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Fuzzy AHP and Modified Fuzzy TOPSIS based Supplier Selection Model

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Abstract

The methods of AHP and Fuzzy AHP provide supports for decision making process, go through normalization procedure, produce different values for decision criteria weights and finally determine decision result. Interestingly both the method produces same decision result in various cases. The model for supplier selection showed by Foriborz Jolai (2011) based on AHP with TOPSIS (Technique for Order of Preference by Similarity to Ideal Solution) is a matter of modification replacing by Fuzzy AHP and Modified Fuzzy TOPSIS methods in order to introduce 'Fuzzification' and 'Defuzzification' which is not available in existing model. The proposed model is verified with an illustrative example and comparing the results generated by both the existing and proposed model in consensus decision making.

Keywords: AHP, Fuzzy, MCDM, TOPSIS, Supplier Selection

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1. Introduction

Fuzzy logic is a tool used to control various processes ranging from medical diagnosis to engineering process control. This tool is used as mode of reasoning underlying approximate not exact. Fuzzy logic is a mode of reasoning that deals with approximate not precise. Let's say, 'Usually rose is red'. Here 'usually' is a fuzzy proportion of how many times snow is seen white and how many times not- neither all the time nor too few times to say. This is called vagueness that is quantified between 'no times it is seen red' and 'each and every time it is seen red' considering 0 and 1 the two extreme levels respectively. The value of 'usually' may reside between 0 [illegible] Let's say we have [illegible] ely and with set notation it can be written as {(F1, 0.45), (F2, 0.55), (F3, 0), (F4, 0.6)}. This process of fuzzification and fuzzy quantification can also be done using triangular fuzzy numbers like (0, 0.25, 0.5), (0.25, 0.5, 0.75) and (0.5, 0.75, 1.0) for describing 'non red', 'slightly red' and 'full red'. In doing such, we need proper linguistics for proper description. We have flexibility to define the linguistics and fuzzy numerical values for the linguistics on case basis. This is suitably applied in Multi Criteria Decision Making (MCDM) by fuzzy quantification using predefined linguistics for different criteria and normalization using some particular mathematical method. Such a method is Analytical Hierarchical Method or AHP in short. A manager needs to make decision when he has some options available like investing in a project A considering criteria c1 and c2 or investing in project B with consideration of same criteria. The manager may have options to decide with possible return r1 and r2 in project A and B respectively with weight values of the criteria of c1 and c2 are w1 and w2 respectively. Such a case can be resolved making a decision tree. If there is options available to project A and B of return possibilities rA and rB respectively with the condition to the investment variation in sub sectors of sA and sB then the decision for maximizing the project return considering the objective criteria can be achieved using Linear Programming. Obviously this is a deterministic approach.

AHP is the first method that deals with Multi Criteria Decision Making (MCDM) introduced by Thomas Saaty (1970) which generates decision from stochastic data. Fuzzy Logic introduced by Lofti A Jadeh (1988) is another decision making tool that can quantify the vagueness. Fuzzy Logic became a powerful tool to deal with the non deterministic cases for determining results. Fuzzy MCDM, combination of Fuzzy Logic and MCDM method, showed its role for decision making became a more preferable method by the decision makers later. A lot more work have been done on both these methods to apply in various disciplines for implementation in decision making problems.

In an extensive case study of selection of ERP solution providers, decision makers' choices were fed into AHP, Fuzzy AHP and ANP methods. They found the same selection of solution provider among several as per assessors' choice of preference and the result determined. A crucial job is the supplier selection in tendering process through a consensus decision making. The assessment of different members of a bidder selection committee can have different level of preferences and assessment for different criteria. In a consensus decision making, a group of decision makers may submit individual choices of preference against various selection criteria. These preference levels can be quantified according to a particular scale with proper linguistics and fed into a mathematical method like AHP. The resulting value may not be same if these choice levels are fed into another mathematical process like Fuzzy AHP. But we are concerned about the desired selection of particular bidder whether it is same for the both the methods or not in special case of Jolai Model (2011) for supplier selection. Jolai's mode is a three phase model discussed in next section.

The aim of this research is to incorporate Fuzzy TOPSIS (Technique for Order of Preference by Similarity to Ideal Solution) method in Jolai Model for supplier selection which is based on AHP and Fuzzy AHP. The aim also includes the verification of the result generated by the proposed model with existing AHP and AHP TOPSIS model. The objective is to feed a set of preference for the decision criteria and supplier choices and determine the results of mathematical simulation for verification.

2. Supplier Selection Model

Foriborz Jolai (2011) presented his model as a three phase model where Phase 0 is for primary selection of bidders meeting the basic requirements, Phase 1 is for determining weights of decision criteria and Phase 2 is for final selection of bidder according to the decision criteria weights and preferences of individual bidder assessed by the decision makers. This model is shown in Figure 1 whereas Figure 2 depicts the same model with substitution of Fuzzy based methods by AHP based methods.

We have got the mathematical modelling from Jolai model (2011) described here in this section. For evaluating suppliers, the decision criteria could be suited for the process of evaluation of bidders. These are: On-time delivery (C1), Closeness of relationship with supplier (C2), Supplier product/service quality (C3), Supplier operational capability (C4), Price/cost (C5). Fuzzy pair wise decision criteria evaluation matrix using all above criteria (C1~C5) are formed in Table 1 and generalized in Table 2

The decision makers place their preferences for decision criteria weight determination using the scale shown in Table 3 which are substituted in a matrix as shown in Table 1 or Table 2. For normalization of such a matrix, eq 1 can be applied in order to form the normalized matrix after substituting the triangular fuzzy numbers as per ratings by Decision Makers using Linguistics in Table 3.

$$a_{jl} = \left(\prod_{k=1}^{K} a_{jl}^{k}\right)^{1/K} \ , j = 1,2,\dots,n \ , \ l = 1,2,\dots,n \qquad (1)$$

