Mathematical Theory and Modeling
ISSN 2224-5804 (Paper) ISSN 2225-0522 (Online)
Vol.10, No.5, 2020
www.iiste.org

Modeling Optimal Control for Mosquito and Insecticide

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ABSTRACT

In this paper we consider a mathematical model of mosquito and insecticide. The aim of this model is to reduce the amount of mosquitoes in the ponds and swamps. Mosquitos are the main cause of malaria disease. We used the optimal spray strategies to minimize the amount of mosquito, we work optimal control framework by applying the Pontryagin's Maximum Principle. A characterization of the optimal control via adjoint variables was established. We obtained an optimality system that we sought to solve numerically by using MATLAB.

1-INTRODUCTION

1.1 GENERAL INTRODUCTION

Mosquito- borne diseases, the best known of which is malaria, are among the leading causes of human deaths worldwide. Vector control is a very important part of the global strategy for management of mosquito-associated diseases, and insecticide application is the most important component in this effort. However, mosquito-borne diseases are now resurgent, largely because of the insecticide resistance that has developed in mosquito vectors and the drug resistance of pathogens.

Insecticides are a quick, powerful way to get rid of mosquitoes around the yard, but, unfortunately, they are only temporary. The effect usually lasts only as long as the insecticide is present, so as soon as it drifts away or dries out, the mosquitoes are back. Mosquito control officials use insecticides only when mosquitoes are especially thick and only in combination with other form of mosquito control. The same should apply to use around the house. By itself, insecticide is not a long-term solution. Two popular insecticides are:

Malathion: an organophosphate often used to treat crops against a wide array of insects. It can be sprayed directly onto vegetation, such as the bushes where mosquitoes like to rest, or used in a 5 percent solution to fog the yard. In the small amounts used for mosquito control it poses no threat to humans or wildlife. In fact, malathion is also used to kill head lice.

Permethrin: one of a group of chemicals called pyrethroids, it is a synthetic form of a natural insecticide found in chrysanthemum flowers. It usually is mixed with oil or water and applied as a mist, about 1/100th of a pound per acre. Like malathion, permethrin kills mosquitoes by disrupting their central nervous systems. Not harmful to people and animals in small amounts, but it is toxic to fish and bees. There are three types of mosquito spraying with insecticides. Home and fog spraying, sprinkling ponds and swamps. In this paper, the lesson will focus on sprinkling ponds and swamps.

1.2 THE MATHEMATICAL MODEL

In our basic optimal control problem for ordinary differential equations, we use $u(t)$ for the control and $x(t)$ for the state. The state variable satisfies a differential equation which depends on the control variable:

$$x'(t) = g(t, x(t), u(t))$$

As the control function is changed, the solution to the differential equation will change. Thus, we can view the control-to-state relationship as a map $u(t) \rightarrow x = x(u)$ (of course, x is really a function of the independent variable t, we write $x\ (u)$ simply to remind us of the dependence on u). Our basic optimal control problem consists of finding a piecewise continuous control $u(t)$ and the associated state variable $x(t)$ to maximize the given objective functional, i.e.

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$$\int_{t_0}^{t_1} f(t, x(t), u(t))dt$$

Subject to $\quad x'(t) = g(t, x(t), u(t))$

$$x(t_0) = x_0 \wedge x(t_1) \quad free.$$

Such a maximizing control is called an optimal control. By $x(t_1)$ free, it ismeant that the value of $x(t_1)$ is unrestricted. For our purposes, f and g will always be continuously differentiable functions in all three arguments. Thus, as the control(s) will always be piecewise continuous, the associated states will always be piecewise differentiable.

