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The Application of Saturated Incidence Rate with Delay and Awareness in Modelling the Spread of Infectious Diseases

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Abstract
The current state as regards the spread of the coronavirus disease 2019 indicates that the major preventive measure is minimizing individual contact with the virus. Consequently, this paper derived a mathematical model using saturated incidence rate with the disease incubating period as a delay parameter along other parameters measuring the inhibitory effects of awareness dissemination from the media, general campaign and individual interactions. The analytical evaluation of the model indicates that the stability of the disease-free and the endemic steady states of the model are determined by the basic reproduction number. In addition, the disease-free steady state is independent of the incubating period, whereas the endemic steady state is initially stable but undergoes Hopf bifurcation as the incubating period exceeds the critical value by exhibiting periodic oscillation and become unstable. The outcome of the numerical simulation of the model via MATLAB application affirmed the results obtained analytically.
Keywords: Infectious Diseases, Model, Incidence rate, Basic reproduction number, Hopf bifurcation

1. Introduction

The advent of the coronavirus disease 2019 (COVID-19) has sprung a lot of scientific and clinical researches in search for means of treating, controlling and/or preventing the spread of the disease. Though there is no specific treatment for the cure of COVID-19 (that is, currently there is no effective vaccine or specific drug for preventing or treating the virus), a number of pharmacologic treatments have been suggested as potential therapies for COVID-19. These treatments are virtually supportive based on the severity of the individual illness. The following are some suggested drugs that have been found to be helpful in controlling the infection: Chloroquine Phosphate (Devaux *et al*., 2020; Colson *et al*., 2020), Hydroxychloroquine Solphate (Gautret *et al*., 2020; Yao *et al*., 2020), Lopinavir/Ritonavir (Cao *et al*., 2020; Costanzo *et al*., 2020), Umifenovir (Costanzo *et al*., 2020), Remdesivir (Al-Tawfiq *et al*., 2020; Dong *et al.*, 2020; Sheahan *et al*., 2020), Favipiravir (Dong *et al.*, 2020), Tocilizumab (Xu *et al*., 2020).

The most common transmission mode of COVID-19 is by inhalation of infected droplets as a result of the susceptible having direct contact with the infected droplets (Singhal, 2020). Consequently, as preventive and control measures, several researchers recommended the used of face masks (Cheng *et al*., 2020; Einkenberry *et al*., 2020; Greenhalgh *et al*., 2020), social distancing (Courtemanche *et al*., 2020; Kissler *et al*., 2020; Singh & Adhikari, 2020), hand washing with soap and water or alcohol based hand sanitizers (Adhikari *et al*., 2020; Malhotra *et al*., 2020; WHO 2020), along with awareness creation and information dissemination (Agaba, 2020; Agaba & Soomiyol, 2020).

Cheng *et al.* (2020) considered masking to be as important as other preventive measures noting that it has large population benefit. They also opined that if worn by most people, it is capable of reducing the reproduction number. Furthermore, Einkenberry *et al.* (2020) states that face masks are able to decrease the effective transmission rate and cause a decrease in epidemic mortality and healthcare system burden when combined with other non-pharmaceutical control measures. While Greenhalgh *et al.* (2020) are of the view that the use of face mask is a good precautionary measure that is simple, cheap and can have a sizeable impact on the transmission of the virus with little impact on social and economic life. They suggested that people be educated consistently on the proper usage of face masks without disregarding other anti-contagion measures.

Kissler *et al* (2020) from their research on the effect of social distancing on the transmission rate of COVID-19 opined that a one-time intervention will not be sufficient enough to maintain the disease; it will only postpone the epidemic peak to a later time. A better way will be by periodic distancing measures which may be maintained into 2022 along with other interventions. The impact of imposed social distancing measures, such as closure of schools and other crowding areas, on COVID-19 growth rate was evaluated by Courtemanche *et al.* (2020) and they discovered daily reduction in the growth rate of confirmed cases. In addition, the Indian society of anesthesiologists recommends frequent hand washing with soap and water or an alcohol based sanitizer (Malhotra, 2020). Similarly, in a technical brief by the World Health Organisation (WHO), frequent hand

hygiene is proposed as one of the most important measures for preventing infection and transmission of COVID-19 (WHO, 2020).

Consequently, wearing of face masks, social distancing and washing of hands with soap and water or sanitizers are measures that inhibit individual contact with the virus. The susceptible uses these measures for protection whereas the infected uses same for preventing or controlling the spread of the disease. In both scenarios, the adherence to these measures is geared by their level of awareness. Hence, the dynamical model for the spread of the disease can be evaluated using the saturated incidence rate similar to other epidemic models. Incidence rate that are either linear or non-linear have been used to reflect the dynamical behaviour of epidemic models.

