Mathematical music theory of embodied acoustics of Ikoro music using beat-class theory

Andrea M. Calilhanna, Stephen G. Onwubiko, and Adebowale O. Adeogun

Citation: Proc. Mtgs. Acoust. **39**, 035006 (2019); doi: 10.1121/2.0001276
View online: https://doi.org/10.1121/2.0001276
View Table of Contents: https://asa.scitation.org/toc/pma/39/1
Published by the Acoustical Society of America

POMA Proceedings of Meetings on Acoustics

Volume 39 http://acousticalsociety.org/

178th Meeting of the Acoustical Society of America
San Diego, California
2-6 December 2019

Musical Acoustics: Paper 4aMU8

Mathematical music theory of embodied acoustics of Ikoro music using beat-class theory

Andrea M. Calilhanna
MARCS Institute for Brain, Behaviour and Development, Western Sydney University, Sydney, New South Wales, 2126, AUSTRALIA; a.calilhanna@gmail.com; a.calilhanna@westernsydney.edu.au

Stephen G. Onwubiko
University of Nigeria, Nsukka, Enugu, 410002, NIGERIA; stephen.onwubiko@gmail.com

Adebowale O. Adeogun
Department of Music, University of Nigeria, Nsukka, Enugu, 410001, NIGERIA; adebowale.adeogun@unn.edu.ng

Through mathematical representation (beat-class theory) of embodied acoustics (psychoacoustics) the predominance of the musical tradition of the *Ikoro* drum with the Igbo's can be traced from the past, into the present and forecasted into the future. The *Ikoro* music tradition has been viewed as an integral and indispensable part of Igbo culture at large (Onwubiko and Neilsen, 2019). The major musical instruments that accompany most Igbo music are percussional, such as, *ichaka* (beaded-gourd rattle), *okpokolo* (wooden claves), and *igba* (membrane drum) and are characterized by successions of rhythmic interchange unlimited to interesting pitch, timbre, rhythm and meter by employing shifted accents, non-accented rhythms and syncopations. In order to understand Ikoro music located in the listener's experience (embodied psychoacoustics), we demonstrate how mathematical music theory (beat-class theory) provides the means to articulate the "mind and body" response to the stimulus of sound. By examining the aural tradition of *Ikoro* music of the Igbo's through visualizations and sonifications of beat-class theory using ski-hill graphs and circular cyclic graphs, "hidden" musical structures are revealed which possess significant cultural significance.

Published by the Acoustical Society of America

 https://doi.org/10.1121/2.0001276

1. Introduction

Our paper functions as one of a set of two papers with *Spectral Analysis of the Ikoro Drum* (Onwubiko, S.G. and Neilson, T., 2019) where the authors present an in-depth acoustical analysis of *Ikoro* music of the Nigerian Igbo tribe focusing on the role, design and construction of the *Ikoro* and Igbo cultural practices. The aim of our paper is to extend the acoustical analysis provided in that paper to demonstrate how a psychoacoustic approach to the preservation of traditional music, specifically in our study, Nigerian *Ikoro* music of the Igbo tribe, contributes further important scientific details towards the process of identification and transmission of cultural heritage of *Ikoro* music for future generations. Psychoacoustics is defined as embodied acoustics, the interdisciplinary and scientific study of the listener's "mind and body" experience of sound. Like most of the world's traditional music, *Ikoro* music is transmitted aurally rather than through notation and for this reason we chose to use an approach which has the capacity to articulate sound-based, rather than notation-based understandings of music.

The music theory we sourced to analyse the *Ikoro* music is based on Richard Cohn's (Yale) psychoacoustical approach to mathematical music theory (MMT) where representation: beat-class theory (Cohn, 2016, 2018b) of embodied acoustics, provides the means to articulate the "mind and body" response to the stimulus of sound. To do this, we use visualizations and sonifications of beat-class theory through ski-hill graphs (Cohn, 2001) and circular cyclic graphs and their computerized versions the SkiHill (2016b) and XronoBeat (2018a) apps to analyze *Ikoro* music. MMT is defined as music theory as understood through the instruments and principles of mathematics.

Our paper illustrates how Western music theory has evolved to now have the capacity to reveal temporal complexities and geometric musical structures which possess significant cultural importance and contribution that might otherwise have remained unnoticed and potentially lost forever in a rapidly changing part of the world: Africa. Thus, through the study of *Ikoro* music we demonstrate how Cohn's psychoacoustic approach to music theory also solves a pressing problem for the study of music which is not notated, to now be included equally alongside Western 'art' music in scholarly discussions (Cohn, 2015; Pressing, 1983).

2. Embodied acoustics of *Ikoro* music (psychoacoustics)

Like the majority of the world's music, traditional *Ikoro* music is not notated and is transmitted aurally through embodied acoustics. The listener internalizes the sound first by the mind and in response then entrains and projects the sound through the body (London, 2012; Hasty, 1997). This same aurally transmitted process occurs wherever music is not notated such as in jazz ensembles, rock music groups, and folk music ensembles.

Using entrainment and projection, the Nigerian musician performing traditional music, such as *Ikoro* music, navigates complexities of rhythms, hemiolas and polymeter, often performed polyphonically in ensembles, that experienced and musically-trained 'Western' musicians often find difficult to perform and bewildering to discuss. Yet, even though traditional music of Nigeria is transmitted aurally and not notated, most Nigerian musicologists, and those from other countries, at some stage, source the notation-based understandings of meter of traditional Western music theory, to analyse and understand rhythm and meter in traditional Nigerian music.

Among many noted music scholars who write on the music of Africa, there is diversity of approaches toward the discussion of meter, for example, Arom (2004) provides a lengthy discussion of mainly notational understandings of meter and rhythm he uses for his transcriptions utilizing marks akin to a tactus; Agawu (2003) refers to a dependence on dance movement to understand meter; Agu's (1999) brief reference to meter has the notational understandings and classifications of traditional Western music theory; Okafor's (2017) study of Igbo folk songs provides transcriptions using traditional Western notation and meter signatures.

