

# Permutation Coding with Injections for Modified PAM System

Opeyemi O. Ogonyanda, Thokozani Shongwe, and Theo G. Swart

**Abstract**—Arriving at a good combination of coding and modulation schemes that can achieve good error correction constitutes a challenge in digital communication systems. In this work, we explore the combination of permutation coding (PC) and pulse amplitude modulation (PAM) for mitigating channel errors in the presence of background noise and jitter. Since PAM is characterised with bi-polar constellations, Euclidean distance is a good choice for predicting the performance of such coded modulation setup. In order to address certain challenges facing PCs, we therefore introduce injections in the coding system, together with a modified form of PAM system. This modification entails constraining the PAM constellations to the size of the codeword's symbol. The results obtained demonstrate the strength of the modified coded PAM system over the conventional PC coded PAM system.

**Keywords**—Injection; Permutation Codes; Power Line Communications; Pulse Amplitude Modulation; Jitter

## I. INTRODUCTION

PERMUTATION coding (PC) is not a new study and its usage for power line communications (PLC) was proposed by Vinck [1]. Afterwards, a number of researches done in this regard have been published [2]–[11]. PC entails mapping binary  $n$ -tuples onto permutation  $M$ -tuples consisting of codewords with non-repetitive symbols from the set  $U = \{1, 2, \dots, M\}$  [2], [3], [12]. In [4], the author identified certain challenges facing PCs, which were addressed by using injections. One of such challenges is the poor understanding of the codebooks  $\mathcal{C}(M, H_{\min})$ , where  $H_{\min}$  is the minimum Hamming distance and  $M$  the codeword length. For instance, as it is difficult finding the existence of a projective plane of order  $M$ , so it is for finding codebooks with  $H_{\min} = M - 1$  for general  $M$  [5]. Injection codes thus offer alternatives for applications with such unpleasant parameters in PCs. Duke's approach in [4] was however used in the context of the conventional permutation  $M$ -tuples symbols in  $U$ .

More so, in [6] the shortcoming of using a conventional Differential phase shift keying (DPSK) modulation with PCs whose codeword length  $M$  is shorter than the number of

constellation points  $M_{DP}$  was identified. This causes the demodulator to be prone to unwanted symbol errors resulting from the appearances of symbols that are not contained in the permutation  $M$ -tuple symbols [9]. As such, the authors introduced a modified form of DPSK which constrains the constellation points to match the codeword length used. The approach was however reported for a PC coded DPSK system.

Pulse amplitude modulation (PAM) is useful for reducing symbol rate and bandwidth of the modulated signal. As such, it is preferable for high speed digital transmission as in optical fibre channel and LAN systems [13]. When PAM symbols are arranged in the form of permutation coding, spectral shaping can be achieved, which is of key importance when reducing the effects of DC-wander in digital recorders and metallic cables [14].

Channel conditions that plague digital communication systems are background noise and timing jitter. Usually, additive white Gaussian noise (AWGN) is used to reproduce background noise. Timing jitter, which is categorised into random jitter and deterministic jitter, can be represented using random distribution and inter-symbol interference (ISI) [15].

The motivation for this work is to optimise constellation point spacing so as to achieve better error correction capability of a coded PAM system under background noise and timing jitter. As such, we shall only be focusing on the concatenation of permutation coding with PAM modulation. We therefore use bi-polar  $M$ -tuple symbols in the set  $U$  to define the PCs involved in this work. These symbols are chosen from the constellations of the PAM system used. This thus warrants the use of Euclidean distance as the yardstick for determining the performance of the codebooks considered. Also, in order to achieve better performance, injections were introduced in the PCs, by employing a similar approach used in [4]. To further improve the performance of the injected PCs, we employ a similar approach used in [6], but in this regard, PAM constellations are constrained to match the codeword length. It is also worth noting here that injected PC is involved in this work, as opposed to [6], where injections are not involved.

In Section II, we give a brief description of how PCs are inculcated with PAM systems, before going to Section III, where PC with injections is discussed. The proposed modified PAM system with injected PC is detailed in Section IV. Simulation setup is discussed in Section V followed by Section

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VI, where performance results are discussed. The paper is concluded in Section VII.