Li & Liu (2014) gave details of the following types: the bilinear incidence rate βSI with β as the transmission rate, the standard incidence rate $\beta SI/N$ with $N=S+I+R$, the Holling incidence rate of the form $\beta SI/(1+\alpha_1 S)$ where α_1 is a positive constant, the saturated incidence rate of the form $\beta SI/(1+\alpha_2 I)$, where α_2 is a positive constant, and the saturated incidence rate of the form $\beta SI/(1+\alpha_1 S+\alpha_2 I)$, where α_1, α_2 are positive constants. Abta *et al.* (2014) also used $\beta SI/(1+\alpha_2 I)$ with delay while Xiao & Ruan proposed the non-linear incidence rate $KSI/(1+\alpha_2 I^2)$ that accounts for the effect of mass media coverage (see Sun *et al*., 2011), whereas Zhao *et al.* (2014) proposed $[\beta_1-\beta_2 I/(m+I)]SI$ with β_1 denoting the maximum contact rate, $\beta_2 I/(m+I)$ the maximum reduced contact rate due to mass media alert and m represents the impact of media coverage. This paper proposes a model using the saturated incidence rate $\beta SI/(\omega+\phi S+\alpha I)$ where ω, ϕ, α, are positive parameters that measure the inhibitory effect of awareness springing from global sources, the susceptible and number of infected cases respectively.

2. Model derivation

An SIR (Susceptible-Infective-Recovered) epidemic model for the spread of infectious diseases is proposed using saturated incidence rate with delay, denoting the incubating period of the disease, and awareness dissemination from both global and local sources of information. The model is derived as follows:

$$\begin{aligned}
\frac{dS}{dt} &= r\left(1-\frac{S(t)}{K}\right)S(t)-\frac{\beta S(t)I(t)}{\omega+\phi S(t)+\alpha I(t)} \\
\frac{dI}{dt} &= \frac{\beta S(t-\tau)I(t-\tau)}{\omega+\phi S(t-\tau)+\alpha I(t-\tau)}-(\varepsilon+\lambda_d+\lambda)I(t) \\
\frac{dR}{dt} &= \varepsilon I(t)-\lambda R(t)
\end{aligned} \tag{1}$$

with the initial conditions

$$S(0)=S_0\geq 0,\ I(0)=I_0>0,\ R(0)=R_0\geq 0,\ S(v)=\varphi(v)\geq 0,\ I(v)=\varphi(v)\geq 0,\ -\tau\leq v<0,$$

defined within the region $\psi=(S,I,R)\in\Re_+^3$. The parameter β represents the transmission rate of the infectious disease from the infective to the susceptible individuals, τ is the incubating period of the disease, while ε is the recovery rate and λ_d is the disease-related death rate. The population is considered to be growing logistically with r as the growth rate and K as the carrying capacity while λ is the natural death rate. The parameter ω denotes the rate at which disease related awareness circulates from global sources such as the media, awareness campaigns and so on whereas α and ϕ are the respective rate at which awareness stems from reported number of infections and the increasing number of aware susceptible also known as local sources of awareness.

3. Model analysis

3.1 Analysis of the steady states

The analysis of the model (1) gave three steady states defined within the region ψ: the trivial steady state obtained as $E_t=(\bar{S},\bar{I},\bar{R})=(0,0,0)$, the disease-free steady state of the model obtained as $E_d=(\tilde{S},\tilde{I},\tilde{R})=(K,0,0)$ and the endemic steady state $E_n=(S^*,I^*,R^*)$ with

$$S^*=Kh_n,\qquad I^*=\frac{Krh_n(1-h_n)}{\varepsilon+\lambda_d+\lambda},\qquad R^*=\frac{Kr\varepsilon h_n(1-h_n)}{\lambda(\varepsilon+\lambda_d+\lambda)} \tag{2}$$

and

$$h_n=\frac{1}{2}\left(1-\frac{\beta-\phi(\varepsilon+\lambda_d+\lambda)}{r\alpha}\right)+\sqrt{\frac{1}{4}\left(1-\frac{\beta-\phi(\varepsilon+\lambda_d+\lambda)}{r\alpha}\right)^2+\frac{\omega(\varepsilon+\lambda_d+\lambda)}{Kr\alpha}} \tag{3}$$

where $0 < h_n < 1$ is true for all values of $\omega > 0$ but for $\omega = 0$, it is satisfied if $\beta < \phi(\varepsilon + \lambda_d + \lambda) + r\alpha$. At the endemic steady state, it is also obtained that

$$\varepsilon + \lambda_d + \lambda = \frac{\beta S^*}{\omega + \phi S^* + \alpha I^*} \qquad \text{and} \qquad r\left(1 - \frac{S^*}{K}\right) = \frac{\beta I^*}{\omega + \phi S^* + \alpha I^*} \tag{4}$$

The disease-free and endemic steady states of the system (1) are determined by the basic reproduction number, R_0 obtained as

$$R_0 = \frac{\beta K}{(\varepsilon + \lambda_d + \lambda)(\omega + \phi K)}. \tag{5}$$

This implies that the system has a disease-free steady state when it is less than one but if greater than one there exists an endemic steady state.