A brief survey of Nigerian journals in African studies, musicology and education such as the Ikenga Journal of African Studies, Institute of African Studies, University of Nigeria (1975 and 1981); the Journal of the Association of Nigerian Musicologists (JANIM)(2011 and 2012); and the Conference of Music Educators (COMEN) in Nigeria publication – the Journal of Nigerian Music Education (2015, 2016 and 2017), demonstrate an absence of the detailed and accurate study of traditional Nigerian music.

For decades, many Nigerian scholars have called for improvements in music education at all levels of the curricula (Adeogun, 2012 & 2018; Agawu, 2003; Agu, 1999 & 2012; Arom, 2004; Onwubiko & Neilson, 2017 & 2018). In response, Nigerian scholars provide Conferences such as the 15th Annual National Conference of Music Educators in Nigeria (COMEN) 2019 'Africa-Sensed Music Education in Nigeria' and the University of Nigeria's 5th Biennial National Conference of Music and the Performing Arts (NACOMPA) 2019 'Decolonializing the Musical Arts in Nigeria'.

With so much confusion about how to write about meter and rhythm in music which is not notated, we are of the view that the perpetuation of unsuitable music theory, through which to discuss and understand Nigerian music, is the main reason for the diminishing quality in the scholarly discussion, appreciation and preservation of traditional Nigerian (Igbo) music.

In Section 6 we demonstrate how traditional Western music theory has evolved through new tools of MMT, the ski-hill graph, cyclic graph, SkiHill and XronoBeat applications, which use visualizations and sonifications of beat-class theory, to have the capacity to represent the listeners' psychoacoustic experience of both music and mathematics from listening to non-notated and notated music such as the music of Nigeria (Cohn, 2016, 2018a, 2018b, 2017, 2015, 2001; Milne and Calilhanna, 2019; Hilton, Calilhanna and Milne, 2018; Hamilton et al. 2018; Milne et al, 2016a; Milne 2018a, 2018b, Milne 2016b; Calilhanna, 2018; Calilhanna and Webb 2018; Calilhanna, 2017 Unpublished; Calilhanna and Onwubiko, 2019a, 2019b; Calilhanna, Onwubiko and Kemewerigha, 2019b; Kulma, 2017).

3. The link between psychoacoustics and MMT

Over the past 40 years or so, scholars mainly from North America, have sought to improve how rhythm and meter is understood due to the absence of a rigorous theory of meter. Over time the work of music theorists and the cognitive sciences have cross-pollinated producing numerous insights into rhythm and meter as sound and as experienced. For instance, Komar (1971, p. 52) introduces the term "time-span" in defining "beat" as "the initial time-point of an equal subdivision of the background structural time-span;" Imbrie (1973, p.51) argued that "two contradictory metrical interpretations of the same event can be simultaneously entertained in the mind of the performer and listener" (p. 51). Maury Yeston (1976) presents a modern theory of rhythm based on rhythmic strata in the context of rhythm as sounded music. Kramer (1981) notes the contribution of "non-Western" music towards showing composers new temporal non-linear understandings of time. Numerous other scholars from music theorists and the cognitive sciences, have contributed towards improving understandings of meter and rhythm (Lewin, 1981; Cohn, 1992; Roeder, 2003; Yeston, 1976; Kramer, 1988; Schachter, 1987; Krebs, 1987, 1999; Anku, 2000; Pressing, 1983; Lerdahl and Jackendoff, 1983; Bamberger, 2003, 2016, 2018; Hasty, 1997; London, 2012; Clayton, 2000; Huron, 2006; Imbrie, 1973; Krumhansl, 1990, 2000; Volk, 2008; Nozaradan, 2011; Povel and Essens, 1985; Large and Snyder, 2009; Patel and Iverson, 2014).

The evolution of traditional Western music theory, through Cohn's (2016, 2018) MMT (beat-class theory), is a prime example of how the field of acoustics and its sub-field psychoacoustics has influenced another scientific field, music (Cohn, 2016, 2018). Cohn's recent article "Meter"

(2018a) for *The Oxford Handbook of Critical Concepts in Music Theory*, is a distillation of the theory of meter he developed through the works of many scholars, mainly from North America, and is intended as an analytical model to understand musical meter. Rather than simply providing classifications and descriptions for the different types of meter and their relations to describe the notation, Cohn's theory of meter also acknowledges research which indicates that musical meter is located in the listener as a "mind and body" response to the stimulus of sound (Cohn, 2015).

Cohn provides a definition of meter: "A meter is an inclusionally related set of distinct, notionally isochronous time-point sets" which prompts new understandings about what meter is and where meter is located and also encourages an interdisciplinary pedagogical approach to meter. Thus, the field of psychoacoustics has prompted the development of music theory to have the capacity to represent, with accuracy, a listener's psychoacoustic experience of listening to music (Cohn, 2018a). From developing through an interdisciplinary and cross-conceptual, or isomorphic pathway, acoustics is influencing how and what is being taught not only in music classes but in other fields such as science, mathematics and physics through MMT (Milne & Calilhanna, 2019; Hamilton et al 2018).

Where spectral analysis and the computerization of sound has led to machine learning to study aspects of music and sound such as the frequency, timbre, number and volume of the onsets, psychoacoustics, on the other hand, has led to the development of instruments of MMT which have the capacity to accurately represent music formed in the listener's imagination as meter and tonality (Cohn, 2018a; Brochard, 2003). These instruments include beat-class theory and its visualization and sonification through the ski-hill graph and cyclic graph and their digital versions the SkiHill and XronoBeat applications.

4. MMT: Beat-class theory

By "MMT" we mean music theory as understood through the instruments and principles of mathematics. Cohn (2016c) p. 237 provides a summary of the different mathematical modes of "exploring, conceiving, and representing musical phenomena" which have contributed towards MMT regaining its central position in music theory today such as the symbolic languages of abstract algebra, especially sets and groups. However, Cohn states that since about 1995 there has been a shift towards "mathematical sub-disciplines that characteristically communicate through visual images: graph theory, geometry, and topology." He notes the current prevalence of musical graphs and geometries "not just polygons, and Cartesian graphs, but also multi-dimensional cylinders, including helices, torii, and complex orbifolds."

