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Passenger Centric Train Timetabling Problem with elastic demand

Ricardo García-Ródenas^a, María Luz López-García^a, Julio Alberto López-Gómez^{b,*}, Luis Jiménez Linares^b

^aDepartment of Mathematics. University of Castilla la Mancha, Paseo de la Universidad 4, 13071 Ciudad Real. Spain ^bDepartment of Technologies and Information Systems. University of Castilla la Mancha, Paseo de la Universidad 4, 13071 Ciudad Real. Spain

Abstract

A wide taxonomy of the adopted design criterion in models for Train Timetabling Problem (TTP), considers two kinds of models: operations-centric models, which are focused on railway operations and assume the demand has been considered and modeled in the design stage of the railway network, and passenger-centric models, which tries to maximize customers satisfaction. The first models have been widely studied since historically, the train operating companies (TOC) had a monopoly in the railway market. Currently, and because of the European directive (EU Directive 91/440) which allows the competition between different TOC, the research interest in the second kind of models is increasing. This paper proposes a mathematical model for TTP which includes the elasticity of the demand against the characteristics of supply. To do that, the proposed model is based on the concept of strategy and by means of the use of radial-basis functions, the demand model allows to estimate the number of passengers in each strategy on the network, taking into account the existing correlation between strategies.

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Keywords: Train Timetabling Problem (TTP); models focused on passengers; elastic demand

1. Introduction

Mathematical models for Train Timetabling Problem (TTP) can be roughly classified in two main groups according to the adopted design criterion: i) *Operations-centric models* and ii) *Passenger-centric models*. The first group assumes the demand has been considered previously when the network design problem was addressed. These models are focused on railway operations, for example, minimizing the operational costs or increasing the robustness of the timetables in order to absorb disruptions (see Fischetti et al. (2009), Liebchen and Stiller (2009)). The second group of models is focused on passengers and the aim is to increase customers satisfaction, for example, maximizing the number of direct links between stations or minimizing the waiting time for passengers (see Wong et al. (2008)).

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^{*} Corresponding author. Tel.: +34-926-295-300 Ext. 3351

E-mail address: julioalberto.lopez@uclm.es

Traditionally, the operations-centric models have attracted the most attention from the research community. It is due to the fact that, historically, train operating companies (TOC) had a monopoly in the national railway market and assumed a captive demand. Currently, many European railway markets allow the competition between TOCs (EU Directive 91/440). The liberalized context increases the motivation in passengers-centric models, since users can switch from one TOC to another, where the timetables and/or prices are more attractive for them.

Passenger-centric models must include the elasticity of the demand against the supply's features. To do that, the key element in these models is a demand forecasting model. The most widely used approach is based on the lines frequency concept. However, these methods are not suitable for railway systems with low frequencies or for problems at an operational level where the users choose explicitly the arrival time at their destination. In these situations, demand forecasting models based on timetables are necessary. Thus, Cascetta and Coppola (2012), Cascetta and Coppola (2014) developed a multimodal assignment model based on timetables considering an elastic demand to high-speed trains.

However, the goals employed in both type of models may come into conflict. For example, including more direct connections will lead to an increase in operating costs. In order to address this situation, Robenek et al. (2016) proposed a linear-integer multiobjective model where the concept of satisfaction was introduced in TTP. The preferred instant to arrive to destination appeared in the conformation of the design criterion, among other aspects. This approach considered an inelastic demand. Later, the same authors proposed a TTP using elastic demand, in Robenek et al. (2018).

Espinosa-Aranda et al. (2015) employed a constrained nested logit model to build a TTP model oriented to the passenger's preferences. This model considered non-linear utilities defined through radial-basis functions. This way, the model included attributes such as the preferred arrival time, price, travel time, etc. The main feature of this model is the explicitly inclusion of the capacity restrictions, which causes a competitive context between passengers and generates an equilibrium point. These authors applied this model in the Spanish high-speed corridor Madrid-Seville.