Where K = number of decision makers, a_{jl} = is the normalized matrix. The normalized matrix is further simplified into a row matrix using eq. 2.

$$e_j = (a_{j1}, a_{j2}, \dots, a_{jn})^{1/n} \qquad (2)$$

Using the fuzzy geometric mean technique, the above row matrix can be transformed into fuzzy weight matrix of

W_j.

$$W_j = e_j.(e_1 \oplus e_2 \oplus \ldots \oplus e_n)^{-1} = e_j.\frac{1}{n}\sum_{j=1}^{n} e_j \quad (3)$$

These above equations eq. (1), (2) and (3) are belonged to Phase 1 of Jolai Model depicted in Figure 1 and lets start to describe Phase 2 computation mathematics of same model. Since (a, b, c) be a triangular fuzzy number, the graded mean integration method represents

$$P(A) = \frac{a+4b+c}{6} \quad (4)$$

The linear scale normalization formulas are used for transformation of the various criteria from linguistic variables to equivalent fuzzy numeric values according to scales into normalized values of matrix which is to be used for normalized fuzzy decision making.

Generally speaking, $\mathbb{R} = [r_{ij}]_{m\times n}$

Where $r_{ij} = \left(\frac{a_{ij}}{c_{ij}}, \frac{b_{ij}}{c_{ij}}, \frac{c_{ij}}{c_{ij}}\right) \quad i = 1,2,\ldots,m$ and $r_{ij}^{-} = \left(\frac{a_{ij}^{-}}{c_{ij}}, \frac{b_{ij}^{-}}{c_{ij}}, \frac{c_{ij}^{-}}{c_{ij}}\right) \quad i = 1,2,\ldots,m$ (5)

which leads to

$r_j^{+} = \max\,[r_{ij}]$ and $r_j^{-} = \min\,[r_{ij}]$ where $i = 1,2, \ldots, m$ and $j = 1,2, \ldots, n$ (6)

In this consequence, fuzzy positive ideal solution and fuzzy negative ideal solution implied the formula in eq 7.

$$D_i^{+} = \sum_{j=1}^{n} W_j.d(r_{ij}, r_j^{+}) \;,\; i = 1,2,\ldots,m \quad (7)$$

$$D_i^{-} = \sum_{j=1}^{n} W_j.d(r_{ij}, r_j^{-}) \;,\; i = 1,2,\ldots,m \quad (8)$$

$$C_i = \frac{D_i^{-}}{D_i^{-} + D_i^{+}} \quad i = 1,2,\ldots,m \quad (9)$$

$$CC_{ii} = \frac{C_{i1}+4C_{i2}+C_{i3}}{6} \;,\; i = 1,2,\ldots,m \quad (10)$$

$$R_i = \frac{CC_{ii} - {CC_{ii}}^{min}}{{CC_{ii}}^{max} - {CC_{ii}}^{min}} \;,\; i = 1,2,\ldots,m \quad (11)$$

In this stage, evaluation of alternative bidders for different criteria is the major important task which is the core job function of this particular model. Table 4 is the tool which describes the linguistic variable and their fuzzy equivalent numeric.

