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SURVEY PAPER

# Research on early childhood mathematics teaching and learning

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**Abstract**
This paper reports an overview of contemporary research on early childhood mathematics teaching and learning presented at recent mathematics education research conferences and papers included in the special issue (2020–4) of *ZDM Mathematics Education*. The research covers the broad spectrum of educational research focusing on different content and methods in teaching and learning mathematics among the youngest children in the educational systems. Particular focus in this paper is directed to what lessons can be drawn from teaching interventions in early childhood, what facilitates children's mathematical learning and development, and what mathematical key concepts can be observed in children. Together, these themes offer a coherent view of the complexity of researching mathematical teaching and learning in early childhood, but the research also brings this field forward by adding new knowledge that extends our understanding of aspects of mathematics education and research in this area, in the dynamic context of early childhood. This knowledge is important for future research and for the development of educational practices.

## 1 Introduction

Early childhood mathematics education is a rich field of study and practice that includes the provision of stimulating activities and learning environments, organized and orchestrated by teachers, care-takers and other professionals with the aim of offering young children experiences that extend their knowledge and development of mathematical concepts and skills. Generally, early childhood mathematics education involves children aged 3–6 years, but in many countries even the youngest toddlers go to early childhood centres. Therefore, contemporary research on early mathematics education focuses on children from birth until they enter formal schooling in the first grade. To develop this field of research, a strong foundation of theory and methodology is necessary, along with consideration of the practical settings of young children's learning as well as the societal needs and relevant educational policy frameworks. Moreover, from a didactical perspective, it also requires consideration of the essence of the mathematics to be taught to young children.

High-quality research grounded in theory is necessary for all areas of mathematics education, in order to move forward and contribute to the generation of new knowledge from which the educational practice can benefit. Since there is much evidence that later development in mathematics is laid in the early years (e.g., Duncan et al. 2007; Krajewski and Schneider 2009; Levine et al. 2010), such high-quality research is especially critical for early childhood mathematics education. Research involving young children entails certain challenges that cannot simply be solved by adopting research designs that are used with older students. The aim of gaining deep knowledge of how young children's mathematical understanding can be fostered places high demands on research methods. As early as 40 years ago, Donaldson (1978) stated that children act differently in their everyday situations than they do in experiment situations, and this has been confirmed by many others since then. Thus, gaining knowledge about teaching and learning mathematics in the early years requires research that is conducted in various learning environments and that acknowledges that these learning environments are complex, multifaceted, and dynamic.

Research in mathematics education is a relatively recent scientific discipline beginning in the last century (Kilpatrick 2014). Investigating young children's mathematical learning and teaching became part of this discipline much later. Early childhood mathematics has long been the research field of

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developmental psychology and cognitive sciences. From the studies of mental abilities and thinking in mathematical problem-solving carried out in these disciplines, we have gained knowledge about the influence of working memory and attention span (e.g., Ashcraft et al. 1992; Passolunghi and Costa 2016; Stipek and Valentino 2015), as well as about the role of innate abilities of numerical awareness in children's mathematical performance (e.g., Butterworth 2005; Wynn 1998). Yet, these studies lack a deeper investigation of the *mathematics* that is performed and how it is developed by children. Neither do such investigations address why certain mathematical competencies are important or why some activities stimulate their development and others do not. Contrary to psychological research, mathematics education research has a didactic perspective, which means that it is linked to the perspective of the learning child, the teaching teacher, and the environment offering learning opportunities in which the teaching and learning take place. Above all, didactic research distinguishes itself from psychological research because it deals explicitly with the question of *what the mathematics is* in early childhood activities, both within and outside formal education.

## 2 A brief overview of the current field of early mathematics education research

As shown by the many publications on teaching and learning of mathematics in early childhood that have been released in the past few years, this area of mathematics education research has increasingly become a mature discipline. The same is reflected by the special interest groups, working groups, and research fora dedicated to mathematics education in the early years. No self-respecting conference today can afford not to pay attention to the area of early mathematics, and there are now also communities and conferences that focus exclusively on early childhood mathematics education. All these communities and conferences are the epicentres where the latest developments in this field are brought together. To set the scene for research on early childhood mathematics teaching and learning, without it being complete, we first provide a brief overview of recently presented and discussed early mathematics education research. As an orientation point for this overview, we used what has recently been presented by researchers at three international meetings.

