_2 > w_1$ . Define  $p$  as the point on the line segment between  $s_1$  and  $s_2$  such that  $\frac{d(p,s_1)}{d(s_1,s_2)} = w_2$ . Then translate the original edge(s) shared between cells with seed schools  $s_1$  and  $s_2$  so that it passes through  $p$ .

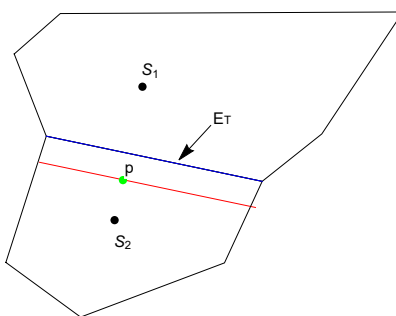


FIGURE 4.1.  $s_1$  and  $s_2$  shared edge in the Euclidean Voronoi model (blue) is translated to pass through  $p$  (red).

In order to redefine the cells affiliated with schools  $s_1$  and  $s_2$  with the new translated edge,  $E_T$ ,  $E_T$  may need to be lengthened or shortened in order to intersect the cell boundary of  $s_2$ . Three possible situations could occur based on the number of intersections between  $E_T$  and the cell boundaries of  $s_2$ . An automated algorithm is created so that new cells can be established algorithmically.

1. If  $E_T$  intersects the cell boundary of  $s_2$  twice, then the segments of  $E_T$  that lie outside the cell are deleted.

2. If  $E_T$  intersects the cell boundary of  $s_2$  only once, the end points of  $E_T$  are extended with an infinite line, and the intersections,  $p_1$  and  $p_2$ , are identified. Next,  $p_1$  and  $p_2$  are joined to the list of defined points of  $E_T$ , and the combined list is ordered based on their  $x$  or  $y$  coordinates depending on the orientation of  $E_T$ . In Figure 4.1, the list would be sorted by their  $x$  coordinates. The positions of  $p_1$  and  $p_2$  are identified in the sorted list and then any points before or after the intersections' positions in the list are dropped.
3. If  $E_T$  does not intersect the cell boundary of  $s_2$ , the end points of the  $E_T$  are extended with an infinite line, the intersections,  $p_1$  and  $p_2$ , are identified, and the intersections are added to the beginning and end of the list of points defining  $E_T$ .

Once  $E_T$  is connected to the other existing cell edges of  $s_2$ , the cell can be split into two polygons along  $E_T$  using the algorithm discussed in the road networking model.

A weighted Voronoi diagram is created based on student population and capacity measures. Recall that the calculated difference of the student population minus the student capacity yields positive value if the school is over capacity and negative value if the school is under capacity. Since the goal is to balance all schools so that the values are close to 0, the weighted module is used to shift the boundary lines to move students from schools that are over capacity to schools that are under capacity. If both numbers are positive, weighted averages are used as weights with the goal of moving  $E_T$  closer to the school that is over capacity. This process is more ambiguous with negative numbers.

Denote the overcapacity factor, population minus capacity of the school, for schools  $s_1$  and  $s_2$  as  $n_1$  and  $n_2$  respectively, where  $n_2 < 0 < n_1$ . Define  $w_1 = \frac{n_1+2|n_2|}{n_1+2|n_2|}$  and  $w_2 = \frac{|n_2|}{n_1+2|n_2|}$ . This yields new polygons that balance both values. For instance, the edge between South Mecklenburg and Myers Park is shifted using weights  $\frac{631}{2292}$  and  $\frac{1661}{2292}$ . The value of overcapacity for South Mecklenburg is updated to 338, and then the edge between Ardrey Kell and South Mecklenburg is moved based on the following weights:  $\frac{663}{988}$  and  $\frac{325}{988}$ . Now instead of Myers Park being overcapacity by 1030, it is only 61 students over capacity and South Mecklenburg and Ardrey Kell are both closer to at capacity, as seen in Figure 4.2. The weighted module is applied to each polygon in the Euclidean Voronoi model along one or more of its edges. The number of students over or under capacity is continually recalculated throughout the process to strategically choose adjacent polygons to shift their shared edge. Because the weighted module can be applied to two cells even if the shared edge is made of multiple line segments, two polygons can be temporarily joined together to complete a combined boundary shift between three polygons. For example, in Figure 4.3, Garinger and East Mecklenburg's cells are temporarily joined to shift the shared edge (red solid line) with Independence's cell to the  $E_T$  (red dotted line) because both Garinger and East Mecklenburg are over capacity and Independence is under capacity in the Euclidean Voronoi model. Figure 4.4 illustrates the final weighted Voronoi diagram based on population weights.

