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Authors: Chicharro J.M., Salazar F. y Bayón A.

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MEASUREMENT OF FIELD-DEPENDENCE ELASTIC MODULUS AND MAGNETOMECHANICAL COUPLING FACTOR BY OPTICAL HETERODYNE INTERFEROMETRY

J.M.Chicharro, A.Bayón, and F.Salazar
Tel. 34-1-3367970 Fax. 34-1-3366952 E-Mail: josema@dfarn.upm.es
Departamento de Física Aplicada a los Recursos Naturales
E.T.S.I.Minas (Universidad Politécnica de Madrid)
c/Ríos Rosas, 21,28003-Madrid(Spain)

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Abstract

Optical heterodyne interferometry is applied to determine the change in elastic modulus (ΔE) with the applied magnetic field and the magnetoelastic coupling coefficient k. The samples used are thin nickel rods and wires up to 1 mm in diameter positioned within the magnetic field of a solenoid. The Young's modulus is determined from the first natural vibration frequency. A dual methodology is evaluated, respectively involving free and forced sample vibrations. Estimations are also made of eddy current-induced magnetic losses of the material. This technique permits point-like detection, involving an illuminated zone approximately 20 μm in diameter, without contact with the sample.

Keywords: magnetostriction, elastic constants, resonance, interferometry, speckle
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1 Introduction

In recent years, considerable importance has been given to the measurement of magnetomechanical properties for posterior application in different fields of applied science. Such applications range from the designing of magnetic circuits to the construction of tension sensors with metallic glass. The characterization of magnetic materials requires measurement of their hysteresis curves (B versus H), magnetostrictive curves (λ versus H), and curves reflecting the change in elastic modulus with magnetization (E versus H).

If tension is applied to a sample, two types of relative deformations result: elastic (ε_{ll}) and magnetoelastic (ε_{ml}), where the relative variation of E equals:

$$\frac{\Delta E}{E} = \frac{E_S - E_D}{E_D} = \frac{\varepsilon_{ml}}{\varepsilon_{ll}}\,. \tag{1}$$

The phenomenon by which the elastic modulus (E) changes with the applied magnetic field H (ΔE) is a special case of the more general "modulus defect". When some mechanism induces extra deformation either above or below the elastic deformation, the resulting Young´s modulus is different to normal. Magnetoelastic deformation produces an effect of this kind. Consequences of magnetostriction in ferromagnetic materials are the magnetic field-dependence of the elastic modulus and coupling of the magnetic and elastic energies. A fundamental relation between these two effects in application to linear deformations is represented by the magnetoelastic coupling coefficient k, which represents the fraction of magnetic energy that may be converted to elastic energy at low frequencies [1]:

$$k^2 = 1 - \frac{E_H}{E_S}\,, \tag{2}$$

where E_S is the Young´s modulus under conditions of magnetic saturation, and E_H is the

Young´s modulus without force application in the presence of a magnetic field H. In this sense, k values close to 1 are desirable for improved functioning of magnetoelastic devices.

The methods most commonly used to measure the dependency of the elastic modulus upon the magnetic field applied (the so-called ΔE effect) and the corresponding magnetomechanical coupling factor k, involve resonance-antiresonance techniques [2], the measurement of ultrasound velocity [3], and optical methods [4]. A recent review of the different magnetomechanical measurement techniques is provided by Squire [5].

The present study describes a new approach for the measurement of ΔE, based on heterodyne speckle interferometry. This technique has already been successfully used to measure elastic constants of different materials [6, 7] and can be applied to characterize rods and wires. The method permits non-contacting measurement of very small displacements. In conventional methods, Young's modulus is usually determined from the measurement of indirect magnetic properties. However, the proposed method enables to determine E from a direct measurement of the displacements, without electromagnetic excitation. In addition, the measurements can be made over the whole range of the applied field. The technique is remote, only requires optical contact with the sample, and so could be used over a wide range of temperatures.

Since there is no interaction with the sample its vibration is free. The natural frequencies are therefore accurately determined from the displacements. The detection system has a broad frequency response and is immune to environmental vibrations. When high-frequency transducers are used to detect the ultrasonic wave velocity, the signals are frequently weak due to attenuation, which could make difficult to understand the nature of the propagating waves.

ΔE effect has been also measured by means of optical methods that usually need reflecting surfaces. Additional components as mirrors have to be placed on the sample, which could distort

the measurement. The method used here is based on speckle phenomenon, which simply requires a scattering surface, and the vibration can be directly detected on the sample. On the other hand, the method has been presently used only for characterizing slender rods and wires, whose diameters are limited by the spot of the laser beam of the order of 20 μm.

