On the Solution of Revenue- and Network-Constrained Day-Ahead Market Clearing under Marginal Pricing—Part II: Case Studies

Ricardo Fernández-Blanco, *Member, IEEE,* José M. Arroyo, *Senior Member, IEEE,* and Natalia Alguacil, *Senior Member, IEEE*

***Abstract*—This paper presents the numerical analysis of the bilevel programming approach for revenue- and network-constrained market clearing developed in its companion paper. The impact of minimum revenue conditions and minimum declared profits on generation and consumption levels as well as on locational marginal prices for energy is examined in detail through three case studies. First, the results from an illustrative example including minimum revenue conditions are comprehensively analyzed. The second case study is based on the IEEE Reliability Test System and considers minimum declared profits. In the third case study, a modified version of the IEEE 118-bus system is tested while accounting for minimum revenue conditions. In addition, the computational behavior of the proposed approach is illustrated with several case studies including the IEEE 300-bus system. Numerical results show the effectiveness of the proposed approach to handle revenue constraints as well as its superiority over the heuristic currently implemented in the Iberian electricity market. Moreover, simulations reveal that, unlike previous works in the literature, generation revenue constraints can be precisely incorporated in day-ahead market clearing while explicitly considering the standard economic-dispatch-based marginal pricing scheme and without requiring price uplifts.**

***Index Terms*—Locational marginal pricing, minimum profit constraints, minimum revenue conditions, revenue-constrained auction design.**

I. Introduction

POOL-BASED electricity markets worldwide [1]–[4] incorporate specific features in their designs in order to fulfill the revenue sufficiency rule [5]–[7]. A relevant practical example is the consideration of generation revenue constraints in the day-ahead auction model such as the so-called minimum income conditions routinely accounted for in several European electricity markets [2], [4]. The yet unresolved challenges posed by the incorporation of generation revenue constraints in the auction process have triggered considerable research effort both under the conventional declared social welfare maximization [8]–[13] and under alternative optimization goals [14]–[19]. In those works, generation revenue constraints not only include minimum revenue constraints but also minimum profit constraints.

The thrust of this paper is to empirically validate the bilevel-programming-based approach, hereinafter referred to as B-RC, described in its companion [20]. B-RC is developed to overcome the difficulties of both current practical solutions and available works in the technical literature for revenue- and network-constrained day-ahead market clearing driven by declared social welfare maximization under marginal pricing. For expository purposes, two instances of generation revenue constraints are analyzed, namely minimum revenue conditions and minimum declared profits.

The performance of B-RC has been assessed with 1) a revenue-unconstrained auction [21], which is formulated as a mixed-integer linear program; 2) the *ad-hoc* heuristic currently implemented by the system operator of the Iberian electricity market [2] for minimum revenue conditions; and 3) the integrality-relaxation-based technique presented in [12] for minimum profit conditions. For quick reference, those methodologies are denoted by No-RC, H-RC, and R-RC, respectively.

The heuristic described in the operational rules of the Iberian day-ahead electricity market [2] is based on iteratively solving a revenue-unconstrained auction and computing the resulting revenue margins for all generating units that are scheduled on. The revenue margin of a generating unit is defined as the difference between the revenue earned and the minimum revenue this unit is willing to earn according to a minimum revenue function that is part of the supply offer. At each iteration, the generating unit with the largest negative revenue margin is scheduled off for all subsequent iterations. The process stops when either all revenue margins are greater than or equal to 0 or when no feasible solution exists for the revenue-unconstrained auction as a result of the iterative reduction in the set of available generating units for scheduling. Unfortunately, this approach does not guarantee optimality and may even terminate without attaining feasibility.

The solution methodology described in [12] is based on 1) relaxing the integrality constraints of the original problem in order to derive its dual counterpart, 2) minimizing the duality gap of both primal and dual problems subject to revenue constraints, and 3) applying the binary expansion approach presented in [22] to linearize bilinear revenue terms. As a consequence, this methodology features three main drawbacks, namely 1) awarded generation and consumption levels may

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Ricardo Fernández-Blanco is with the Department of Electrical Engineering, University of Washington, Seattle, WA 98195-2500, USA (e-mail: rfbc85@uw.edu).

José M. Arroyo and Natalia Alguacil are with the Departamento de Ingeniería Eléctrica, Electrónica, Automática y Comunicaciones, E.T.S.I. Industriales, Universidad de Castilla-La Mancha, Ciudad Real E-13071, Spain (e-mail: JoseManuel.Arroyo@uclm.es; Natalia.Alguacil@uclm.es).

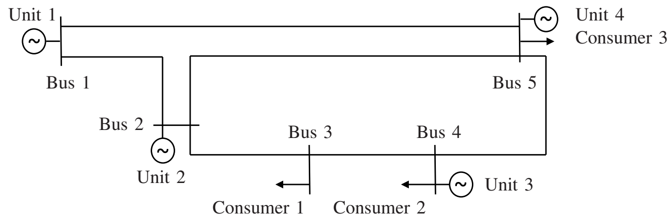

Fig. 1. One-line diagram for the illustrative example.