## II. PC-PAM SYSTEM

The conventional PC codewords can be directly modulated using modulators such as phase shift keying (PSK) or DPSK. The Hamming distance between two codewords  $u$  and  $v$  of the same length  $M$  denoted as  $H(u, v)$ , is the number of positions in which symbols in  $u$  and  $v$  disagree. Codebooks with smaller minimum Hamming distance  $H_{\min}$  exhibits poorer performance as compared to those with larger  $H_{\min}$ . In general, the cardinality  $|\mathcal{C}|$  of a permutation codebook is bounded by [12]

$$|\mathcal{C}| = \frac{M!}{(H_{\min} - 1)!}. \quad (1)$$

However, when PAM system is involved, the  $M$ -tuples symbols permuted are bi-polar, due to the constellation arrangement of PAM modulation. Assuming all the symbols are equally likely, the average energy  $d_{\text{av}}^E$  of a PAM system is given by [16, Chapter 5]

$$d_{\text{av}}^E = \frac{M_{\text{DP}}^2 - 1}{3} E, \quad (2)$$

where  $E$  is the energy of the lowest amplitude signal, and  $M_{\text{DP}}$  is the modulation order.

For instance 4-ary PAM has  $d_{\text{av}}^E = 5E$ , and after normalization, its constellation is as illustrated in Fig. 1, where  $E_b$  is the energy per bit. Hence the permutation  $M$ -tuple symbols for such 4-PAM system will be from the set  $U = \{-3, -1, 1, 3\}/\sqrt{5}$ . Therefore, to define the relationship between every unique codeword, Euclidean distance is best used.

Let us consider a codebook with codeword length  $M = 5$ . For a PAM system, the appropriate modulator to be used for modulating the codewords will be an 8-PAM system, whose average energy is  $d_{\text{av}}^E = 21E$ . As such, the permutation  $M$ -tuple symbols will be selected from  $U = \{-7, -5, \dots, 5, 7\}/\sqrt{21}$ . Let us consider  $\{-7, -5, -3, -1, 1\}/\sqrt{21}$  for example. By permuting these five symbols selected from  $U$ , 20 codewords are possible, according to (1), without any repetitive symbol, with  $H_{\min} = 4$ . With these, only  $n = 4$  information bits can be mapped onto the  $M$ -tuple symbols. This means only  $2^n = 16$  codewords

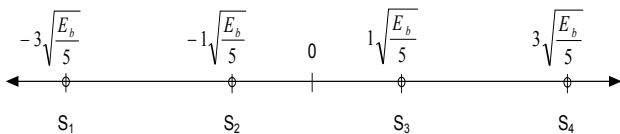


Fig. 1. Constellation plot for 4-PAM modulation

are useful. These selected codewords are as given in the codebook

$$\left\{ \begin{array}{l} -5-3-1+1-7, -5-1+1-7-3, -5+1-7-3-1, \\ -5-7-3-1+1, -3-1-7-5+1, -3-7-5+1-1, \\ -3-5+1-1-7, -3+1-1-7-5, -1-5-7+1-3, \\ -1-3-5-7+1, -1+1-3-5-7, -1-7+1-3-5, \\ -7-3+1-5-1, -7-1-3+1-5, -7-5-1-3+1, \\ -7+1-5-1-3 \end{array} \right\} / \sqrt{21}. \quad (3)$$

The codebook in (3), yields a minimum Euclidean distance  $d_{\min}^E = 0.8729$  and a transmission rate  $r = 4/5$  in bits per symbol. Of course it is possible to achieve 32 codewords from the symbols in  $U$ , thereby enhancing higher transmission rate  $r = 1$ , (i.e. mapping  $n = 5$  bits onto  $M = 5$  symbols). However, this will be at the expense of lower  $d_{\min}^E$  and poorer codebook performance. This is because the lower the value of  $d_{\min}^E$ , the poorer the codebook's performance, if  $M$  remains unchanged. Hence, the trade off stands between sacrificing either  $r$  or  $d_{\min}^E$ . By introducing injections, this trade off becomes worthwhile.

