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MINLP optimization of structures

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Abstract. The paper discusses the mixed-integer non-linear programming (MINLP) approach to the optimization of structures. MINLP is an optimization technique capable of solving non-linear and discrete optimization problems. It calculates continuous variables (loads, dimensions, stresses, deflections, costs) and discrete variables (topology, standard sections, material grades). The MINLP optimization model of a structure should be developed. In the model, an objective function is subjected to structural analysis and dimensioning constraints to meet ultimate and serviceability limit states according to Eurocodes. Suitable MINLP algorithms and strategies are used to solve the defined MINLP problem. Three numerical examples are presented at the end of the paper.

Keywords: Structural Optimization, Discrete optimization, Mixed-Integer Non-Linear Programming, MINLP

1 Introduction

The paper discusses the mixed-integer non-linear programming (MINLP) approach to the optimization of structures in civil engineering. Since the MINLP performs continuous and discrete optimization simultaneously, it deals with continuous and discrete variables. While the continuous variables are defined for the continuous optimization of parameters (loads, stresses, deflections, weights, costs, etc.), the discrete variables are used to express discrete decisions, i.e. the existence or non-existence of structural elements within the defined structure. Extra discrete binary 0-1 variable y is assigned to each structural element. The element is selected by the optimization process to construct the structure if its binary variable takes the value one (y=1), otherwise it is removed from the structure (y=0). Different discrete material grades, standard cross-sections and rounded dimensions can also be defined as discrete alternatives.

The MINLP optimization approach requires that a structure is generated as an MINLP superstructure, which is composed of different structure/topology and design alternatives, all of which are candidates for a feasible and optimal solution. While the topology alternatives represent different selections and interconnections of the corresponding structural elements, the design alternatives include different material grades, standard sections and rounded dimensions. The main goal of the optimization is to find a feasible structure within the given superstructure that is optimal with respect of manufacturing cost (or structural mass), topology, material, standard dimensions and rounded dimensions.

For MINLP optimization, the MINLP optimization model of a structure must be developed in which the cost or mass objective function of a structure is subjected to design, structural analysis and dimensioning constraints in order to fulfill the ultimate and serviceability limit state conditions. While structural analysis constraints are used to calculate internal forces and deflections, dimensioning constraints are defined according to Eurocode specifications or other standards.

Many different methods to solve MINLP problems have been developed in the near past. This paper reports on the experience gained in MINLP problems using the Modified Outer-Approximation /Equality-Relaxation (Modified OA/ER) algorithm by Kravanja & Grossman [1]. The algorithm was adapted by Kravanja et al. [2-5] and applied in structural optimization. The Linked Multi-level Hierarchical Strategy (LMHS) was developed to accelerate the convergence of the mentioned algorithm.

2 MINLP model formulation

The optimization problems in the field of structural optimization are usually non-linear, non-convex, continuous and discrete. The MINLP is therefore selected for the optimization. A general MINLP model formulation can be formulated as follows:

$$\min \quad z = f(\mathbf{x},\mathbf{y}) \tag{1}$$

$$\text{subjected to:} \quad \mathbf{g}(\mathbf{x},\mathbf{y}) \leq 0 \tag{2}$$

$$\mathbf{x} \in \mathrm{X} = \{\mathbf{x} \in \mathrm{R}^{\mathrm{n}}:\ \mathbf{x}^{\mathrm{LO}} \leq \mathbf{x} \leq \mathbf{x}^{\mathrm{UP}}\} \tag{3}$$

$$\mathbf{y} \in \mathrm{Y} = \{0,1\}^{\mathrm{m}} \tag{4}$$

where **x** is a vector of continuous variables and **y** is a vector of discrete binary (0,1) variables. The function $f(\mathbf{x},\mathbf{y})$ is the objective function subjected to the (in)equality constraints $\mathbf{g}(\mathbf{x},\mathbf{y})$. At least one function must be non-linear. All functions must be continuous and differentiable.

In structural optimization, the continuous variables x define dimensions, strains, stresses, costs, etc., and the binary variables y represent the potential existence of structural elements within the defined superstructure and the choice of discrete material grades, standard sections and rounded dimensions (continuous dimensions are rounded up to whole values in cm or mm). Non-linear equality and inequality constraints and the bounds of continuous variables represent the strict system of design, loading, resistance and deflection constraints known from structural analysis and dimensioning.

