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# Torsional vibration solution of tapered pile considering stress diffusion effect of pile end soil

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**Abstract:** Based on the torsional plane strain model and tapered fictitious soil pile model, the torsional vibration characteristics of tapered pile considering stress diffusion effect of pile end soil is investigated theoretically. Considering the coupling conditions of pile-soil system, the torsional impedance at the tapered pile head is obtained by means of Laplace transform and impedance transfer method. A parametric study is performed, the rationality of the tapered fictitious soil model is verified and the influence of the cone angle of tapered fictitious soil pile on the torsional vibration characteristics of tapered pile embedded in layered soil is studied. The results show that: the cone angle of tapered fictitious soil pile has obvious effect on the torsional vibration characteristics of tapered pile with the shorter length.

## 1. Introduction

The dynamic interaction between pile and pile end soil is one of the hot issues in the research of pile foundation dynamics, which has attracted the attentions of many researchers and made many achievements. The existing dynamic interaction model of pile-pile end soil system falls into the following three main categories: 1) The simplified dynamic Winkler model [1]; 2) The second model is that the pile bottom boundary can be assumed to be fixed or free[2];3) The third model is that by superimposing virtual rod in elastic half-space medium[3-5]. But none of the above models can reflect the influence of the delamination, the thickness, the construction disturbance and pile-end sediment of the pile end soil. In order to make up for the shortcomings of the above models, Wang Kuihua's team proposed the conception of the fictitious soil pile model, which assumes that the soil layer between the tapered pile and the bedrock as a soil pile. Then, that model was applied to study the settlement characteristics of single pile[6],the longitudinal vibration problem of rigid circular plate[7] and piles[8]. And it was proved that the fictitious soil pile model is a relatively rigorous theoretical model. According to the solution of Mindlin in elastic semi-space, stress diffusion in foundation exists objectively[9]. However, the original fictitious soil pile model ignored the stress diffusion effect in the pile end soil. Then, Wang kuihua et al.[10] assumes that the soil between the pile and bedrock as a tapered soil pile, the cone angle of the tapered soil pile reflects the degree of stress diffusion effect, the larger cone angle of the tapered fictitious soil pile indicates that the stress diffusion effect is obvious. Wang kuihua, Yang dongying et al.[11-12] further expanded the applicable scope of the tapered fictitious soil pile model.

The above researches are mainly aimed at the pile with constant section. In recent years, due to the tapered pile can not only save material but also improve the bearing capacity, the tapered pile has attracted attentions from many researchers all around the world[13-16]. But there is few research on the torsional dynamic characteristics of tapered pile. In this paper, based on the tapered fictitious soil pile model and plane strain model, the torsional vibration solution of tapered pile considering stress diffusion effect of pile end soil is obtained.