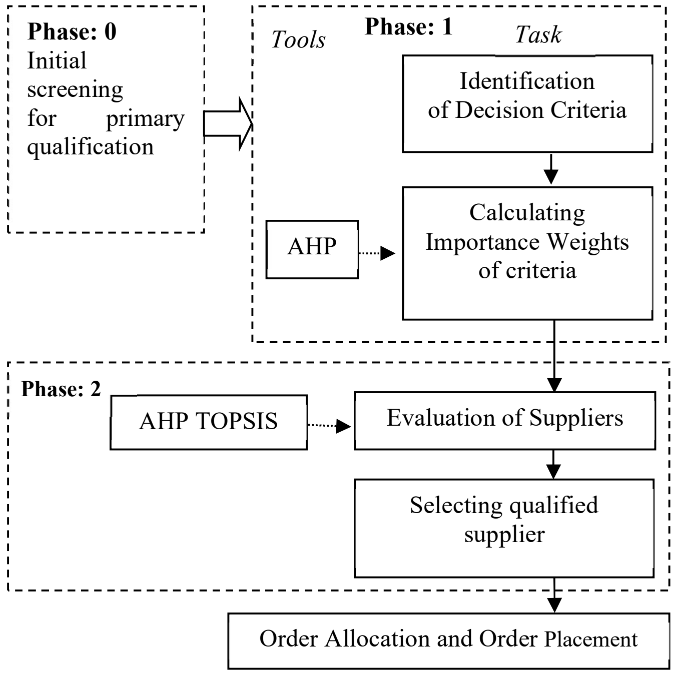

Figure 1. Jolai Model with AHP and TOPSIS approach

3. Proposed Model

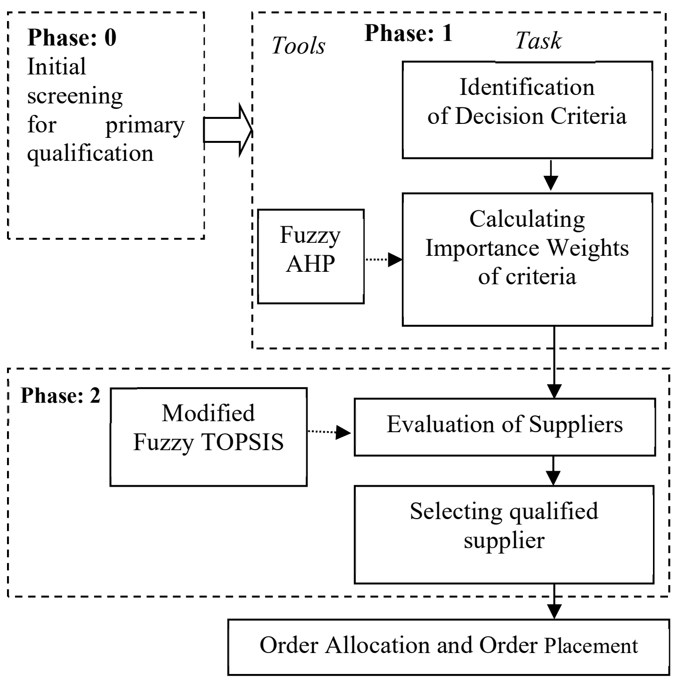

Figure 2. Jolai Model (2011) with Fuzzy AHP and Fuzzy TOPSIS approach

In proposed model, Fuzzy AHP is introduced replacing pure AHP in Phase 1 as a tool in order to determine the values of weights of selection criteria. Besides, Modified Fuzzy TOPSIS is incorporated replacing AHP TOPSIS method in Phase for evaluating the suppliers. The input to the proposed model is nothing but the Fuzzy ratings or Fuzzy values as described in the introduction whereas in the older method there was no scope of Fuzzy input. Now, let us have an illustrative example.

4. Illustrative Example

In a case of procurement, four bidders (B1, B2, B3 & B4) have been primarily selected for final selection of one particular bidder for awarding the contract. In this procurement process, three members committee is considered for decision making. The decision makers are DM1, DM2 and DM3. Firstly, they put their preference weights for each decision criteria according to linguistics and their equivalent numeric in Table 3 then substitute in pair wise comparison matrix shown in Table 5, Table 6 and Table 7. The values of e_j and W_j for the criteria C1 to C5 are calculated using eq 1 and summarized in Table 9. The data set used for illustration is taken from Jolai (2011) illustration.

In phase 2, the decision makers are supposed to put the grading for primarily selected four bidders (B1, B2, B3 & B4) to determine the ranking value of each bidder to finally select a single one. The bidders are evaluated as per assessments of decision makers in the same way in fuzzy comparison matrixes. The decision makers' ratings are shown in Table 10, Table 13, Table 16, Table 19 and Table 22 for criteria C1, C2, C3, C4 and C5 respectively using the scale described in Table 4. Three decision makers' ratings are quantified in Table 11, Table 14, Table 17, Table 20 and Table 23 which are then aggregated in Table 12, Table 15, Table 18, Table 21 and Table 24 respectively.

Each bidder's aggregated fuzzy ratings obtained are summarized in Table 25 for each criteria C1 ~ C5. After that Table 25 is normalized and formed Table 26 and Table 26 to determine the values of r_j^+ and r_j^- which will lead to values of D_i^+ and D_i^- that has provided the value for C_i and CC_i. Hence finally the ranking of the bidders have been obtained with R_i.

The result of bidders' ranking and selected bidder with highest rank shown in Table 31 is determined through the model described in Figure 1. Now using the Jolai Model with AHP and TOPSIS method in Figure 2, we have got the aggregated pair wise comparison matrix for criteria C1~C5 is shown in Table 32. The matrix in Table 32 is normalized by dividing each element by the each respective column sum and a new normalized matrix in Table 34 is obtained. The row averages of Table 34 are mentioned in the same table in additional row and also listed in Table 33 for comparing the Wj values determined in Table 9 of Fuzzy based approach. The comparison of both the approach results are graphically shown in Figure 3, Figure 4 and Figure 5 and found least difference between Fuzzy MCDM and AHP normalization results in determining W_j.

For Phase 2 calculation in Figure 2 model, we have considered the same ratings of the decision maker for four selected bidders whose were primarily selected which are shown in Table 10, Table 13, Table 16, Table 19, Table 22 and the aggregated ratings for all criteria of them for evaluation are in Table 34. Each value of Table 26 is multiplied by AHP generated Wj values {(0.185882, 0.188482, 0.209408), (0.047728, 0.04175, 0.039353), (0.375054, 0.400244, 0.38042), (0.060358, 0.056896, 0.059698), (0.330974, 0.312624, 0.28624)} for C1, C2, C3, C4 and C5 respectively and new matrix is formed in Table 34. New matrix in Table 34 is further normalized dividing each element by the respective column sum and taking the row averages determined the values of AHP Ci. AHP Ci values are converted into AHP CCi using eq. 10 and Ranking values of AHP Ri using eq. 11. Finally AHP and Fuzzy Ranking values are compared with each other as in Table 35 and Table 31 and all graphical interpretations are shown in Figure 6, Figure 7, Figure 8, Figure 9 and Figure 10 successively.

5. Conclusions and Recommendations

Foriborz Jolai (2011) showed the application of Fuzzy MCDM with Fuzzy TOPSIS method very successfully. In this research paper, we have substituted AHP and AHP TOPSIS method by Fuzzy MCDM and Fuzzy TOPSIS method respectively in Forborz Jolai's model for supplier selection and compared the result for the same set of input values both the cases. According to Jolai Model using Fuzzy MCDM and its mathematical outline, the result shows that Bidder B2 is mostly preferred by decision makers and Bidder B3 is the least preference by the decision makers as in Figure 10. Fuzzy MCDM has been applied instead of AHP to method fit into Jolai Model, differences in selection values have been found but same bidder is selected in this case too i.e. Bidder B2 is the selection result of both AHP and Fuzzy MCDM method both. If we like to select one bidder among many, we should select one with highest ranking or top ranked bidder. According to AHP method, Bidder B2 is selected and also the same selection (B2) is seen using Fuzzy MCDM method incorporated in Jolai Model i.e. Fuzzzy MCDM has successfully produce same result as produced by AHP method. Besides Jolai Model computes the values of Di+ and Di- which defines a particular range where optimized selection is resided and then the ranking is determined though human assessments can be varied and fluctuated in greater range. The strong feature of proposed model can clearly be stated that it generates a reliable selection result of supplier and it is independent of available mathematical methods incorporated. Hence proposed model proves its versatility.