MMT and meter-related research in pedagogy continues to increase through the work of scholars (Cohn, 2001 2015, 2016, 2017, 2018a, 2018b; Douthett, Clampitt and Carey, 2018; Clough and Douthett, 1991; Douthett et al, 2008; Montiel, 2018; Chew and Volk, 2008; Milne, 2018; Montiel, 2018; Gómez, 2018; Wilhelmi, 2018; Hall, 2018; Hughes, 2018; Johnson, 2018; Kochavi, 2018; Clampitt, 2008; Mannone, 2018; Milne and Calilhanna, 2019; Hamilton et al, 2018; Hilton, Calilhanna and Milne, 2018; Calilhanna 2018; Milne et al, 2016a; Milne 2018a, 2018b, Milne 2016b; Calilhanna and Webb 2018; Calilhanna, 2017 Unpublished; Calilhanna and Onwubiko, 2019a, 2019b; Calilhanna, Onwubiko and Kemewerigha, 2019b; Amiot, 2019; Chiu, 2018) and through international conferences such as the Society for Mathematics and Computation in Music (SMCM). Teaching MMT with school-age students has also been piloted in secondary schools in Sydney, Australia through Richard Cohn's music theory and Andrew Milne's software applications (MARCS Institute for Brain Behaviour and Development, Western Sydney University) in the project *Teaching Mathematics with Music and Music with Mathematics* (Sydney 2017- ongoing).

In our discussion of *Ikoro* music we demonstrate how mathematics is sourced as a universal language through which to represent music. As a result of applying MMT, a truly level playing field is established for the ethical, inclusive and scientific study of the music of a diversity of the world's cultures. In applying a psychoacoustic approach to MMT, it is now possible to uncover the hidden structural characteristics of music including those of the *Ikoro* music of the Igbo tribe in Nigeria. For instance, not only were we able to explore how the music is internalized by the performer(s) as embodied acoustics, this approach also enabled the articulation of findings from listening through new music theory.

Through using a psychoacoustical approach to music theory, solutions are provided for the scholarly discussion of music which is non-notated, such as, the traditional music of the Igbo's discussed further in this paper. For instance, in the discussion, we demonstrate how traditional Western music theory has evolved through psychoacoustics, to also solve the problems of the labelling used in the American numerical system to describe meter signatures and note values. This occurs because Cohn's beat-class theory provides music theory for listeners to make observations about meter and rhythm by representing cyclic universes of different sizes, such as C6, C8 or C12, which represent the number of elements in a cycle and the cardinality of each cycle, through mapping selected timepoint sets (see Cohn, 2016 and 2018b). The XronoBeat app (Milne, 2018) is the world's first computerized version of the cyclic graph which uses both visualisations and sonifications to represent meter as sets of timepoints using numbers for both small and large cyclic universes. See Figure 1:

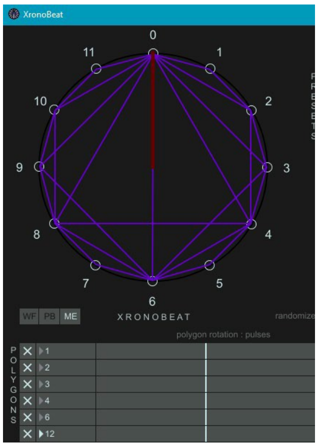


Figure 1. Cyclic graph XronoBeat representing C12 and subsets.

Cohn's beat-class theory uses modular arithmetic to enable listeners to articulate their observations about the sets that form on cyclic graphs from their mapping of rhythms as points and polygons from listening to music (see also Calilhanna, 2018). Unlike other computer-based models of a cyclic graph, the XronoBeat app (Milne, 2018), allows the listener to experience in a perceptually immediate way, the theoretical information about music and mathematics they gather from their observations of cyclic universes through applying beat-class theory to a musical experience (Cohn, 2016 and 2018; Hilton, Calilhanna, Milne 2018 p.220). In other words, through beat class theory and cyclic graphs, listeners learn more about the music and mathematics they embody through their experience of cycles when listening to, performing or composing music.

Listeners map rhythms, meter and pitch to evenly distributed timepoints numbered from zero around the circumference. A cyclic graph with 12 points, also known as C12, or clockface, displays the numbers 0-11 (Cohn, 2016, 2018b and 2017 Unpublished; Milne and Calilhanna, 2019). Onsets mapped to points on the cyclic graph are labelled using Cohn's beat-class ordered set notation. For instance, c represents the number of elements or subsets of a larger cyclic universe known as C; d represents the cardinality of a pitch-class set, thus labelled (c, d). (12, 7) represents the notation for a major scale in a chromatic universe and (7, 3) represents a triad in a diatonic universe. The 64 different permutations or sets possible for C6 can be represented as 6 singles (6,1), 15 pairs (6, 2), 20 triplets (6, 3) and so on, which can then be explored through other practical musical activities. To learn about C6 a lesson might involve studying music notated in 6/8 where six quavers [012345] divide into two dotted crotchets are mapped as [03] and a measure notated in 3/4 where six quavers divide into three crotchets mapped as [024] (Cohn, 2018a). Cohn (2018b p. 132) also provides a useful table of the beat-class for each cycle (see also Milne and Calilhanna, 2019).

5. The ski-hill and cyclic graph to represent meter and rhythm

The ski-hill graph, which is a two-dimensional matrix designed by Cohn (2001) to represent hemiolas, is arguably the most compact and efficient instrument of music theory for the listener to quantify meter. The nodes of the ski-hill graph are empty to represent that meter is understood as sound experienced rather than as the notation. The listener has the option to choose to map the quantified results of their psychoacoustical embodiment of sound through mapping the pulses they hear that form meter as empty nodes, traditional notation, fractions, polygons, and a choice of sonifications. See Figure 2

Figure 2. Computerized version of the ski-hill graph the SkiHill app representing meter (polygons not shown)

Unlike linear-notation, by mapping adjacent pulses in a relation of inclusion in a ratio of 2:1 duple meter to the left direction and adjacent pulses in a relation of inclusion in a ratio of 3:1 triple meter to the right direction, the two different metric pathways or subdivisions provide a useful distinction visually and through sonifications (in the computerized version). Minimal meters are formed in any music where there are duple and triple minimal meters such as those heard in the *Ikoro* music of the following analysis. Meter is represented as ordered set notation where minimal duple meter is represented as <2>, minimal triple meter as <3> and deep meter as, for example <232>.