TABLE I
ILLUSTRATIVE EXAMPLE – GENERATION DATA

Unit	$\underline{P}^{g}_{it}$ (MW)	$\overline{P}^{g}_{it}$ (MW)	C^{su}_{it} ($)	C^{g}_{1it} ($/MWh)	M^{f}_{i} ($)	M^{v}_{1it} ($/MWh)
1	60	600	6000	10	5000	10
2	15	210	3000	15	800	15
3	20	280	360	30	0	30
4	10	200	150	35	180	35

not correspond to an economic dispatch solution, thereby jeopardizing the efficiency of the use of energy resources; 2) the resulting vector of locational market-clearing prices may not be consistent with the theory of marginal pricing, which is the worldwide accepted standard; and 3) the solution may be suboptimal or even infeasible as well as time consuming due to the use of an approximate and computationally expensive linearization scheme.

In order to comprehensively analyze the impact of marginal-pricing-based revenue constraints on the market clearing, B-RC is first applied to a five-bus, six-line, three-demand, four-generator system including minimum revenue conditions. The resulting schedule, dispatch, and locational marginal prices are compared to those obtained by the revenue-unconstrained auction model No-RC. This illustrative example is also useful to substantiate the superiority of the proposed approach over the suboptimal or even infeasible H-RC, which corresponds to industry practice. Subsequently, the IEEE Reliability Test System (RTS) serves as a benchmark to illustrate how the optimal revenue-constrained solution identified by B-RC overcomes the pricing inconsistency of the solution attained by R-RC when minimum declared profits are considered. For this particular example, the optimal solution to No-RC is useful to show this pricing inconsistency. Subsequently, the IEEE 118-bus system is solved under minimum revenue conditions. As done in the illustrative example, results from H-RC and No-RC are also reported. Finally, the computational behavior of the proposed approach is illustrated with several case studies including the IEEE 300-bus system. B-RC, No-RC, and H-RC have been implemented on a Dell PowerEdge R910X64 with four Intel Xeon E7520 processors at 1.866 GHz and 32 GB of RAM using CPLEX 12.5 [23] under GAMS 24.1 [24].

The remainder of this paper is organized as follows. Section II is devoted to examining in depth the illustrative example. Section III reports the results from the case based on the IEEE RTS. The analysis of a modified version of the IEEE 118-bus system is presented in Section IV. The computational performance of the proposed approach is discussed in Section V. Finally, conclusions are drawn in Section VI.

TABLE II
ILLUSTRATIVE EXAMPLE – PRICES OF DEMAND BID BLOCKS ($/MWh)

	Block		
Consumer	1	2	3
1	40	32	25
2	40	18	12
3	40	20	18

TABLE III
ILLUSTRATIVE EXAMPLE – MAXIMUM DEMAND BID LEVELS (MW)

	Hour		
Consumer	1	2	3
1	485	500	470
2	285	300	270
3	385	400	370

TABLE IV
ILLUSTRATIVE EXAMPLE – GENERATION AND COMSUMPTION LEVELS (MW)

	No-RC			B-RC			H-RC		
	Hour			Hour			Hour		
	1	2	3	1	2	3	1	2	3
p^{g}_{1t}	590.7	590.7	590.7	546.5	543.3	600.0	0.0	0.0	0.0
p^{g}_{2t}	210.0	210.0	210.0	210.0	210.0	0.0	210.0	210.0	210.0
p^{g}_{3t}	49.5	79.1	20.0	182.5	221.7	280.0	280.0	280.0	280.0
p^{g}_{4t}	88.8	95.2	82.4	0.0	0.0	0.0	200.0	200.0	200.0
p^{d}_{1t}	436.5	450.0	423.1	436.5	450.0	400.0	187.5	165.0	210.0
p^{d}_{2t}	213.8	225.0	202.5	213.8	225.0	202.5	213.8	225.0	202.5
p^{d}_{3t}	288.7	300.0	277.5	288.7	300.0	277.5	288.7	300.0	277.5

II. ILLUSTRATIVE EXAMPLE

This example considers four generating units, three consumers, and a transmission network comprising five buses and six lines, as depicted in Fig. 1, over a three-hour time span. Reactances of lines 1-2, 1-5, 2-3, 2-5, 3-4, and 4-5 are 0.0064 p.u., 0.0297 p.u., 0.0281 p.u., 0.0304 p.u., 0.0108 p.u., and 0.0297 p.u., respectively, on a 100-MVA base. Line capacities are all equal to 400 MW except for line 1-5, the flow of which is limited to 240 MW. Generation data are shown in Table I, where single-block energy offers and linear minimum revenue functions are considered, while shut-down and no-load offers are disregarded. All generating units are initially scheduled off. For the sake of simplicity, generation data and offers remain unchanged along the time span, and inter-temporal operational constraints are neglected. Consumers submit three-block demand bids priced as indicated in Table II with upper limits $\overline{P}^{d}_{1jt}$, $\overline{P}^{d}_{2jt}$, and $\overline{P}^{d}_{3jt}$ respectively equal to 75%, 15%, and 10% of the corresponding maximum demand bid level $\overline{P}^{d}_{jt}$. The values of $\overline{P}^{d}_{jt}$ are listed in Table III, whereas minimum consumption levels for all consumers are equal to zero. Similar to suppliers, consumers do not modify the prices of the demand bid blocks throughout the time span.