Software development and implementation is badly necessary for this model and study in feedback analysis can help to understand the nature of the requirement fulfilment of the real cases in practice. Any changes necessary in decision criteria parameters, or the selection really can meet the satisfaction of practicing organization is the subject to be studied and modification of the model if found is recommended. For order allocation and distribution, Foriborz Jolai suggested for enhancing the model with incorporation of Fuzzy Goal programming approach. Both Fuzzy goal programming and normal goal programming approach can be tested in order allocation process and compared the results for verification. Most important issue is to incorporate the Foriborz Jolai's supplier selection model in e-GP Access Model which is a world recognized model of electronic procurement. e-GP Access model is recognized by World Bank and implemented in various countries that ensures transparency in public procurement cases that brings huge benefits for the underdeveloped countries. But the e-GP Access model does have the lacking of consensus decision making for selection of bidders participating in public tenders. Incorporation of supplier selection model in e-GP will make it more methodological and transparent in procurement practice.

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Notes

Note 1. Donegan H. A. and Dodd F. J. (1991) 'A Note on Saaty's Random Indexes', *Mathl. Comput. Modelling* Vol. 15, No. 10, pp. 135-137, 1991

Table 1. Fuzzy pair wise decision criteria evaluation matrix

	C1	C2	C3	C4	C5
C1	W11	W12	W13	W14	W15
C2	W21	W22	W23	W34	W25
C3	W31	W32	W33	W34	W35
C4	W41	W42	W43	W44	W45
C5	W51	W52	W53	W54	W55

Table 2. Fuzzy pair wise decision criteria evaluation matrix for n criteria

	C1	C2	………	Cn
C1	W11	W12	………	W1n
C2	W21	W22	………	W2n
. . .	. .	. .	. .	. .
Cn	Wn1	Wn2	………	Wnn

Table 3. Linguistic variables for pair wise comparisons of decision criteria and their triangular fuzzy values

Linguistic Variable	**Three variable fuzzy numbers**
Equal importance	(1,1,3)
Moderately more important	(1,3,5)
Strongly more important	(3,5,7)
Very strongly important	(5,7,9)
Extremely more important	(7,9,9)

Table 4. Linguistic variables for rating of alternatives with respect to each criterion

Linguistic Variable	Three Variable Fuzzy Number
Very Poor (VP)	(0, 0, 1)
Poor (P)	(0, 1, 3)
Medium Poor (MP)	(1, 3, 5)
Fair (F)	(3,5,7)
Medium Good (MG)	(5, 7, 9)
Good (G)	(7, 9, 10)
Very Good (VG)	(9, 10, 10)

Table 5. The fuzzy pair wise comparision matrix of cirteria (decisin maker 1)

	C1	C2	C3	C4	C5
C1	(1,1,1)	(5,7,9)	(1/5,1/3,1)	(3,5,7)	(1/3,1,1)
C2	(1/9,1/7,1/5)	(1,1,1)	(1/9,1/9,1/7)	(1/5,1/3,1)	(1/9,1/7,1/5)
C3	(1,3,5)	(7,9,9)	(1,1,1)	(3,5,7)	(1,3,5)
C4	(1/7,1/5,1/3)	(1,3,5)	(1/7,1/5,1/3)	(1,1,1)	(1/7,1/5,1/3)
C5	(1,1,3)	(5,7,9)	(1/5,1/3.1)	(3,5,7)	(1,1,1)

Table 6. The fuzzy pair wise comparision matrix of cirteria (decisin maker 2)

	C1	C2	C3	C4	C5
C1	(1,1,1)	(3,5,7)	(1/5,1/3,1)	(5,7,9)	(1/5,1/3,1)
C2	(1/7,1/5,1/3)	(1,1,1)	(1/9,1/7,1/5)	(1,1,3)	(1/9,1/7,1/5)
C3	(1,3,5)	(5,7,9)	(1,1,1)	(5,7,9)	(1/3,1,1)
C4	(1/9,1/7,1/5)	(1/3,1,1)	(1/9,1/7,1/5)	(1,1,1)	(1/7,1/5,1/3)
C5	(1,3,5)	(5,7,9)	(1,1,3)	(3,5,7)	(1,1,1)

Table 7. The fuzzy pair wise comparision matrix of cirteria (decisin maker 3)

	C1	C2	C3	C4	C5
C1	(1,1,1)	(5,7,9)	(1/7,1/5,1/3)	(1,3,5)	(1/5,1/3,1)
C2	(1/9,1/7,1/5)	(1,1,1)	(1/9,1/7,1/5)	(1/3,1,1)	(1/9,1/7,1/5)
C3	(3,5,7)	(5,7,9)	(1,1,1)	(3,5,7)	(1,1,3)
C4	(1/5,1/3,1)	(1,1,3)	(1/7,1/5,1/3)	(1,1,1)	(1/9,1/7,1/5)
C5	(1,3,5)	(5,7,9)	(1/3,1,1)	(5,7,9)	(1,1,1)

Table 8. Aggregated Fuzzy pair wise comparison matrix of criteria (over all decision makers)

	C1	C2	C3	C4	C5
C1	(1.0,1.0,1.0)	(4.16, 6.14, 8.10)	(0.18, 0.28, 0.70)	(2.44, 4.65, 6.67)	(0.24, 0.48, 1.0)
C2	(0.12, 0.16, 0.24)	(1.0,1.0,1.0)	(0.11, 0.13, 0.18)	(0.41, 0.70, 1.44)	(0.11, 0.15, 0.20)
C3	(1.44, 3.51, 5.50)	(5.50, 7.46, 8.80)	(1.0,1.0,1.0)	(3.51, 5.50, 7.46)	(0.70, 1.44, 2.44)
C4	(0.15, 0.22, 0.41)	(0.70, 1.44, 2.44)	(0.13, 0.18, 0.28)	(1.0,1.0,1.0)	(0.13, 0.18, 0.28)
C5	(1.44, 2.44, 3.51)	(4.92, 6.87, 8.80)	(0.41, 0.70, 1.44)	(3.51, 5.50, 7.46)	(1.0,1.0,1.0)