The cyclic graph and its computerized version Xronobeat also measures the mathematical properties of meter but in a circular graph representing also rhythm and pitch with integers, polygons and sonifications. Rhythm is defined here as timepoints selected from the metric hierarchy and pitch is defined as frequency. Meter, rhythm and pitch are all presented as sets on cyclic graphs in visualisations of beat-class theory as integers, polygons, and sonifications. In this way rhythm, meter and pitch are quantized through the listener's own reporting of these elements as sets using beat-class and pitch-class theory and spectral analysis in visualizations and sonifications.

Cohn's beat-class theory represents modern meter theory instead of, for instance, propagating notation-based understandings of meter and rhythm, which promote inaccurate quantifications of the listener's embodied experience of meter. Traditional, notation-based meter theory doesn't recognize all metric levels and only two pulses are represented by the meter signature. Cohn's modern meter theory, on the other hand, recognizes all of the pulses the listener hears because it is based on the understanding that meter is “an inclusionally related set of distinct, notionally isochronous time-point sets” and it acknowledges research which indicates that musical meter is located in the listener as an embodied psychoacoustic or “mind and body” response to the stimulus of sound.

6. Application

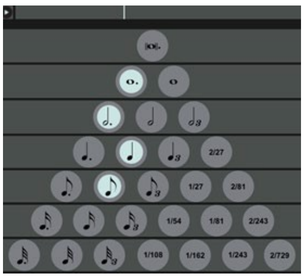
In this section we have documented details of what first appeared as a simple and short *Ikoro* drumming improvisation, however, through taking a psychoacoustical approach to the study of rhythm, meter and pitch this piece proved more complex than expected. It was only through studying the mathematical underpinnings, discoverable through embodied psychoacoustics, that we were able to collect accurate data which included otherwise hidden geometric musical structures. We achieved this through using - beat-class theory - and visualizing and sonifying it with new tools of music theory – Cohn’s (2001) ski-hill graph, cyclic graphs and Milne’s (2016 and 2018b) SkiHill and XronoBeat applications.

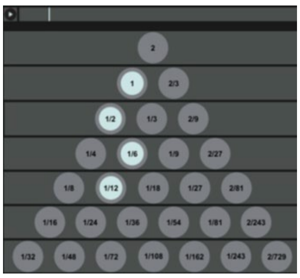
According to the SkiHill graphs, shown in Figure 3a) and b), the *Ikoro* drum music exhibits the meter <232> with minimal duple meter mapped to the left pathways in a ratio of 2:1, adjacent pulses in a relation of inclusion in a ratio of 2:1, and minimal triple meter mapped to the right pathways, adjacent pulses in a relation of inclusion in a ratio of 3:1. Figure 3a) represents the meter <232> with traditional staff notation (and sonifications) and Figure 3b) represents a hearing of the meter through fractions.

The beat-class theory forming the meter <232> mapped to the ski-hill graphs in Figure 3a) and b) is represented in Figure 3c) as beat-class theory representing sets and subsets of integers in the cyclic universe U^{12} with 12 elements {0-11}. Figure 3d) represents the results from XronoBeat through visualizing and sonifying the U^{12} sets of integers {0-11} in Figure 3c) as polygons in a C12 cycle to examine the divisions, periodicity, and sonifications forming the meter <232>.

Figure 3 e)-f) represent the rhythms the listener experienced which initiated meter as sets using beat-class theory with XronoBeat. The structure of this rhythm can be defined by the cyclic operations in C=3 of the maximally even universal set {0, 1, 2}. The set {0, 1, 2} is rotated once $T^{-}1$ to the set {2, 0, 1} from which the subset {2, 0} C3 and E3 which is heard five times before the set is ‘reversed’ {1, 0, 2} after a metric displacement of an ‘extra’ pulse. In this case, the rhythm which initiates meter, can be observed ‘close up’ to measure and document the details of pulses and their relations underpinning the listener’s mathematical experience of C12, that is, ‘the feel’.

The beat-class theory illustrated in Figure 3g) represents together both the meter <232> , the rhythm, and pitch {0, 2, 4, 5, 7, 9} CDEFGA also known also as the Guidonian hexachord represented as a cycle using beat-class theory:

a) 

b) 

c)

U^{12}{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10,11}

12,1 {0}

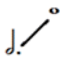
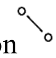
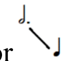
12,2 {0, 6}

12,6 {0, 2, 4, 6, 8, 10}

12,12 {0, 1, 2, 3, 4, 5, 6, 7, 8, 9,10,11}

Figure 3. ***Representations of the deep meter <232> a)-b) ski-hill graphs; c) beat-class theory for U*** ^{12}***; d) cyclic graph with integers and polygons using beat-class theory to represent the divisions of the 12-cycle experienced psychoacoustically by the listener; e)-f) the rhythm as C3 in C12 with beat-class theory; g) meter, rhythm and pitch represented as C12 using beat-class theory***

Notably through analyzing the mathematics of the listener's psychoacoustic experience of the *Ikoro's* sound of the rhythm, meter and pitch it was observed that the music's structure metaphorically forms a temporal revolution or palindrome, due to rotational invariance.

The selected nodes and their edges in a) and b) form a metric pathway to represent adjacent pulses in a relation of inclusion in sets (a set refers to pairs of pulses where one of the notes is either two or three times faster or slower than the other) of minimal duple meter in a ratio of 2:1 sloping to the left direction, such as ,  or  and minimal triple meter in a ratio of 3:1 sloping to the right direction  or . Unlike pulse stacks in a linear-vertical orientation where the minimal meters can be identified in a straight line, mapping the pulses duple meter to the left and triple meter to the right pathways provides an additional benefit of a representation which differentiates the meters visually as distinct duple and triple meters and pathways.