When passenger-centric TTP is addressed, the utility or satisfaction of passengers is a relevant dimension in the definition of the design criterion. In a competitive context between TOCs and transport modes, it will be necessary to model explicitly traveler's behaviour. At tactical planning, the concept of *strategy* appears in passengers assignment models for urban public transport (see Spiess and Florian (1989), Cominetti and Correa (2001), Cepeda et al. (2006), Codina (2013)) as a mechanism to describe the decisions of the passengers on the transit network. A strategy is any selection of a set of lines, including transfers, and conditional decisions depending on the vehicle that arrives first at the stop, such that it allows the passengers reach their destination. The optimal strategy is that do the passenger travel at minimum expected cost.

In this paper, a TTP model which includes the elasticity of the demand against features supply is proposed. The proposed model extends the concept of strategy, which was developed for frequency-based public transport assignment models, to schedule-based approaches.

The rest of the paper is structured as follows: Section 2 defines the problem and formulates the proposed model, Section 3 details the preliminary numerical experiments carried out, and finally, Section 4 shows the conclusions and future lines of research.

2. Problem and formulation of the model

In this paper, a railway deregulated market is assumed. In it, the competence between Train Operating Companys (TOCs) exists. Each of these companies operate a set of railway lines and offer a set of services with different features such as timetables, travel times, prices, etc. The TOCs compete with each other in order to increase their market share and benefit. Regarding passengers, they observe the so-called *service network* (see Marín and García-Ródenas (2009)), it means, the set of all possible alternatives to satisfy their trip, considering a traveller can combine services

from different TOCs to make his/her trip.

The general problem incorporates a Nash equilibrium between the passengers which compete with each other for the capacity of the best services (best timetables, the cheapest, so on). Modeling both equilibria, TOC equilibrium and passenger equilibrium, is complex. For that reason, a simplification of the problem will be effected in this work. We assume the point of view of a certain TOC that will be called *reference* TOC. The general problem can be stated as finding the optimal planning for the reference TOC to face the competence. This strategy may combine pricing policy, seat allocations, timetables among others factors. For sake simplicity, we consider in this paper that the reference TOC only wishes to design the timetables and the other attributes are fixed. To avoid the modeling of both equilibria, we assume that the supply provided by the others TOCs is fixed and there exist an unlimited capacity of the trains. The unlimited capacity is assumed at tactical level to solve TTP, since this hypothesis will be relaxed in an operational context when rolling stock assignment is addressed using a microscopic point of view. The goal of the reference TOC is to schedule its own supply of railway services assuming both hypothesis. The different components of the proposed model will be discussed.

2.1. Supply of railway services

Let \mathcal{L} be the whole set of railway lines operated on a railway network \mathcal{G} . The lines \mathcal{L} are managed by the different TOCs. We assume that the reference TOC operates a subset of lines $\widehat{\mathcal{L}} \subseteq \mathcal{L}$. Moreover, the whole set of trains in the railway network is denoted as \mathcal{T} and the subset of trains managed by the reference TOC as $\widehat{\mathcal{T}}$. We also assume that the reference TOC assigned each train $T \in \widehat{\mathcal{T}}$ to the lines $L \in \widehat{\mathcal{L}}$.

Furthermore a railway service can be defined as a pair $\ell = (L, T)$ where $L \in \mathcal{L}$ and $T \in \mathcal{T}$. We denote as S the set of all railway services. Note that the railway services depend on time which are operated and if the same pair (L, T) is run in two different instants, the associated railway services $\ell = (L, T)$ and $\ell' = (L, T)$ are treated as two different elements of S.