## 2. Mathematical model

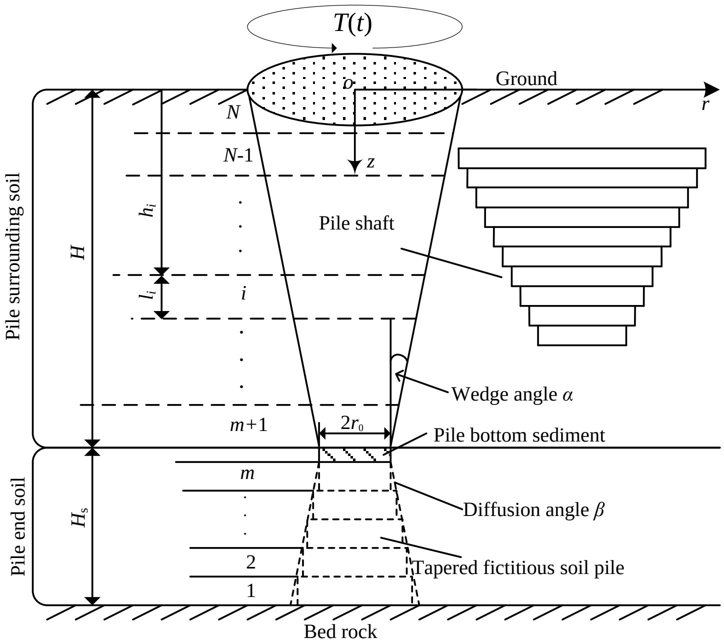


Figure 1. Dynamic interaction model of pile-soil system

Figure 1 shows the schematic of pile soil interaction model, in which the finite soil layers underlying the pile end are assumed to be a tapered soil pile and the cross-sectional area of tapered fictitious soil pile head is same as that of tapered pile end. Allowing for the variation of soil properties, the pile-soil system is divided into a total number of $N$ along pile shaft in the vertical direction, in which the soil underlying the pile end is divided into $m$ layers and the tapered pile is divided into $k$ layers. The total layers are numbered by 1,2,…,$i$,…, $m$ , $m$+1,…, $N$ ($N$=$m$+$k$)from the tapered fictitious soil pile toe to the tapered pile head. The properties of the tapered pile and soil are assumed to be homogeneous in each layer, but may vary from layer to layer. Each layer's shape of pile (including the tapered fictitious soil pile) is assumed to be cylindrical when the number of pile segments is big enough. The thickness of the $i$-th layer is denoted by $l_i$ and the depth of the $i$-th layer top is denoted by $h_i$, respectively. $H$ and $\alpha$ represent the length and the wedge angle of the tapered pile, respectively; the radius at the tapered pile toe is $r_0$. The length and the cone angle of the tapered fictitious soil pile are denoted by $H_s$ and $\beta$, respectively. In the $i$-th pile (including the tapered fictitious soil pile) layer, the radius is denoted by $r_i$ ($r_i=r_0+\tan\beta(h_i-H)(i\le m)$, $r_i= r_0+ \tan\alpha(H-h_{i-1})(m< i\le N)$). The tapered pile head subjected to a torsional loading expressed by $T(t)$.

The following assumptions are adopted in this paper:

1. The pile surrounding soil is layered, isotropic and linear viscoelastic medium. The free surface of soil has no normal stress and shear stress, and the soil is infinite in the radial direction. The bottom of the fictitious soil pile is bedrock and is assumed to be rigid.

2. Only the tangential displacement is considered during the torsional vibration.

3. The pile-soil system is subjected to small deformations, thus the pile and soil remains in perfect contract during the torsional vibration.

4. Tapered pile is a viscoelastic, vertical, inverse cone bar.

## 3. Dynamic equations

### *3.1 Torsional dynamic equation of pile surrounding soil*

According to the torsional plane strain model proposed by Naggar et al.[17]. The tangential displacement of the soil at any point in the *i*-th soil layer is denoted by $U_i$. The dynamic equation of pile surrounding soil can be expressed as:

$$r^2\frac{\mathrm{d}^2U_i(r)}{\mathrm{d}r^2}+r\frac{\mathrm{d}U_i(r)}{\mathrm{d}r}-(\eta_i^2r^2+1)U_i(r)=0\ (i=1,2,3\ldots,N) \tag{1}$$

in which, $\eta_i = j\omega/(v_{si}\sqrt{1+jD_{si}})$; $v_{si}=\sqrt{G_{si}/\rho_{si}}$, $\rho_{si}$, $D_{si}$, $G_{si}$ are the shear wave velocity, mass density, material damping coefficient and shear modulus of pile(including the tapered fictitious soil pile) surrounding soil, respectively. $\omega=2\pi f$ is the angular frequency, $f$ is the natural frequency and $j=\sqrt{-1}$ is imaginary unit.

### *3.2 Torsional dynamic equation of tapered pile*

In the *i*-th pile(including the tapered fictitious soil pile) layer, the twist angle of pile(including the tapered fictitious soil pile) is denoted by $\varphi_i(z,t)$. Based on line viscoelastic assumption, the dynamic equilibrium equation for the *i*-th pile(including the tapered fictitious soil pile) layer can be written as:

$$G_iI_{\mathrm{p}i}\frac{\partial^2\varphi_i(z,t)}{\partial z^2}+\delta_i\frac{\partial^3\varphi_i(z,t)}{\partial z^2\partial t}-2\pi r_i^2f_i(z,t)=\rho_iI_{\mathrm{p}i}\frac{\partial^2\varphi_i(z,t)}{\partial t^2}\ (i=1,2,3\ldots,N) \tag{2}$$

where, $G_i$ , $\rho_i$ , $I_{\mathrm{p}i}=\pi r_i^4/2$, $\delta_i$ represent the shear modulus, mass density, polar moment of inertia and material damping coefficient of the *i*-th pile(including the tapered fictitious soil pile) layer, respectively. And $f_i(z,t)$ is the frictional resistance of *i*-th surrounding soil layer acting on the wall of the *i*-th pile (including the tapered fictitious soil pile)layer in unit area.