2 Experimental procedure

The experimental arrangement is shown in Figure 1. The sample used is either a thin cylindrical rod or a wire composed of the ferromagnetic material under study; the sample is positioned horizontally on a central thin rubber support. The sample is the "nucleus" of a straight solenoid powered by a direct current source that regulates and stabilizes the current intensity within the circuit. This system makes it possible to generate magnetic fields within the solenoid of up to 320 oersted - sufficient to saturate all the samples used in our experiments. The magnetic field is regulated by the current intensity in the solenoid, and is kept constant in each experimental stage where vibration frequency measurements are made. In addition, the dimensions of the sample and the experimental arrangement used cause the field to remain approximately homogeneous in the region where the sample is positioned.

Sample longitudinal vibration is induced by means of a brief impact perpendicular to the base of the cylinder and at its center. To this effect we use a pendulum consisting of a thread and quartz sphere with a diameter ranging from 1.80 to 4.40 mm, depending on the frequencies to be excited. The pendulum causes the rod or wire to vibrate freely at its natural frequencies, for once the impact has terminated vibration is not due to the continuous action of external forces. Moreover, this type of excitation exhibits a broad-band spectrum, with the simultaneous excitation of several vibration modes. The type of support used does not restrict sample

movement in any way. In addition, the center of the sample constitutes a node for the first longitudinal vibration mode. The longitudinal oscillations generated by impact delivery at the center of the base are detected at the center of the opposite base, where the normal components of the displacements are registered. Posteriorly, the displacements detected are subjected to spectral analysis to establish the natural vibration frequencies.

In order to detect sample vibration at a point we use a heterodyne optic interferometer (Ultra-Optec Inc. OP-35-I/O). The working principle is based on the speckle phenomenon [8, 9]. This effect consists of an intensity distribution formed by bright and dark speckles observed when coherent light is scattered by a rough surface at the laser wavelength scale. A light phase shift occurs due to the optical trajectory variations induced by vibration at a point of the sample. The interferometer is able to sequentially measure both displacement components at the same detection point, i.e., that contained in the plane of the sample (the in-plane component), and the perpendicular (or out-of-plane) component. The bandwidth is 1 kHz to 35 MHz, and the smallest detectable displacement is about 1 nm. Due to the working principle of the interferometer, the sample must possess a scattering surface.

The optical unit is equipped with a laser source (He-Ne, 10 mW). The coherent beam is split in two by a Bragg acousto-optic cell - one component with a frequency equal to that of the original beam, and the other with a frequency increased by 40 MHz. In the out-of-plane configuration used in our experiments, the unaltered frequency beam is focused onto the surface of the sample, the illuminated zone measuring approximately 20 μm in diameter. The dispersed light is in turn collected in the direction symmetrical to the incident beam with respect to the normal to the sample surface, and is directed towards the beam mixer where it interferes with the changed-frequency reference beam. Lastly, the resulting interference is directed towards the detector. This configuration is sensitive to displacements perpendicular to the sample plane, so

that an out-of-plane displacement of the observation point δ_0, originates a change in the optical trajectory, resulting in a phase shift of $4\pi \cos\theta\delta_0/\lambda$, where λ is the wavelength and θ the angle between the normal to the surface and the direction of the incident or observation beam.

The signal associated to the interference obtained at the output of the photodiode is a 40 MHz carrier frequency phase modulated by the displacement of the point on the surface of the study sample. The signal is posteriorly demodulated by the electronic unit, where a signal proportional to the displacement is obtained. This heterodyne system is not sensitive to low frequency vibrations. In addition, it detects vibrations without requiring contact with the sample, thereby avoiding the introduction of correcting factors.

Lastly, the displacement associated signal is digitized by an HP-54504A oscilloscope. A sampling frequency more than double the expected maximum frequency must be selected. When determining the natural vibration frequencies of the sample, the signal is transferred to a computer to determine its frequency spectrum, following calculation of the fast Fourier transform (FFT) of the signal. The natural frequencies correspond to the maximum amplitudes of the spectrum. For the wire a Tektronic TDS-430A oscilloscope with the FFT option was used.

For ideal samples we can apply the simplified theory of longitudinal wave propagation in thin rods. This provides the following equation [10], which allows us to express the natural vibration frequency f as a function of the Young´s modulus E, the length of the rod l, the sample density ρ, and the integer n associated to each vibration mode:

$$f = \frac{n}{2l}\left(\frac{E}{\rho}\right)^{1/2}. \tag{3}$$

This expression allows us to determine the Young´s modulus by determining the frequency

f, which moreover depends on the magnetization of the sample. In the concrete case of the first vibration mode, $E = 4\rho l^2 f^2$.