It should be noted that regardless of how the 5 symbols are selected from  $U$ , the same  $H_{\min} = 4$  is achievable. However,  $d_{\min}^E$  can further be improved in this regard. One combination that gives the best  $d_{\min}^E = 1.3801$  is from  $U = \{-7, -5, 1, 3, 7\}/\sqrt{21}$ . This choice of  $U$  results in a PC codebook that is given by

$$\left\{ \begin{array}{l} -5+3-1+7-7, -5-1+7-7+3, -5+7-7+3-1, \\ -5-7+3-1+7, +3-1-7-5+7, +3-7-5+7-1, \\ +3-5+7-1-7, +3+7-1-7-5, -1-5-7+7+3, \\ -1+3-5-7+7, -1+7+3-5-7, -1-7+7+3-5, \\ -7+3+7-5-1, -7-1+3+7-5, -7-5-1+3+7, \\ -7+7-5-1+3 \end{array} \right\} / \sqrt{21}. \quad (4)$$

## III. PC-PAM SYSTEM WITH INJECTIONS

Given that  $U = \{x_1, x_2, \dots, x_M\}$  and  $U' \subseteq U$ , with the length of  $U'$  being  $M'$ , then an  $M'$ -arrangement of  $U$  is an injection of  $U'$  into  $U$ , provided that  $M \geq M'$ . This implies that a PC has all its codewords from the permutation of all the symbols in  $U$ , while for an injection code, all the codewords are derived by permuting a subset of symbols from  $U$ . This is best understood using an example.

### Example 1:

Let us derive a new set of codebook from (3) by truncating the 5th symbol from each codeword. This process is as illustrated in Fig. 2. The codewords on the left hand side of Fig. 2 consist of all elements in the set  $U$  and the codewords on the right hand side are a subset of  $U$ , which we represent by  $U'$ .

By examining the new set of codebooks derived from (3), it is seen that each codeword is a subset of  $U$ . These two types

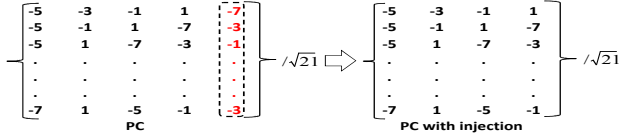


Fig. 2. Illustrating PC to injection code

of codebooks have the same cardinality but different codeword lengths (i.e.  $M$  for ordinary PCs and  $M'$  for injected PCs). More so, the injected code has a shorter minimum Euclidean distance  $d_{\min}^E = 0.7559$ . However, its advantage is that an increased transmission rate  $r' = 1$  is achieved, because  $n = 4$  bits are mapped onto  $M' = 4$  symbols.

Similar to the notation of an ordinary PC codebook, we can also denote an injected PC codebook by  $\mathcal{C}'(M', d_{\min}^E)$ . Let us denote the cardinalities of a PC and an injected PC as  $|\mathcal{C}|$  and  $|\mathcal{C}'|$ , respectively. Therefore, a PC and an injected PC with the same cardinality  $|\mathcal{C}| = |\mathcal{C}'|$ , can be simply related by:

$$\left. \begin{array}{l} d_{\min}^E < d_{\min}^E, \quad M' \leq M, \\ U' \subseteq U \quad \text{and} \\ (r' = n/M') > (r = n/M) \end{array} \right\}, \quad \text{if } |\mathcal{C}| = |\mathcal{C}'|. \quad (5)$$

where  $\{\}'$  is used to denote any variable associated with injected PC. If  $M' = M$ , that means the codebook is an ordinary PC without injections.

For the example in Fig. 2, one may question why not just obtain direct PC from the set  $\{-3, -1, 1, 3\}/\sqrt{5}$ . This will however imply a shorter  $d_{\min}^E$ , which will yield poorer performance, when compared to the injection code. It should also be noted that more than one symbol can be truncated in a PC codebook to produce a corresponding injection codebook with higher rates.