### 2.1 CERME 11 thematic working group (TWG) on early years mathematics

A conference that already has a considerable track record for including early childhood mathematics as a fixed part of its programme is the biennial conference of the European Society for Research in Mathematics Education (ERME). This conference started in 2009 with a Thematic Working Group (TWG) on Early Years Mathematics. Since then, the number of participants in this group has grown consistently. In 2019, this TWG (that is, TWG13) consisted not only of European researchers but also attracted participants from Canada, Japan, and Malawi. The most dominant theme presented there involved studies of children's emerging number knowledge. Many of these presentations were traditional in design, including giving children tasks that had to be solved both individually when the children were interviewed and when they worked in groups in a classroom setting. Based on these studies, researchers formulated descriptions of the children's knowledge. Sometimes, learning trajectories could be generated from these empirical observations. However, within this TWG several examples of studies with more innovative designs and research settings were also presented, including different modes of exploring and expressing numbers, which can extend our knowledge of early childhood mathematics education. An example of such research is Bjørnebye's (2019) study, in which a dice game including elements of multiple representations and embodiment of counting strategies opened up the possibility of observing how children's actions and responses reflect their understanding. Other studies investigated how affordances of manipulatives and applications encouraged children to develop new ways of thinking about numbers either by working in a digital environment (Bakos and Sinclair 2019) or by using their fingers to represent numbers (Lüken 2019; Björklund and Runesson Kempe 2019).

A characteristic of the research community gathered at CERME11 TWG13 is that the participants generally had in common an interest in better understanding the mathematical thinking of the child. Therefore, it was considered crucial that research establish clues for how to recognize mathematical thinking in the early years. For this purpose, Sprenger and Benz (2019) used eye-tracking data, as this platform was considered to contribute to the analysis of children's perception of structure in the process of determining quantities. Yet, what Sprenger and Benz discovered is that data from technological devices still need to be interpreted, and that other expressions of children's perceptions and reasoning are necessary assets for drawing valid conclusions.

A further important issue that was present at CERME11 TWG13 was related to teaching practice. Specifically, several presentations addressed the questions of how mathematics education should be orchestrated in early childhood education and what opportunities to learn should be offered to children. For example, Breive (2019) investigated the link between inquiry-based education and open-ended problem-solving, and the role of the teacher in orchestrating such conditions for mathematical exploration. In her paper, Breive described the teachers' behaviour in terms of

the degrees of freedom offered to the children with respect to their actions related to the mathematical content and context. Based on the data she collected, Breive concluded that teachers' ways of acting, and the accompanying learning opportunities, should be given more attention within early mathematics education research. Similarly, Vogler (2019), who observed teacher–child group interactions, concluded that so-called indirect learning (which can be found as a common approach in many preschool settings) may induce an obstacle to learning mathematics embedded in activities if there is not a mutual understanding of what learning content is the aim of the activity. In line with these two studies, other researchers who focused on teachers' interactions with children also highlighted critical issues for educational practice and supported further research inquiries.

### 2.2 POEM4

Another source for learning about the latest developments in early childhood mathematics education research is the POEM conferences (Mathematics education perspective on early mathematics learning between the poles of instruction and construction). The latest conference, POEM4, was held in 2018. The presentations published in the conference proceedings (Carlsen et al. 2020) all, in one way or another, reflect the question "In what way—and how much—should children be 'educated' in mathematics before entering primary school?" This was also the recurring question in the discussions between the participating scholars. Among the contributions, three themes stood out: children's mathematical reasoning, early mathematics teaching, and parents' role in children's mathematical development. There was a strong interest in children's reasoning abilities and strategies in problem-solving. For example, Tsamir et al. (2020) investigated how children express their understanding of patterning. For this purpose, the researchers provided preschoolers with patterns to be copied and compared, while observing their strategies. Children's strategy use was also observed in relation to play situations. Bjørnebye and Sigurjonsson (2020) observed them in teacher-led outdoor games, while Lossius and Lundhaug (2020) observed child-initiated play activities. Some researchers used their observations of children's encounters with mathematical content for theoretical discussions on how to understand children's meaning-making, for example by taking the semiotic mediation perspective (e.g., Bartolini Bussi 2020) or through the lens of attentional processes (Verschaffel et al. 2020).

With respect to early mathematics teaching, at POEM4 it was discussed that teachers' educational work largely concerns how to empower children in the learning process, assuming that children have agency in their learning (Radford 2020). Some of the presented studies (e.g., Palmér and Björklund 2020) specifically chose children's perspectives and problematized how seriation was made a content for learning in a children's story. They showed how different manipulatives and tools used in teaching have different implications for what is made possible for the children to learn. A critical but essential notion was expressed by Tzekaki (2020), who underlined that whether children act and think mathematically and learn mathematical concepts depends on what is defined to be mathematical thinking and acting. In line with this perspective, Keuch and Brandt (2020) and Bruns et al. (2020) also raised the issue that teachers' and student teachers' knowledge of mathematics in early childhood education affects their readiness to exploit the content in ways that facilitate children's mathematical learning.

The issue of the knowledge of mathematics in early childhood was also addressed in papers on the role of parents in children's learning of mathematics. Parents are recognized as young children's first educators, contributing to their mathematical understanding and skills. One example of this research focus is Lembrér's (2020) study. In order to know what experiences children bring with them into preschool education and thus might inform their encounter with mathematics, she investigated what parents value in the mathematics activities in which their children are engaged at home.