3 Experimental results

The samples studied were thin nickel rods and a wire. The rods are 8 and 2 mm in diameter and 10 and 5 cm in length, respectively. The metallographic analysis revealed a nickel purity of 99.25% and 99.95% for the longer and shorter rods, respectively. Both rods were hot formed and soft annealed. The wire is 1 mm in diameter and 10 cm in length with a percentage of nickel of 99.99% and a hard drawn treatment. The samples were demagnetized exceeding the Curie point by heating in an electric oven. The results obtained using the method described above, involving impact delivery with a quartz pendulum, are summarized in Table 1. The experimental results shown are for samples in demagnetized and technical saturation states. In both states, the demagnetizing effect has hardly any influence on the results.

In turn, Figure 2 shows the spectrum of a signal recorded with a demagnetized rod 8 mm in diameter subjected to impact with a quartz pendulum 2 mm in diameter. The spectrum shows the first two vibration modes; only the first mode is used to calculate E.

The Table shows the frequencies f, measured with the free excitation method under both demagnetized and saturated conditions, with the corresponding uncertainty values $u(f)$. In order to calculate these uncertainty values we employed the recommendations of the ISO guide for expressing measurement uncertainty [11]. Accordingly, the greatest uncertainty in the frequency is introduced by the digital oscilloscope used to record the signal. The frequencies resolution obtained with the Fourier analysis was f_s/N, where f_s is the sampling frequency and N the

number of points; f_s must be at least double the maximum frequency analyzed, and N is limited by the recording capacity of the oscilloscope. In our case for the rods we used a f_s of 100 ksamples/s and 2048 points - the resulting uncertainty being 25 Hz. This is the minimum uncertainty that can be achieved with the digital oscilloscope HP-54504A employed in our experiments with rods, though the value could be further reduced by using a digital oscilloscope with a larger N (yielding less error on performing the Fourier transform). With the wire 1 mm in diameter, we employed the digital oscilloscope TDS-430A to improve the uncertainty of the method. In this case, with f_s =100 ksamples/s and 10000 points, the resulting uncertainty is 10 Hz, which is 2.5 times smaller.

The values of the elasticity modulus E were calculated from equation 3. The results obtained show a difference in the uncertainty of E between the 8 mm and 2 mm rods attributable to the different sample density precision involved - the uncertainty of the frequency measurements being the same in both cases. In application to the longer nickel rod, we used the method described by Pratten [12], to secure minimum measurement uncertainty. In the case of the 2 mm rod, we used the data provided by the sample manufacturer. The uncertainty $u(E)$ for the 1 mm rod is 2.2 times smaller than the uncertainty for the 2 mm rod, which is an improvement due to the accuracy of the oscilloscope -the uncertainty of the density being the same in both cases. Assuming that the uncertainty of $u(E)$ comes from $u(f)$, the only one originating from our method, we would have for the wire a $u(E) = 0.17$ GPa and $u(E)/E = 0.08\%$. In this case, the uncertainty of the density is about 20% of the actual value of $u(E)$. Table 1 also shows the variation in elastic modulus (ΔE) that is expressed in absolute terms and as a percentage with respect to the demagnetized state. Finally, we calculated the magnetoelastic coupling coefficient k, for a field $H = 0$, according to equation 2.

Figure 3 shows an experiment (E versus H) involving a nickel rod 2 mm in diameter in

transition from a demagnetized state to saturation with the free excitation method. A quartz pendulum 1.8 mm in diameter was used to this effect. The demagnetizing factor has been considered [13]. The effective field H inside the sample has been calculated by subtracting the demagnetizing field from the external field. The value of the demagnetizing factor for this rod, with a dimensional ratio of 25, is 4.67×10^{-3}. A demagnetizing field of 28.77 Oe is obtained in saturation (being the intensity of magnetization in saturation 0.619 T). This effect can be neglected in the case of samples whose ratio length-to-diameter was greater than 100, i.e., for the wire of 1 mm. The test shows how E increases with H from the demagnetized state to saturation of the sample; the resulting ΔE was 3.4 GPa.

4 Other experiments

An alternative method proposed for determining ΔE is based on the measurement of the resonance frequency in the case of the forced vibration of the sample. The purpose of this study is to compare the results obtained with both methods, and to contribute to estimate losses attributable to the generation of eddy currents.