Representation of a deep meter on a ski-hill graph such as <232> means the relationships between the pulses forming duple and triple meters can be observed more closely and studied as isomorphic representations of the embodied experience of mathematics. Figure 3a)-g) represents the integers <232> or the depth of the meter in the metric hierarchy experienced from listening to this piece. In the deep meter <232> the span pulse is worth two of its next fastest (adjacent) pulse in the metric hierarchy, six of the next fastest pulse, and twelve unit or the fastest pulse. Thus, the ski-hill graph enables the listener to articulate the embodied mathematical experience of musical meter without having to default to traditional Western notational understandings of meter.

12/8 would likely be the obvious choice for notation into traditional Western notation, however, using an approach which recognizes embodied acoustics through MMT, 12/12 provides a more meaningful notation, one which is based on an understanding of meter as sound experienced psychoacoustically rather than as notation. Thus in rendering this seemingly simple, and short Igbo *Ikoro* drumming improvisation solely into traditional notation, the music's deepest structural qualities would remain hidden and potentially lost forever.

A summary of the temporal revolution of the 12 seconds *Ikoro* drum recording is shown in the timeline in Figure 4. The data documented in Figure 4 (bottom to top) represents observations of the following elements: the rhythm in traditional notation; integers for C3 (beat-class theory); a timeline; the onsets as ten sets of two pulses as X's; the listener's subjective accents; an indication of the two pitches as L (Low) and H (High); the numbers for counting meter are based on the number of pulses heard for each timepoint (as indicated by the pulses stacked above in the metric hierarchy section); the top four lines are the pulses a listener can observe from the fastest (bottom) to slowest (top) in the metric hierarchy. Above the lines is an indication of the metric displacement the listener hears and the minimal meters are listed to the left as pairs of pulses in a vertical stack. The metric hierarchy is represented through an indication of the minimal meters through a pulse stack with 2 representing duple meter and 3 representing triple meter and numbers for counting meter. An approximation of the pitch is illustrated (L is approximately C3 and H is approximately E3). Notionally isochronous time point sets (pulses) in the metric hierarchy section can be observed from hearing the repetition of the rhythm of the *Ikoro* drumming. Through projection and entrainment the parallelism (Lerdahl and Jackendoff, 1983) experienced through listening to the rhythms, initiates inclusionally-related sets of pulses in ratios of 2:1 or 3:1, to form minimal and deep meters.

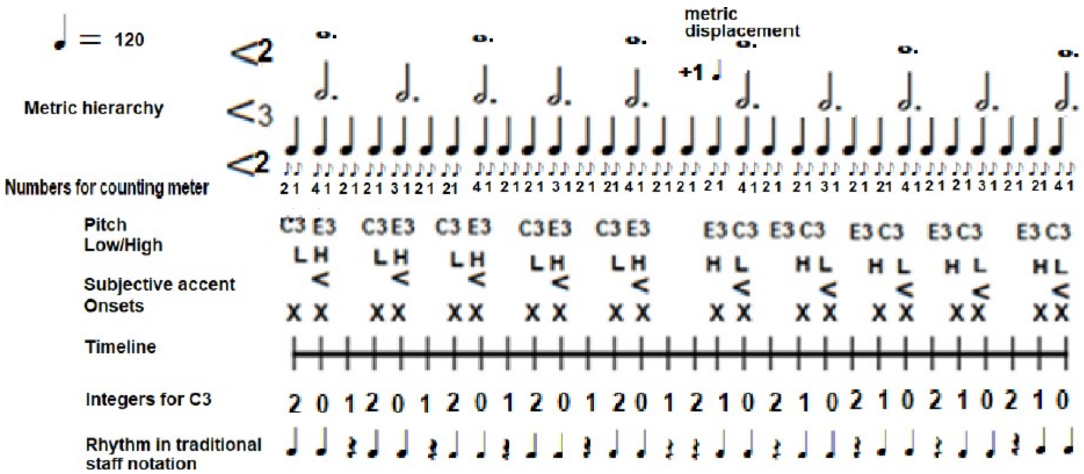


Figure 4. Timeline and initial analysis ('note-taking') for audible clues to analyse the rhythm, meter and pitch

From listening closely to the *Ikoro* drumming solo and quantifying the rhythms, meter and pitch through visualizations and sonifications of Cohn's beat-class theory, many important details of potential cultural significance were discovered that would normally go unnoticed. For instance, through ski-hill graphs it was possible for a listener to map all of the pulses that formed part of their psychoacoustic experience of meter and mathematics, critically, including in the analysis representations of those experienced in the imagination.

From quantifying the pulses through pairing in 2:1 or 3:1 ratios, minimal meters could be mapped to a metric hierarchy in distinct duple and triple metric pathways to form deep meter. The meter, rhythms and pitch were then mapped to cyclic graphs so as to further quantify the *Ikoro* drumming solo through examining the cyclical hierarchy, periodicity, sets, ratios, and divisions through polygons to reveal an otherwise hidden metric and tonal structure.

The value and potential of Cohn's MMT to quantify details about Igbo culture, including music and everyday sounds, is immense. In this way Igbo cultural practices and contributions, which are at risk of disappearing through a rapidly changing world, can be preserved for future generations. Recordings, spectral analysis, written accounts and photographs are all important for the preservation of a culture, however, the importance of the role of the listener in preserving the aural tradition through quantification of music and sound for future generations, should no longer be overlooked.

The musical characteristics of the *Ikoro* drum include the meter, rhythm, and pitch. If this rhythm were to be analysed and notated solely using traditional staff notation, or, as a linear arrangement, a number of important structural elements would go unnoticed. Cohn's beat-class theory, therefore, can provide more accurate accounts of music such as this study of the music of the Igbo's of Nigeria. In a mathematical environment it was possible to study both meter and rhythm with pitch where before none of the music was literally notated only 'felt' or projected and entrained to. Most of the pulses were experienced in the imagination unseen and in a sense unheard but 'felt'.