3.2 Stability and Hopf bifurcation analyses of the model

The linearization of the model (1) near any steady state $E_a = (\bar{S}, \bar{I}, \bar{R})$ gives the system of equations

$$\begin{aligned}
\frac{dy_1}{dt} &= r\left(1 - \frac{2\bar{S}}{K}\right) y_1(t) - \frac{\beta\bar{I}(\omega + \alpha\bar{I})}{(\omega + \phi\bar{S} + \alpha\bar{I})^2} y_1(t) - \frac{\beta\bar{S}(\omega + \phi\bar{S})}{(\omega + \phi\bar{S} + \alpha\bar{I})^2} y_2(t) \\
\frac{dy_2}{dt} &= \frac{\beta\bar{I}(\omega + \alpha\bar{I})}{(\omega + \phi\bar{S} + \alpha\bar{I})^2} y_1(t - \tau) + \frac{\beta\bar{S}(\omega + \phi\bar{S})}{(\omega + \phi\bar{S} + \alpha\bar{I})^2} y_2(t - \tau) - (\varepsilon + \lambda_d + \lambda) y_2(t) \\
\frac{dy_3}{dt} &= \varepsilon y_2(t) - \lambda y_3(t)
\end{aligned} \tag{6}$$

where $y_1 = S - \bar{S}$, $y_2 = I - \bar{I}$ and $y_3 = R - \bar{R}$.

Theorem 1: *The trivial steady state of the model equation (1) is unstable and independent of* τ.

Proof: Considering the model (1), the linearization near the trivial steady state $E_t = (0,0,0)$ reduces the system (6) to

$$\begin{aligned}
\frac{dy_1}{dt} &= r y_1(t) \\
\frac{dy_2}{dt} &= -(\varepsilon + \lambda_d + \lambda) y_2(t) \\
\frac{dy_3}{dt} &= \varepsilon y_2(t) - \lambda y_3(t)
\end{aligned}$$

which indicates that the system is independent of τ and gives the Jacobian matrix

$$J_t = \begin{bmatrix} r & 0 & 0 \\ 0 & -(\varepsilon + \lambda_d + \lambda) & 0 \\ 0 & \varepsilon & -\lambda \end{bmatrix}$$

whose diagonal entries represent the eigenvalues of the characteristic equation. Hence, $\mu_1 = r$, $\mu_2 = -(\varepsilon + \lambda_d + \lambda)$ and $\mu_3 = -\lambda$ which implies that the trivial steady state is unstable since $\mu_1 > 0$. □

Theorem 2: *The model equation (1) has a disease-free steady state that is linearly asymptotically stable if* $R_0 < 1$ *for all* $\tau \geq 0$ *and unstable for* $R_0 > 1$, *where* R_0 *is as defined in (5).*

Proof: For the disease-free steady state $E_d = (K,0,0)$ of the model (1), the linearized system (6) reduces to

$$\begin{aligned}
\frac{dy_1}{dt} &= -r y_1(t) - \frac{\beta K}{\omega + \phi K} y_2(t) \\
\frac{dy_2}{dt} &= \frac{\beta K}{\omega + \phi K} y_2(t - \tau) - (\varepsilon + \lambda_d + \lambda) y_2(t) \\
\frac{dy_3}{dt} &= \varepsilon y_2(t) - \lambda y_3(t)
\end{aligned}$$

Consequently, from the Jacobian matrix the characteristic equation is generated as

$$(r+\mu)(\lambda+\mu)\left(\frac{\beta K}{\omega+\phi K}e^{-\mu\tau}-(\varepsilon+\lambda_d+\lambda)-\mu\right)=0$$

from which the first two eigenvalues are obtained as $\mu_1=-r$ and $\mu_2=-\lambda$. Indicating that the stability of the disease-free steady state is determined by the equation

$$\mu+\varepsilon+\lambda_d+\lambda-\frac{\beta K}{\omega+\phi K}e^{-\mu\tau}=0 \tag{7}$$

For $\tau=0$, the third eigenvalue is obtained from (7) as

$$\mu=\frac{\beta K}{\omega+\phi K}-(\varepsilon+\lambda_d+\lambda)$$

which shows that the disease-free steady state is stable for $\tau=0$ provided

$$R_0=\frac{\beta K}{(\varepsilon+\lambda_d+\lambda)(\omega+\phi K)}<1$$

whereas for $R_0>1$, the disease-free steady state is unstable for $\tau=0$ and remains unstable for $\tau>0$. Hence, it is unstable for all $\tau\geq 0$.