An injected PC version of (4) can be derived by truncating its 5th column as follows:

$$\left\{ \begin{array}{l} -5 + 3 - 1 + 7, -5 - 1 + 7 - 7, -5 + 7 - 7 + 3, \\ -5 - 7 + 3 - 1, +3 - 1 - 7 - 5, +3 - 7 - 5 + 7, \\ +3 - 5 + 7 - 1, +3 + 7 - 1 - 7, -1 - 5 - 7 + 7, \\ -1 + 3 - 5 - 7, -1 + 7 + 3 - 5, -1 - 7 + 7 + 3, \\ -7 + 3 + 7 - 5, -7 - 1 + 3 + 7, -7 - 5 - 1 + 3, \\ -7 + 7 - 5 - 1 \end{array} \right\} / \sqrt{21}. \quad (6)$$

#### IV. MODIFIED PAM SYSTEM

From the mapping representation in (3) and (4),  $n = 4$  message bits can be mapped onto  $M = 5$ -tuple symbols. In order to map such symbols into PAM constellations, the appropriate modulation order to be used is  $M_{\text{DP}} = 8$ . The implication of this is that only 5, out of the 8 available

constellations will be used by the  $M$ -tuple symbols. As such, there is probability that 3 non-transmitted PAM symbols feature in the received symbols, due to channel effect. We therefore modify the PAM modulator and demodulator by restricting the input and output symbols to match the alphabet size from the codebook. For the codebook in (3) and (4), we can term the PAM system a 5-PAM system. This system has the same modulation rate with an 8-PAM system. The advantage of the 5-PAM is that only 5 constellations, which are evenly spaced in the Euclidean space are involved, thereby increasing the Euclidean distance between each constellation point. Employing (2), a 5-PAM has  $d_{\text{av}}^E = 8$ . Its constellations are as depicted in Fig. 3.

Based on the constellation spacing depicted in Fig. 3, one could imply that the  $M$ -tuple symbols permuted are taken from  $U = \{-4, -2, 0, 2, 4\}/\sqrt{8}$ . Therefore, by adopting this 5-PAM system in the injected PC system, the following codebook can be defined:

$$\left\{ \begin{array}{l} -20 + 2 + 4, -2 + 2 + 4 - 4, -2 + 4 - 40, -2 - 40 + 2, \\ 0 + 2 - 4 - 2, 0 - 4 - 2 + 4, 0 - 2 + 4 + 2, 0 + 4 + 2 - 4, \\ +2 - 2 - 4 + 4, +20 - 2 - 4, +2 + 40 - 2, +2 - 4 + 40, \\ -40 + 4 - 2, -4 + 20 + 4, -4 - 2 + 20, -4 + 4 - 2 + 2 \end{array} \right\} / \sqrt{8}. \quad (7)$$

Other conventional PAM modulation systems have modulation orders in powers of 2. The approach of this modified PAM system can be employed for other injected PC systems whose alphabet sizes are not necessarily in powers of 2.

#### V. SIMULATION SETUP

The simulation model used in this work is as presented in Fig. 4.

In order to validate the significance of the proposed modified PAM system with injected PC, four different schemes were simulated in this work. The first scheme termed Scheme A is an ordinary  $M = 5$  PC scheme whose codebook is presented in (4). This was used in an ordinary 8-PAM system with 8 constellations. Scheme B, whose codebook is shown in (6), is an  $M' = 4$  injected version of the codebook in (4). This was first used in an ordinary 8-PAM system in order to examine the strength of an injected PC with higher rate, but without modifying the modulator. For Scheme C, the codebook in (7) is used in the proposed 5-PAM system. The purpose of this scheme is to showcase the significance of an injected PC, when a stronger Euclidean distance is achieved using the 5-PAM system. Scheme D is an ordinary  $M = 4$  PC scheme

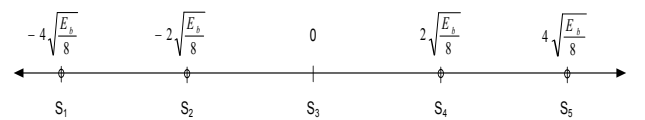


Fig. 3. Constellation plot for 5-PAM modulation

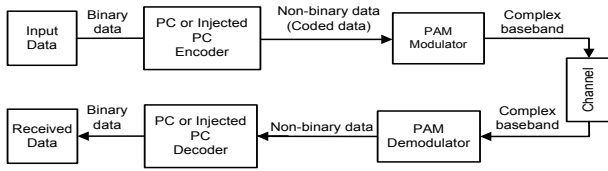


Fig. 4. Simulation model

modulated by a 4-PAM system. The codebook is as shown below:

$$\left\{ \begin{array}{l} +3 - 1 - 3 + 1, -1 + 3 + 1 - 3, -3 + 1 + 3 - 1, \\ +1 - 3 - 1 + 3, +1 - 3 + 3 - 1, +1 + 3 - 3 - 1, \\ +1 + 3 - 1 - 3, -3 - 1 + 1 + 3, -3 - 1 + 3 + 1, \\ -3 + 3 - 1 + 1, -3 + 3 + 1 - 1, +1 - 1 - 3 + 3, \\ +1 - 1 + 3 - 3, -1 + 1 - 3 + 3, -1 + 1 + 3 - 3, \\ -1 - 3 + 1 + 3 \end{array} \right\} / \sqrt{5}. \quad (8)$$