In approaching musical analysis through visualization and sonifications of Cohn's beat-class theory the listener articulates their own quantifiable experience of music through the scientific examination of music, such as in our study of the music of the Igbo's *Ikoro* music of Nigeria. The capacity for the ski-hill graph, to delineate between duple and triple meters provides a powerful reason to include ski-hill graphs in the analysis of any music wherever duple and triple meter are evidenced in the psychoacoustical experience of the listener. We would argue that Cohn's MMT should be included in classroom music textbooks and online resources so that the cultural significance of music such as the Igbo's can be appreciated and preserved for future generations.

7. CONCLUSION

Through our study of *Ikoro* music we have demonstrated how Western music theory has evolved to now have the capacity to discuss and understand music which is not notated. By applying a psychoacoustical approach, through the listener's embodied acoustics and specially-designed computerized tools of MMT: Cohn's ski-hill graph, cyclic graphs and their computerized versions the SkiHill and XronoBeat apps, to visualize and sonify the psychoacoustic experience of beat-class theory, our results revealed surprising geometric musical structures in the music of the Igbo tribe.

What seemed, at first, to be a simple and short *Ikoro* drumming improvisation proved more complex than expected and it was only by studying the mathematical underpinnings, discoverable through visualizing and sonifying the beat-class theory of embodied acoustics was it possible to collect accurate data about the piece. This occurs because the approach presented in this paper can be applied to the embodied psychoacoustic experience of cycles of different size universes initiated from listening to music from around the world and not limited to notated music from the Western canon.

In addition, through applying Cohn's beat-class theory, ski-hill and cyclic graphs to the analysis of Igbo *Ikoro* music we provide compelling evidence to support the reassessment of the numerical labels given to note values and meter signatures. As demonstrated by the analysis in Section 6 Cohn's psychoacoustic approach to music theory provides a rigorous theoretical framework and practical tools for the representation of meter, rhythm and pitch through enabling the listener to experience a more meaningful understanding of music and mathematics its abstract representations such as music notation. In this way, Cohn's psychoacoustic approach to music theory is also the first interdisciplinary music theory which is holistic and listener-centered in the broader study of acoustics - an approach which recognizes the importance of the aural tradition for a healthier and more inclusive society.

Our paper presents qualitative research as to why mathematical music theory: beat-class theory, the ski-hill and cyclic graphs and their computerized versions, the SkiHill and XronoBeat applications, should be included in the curricula of universities, schools and studios wherever acoustics is taught. In this way, music from diverse cultures such as Nigeria (Igbo) music, can be appreciated, and included, in the study of music and acoustics alongside music of the Western canon. For the reasons detailed in our paper, Cohn's psychoacoustic approach to understanding and discussing music is a universal theory of music in the fullest sense.

REFERENCES

Adedeji, Femi. Ed. (2015). *Journal of Nigerian Music Education* (JONIMED). Nigeria: The Conference of Music Educators in Nigeria. Department of Music, Obafemi Awolowo University, (7).

Adedeji, Femi. Ed. (2016). *Journal of Nigerian Music Education* (JONIMED). Nigeria: The Conference of Music Educators in Nigeria. Department of Music, Obafemi Awolowo University, (8).

Adeogun, A. O. (2012). *Music education in Nigeria, 1842-2001: Policy and content evaluation, Towards a new dispensation*. Germany: Lambert Academic Publishing.

Adeogun, A. O. (2018). A historical review of the evolution of music education in Nigeria until the end of the twentieth century. Journal of the Musical Arts in Africa,15, 1–18.

Afigbo, A.E. Ed. (1975). Ikenga Journal of African Studies, Institute of African Studies, University of Nigeria, 3(1&2).

Agu, D.C.C (1999). *Form and Analysis of African Music.* Nigeria: New Generation Books.

Amiot, E. (2019) "Concérferences": of Music and Maths, for the Audience's Delight. In Montiel, M., Gómez-Martín, F., and Agustín-Aquino, O. A., editors, *Mathematics and Computation in Music. Springer.*

Anku, W. (2000). Circles and time. A theory of structural organization of rhythm in African music. Music Theory Online, 6(1), 1-8.

Bamberger, J. (2003). Music as embodied mathematics: A study of a mutually informing affinity. *International Journal of Computers for Mathematical Learning,* 8 (2), 123-160.

Bamberger, J. (2016). Action and Symbol: An Essential Tension. In *Mathemusical Conversations: Mathematics and Computation in Music Performance and Composition* (pp. 189-207).

Bamberger, J. (2018). Action knowledge and symbolic knowledge. The computer as mediator/ Conocimiento basado en la acción y conocimiento simbólico. El equipo informático como mediador. *Infancia y Aprendizaje*, *41*(1), 13-55.

Brochard, R., D. Abecasis, D. Potter, R. Ragot, and C. Drake. (2003). "The "Ticktock" of Our Internal Clock: Direct Brain Evidence of Subjective Accents in Isochronous Sequences." *Psychological Science* 14, no. 4: 362–66.

Calilhanna, A. (2018). *Teaching musical meter to school-age students through the ski-hill graph* (Master's thesis, University of Sydney). Retrieved from The Sydney eScholarship Repository. (http://hdl.handle.net/2123/19791).

Calilhanna, A.M., Onwubiko, S.G., Kemewerigha, T. E. (2019). Mathematical Music Theory and the Representation of Igbo Music. 15th Annual National Conference of Music Educators in Nigeria (COMEN) 'Africa-Sensed Education in Nigeria.'

Calilhanna, A.M., Onwubiko, S. G., & Kemewerigha, T. (2019b) Ikorodo Music Analyzed Through Visualizations and Sonifications of Beat-Class Theory. 23rd International Conference on Acoustics, Aachen.

Calilhanna, A.M.; Onwubiko, S. G.; Kemewerigha, T. E. (2019c). Decolonizing African Music with Visualizations and Sonifications using Beat-Class Theory. 5th Biennial National Conference of Music and the Performing Arts (NACOMPA).

Calilhanna, A., & Onwubiko, S. G. (2019a). Mathematical Music Theory of embodied acoustics of *Ikoro* music using beat-class theory. *The Journal of the Acoustical Society of America*, *146*(4), 2999-2999.