The computing time required to achieve the optimal solutions to No-RC and B-RC was less than 1 s. Likewise, H-RC terminated in less than 1 s. Results from all models are summarized in Tables IV–VIII. Generation and consumption

TABLE V
ILLUSTRATIVE EXAMPLE – LOCATIONAL MARGINAL PRICES ($/MWh)

	No-RC			B-RC			H-RC		
	Hour			Hour			Hour		
Bus	1	2	3	1	2	3	1	2	3
1	10.0	10.0	10.0	10.0	10.0	32.0	40.0	40.0	40.0
2	15.2	15.2	15.7	17.0	17.0	32.0	40.0	40.0	40.0
3	28.2	28.2	25.0	26.4	26.4	32.0	40.0	40.0	40.0
4	30.0	30.0	27.7	30.0	30.0	32.0	40.0	40.0	40.0
5	35.0	35.0	35.0	39.9	39.9	32.0	40.0	40.0	40.0

TABLE VI
ILLUSTRATIVE EXAMPLE – ECONOMIC RESULTS FROM NO-RC ($)

Unit	r_i	r_i^{min}	Δr_i
1	17719.6	22719.6	−5000.0
2	9687.1	10250.0	−562.9
3	4413.7	4460.4	−46.7
4	9326.6	9506.6	−180.0
Social welfare		60470.2	

TABLE VII
ILLUSTRATIVE EXAMPLE – ECONOMIC RESULTS FROM B-RC ($)

Unit	r_i	r_i^{min}	Δr_i
1	30098.2	21898.2	8200.0
2	7130.5	7100.0	30.5
3	21085.3	20525.3	560.0
4	0.0	0.0	0.0
Social welfare		57114.5	

TABLE VIII
ILLUSTRATIVE EXAMPLE – ECONOMIC RESULTS FROM H-RC ($)

Unit	r_i	r_i^{min}	Δr_i
1	0.0	0.0	0.0
2	25200.0	10250.0	14950.0
3	33600.0	25200.0	8400.0
4	24000.0	21180.0	2820.0
Social welfare		23640.0	

levels are provided in Table IV, whereas locational marginal prices are shown in Table V. Tables VI–VIII, respectively associated with No-RC, B-RC, and H-RC, list the major economic results including 1) generation revenues, r_i; 2) minimum revenues, r_i^{min}, which, in general, are computed as $r_i^{min} = M_i^f z_i + \sum_{o \in \mathcal{O}_i} \sum_{t \in \mathcal{T}} M_{oit}^v p_{oit}^g$, and which, for the single-block linear minimum revenue functions considered in this example, are computed as $r_i^{min} = M_i^f z_i + \sum_{t \in \mathcal{T}} M_{1it}^v p_{it}^g$; 3) revenue margins, Δr_i, where $\Delta r_i = r_i - r_i^{min}$; and 4) levels of declared social welfare.

Under No-RC, all units are scheduled on at all hours (Table IV). At hours 1 and 2, as a consequence of the congestion of lines 1-5 and 2-3, the cheapest generator 1 limits its production to 590.7 MW, generator 2 is dispatched at its rated capacity, and the more expensive units 3 and 4 are dispatched within their production limits. At hour 3, the dispatch is similar for all units except for unit 3 that operates at its minimum power output. Regarding consumption, the following bid blocks are accepted: 1) the first and second blocks of consumer 1 at all hours, 2) the third block of consumer 1 at hour 3, and 3) the first block of consumers 2 and 3 at all hours. Hence, in each period, all locational marginal prices are different (Table V) and the maximum attainable declared social welfare is equal to $60470.2 (Table VI). Note that this solution yields negative revenue margins for all units. Consequently, no generator would be willing to participate according to the minimum revenue function submitted as part of its generation offer. Thus, the optimal solution to No-RC is infeasible in terms of revenues.

Under B-RC, units 1–3 are scheduled on in all periods except for unit 2 at hour 3, whereas unit 4 is scheduled off along the time span (Table IV). The differences in the generation schedule with respect to No-RC are due to two reasons: 1) unit 4 imposes the most restrictive minimum revenue constraint as per the values for parameters M_i^f and M_{1it}^v (Table I), and 2) scheduling off unit 2 at hour 3 prevents the congestion of line 2-3 and yields an increase in the energy price at bus 1 that allows unit 1 to meet its minimum revenue condition. As a consequence of the different schedule and the reduced capacity of line 1-5, the generation dispatch also differs from that attained by No-RC. As an example, the total power output of unit 3 is increased by 360.4% with respect to that resulting from No-RC in order to compensate for the reduction in the production of unit 4, which is not scheduled under B-RC, and unit 2, which is shut down at hour 3. Unit 2 at hours 1 and 2 as well as units 1 and 3 at hour 3 are dispatched at their rated capacity. Moreover, the significant increase in the production of unit 3 at hour 3 leads to the compliance with the capacity of line 1-5, thereby allowing the satisfaction of minimum revenue constraints.

In terms of consumption levels, the only difference between B-RC and No-RC is found at hour 3 for consumer 1, whose consumption is decreased by 23.1 MW. Hence, at hours 1 and 2, the network is congested, thereby leading to different locational marginal prices (Table V). Note that, in both periods, the values of λ_{2t}, λ_{3t}, and λ_{5t} are increased by 11.8%, −6.4%, and 14.0% over those achieved by No-RC. Thus, no relationship between both price profiles can be inferred. At hour 3, the network is uncongested and all prices are set equal to $32/MWh, which is the price of the second block bid by consumer 1.

