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Analysis of Orthotropic RC Rectangular Slabs Supported on Two Adjacent Edges - A Simplistic Approach

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Abstract

The design of reinforced concrete slabs supported on two adjacent edges involves complex formulations. In this paper, a simplistic approach is presented for designing orthotropic slabs supported on two adjacent edges. Slab supported on two adjacent edges (existing slab) is transformed into a slab supported on three edges (equivalent slab) by taking a mirror image of the yield line pattern of two adjacent edges supported RC slabs about its unsupported edges to get the exact collapse mechanism for the slabs supported on three edges. The equivalent aspect ratio can be used in the equations already developed for the slabs supported on three sides. Ultimate moment carrying capacity of the slab carrying uniform load can be evaluated by using the available analytical formulations of the slab supported on three edges. So, the present approach gives a simplified method to analyse and design the orthotropic RC rectangular slab supported on two adjacent edges using the equations available for slab supported on three adjacent edges. Hence, the simplistic approach will be very helpful for structural designers dealing with analysis and design of slabs supported on two adjacent edges.

Keywords: Orthotropic; Concrete; Two Adjacent Edge Supported; Three Adjacent Edge Supported; Slabs; Ultimate Moment; Simplistic Approach.

1. Introduction

Slab is an important structural component, the integrity of the same effects the structural analysis and design concepts. It has been observed that while analyzing the slabs, boundary constraints plays a vital role. The design aids given by the design codes [1-3] are limited to the slabs supported along its all four edges carrying uniform distributed load. The analysis given in these codes is either based on elastic plate theory or theory of plasticity. The design aids of the three edge supported RC slabs for different boundary conditions have been given by Gupta and Singh [4]. Though, the formulations for three-side supported slabs have been presented by Demsky et al. [5] and Singh et al. [6], it is not pertinent to use these formulations for slabs supported along two adjacent edges because of the higher bending moment in these slabs.

In general, yield line analysis is used to analyze the reinforced concrete slabs. It is the limit analysis of the slabs to determine the ultimate moment carrying capacity of the slab just prior to the failure. The ultimate moment carrying capacity of the slab can be examined either by satisfying the yield creation at all points of the slab or by observing the yield line pattern equivalent to lines of maximum curvature and it will characterize the true collapse mechanism. However, it will be easier to examine the possible collapse mechanisms rather than to examine a yield creation at all

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points of the slabs. Collapse mechanisms depend on boundary restraints and yield creation. Therefore, variation in these parameters substantially effects the ultimate load carrying capacity of the slab. The mechanism which gives ultimate load will be considered as true mechanism [5]. In the present research work, the existing slab has been transformed into equivalent slab while considering the collapse mechanism. When existing slab was transformed into equivalent slab, these conditions has been taken into account.

The yield line theory was first proposed by Ingerslev [7]. After that Johansen [8-10] carried out an extensive work in this field by introducing geometrical parameters and published the analytical formulations for analysis of RC slabs using yield line theory. Kemp [11] presents the yield line creation for orthotropic reinforced concrete slabs. This yield creation was developed in form of principle moments irrespective of direction relative to reinforcement axis. Jones and Wood (1967), Nielsen (2003) and Kennedy (2005) worked on the mathematical formulations to predict the collapse load of the reinforced concrete slabs with different combinations of boundary conditions and loading conditions of reinforced concrete slabs [12-14]. Kennedy [15] gives analysis of flat slab, solid slab and raft using yield line theory for simplified design of the same. Megson (2005), Pillai (2011) and Quintas (2003) carried out an extensive study to obtain an ultimate moment carrying capacity of the RC slabs under different loading using yield line theory at ultimate state by establishing suitable collapse mechanisms [16-18].

Abdul-Razzaq et al. [19] studied the behavior of post-tensioned two way slab under flexure with different layouts of tendons. Parametric study was conducted on nineteen different post-tensioned concrete slab models and shows that load carrying capacity of the slab having tendons placed in two directions is greater than tendons placed in one direction. Colombo et al. [20] presents a strip method considering tensile membrane action to obtain ultimate load carrying capacity of reinforced concrete two way slabs. The analytical model was verified for laterally restrained strips and slabs supported on four laterally unrestrained edges.