Table 9. e_j values and fuzzy importance weights W_j

Criteria	e_j	W_j
C1	(0.85, 1.31, 2.07)	(0.13, 0.18, 0.20)
C2	(0.23, 0.29,0.42)	(0.03, 0.04, 0.04)
C3	(1.81, 2.91, 3.88)	(0.27, 0.40, 0.38)
C4	(2.28, 0.40, 0.60)	(0.34, 0.06, 0.06)
C5	(1.59, 2.30, 3.19)	(0.24, 0.32, 0.32)

Table 10. The rating of four selected bidders by decision makers for criteria C1

Alternative bidders	Decision Makers (DM)		
	DM 1	DM 2	DM 3
B1	G	VG	VG
B2	VG	G	F
B3	F	MG	G
B4	MG	F	G

Table 11. Three variable fuzzy rating for criteria C1

Alternative bidders	Decision Makers (DM)		
	DM 1	DM 2	DM 3
B1	(7, 9, 10)	(9, 10, 10)	(9, 10, 10)
B2	(9,10,10)	(7, 9, 10)	(3, 5, 7)
B3	(3,5,7)	(5, 7, 9)	(7, 9, 10)
B4	(5, 7, 9)	(3, 5, 7)	(7, 9, 10)

Table 12. Three variable fuzzy rating for criteria C1

Alternative bidders	DM ratings aggregated
B1	(8.10, 9.44, 9.77)
B2	(5.64, 7.51, 8.69)
B3	(4.65, 6.67, 8.39)
B4	(4.65, 6.67, 8.39)

Table 13. The rating of four selected bidders by decision makers for criteria C2

Alternative bidders	Decision Makers (DM)		
	DM 1	DM 2	DM 3
B1	F	VG	F
B2	VG	MG	VG
B3	MG	G	VG
B4	G	MG	F

Table 14. Three variable fuzzy rating for criteria C2

Alternative bidders	Decision Makers (DM)		
	DM 1	DM 2	DM 3
B1	(3, 5, 7)	(9, 10, 10)	(3, 5, 7)
B2	(9, 10, 10)	(5, 7, 9)	(9, 10, 10)
B3	(5, 7, 9)	(7, 9, 10)	(9, 10, 10)
B4	(7, 9, 10)	(5, 7, 9)	(3, 5, 7)

Table 15. Three variable fuzzy rating for criteria C2

Alternative bidders	DM ratings aggregated
B1	(4.26, 6.18, 7.72)
B2	(7.25, 8.69, 9.44)
B3	(6.67, 8.39, 9.44)
B4	(4.65, 6.67, 6.98)

Table 16. The rating of four selected bidders by decision makers for criteria C3

Alternative bidders	Decision Makers (DM)		
	DM 1	DM 2	DM 3
B1	MG	F	VG
B2	VG	G	MG
B3	G	G	G
B4	MG	VG	G

Table 17. The rating of four selected bidders by decision makers for criteria C3

Alternative bidders	Decision Makers (DM)		
	DM 1	DM 2	DM 3
B1	(5, 7, 9)	(3, 5, 7)	(9, 10, 10)
B2	(9, 10, 10)	(7, 9, 10)	(5, 7, 9)
B3	(7, 9, 10)	(7, 9, 10)	(7, 9, 10)
B4	(5, 7, 9)	(9, 10, 10)	(7, 9, 10)

Table 18. Three variable fuzzy rating for criteria C3

Alternative bidders	DM ratings aggregated
B1	(5.05, 6.91, 8.39)
B2	(6.67, 8.39, 9.44)
B3	(6.87, 8.80, 9.77)
B4	(6.67, 8.39, 9.44)

Table 19. The rating of four selected bidders by decision makers for criteria C4

Alternative bidders	Decision Makers (DM)		
	DM 1	DM 2	DM 3
B1	P	F	G
B2	VG	F	G
B3	G	G	F
B4	F	P	P

Table 20. The rating of four selected bidders by decision makers for criteria C4

Alternative bidders	Decision Makers (DM)		
	DM 1	DM 2	DM 3
B1	(0, 1, 3)	(3, 5, 7)	(7, 9, 10)
B2	(9, 10, 10)	(3, 5, 7)	(7, 9, 10)
B3	(7, 9, 10)	(7, 9, 10)	(3, 5, 7)
B4	(3, 5, 7)	(0, 1, 3)	(0, 1, 3)

Table 21. Three variable fuzzy rating for criteria C4

Alternative bidders	DM ratings aggregated
B1	(0.0, 3.51, 5.84)
B2	(5.64, 7.51, 8.69)
B3	(5.19, 7.25, 8.69)
B4	(0.0, 1.70, 3.92)

Table 22. The rating of four selected bidders by decision makers for criteria C5

Alternative bidders	Decision Makers (DM)		
	DM 1	DM 2	DM 3
B1	MG	MG	F
B2	G	VG	VG
B3	G	G	MG
B4	VP	P	F

Table 23. The rating of four selected bidders by decision makers for criteria C5

Alternative bidders	Decision Makers (DM)		
	DM 1	DM 2	DM 3
B1	(5, 7, 9)	(5, 7, 9)	(3, 5, 7)
B2	(7, 9, 10)	(9, 10, 10)	(9, 10, 10)
B3	(7, 9, 10)	(7, 9, 10)	(5, 7, 9)
B4	(0, 0, 1)	(0, 1, 3)	(3, 5, 7)