Next, using the transcendental equation (7) to solve for $\mu=ix$ when $\tau>0$ gives

$$ix+\varepsilon+\lambda_d+\lambda=\frac{\beta K}{\omega+\phi K}e^{-ix\tau}=\frac{\beta K}{\omega+\phi K}[\cos(x\tau)-i\sin(x\tau)]$$

Equating real and imaginary parts of this equation generate

$$\begin{aligned}\varepsilon+\lambda_d+\lambda&=\frac{\beta K}{\omega+\phi K}\cos(x\tau)\\ x&=-\frac{\beta K}{\omega+\phi K}\sin(x\tau)\end{aligned} \tag{8}$$

Squaring and adding both equations in (8) produce

$$x^2+\left(\varepsilon+\lambda_d+\lambda+\frac{\beta K}{\omega+\phi K}\right)\left(\varepsilon+\lambda_d+\lambda-\frac{\beta K}{\omega+\phi K}\right)=0$$

Therefore, with $R_0<1$ there exists no positive real root x for which $\mu=ix$ and this indicates that the disease-free steady state is stable for all values of τ greater than zero. □

The linearization of the model (1) around the endemic steady state $E_n=(S^*,I^*,R^*)$ reduces (6) to

$$\begin{aligned}\frac{dy_1}{dt}&=-\frac{rS^*}{K}y_1(t)+\frac{\beta\phi S^*I^*}{(\omega+\phi S^*+\alpha I^*)^2}y_1(t)-\frac{\beta S^*(\omega+\phi S^*)}{(\omega+\phi S^*+\alpha I^*)^2}y_2(t)\\ \frac{dy_2}{dt}&=\frac{\beta I^*(\omega+\alpha I^*)}{(\omega+\phi S^*+\alpha I^*)^2}y_1(t-\tau)+\frac{\beta S^*(\omega+\phi S^*)}{(\omega+\phi S^*+\alpha I^*)^2}y_2(t-\tau)-(\varepsilon+\lambda_d+\lambda)y_2(t)\\ \frac{dy_3}{dt}&=\varepsilon y_2(t)-\lambda y_3(t)\end{aligned}$$

which gives the Jacobian matrix

$$J_n=\begin{bmatrix}-a_1 & -a_2 & 0\\ a_3a_\tau & a_2a_\tau-m & 0\\ 0 & \varepsilon & -\lambda\end{bmatrix}$$

where

$$a_1=\frac{rS^*}{K}-\frac{\beta\phi S^*I^*}{(\omega+\phi S^*+\alpha I^*)^2},\ a_2=\frac{\beta S^*(\omega+\phi S^*)}{(\omega+\phi S^*+\alpha I^*)^2},\ a_3=\frac{\beta I^*(\omega+\alpha I^*)}{(\omega+\phi S^*+\alpha I^*)^2},\ a_\tau=e^{-\mu\tau},\ m=\varepsilon+\lambda_d+\lambda \tag{9}$$

and μ is the eigenvalue. The characteristic equation is obtained as

$$\mu^2+(a_1+m)\mu+a_1m-a_2(\mu+a_1-a_3)e^{-\mu\tau}=0 \tag{10}$$

For $\tau=0$, the transcendental equation (10) become

$$\mu^2+(a_1+m-a_2)\mu+a_2a_3+a_1(m-a_2)=0$$

with negative real roots if and only if $a_1+m-a_2>0$ and $a_2a_3+a_1(m-a_2)>0$. Using (4) and (9) indicate

$$\begin{aligned}a_2a_3+a_1(m-a_2)&=\frac{\beta^2S^*I^*(\omega+\phi S^*)(\omega+\alpha I^*)}{(\omega+\phi S^*+\alpha I^*)^4}+\frac{\beta S^*\alpha I^*}{(\omega+\phi S^*+\alpha I^*)^2}\left[\frac{rS^*}{K}-\frac{\beta\phi S^*I^*}{(\omega+\phi S^*+\alpha I^*)^2}\right]\\&=\frac{\beta r\alpha I^*(S^*)^2}{K(\omega+\phi S^*+\alpha I^*)^2}+\frac{\beta^2S^*I^*}{(\omega+\phi S^*+\alpha I^*)^4}\left[(\omega+\phi S^*)(\omega+\alpha I^*)-\phi\alpha S^*I^*\right]\\&=\frac{\beta r\alpha I^*(S^*)^2}{K(\omega+\phi S^*+\alpha I^*)^2}+\frac{\beta^2\omega S^*I^*}{(\omega+\phi S^*+\alpha I^*)^3}>0\end{aligned}$$

and

$$\begin{aligned}a_1+m-a_2&=\frac{rS^*}{K}-\frac{\beta\phi S^*I^*}{(\omega+\phi S^*+\alpha I^*)^2}+\frac{\beta S^*}{\omega+\phi S^*+\alpha I^*}-\frac{\beta S^*(\omega+\phi S^*)}{(\omega+\phi S^*+\alpha I^*)^2}\\&=\frac{rS^*}{K}+\frac{\beta S^*I^*(\alpha-\phi)}{(\omega+\phi S^*+\alpha I^*)^2}\end{aligned}$$

Let $(H_1):a_1+m-a_2>0$ and $(H_2):a_1+m-a_2<0$ then if the condition (H_1) is satisfied the endemic steady state is linearly asymptotically stable for $\tau=0$ but unstable when (H_2) is satisfied. Therefore, when the condition (H_2) is satisfied, it will remain unstable for all $\tau\geq0$ (see Figures 1(b) and 3).