Al-Ahmed et al. [21] carried out the limit analysis of orthotropic RC slabs supported on four sides carrying uniform distributed load using yield line theory and prepared the design coefficients for the same. Results obtained were compared with ACI code recommendations and it was observed that results from simple yield line theory gives less design bending moment than ACI recommendations which will reduce the cost of construction. Panda et al. [22] carried out an experimental study on two way RC slabs simply supported on three edges and having one edge free under concentrated load. The yield line pattern or failure pattern observed in the experiment matches with Johansen theoretical yield line pattern [9]. Jain and Kumar [23] carried out yield line analysis of RC slabs simply supported on two parallel sides having other two sides as free supporting concentrated load at different locations. Design aids were also prepared for different aspect ratios for correct/true failure mechanism.

Sometimes, while designing the structures either masonry or reinforced concrete, designers may encounter the problem of two adjacent edge supported slabs. It is clear from the above that for the analysis of RC slab supported on two adjacent discontinuous edges the formulations given in the literature are to complex to use in daily routine which takes significant amount of time for analysis. Therefore, a simplistic approach is presented in this paper to easily predict an ultimate moment carrying capacity of the slab irrespective of support conditions i.e. equations used for analysis of slabs supported on 3-edges are also applicable to slabs supported along their two adjacent edges with some minor modifications related to aspect ratio.

2. Methodology

The purpose of this simplistic approach lies in providing some relationships which are of interest, to analyze the behavior of the existing slab at an ultimate state using some few procedural steps rather than carrying out a comprehensive analysis. It can be simply achieved by establishing the relationship between the aspect ratios of existing slab and equivalent slab to transform the existing slab into equivalent slab. It has been assumed that load-deformation behavior of the RC slab is rigid perfectly plastic at an ultimate state, which demands adequately ductile slab section. The overall methodology is shown in flowchart as given in Figure 1.

Let us consider a rectangular RC slab having unyielding simple supports on two adjacent edges having longer edge L_x and shorter edge L_y thereby giving aspect ratio (r) of the slab as L_y / L_x. The positive yield line pattern of the existing slab carrying uniform distributed load (w) at top entire face of the slab is shown in Figure 2. The slab is orthotropic i.e. area of steel placed on tensile face of the slab is different in both directions which gives moment of resistance in x direction as m_x and moment of resistance in y direction as m_y thereby giving the orthotropic coefficient (μ) as ratio of m_y / m_x. The orthotropic of the slab can be obtained from the concept of theory of plates given in Reddy J.N. [24] and Timoshenko and Krieger [25].