### 2.3 ICME-13 monograph "Contemporary research and perspectives on early childhood mathematics education"

The ICME-13 Monograph "Contemporary research and perspectives on early childhood mathematics education" (Elia et al. 2018) is the third source for becoming informed about the state of the art in the field of teaching and learning mathematics in early childhood. This book, which has its foundations in the ICME-13 (International Congress on Mathematical Education) Topic Study Group 1 (TSG1) "Early childhood mathematics education" held in 2016, contains chapters on a broad range of topics grouped into five key themes: pattern and structure, number sense, embodied action and context, technology, and early childhood educators' professional issues and education.

Within these themes, the domain-overarching theme of *pattern and structure* played a prominent role. As Mulligan and Mitchelmore (2018) showed in a series of studies, children's awareness of mathematical structures turned out to be crucial for acquiring mathematical competence. Particularly children's structuring skills were found to be critical to developing coherent mathematical concepts and relationships. These findings are in line with Lüken and Kampmann's (2018) intervention study with first graders, in which 5 months of explicit teaching of pattern and structure during regular mathematics lessons resulted in significant

differences between pre- and post-test arithmetic achievement scores in the intervention group. Moreover, the intervention was most beneficial to the low-achieving children.

The research within the theme *number sense* examined a large variety of different aspects of number development. For example, there was a study about children's enumeration skills when making lists for designating and representing collections of objects (Dorier and Coutat 2016). Also, attention was paid to the use of numerical finger gestures and other bodily-based communication in order to facilitate the learning process (Rinvold 2016), children's spontaneous focusing on numerosity (SFON) (Rathé et al. 2018; Bojorque et al. 2018), and the link between writing skills and number development (Adenegan 2016). Furthermore, an exploration of children's ability to operate with numbers revealed that 5-year-olds were able to solve multiplication and division problems when they were presented in familiar contexts (Young-Loveridge and Bicknell 2016).

In the theme *embodied action and context*, Karsli's (2016) video-ethnographic research in a pre-kindergarten classroom showed that young children's hand and body movements hold rich potential for engaging them in mathematics, and underlined the importance of early childhood teachers' attention to the embodied ways in which children engage with mathematics, with potential for creating teachable moments. Other studies investigated children's engagement in the context of play. In Henschen's (2016) study free play was examined, while Nakken et al. (2016) compared free with guided play, of which the latter resulted in the children exhibiting deeper mathematical thinking, and engagement with more specific mathematical concepts. Anderson and Anderson (2018) broadened the scope by investigating children's learning of mathematics in their home environment. Thom's (2018) and Elia's (2018) research on geometrical and spatial thinking in early childhood offered further insights into the crucial role of the body and other semiotic resources (language, drawings, and artefacts) by which young children develop and communicate spatial-geometrical thinking. A general conclusion within this theme was that the limited ways in which young children are invited to engage with geometrical, spatial, and measurement concepts undervalue the embodied, gestural, in-context nature of their mathematical thinking.

The theme *technology* specifically addressed the integration of technology into early childhood mathematics teaching and learning both at school and at home. The focus was mostly on touch-screen tablet-based applications. Because this new technology significantly differs from the traditional physical aid materials, professional development is needed to help educators identify and implement effective uses of these applications. To learn more about the role of the educator (teacher or parent) in the child's interaction with the software, Baccaglini-Frank (2018) carried out an analysis of student-software-teacher relations, revealing how the teacher's goal of helping the children experience success actually limited their development of numerical abilities. The use of technology also opened a window to a new perspective in early childhood mathematics, namely by exposing young children to advanced mathematics such as understanding symmetric transformation (Fletcher and Ginsburg 2016) and dealing with large numbers (also in symbolic form) and ordinality (Sinclair 2018).

In the theme *early childhood educators' professional issues and education*, Cooke and Bruns (2018) provided a comprehensive overview of the various contributions in TSG1, for which they proposed to distinguish conditions at three levels that influence opportunities for young children to develop mathematical understanding and skills. At the macro level, curricula provide a framework (aims, content to learn, and activities) for mathematics teaching and learning in early childhood, with varying views. Several papers mentioned the tensions regarding new curricula and frameworks that may impose mathematical content rather than allowing the child to develop understanding of mathematical concepts through play. At the meso level, with focus on the teachers' competence, all involved papers agreed as to the importance that the teacher possess a fundamental understanding of mathematics as the basis for high-quality early mathematics education. However, different studies used different conceptualizations and instruments to measure teachers' mathematical competence. The micro level refers to the mathematics educational programmes and materials, as well as to the required training for teachers to develop their ability to effectively select and implement such programmes that address children's mathematical needs (Fritz-Stratmann et al. 2016).