3 Solution of the MINLP optimization problem

Once the MINLP model of a structure is developed, the defined MINLP problem is solved by using an appropriate MINLP algorithm and strategy. In principle, a general class of MINLP optimization problems can be solved by the following algorithms and their extensions:

- Generalized Benders Decomposition (GBD), presented by Benders [6] and Geoffrion [7],
- Non-linear Branch and Bound (NBB), proposed and used by many authors, e.g. Beale [8], and Gupta and Ravindran [9],
- Outer-Approximation (OA), presented by Duran and Grossmann [10],
- Feasibility Technique (FT) by Mawengkang and Murtagh [11],
- Sequential Linear Discrete Programming (SLDP), presented by Olsen and Vanderplaats [12], Bremicker et al. [13],
- LP/NLP based Branch and Bound (LP/NLP BB), presented by Quesada and Grossmann [14],
- Extended Cutting Plane (ECP) by Westerlund and Pettersson [15].

The extension of the OA, the Outer-Approximation /Equality Relaxation (OA/ER) algorithm, developed by Kocis and Grossmann [16] to handle equality constraints, seems to be one of the most efficient algorithms for solving large MINLP problems when NLP sub-problems are expensive and difficult to solve. The OA/ER algorithm consists of the solution of an alternative sequence of optimization sub-problems of Non-linear Programming (NLP) and Mixed-Integer Linear Programming (MILP) main problems, see Kravanja et al. [3]. The former corresponds to a continuous optimization of parameters for a mechanical structure with a fixed topology, material grades and standard sections, and results in an upper bound to the objective to be minimized. The latter involves a global linear approximation to the superstructure of alternatives where a new topology, material grades and standard sections are identified so that its lower bound does not exceed the current best upper bound. The search for a convex problem is terminated if the predicted lower bound exceeds the upper bound, otherwise it is terminated if the NLP solution cannot be improved.

The OA/ER algorithm and all other MINLP algorithms mentioned do not generally guarantee that the solution found represents the global optimum. This is due to the presence of nonconvex functions in the models that can cut off the global optimum. To reduce the undesirable effects of non-convexity, the Modified OA/ER algorithm of Kravanja and Grossmann [1] was proposed.

The optimal solution of a comprehensive, non-convex and non-linear MINLP problem with a high number of discrete decisions is generally very difficult to achieve. For this purpose, the MINLP Linked Multi-level Hierarchical Strategy (LMHS) was developed to accelerate the convergence of the Modified OA/ER algorithm. With the LMHS strategy we hierarchically decompose the original integer space and the original MINLP problem into several subspaces and corresponding MINLP levels, which significantly improves search efficiency. Decision levels are hierarchically classified as:

- Level of discrete topology and material alternatives (the highest level),
- Level of discrete standard dimension decisions (the middle level),
- Level of rounded dimension decisions (the lower level).

Higher levels give lower bounds to the original objective function to be minimized, while lower levels give upper bounds. The MINLP sub-problems are iterated at each level until the NLP solution has no more improvements. For more on the multi-level strategies see [17, 18].

5 Numerical examples

To demonstrate the applicability of the MINLP optimization approach, three numerical examples are presented. The optimizations are performed with the MINLP computer package MIPSYN [19]. The Modified OA/ER algorithm and the LMHS strategy are applied, using CONOPT4 (Generalized reduced-gradient method) [20] for the solution of NLP sub-problems and CPLEX 12.7 (Branch and Bound method) [21] for MILP main problems. GAMS (General Algebraic Modelling System) [22] is used for modelling.

5.1 Composite floor system

The first example shows the simultaneous cost, material and standard dimension optimization of an I-beam composite floor system with a span of 30 m, which is exposed to its own weight and the uniformly distributed imposed load of 6 kN/m^2.

The composite floor system consists of a reinforced concrete slab and symmetrically welded steel I-beams. A complete shear connection between the concrete slab and the steel profiles is taken into account. The optimization model COMBOPT is developed in the environment GAMS, see [23-24]. The material and labour production costs for the composite beams are considered in the economical type of the objective function, subjected to the given design, material, resistance and deflection constraints defined in accordance with Eurocode 4 [25]. The design plastic moment resistance of the composite section is considered.