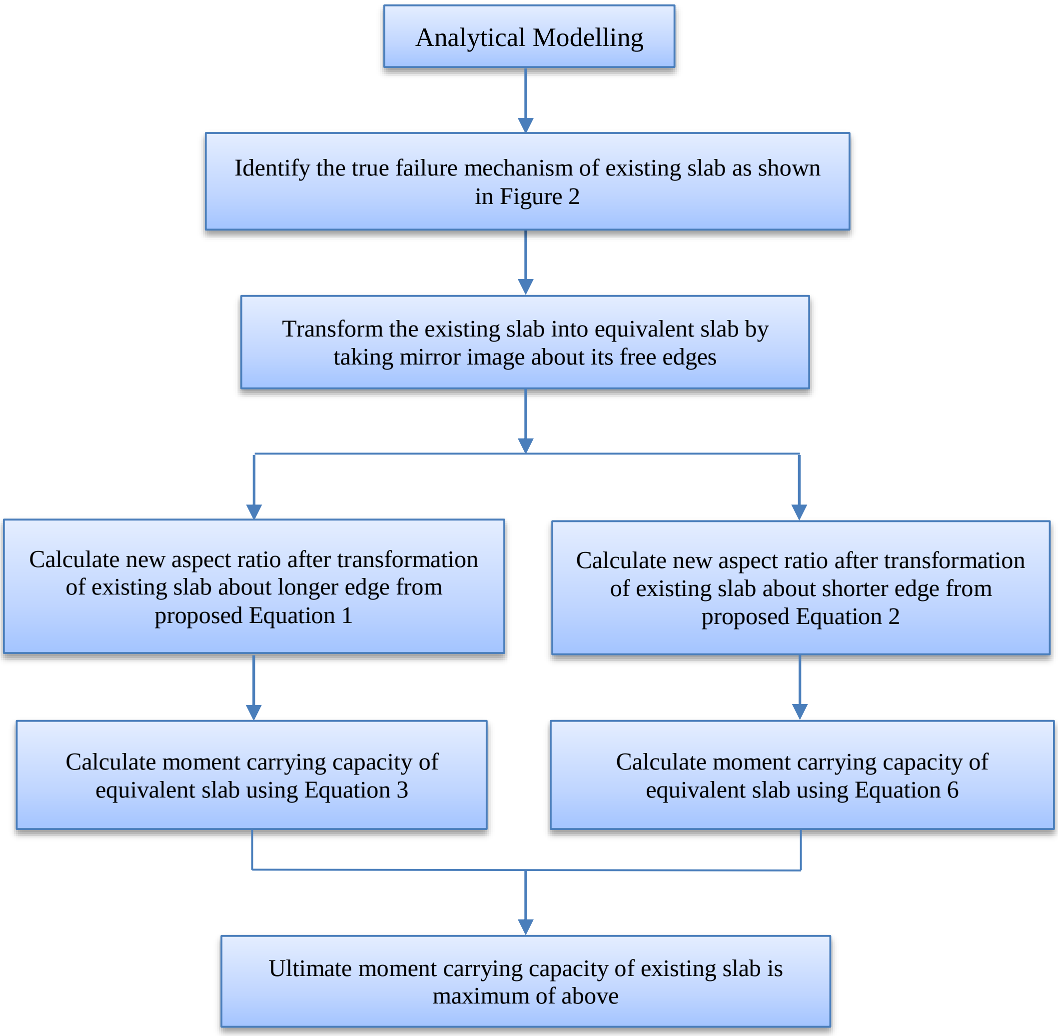

Figure 1. Flow chart of research methodology

Analysis of the existing slab as shown in Figure 2 has been carried out by simply transform of the existing slab into equivalent slab by taking mirror image of the existing slab about its unsupported edges. The slab has two adjacent unsupported edges so, we have to take mirror image about its both unsupported edges separately to get two collapse mechanisms. Both collapse mechanisms should be considered to procure an ultimate moment carrying capacity of the slab.

Mirror image of existing slab about its longer edge L_x is shown in Figure 3 which depicts the positive yield line pattern of the rectangular slab supported on three un-yielding edges when yield lines meet perpendicular to its unsupported edge/free edge. In case of slabs supported on 3-edges, L_x is parallel to unsupported edge and L_y is perpendicular to unsupported edge. As shown in Figure 3, span L_x of the equivalent slab is twice the shorter span of the existing slab L_y. The aspect ratio of equivalent slab is denoted by (R) and can be derived in terms of aspect ratio of existing slab (r) from Equation 1.

$$R = \frac{L_y}{L_x} = \frac{L_x}{2L_y} = \frac{1}{2r} \quad (1)$$

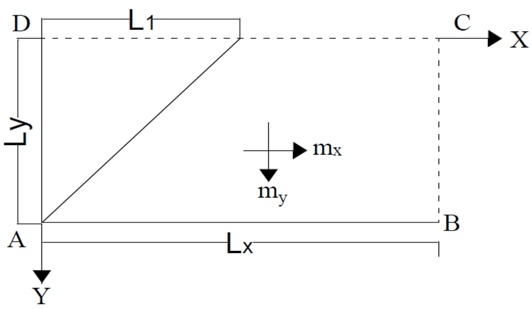

Figure 2. Two adjacent edge supported slab (Existing slab)

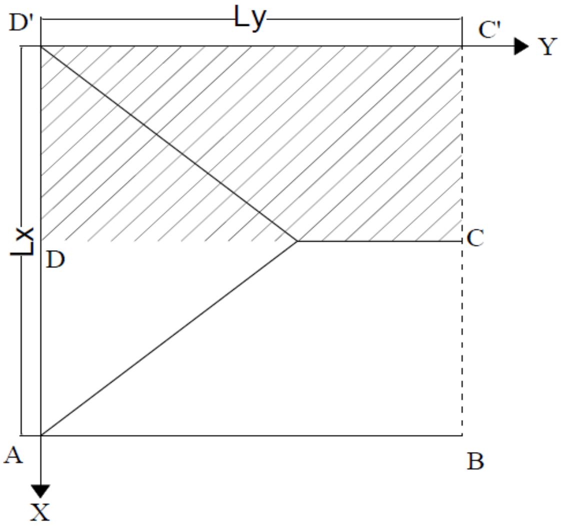

Figure 3. Equivalent slab supported on three sides (Mirror about long edge)