Analysing the stability of the endemic steady state to check if it will lose its stability for $\tau>0$ when the condition (H_1) is satisfied implies setting $\mu=ix$ since for $\tau=0$ it is clear that $\mu=0$ is not a solution of the characteristic equation (10). This generates the equation

$$\begin{aligned}-x^2+i(a_1+m)x+a_1m&=a_2(ix+a_1-a_3)e^{-ix\tau}\\&=a_2(ix+a_1-a_3)[\cos(x\tau)-i\sin(x\tau)]\end{aligned}$$

Therefore, equating real and imaginary parts give

$$\begin{aligned}-x^2+a_1m&=a_2(a_1-a_3)\cos(x\tau)+a_2x\sin(x\tau)\\(a_1+m)x&=a_2x\cos(x\tau)-a_2(a_1-a_3)\sin(x\tau)\end{aligned}\tag{11}$$

and summing the squares of the two equations in (11) generates

$$x^4+[a_1^2+(m+a_2)(m-a_2)]x^2+[a_2a_3+a_1(m-a_2)][a_1m+a_2(a_1-a_3)]=0$$

The following equations are the results of expressing (11) in terms of $\sin(x\tau)$ and $\cos(x\tau)$:

$$\begin{aligned}\sin(x\tau)&=\frac{x[a_3m-a_1(a_1-a_3)]-x^3}{a_2[x^2+(a_1-a_3)^2]}\\\cos(x\tau)&=\frac{x^2(a_3+m)+a_1m(a_1-a_3)}{a_2[x^2+(a_1-a_3)^2]}\end{aligned}\tag{12}$$

which gives

$$\tau_n=\frac{1}{x}\left[\cos^{-1}\left(\frac{x^2(a_3+m)+a_1m(a_1-a_3)}{a_2[x^2+(a_1-a_3)^2]}\right)+2\pi n\right],\qquad n=0,1,2,\cdots$$

Let $z_1=a_1^2+(m+a_2)(m-a_2)$ and $z_2=[a_2a_3+a_1(m-a_2)][a_1m+a_2(a_1-a_3)]$ then the Hopf frequency equation become

$$f(x)=x^4+z_1x^2+z_2=0$$

$$\Rightarrow\ f'(x)=4x^3+2z_1x=2x(2x^2+z_1)$$

and from (4) and (9) it is obvious that $m-a_2>0$. Hence, $z_1>0$ which implies that $f'(x)>0$. Next, considering τ as the Hopf parameter, without any loss of generality, it is assumed that there are four distinct positive real roots, x_i, $i=1,\cdots,4$, which implies that for each x

$$\tau_{j,n} = \frac{1}{x_j}\left[\cos^{-1}\left(\frac{x_j^2(a_3+m)+a_1m(a_1-a_3)}{a_2[x_j^2+(a_1-a_3)^2]}\right)+2\pi(n-1)\right], \qquad j=1,\cdots,4, \quad n\in\aleph$$

and

$$\tau_0 = \tau_{j_0,n_0} = \min_{1\le j\le 4, n\ge 1}\{\tau_{j,n}\}, \quad x_0 = x_{j_0} \tag{13}$$

Investigating if the endemic steady state actually undergoes Hopf bifurcation at $\tau=\tau_0$, the transcendental equation (10) is differentiated with respect to τ in order to obtain the sign of $d[\mathrm{Re}(\mu)]/d\tau$. This gives

$$[2\mu+(a_1+m)]\frac{d\mu}{d\tau} = \left[a_2e^{-\mu\tau}-\tau a_2(\mu+a_1-a_3)e^{-\mu\tau}\right]\frac{d\mu}{d\tau} - \mu a_2(\mu+a_1-a_3)e^{-\mu\tau}$$

$$\Rightarrow \left(\frac{d\mu}{d\tau}\right)^{-1} = \frac{a_2e^{-\mu\tau}-2\mu-(a_1+m)}{a_2[\mu^2+\mu(a_1-a_3)]e^{-\mu\tau}} - \frac{\tau}{\mu}$$