In sum, the common themes that stand out from the three international meetings are children's learning through play, and concerns regarding how to apply content-focused teaching, with or without technology. We found that a great deal of the research is on children's mathematical thinking and learning, including two main areas concerning children's emerging number knowledge and children's learning of patterns. It is noteworthy that in both areas, how children perceive structure or how they manifest structuring abilities were analysed in several of the studies. There were also a number of studies that focused on how finger patterns, gestures, or bodily-based communication may facilitate children's learning of numbers.

Children's learning through free or guided play is also a main issue that was discussed. Teachers' guiding interaction with children in play was shown to contribute more to deeper mathematical thinking and engagement with specific mathematical content. How teaching affects children's learning opportunities in preschool was furthermore of great concern in several of the studies. A conclusion drawn from

this research is that teachers' ways of acting and the learning opportunities created for children should be given more attention. In what way, and how much, children should be educated before entering primary school remains a central issue.

## 3 The contributions of this special issue

In this special issue of *ZDM Mathematics Education* (Issue 2020–4), contemporary research on early childhood mathematics teaching and learning is discussed by researchers from all over the world. The initiative emanated from the 42nd PME conference in Umeå, Sweden (July 2018), where we had the opportunity to organize a Research Forum in which researchers involved in the field of early childhood mathematics education gathered to present and discuss theoretical and methodological challenges and outcomes of studies on learning and teaching arithmetic skills in early years (Björklund et al. 2018; Van den Heuvel-Panhuizen 2018). The conclusion of the Research Forum was that early childhood mathematics education research is key, but that more efforts are needed to bring together the state of the art within this field as a foundation for moving early childhood mathematics education research forward. This special issue again provides a window into the contemporary field of research on early childhood mathematics teaching and learning. To discuss what this special issue adds to this field and reflect on the challenges that lie ahead for research on early childhood mathematics education, in the next section we synthesize the themes that emerge from the 15 papers included in this special issue. Each theme highlights the papers' shared knowledge and contributions to research methods. Many papers are related to several themes, but for our discussion we chose those papers that predominantly belong to a particular theme. In total, we identified three recurring themes: the early interventions and their effects, the facilitating factors for learning and development, and the mathematical key concepts that can be observed in children. Together, these themes bring to the fore aspects that are essential for understanding the learning and teaching of mathematics in the early years.

### 3.1 What lessons can be drawn from interventions?

Research shows that children's development of mathematical skills and knowledge is often influenced by socio-economic and curricular factors, and by social interaction in both short- and long-term perspectives (Pruden et al. 2011). Thus, there is a raised awareness of the impact early childhood education may have on reducing differences in conditions for learning and on increasing and securing equal opportunities for a good foundation in learning for all children. Based on their meta study of early mathematics education research, Duncan et al. (2007) stated that early intervention counts and numerous references to the same study indicate that this is an important standpoint in research. Why else indulge in the challenging task of researching learning among the youngest in our education systems, if one does not believe that efforts made through teaching are significant for children's wellbeing and lifelong learning path?

Research on teaching and learning mathematics often shares a common research design in which interventions are implemented (designed, conducted, and the outcomes assessed) with the aim of finding ways to improve teaching practice for the benefit of the learning child, and often to reduce socio-economic inequality. Intervention studies can be objects of research in different ways, focusing on the children's learning outcomes or the teaching practices. Nevertheless, the goal is to enhance learning through improved teaching. In the papers in this special issue we find efforts to implement well-designed interventions, explicitly focusing on how to teach. Some implement and analyse fine-grained differences in (teaching) actions and the effects on children's attention to certain content (Paliwal and Baroody 2020; Mulligan et al. 2020), while others study the effects of attentiveness to children's experiences and knowledge and the related choices of tasks (Clements et al. 2020; Grando and Lopes 2020). Nevertheless, essential to studying intervention success or failure is how learning outcomes are measured and interpreted, which is also an important aspect of early childhood mathematics education research (Li et al. 2020).

How teaching is framed to present mathematical content to young children, in order for it to be meaningful to them, and in order to be attentive to children's experiences and knowledge, is investigated and discussed by Grando and Lopes (2020). Through narratives provided by early childhood teachers, they find insights into how teachers chose to frame the subjects of statistics and probability in ways that engaged children and were responsive to the children's own experiences, rather than using materials provided by textbooks. Unconventional teaching methods whereby teachers turned their mathematics classroom into a space of creative insubordination are discussed in this paper in relation to the opportunities they offer children to become equipped with critical thinking. The authors argue that the specific content—statistics and probability—demands problematizing activities and experimentation with uncertain outcomes of problems in order to develop probabilistic thinking. This study highlights an essential issue in didactical research: that the content to be taught is not indifferent to how the teaching is designed. The study particularly raises concerns that the design of teaching cannot be random but rather has to be linked to the educational environment and the students attending that particular environment. Consequently, the

generalizability of intervention programmes and teaching methods has to be taken into serious consideration if they are to be implemented in different educational settings.