The experimental arrangement is similar to that shown in Figure 1. The sample excitation is achieved by means of an electromagnetic vibrator. The vibrator is positioned parallel to one of the rod bases, at a distance of approximately 1 mm - its revolution axis coinciding with that of the rod. When a harmonic potential difference is applied to the vibrator, the associated magnetic field is of the type $B_1 + B_0 \sin(2\pi\nu)$, where B_0 is less than B_1, so that the resulting magnetic field exhibits the same sense. Thus, the base of the rod is fundamentally subjected to a constant longitudinal force plus a same frequency harmonic causing forced vibration of the sample. The resulting vibration is detected at the opposite base of the rod by means of the above

mentioned interferometer. Neither excitation nor detection involve contact with the sample.

Following the previously described methodology, we determined the vibration amplitude and phase according to the excitation frequency. The functions generator used to excite the vibrator is an HP-3324H with a peak-to-peak maximum potential difference of 10V. The normal component of the displacements is recorded at the center of the rod base by means of the interferometer, and the resulting harmonic signal is visualized with an oscilloscope synchronized to the generator.

Figure 4 shows the vibration amplitude and phase at the detection point corresponding to different frequency values close to resonance, for the thin nickel rod 8 mm in diameter. In this case the field H applied to the sample is zero, i.e., the demagnetized rod is evaluated. Maximum amplitude is recorded for a frequency of 24829 Hz, with a phase change of π for a frequency of 24830 Hz. The variations in amplitude and phase close to resonance are very violent, though the functions are not discontinuous as would be expected in an ideal situation without losses; this demonstrates the existence of damping. Nevertheless, the resonance peak is seen to be narrow, with a rapid phase change - i.e., damping is weak. The apparent width and phase change are amplified by the scale employed.

The previously mentioned method was used to determine the ΔE of the nickel rods referred to above. In each experimental stage the field H in which the sample was introduced was kept constant, and the vibrator excitation frequency was varied to study the response. For each field H we determined amplitude and phase as a function of frequency to establish the corresponding resonance frequency. The experimental results obtained for the demagnetized and saturated samples are reflected in Table 2. The ΔE variations were 1.5% and 1.2%, with relative E uncertainties of 0.46% and 0.05% corresponding to the 99.25% and 99.95% nickel purity rods,

respectively. The improvement in the uncertainty is afforded by the possibility of conducting a practically continuous study of system response for each frequency.

On comparing these results with those obtained in the free vibration study, the difference in values observed can be attributed to the different frequencies taken in calculating E - as warranted by the associated uncertainty values. The final frequency uncertainty with the forced excitation method is less than in the case of the free vibration study, since the HP-3324H signals generator allows individual Hz-by-Hz selection of the desired excitation frequency - in contrast to the free excitation method, where variations in frequency in the signal spectrum are performed on a stepwise basis involving 25 or 10 Hz intervals. With this method the final frequency uncertainty $u(f)$, predominates the variations observed between different test. This uncertainty is difficult to reduce, since the functions generator introduces only minor uncertainty values (100 mHz).

One of the characteristics of the method described is the dual recording of amplitude and phase, which allows us to estimate possible losses in the material caused by eddy currents. In the case studied, and for a constant magnetic field, the equation of motion for the vibrating medium includes (for the simple model analyzed) an additional body force related to the applied magnetic field [14], and which we suppose to be proportional to the velocity of the portion of the rod studied. These losses per unit volume are due to eddy currents generated at the frequencies used [10]. The differential equation regulating vibration of a portion of the rod is expressed as follows:

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{\frac{E}{\rho}} \frac{\partial^2 u}{\partial t^2} + \gamma \frac{\partial u}{\partial t}\,, \tag{4}$$

where u is the longitudinal displacement associated to the vibration propagating in the direction of the rod axis, γ is the proportionality coefficient relating volume force and velocity, $\sqrt{\frac{E}{\rho}} = v$, represents the phase velocity in the case that no additional term exists associated to loss. If

we assume displacement of the type $u = U_0 \exp j(\omega t - kx)$, the following equation is obtained, applicable to each section of the rod and defining its behavior over time:

$$\frac{1}{v^2}\frac{\partial^2 u}{\partial t^2} + \gamma \frac{\partial u}{\partial t} + k^2 u = 0\,. \tag{5}$$

This equation predicts that the vibration of each section is determined by the product of a decreasing exponential function, with a damping coefficient associated to the resistive term, and a harmonic function whose vibration frequency is less than that obtained in an ideal setting similar to that analyzed, but without losses.