Figure 1 shows four lines on a railway network. Assuming there are four trains T_1, T_2, T_3, T_4 , a set of railway services may consists of

$$S = \{\ell_1 = (L_1, T_1), \ell_2 = (L_2, T_2), \ell_3 = (L_2, T_3), \ell_4 = (L_3, T_4), \ell_5 = (L_4, T_4)\}$$



Fig. 1. Example of railway network

In the proposed model, an strategy is defined in the following way: Let o and d be two stations in the railway network. Consider the origin-destination pair $\omega = (o, d)$. We say that a sequence of railway services $j = \{\ell_1, \dots, \ell_r\}$ is a strategy for ω if an user who begins his/her trip at the station o can achieve the station d, using all railway services. We denote the set of strategies j in S for the origin-destination pair ω as J_{ω} . Two strategies j and j' are different if the set of their railway services or the order in which the train services are used are different.

Consider the origin-destination pair $1 \rightarrow 3$ shown in Figure 1, a service network may have the following strategies.

Strategy	Definition
$I = \{j_1 = \{\ell_1\},\$	$1\overset{L_1(T_1)}{\rightarrow}3$
$j_2 = \{\ell_2\},$	$1\stackrel{L_2(T_2)}{\rightarrow}2\stackrel{L_2(T_2)}{\rightarrow}3$
$j_3 = \{\ell_3\},$	$1\stackrel{L_2(T_3)}{\rightarrow}2\stackrel{L_2(T_3)}{\rightarrow}3$
$j_4=\{\ell_4,\ell_3\},$	$1\stackrel{L_3(T_4)}{\rightarrow}2\stackrel{L_2(T_3)}{\rightarrow}3$
$j_5=\{\ell_2,\ell_5\},$	$1\stackrel{L_2(T_2)}{\rightarrow}2\stackrel{L_4(T_4)}{\rightarrow}3$
$\{t_3, t_5\}, Not$	temporarily consistent
$\{\ell_4, \ell_2\}$ Not	temporarily consistent

2.2. Demand-forecasting model

The main feature of the proposed demand-forecasting model is it will take into account the correlations among strategies. This property is essential in a deregulated market due to it captures the effect of the changes in the features of a railway service on the existing service network. For sake clarity, we omit the index ω associated with the origin-destination pairs.

The reference TOC states the features of their services $\widehat{S} \subseteq S$. These decisions modify the structure of the space of strategies due to some connections may be added/deleted, as well as the characteristics of the own strategies. We denote as *i* a feasible decision of the reference TOC. In short, we refer a *i* as the supply. The set of feasible supplies for the reference TOC is denoted by *I*.

Furthermore, we assume a strategy *j* is characterized by a vector of attributes $x \in \mathbb{R}^P$ such as arrive time, monetary cost, in-vehicle time, waiting time, number of transfers, etc. Reciprocally, we assume that there exists a set $X \subset \mathbb{R}^P$ which represents all feasible strategies. That is, a given vector $x \in X$ is associated with a potential strategy.

Let's suppose we are planning the supply *i* for a certain temporal interval. The *potential demand* \bar{g} for this period is the generated demand if all the alternatives in the region X are available to passengers. Obviously, X is a continuous set and it has infinite alternatives. Besides, we consider the so-called *potential demand intensity function* D(x) which measures the induced demand for any set of strategies x. Actually, the service network has a finite set of alternatives, according to the proposed supply *i* by the reference TOC. We also consider there is *effective demand intensity function* for each supply *i*, and it is denoted by $D_i(x)$. We distinguish between potential demand, which is the maximum demand that can be generated in the railway network, and the effective demand which is induced by the supply *i*. They are computed through the expressions:

$$\bar{g} = \int_{\mathcal{X}} D(\xi) \mathrm{d}\xi, \quad \bar{g}_i = \int_{\mathcal{X}} D_i(\xi) \mathrm{d}\xi \tag{1}$$

The model assumes that D(x) is known and the effective demand intensity function $D_i(x)$ is derived from D(x). Both intensity functions can be related in the following way (see García-Ródenas and López-García (2015): let \mathcal{H} be a Reproducing Kernel Hilbert Space (RKHS). The space of functions \mathcal{H} is univocally defined by the so-called *Mercer kernel* $K(x, \tilde{x})$ and we state this fact by adding the suffix K, i.e, \mathcal{H}_K . The family of functions $\{K(x, \tilde{x})\}_{\tilde{x} \in \mathcal{X}}$ constitutes a basis of the vector space \mathcal{H}_K .