As can be seen in Table VII, revenue margins are all nonnegative. It should be noted that, compared to the results from No-RC, the values of r_i^{min} for units 1 and 2 respectively decrease by 3.6% and 30.7%, whereas a 360.2% increase is experienced by unit 3 due to its larger power output. Such optimal compliance with generation revenue constraints leads to a 5.5% reduction in declared social welfare over the maximum attainable level associated with the optimal albeit revenue-infeasible solution achieved by No-RC.

This illustrative example is relevant to evidence the suboptimality of H-RC. According to the iterative procedure described in [2], the first iteration of H-RC yields the same solution as No-RC. Since unit 1 incurs the largest violation of the minimum revenue condition, i.e., −$5000.0 (Table VI), this unit is no longer considered for scheduling in subsequent iterations (Table IV). As a consequence, at the second iteration, generators 2–4 are dispatched at their maximum capacity

TABLE IX
RTS-BASED CASE – RESULTS

	No-RC				B-RC				R-RC			
Unit	v_{i1}	p^g_{i1} (MW)	$\lambda_{n(i)1}$ (\$/MWh)	Ψ_i (\$)	v_{i1}	p^g_{i1} (MW)	$\lambda_{n(i)1}$ (\$/MWh)	Ψ_i (\$)	v_{i1}	p^g_{i1} (MW)	$\lambda_{n(i)1}$ (\$/MWh)	Ψ_i (\$)
1	1	16.0	11.93	13.3	1	16.0	13.00	30.5	1	16.0	12.89	28.7
2	1	16.0	12.88	28.6	1	16.0	13.00	30.5	1	16.0	13.37	36.4
3	0	0.0	11.91	0.0	0	0.0	13.00	0.0	0	0.0	12.88	0.0
4	1	25.0	12.94	8.5	1	25.0	13.00	10.0	1	27.2	13.40	14.1
5	1	68.9	9.60	−33.2	1	197.0	13.00	442.3	1	73.3	11.72	117.7
6	1	2.4	13.00	6.2	1	2.4	13.00	6.2	1	2.7	13.43	6.4
7	1	155.0	12.04	251.1	1	155.0	13.00	399.5	1	155.0	12.95	391.6
8	1	400.0	9.60	653.8	0	0.0	13.00	0.0	1	400.0	11.72	1502.6
9	1	50.0	11.93	596.3	1	50.0	13.00	650.0	1	50.0	12.89	644.5
10	0	0.0	12.04	0.0	0	0.0	13.00	0.0	0	0.0	12.95	0.0
11	1	16.0	13.00	30.5	1	16.0	13.00	30.5	1	16.0	13.43	37.5
12	0	0.0	11.78	0.0	0	0.0	13.00	0.0	0	0.0	12.82	0.0
13	1	25.0	11.91	−17.3	1	25.0	13.00	10.0	1	27.2	12.88	0.0
14	0	0.0	11.95	0.0	1	197.0	13.00	142.3	0	0.0	12.90	0.0
15	1	2.4	12.94	6.1	1	2.4	13.00	6.2	1	2.7	13.40	6.4
16	0	0.0	12.94	0.0	0	0.0	13.00	0.0	0	0.0	13.40	0.0
17	1	400.0	12.88	2967.3	1	400.0	13.00	3014.6	1	400.0	13.37	3163.1
18	1	50.0	9.60	370.9	1	50.0	13.00	550.0	1	50.0	11.72	486.0
Social welfare (\$)	8542.1				7019.1				8519.4			

in all periods and consumer 1 reduces its consumption by 57.0%, 63.3%, and 50.4% at hours 1–3, respectively, over the consumption levels provided by No-RC. As a result, the network is uncongested and all prices are set to \$40/MWh in all periods (Table V), corresponding to the price of the first block bid by consumer 1. Table VIII shows that all revenue margins are nonnegative, i.e., a feasible solution is attained and the iterative process terminates. However, the declared social welfare drops by 58.6% over the optimal revenue-constrained level achieved by B-RC. This result reveals that B-RC clearly outperforms H-RC.

This test system is also useful to illustrate that H-RC may even fail to find a feasible solution. This major shortcoming arises if the minimum consumption levels were set equal to the corresponding power bid in the first block by consumers, i.e., $\underline{P}^d_{jt} = \overline{P}^d_{1jt} = 0.75\overline{P}^d_{jt}$. This change in the input data does not affect B-RC, and hence the optimal revenue-constrained solution is that reported in Tables IV, V, and VII. In contrast, the new values of $\underline{P}^d_{jt}$ significantly impact on H-RC. At the second iteration of H-RC, the total available generation capacity $\left(\sum_{i\in\mathcal{I}} \overline{P}^g_{it}\right)$ would be equal to 690 MW for all periods once unit 1 had been withdrawn at the first iteration. This value would be less than the total minimum consumption levels $\left(\sum_{j\in\mathcal{J}} \underline{P}^d_{jt}\right)$ for periods 1–3, which amount to 866.3 MW, 900.0 MW, and 832.5 MW, respectively. Therefore, H-RC would be unable to solve the problem, thereby corroborating the superiority of B-RC.