1.3 PONTRYAGIN'S MAXIMUM PRINCIPLE

$$H(t, x, u, \lambda) = f(t, x, u) + \lambda g(t, x, u)$$

= integrand + adjoint * RHS of DE:

We are maximizing *H* with respect to *u* at *u**, and the above conditions can be written in terms of the Hamiltonian:

$$\frac{\partial H}{\partial u} = 0 \quad at \quad u^* \Rightarrow f_u + \lambda g_u \qquad (Optimality\ condition).$$

$$\lambda' = \frac{-\partial H}{\partial x} \Rightarrow \lambda' = -(f_x + \lambda g_x) \qquad (adjoint\ equation).$$

$$\lambda(t_1) = 0 \qquad (transversality\ condition)$$

We are given the dynamics of the state equation:

$$x' = g(t, x, u) = \frac{\partial H}{\partial \lambda}, x(t_0) = x_0.$$

THEOREM

Consider

$$J(u) = \int_{t_0}^{t_1} f(t, x(t), u(t))dt$$

Subject to $x'(t) = g(t, x(t), u(t)), x(t_0) = x_0$

Suppose that $f(t, x, u) \wedge g(t, x, u)$ are both continuously differentiable functions in their three arguments and concave in x and u. suppose u^* is a control, with associated state x^*, and λ a piecewise differentiable function, such that u^*, x^*, and λ together satisfy on $t_0 \leq t \leq t_1$:

$$f_u + \lambda g_u = 0\,,$$

$$\lambda' = -(f_x + \lambda g_x)\,,$$

$$\lambda(t_1) = 0$$

$$\lambda(t) \geq 0.$$

Then for all controls u, we have

$$J(u^*) \geq J(u).$$

PROOF

Let u be any control, and x its associated state. Note, as $f(t, x, u)$ is concave in both the x and u variable, we have by the tangent line property

$$f(t, x^*, u^*) - f(t, x, u) \geq (x^* - x) f_x(t, x^*, u^*) + (u^* - u) f_u(t, x^*, u^*)$$

This gives

$$J(u^*) - J(u) = \int_{t_0}^{t_1} f(t, x^*, u^*) - f(t, x, u)$$

$$\geq \int_{t_0}^{t_1} (x^*(t) - x(t)) f_x(t, x^*, u^*) + (u^*(t) - u(t)) f_u(t, x^*, u^*) dt \ldots\ldots\ldots.(1)$$

Substituting

$$f_x(t, x^*, u^*) = -\lambda'(t) - \lambda(t) g_x(t, x^*, u^*) \;\; and \;\; f_u(t, x^*, u^*) = -\lambda(t) g_u(t, x^*, u^*)$$

as given by the hypothesis, the last term in (1) becomes

$$\int_{t_0}^{t_1} (x^*(t) - x(t))(-\lambda'(t) - \lambda(t) g_x(t, x^*, u^*)) dt + (u^*(t) - u(t))(-\lambda(t) g_u(t, x^*, u^*)) dt.$$

Using integration by parts, and recalling $\lambda(t_1) = 0$ and $x(t_0) = x^*(t_0)$, we see

$$\int_{t_0}^{t_1} -\lambda'(t)(x^*(t)-x(t))dt = \int_{t_0}^{t_1} \lambda(t)(x^*(t)-x(t))'dt$$

$$= \int_{t_0}^{t_1} \lambda(t)\left(g\big(t,x^*(t),u^*(t)\big) - g\big(t,x(t),u(t)\big)\right) dt$$

Making this substitution,

$$J(u^*) - J(u) \geq \int_{t_0}^{t_1} \lambda(t)[g(t,x^*,u^*) - g(t,x,u) - (x^*-x)g_x(t,x^*,u^*) + (u^*-u)g_u(t,x^*,u^*)]dt$$