### *3.3 Boundary and initial conditions of the pile-soil system*

The tangential shear stress for *i*-th soil layer acting on the wall of *i*-th pile(including the tapered fictitious soil pile)layer is expressed by $\tau_i(r_i)$.

(1)The boundary condition at the tapered pile head:

$$\left[G_NI_{\mathrm{p}N}\frac{\partial\varphi_N(z,t)}{\partial z}+\delta_N\frac{\partial^2\varphi_N(z,t)}{\partial z\partial t}\right]\bigg|_{z=0}=-T(t) \tag{3}$$

(2)The boundary condition at the tapered fictitious soil pile end($z=h=H+H_s$):

$$\varphi_1(z,t)\big|_{z=h}=0 \tag{4}$$

(3)The continuity of the stress and displacement at the interface of pile layers(including the tapered fictitious soil pile layers):

$$\begin{cases}\varphi_i(z,t)\big|_{z=h_i}=\varphi_{i+1}(z,t)\big|_{z=h_i}\\ \left[G_iI_{\mathrm{p}i}\frac{\partial\varphi_i(z,t)}{\partial z}+\delta_i\frac{\partial^2\varphi_i(z,t)}{\partial z\partial t}\right]\bigg|_{z=h_i}=\left[G_{i+1}I_{\mathrm{p}i+1}\frac{\partial\varphi_i(z,t)}{\partial z}+\delta_i\frac{\partial^2\varphi_i(z,t)}{\partial z\partial t}\right]\bigg|_{z=h_i}\end{cases} \tag{5}$$

(4)The initial condition of the pile-soil system:

$$\left.\begin{aligned}\varphi_i(z,t)\big|_{t=0}&=0\\ \frac{\partial\varphi_i(z,t)}{\partial t}\bigg|_{t=0}&=0\end{aligned}\right\} \tag{6}$$

(5)The continuity of the stress and displacement at the interface of the soil and pile (including the tapered fictitious soil pile):

$$\begin{cases} U_i(r_i) = \varphi_i(z,t) r_i \\ f_i(z,t) = -\tau_i(r_i) \end{cases} \tag{7}$$

## 4. The solutions of dynamic equations

### *4.1 The solution of the soil equation*

Equation (1) is a Bessel function, and its solution can be written as:

$$U_i(r) = A_i \mathrm{K}_1(\eta_i r) + B_i \mathrm{I}_1(\eta_i r) \quad (i=1,2,3\ldots,N) \tag{8}$$

in which, $\mathrm{I}_1(\eta_i.r)$ and $\mathrm{K}_1(\eta_i.r)$ are the modified Bessel functions of first order of the first and second kinds, respectively. $A_i$ and $B_i$ are arbitrary constants. Based on the assumption that the stresses and displacements of soil approach zero at an infinite radial distance from the tapered pile, equation (8) yields $B_i=0$.

The tangential shear stiffness of the $i$-th soil layer around the $i$-th pile segment can be expressed as:

$$K_{\mathrm{p}i} = \frac{-G_{\mathrm{s}i}^*\left(\frac{\mathrm{d}U_i(r)}{\mathrm{d}r} - \frac{U_i(r)}{r}\right)}{U_i(r_i)} = G_{\mathrm{s}i}^*\eta_i \frac{\mathrm{K}_2(\eta_i r_i)}{\mathrm{K}_1(\eta_i r_i)} \quad (i=1,2,3\ldots,N) \tag{9}$$

where $G_{si}^*=G_{si}(1+jD_{si})$, $\mathrm{K}_2(\eta_i r)$ is the modified Bessel function of second order of the second kind.