III. RTS-BASED CASE

The second case study is based on the modified version of the IEEE RTS analyzed in [12]. This test system comprises 24 buses, 38 transmission lines, 18 generating units, and 17 consumers in a single-period setting. Generation and consumption data are provided in [12]. Demand bid blocks and their respective prices correspond to the peak-demand case described in [12]. For this case study, revenue constraints model nonnegative generation declared profits, i.e., $\underline{\Psi}_i = 0$, $\forall i \in \mathcal{I}$.

No-RC and B-RC required less than 1 s to attain the optimal solutions summarized in Table IX. This table also lists the optimal results reported for R-RC in [12]. Table IX includes 1) generation schedules, v_{i1}; 2) generation levels, p^g_{i1}; 3) market-clearing prices at the buses where generating units are located, $\lambda_{n(i)1}$; 4) generation declared profits, Ψ_i; and 5) the levels of declared social welfare.

Under No-RC, generating units 3, 10, 12, 14, and 16 are scheduled off. This solution causes congestion in the network and hence all locational marginal prices are different. As a result, the maximum attainable declared social welfare is equal to \$8542.1 and units 5 and 13 incur negative declared profits. Therefore, the optimal solution attained by No-RC is infeasible in terms of generation declared profits.

In contrast, B-RC achieved the optimal solution complying with profit bounds. As can be observed in Table IX, the generation schedule differs from that obtained by No-RC for units 8 and 14, which are located at buses 23 and 21, respectively. Unit 8 is scheduled off because, otherwise, its large start-up offer price would preclude the attainment of an economical profit-feasible solution. On the other hand, unit 14 is scheduled on to meet the capacity of line 12-23. As compared to the generation dispatch determined by No-RC, the most significant difference is the increased production of unit 5. This increased generation level partially compensates for the lack of production from unit 8, which is located at the same bus but is scheduled off under B-RC. As a consequence, the network is uncongested and locational marginal prices are all equal to the price of the demand block bid by consumer 7, i.e., \$13/MWh. For this case study, the value of this bid price is greater than or equal to the locational marginal prices provided by No-RC. As can be seen, the optimal profit-constrained solution achieved

TABLE X
118-BUS SYSTEM – VALUES OF M_i^f DIFFERENT FROM 0 ($)

Unit	M_i^f
1	1583.5
2	1583.5
3	1583.5
4	13560.0
5	13560.0
6	1583.5
7	101.5
8	1583.5
9	1583.5
16	20300.0
19	507.5

TABLE XI
118-BUS SYSTEM – HOURLY SCALING FACTORS FOR DEMAND

Hour	Factor	Hour	Factor
1	0.63	13	0.93
2	0.62	14	0.70
3	0.60	15	0.75
4	0.58	16	0.80
5	0.59	17	0.85
6	0.65	18	0.92
7	0.72	19	0.96
8	0.85	20	0.98
9	0.95	21	0.96
10	0.99	22	0.90
11	1.00	23	0.80
12	0.99	24	0.70

by B-RC yields a 17.1% reduction in declared social welfare with respect to the maximum attainable level associated with the optimal albeit profit-infeasible solution resulting from No-RC.

Finally, the optimal solution attained by R-RC is profit feasible at the expense of slightly decreasing the declared social welfare by 0.3% over the maximum attainable level determined by No-RC. Hence, the level of declared social welfare attained by R-RC is considerably larger than that provided by B-RC. However, this result is associated with the inconsistency of R-RC with the standard economic-dispatch-based marginal pricing, which is crucial for practical implementation purposes. As can be observed in Table IX, the optimal generation schedules identified by both R-RC and No-RC are identical. Therefore, the optimal economic-dispatch solution and the locational marginal prices associated with the generation schedule achieved by R-RC are equal to the production levels and prices yielded by No-RC, which are listed in columns 3 and 4 of Table IX, respectively. The comparison of those results with the generation dispatch and prices provided by R-RC, which are reported in columns 11 and 12 of Table IX, reveal that profit feasibility is achieved by R-RC by slightly albeit artificially increasing the generation levels awarded to units 4–6, 13, and 15, and by substantially rising locational market-clearing prices, which no longer constitute marginal prices. Note that such increases in nodal prices range between 3.3% for bus 7, where units 6 and 11 are located, and 22.1% for bus 23, which is the location for units 5, 8, and 18.

Thus, although profit constraints are met, the optimal solution to R-RC does not constitute an equilibrium solution and

TABLE XII
118-BUS SYSTEM – PRICES OF DEMAND BID BLOCKS ($/MWh)

	Block		
Consumer	1	2	3
1, 11–20	50	20	18
2, 21–30	50	16	14
3, 31–40	50	12	10
4, 41–50	50	8	6
5, 51–60	50	7	5
6, 61–70	50	28	26
7, 71–80	50	25	20
8, 81–86	50	23	13
9, 87–91	50	22	15
10	50	13	8

market-clearing prices do not correspond to shadow prices, i.e., marginal levels of declared social welfare. Given the widespread use of both economic dispatch and marginal pricing in currently implemented electricity markets, modifying not only the scheduling model but also the pricing model, as required by R-RC, may be considered as a deterrent factor for the practical adoption by market agents of the methodology described in [12] to deal with revenue constraints in an auction design. In contrast, the alternative scheduling solution attained by B-RC is profit feasible, it results from an economic dispatch, and it complies with marginal pricing, thereby being suitable for current pool-based electricity markets.