The next model constructed for this research adapts the Euclidean Voronoi model to incorporate three major roads within the CMS district. The major roads are considered as possible boundaries in these models, since they, like natural boundaries, tend to be used as attendance zone boundaries. Additionally, by using large roads for these boundaries, neighborhoods will be more likely to remain together within a school attendance model. The three major interstates are I-77, which runs north to south; I-85, which runs northeast to west; and I-485, which roughly forms an ellipse shape around the city center. However, I-485 is not considered because the interstate is closely located

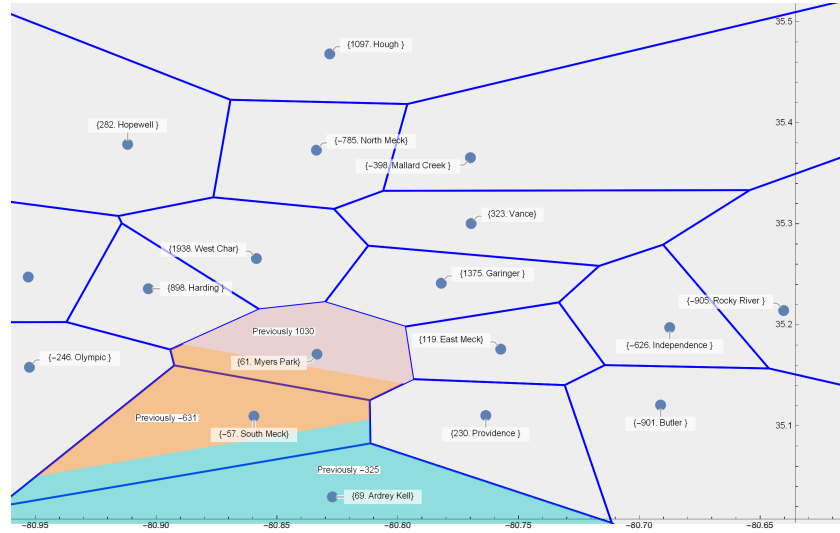


FIGURE 4.2. The boundary lines between Myers Park, South Mecklenburg, and Ardrey Kell are moved using the weighted module to create new pink, orange, and blue polygons that change all three schools to be closer to the “at capacity” value of 0.

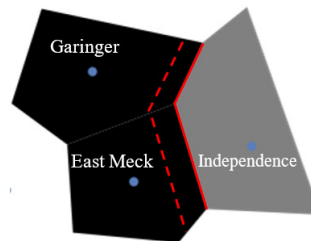


FIGURE 4.3. The weighted module can shift a shared edge even if it is made up of multiple line segments as seen with Garinger, East Mecklenburg, and Independence.

to seven schools out of the 11 school polygons it cuts through. Further, I-485 cuts through two other cells outside the district lines. Thus, another major road, Highway 16/74, is chosen that goes through the middle of the district with a different cardinal direction, northwest to southeast, than I-77 and I-85. The geographical location (latitude and longitude marks) of each road was collected from the Google Maps web mapping system.

This model splits Euclidean Voronoi cells into two or more smaller polygons along the road lines that intersect the cell, if any. Then with consideration of the current percentage of overcrowdedness for each school in the Euclidean Voronoi model, one or more of the smaller polygons created by the road intersection(s) is moved from the current cell to a neighboring cell.

In order to split the cells with road intersections into two smaller polygons along the road, the section of the road that intersects the cell and then the points where the road enters and exits the cell must be identified. The cardinal direction of the interstates/highways is used to order the

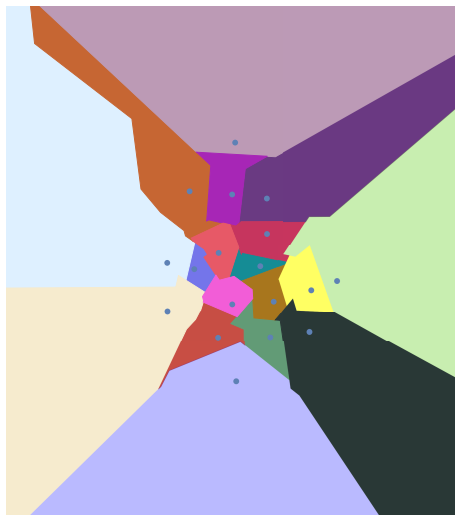


FIGURE 4.4. Weighted Population Voronoi Diagram of Charlotte-Mecklenburg School District.