We assume the effective intensity function $D_i(x) \in \mathcal{H}_K$ and this element can be expressed as a linear combination of any basis, in particular

$$D_i(x) = \sum_{\tilde{x} \in X_i \subset X} \alpha_{\tilde{x}} K(x, \tilde{x})$$
(2)

Note that in the expression (2) the kernel function $K(x, \tilde{x})$ is provided by the planner and it is known but the vector of parameters $\alpha_{\tilde{x}} = (\cdots, \alpha_{\tilde{x}}, \cdots)$ and the set of points X_i are unknown. The induced demand from a supply *i* should be expressed as a function of the generated alternative set J_i . For this reason, we impose that the set X_i consists of the set of attribute vectors x_i of the current strategies $j \in J_i$.

$$D_i(x) = \sum_{j \in J_i} \alpha_j K(x, x_j)$$
(3)

The kernel function $K(x, x_j)$ measures the similarity between the desired railway service x and the established railway service x_j . It allows to assess if a passenger which wishes to make his/her trip in the service x has or has no a close alternative generated by the supply *i*.

The definition of D(x) is based on the assumption that there exists a railway service for any vector of features $x \in X$. The functions D(x) and $D_i(x)$ should exhibit a similar shape around the railway services of the *i* supply. This fact is used to compute the vector of parameters $\alpha_i = (\cdots, \alpha_j, \cdots)^{\top}$ where $j \in J_i$, we impose that the $D_i(x)$ functions should interpolate to the function D(x) in the alternatives generated by the supply *i*,

$$D(x_j) = D_i(x_j) = \sum_{j' \in J_i} \alpha_{j'} K(x_j, x_{j'}) \text{ with } j \in J_i.$$

$$\tag{4}$$

The interpolation problem (4) leads to approximate the vector of parameters $\alpha_i = (\dots, \alpha_j, \dots)$ as the solution to the linear equation systems.

$$\mathbf{K}_i \boldsymbol{\alpha}_i = \mathbf{D}_i, \tag{5}$$

where \mathbf{K}_i is the Gram matrix $\mathbf{K}_i = K(x_j, x_{j'}), j, j' \in J_i$ and $\mathbf{D}_i = (\cdots, D(x_j), \cdots)^{\top}$ where $j \in J_i$.

Once the parameters vector α_i is computed, the split of the induced demand by alternatives can be computed as follows:

$$\bar{g}_i = \int_{\mathcal{X}} D_i(\xi) \mathrm{d}\xi = \int_{\mathcal{X}} \sum_{j \in J_i} \alpha_j K(\xi, x_j) \mathrm{d}\xi = \sum_{j \in J_i} \alpha_j \int_{\mathcal{X}} K(\xi, x_j) \mathrm{d}\xi = \sum_{j \in J_i} \bar{g}_{i,j}$$
(6)

where

$$\bar{g}_{i,j} = \alpha_j \int_{\mathcal{X}} K(\xi, x_j) \mathrm{d}\xi, \forall j \in J_i$$
(7)

Once the demands $\bar{g}_{i,j}$ for all $j \in J_i$ have been computed, the demand for the services $\ell \in \widehat{S}$ of the reference TOC can be also calculated. Note that i) if the reference TOC operates the railway network on an exclusive basis then it is not mandatory to split the demand by alternatives to compute the attracted demand by the reference TOC, it is sufficient to evaluate the single integral given in Eq. (6) instead of assessing the integrals given in Eq. (7), and ii) there is no guarantee to ensure that the parameters α_j were not negative.