The superstructure comprises 7 different concrete strengths (C20/25, C25/30, C30/37, C35/45, C40/50, C45/55, C50/60), 3 different structural steel grades (S 235, S 275, S 355), 9 different standard iron sheet thicknesses (from 8 mm to 40 mm) for webs and flanges separately, 25 different standard reinforcing meshes and 27 different rounded dimensional alternatives in a whole cm for slab depth (from 4 to 30 cm). The combination between the above-mentioned discrete alternatives results in $1.148 \cdot 10^6$ different structural alternatives. One of these is the optimum variant.

The optimal result of 112.08 EUR/m^2 is achieved, see Figure 1. In addition to the optimal self-production costs, the optimal concrete strength C50/60, steel grade S 355, the concrete slab depth, standard reinforcing wire mesh and the optimal standard thicknesses of steel webs and flanges are achieved.

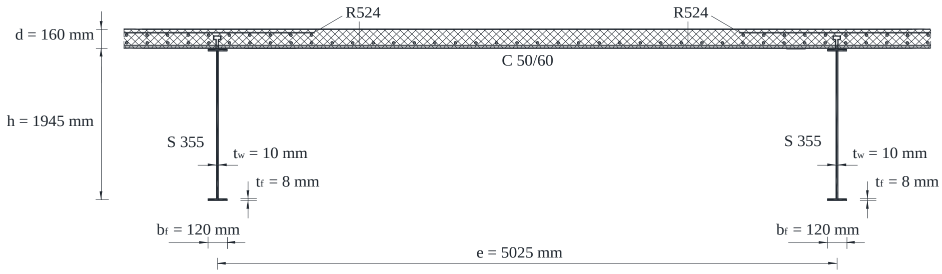

Fig. 1. Optimal composite I beam floor system.

5.2 High-pressure steel penstock Kozjak

The second example deals with the mass optimization of a high-pressure steel penstock, planned to be manufactured for the Kozjak pumped storage hydroelectric power plant in Slovenia, 15 km from the city of Maribor. The Kozjak power plant consists of an already constructed water reservoir of 3 million m^3, a gross water head of 743 m and a net capacity of 2 x 220 MW of 2 Francis-reversible turbines.

The company IBE from Ljubljana prepared the design for the Kozjak power plant and carried out a dimensioning of the penstock. Different variants were planned, including the alternatives of the inclined and vertical penstock. The variants of the inclined penstock were designed in 2011 [26]. The optimization of the penstock variants was then performed at the Faculty of Civil Engineering, University of Maribor.

The optimization model PIPEOPT was developed and applied, see Kravanja [27]. The model includes the mass objective function of the longitudinal pipe sections. The constraints for the dimensioning of the penstock are defined according to the C.E.C.T. recommendations [28]. Two load cases on the pipe are considered. The first case is the internal water pressure caused by filling the penstock with water. The dynamic effect of water hammer is also taken into account. The second load case represents the external water pressure, which is calculated so that it is equal to the height of the external groundwater.

The optimization of the longest penstock variant, variant 1, with the length of 2471.03 m is shown in the paper, see Figure 2. This variant consists of the steel lining without stiffening rings, designed from the high-strength steel S 690Q.

The inner diameter of the pipe varied from 3.9 m to 4.3 m, the various defined pipe length sections are from 144.10 m to 619.78 m long, the corrosion allowance taken into account is 2 mm, the maximum defined internal water pressure is 102.83 bar and the maximum external water pressure taken into account is 46.38 bar. While the safety factor in relation to the internal water pressure is 1.50, the safety factor in relation to the external water pressure is 1.80.