Considering the number of bits mapped to the  $M$ -tuple symbols in each scheme, the effective ratio of their coding rates is given by

$$r_A : r_B : r_C : r_D = 4 : 5 : 5 : 5. \quad (9)$$

This is because Scheme A maps 4 bits onto 5 symbols, while the rest Schemes B to D map 4 bits onto 4 symbols. Since all the schemes evaluated are not of the same rate, we have ensured rate compensation in all simulations done so as to ensure fair comparisons.

The communication channels involved in this simulation include AWGN and timing jitter. AWGN was modelled using a probability distribution of zero-mean and a standard deviation  $\sigma = 1$ . The random jitter used was also modelled as a zero-mean Gaussian random distribution, but with a specified standard-deviation  $\sigma_j$ . As per deterministic jitter, only ISI is considered and this was modelled as a train of Dirac functions of equal amplitude. Details of these noise models can be accessed in [17].

## VI. PERFORMANCE RESULTS AND DISCUSSION

The results presented in Fig. 5 is obtained using an ordinary AWGN channel. The PC with injection (i.e., Scheme B) exhibits a relatively overlapping performance with Scheme A. This is unlike the case in [9], where an injected PC performs better than ordinary PC scheme in a differential phase shift keying (DPSK) modulation. The strength of the injected PC becomes evident, when it is modulated using the proposed 5-PAM modulation in Scheme C. At  $P_s = 10^{-4}$ , Scheme C is about 4 dB better than B and A. Moreover, because Scheme C has a better  $d_{\min}^E$  than D, it exhibits a better performance and it is seen to be about 1.2 dB better than D at  $P_s = 10^{-4}$ .

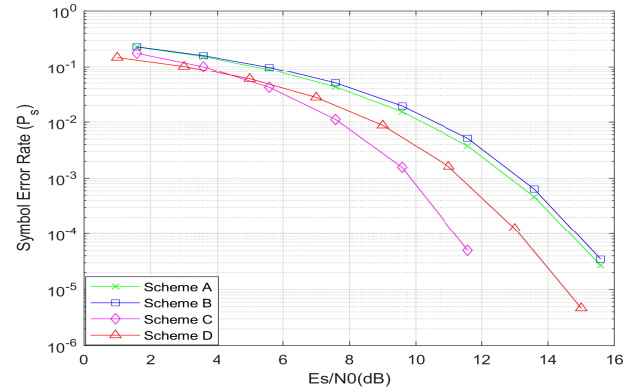


Fig. 5. Symbol error rate curves for Schemes A to D, in the presence of AWGN

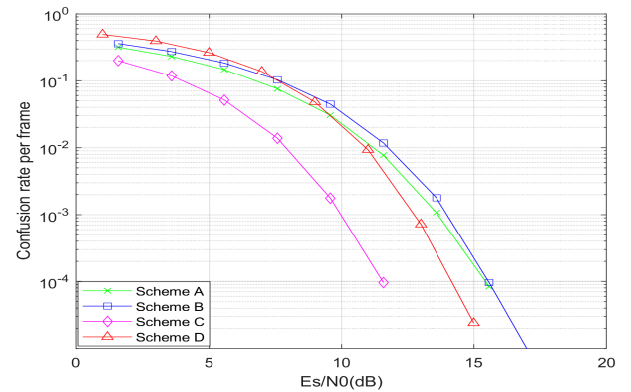


Fig. 6. Confusion rate curves for Schemes A to D, in the presence of AWGN

Decoders are prone to confusions, when a received codeword happens to have the same metric with more than one codeword in the registered codebook. A better codebook naturally exhibits lesser confusion rate than others. Based on this phenomenon, we further examine the characteristics of Schemes A to D. Fig. 6 shows confusion rate curves for Schemes A to D under AWGN channel. Here, Scheme C exhibits the least rate of confusion, followed by Schemes D, A and B respectively. Scheme A's confusion rate is slightly better than B at  $E_s/N_0 < 15$  dB. Schemes C and D are less confused due to the fact that their codebook symbols are of the same order with their constellation points, unlike Schemes A and B, whose constellations are not the same orders as their codeword symbols. This therefore causes Schemes A and B to be prone to unwanted symbol errors resulting from the occurrences of symbols that were not in the set  $U$ , but were demodulated due to channel error [9]. This thus causes them to have more confusion rates.