Calilhanna, A., & Onwubiko, S. G. (2019b). Decolonizing African music with visualizations and sonifications using beat-class theory. *The Journal of the Acoustical Society of America*, *146*(4), 2908-2908.

Calilhanna, A. (2017, Unpublished). Understanding and Teaching Meter Survey Report. Sydney Conservatorium of Music.

Calilhanna, A. and Webb, M. (2018, Unpublished). Teaching time: A survey of music educators' approaches to meter, presented at the Meter Symposium 3, Sydney, 2018.

Chew, E., & Volk, A. (2008). *Reconsidering the Affinity between Metric and Tonal Structures in Brahms's Capriccio Op. 76, No. 8*. 15, 138-171.
Chiu, M. (2018). *Form as meter: metric forms through Fourier space*. *Open.bu.edu*. Retrieved 18 August 2018, from https://open.bu.edu/handle/2144/30655.
Clayton, M. (2000). *Time in Indian Music: Rhythm, Metre, and Form in North Indian Rag Performance*. New York: Oxford University Press.
Clough, J. & Douthett, J. (1991). Maximally even sets. *Journal of Music Theory*, 35 (1/2), 93 – 173.
Cohn, R. (1992e). Transpositional combination of beat-class sets in Steve Reich's Phase-Shifting Music. *Perspectives of New Music*, 30 (2), 146 – 177.
Cohn, R. (2001). Complex Hemiolas, Ski-Hill Graphs and Metric Spaces. *Music Analysis*, 20 (3), 295-326.
Cohn, R. (2015). Why we don't teach meter, and why we should. *Journal of Music Theory Pedagogy,* 29, 1-19.
Cohn, R. (2016). Teaching atonal and beat-class theory, modulo small. *Brazilian Journal of Music and Mathematics*, 1 (1), 15–24.
Cohn, R. (2016c). Graph-theoretic and geometric models of music. In Jordan, B.L., Smith, E., & Chew, G.A. (eds). *Mathemusical Conversations: Mathematics and Computation in Music Performance and Composition* (Lecture Notes Series, Institute for Mathematical Sciences, National University of Singapore) (pp. 240-243). Singapore: World Scientific Publishing Company.
Cohn, R. (2017, April 26). Musicology Colloquium series, Sydney Conservatorium of Music. Lecture: Poetic and empirical theories of musical meter: Cognitive dissonance in the historical archive, the laboratory, and the modern conservatory classroom.
Cohn, R (2017) *Teaching Mathematics with Music and Music with Mathematics*, project notes. (Unpublished).
Cohn, R. (2018a). Meter. Rehding, A. & Rings, S. (eds.) *The Oxford handbook of critical concepts in music theory.* Oxford: Oxford University Press.
Cohn, R. (2018b). Scaling up to Atonality: The Pedagogy of Small Cyclic Universes. In Montiel, M. & Gómez, F. (Eds.), *Theoretical and Practical Pedagogy of Mathematical Music Theory. Music for Mathematics and Mathematics for Musicians, From School to Postgraduate Levels* (pp. 127-149). World Scientific.
Dalby, B. (2005) Toward an Effective Pedagogy for Teaching Rhythm: Gordon and Beyond. *Music Educators Journal*, *92*(1) 54-60
Dalby, B. (2015) Teaching Movable Du: Guidelines for Developing Enrhythmic Reading Skills. *Music Educators Journal*, *101*(3), 91-99.
Dirk-Jan, P. & Essens, P. (1985). Perception of temporal patterns. *Music Perception*, 2 (4), 411–40.
Douthett, J., Hyde, M. M., & Smith, C. J. (2008). *Music theory and mathematics: Chords, collections, and transformations*. Rochester: University of Rochester Press.
Douthett, J., Clampitt, D., & Carey, N. (2018). From Musical Chords to Twin Primes. *Mathematical Music Theory: Algebraic, Geometric, Combinatorial, Topological and Applied Approaches to Understanding Musical Phenomena*, 1.
Gordon, E. (1989). *Learning sequences in music: Skill, content, and patterns*. Chicago: GIA Publications.
Gordon, E.E. & Woods, D.G. (2001). *Jump right in: The music curriculum*. Chicago: GIA Publications, Inc.
Gómez, F. (2018). Montiel, M. & Gómez, F. (eds). *Mathrhythm*: Lessons on Divisibility and Rhythm *(Theoretical and Practical Pedagogy of Mathematical Music Theory. Music for Mathematics and Mathematics for Musicians, From School to Postgraduate Levels).* World Scientific Press.
Hall, R. (2018). Montiel, M. & Gómez, F. (eds). Rhythm and Transformation *Theoretical and Practical Pedagogy of Mathematical Music Theory. Music for Mathematics and Mathematics for Musicians, From School to Postgraduate Levels).* World Scientific Press.
Hamilton, T.J., Doai, J., Milne, A.J., Saisanas, V., Calilhanna, A., Hilton, C., Goldwater, M., Cohn, R. "Teaching Mathematics with Music: a pilot study." Accepted for presentation at the *IEEE International Conference on Teaching, Assessment, and Learning for Engineering (TALE): Engineering Next-Generation Learning*, Australia, 4 - 7 December 2018.

Hasty, C. (1997). *Meter as rhythm*. Oxford: Oxford University Press.

Hilton, C., Calilhanna, A., and Milne, A. J. (2018). Visualizing and sonifying Mathematical Music Theory with software applications: Implications of computer-based models for practice and education. In Montiel, M. and Gómez, F. (Eds.), *Theoretical and Practical Pedagogy of Mathematical Music Theory: Music for Mathematics and Mathematics for Musicians, From School to Postgraduate Levels* (201-236). World Scientific.

Hughes, J. R. (2018).Visualizing and sonifying Mathematical Music Theory with software applications Implications of computer-based models for practice and education. In Montiel, M. and Gómez, F. (Eds.), *Theoretical and Practical Pedagogy of Mathematical Music Theory: Music for Mathematics and Mathematics for Musicians, From School to Postgraduate Levels* (201-236). World Scientific.