It is to be expected that when the rod is subjected to forced harmonic vibration at one end, the maximum vibration frequency corresponds to a frequency slightly lower than that observed in the ideal setting without losses - though the phase shift would be detected for the resonance frequency f_0, in the ideal situation without resistive term.

On applying the above considerations to our model, and considering that the method applied provides a dual recording of amplitude and phase shift versus frequency, an analysis of the results based on the frequency differences obtained corresponding to maximum amplitude and phase would allow us to estimate the possible existence of losses caused by the generation of eddy currents. The logarithmic decrement δ of the solution to equation 5, equal to $\gamma/2f_0$, relates the energy stored in the system W, to the energy dissipated per second P, as defined in [10]:

$$\delta = \frac{P}{2f_0 W}\,. \tag{6}$$

For this rod, involving 99.25% nickel, in the particular case of its demagnetized state, a difference of 1 Hz between the frequency of the phase shift and maximum amplitude is obtained.

Damping δ, estimated with these data equals 0.0526, with an uncertainty $u(\delta)$ of 7.9×10^{-3}, thus showing the losses in our case to be small. These results agree with those reported elsewhere [10].

5 Conclusions

The present study determines the change in elastic modulus with the applied magnetic field, and the magnetoelastic coupling coefficient of nickel samples based on the analysis of thin rod vibrations. Vibration at a point of the sample is registered with a heterodyne laser interferometer. Free vibration of the rod or wire is produced by a broad spectrum impact, and Fourier analysis of the vibration provides several vibration modes - though only the first is used for calculating E. The different E values are obtained with the sample positioned within different magnetic fields H until sample saturation is reached; the relative variations of E in the saturated state are around 2.0% for the rods and 3.1% for the wire. The relative maximum uncertainty of the method is about 0.6%, though this figure may be improved by using a digital oscilloscope with a larger number of sample points for the same sampling frequency. The results obtained show the method to be good for the evaluation of ΔE, due to the absence of contact with the sample - thereby preventing external factors from altering the frequency. Moreover, the method facilitates the simple, precise and rapid characterization of rods and wires.

An alternative method is also used to evaluate ΔE based on the forced vibration of a sample. The results corresponding to ΔE obtained for nickel samples are around 1.5%, with a relative maximum uncertainty of 0.5%. This system offers great precision by allowing a high-resolution analysis of the frequency response. In addition, it is possible to estimate the magnetic losses caused by the generation of eddy currents in the sample. The method is scantly operative,

however, since each measurement requires considerable time; as a result, the interferometry-sample setup tends to become dealigned, requiring repeat calibration of the system.

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Figure captions

Figure 1. Experimental arrangement showing (in blocks) all the major elements: S-sample, SDC-solenoid driving circuitry, EA-exciting arrangement, OU-optical unit, EU-electrical units, IS-interferometric set, DO-digitizing oscilloscope, PC-personal computer, FS-frequency spectrum.

Figure 2. Signal spectrum recorded following the impact of a quartz pendulum.

Figure 3. Change in elastic modulus E, with the applied magnetic field H, using the free excitation method.

Figure 4. Amplitude and phase shift curves with the forced excitation method.

Tables

Table 1. Results obtained with the free excitation method.

Table 2. Results obtained with the forced excitation method.

Table 1: Results obtained with the free excitation method.

Sample	State	$\overline{f}$ (Hz)	$u(f)$ (Hz)	$\overline{E}$ (GPa)	$u(E)$ (GPa)	ΔE (GPa)	$k(H=0)$
$\emptyset = 8mm$	demagnetized	24804	25	204.60	1.24	4.08(2.0%)	0.14
	saturated	25050	25	208.68	1.26		
$\emptyset = 2mm$	demagnetized	48150	25	213.71	0.45	3.44(1.7%)	0.13
	saturated	48536	25	217.15	0.46		
$\emptyset = 1mm$	demagnetized	24890	10	220.84	0.21	6.98(3.1%)	0.17
	saturated	25280	10	227.82	0.21		

Table 2: Results obtained with the forced excitation method.

Sample	State	$\overline{f}$ (Hz)	$u(f)$ (Hz)	$\overline{E}$ (GPa)	$u(E)$ (GPa)	ΔE (GPa)	$k(H=0)$
$\emptyset = 8mm$	demagnetized	24830	2.5	204.81	0.95	2.91(1.5%)	0.12
	saturated	25006	2.5	207.72	0.98		
$\emptyset = 2mm$	demagnetized	48202	2.6	214.05	0.12	2.47(1.2%)	0.11
	saturated	48520	2.6	216.52	0.12		