Table 24. Three variable fuzzy rating for criteria C5

Alternative bidders	DM ratings aggregated
B1	(4.16, 6.14, 8.10)
B2	(8.10, 9.44, 9.77)
B3	(5.50, 8.10, 9.44)
B4	(0.0, 0.0, 2.73)

Table 25. Aggregated fuzzy ratings for all criteria of selected bidders for fuzzy evaluation

	C1	C2	C3	C4	C5
B1	(8.10, 9.44, 9.77)	(4.26, 6.18, 7.72)	(5.05, 6.91, 8.39)	(0.0, 3.51, 5.84)	(4.16, 6.14, 8.10)
B2	(5.64, 7.51, 8.69)	(7.25, 8.69, 9.44)	(6.67, 8.39, 9.44)	(5.64, 7.51, 8.69)	(8.10, 9.44, 9.77)
B3	(4.65, 6.67, 8.39)	(6.67, 8.39, 9.44)	(6.87, 8.80, 9.77)	(5.19, 7.25, 8.69)	(5.50, 8.10, 9.44)
B4	(4.65, 6.67, 8.39)	(4.65, 6.67, 6.98)	(6.67, 8.39, 9.44)	(0.0, 1.70, 3.92)	(0.0, 0.0, 2.73)

Table 26. Normalized matrix of fuzzy ratings for all criteria of selected bidders

	C1	C2	C3	C4	C5
B1	(0.81, 0.94, 0.98)	(0.43, 0.62, 0.77)	(0.51, 0.69, 0.84)	(0.0, 0.35, 0.58)	(0.42, 0.61, 0.81)
B2	(0.56, 0.75, 0.87)	(0.73, 0.87, 0.94)	(0.67, 0.84, 0.94)	(0.56, 0.75, 0.87)	(0.81, 0.94, 0.98)
B3	(0.47, 0.67, 0.84)	(0.67, 0.84, 0.94)	(0.69, 0.88, 0.98)	(0.52, 0.73, 0.87)	(0.55, 0.81, 0.94)
B4	(0.47, 0.67, 0.84)	(0.47, 0.67, 0.70)	(0.67, 0.84, 0.94)	(0.0, 0.17, 0.39)	(0.0, 0.0, 0.27)
W_j	(0.13, 0.18, 0.20)	(0.03, 0.04, 0.04)	(0.27, 0.40, 0.38)	(0.34, 0.06, 0.06)	(0.24, 0.32, 0.32)

Table 27. r_{ij} matrix

	C1	C2	C3	C4	C5
B1	0.83, 0.96, 1.0	0.56, 0.81, 1.0	0.61, 0.82, 1.0	0.0, 0.60, 1.0	0.52, 0.75, 1.0
B2	0.64, 0.86, 1.0	0.78, 0.93, 1.0	0.71, 0.89, 1.0	0.64, 0.86, 1.0	0.83, 0.96, 1.0
B3	0.56, 0.80, 1.0	0.27, 0.10, 1.0	0.70, 0.90, 1.0	0.60, 0.84, 1.0	0.59, 0.86, 1.0
B4	0.56, 0.80, 1.0	0.67, 0.96, 1.0	0.71, 0.89, 1.0	0.0, 0.44, 1.0	0.0, 0.0, 1.0

$$r_j^+ = \max[r_{ij}] = (0.83, 0.96, 1.0)$$
$$r_j^- = \min[r_{ij}] = (0.0, 0.0, 1.0)$$

Table 28. r_{ij} and W_j

r_j^+	(0.83, 0.96, 1.0)	(0.78, 0.96, 1.0)	(0.71, 0.90, 1.0)	(0.64, 0.86, 1.0)	(0.83, 0.96, 1.0)
	r_j^+ = (0.83, 0.96, 1.0)				
r_j^-	(0.56, 0.80, 1.0)	(0.27, 0.10, 1.0)	(0.61, 0.82, 1.0)	(0.0, 0.44, 1.0)	(0.0, 0.0, 1.0)
	r_j^- = (0.0, 0.0, 1.0)				
W_j	(0.13, 0.18, 0.20)	(0.03, 0.04,0.04)	(0.27, 0.40,0.38)	(0.34, 0.06, 0.06)	(0.24, 0.32, 0.32)

Table 29. D_i^+ calculation

	C1	C2	C3	C4	C5
B1	(0,0,0)	(0.0081, 0.006, 0)	(0.0594, 0.056, 0)	(0.2822, 0.0216, 0)	(0.0744, 0.0672, 0)
B2	(0.0247, 0.018, 0)	(0.0015, 0.0012, 0)	(0.0324, 0.028, 0)	(0.0646, 0.006, 0.01)	(0,0,0)
B3	(0.0351, 0.0288, 0)	(0.0168, 0.0344, 0)	(0.0351, 0.024, 0)	(0.0782, 0.0072, 0)	(0.0576, 0.032, 0)
B4	(0.0351, 0.0288, 0)	(0.0048, 0,0)	(0.0324, 0.028, 0)	(0.2822, 0.0312, 0)	(0.1992, 0.3072, 0)

Table 30. D_i^- calculation

	C1	C2	C3	C4	C5
B1	(0,0,0)	(0.00024, 0.00024, 0)	(0.01604, 0.0224, 0)	(0.09595, 0.0013, 0)	(0.01786, 0.0215, 0)
B2	(0.00321, 0.00324, 0)	(0.00005, 0.00005, 0)	(0.00875, 0.0112, 0)	(0.02196, 0.00036, 0)	(0, 0, 0)
B3	(0.00456, 0.00518, 0)	(0.0005, 0.00138, 0)	(0.00948, 0.0096, 0)	(0.02659, 0.00043, 0)	(0.01382, 0.01024, 0)
B4	(0.00456, 0.00518, 0)	(0.00014, 0, 0)	(0.00875, 0.0112, 0)	(0.09595, 0.00187, 0)	(0.04781, 0.0983, 0)