In order to observe the strength of the proposed scheme over a multi-carrier modulation scheme, PAM-OFDM modulation was used in place of the ordinary PAM modulator in the model presented in Fig. 4. With this, we are able to observe how each scheme performs under frequency disturbance.

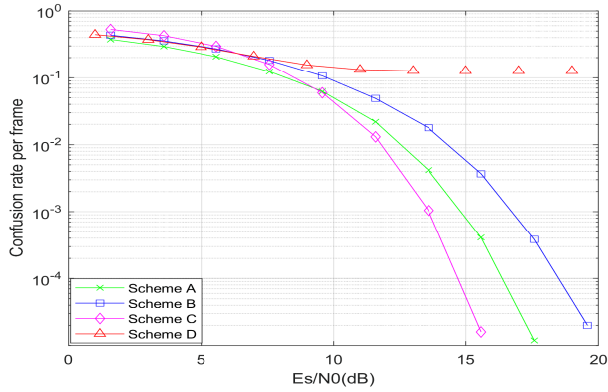


Fig. 7. Confusion rate curves for Schemes A to D in an OFDM system, under permanent frequency noise and AWGN. Frequency 1 was affected.

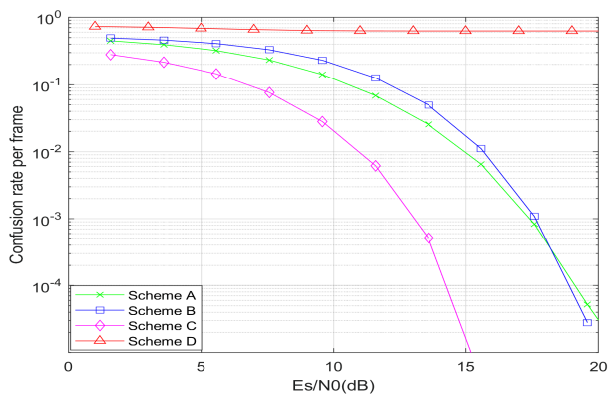


Fig. 8. Confusion rate curves for Schemes A to D in an OFDM system, under permanent frequency noise and AWGN. Frequency 3 was affected.

Figs. 7 and 8 show confusion rate curves for Schemes A to D under PAM-OFDM system. Here, frequencies 1 and 3 are respectively selected to be affected by a permanent frequency error in addition with AWGN. The proposed Scheme C turns out to be the best performing scheme under these two scenarios, while Scheme D is the worst. This is because Scheme C is able to avoid unwanted symbol errors due to the modified modulator used. In Scheme B, one symbol out of every 4 symbols is affected by permanent frequency, while in Scheme A, one out of every 5 is affected. That is why Scheme B turns out to be more confused than A in these two scenarios. It should be noted that there is nothing specific about frequencies 1 and 3. They were only considered for the purpose of random comparisons.

Figs. 9 and 10 show eye diagrams of Scheme B without noise and after introducing AWGN and random jitter at 30 dB. The eyes of the jittered signal are seen to be considerably narrower than the eyes of the non-jittered signal.

As displayed in Fig. 11, the proposed Scheme C is seen to be the most robust when AWGN is combined with random jitter. A similar situation is observed with combined AWGN, random jitter and deterministic jitter, as shown in Fig. 12.