Therefore, evaluating at $\tau=\tau_0$ and $\mu=ix_0$ gives

$$\left(\frac{d\mu}{d\tau}\right)^{-1}\Big|_{\tau=\tau_0} = \frac{a_2e^{-ix_0\tau_0}-2ix_0-(a_1+m)}{a_2[ix_0(a_1-a_3)-x_0^2]e^{-ix_0\tau_0}} - \frac{\tau_0}{ix_0}$$

$$= \frac{a_2[\cos(x_0\tau_0)-i\sin(x_0\tau_0)]-2ix_0-(a_1+m)}{a_2[ix_0(a_1-a_3)-x_0^2][\cos(x_0\tau_0)-i\sin(x_0\tau_0)]} - \frac{\tau_0}{ix_0}$$

Collecting the real part and substituting (12) gives the following result after simplification:

$$\mathrm{Re}\left(\frac{d\mu}{d\tau}\right)^{-1}\Big|_{\tau=\tau_0} = \frac{2x_0^5+[z_1+2(a_1-a_3)^2]x_0^3+z_1(a_1-a_3)^2x_0}{a_2^2x_0[x_0^2+(a_1-a_3)^2]^2}$$

which implies

$$\mathrm{Re}\left(\frac{d\mu}{d\tau}\right)^{-1}\Big|_{\tau=\tau_0} = \frac{2x_0(2x_0^2+z_1)}{2x_0a_2^2[x_0^2+(a_1-a_3)^2]} = z_x f'(x_0)$$

where $z_x=[2x_0a_2^2(x_0^2+(a_1-a_3)^2)]^{-1}>0$. Consequently,

$$\mathrm{sign}\left\{\frac{d[\mathrm{Re}(\mu)]}{d\tau}\right\}\Big|_{\tau=\tau_0} = \mathrm{sign}\left\{\mathrm{Re}\left(\frac{d\mu(\tau_0)}{d\tau}\right)^{-1}\right\} = \mathrm{sign}\{z_x f'(x_0)\} = \mathrm{sign}\{f'(x_0)\}.$$

Hence, the result can be summarized as the following theorem:

Theorem 3: *For $R_0>1$, let τ_0, x_0 be as defined in (13) and the condition (H_1) satisfied, then the endemic steady state of the model (1) is linearly asymptotically stable for $\tau<\tau_0$, undergoes Hopf bifurcation at $\tau=\tau_0$ by oscillating periodically and is unstable for $\tau>\tau_0$. Whereas for the condition (H_2), the endemic steady state is unstable for all $\tau\ge 0$.*

4. Numerical simulation

The numerical study for the stability region of the model (1) using some varied parameter values with respect to the incubating period of the disease was carried out via MATLAB application. The extracted results were then applied on GNUPLOT software for smoother output. The outcome generated is captured in Figure 1 while a magnified replicates of Figure 1(a) and (d) for smaller range of the parameter values are represented in Figure 2(a) and (b) respectively. The results show the different stability regions of the model (1) which are distinguished by colours displayed using the colour bar denoting the values of the eigenvalues generated from solving the model around the endemic steady state. The dark blue and blue colours indicate stable region for the endemic steady state with $R_0>1$, while the unstable region is captured by the range of colours, green to dark red as shown on the colour bar. The portions with white colour signify region of non-existence of the endemic steady state, that is $R_0<1$, whereas the disease-free steady state exists and is stable.

Figure 1 indicates that with very small values of ω, ϕ and λ_d (that is, the inhibitory parameters measuring the impact of awareness stemming from global sources such as the media, awareness campaigns, and the awareness springing from the susceptible respectively, and the disease-related death rate), the endemic steady state is stable for small incubating period τ and become unstable as the incubating period increases. But as these parameter values increases respectively the system remains stable for all values of τ and eventually the disease is gradually eradicated. In the contrast, with little value of the inhibitory parameter α, measuring the influence of awareness emerging from the number of infected cases, and ε denoting the recovery rate, the endemic steady state is unstable for all values of τ but become stable as the values of α and ε increases. Eventually the disease is eradicated as the recovery rate becomes large enough which is in contrast to α. As regards the gradual increment in the transmission rate of the disease, β the system has an initial stable disease-free steady state which eventually transit to an endemic steady state that is stable for all values of τ. But as β increases the system become unstable signifying increase in the spread of the disease.