points on the road. For example, since I-77 runs north and south, the points are arranged by their y-coordinate. One must also identify which edges of the cell the road intersects. One way to do this is to separate the cell border into individual line segments and individually testing whether or not one of the intersection points lie on that edge. For example, Figure 4.5 shows North Mecklenburg High School's Voronoi cell; the section of I-77 that intersects the cell, and the points on I-77 that intersects the cell border are labeled with black dots. In this case, the intersection points lie on boundary edges  $L_2$  and  $L_5$ .

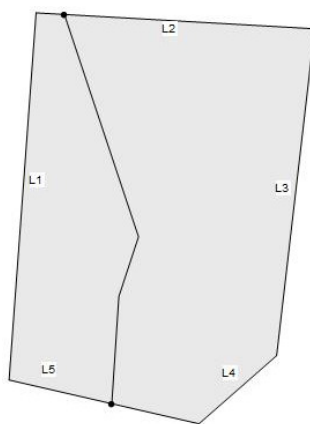


FIGURE 4.5. I-77 intersects North Mecklenburg's Voronoi cell boundary on edges  $L_2$  and  $L_5$ .

Multiple cells on the Euclidean Voronoi diagram are intersected by more than one of the three major roads, so the next goal is to split the cell into smaller polygons created by all the roads intersections simultaneously. As seen in Figure 4.6, the West Charlotte school district can be divided into seven smaller polygons because of the way all three roads intersect its cell. The previous code splits the cell into two polygons along each road. Since all three roads intersect West Charlotte, six



polygons can be defined with the algorithm discussed above ( $R_1$  polygon west of I-77,  $R_2$  polygon east of I-77,  $R_3$  polygon north I-85,  $R_4$  polygon south I-85,  $R_5$  polygon east of Highway 16/74,  $R_6$  polygon west of Highway 16/74). Each of the smaller seven polygons pictured in Figure 4.6 must be identified based on the intersections of regions  $R_1, R_2, \dots, R_6$ .

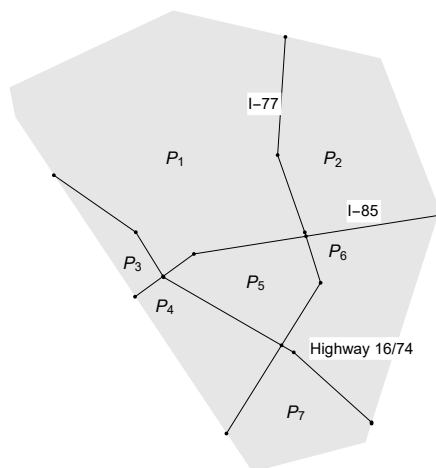


FIGURE 4.6. The major road intersections in West Charlotte Euclidean Voronoi cell splits the cell into seven smaller polygons.

In the interest of addressing the distance factor and population factor simultaneously, the amount of overcrowding for each school in the Euclidean Voronoi model is considered; if moving one or more of the smaller polygons created by the road intersection(s) to a neighboring cell alleviates overcrowding issues, then the adjustment is made. For example, West Charlotte High School (Figure 4.6) is overcapacity by 1,938 in the Euclidean Voronoi model, so  $P_2$  is joined to Vance High School and polygon  $P_3$  is joined to Harding High to decrease the student population at West Charlotte. Figure 4.7 illustrates the full road network model of CMS.

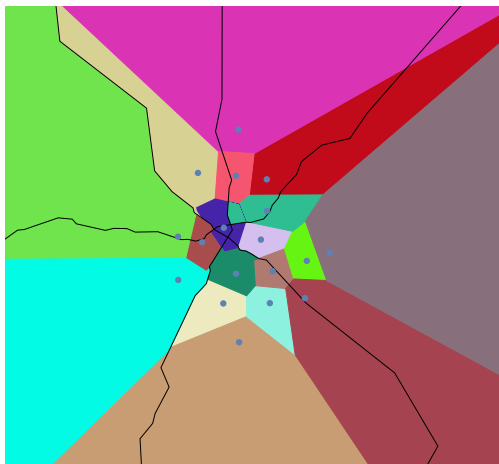


FIGURE 4.7. Road Network Model of Charlotte-Mecklenburg School District.