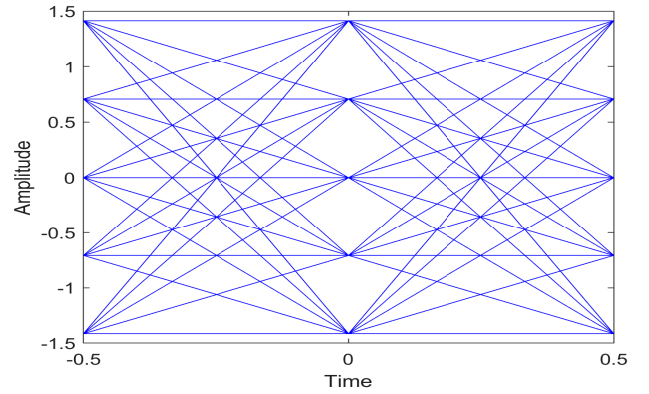


Fig. 9. Eye diagram of Scheme B with no noise

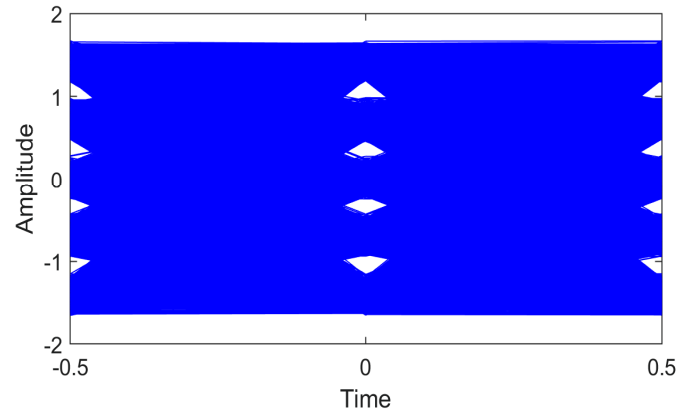


Fig. 10. Eye diagram of Scheme B with AWGN and random jitter at 30 dB  $E_s/N_0$

## VII. CONCLUSION

We have reported a special form of permutation coded PAM modulation scheme, which involves the injection of selected symbols from the conventional PC codebooks. The injected PC is able to attain higher rate with a slight reduction in  $d_{\min}^E$ . The PAM system used was also modified so as to make its constellations match the symbol size of the injected PC codebook. This thus aided the better performance of the codebook when compared with conventional PC codebooks. An added advantage of the proposed scheme is that it can be extended to every injected PCs whose symbol orders are not necessarily in the order of powers of 2. As such, the corresponding PAM system to be used can be modified by constraining its constellations to match the codebook's symbol size. The proposed scheme is a good candidate in applications such as LAN systems, digital recordings and in fibre channels.

## REFERENCES

- [1] A. J. H. Vinck and J. Häring, "Coding and modulation for power-line communications," in *Power Line Commun. and Its Applicat. (ISPLC), 2000 4th IEEE Int. Symp.*, Limerick, Ireland, Apr. 2000, pp. 265–271.
- [2] T. G. Swart, I. De Beer, H. C. Ferreira, and A. J. H. Vinck, "Simulation results for permutation trellis codes using M-ary FSK," in *Int. Symp. Power Line Commun. and Its Applicat. (ISPLC), 2005 IEEE Int. Symp.*, Vancouver, CA, Apr. 2005, pp. 317–321.



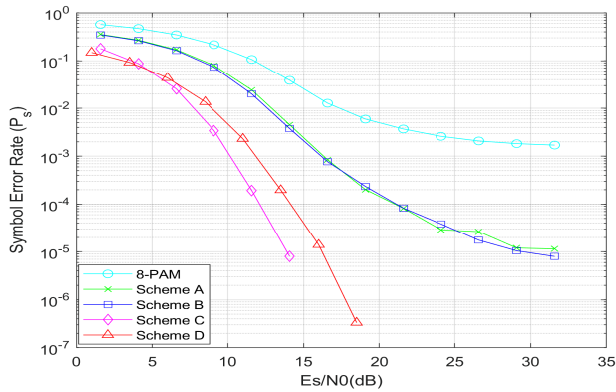


Fig. 11. Symbol error rate curves for Schemes A to D, in the presence of AWGN and random jitter