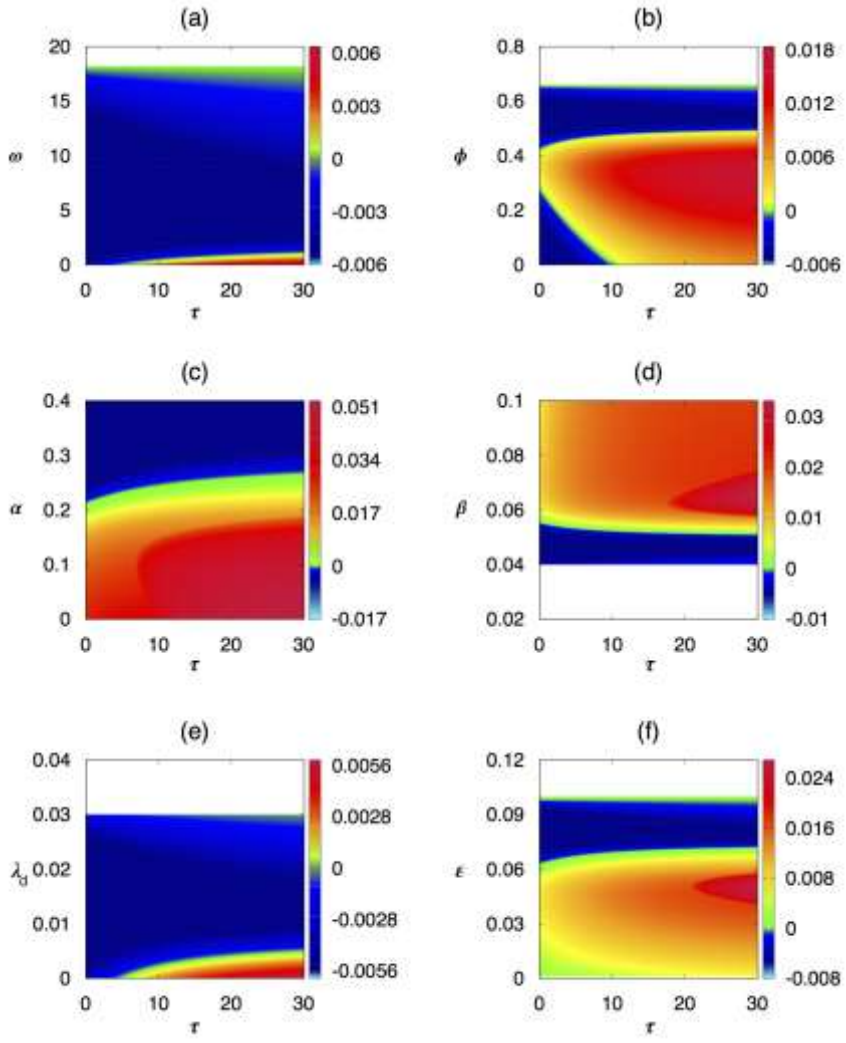

Figure 1: Stability region for the endemic steady state of the model (1) using some parameters against the period of incubation, τ. Parameter values used are $r=0.084$, $\beta=0.052$, $\omega=0.6$, $\phi=0.48$, $\alpha=0.25$, $\varepsilon=0.07$, $\lambda_d=0.003$, $\lambda=0.0056$, $K=100$. Whereas varied parameters are captured respectively in (a)-(f). The dark blue and blue colours indicate stable region, while unstable region is captured by the range of colours; green to dark red as shown on the colour bar representing the eigenvalues. The portions with white colour signify region of non-existence.

In Figure 3, the results obtained from the dynamics of the model (1) using different values of ϕ for some specific values of τ confirmed the statement in Theorem 2 and Theorem 3 and also the result in Figure 1(b). For $\phi=0.1$, $a_1+m-a_2=0.0203$ and $R_0=6.2413$ which implies that the condition (H_1) is satisfied and the stable endemic steady state exhibits Hopf bifurcation at a certain time within the incubating period (the critical value) and thus become unstable. Similarly, for the value of $\phi=0.5$ the condition (H_1) is also satisfied with $a_1+m-a_2=0.0198$ and $R_0=1.3075$ but the endemic steady state remains stable for all $\tau\geq 0$. Whereas for

$\phi = 0.35$, $a_1 + m - a_2 = -0.0042$ and $R_0 = 1.8584$ therefore the condition (H_2) is satisfied while $\phi = 0.7$ gave a disease-free steady state with $R_0 = 0.9371$ and is stable for all $\tau \geq 0$ hence the outcome in Figure 3 which confirms the result in Figure 1(b). The dynamic of the model (1) for the endemic steady state with $R_0 > 1$ is captured by Figure 4 for different values of the parameter τ while the result in Figure 5 represents the outcome for the disease-free steady state with $R_0 < 1$.

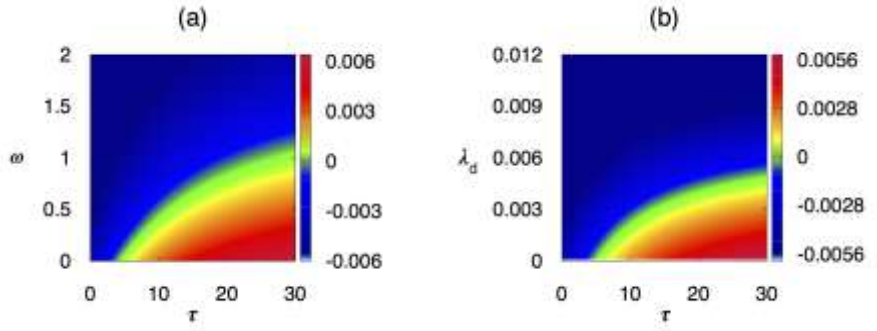

Figure 2: Magnified replicates of Figure 1(a) and 1(d) are represented in (a) and (b) respectively for smaller range of values. The other parameter values used are as defined in Figure 1.