IV. 118-BUS SYSTEM

The third case study is based on the IEEE 118-bus system [25], [26] and comprises 54 generating units, 91 consumers, and 186 transmission lines over a 24-hour time span. Generation data can be found in [26]. Three-block energy offers are obtained from the linearization of the quadratic production costs. Start-up offer prices are set equal to the start-up cost coefficients, shut-down offers are neglected, and no-load offers are set equal to the constant terms of the quadratic production costs. Producers include in their offers minimum revenue functions characterized by $M_{oit}^v = C_{oit}^g$ and M_i^f all equal to 0 except for those listed in Table X. It should be noted that generation data and offers remain unchanged over the scheduling horizon. Prior to the first period of the time span, units 1–9 and 31–54 have been scheduled off for a number of hours equal to their corresponding minimum down times. In contrast, units 10–30 are initially scheduled on with $P_{i0}^g = \overline{P}_{it}^g/2$ and with no limitation regarding their minimum up times. For illustration purposes, maximum demand bids are based on 1) the nodal peak demands available in [25], which have been increased by 35%; and 2) the hourly scaling factors given in Table XI. Moreover, it is assumed that consumers submit three-block demand bids priced as listed in Table XII, with $\overline{P}_{1jt}^d$, $\overline{P}_{2jt}^d$, and $\overline{P}_{3jt}^d$ respectively equal to 90%, 5%, and 5% of the corresponding maximum demand bid level. For the sake of simplicity, minimum demand bid levels for all consumers are set equal to 0. Similar to producers, consumers do not modify the prices of their respective demand bid blocks over the time span.

For this case study, the stopping criterion for B-RC is based on a 1% optimality gap. Results from No-RC, B-RC, and

TABLE XIII
118-BUS SYSTEM – REVENUE MARGINS FOR SCHEDULED UNITS ($)

Unit	No-RC	B-RC	H-RC
1	0.0	0.0	5774.0
2	0.0	0.0	7599.0
3	0.0	0.0	6301.3
4	−5374.6	3112.6	0.0
5	−4872.1	3510.2	0.0
6	0.0	0.0	4740.9
7	0.0	0.0	7200.1
10	7517.8	8924.4	9407.1
11	40015.9	47566.7	40671.1
14	13.5	13.5	1442.7
20	22020.7	28022.2	22839.1
21	22013.7	27988.0	22585.8
24	11732.4	16495.6	11943.2
25	11729.0	16478.9	11774.2
27	39266.7	49093.6	39077.1
28	39259.3	49057.6	38725.4
29	8348.3	6422.0	8292.1
36	7758.5	0.0	7872.6
39	34870.9	41518.4	35139.5
40	8426.6	12674.6	8388.7
43	8255.9	14309.9	8234.3
44	8258.2	14326.7	8229.0
45	8257.2	14319.5	8231.2
53	0.0	0.0	2568.8

TABLE XIV
118-BUS SYSTEM – LEVELS OF DECLARED SOCIAL WELFARE AND COMPUTING TIMES

	No-RC	B-RC	H-RC
Social welfare ($)	3175596.1	3163283.9	3110302.4
Computing time (s)	9.1	792.6	489.5

H-RC are summarized in Tables XIII and XIV, and Figs. 2 and 3. In Table XIII, generation revenue margins are listed for all units except for those that are scheduled off for all models, with revenue margins all equal to $0. Table XIV provides the levels of declared social welfare as well as the computing times. Fig. 2 depicts the consumption profiles and the load-weighted average locational marginal prices for energy. The load-weighted average locational marginal price for time period t, denoted by λ_t^{av}, is computed as the sum over all buses of the products of the corresponding locational marginal price (LMP) and nodal consumption level divided by the total awarded consumption level in that period, i.e., $\lambda_t^{av} = \frac{\sum_{n\in\mathcal{N}} \lambda_{nt}(\sum_{j\in\mathcal{J}_n} p_{jt}^d)}{\sum_{j\in\mathcal{J}} p_{jt}^d}$. Finally, in order to examine the impact of transmission congestion, Fig. 3 shows the locational marginal prices for all generating units in two representative periods, namely off-peak hour 4 and peak hour 20.

No-RC required 9.1 s to attain the optimal solution (Table XIV), which is infeasible in terms of revenues since generating units 4 and 5 feature negative revenue margins (Table XIII). The maximum attainable level of declared social welfare is equal to $3175596.1 (Table XIV) and the load-weighted average LMPs follow a similar pattern as the awarded consumption level curve (Fig. 2).