Mirror image of the existing slab about its shorter edge L_y is shown in Figure 4 which depicts the positive yield line pattern of the rectangular slab supported on three un-yielding edges when yield lines meet at some angle with its unsupported edge. As shown in Figure 4, span L_x of the equivalent slab is twice the longer span of the existing slab L_x. The aspect ratio of equivalent slab is denoted by (R) and can be derived in terms of aspect ratio of existing slab (r) from Equation 2.

$$R = \frac{L_y}{L_x} = \frac{L_y}{2L_x} = \frac{r}{2} \tag{2}$$

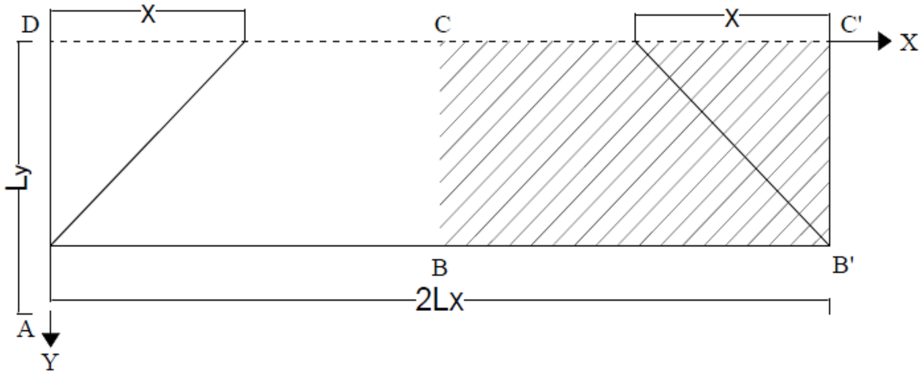

Figure 4. Equivalent slab supported on three sides (Mirror about short edge)

3. Available Formulations for 3-edge Supported Slab

An analytical formulation for slabs supported along three-edges having one edge free carrying uniform distributed load (w) at top face of the slab has been proposed by Gupta and Singh [4]. These formulations have been used in this research work to analyze the slab supported on two adjacent edges by proposing some minor amendments in the equations of aspect ratio as suggested in Equations 1 and 2.

These formulations were based on virtual work method used in yield line theory to establish an equilibrium between internal forces and external forces. There are two failure mechanisms for slabs supported on 3-edges as shown in Figures 3 and 4. To get the true collapse load, both cases should be considered and the mechanism which gives minimum load or maximum moment capacity was taken as a true collapse mechanism.

In these formulations, span L_x has been considered parallel to free edge and span L_y perpendicular to free edge thereby, gives an aspect ratio (R) equals to L_y/L_x. Moments m_x and m_y are positive moments in x and y direction respectively thereby giving an orthotropic (μ) expressed as m_y / m_x. Negative moments in x direction and y direction were denoted by m'_x and m'_y respectively thereby giving *continuity factors* i_1 and i_2 expressed as,

$$i_1 = \frac{m'_y}{m_y} \text{ and } i_2 = \frac{m'_x}{m_x}$$

Case 1. Equivalent slab (Mirror about longer edge)

Moment carrying capacity of the equivalent slab correspond to failure mechanism as shown in Figure 3 can be procured from Equation 3. For the case of slab discontinuous/ simple support on its three supported edges, put value of i_1 and i_2 equals to zero. This case is valid only if value of β_1 is less than 1.

$$m_x = \frac{wL_x{}^2}{24}\frac{(3\beta_1 - \beta_1{}^2)}{(1+ i_2)\left[\beta_1 + \frac{1}{S_1}\right]} \tag{3}$$

In Equation 3, β_1 can be determined by Equation 4.

$$\frac{y}{L_y} = \frac{\sqrt{1+3S_1} - 1}{S_1} = \beta_1 \tag{4}$$

Where, S_1 can be determined by Equation 5:

$$S_1 = \frac{4R^2(1+ i_2)}{\mu(1+ i_1)} \tag{5}$$

Case 2. Equivalent slab (Mirror about shorter edge).