Combining equation (7) and equation (9), the resistance $f_i(z,t)$ can be obtained as:

$$f_i(z,t) = -\tau_i(r_i) = K_{\mathrm{p}i} U_i(r_i) = K_{\mathrm{p}i}\varphi_i(z,t) r_i \tag{10}$$

### *4.2 The solution of the tapered pile equation*

Utilizing the initial condition given in equation (6), substituting equation (10) into equation (2) and performing the Laplace transform for equation (2)(two-side) yields:

$$v_{\mathrm{p}i}^2\left(1+\frac{s\delta_i}{G_i I_{\mathrm{p}i}}\right)\frac{\partial^2\phi_i(z,s)}{\partial z^2} - \left(s^2 + \frac{4K_{\mathrm{p}i}}{\rho_i r_i}\right)\phi_i(z,s) = 0 \quad (i=1,2,3\ldots,N) \tag{11}$$

where, $\phi_i(z,s) = \int_0^\infty \varphi_i(z,t)\mathrm{e}^{-st}\mathrm{d}t$ , is the Laplace transform of $\varphi_i(z,t)$ with respect to time $t$, $s$ is the Laplace transform parameter; $v_{\mathrm{p}i}$ is the shear wave velocity for $i$-th pile(including the tapered fictitious soil pile) layer.

The general solution of equation (11) can be acquired as:

$$\phi_i(z,s) = D_{1i}\cos\left(\frac{\lambda_i}{l_i}z\right) + D_{2i}\sin\left(\frac{\lambda_i}{l_i}z\right) \tag{12}$$

where, $\lambda_i = t_i\left[-T_\mathrm{c}^2\left(s^2+\frac{4K_{\mathrm{p}i}}{\rho_i r_i}\right)\Big/\left(1+\frac{s\delta_i}{G_i I_{\mathrm{p}i}}\right)\right]^{1/2}$ , is dimensionless eigenvalue, in which, $T_\mathrm{c} = \sum_{i=m+1}^{N} l_i/v_{\mathrm{p}i}$ is the time of shear wave propagating from the tapered pile head to the tapered pile end and $t_i = l_i/(v_{\mathrm{p}i}T_\mathrm{c})$ is the time of shear wave propagating in the $i$-th pile layer. $D_{1i}$ and $D_{2i}$ are arbitrary constants which can be obtained by boundary conditions.

According to the torsional impedance function defined by Militano & Rajapakse[18], and by virtue of the impedance transfer method which was employed in Wu's research[13], the torsional impedance function at the tapered pile head can be expressed as:

$$Z_N(s)\big|_{z=0} = -\rho_N v_{\mathrm{p}N} I_{\mathrm{p}N}\left(1+\frac{s\delta_N}{I_{\mathrm{p}N}G_N}\right).\frac{\lambda_N}{t_N T_\mathrm{c}}\tan(\lambda_N-\theta_N) \tag{13}$$

where, $\theta_N = \arctan\left(Z_{N-1}(s)t_N T_\mathrm{c}\Big/\left[\rho_N v_{\mathrm{p}N} I_{\mathrm{p}N}\left(1+\frac{s\delta_N}{I_{\mathrm{p}N}G_N}\right)\lambda_N\right]\right)$ .

The real and imaginary parts of $Z_N(s)$ denote the real dynamic stiffness and dynamic damping, respectively. Thus, $Z_N(s)$ can be rewritten as:

$$Z_N(s) = K_\mathrm{k} + jC_\mathrm{k} \tag{14}$$

## 5. Analysis

Referring to the Wang's[10] research, the cone angle of tapered fictitious soil pile can varies from 0° to 45°. When $\beta$=0°, there is no stress diffusion effect in the pile end soil. Therefore, in the subsequent analysis, the cone angle of tapered fictitious soil pile can be set as $\beta$=0°, 15°, 30°, 45°.

Unless otherwise specified, the parameters of the pile-soil system are set as follows: The length, shear wave velocity, mass density and cone angle of tapered pile are 10m, 2200m/s, 2500kg/m$^3$ and 1°, respectively. And the radius of tapered pile end is set as $r_0$=0.5m. The shear wave velocity and mass density of pile surrounding soil are 100m/s and 2000kg/m$^3$, respectively. The height, shear wave velocity, and mass density of pile end soil are 5m, 180m/s and 2000kg/m$^3$, respectively. The material damping coefficient of pile (including the tapered fictitious soil pile) is 0. When the number of tapered pile segments reached 200(i.e. $k$=200 in this paper)[14] and the number of pile end soil layers reached 100(i.e. $m$=100 in this paper) [10], the precision requirements of longitudinal division of the tapered pile and the tapered fictitious soil pile can be fully met, respectively.