### 5. Tackling Poverty

Some scholars assert that school district leaders should strive for balanced demographics in their district’s attendance plans because diversity can influence children’s academic success. Phillips (2014) argues that working with people of different races and ethnicities increases creativity and diligence, and asserts that engaging in diverse environments enhances critical thinking and problem solving. Legally, the N.C. Gen. Stat. § 115C-367 (Law, 2020) prohibits the assignment of students to schools based on race in order to prevent racial gerrymandering, however schools segregated on the basis of race are usually segregated along other features, including poverty (Orfield and Lee, 2005). Research shows that individual student achievement is impacted by school wide poverty; a 1993 study reported that as the percentage of students in poverty increases, students’ scores decline (Puma et al., 1993). While there are many high performing high poverty schools, school-wide poverty does have negative consequences on achievement growth (Rumberger and Palardy, 2005). Thus this model will focus on balancing student poverty percentages.

The CMS district was chosen because its current attendance plan is saturated with racial segregation, income segregation, and overcrowdedness. The district’s student population is 40% African American, 29% white, and 22% Hispanic, yet 61% of the white student population attends 39 of the 168 schools in the district (Helms, 2018). Thirty-nine percent of the high school student population is economically disadvantaged in CMS, yet six of the 18 high schools’ student populations are over 50% economically disadvantaged, and three high schools’ student populations are under 15% economically disadvantaged.

In addition to a weighted population Voronoi diagram, the weighted module is used to produce a weighted Voronoi diagram based on student poverty percentages. Calculating the weights to input into the module to produce a more balanced percentage of impoverished students across CMS high schools is more challenging compared to the population data. Depending on the percentage of students in poverty in the census tracts near the boundary lines, sometimes it is better to move the edge closer to the school with a higher percentage, and in other cases it is better to shift the edge away from the school with the higher percentage.

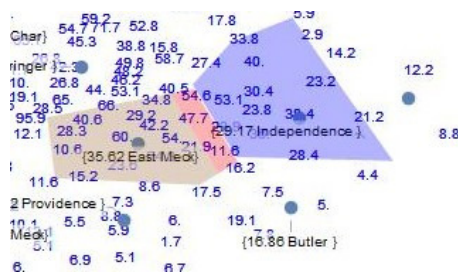


FIGURE 5.1. The red polygon,  $P_s$ , is shifted from East Mecklenburg to Independence’s cell using the weighted module. The centroids of the tracts are marked with blue numbers that represent the percentage of students in poverty in the census tract.

Let  $s_1$  and  $s_2$  be schools whose polygons,  $P_{s_1}$  and  $P_{s_2}$ , are adjacent. Let the percentage of students in poverty at  $s_1$  and  $s_2$  be  $x$  and  $y$  respectively where  $x < y$ . Define  $P_s$  as the polygon formed

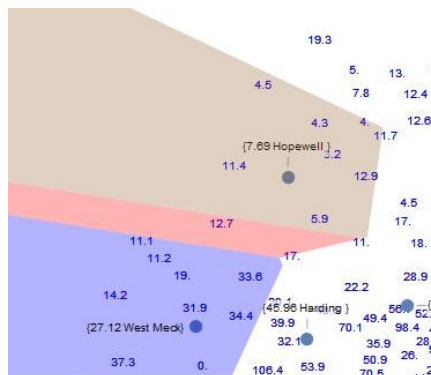


FIGURE 5.2. The red polygon shifted from Hopewell to West Mecklenburg since the red polygon's poverty (12.7%) is greater than Hopewell's (7.69%) and less than West Mecklenburg's (27.12%).

by shifting the shared edge,  $E_T$ , between  $P_{s_1}$  and  $P_{s_2}$ .

Define  $z$  as the percentage of students in poverty in  $P_s$ . If  $x < z < y$ , then both percentages will decrease by joining the  $P_s$  with  $s_2$ , the school with a higher rate of poverty (Figure 5.2). For example, in Figure 5.2, Hopewell and West Mecklenburg's poverty percentages are 7.69% and 27.12% respectively. Since  $P_s$  has a poverty percentage of 12.7%,  $P_s$  should be joined to West Mecklenburg, the school with the higher poverty percentage. If  $z < x$ , then the percentage affiliated with  $s_2$  will decrease and that affiliated with  $s_1$  will increase by joining  $P_s$  to  $s_2$  (Figure 5.3). Finally, if  $y < z$ , then the percentage affiliated with  $s_2$  will decrease and that affiliated with  $s_1$  will increase by joining  $P_s$  to  $s_1$ .