Moment carrying capacity of the equivalent slab correspond to failure mechanism as shown in Figure 4 can be procured from Equation 6. For the case of slab discontinuous on its three supported edges, put value of i_1 and i_2 equals to zero. This case is valid only if value of β_2 is less than 1.

$$m_x = \frac{wL_x{}^2}{12}\frac{(3\beta_2 - 2\beta_2{}^2)}{(1+ i_2)\left[1 + \frac{S_2\beta_2{}^2}{3}\right]} \tag{6}$$

In Equation 6, β_2 can be determined by Equation 7.

$$\frac{x}{L_x} = \frac{\sqrt{4+3S_2} - 2}{S_2} = \beta_2 \tag{7}$$

Where,

S_2 can be determined from Equation 8:

$$S_2 = \frac{3\mu(1+ i_1)}{R^2(1+ i_2)} \tag{8}$$

Moment carrying capacity of the two adjacent edge supported slab carrying uniform distributed load at top face of the slab can be determined using equations of equivalent slab i.e. Equations 3 and 6.

4. Results and Discussion

The moment-field (moment carrying capacity) of the orthotropic reinforced concrete slab supported on two adjacent un-yielding edges can be computed with the help of simplistic approach proposed in this paper. The obtained results were validated with available formulations and are quite satisfactory. The analysis of results are illustrated in the following example in which selection of governing failure mechanism is also a part of discussion.

4.1. Examples

Example 1. Determine an ultimate moment carrying capacity of the reinforced concrete rectangular slab 3×1.5 m supported on its two adjacent edges carrying uniformly distributed load (w) of 10 kN/m^2 at top face of the slab. Take coefficient of orthotropic of the slab as 0.688.

Aspect ratio of existing slab is given by $r = \frac{L_y}{L_x} = \frac{1.5}{3} = 0.5$

Case 1. Equivalent slab (Mirror about longer edge).

After transformation, $L_x = 3$m and $L_y = 3$m

$R = \frac{1}{2r} = 1.0$

$S_1 = \frac{4R^2(1+i_2)}{\mu(1+i_1)} = 5.814$

From Equation 4,

$\frac{y}{L_y} = \frac{\sqrt{1+3S_1}-1}{S_1} = \beta_1 = 0.567$

From equation (3),

$m_x = \frac{wL_x^2}{24}\frac{(3\beta_1-\beta_1^2)}{\left[\beta_1+\frac{1}{S_1}\right]} = 7.00$ kNm/m

Case 2. Equivalent slab (Mirror about short Edge).

After transformation, L_x = 6m and L_y = 1.5m

$R = \frac{L_y}{L_x} = \frac{r}{2} = 0.25$

$S_2 = \frac{3\mu(1+i_1)}{R^2(1+i_2)} = 33.024$

From Equation 7,

$\frac{\boldsymbol{x}}{\boldsymbol{L_x}} = \frac{\sqrt{\boldsymbol{4+3S_2}}\boldsymbol{-2}}{\boldsymbol{S_2}} = \beta_2 = 0.247$

From Equation 6,

$m_x = \frac{wL_x^2}{12}\frac{(3\beta_2-2\beta_2^2)}{\left[1+\frac{S_2\beta_2^2}{3}\right]} = 11.109$ KNm/m

$m_{ux} = \begin{bmatrix} 7.000 \\ 11.109 \end{bmatrix}$ kNm/m

(Whichever is greater).

Example 2. Determine the ultimate moment carrying capacity of the reinforced concrete rectangular slab 2.7×3 m supported on its two adjacent edges carrying uniform distributed load (w) of 10 kN/m^2 at top face of the slab. Take coefficient of the orthotropic of the slab as 0.94.

Aspect ratio of the existing slab is given by $r = \frac{\boldsymbol{L_y}}{\boldsymbol{L_x}} = 0.9$

Case 1. Equivalent slab (Mirror about longer edge).

After transformation, L_x = 5.4m and L_y = 3m

$R = \frac{\boldsymbol{1}}{\boldsymbol{2r}} = 0.56$

From Equation 3,

$m_x = \frac{wL_x^2}{24}\frac{(3\beta_1-\beta_1^2)}{\left[\beta_1+\frac{1}{S_1}\right]} = 13.829$ KNm/m

Case 2. Equivalent slab (Mirror about short Edge).

After transformation, L_x = 6m and L_y = 2.7 m

$R = \frac{L_y}{L_x} \;= \frac{r}{2} = 0.45$

From Equation 6,

$m_x = \frac{wL_x{}^2}{12} \frac{(3\beta_2 - 2\beta_2{}^2)}{\left[1 + \frac{S_2\beta_2{}^2}{3}\right]} = 15.401$ kNm/m

$m_{ux} = \begin{bmatrix} 13.829 \\ 15.401 \end{bmatrix}$ kNm/m

(Whichever is greater).