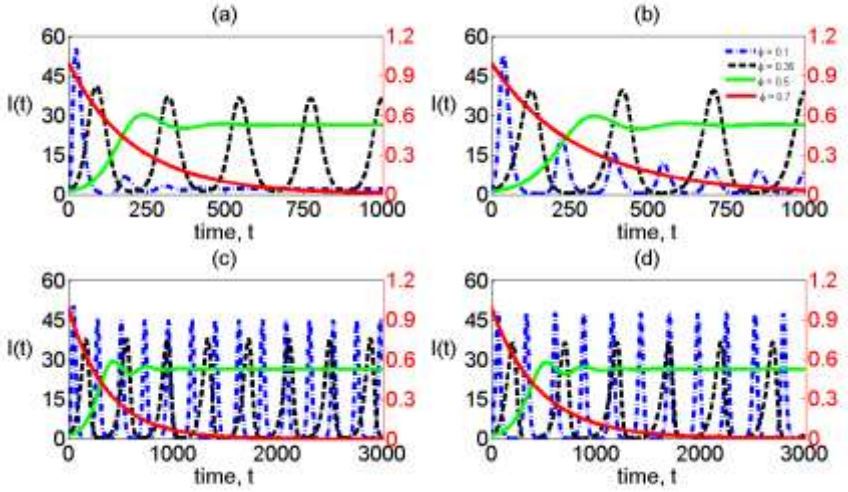

Figure 3: Showing the dynamics of the model (1) for the infective individuals with respect to different values of ϕ using (a) $\tau = 1$ (b) $\tau = 6$ (c) $\tau = 12$ (d) $\tau = 18$. The values for other parameters used are as defined in Figure 1.

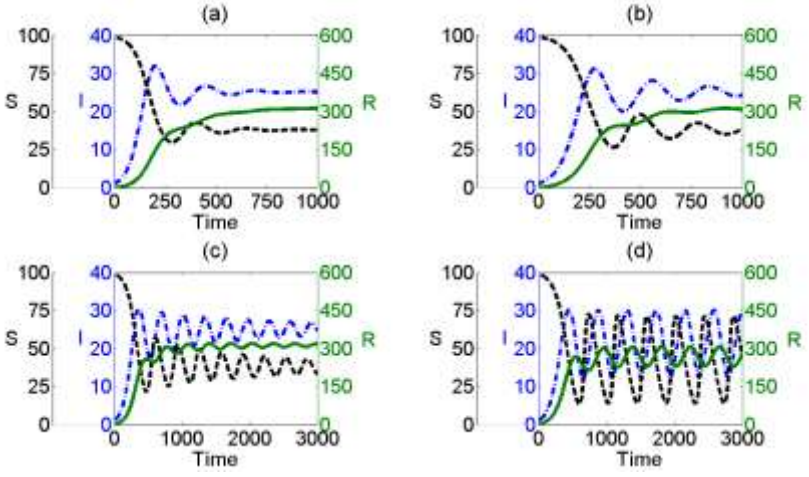

Figure 4: Dynamics of the model (1) for the endemic steady state with $R_0 = 1.3613$ using (a) $\tau = 1$ (b) $\tau = 6$ (c) $\tau = 12$ (d) $\tau = 18$. The parameter values are same as those defined in Figure 1.

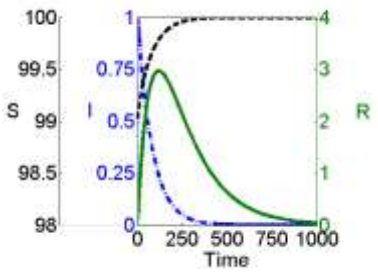

Figure 5: Dynamics of the model (1) showing the disease-free steady state when $R_0 = 0.8377$. The parameter values used are same as the ones defined in Figure 1 except that $\beta = 0.032$.

5. Conclusion

The study has derived a mathematical model for analysing the spread of infectious disease using saturated incidence rate to evaluate the impact of awareness dissemination alongside the incubating period of the disease. Based on the outcome of the study, the impact of awareness generally on the incidence rate indicates that increasing influence from awareness dissemination, especially the susceptible individuals adhering to available information, tends to minimize the spread of the disease with a likelihood of disease eradication. Consequently, it is pertinent to intensify the creation of awareness and to ensure that the populace adhere to the preventive measures such as wearing of face masks, social distancing and regular hand washing in order to curtail the spread and possibly eradicate the infectious disease.

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