Huron, D.B. (2006). *Sweet anticipation: Music and the psychology of expectation*. Massachusetts: MIT press.

Idolor, G.E. Ed. (2011). Journal of the Association of Nigerian Musicologists. Department of Music, Delta State University, (5).

Idolor, G.E. Ed. (2012). Journal of the Association of Nigerian Musicologists. Department of Music, Delta State University, (6).

Imbrie, A. (1973). "Extra" measures and metrical ambiguity in Beethoven. In A. Tyson (ed). *Beethoven Studies* (pp. 45-66). New York: Princeton University Press.

Kramer, J. D. (1981). New temporalities in music. *Critical Inquiry*, *7*(3), 539-556. Kramer, J. D. (1988). *The Time of Music New Meanings, New Temporalities, New Listening Strategies*. New York: Schirmer Books.

Krebs, H. (1987). Some extensions of the concept of metrical consonance and dissonance. *Journal of Music Theory*, 31 (1), 99-120.

Krebs, H. (1999). *Fantasy pieces: Metrical dissonance in the music of Robert Schumann*. Oxford: Oxford University Press.

Krumhansl, C.L. & Palmer, C. (1990). Ohio State University mental representations for musical meter. *Journal of experimental Psychology*, 16 (4), 728-741.

Krumhansl, C. L. (2000). Rhythm and pitch in music cognition. *Psychological bulletin*, *126*(1), 159.

Kulma, D. (2017). Music theory: March v. Waltz - A short intro to meter and ski-hill graphs. *Music Corner Breve*. Retrieved on May 8, 2018 at: https://www.youtube.com/watch?v=0yxba7yoMSk

Large, E. W., & Snyder, J. S. (2009). Pulse and meter as neural resonance. *Annals of the New York Academy of Sciences*, *1169*(1), 46-57.

Lerdahl, F. & Jackendoff, R. (1983). *A generative theory of tonal music*. Cambridge MA: MIT Press.

Lewin, D. (1981). On harmony and meter in Brahms's op. 76, no. 8. *Nineteenth-Century Music*, 4 (3), 261-265.

London, J. (2012). *Hearing in time: Psychological aspects of musical meter,* 2nd edn. Oxford: Oxford University Press.

Milne, A. J., Herff, S., Bulger, D., Sethares, W A., & Dean, R. (2016a). XronoMorph: Algorithmic generation of perfectly balanced and well-formed rhythms. Proceedings of the 2016 International Conference on New Interfaces for Musical Expression (NIME 2016) (pp. 388–393). Brisbane: Griffith University.

Milne, A. J. (2016b) SkiHill application (beta version and currently under development) Computer software retrieved from https://www.dynamictonality.com/skihillapp.htm

Milne, A. J. (2018a). XronoBeat application (beta version and currently under development) Computer software retrieved from https://www.dynamictonality.com/xronobeat.htm

Milne, A. J. (2018b). Linking sonic aesthetics with mathematical theories. *The Oxford Handbook of Algorithmic Music, 155-180.*

Milne, A. J. and Calilhanna, A. M. (2019). Teaching music with mathematics: A pilot Study. In Montiel, M., Gómez-Martín, F., and Agustín-Aquino, O. A., editors, *Mathematics and Computation in Music. Springer.*

Montiel, M. & Gómez, F. (eds). (2018). *Theoretical and Practical Pedagogy of Mathematical Music Theory. Music for Mathematics and Mathematics for Musicians, From School to Postgraduate Levels.* World Scientific Press.

Nozaradan, S., Peretz, I., Missal, M. & Mouraux, A. (2011). Tagging the neuronal entrainment to beat and meter. *Journal of Neuroscience,* 31 (28), 10234-10240.

Nwamara, A-I. Ed. (2017). *Journal of Nigerian Music Education* (JONIMED). Nigeria: The Conference of Music Educators in Nigeria. Department of Music, Nnamdi Azikiwe University, (9). Nwoga, D.I. Ed. (July, 1981). Ikenga Journal of African Studies, Institute of African Studies, University of Nigeria, 5, (1).

Okafor, R.C. (2017) *A Study of Igbo Folk Music*. Nigeria: Academic Publishing Company. Onwubiko, S. G., & Neilsen, T. B. (2017). Music a scientific art: A call for review of Department of Music, University of Nigeria Nsukka's curriculum and course outline. *The Journal of the Acoustical Society of America, 142(4), 2540-2540.*

Onwubiko, S. G., & Neilsen, T. B. (2019). Non-destructive correlation of Nigerian drumbeat-pattern and pitch to detect a ripen watermelon. *The Journal of the Acoustical Society of America*, *145*(3), 1708-1708.

Patel, A. D., & Iversen, J. R. (2014). The evolutionary neuroscience of musical beat perception: the Action Simulation for Auditory Prediction (ASAP) hypothesis. *Frontiers in systems neuroscience*, *8*, 57.

Povel, Dirk-Jan & Peter Essens. (1985). "Perception of Temporal Patterns." *Music Perception* 2, no. 4: 411–40.

Pressing, J. (1983). Cognitive isomorphisms between pitch and rhythm in World musics: West Africa, the Balkans and western tonality. *Studies in Music*, 17, 45.

Rahn, J. (1998). A theory for all music: Problems and solutions in the analysis of non-Western forms. University of Toronto Press.

Repp, B. H., & Su, Y. H. (2013). Sensorimotor synchronization: a review of recent research (2006–2012). *Psychonomic bulletin & review*, *20*(3), 403-452.

Roeder, J. (2003) Beat-class modulation in Steve Reich's music. *Music Theory Spectrum* 25(2), 275-304.

Schachter C. Rhythm and Linear Analysis: Aspects of Meter." The Music Forum 6. New York: Columbia University Press, 1987, 1–60.

Volk, A. (2008) The Study of Syncopation Using Inner Metric Analysis: Linking Theoretical and Experimental Analysis of Metre in Music, *Journal of New Music Research*, 37:4, 259-273.

Yeston, M. (1976). *The stratification of musical rhythm*. Yale University: Yale University Press.