### *5.1 Verification of the fictitious soil pile model*

In this section, the rationality of the fictitious soil model was verified by comparing with Xie's solution[16]. The basic parameters of the pile-soil system are the same as the Xie's research, and the value $\beta$ of the present solution in this paper is set as $\beta$=0°. As shown in figure 2, the change trend of the curves of the present solution in this paper are completely consistent with that of Xie's solution, which indicated that the fictitious soil model is suitable for stimulating the torsional dynamic interaction between the tapered pile and the pile end soil.

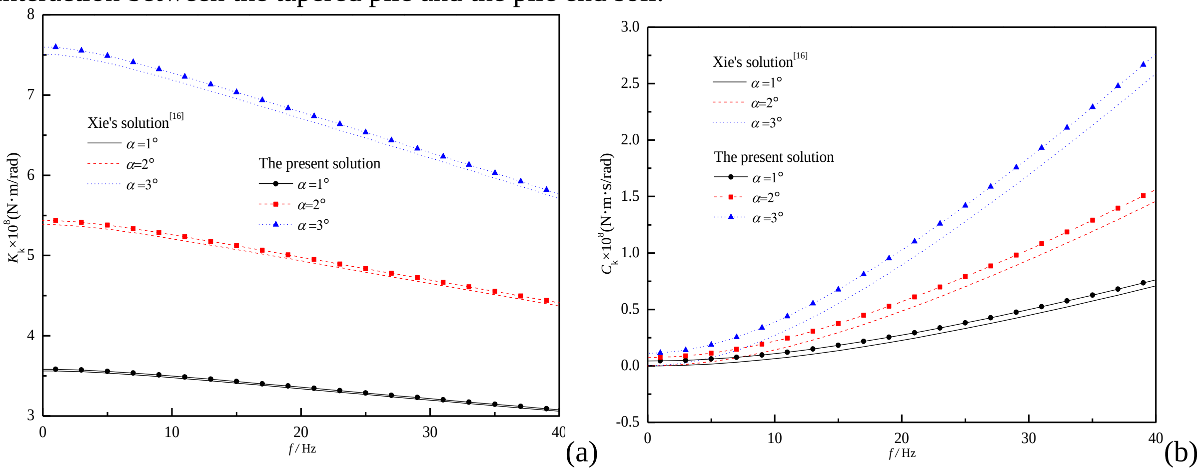


Figure 2. Comparison of the present solution and Xie's solution[16]:(a)Dynamic stiffness curve; (b)Dynamic damping curve

### *5.2 Influence of the cone angle of tapered fictitious soil pile on the torsional vibration characteristics of tapered pile with different length of tapered pile*

As illustrated in figure 3, the influence of the cone angle of tapered fictitious soil pile on the torsional vibration characteristics of tapered pile with different length within the low frequency range considered in dynamic foundation design. And the length of tapered pile is set as $H$=5m, 10m, 15m. It is noted from figure 3 that as the cone angle of the tapered fictitious soil pile increases, the dynamic stiffness of tapered pile head increases and the dynamic damping of tapered pile head decreases, respectively.

It can also be observed that as the length of tapered pile decreases, the influence of the cone angle of tapered fictitious soil pile on the torsional vibration characteristics of tapered pile become more obvious. That phenomenon may be explained as follows: the pile-end soil system is in a state of dynamic equilibrium subjected to dynamic load, the upper dynamic load is balanced by the soil resistances at the tapered pile side and tapered pile end. As the length of the tapered pile increase, the

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soil resistance around the pile shaft increases and the soil resistance at the pile end decreases. When the length of tapered pile is long enough, the upper dynamic load is mainly balanced by soil resistance at the pile side, the resistance at the pile end is small. The results show that the shorter length of tapered pile leads to obviously stress diffusion effect of pile end soil , and there is a critical length of tapered pile, in this paper, $H$=15m. When the length of tapered pile is bigger than the critical value, the stress diffusion effect of pile end soil can be neglected.