The mechanism which gives maximum bending moment shall considered as true/exact collapse mechanism. The corresponding moment is considered as ultimate moment carrying capacity (m_u).

4.2. Validation of Results with Available Literature

The failure/collapse mechanism is an integral property of the reinforced concrete slabs. It generally depends on boundary constraints and type of loading. For slabs supported on two adjacent simple supports, there are only two possible failure mechanisms suggested by Kennedy (2003) and Al-Ahmed (2020) [15, 21]. The ultimate moment of resistance of the slab supported on two adjacent edges can be determined using Equation 10 for the failure mechanism as shown in Figure 2 and Equation 11 for the Failure mechanism as shown in Figure 5. For the slabs supported on simple supports, i-values will be kept zero on supported edges.

For example 1, L_x = 3m, L_y = 1.5m, μ = 0.688 and r = 0.5.

$$\frac{L_1}{L_X} = \frac{\sqrt{1 + 3K_1} - 1}{K_1} \tag{9}$$

where $K_1 = \mu \frac{(3 + i_2)}{(1 + i_1)} \left[\frac{1}{r}\right]^2 = 8.256$

$\frac{L_1}{L_X} = 0.493$

$$w_u = \frac{6\,(1 + i_1)\, m_x}{L_1{}^2} - \frac{6\, m_y}{L_y{}^2} \tag{10}$$

Therefore, from Equation 10,

m_x = 11.109 KNm/m.

For the case of cantilever failure as shown in Figure 5, moment of resistance can be determined from Equation 11 proposed in Park (2000) [26].

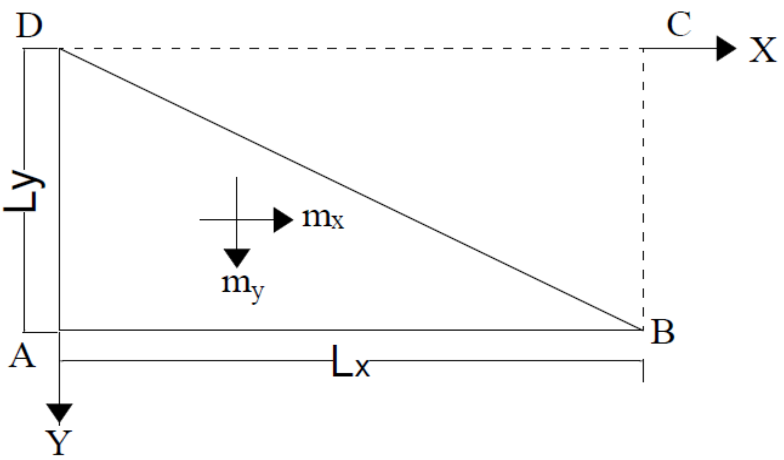

Figure 5. Cantilever failure for two adjacent edge supported slab

$$m_x = \frac{w\,L_x^{\,2}}{6\left[i_1 + \frac{\mu\, i_2}{r^2}\right]} \qquad (11)$$

Solving above equation we get,

m_x = 3.99 KNm/m

$m_{ux} = \begin{bmatrix} 11.109 \\ 3.99 \end{bmatrix}$ kNm/m

(Whichever is greater)

For example 2, L_x = 3m, L_y = 2.7m, μ = 0.94 and r = 0.9.

Therefore, for failure mechanism as shown in Figure 2, moment carrying capacity can be obtained from Equation 10 which comes out be 15.401 kNm/m. For the case of cantilever failure as shown in Figure 5, moment of resistance can be determined from Equation 11 proposed in Al-Ahmed (2020) [21] and comes out to be 6.94 kNm/m.

$m_{ux} = \begin{bmatrix} 15.401 \\ 6.94 \end{bmatrix}$ kNm/m

(Whichever is greater).