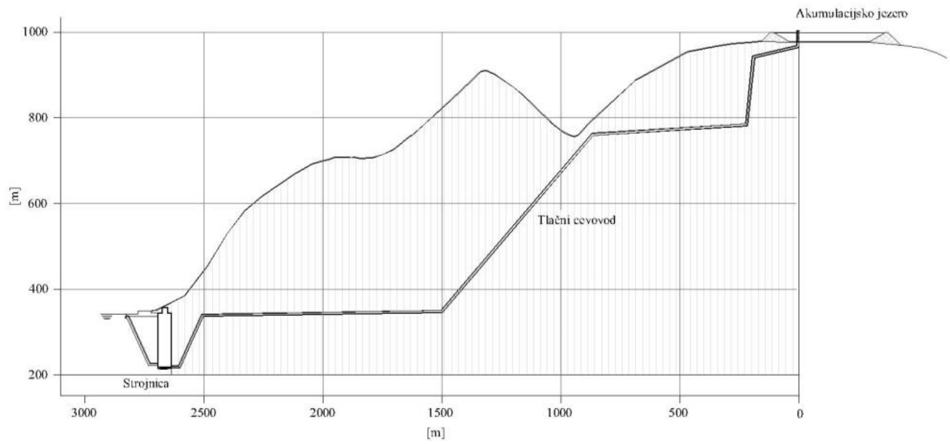

Fig. 2. Longitudinal section of the steel penstock Kozjak.

The superstructure of the treated penstock variant comprises 70 different rounded wall thickness alternatives in mm (from 10 to 80 mm) for all 6 longitudinal penstock sections separately, resulting in 420 different structural alternatives. One of them is the optimal one.

Six different pipe longitudinal sections of different diameters and lengths are optimized. The optimal result is the minimal mass of the obtained steel penstock of 9967.40 tons and the calculated wall thicknesses of 20 mm to 63 mm.

5.3 Underground gas storage Senovo

The third example shows the MINLP optimization of the investment costs of the underground gas storage facility (UGS) in Senovo, Slovenia. The project comprises four equal lined rock caverns (LRC) for the storage of 4x5.56=22.24 million m^3 of natural gas, see Žlender and Kravanja [29].

The optimization model UGSOPT was developed. The cost items and prices defined in the objective function are the same as those used in the project. The optimization model includes geotechnical boundary conditions that ensure that the strength of the rock mass is sufficient, that the lifting of the rock over the cavern is prevented, that the collapse of the rock between the caverns is prevented and that the deformations of the concrete wall and the steel lining are limited.

The LRC superstructure comprises 201 different alternatives with rounded dimensions for the internal diameter of the cavern, 2001 alternatives for the depth of the cavern, 301 alternatives for the height of the cavern tube, 31 alternatives for the thickness of the concrete cavern wall and 201 discrete alternatives for the internal gas pressure. 2735 binary variables are defined. In this way, the combination between the given discrete alternatives for the gas storage gives $7.543 \cdot 10^{11}$ different LRC structural alternatives. One of them is the optimum variant.

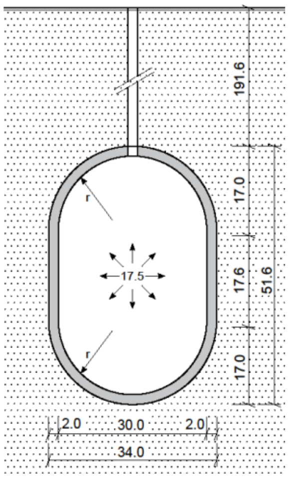

Fig. 3. The optimized lined rock cavern.

The DICOPT program from Grossmann and Viswanathan [30] was selected for optimization. The optimal result is the achieved minimum investment costs of 72.88 million EUR. Figure 3 shows the vertical cross-section of the optimized lined rock cavern with the calculated optimal dimensions and the internal gas pressure.

6 Conclusion

The paper handles with the MINLP approach (Mixed-Integer Non-linear Programming) for structural optimization. MINLP is a combined discrete and continuous optimization technique. It requires that a structure is generated as a MINLP superstructure, consisting of different topology and design alternatives that can be considered for an optimal solution. For each structure an MINLP optimization model is modelled. This model defines the cost or mass objective function of a structure that is subject to design, structural analysis and dimensioning constraints. The Modified Outer-Approximation/Equality-Relaxation algorithm, the Linked Multi-level Hierarchical Strategy and the MINLP computer packages MIPSYN or DICOPT are used for the optimization. MINLP has proven to be a successful optimization technique for solving large-scale, non-linear and discrete optimization problems of structures in civil engineering.

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