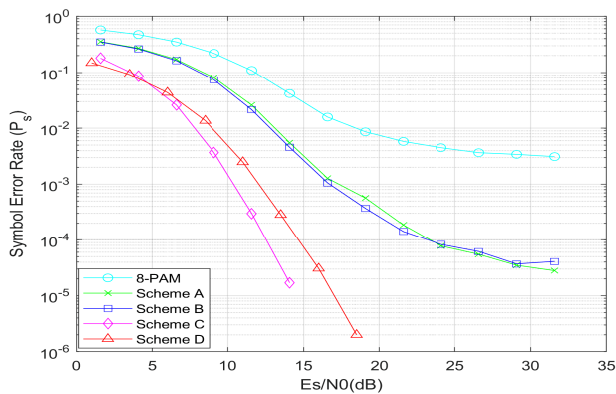


Fig. 12. Symbol error rate curves for Schemes A to D, in the presence of AWGN, random and deterministic jitter

[3] T. G. Swart and H. C. Ferreira, "Decoding distance-preserving permutation codes for power-line communications," in *AFRICON 2007*, Windhoek, Namibia, Sept. 2007, pp. 1–7.

[4] P. J. Dukes, "Coding with injections," *Designs, Codes and Cryptography*, vol. 65, no. 3, pp. 213–222, 2012.

[5] C. J. Colbourn, T. Klove, and A. C. H. Ling, "Permutation arrays for powerline communication and mutually orthogonal latin squares," *IEEE Trans. Inf. Theory*, vol. 50, no. 6, pp. 1289–1291, 2004.

[6] K. Ogunyanda, A. D. Familua, T. G. Swart, H. C. Ferreira, and L. Cheng, "Permutation coding with differential quinary phase shift keying for power line communication," in *The 5th IEEE PES Innovative Smart Grid Technol. (ISGT) European 2014 Conference*, Istanbul, Turkey, Oct. 2014, pp. 1–6.

[7] T. G. Swart and H. C. Ferreira, "Analysis of permutation distance-preserving mappings using graphs," in *Proceedings of the International Symposium on Communication Theory and Applications*. IEEE, 2007, p. 6 pages.

[8] F. Gologlu, J. Lember, A. E. Riet, and V. Skachek, "New bounds for permutation codes in Ulam metric," in *The 2015 Int. Symp. Informat. Theory and its Applications*, Hong Kong, Jun. 2015, pp. 1726–1730.

[9] K. Ogunyanda, A. D. Familua, T. G. Swart, H. C. Ferreira, and L. Cheng, "Evaluation of mixed permutation codes in PLC channels, using hamming distance profile," *Telecommun. Syst.*, vol. 65, no. 1, pp. 1–11, 2016.

[10] T. Shongwe, T. G. Swart, and H. C. Ferreira, "Distance-preserving mapping with euclidean distance for 4-ary PAM," in *2018 IEEE Int. Symp. on Commun. System, Networks and Digital Signal Processing (CSNDSP)*, Budapest, Hungary, Jul. 2018, pp. 1–6.

[11] T. Shongwe, "Trellis coded 4-ary PAM using distance-preserving mapping," *International Journal of Electronics and Telecommunications*, vol. 64, no. 4, pp. 527–533, Oct. 2018.

[12] H. C. Ferreira, D. A. Wright, and A. Nel, "Hamming distance preserving mappings and trellis codes with constrained binary symbols," *IEEE Trans. Inform. Theory*, vol. 35, pp. 1098–1103, Nov. 1989.

[13] K. Ouahada, T. G. Swart, and H. C. Ferreira, "Permutation sequences and coded PAM signals with spectral nulls at rational submultiples of the symbol frequency," *Cryptography and Commun.*, vol. 3, no. 2, pp. 87–108, Jun. 2011.

[14] K. T. Ouahada, "Spectral shaping and distance mappings with permutation sequences," Ph.D. dissertation, Univ. Johannesburg, Johannesburg, Gauteng, 2009.

[15] N. Ou, T. Farahmand, A. Kuo, S. Tabatabaei, and A. Ivanov, "Jitter models for the design and test of Gbps-speed serial interconnects," *IEEE Des. Test of Comput.*, vol. 21, no. 4, pp. 302–313, July 2004.

[16] J. R. Barry, E. A. Lee, and D. G. Messerschmitt, *Digital Communication*, 3rd ed. Springer, 2004.

[17] R. Stephens, *Jitter Analysis: The dual-Dirac Model, RJ/DJ, and Q-Scale*. Agilent Technologies, 2004. [Online]. Available: [http://www. Agilent.com](http://www.Agilent.com)