Clements et al. (2020) set out to investigate the efficacy of implementing an intervention programme in which instructions and progression are grounded in a research-based learning trajectory. Even though the programme itself had previously been found to have positive outcomes for preschool children's mathematics learning, the goal of the current study was to investigate how to teach in the most successful way. For this purpose, the authors used the same programme but adapted the choices of the tasks' difficulty level to the children's current knowledge levels. How to teach was then related to what to teach individual children. Results indicate that skipping difficulty levels to shorten the steps to the learning goals was not successful. This thorough investigation of teaching by adapting the complexity of the content to the child's ability to learn best what is intended draws attention to the delicate work of teaching in early childhood education. The study supports child-centred approaches that are sensitive to the individual needs and potential of the child, while simultaneously aiming for the learning goals set by the curriculum.

While Clements et al. investigated the effects of an intervention programme covering broader numerical knowledge, Paliwal and Baroody (2020) aimed to investigate what conditions for learning the cardinality principle are most effective and how subitizing abilities impact on cardinality knowledge achievement. Their efforts were directed towards a fine-grained analysis of how to teach this aspect of the number concept, and what learning processes different approaches elicit in children. What stands out in their study is that they used a highly advanced research design, which allowed them to examine the effects of different ways of directing children's attention to seeing numbers' cardinality. In their paper, they point out the importance of directing children's attention to various ways of seeing numbers' cardinality, as follows: as a constructing act by adding units to get a number; as an act starting from naming the whole set with a counting word and then differentiating the added units by counting; and a third condition, attending only to single units in a counting act. Thus, their intervention was designed with explicit rigour as to what was made possible for the children to experience, and their investigation concerned the learning outcomes of the different conditions. While this attention in Paliwal and Baroody's study to the different conditions can at first glance be considered subtle and far from the instruction children encounter in their mathematics education, the study offers insight into the importance of teachers' awareness of their way of directing children's attention to certain meanings of the content.

In another paper focusing on the effects of an intervention programme, Mulligan et al. (2020) analysed children's written answers to pattern tasks in order to identify differences and changes in their structural awareness. They found a positive effect on the children's development of awareness of mathematical pattern and structure (AMPS), and showed how the levels changed as an effect of a 37-week intervention programme. Mulligan et al. add to the field of early childhood mathematics knowledge of a particular ability (structural awareness), how it can be identified among young children, and also how the ability changes over a prolonged period of time (during an intervention), which may provide insight into what children actually learn while taking part in an intervention programme.

Children's learning is of course at the centre of attention in intervention studies, and Li et al. (2020) pay explicit attention to how to interpret results from a pre- and post-diagnostic test. In their study, Li et al. investigated the development of mathematics problem-solving skills among kindergarteners by analysing their responses to a cognitive diagnostic test. As in most large-scale analyses, it can be shown in quantitative terms how children develop in producing correct answers that indicate growth in knowledge within certain domains that are tested for. However, Li et al. take a step further in their inquiry and illustrate how two children who scored similarly on the cognitive diagnostic test before an intervention had made different progress during the intervention period. Li et al. suggest that the reason for this difference may lie in how children understand and approach tasks, indicating different understanding even though similar answers are produced. Quantitative measures alone do not reveal such differences. The study thus shows the significance of paying attention to how children reason in order to solve a task. Based on their study, Li et al. recommend that children's learning outcomes from participating in interventions be seen in the light of how the effects of interventions are measured, as it is observed that some developed skills do not endure over time and similar outcomes among children may conceal different learning paths.

### 3.2 What facilitates children's learning and development?

Today, it is undisputed that the development of mathematical skills and the teaching of emerging skills in the early years are essential for mathematics education and developmental progress in the long term (Aunio and Niemivirta 2010; Duncan et al. 2007; Krajewski and Schneider 2009). However, in contrast to this perspective, a recent overview of the long-term effects of preschool mathematics education and interventions (Watts et al. 2018) challenges this almost taken-for-granted assumption, as most early interventions have a substantial fadeout effect. Thus, there is a need to revisit our current knowledge of teaching and learning, and scrutinize what seems to make a difference. Some of the

papers in the special issue particularly consider this issue in their efforts to ascertain what facilitates children's mathematical learning and development, and focus on influential aspects found in play settings (Reikerås 2020; Tirosh et al. 2020), verbal communication in teaching practices (Hundeland et al. 2020), and the home numeracy environment (Rathé et al. 2020).

Hundeland et al. (2020) raise the question of how children learn to use and understand the canonical language of mathematics, and study this aspect in terms of mathematical discourses taking place in kindergarten teaching sessions. They take a sociocultural stance (see Vygotsky 1987), seeing communication as the link between internal communication (thinking) and external communication (interaction). Therefore, children's opportunities to contribute ideas and arguments are vital for their (mathematical) learning processes. Earlier research has also shown that care-takers' talk influences not only children's vocabulary but also, for instance, their spatial problem-solving (Pruden et al. 2011). The deeper knowledge that the study by Hundeland et al. (2020) provides regarding the quantity and quality of mathematical talk in which children are involved, offers us better opportunities also to organize supportive and stimulating conditions for knowledge growth.