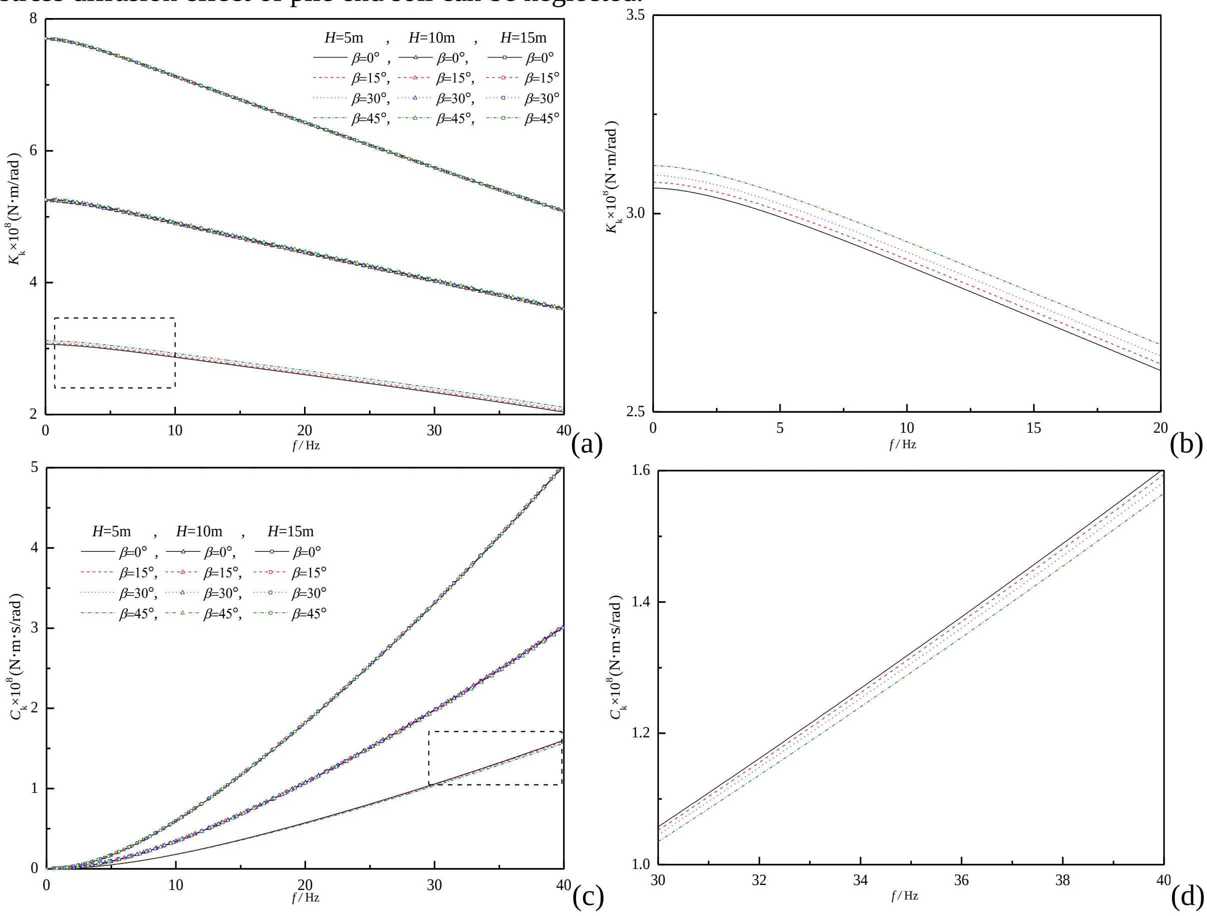


Figure 3. Influence of the cone angle of tapered fictitious soil pile on the torsional vibration characteristics of tapered pile with different length:(a)Dynamic stiffness curve;(b)Local enlargement of the dynamic stiffness curve;(c)Dynamic damping curve;(d)Local enlargement of the dynamic damping curve

## 6. Conclusions

By using the tapered fictitious soil pile model to investigate the stress diffusion effect of tapered pile end soil on the torsional dynamic response of tapered pile in layered soil, the results show that the influence of the cone angle of tapered fictitious soil pile on the torsional impedance of the pile top is obvious. Two main conclusions are obtained which provide theoretical basis for the dynamic design and seismic design of pile foundation.

1. Comparison of the present solution and the existing solution proved that the tapered fictitious soil pile model is suitable for reflecting the stress diffusion effect of pile end soil.

2. There is a critical length of tapered pile, when the length of tapered pile is less than the critical length, as the length of tapered pile decrease, the influence of the cone angle of tapered fictitious soil pile on the torsional vibration characteristics of tapered pile become more obvious. When the length of tapered pile is bigger than the critical value, the stress diffusion effect of pile end soil can be neglected.

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