Under the aforementioned stopping criterion, the proposed B-RC provided a high-quality near-optimal revenue-constrained solution (Table XIII) in 792.6 s (Table XIV). According to current industry practice [27]–[29], achieving a

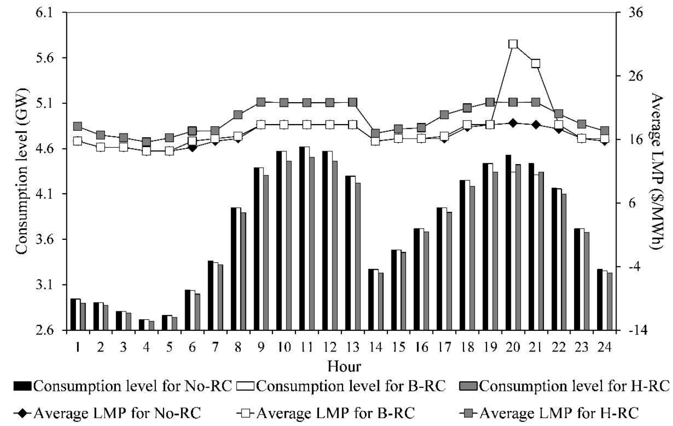

Fig. 2. Load-weighted average locational marginal prices and awarded consumption levels for the IEEE 118-bus system.

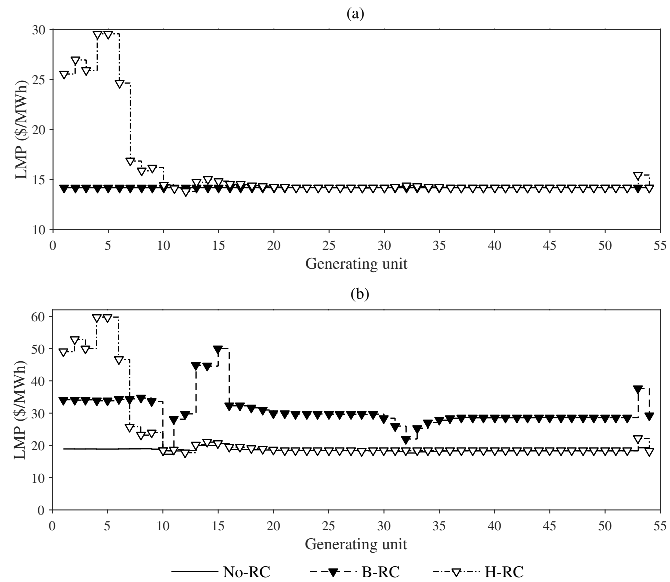

Fig. 3. Locational marginal prices for the generating units of the IEEE 118-bus system: (a) off-peak hour 4, (b) peak hour 20.

solution within a 1% optimality gap in less than 15 minutes reveals an acceptable computational performance since the proposed algorithm is suitable to provide a solution that is very close to the optimum within a practical amount of time for a reasonably-sized problem. Note that B-RC reduces the total declared social welfare by 0.4% over the maximum attainable level associated with the revenue-infeasible solution achieved by No-RC (Table XIV).

The different generation schedule and dispatch identified by B-RC as compared to those provided by No-RC are the primary responsible factors for the compliance with revenue constraints. As can be seen in Table XIII, the main difference with respect to the solution resulting from No-RC is the status of unit 36, which is scheduled off. The schedules of units 4 and 29 also experience significant changes. As a consequence, the cheaper units 27 and 28 are dispatched at substantially larger generation levels, whereas the awarded consumption levels are almost identical in most periods (Fig. 2). As a result, load-weighted average LMPs differ from those obtained by the revenue-unconstrained solution at hours 6–8, 17, 18, 20–22, and 24, for which prices increase. Moreover, the average LMP profile for B-RC follows its associated awarded consumption curve, as is expected under marginal pricing. As can be observed in Fig. 3, at off-peak hour 4, the network is uncongested and marginal prices resulting from B-RC are identical for all generating units. Furthermore, such prices are also equal to those provided by No-RC. In contrast, at peak hour 20, the congestion of the network yields different locational marginal prices for B-RC, which, for most generating units, are greater than those identified by No-RC.

Finally, a suboptimal revenue-constrained solution was attained by H-RC after 4 iterations and 489.5 s (Table XIV). It can be observed that, unlike in the solutions provided by No-RC and B-RC, generating units 1–3, 6, 7, and 53 are scheduled on, whereas generating units 4 and 5 are scheduled off (Table XIII). The economically inefficient generation schedule identified by H-RC leads to average LMPs that, in most periods, are greater than those provided by B-RC, as can be seen in Fig. 2. For the off-peak and peak periods shown in Fig. 3, this price increase particularly affects generating units 1–6, which are located at buses 1, 4, 6, 8, 10, and 12, respectively. Moreover, it is worth noting that the associated declared social welfare, which is equal to \$3110302.4, is a 1.7% lower than that provided by B-RC (Table XIV). Therefore, B-RC clearly outperforms H-RC in terms of solution quality while featuring an acceptable computational behavior.

V. Computational Issues

In current industry practice [27], system operators must provide day-ahead market-clearing solutions within a few hours upon receiving the data from market agents. As an example, the maximum allotted time for the operator of the Greek system is two hours [29]. As described in the companion paper [20], the inherent nonconvexity of revenue-constrained market clearing results in more complex optimization problems as compared to revenue-unconstrained models. Note that locational marginal prices for energy are not present in the mathematical formulation of such revenue-unconstrained auctions. The characterization of such marginal prices requires the use of dual variables and dual constraints, thereby increasing the size of the resulting problem. Thus, the computational performance is a critical issue for the purposes of practical implementation of revenue-constrained auction designs.