Taking into account $\lambda(t) \geq 0$ and that g is concave in both *x* and *u*, this gives the desired result

$$J(u^*) - J(u) \geq 0.$$

2-RESULTS AND DISCUSSION

2.1 THE OPTIMAL CONTROL PROBLEM

Let *x* (*t*) be a population concentration at time *t*, and suppose we wish to reduce the population over a fixed time period. We will assume *x* has a growth rate *r* and carrying capacity *M*. The application of a substance is known to decrease the rate of change of *x*, by decreasing the rate in proportion to the amount of *u* and *x*. Let *u*(*t*) be the amount of this substance added at time *t*. For example, the population could be an infestation of an insect, or a harmful microbe in the body. Here we view *x*(*t*) as the concentration of a mosquito and *u*(*t*) an insecticide known to kill it. The differential equation representing the mold is given by

$$x'(t) = r\big(M - x(t)\big) - u(t)x(t), \qquad x(0) = x_0$$

Where $x_0 > 0$ is the given initial population size. Note the term *u*(*t*)*x*(*t*) pulls down the rate of growth of the mosquitoes. The effects of both the mosquitoes and insecticide are negative for individuals around them, so we wish to minimize both. Further, while a small amount of either is acceptable, we wish to penalize for amounts too large. Hence, our problem is as follows

$$\int_0^T Ax + u(t)^2 dt$$

$$\text{Subject to } \; x'(t) = r\big(M - x(t)\big) - u(t)x(t), \qquad x(0) = x_0$$

The coefficient *A* is the weight parameter, balancing the relative importance of the two terms in the objective functional.

2.2 NUMERICAL SOLUTION

Before writing the code we develop the optimality of this problem by first noting the Hamiltonian is

$$H = Ax + u^2 + \lambda r(M - x) - \lambda xu$$

Using the optimality condition

$$0 = \frac{\partial H}{\partial u} = 2u - \lambda x \quad at \quad u^* \Rightarrow u^* = \frac{\lambda x}{2}$$

The adjoint equation is

$$\lambda'(t) = \frac{-\partial H}{\partial x} = -A + \lambda r + \lambda u$$

$$= -A + \lambda r + 0.5\lambda^2 x$$

$$x'(t) = Mr - x(r + u) \;\;, \quad x(0) = x_0$$

$$\lambda'(t) = -A + \lambda r + 0.5\lambda^2 x \;, \quad \lambda(T) = 0$$

Using these two differential equations and the representation of u^*, we generate the numerical code as described above, written in MATLAB [5].

Using the Runge- kutta sweep method solving $x^{\rightarrow}$ forward in time

```
For i = 1:N
k1 = M*r - x(i)*(r + u(i));
k2 = M*r-(x(i) + h2*k1)*(r + 0.5*(u(i) + u(i+1)));
k3 =  M*r-(x(i) + h2*k2)*(r + 0.5*(u(i) + u(i+1)));
k4 = M*r - (x(i) + h*k3)*(r + u(i+1));
x(i+1) = x(i) + (h/6)*(k1 + 2*k2 + 2*k3 + k4);
```

Using the Runge- kutta sweep method solving $u^{\rightarrow}$ backward in time

```
for i = 1:N
j = N + 2 - i;
k1 = -A - lambda (j)*r +0.5*(lambda (j))^2 * x(j);
k2=-A - (lambda (j)-h2*k1)*r + 0.5*(lambda (j) - h2*k1)^2*0.5*(x(j)+x(j-1)) ;
k3=-A-(lambda(j)-h2*k2)*r + 0.5*(lambda(j)- h2*k2)^2 * 0.5*(x(j)+x(j-1)) ;
k4 = (lambda (j)-h*k3)*r + 0.5*(lambda (j) - h*k3)^2 * x(j-1) ;
```



```
lambda (j-1) = lambda(j) - ...
                    (h/6)*(k1 + 2*k2 + 2*k3 + k4);
```