The approach proposed in this paper precisely predict the moment-field of the slab only if moment carrying capacity obtained from failure mechanism shown in Figure 2 is greater than moment carrying capacity obtained from failure mechanism shown in Figure 5 which is fulfil in above stated example. It has been observed that generally, failure mechanism shown in Figure 2 is a governing mechanism however, in this paper a relationship has been developed between these failure mechanisms to check the applicability of the approach. From Equations 10 and 11:

$$\frac{w\,L_x^{\,2}}{6\left[\left[\frac{L_x}{L_1}\right]^2 - \frac{\mu}{r^2}\right]} \geq \frac{w\,L_x^{\,2}}{6\left[1 + \frac{\mu}{r^2}\right]} = \left[\left[\frac{L_x}{L_1}\right]^2 - \frac{\mu}{r^2}\right] \leq \left[1 + \frac{\mu}{r^2}\right] \qquad (12)$$

In Equation 12, $\frac{L_x}{L_1}$ can be determined from Equation 13 which is obtained by simplifies the Equation 9,

$$\frac{L_1}{L_X} = \frac{\sqrt{1+3K_1}-1}{K_1} = \frac{\sqrt{1+3\frac{3\mu}{r^2}}-1}{\frac{3\mu}{r^2}} \;\; or \;\; \frac{L_x}{L_1} = \frac{\frac{3\mu}{r^2}}{\sqrt{1+3\frac{3\mu}{r^2}}-1} = \frac{3\mu}{r^2\left[\sqrt{\frac{r^2+9\mu}{r^2}}-1\right]} \;\; or \;\; \frac{L_x}{L_1} = \frac{\sqrt{r^2+9\mu}+r}{3r} \qquad (13)$$

To check the applicability of the above simplified equation, results obtained from Equation 9 proposed by Park (2000) [26] and Equation 13 has been observed and it found to be equal. Put Equation 13 in Equation 12 to procure a relationship between failure mechanisms. This has been given below:

$$\frac{\left[\sqrt{r^2+9\mu}+r\right]^2}{9} - \mu \leqslant r^2 + \mu \qquad (14)$$

In above stated example 1, orthotropic coefficient (μ) = 0.688 and aspect ratio (r) = 0.5 which fulfils the condition given in Equation 14 as follows:

0.337 < 0.938

In above stated example 2, orthotropic coefficient (μ) = 0.94 and aspect ratio (r) = 0.9 which also fulfils the condition given in Equation 14 as follows:

0.788 < 1.75

Furthermore, to check the applicability of the approach for different aspect ratios, let us consider one more case of slab 0.75×3 m supported on two adjacent sides thereby, giving aspect ratio of 0.25. The orthotropic coefficient of the slab is 0.56. It also fulfils the condition given in Equation 14 as follows:

0.139 < 0.6225

The fulfilment of the above condition shows that failure mechanism assumed while transforming the existing slab into equivalent slab is the correct/true failure mechanism. The moment carrying capacity procured from this failure mechanism is the ultimate moment carrying capacity of the slab.

Table 1. Comparison of results with previous studies

Ultimate Moment Carrying Capacity	Simplistic Approach		Available Formulations in Literature, [26]	
	Example 1	Example 2	Example 1	Example 2
In x direction, m_{ux} (KNm/m)	11.109	15.401	11.109	15.401
In y direction, m_{uy} (KNm/m)	7.642	14.476	7.642	14.476

Table 1 indicates that true/ultimate moment carrying capacity of the existing slab obtained from simplistic approach is well comparable with the available formulations suggested in Park (2000) [26]. However, the suggested approach simplifies the analysis of the slabs. Designers can use Equations 3 and 6 for analysis of both 3-edge supported slab and slab supported along two adjacent edges carrying uniformly distributed load at entire top surface of the slab. It is clear that the proposed simplistic approach can be used to evaluate the ultimate moment carrying capacity of the reinforced concrete single panel rectangular slab supported on two adjacent discontinuous edges for any aspect ratio and orthotropic coefficients

5. Conclusions

In present research work, a simplistic approach is presented by the authors to predict an ultimate moment carrying capacity of the two adjacent edge supported reinforced concrete slab. This simplistic approach is also validated with available formulations given in literature and results are quite satisfactory. However, maximum percentage difference is zero percent. This analytical approach is conducive to precisely analyse the existing slab (slab supported along its two adjacent edges) with minor modifications in equations suggested by the authors for equivalent slab (slab supported along its three edges).

Every rectangular slab supported on its two adjacent rigid edges, called an existing slab, can be transformed into a rectangular slab supported on three sides, called as equivalent slab, by taking mirror image of the yield line pattern of the existing slab about its free edges. These transformations defines the shape of yield line patterns at collapse which can be used in daily routine to determine the ultimate moment carrying capacity of the slab and thus save computational time

6. Conflicts of Interest

The authors declare no conflict of interest.

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