The computational behavior of the proposed B-RC is summarized in Table XV, which provides the computing times required to solve several case studies of increasing size under the aforementioned stopping criterion relying on a 1% optimality gap. In all case studies, minimum revenue constraints have been considered. The suite of benchmarks comprises the three test systems presented in Sections II–IV, respectively, which are analyzed over a 24-hour time span. Using the same day-ahead framework, a larger test system is also examined, namely the IEEE 300-bus test system [25], which is representative of a practical system bearing in mind that it is larger than most national power systems in the European Union. For the sake of reproducibility, data for all case studies can be downloaded from [30]. As can be observed in Table XV, the computational effort grows with the dimension of the test system, albeit being acceptable for a practical operating setting.

TABLE XV
Computing Times Required by B-RC (s)

5-bus system	24-bus system	118-bus system	300-bus system
9.0	45.9	792.6	3459.5

Admittedly, the dimensionality of the proposed B-RC can be significant for larger test systems. As a consequence, the state-of-the-art branch-and-cut algorithm for mixed-integer linear programming may find difficulties in proving optimality within a practical time frame. Decomposition techniques such as Benders decomposition [31] may be promising strategies to deal with larger systems by keeping tractability of the constraint and variable sets, and by taking advantage of their inherent parallelism. Moreover, a tighter representation of the minimal convex set of the mixed-integer linear program may lead to improved computational performance. These aspects are currently under investigation.

VI. Conclusion

This paper has presented numerical experience for the revenue- and network-constrained market-clearing procedure proposed in the companion paper within the context of pool-based day-ahead electricity markets under marginal pricing. Both minimum revenue conditions and minimum profit constraints have been examined.

An illustrative example is first considered to comprehensively analyze the performance of the proposed approach when minimum revenue conditions are imposed. The major findings revealed by this case study are: 1) nonnegative revenue margins are achieved for all scheduled units at the expense of slightly decreasing the declared social welfare with respect to the maximum attainable level associated with the optimal revenue-unconstrained solution, 2) different dispatch and scheduling decisions are attained with respect to those resulting from a revenue-unconstrained market-clearing procedure, and 3) the proposed approach outperforms the heuristic currently used in industry practice.

The second case study, which is based on the IEEE Reliability Test System, shows the impact of nonnegative generation declared profits on a single-period market-clearing procedure. Results from this case study highlight the capability of the proposed approach to optimally clear the profit-constrained market while complying with the standard economic-dispatch-based marginal pricing scheme, unlike a recently reported relaxed technique.

The proposed bilevel-programming-based method has also been tested on the IEEE 118-bus system with minimum revenue conditions. Numerical results from this case study lead to the following remarks: 1) a high-quality near-optimal revenue-constrained solution is attained with moderate computational effort, and 2) in terms of declared social welfare, a significant improvement is achieved upon the solution provided by the practical heuristic used for assessment purposes.

Furthermore, the computational performance of the proposed approach has been discussed based on the results from several case studies including the realistically-sized IEEE 300-bus system. It is worth noting that 1) as expected, computing times increase as the size of the test system grows, and 2) the computational effort is acceptable for current industry practice.

Ongoing research is focused on the incorporation of uncertainty through scenario-based stochastic programming, robust optimization, and interval optimization. Further research will be devoted to analyzing the strategic behavior of market agents under revenue-constrained market clearing and to examining the long-term effects of this auction design. Two potential extensions of this research are the examination of electricity auctions jointly clearing energy and reserves, and the incorporation of revenue constraints in a consumer payment minimization framework. Another interesting avenue of research is the consideration of practical modeling aspects such as line switching and nonconvex generation offers. Finally, research will also be conducted to explore alternative solution approaches and to cope with the existence of solution multiplicity.

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Ricardo Fernández-Blanco (S'10–M'15) received the Ingeniero Industrial degree and the Ph.D. degree in electrical engineering from the Universidad de Castilla-La Mancha, Ciudad Real, Spain, in 2009 and 2014, respectively.

He is currently a postdoctoral researcher at University of Washington, Seattle, WA, USA. His research interests are in the fields of energy and power systems, bilevel programming, and electricity markets.

José M. Arroyo (S'96–M'01–SM'06) received the Ingeniero Industrial degree from the Universidad de Málaga, Málaga, Spain, in 1995, and the Ph.D. degree in power systems operations planning from the Universidad de Castilla-La Mancha, Ciudad Real, Spain, in 2000.

From June 2003 through July 2004 he held a Richard H. Tomlinson Postdoctoral Fellowship at the Department of Electrical and Computer Engineering of McGill University, Montreal, QC, Canada. He is currently a Full Professor of electrical engineering at the Universidad de Castilla-La Mancha. His research interests include operations, planning, and economics of power systems, as well as optimization.

Natalia Alguacil (S'97–M'01–SM'07) received the Ingeniero en Informática degree from the Universidad de Málaga, Málaga, Spain, in 1995, and the Ph.D. degree in power systems operations and planning from the Universidad de Castilla-La Mancha, Ciudad Real, Spain, in 2001.

She is currently an Associate Professor of electrical engineering at the Universidad de Castilla-La Mancha. Her research interests include operations, planning, and economics of power systems.