Table 31. Final evaluation of alternatives

	D_i^+	D_i^-	$C_i = \frac{D_i^-}{D_i^- + D_i^+}$	CC_i	R_i	Rank
B1	(0.4241, 0.1508, 0)	(0.13009, 0.02328, 0)	(0.2347389, 0.1543767, 0)	0.142041	0.0526	3
B2	(0.1232, 0.0532, 0)	(0.03397, 0.01485, 0)	(0.7338518, 0.21822189, 0)	0.267790	1.0	1
B3	(0.2228, 0.1264, 0)	(0.05495, 0.02683, 0)	(0.197839, 0.15323, 0)	0.135127	0.0	4
B4	(0.5537, 0.3952, 0)	(0.15721, 0.11655, 0)	(0.221139, 0.227748)	0.188688	0.406	2

Table 32. Aggregated pair wise comparison matrix of criteria (over all decision makers) normalized by AHP Method

	C1	C2	C3	C4	C5
C1	(0.24096, 0.13643, 0.09381)	(0.25553, 0.26801, 0.27797)	(0.09836, 0.12227, 0.19444)	(0.22447, 0.26801, 0.27757)	(0.11009, 0.14769, 0.20325)
C2	(0.02892, 0.02183, 0.02251)	(0.06143, 0.04365, 0.03432)	(0.06011, 0.05677, 0.05)	(0.03772, 0.04035, 0.05993)	(0.05046, 0.04615, 0.04065)
C3	(0.34698, 0.47885, 0.51595)	(0.33784, 0.32562, 0.30199)	(0.54644, 0.43668, 0.27778)	(0.32291, 0.317, 0.31045)	(0.3211, 0.44307, 0.49593)
C4	(0.03614, 0.03001, 0.03846)	(0.04299, 0.06285, 0.08373)	(0.07104, 0.0786, 0.07778)	(0.09199, 0.05764, 0.04161)	(0.05963, 0.05538, 0.05691)
C5	(0.34699, 0.33288, 0.32927)	(0.30221, 0.29987, 0.30199)	(0.22404, 0.30568, 0.4)	(0.32291, 0.317, 0.31045)	(0.45872, 0.30769, 0.20325)

Table 33. Comparison of decision criteria weight values

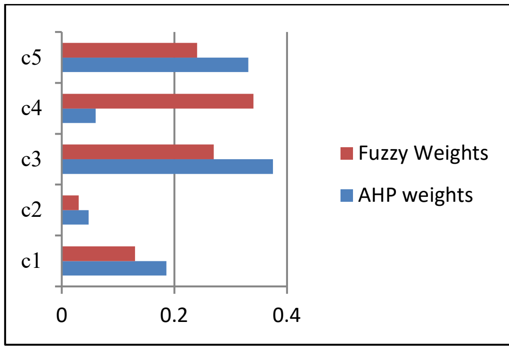
Criteria	AHP Weights	Fuzzy Weights
C1	(0.185882, 0.188482, 0.209408)	(0.13, 0.18, 0.2)
C2	(0.047728, 0.04175, 0.039353)	(0.03, 0.04, 0.04)
C3	(0.375054, 0.400244, 0.38042)	(0.27, 0.4, 0.38)
C4	(0.060358, 0.056896, 0.059698)	(0.34, 0.06, 0.06)
C5	(0.330974, 0.312624, 0.28624)	(0.24, 0.32, 0.32)

Table 34. Normalized matrix of decision maker's ratings for all criteria of selected bidders by AHP Method

	C1	C2	C3	C4	C5
B1	(0.1505644, 0.1771731, 0.2052198)	(0.020523, 0.025885, 0.03030143)	(0.1912775, 0.2761684, 0.319553)	(0, 0.0199136, 0.0346248)	(0.1390091, 0.1907006, 0.231854)
B2	(0.1040939, 0.1413615, 0.182185)	(0.0348414, 0.036323, 0.03699135)	(0.2512862, 0.336205, 0.357595)	(0.0338005, 0.042672, 0.0519373)	(0.2680889, 0.2938666, 0.280515)
B3	(0.0873645, 0.1262829, 0.1759027)	(0.0319778, 0.03507, 0.03699135)	(0.2587873, 0.3522147, 0.372812)	(0.0313862, 0.0415341, 0.0519373)	(0.1820357, 0.2532254, 0.269066)
B4	(0.0873645, 0.1262829, 0.1759027)	(0.0224322, 0.027973, 0.02754675)	(0.2512862, 0.336205, 0.357595)	(0, 0.0096723, 0.0232822)	(0, 0, 0.077285)
W_j values					
W_j	(0.185882, 0.188482, 0.209408)	(0.047728, 0.04175, 0.039353)	(0.375054, 0.400244, 0.38042)	(0.060358, 0.056896, 0.059698)	(0.330974, 0.312624, 0.28624)

Table 35. Final evaluation of alternatives after AHP normalization

Bidder	AHP Ci	AHP CCi	AHP Ri	Rank
B1	(0.5013741, 0.689841, 0.8215533)	0.6803817	0.53493	3
B2	(0.692111, 0.850428, 0.9092236)	0.8338408	1	1
B3	(0.5915514, 0.808327, 0.9067085)	0.7885948	0.862878	2
B4	(0.3610829, 0.500133, 0.6616113)	0.5038708	0	4