What differs in the study by Hundeland et al. compared to most others with similar research questions is their focus on the kind of interaction that the mathematical discourse induces, which, based on the chosen sociocultural theoretical framework, should be crucial for positive learning outcomes. However, what they study and compare is the impact on the mathematical discourse that a certain in-service training has. This places mathematics in the spotlight of mathematics education research. While psychological and cognitive research provides us with important knowledge of mental processes and developmental advancement, studies like the one by Hundeland et al. have a clear direction towards understanding, and not least improving, the conditions for children's learning and development, either by implementing teachers' professional development or through curriculum improvements.

It is commonly agreed that young children's learning is often situated in play. In a large-scale observation study, Reikerås (2020) conducted a thorough examination of the kind of play in which toddlers engage, for the purpose of learning how play skills may be related to early mathematical skills. It was found that competencies that allow the child to be active in solitary and parallel play, as well as children's ability to initiate and remain in a play activity, correlated positively with the toddlers' mathematical skills. The kind of play skills that showed the highest correlation with mathematical skills was their competence to interact in play. General social play skills thus seem to have an impact on mathematical learning, but Reikerås' study cannot reveal how these are connected or any causal effects. An effort to better understand the interaction going on in toddlers' play is made by Tirosh et al. (2020), investigating the challenges toddlers may face as they practise one-to-one correspondence in a playful context, and how different individuals participate in the playful mathematical context. Here, interaction and social skills become one issue with an impact on the learning opportunities arising in the play.

In many cases, the messy context of children's play is a methodological challenge. It is not possible to control influencing variables to the same extent as in an experimental design. On the other hand, findings from the messy settings are more likely to bring to the fore aspects that were not anticipated, which raises new questions for research and theory development. Design research supports this kind of knowledge contribution, as several cycles are conducted, each developed based on insights from the previous cycle. These cycles adhere to children's initiatives such as practising one-to-one correspondence in a setting the table task by putting one spoon inside each cup instead of placing one spoon beside each cup (see Tirosh et al. 2020); thus, the child is expressing an understanding of the concept, but is expressing it differently than how the task suggests. This highlights the importance of directing attention to instructions used in research studies, and particularly to the language of mathematics and the spatial aspects of props used in a task, related to the possibilities involved as young children interpret and execute a task.

Children take part in cultural life, where today numerical aspects are an inevitable part of the everyday environment. Nevertheless, there are differences in the extent to which children attend to these aspects, and consequently in how they learn the meaning of numbers, graphical representations of numbers, and how to use numbers. A common assumption is that home numeracy environment is a strong factor (LeFevre et al. 2009; Skwarchuk et al. 2014), which is reflected not least in the abundance of studies regarding socio-cultural background and demographic factors as a pre-cursor for learning progress. Rathé et al. (2020) put the common assumption to the test—that home environment has an influence on children's progress in mathematical development—by comparing young children's tendency to focus spontaneously on numeracy and numerical symbols in their home numeracy environment. Concerning this specific directionality to numbers, which is assumed to have an impact on children's arithmetic skills in later years (see McMullen et al. 2015), based on their study they propose that the home numeracy environment does not seem to have any significant impact.

### 3.3 What mathematical key concepts can be observed in children?

A great deal of research in the field of early childhood mathematics education studies what mathematics children understand and how this understanding evolves. This knowledge is crucial in designing teaching that contributes to more advanced thinking and problem-solving strategies that support conceptual growth. Therefore, children's utterances and how they act are the centre of interest for many researchers. Also, in this special issue, much attention is paid to the mathematical key concepts that can be attributed to children's thinking, resulting in papers addressing children's understanding of similarity in mathematical objects (Palmér and Van Bommel 2020), their understanding and use of structures (Sprenger and Bentz 2020; Kullberg and Björklund 2020), their understanding of the concept of cardinality and ordinality (Askew and Venkat 2020), and the underlying structure of their quantitative competencies (Van den Heuvel-Panhuizen and Elia 2020).

Children's expressions, and how they are allowed to express themselves, are critical for our understanding of the learning of mathematics. Children's problem posing is one aspect that can tell us about their understanding of mathematics (Cai et al. 2015). In the special issue, this is particularly addressed in the paper by Palmér and Van Bommel (2020), who investigated children's understanding of similarity in mathematical objects. They analysed how children themselves created tasks in three-dimensional geometry that were similar to a previous problem-solving task they had worked on. It is suggested that this finding sheds light on the children's interpretation of the specific mathematical features of the original task.