For example, in Figure 5.1, the percentage of poverty of  $P_s$  seems to be higher than both East Mecklenburg and Independences' percentages. Joining  $P_s$  to Independence's cell decreases East Mecklenburg's poverty percentage even though this adjustment also increases Independence' poverty percentage. Thus, weights are chosen so  $P_s$  will (1) have a percentage that is in between the two schools percentages so both schools percentages decrease when  $P_s$  is moved or (2) will join  $P_s$  to the school that is under capacity so one school's percentage decreases and the population numbers are balanced in the process.

Because of the clustering of poverty in the middle of the CMS district due to residential segregation, the poverty percentages at the schools could only be raised or lowered by a small amount without jeopardizing the distance factor and/or completely restructuring the initial polygons. Figure 5.4 illustrates the final weighted Voronoi diagram based on poverty weights.

## 6. Comparative Results

Recall that the goal in constructing models for the CMS high school attendance zone plan was to incorporate the following factors: average school-to-home distance, the socioeconomic diversity of the student population, and the utilization of the school compared to its capacity. This discussion section takes a comparative look at the current model, Euclidean Voronoi model, taxicab model, road network model, weighted population model, and weighted poverty model with these factors

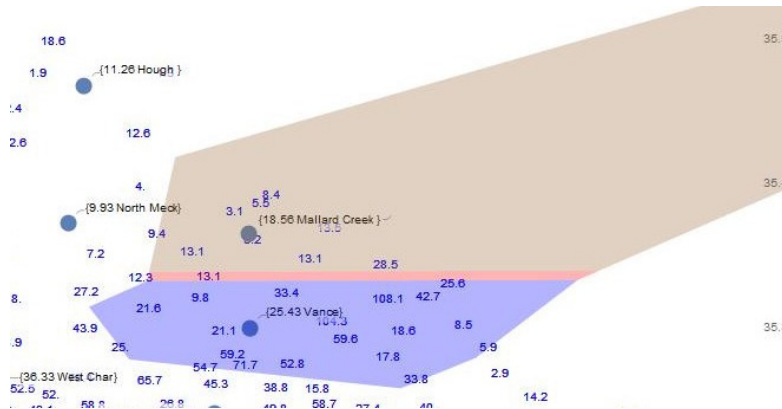


FIGURE 5.3. The red polygon is shifted from Mallard Creek’s cell to Vance’s cell because the red polygon’s poverty percentage (13.1%) is less than both Mallard Creek and Vance’s percentage (18.56% and 25.43%).

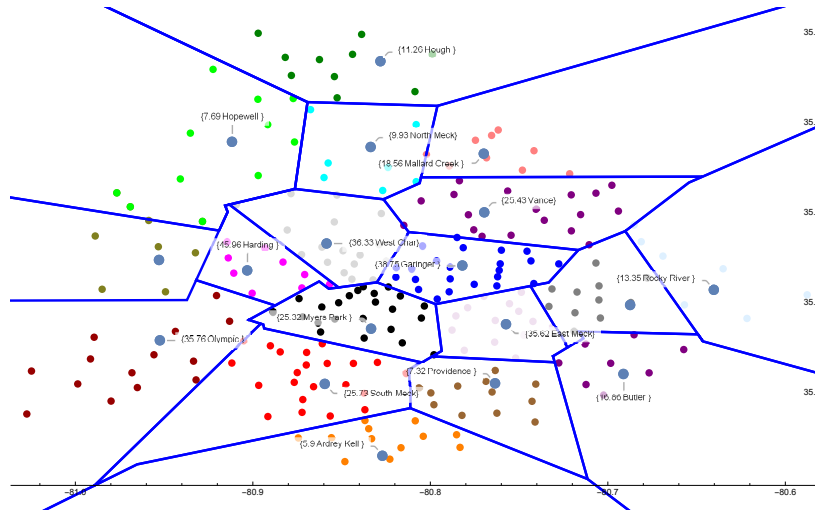


FIGURE 5.4. The Weighted Poverty Voronoi Diagram of CMS District.

in mind.