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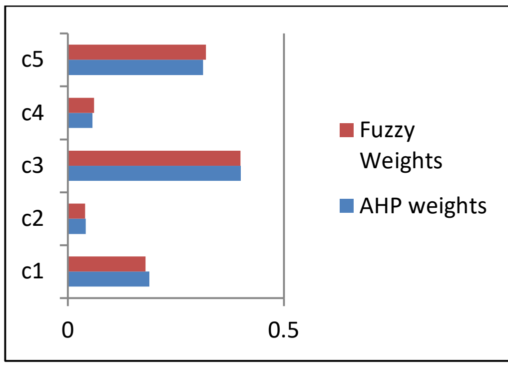
values (a of Triangle a,b,c)



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values (b of Triangle a,b,c)

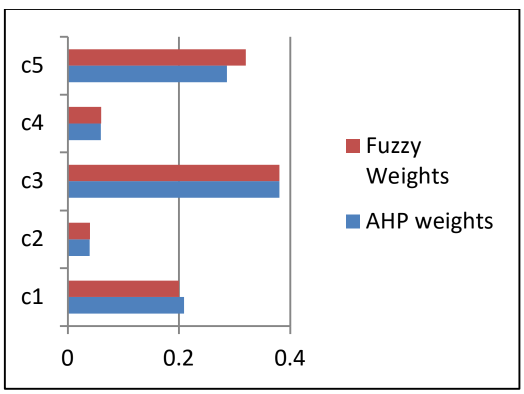

Figure 5. Comparison of decision criteria weight values (c of Triangle a,b,c)

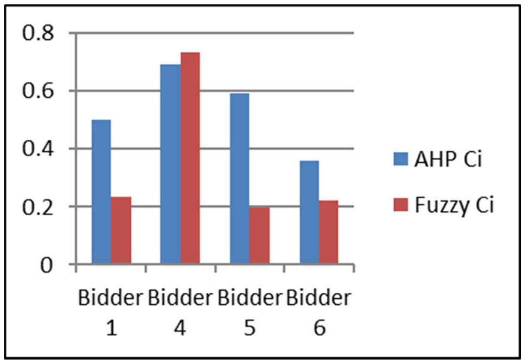

Figure 6. Comparison of Bidders' intermediate score determined by AHP and Fuzzy MCDM (a of triangle a,b,c)

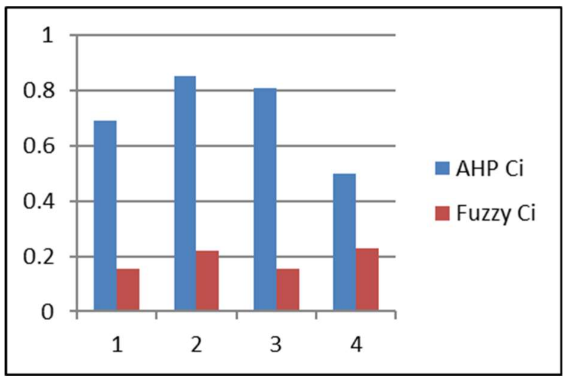

Figure 7. Comparison of Bidders' intermediate score determined by AHP and Fuzzy MCDM (b of triangle a,b,c)

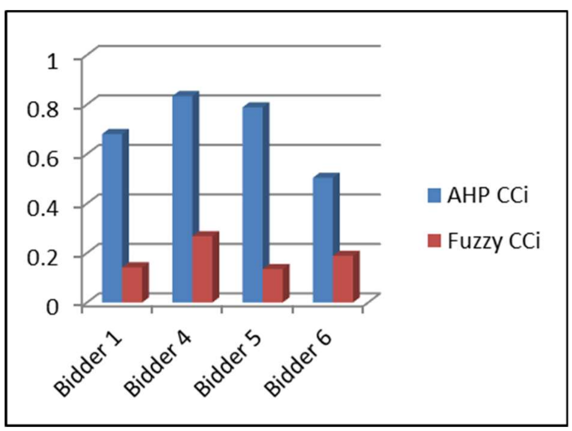

Figure 8. Comparison of Bidders' intermediate score determined by AHP and Fuzzy MCDM (c of triangle a,b,c)

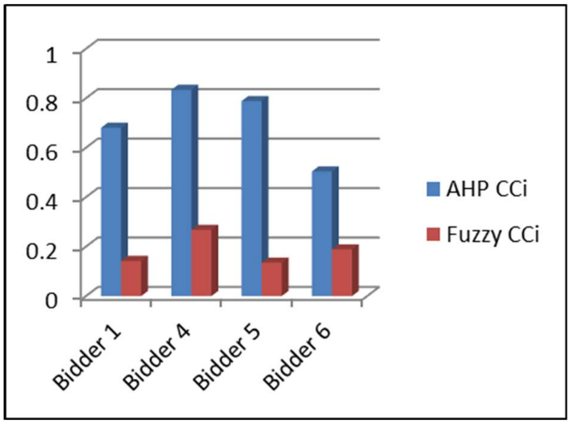

Figure 9. Comparison of intermediate ranking of AHP and Fuzzy MCDM method

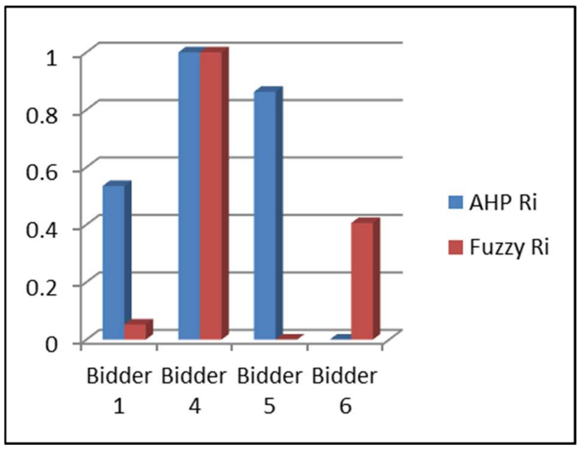

Figure 10. Comparison of final ranking of AHP and Fuzzy MCDM method