How children perceive structure has been shown to play an important role in how they, for example, determine a number of objects or solve an arithmetic problem (Ellemor-Collins and Wright 2009; Resnick 1983). In line with these earlier studies, Sprenger and Bentz (2020) investigated how 5-year-olds perceive structures in visually presented sets. By having the children determine the number of eggs in a 10-egg box while using an eye-tracking device (and recording the children's utterances and gestures), they were able to analyse the children's gaze when determining the cardinality of the set, and thereby gain insight into the process of perception. The eye-tracking data showed, for example, that many of the children were able to see structures (e.g. $4+1$ or $3+2$) and use them to determine a quantity without having to count all the objects. The authors argue that children's ability to perceive structures in sets and use them to determine cardinality is central for their further arithmetic learning, as how children perceive sets (e.g., as individual objects, as a composite whole, or in structured part-whole relations) affects the strategies they use for solving arithmetic tasks.

Similar ideas are found in the study by Kullberg and Björklund (2020), who studied 5-year-olds' use of finger patterns to structure number relations while solving an arithmetic problem. They identified two major ways of structuring the task: only structuring, and counting and structuring. In the group that both structured using their fingers and counted on some fingers, some ways were found to be more powerful. Children who solved the arithmetic task ($3+\_=8$) by creating a finger pattern of eight raised fingers and simultaneously identifying ('seeing') the missing part (5) on two hands ($3+(2+3)=8$) were more successful in solving arithmetic tasks, even in a later follow-up assessment. It is suggested that a possible reason for this later success is that these children were able to see numbers as parts included in other numbers, which has been found in earlier research (Resnick 1983) to be important for developing arithmetic skills.

Baccaglini-Frank et al. (2020) also argue that the appropriate use of fingers can contribute to developing children's number sense. They studied how 4-year-olds interacted (verbally and using finger patterns) when using the application TouchCounts. The app combines multi-touch with audile, visual, and symbolic representation, and several solution strategies are possible, affording the simultaneous experience of, for example, finger patterns on the screen, with the number both seen and spoken. In their paper the authors emphasize how multimodal affordances may encourage children to use different strategies in response to different tasks, and thus experience a broad range of abilities related to number sense, including both cardinality and ordinality.

Askew and Venkat (2020) examined children's understanding of the concept of cardinality and ordinality in connection with their awareness of additive and multiplicative number relations. To investigate this topic, first graders (6- and 7-year-olds) in South Africa were asked to position the numerals 1–9 on a bounded 0–10 number line. The children were able to do this in the correct order, with the fewest errors at the upper and lower ends of the number range. Furthermore, evidence was found that awareness of ordinality and that of cardinality develop alongside each other. However, the logarithmic scale, predicted in earlier research, which is considered to indicate a multiplicative structuring of number relationships, was not confirmed in the South African data. Instead, when the numerals grew larger the intervals became more stretched out rather than compressed. In fact, the children's responses were closer to the linear model, which is considered to indicate an additive structuring of number relationships. Also, the use of unit sizes that did not take into account the length of the number line, together with the underestimation of the position of 5 on the 0–10 line, offered limited evidence of the children's awareness of the multiplicative structure of the cardinality of numbers. More research is needed to disclose the deep

interconnections between children's understanding of cardinality and ordinality, and their understanding of multiplicative and additive number relations.

Another effort to unravel the complex nature of children's early number understanding was carried out by Van den Heuvel-Panhuizen and Elia (2020), investigating the structure of the quantitative competence repertoire of kindergartners. Based on a literature review, they arrived at a model consisting of two constituent parts: quantification (the ability to connect a number to a given collection of objects) and quantitative reasoning (the ability to think and operate with quantities). Quantification was split up into counting and subitizing, and quantitative reasoning into additive and multiplicative reasoning. Although this model is partly in line with models found in earlier research, it also extends previously developed models by including multiplicative reasoning. Data were collected in the Netherlands and Cyprus. A series of confirmatory factor analyses showed that the hypothesized four-factor model fitted the empirical data of the Netherlands, but not those of Cyprus, which clearly challenges the model's generalizability. A comparison of the component performances in the Dutch sample revealed that, in accordance with other studies, the lowest scores were found for multiplicative reasoning and that the competence of subitizing seems to develop before counting. This was partly confirmed by a statistical implicative analysis at item level. Although this analysis resulted in different implicative chains in the two countries, in both samples the multiplicative reasoning and conceptual subitizing items were found at the top of the chain and the counting and perceptual subitizing items at the end. Also, more research is necessary here, particularly concerning the generalizability of the model to other countries.

## 4 Future directions for research on early mathematics teaching and learning

After the Research Forum at PME42 we concluded that to move early childhood mathematics education research forward, more efforts are needed to bring together the state of the art within this field. Thus, we proposed a special issue on the theme *Research on early childhood mathematics teaching and learning* for the purpose of opening up further discussion and inquiry. In this article, the 15 papers included in the special issue are synthesized and discussed in terms of their contribution to the current field of research in early mathematics teaching and learning along with recent research presented at international mathematics education research conferences. Naturally, these do not cover the worldwide field of research, but they at least give a general idea of the current research interests and challenges.