For each of the models, the distribution of the average traveling distance (ATD) in miles for all 18 schools is shown in Figure 6.1. The median ATD for the current model, 3.93 miles, is higher than all other models. The maximum ATD in the current model is 6.47 miles, over one mile higher than all the other maximum values. The best results regarding ATD occur with the taxicab and Euclidean models. This is to be anticipated given that the taxicab model and Euclidean Voronoi model solely construct boundary lines to minimize distance measures. For the Voronoi with and without roads, the median is consistent but the interquartile range (IQR) drops from 1.31 miles to 0.45 miles when the three major roads are incorporated. Because of the small IQR, the road network model is the most effective model at balancing the commute times from school to school.

The distributions of schools’ overcrowdedness for the current, Euclidean, road network, weighted population, and weighted poverty models are shown in Figure 6.2. For the current attendance

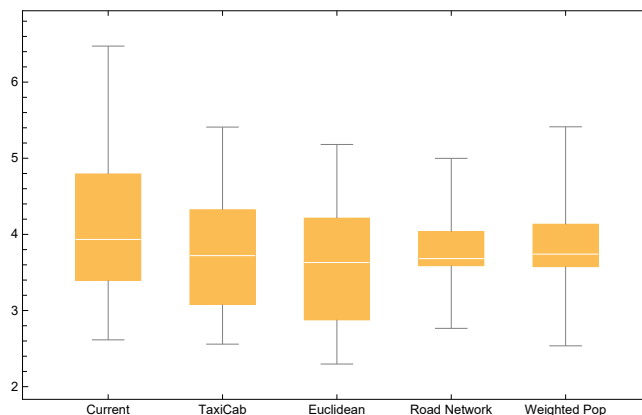


FIGURE 6.1. Side-by-side boxplots of the average traveling distance in miles for all 18 schools for each model.

zone plan, 50% of the schools are between -63 and 454. This spread is much wider for both the Euclidean Voronoi model and the weighted poverty model with an IQR of -626 to 898 and -548 to 713 respectively. The road network model has a smaller IQR, -398 to 637, compared to the Euclidean model, but the weighted population model has the smallest IQR from 20 to 321. While the weighted population model has a positive median value, unlike the Euclidean Voronoi and weighted poverty models, the trade-off of the weighted population model is that more of the schools are concentrated near 0, which means school buildings are not being over or underutilized. More impressively, 25% of the schools in this model fall between 20 and 68 students overcapacity. The current model has three schools that are overcrowded by 763, 758, and 1405 students whereas the weighted population model most overcrowded schools are 487, 513, and 1044. Because overcrowded schools are negatively correlated with student academic success, the weighted population model is ideal if considering population exclusively.

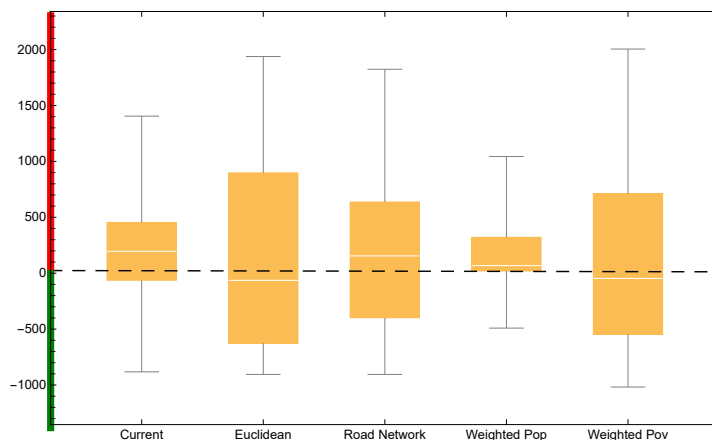


FIGURE 6.2. Side-by-side boxplots of the amount of overcrowdedness for all 18 schools in CMS for each model.

The last factor that is considered and analyzed is poverty. Figure 6.3 illustrates the percentages of the high schools' impoverished student population for the current model, Euclidean model, weighted population model, and weighted poverty model. The CMS district leaders also aim to

reduce high concentrations of poverty in schools (Charlotte-Mecklenburg Schools, 2019). Ideally, the distribution of the percent of student poverty is condensed to avoid heavy concentration of poverty and students of similar demographics in certain schools and not in others. The IQR is fairly consistent across all models, but the maximum value varies. The current model has a maximum poverty of 54.92% while the weighted poverty model has the lowest maximum of 45.96%. Therefore, the weighted poverty and weighted population models are slightly preferred models in regards to the poverty factor.