2.3 NUMERICAL RESULTS

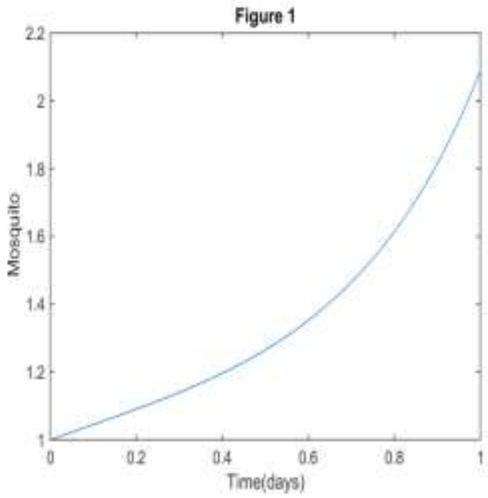
Here we consider a general mosquito and insecticide model and all the parameter values are chosen hypothetically. Enter the values

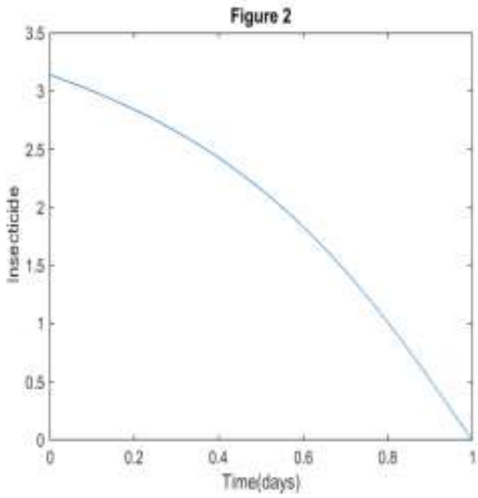
$$r = 0.4, \quad M = 10, \quad A = 6, \quad X_0 = 1 , \; C = 7.$$

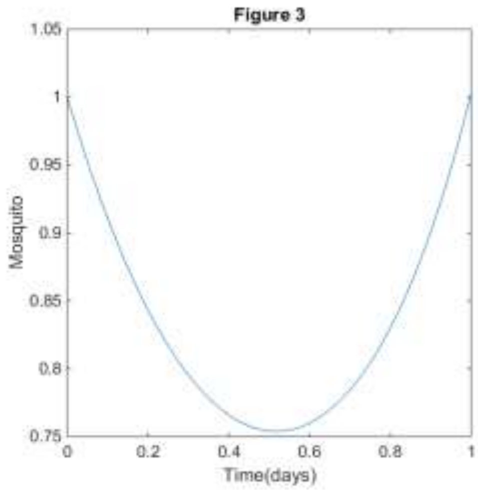
The state never decreases, with growth at the beginning and end of the interval Figure1. The control initially increases then decreases to zero Figure2. Enter the values

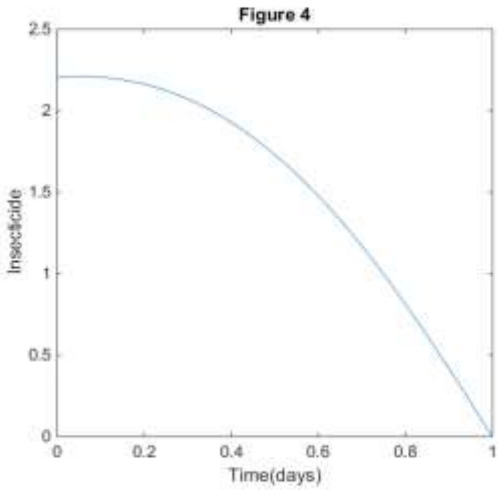
$$r = 0.3, \quad M = 5, \quad A = 10, \quad X_0 = 1 , C = 7.$$

The state decreases, since beginning and constant in the middle with growth at the end Figure3. The control initially increases, and then levels off to become constant. The control eventually begins decreasing again, going all the way to 0 Figure4.









3-Conclusions

In this paper, we cannot completely get rid of mosquitoes and their different phases in ponds and swamps, but by optimal control, we reduced it to a large extent with respect to our model in seven days, and therefore we have reduced the spread of disease malaria.

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