All the papers in this special issue address aspects of early mathematics education and its underlying theories and research methodologies. They share common interests and challenges concerning how to gain knowledge of the youngest children's mathematical development, and they identify prosperous teaching approaches. Our appeal to researchers participating in the special issue was to cover the broad span of mathematical ideas that are relevant in early childhood education. Nevertheless, we see a strong direction towards research on the learning and teaching of number concepts and basic arithmetic. This is in line with Alpaslan and Erden (2015) review of early mathematics research published in 2000–2013 in high-ranked scientific journals in the field of mathematics education, in which the most frequently reported research topics were number systems and arithmetic. The same trend is also found in the research addressed in the latest meetings of ICME, ERME, and POEM. We believe further research should widen this scope, and consider and investigate mathematical topics that are currently less highlighted. There is a need for deeper insight into what mathematics means to young children, and also how the foundations can be laid for the domains of spatial and geometric thinking and measurement, as well as for the domains of structures and patterns, data handling, problem-solving and mathematical reasoning.

Moving an educational field forward, however, is not solely based in covering a broad field of content. To strengthen the field, we need to scrutinize the research designs and methods that are used and the knowledge that is generated. Here, new technologies may open up opportunities for designing tools for investigating children's competencies. However, this initiative goes beyond choosing digital tools or concrete building blocks; it concerns children's opportunities to express themselves within different environments and make use of tools and manipulatives that may reveal new insights into their competencies and open up for innovative research questions to be posed. What is made available to experience surely has an impact on children's expressions of knowledge. And expressions in both words and gestures are important keys here to interpreting the youngest children's knowledge and skills. We can see this in the recent ICME, ERME, and POEM meetings' presentation of a large variety of research designs and in the papers of this special issue. Many innovative research designs have been developed that allow thorough investigation of children's mathematical competence and understanding. What we see, for example, is that subtle differences in expression (e.g. gaze, finger use, or ways of posing questions) reveal new and important insights for developing knowledge of children's mathematical learning. These innovations in methodology allow for the thorough investigation of key features of learning mathematics that go beyond the broad content areas and highlight how mathematical aspects such as cardinality,

ordinality, and number structure are experienced by children. Several of the papers in the special issue particularly attend to these aspects, and do so by creating and using new methodologies and technologies.

The consensus in the field of early mathematics education, reflected in the papers and conference presentations, is strong concerning the impact of early interventions on children's opportunities to thrive as mathematics learners. From longitudinal studies, we know that early knowledge and skills seem to follow through the child's development; that is, weak mathematical skills in early childhood years are likely to predict weak mathematics performance in later school years (Reikerås and Salomonsen 2019; Hannula-Sormunen et al. 2015). This means that early intervention and knowledge of how to offer all children a good start for their mathematical learning are essential to the field of early childhood mathematics education. However, it cannot be assumed that simply participating in education, whether it is framed as free or guided play or problem-solving, or stimulating interactive environments, will result in successful learning outcomes, even though most interventions *do* have a positive impact and most children develop their knowledge to some extent (Wang et al. 2016). Common research objectives, therefore, concern intervention implementation, and analyses of children's learning outcomes from participating in differently designed activities. These studies are of high importance, as they connect the teaching to the learning and provide insights into what seem to be key aspects in the teaching practice. Nevertheless, researching interventions is delicate work, and it is essential to maintain scientific rigor in the design and analysis. Because early childhood education most often takes place in dynamic settings, the conditions under which children learn vary greatly. This diversity is observed in many studies in which children's engagement in play, both self-initiated and guided, is used as data for analysing their mathematics competencies and learning of mathematics. This phenomenon means that the conditions offered to explore mathematical concepts and principles should be critically examined, along with how learning from interventions is measured and valued. There is a need to determine what works, what seems critical, and what aspects serve as particular challenges. In research, also special attention has to be given to the nature of the teaching practices. What we learn from intervention studies, both those included in the special issue and those in other contemporary research, is the importance of situating research in the current field of knowledge and the context in which the research is conducted. Each study broadens the picture of the teaching–learning relationship, which is by no means one-directional. There are many aspects to consider that potentially influence this relationship, and all of them cannot be included in one study alone.

Early childhood mathematics education research often attends to the opportunities and conditions that are offered for learning. There is no doubt that children's activities and interaction with others, already from an early age, offer many opportunities to learn mathematical concepts and basic principles, but our ability to discern what children actually learn from the mathematical learning environments offered to them places high demands on the interpretation process. How to understand the processes going on in play and interaction, and what impacts the children's learning outcomes—what is made possible to learn—often remains an unsolved issue, as the interaction between teacher and children is dynamic, and particularly as play is multidirectional in nature. Studies of interaction in both formal and informal contexts are nevertheless important, as they are conducted in the complex of social and cultural settings that do influence, through norms and individuals' experiences, what is possible for children to learn.

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