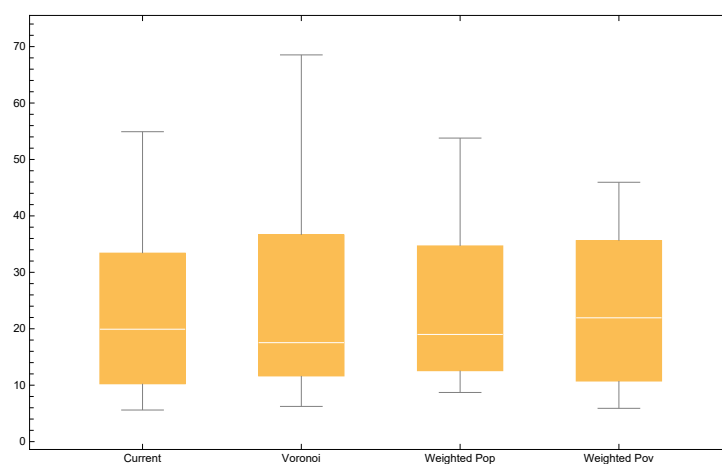


FIGURE 6.3. Side-by-side boxplot of the percentage of students in poverty for all 18 schools in CMS for each model.

## 7. The Real Story and Further Research

As we indicated, school districts cannot legally create attendance zones based on race; however, residential segregation continues to proliferate school segregation. Figures 7.1 and 7.2 show the map of race and ethnicity by block group in the CMS area (Cedar Lake Ventures, Inc., 2018) highlighting the schools in poverty and struggling with overcrowdedness in both the current attendance zones and the weighted Voronoi diagram model based on poverty presented in this paper. The models created in this paper are not meant to be final solutions to the overcrowdedness or poverty issues in the CMS schools, for there are additional fixed restrictions like the number of schools and uneven population density. Nor will the models be a final solution to diversity issues in the CMS schools because there are societal factors that complicate the problem such as housing segregation and school choice. Unfortunately, changing school attendance zones does not guarantee students with different demographics will interact, as tracking and specialized programs continue to keep students from different demographics apart.

Further research could investigate other models to make a larger impact on the distribution of poverty percentages and other social inequalities. Research on attendance zone plan modeling might extend and include other important factors such as feeder patterns with elementary and middle schools and projected population growth. In addition, future research could explore how the models can be used to strategically choose the placement of new schools in districts that can no longer control social inequalities by merely changing attendance zones like CMS.

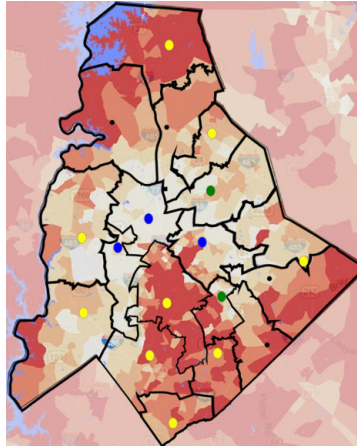


FIGURE 7.1. Race and Ethnicity by Block Group with Red Representing White (nonhispanic). Current School Attendance Zones with School colors representing Overcapacity by more than 100 students (Yellow) , Poverty above 40% (Blue) , Both (Green).

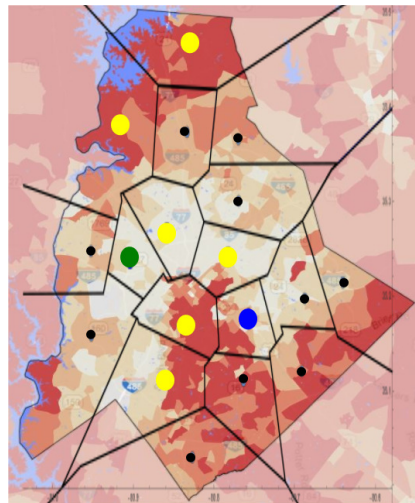


FIGURE 7.2. Race and Ethnicity by Block Group with Red Representing White (nonhispanic). Weighted Voronoi Model Based on Poverty with School colors representing Overcapacity by more than 100 students (Yellow) , Poverty above 40% (Blue) , Both (Green).

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