2.3. Train Timetabling Model

The reference TOC has a set of feasible decisions I on its supply of railway services. These decisions define the attributes of the railway services, such as departure time, capacity, fares, among others. The changes in the features of the railway services modify the vector of attributes x_j of the alternatives, and it make variations on the demands $\bar{g}_{i,j}$. In a competitive market, the reference TOC wants to maximize the capture of passengers or alternatively to maximize the revenue. In this paper, the first criterion is adopted. To compute properly the demand attended by the reference TOC, all TOCs operating in railway network must be simultaneously analyzed. The inclusion of these TOCs expand the choice set of the passengers by adding new alternatives j and these passengers $g_{i,j}$ will be attended by the non-reference TOCs.

If the same user employs two or more railway services of the reference TOC, this passenger is counted as the number of bought tickets. Mathematically, the TTP fot the reference TOC problem is stated as:

[**TTP**]: Maximize
$$\sum_{j \in J_i} g_{i,j}$$
 (8)

This problem is enough general to tackle simultaneously pricing, timetabling, and other problems in an unified way. In this paper, we focus on the TTP problem. Note that the index associated with the origin-destination pair ω has been omitted. The extended formulation the problem exhibits the following form:

$$[\mathbf{TTP}]: \quad \text{Maximize}_{i \in I} \sum_{\omega \in W} \sum_{j \in J_i^{\omega}} g_{i,j}^{\omega}$$
(9)

where J_i^{ω} indicates that the set strategies for the o-d pair ω depends on the characteristics of the supply *i*. Note that if the departure time of the railway services is changed, it is possible certain connections are no longer feasible, or on the contrary, new connections appear. It implies that the modification of the supply *i* transforms the set of strategies J_i .

TTP is addressed in the tactical planning and the railway conflicts are overlooked at a tactical level. After solving TTP, when routing of trains is addressed in a later stage, conflicts will be handled using a microscopic approach. The proposed model assumes that the trains go maximum speed and this assumption leads to the decision variables are the *departure time* of the trains at the head stations, i.e. t_{ℓ} . This assumption can be relaxed by the introduction of dwell times to allow certain connections between train routes. Due to the key issue of the model is the modeling of passenger's behavior with respect to the scheduling decisions, we adopt the maximum simplification. Thus, only the rolling stock constraints are considered. We assume that the set of train services \widehat{S} to be operated is known and let $S_T = \{\ell_1, \ell_2, \dots, \ell_r\}$ be the set of train services operated by the same train *T*. The order in which the train routes are operated is assumed to be known, i.e. $\ell_1 < \ell_2 < \dots \ell_r$. This sequence imposes implicitly that the destination of ℓ_s matches with the origin of the next train route ℓ_{s+1} . Two train routes $\ell_s, \ell_{s+1} \in S_T$ hold the constraint

$$t_{\ell_s} + ta_{\ell_s} \le t_{\ell_{s+1}}, \quad s = 1, \cdots, r-1;$$
(10)

where t_{ℓ_s} , $t_{\ell_{s+1}}$ are the departure times of the railway service ℓ_s , ℓ_{s+1} and ta_{ℓ_s} is the adding of the travel time to complete the trip of ℓ_s plus the (minimal) turn-around time for ℓ_s .

3. Numerical experiments

In this section, the numerical experiments carried out to assess the proposed model are detailed. To do that, the network depicted in Figure 1 has been used. This railway network consists of 3 stations, 4 lines and only one (reference) TOC. In this example, the potential demand function used is

$$D(\tau, C, a) = \max\{-VOT \cdot \tau - C + \phi(a), 0\}$$
(11)

where the attributes analised for the feasible space of strategies are

- $\tau \equiv The generalized travel time.$ The generalized travel time of a stragey is computed as $\tau = t_{train} + 2.5 t_{waiting} + \frac{n_{transfer}}{6}$ where t_{train} is the in-vehicle time (in hours) that passengers spends within the trains, $t_{waiting}$ is the time (in hours) that passangers spend waiting between connections and $\frac{n_{transfer}}{6}$ is a penalization associated with the number of transfer(s) $n_{transfer}$ used for the passengers. The parameter VOT is the value of the time, i.e. it is the willing-to-pay for travel time savings. We set $VOT = 25.3 \frac{e}{h}$ as is used in the Robenek et al. (2016).
- $C \equiv$ The total cost of tickets.
- $a \equiv$ The arrive time at the destination, in this example we've considered the function $\phi(a) = \max\{40, 200 40 * (a 8)^2, 200 40 * (a 16)^2\}$ which indicates that the passengers want to arrive at around 8 am or 4 pm.

The feasible region of the strategies is the domain $x = (\tau, C, a) \in X = [0.5, 5] \times [2, 10] \times [5, 19]$ and thus the total potential demand is computed as $\bar{g} = \int_X D(\xi) d\xi = 10150.69$. In this example a Gaussian kernel $K(x, x_j) = \exp(-\theta ||x - x_j||^2)$ with a $\theta = 0.4$ is used. Moreover, other parameters are the waiting time of the trains at the stations which is set 2 minutes and the time that a passenger needs to do a transfer between two trains is 3 minutes. These parameters are used to set the feasible connections.

Figure 2 shows the elasticity of the demand for the three o-d pairs when the number of trains in each line is increased. It is assumed that the departure time of the trains is uniformly distributed on the planning time and the demand-forecasting model estimates the number of passengers.



Fig. 2. Elasticity of demand in Network 1

Finally, the TTP model is assessed. It is non linear and non-concave (it is posed in the maximization form). Thus, it is required global optimization methods. First, we assume that all railway services are operated with different trains and thus, the rolling stock constraints are not dealt with. The decision variables to be optimized are the departure times of the all railway services. The number of of railway services and the assigned line are fixed by the railway planner. We try to evaluate the optimization method with respect to the performance and the convergence to optimum.

In this test Genetic Algorithms (GA), Particle Swarm Optimization (PSO), CMA-ES and interior point algorithm (IP) have been considered. To do that, the Matlab functions ga, particleswarm, ocmaes, fmincon, have been used. The motivation of using metaheuristics is to avoid the local optima and the exact method is to test its convergence.

The optimal timetable for this railway network is shown in Table 1 and Figure 3 shows the convergence curves of the selected algorithms. It can be noticed that there are big differences between arrivals and departures times. This fact is motivated by the number of services to be operated, which is imposed to be small and, henceforth, it makes the trains spend large time in head stations. The computational burden and the convergence of the exact method to local optima make advisable the use of memetic approaches (see García-Ródenas et al. (2019)).

Railway service	In-vehicle time (in h.)	Price of ticket (in h.)	Departure time	Departure time
	(per segment)	(per segment)	\rightarrow (am)	← (am)
$L_1 = 1 - 3$	0.83	3.5	6.4,13.4	17.2,24
$L_2 = 1 - 2 - 3$	0.92	3.5	15.2	22.7
$L_3 = 1 - 2$	1.50	2	6.6,13.7	1.3,5.8
$L_4 = 2 - 3$	1.50	2	6.5,13.5	13.5,15.1

Table 1. Features of the optimal timetable for Network 1

4. Conclusions and further works

In this work, a passenger-centric TTP model is proposed. It includes the elasticity of the demand against the features of the supply. To do that, an interpolation approach based on radial basis functions is employed. Besides, the



Fig. 3. Optimization alogrithms and its progresses

user's behaviour has also been considered through the concept of strategy. The kernel functions gauge the correlation between strategies and the use of integrals joint to the interpolation approach allow the spliting of the effective demand by alternatives.

As future work, it is intended to carry out more experiments in real railway networks in order to validate more accurately the presented model. Finally, it is pursued to enrich the model adding other operational constraints and facing the general case of TOC equilibrium instead of considering only the reference TOC.

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