Fault Detection in Dynamic Systems:
From State Estimation to Direct Input Reconstruction Methods

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by

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1. Fault detection and isolation 2. Nonlinear systems 3. Dynamic inversion 4. Geometric system theory

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And the future is certain
Give us time to work it out

For
Moira
Miranda
Montealegre

Preface

Modern technology has increasingly created highly complex dynamical systems where the issues of systems' availability and operational safety have become one of the main problems: dependability and reliability became major concerns in the design of modern technical control systems. In engineering the term *'safety intensive'* is used for denoting and characterizing these systems more closely. The scope of the theory and application of the technical and operational management of safety intensive systems is vast and increasing. This ranges from the most common issues of operational safety such as condition monitoring, fault detection and isolation, fault diagnosis, fault management and fault tolerance to some more general questions of the operation such as the effect of human factor, mentioning just a few. Indeed, faults may have a considerable impact on the achievement of the system's technical goal, on the security of human operators, on the integrity or on the profitability of the system itself, or on the integrity of the system's environment. Practically, for safety intensive applications, every kind of system is concerned: embedded systems (for civil or military applications), or huge complex installations, ranging from domestic systems to transportation, communication, industrial processes. Their design must take into account safety and dependability considerations, for economic, sociologic and human reasons.

Incorporating safety issues into the design process is a rather contradictory problem, however. Introducing dependability considerations into the design and the exploitation of systems has a visible cost, while the production losses, quality losses, performance degradations or even accidents which they would help to avoid are obviously not so apparent. In other words, selling safety is not an easy problem in modern economy where the actors of the economy are mostly interested in maximizing their profits. In order to push decision makers of the economy towards the acceptance of safety regulations, and to encourage a volunteer approach to the consideration of those problems, it is necessary to develop techniques and application methodologies which produce safe and secure systems at affordable prices, and in parallel, to develop analysis and evaluation tools in order to quantify, prove and certify the abovementioned systems performances, that is to say, to increase the confidence measures of the application of the new safety intensive technologies.

Fault detection and isolation (FDI) issues, associated with fault tolerant control, have first been developed in the United States and in the former Soviet Union, mainly in the frame of defense and aerospace applications. Modern approaches of fault detection and isolation credited precursors from the early 70's of a series of C. S. Draper Laboratory Studies at the

MIT, Massachusetts, USA, dealing with different aeronautic and navigation applications. Civil applications have been considered later, and mainly associated with huge programmes like nuclear power plants, chemical and petrochemical processes (following the occurrence of a number of technical disasters and industrial catastrophes). In this second period, European research laboratories have gained a very good expertise and have developed a real leadership in this area, because of the decrease of funding in the US, the political-economic crisis in Russia, and the recognition of the safety topic as a definitely important issue, which led to the funding of dedicated projects by the European Science Foundation (ESF), and of many projects and networks initiated by the European Commission (EC) within the international research programmes FP 4 and FP 5. The research is now open for the FP 6 framework initiatives whose individual projects were launched or were ready to be launched by 2004-2005, and, in April 2005, the EC published its proposals for the forthcoming FP7.

The increasing importance of the topic has been recognized by the creation of specific Technical Committees (TC) by the International Federation of Automatic Control (IFAC), in addition to existing committees in which the dependability considerations were already taken into account like the TC's on *Supervision in Chemical Industries*, and *Components and Instruments*. The most specific TC, aimed at *Fault Detection, Supervision and Safety for Technical Processes*, was created around the activity of the triennial symposia series SAFEPROCESS, which is now supported by approximately 50 members. It is meaningful to note that the first four SAFEPROCESS conferences have been taken place in Europe, including the 4th meeting, which was held in Budapest, Hungary, in the year 2000[1]. SAFEPROCESS 2003 went outside of Europe and was held in Washington D.C., USA. These events summarized perhaps the best recent advances and activities in the area of safety intensive systems.

Another important milestone of the progress was that *security* and *safety* became closely coupled notions within automation and control: security, both in hardware and software technology, has emerged as a high priority in many countries, especially after the shocking day of September 11, 2001, with its significant social and economic implications.

The major objective of the application of a fault detection methodology for dynamic systems is to detect incipient faults and other disturbing patterns of the operation by isolating the failed system components in an attempt to prevent the development of (possibly global) malfunctions of the system liable to cause performance degradation and/or destruction of the equipment. Early detection of component malfunctions plays a fundamental role in advanced corporate management and in predictive maintenance planning.

The most common approach to FDI is the use of hardware redundancy, where measurements from multiple sensors are compared, and the existence of a failure is determined by implementing a voting mechanism. In many situations, however, hardware redundancy may not be possible or desirable, since it imposes a penalty in terms of volume, weight and cost, etc. In other situations, such as with actuators, the access to direct measurements is often not

[1] The author is a member of the SAFEPROCESS TC. He was the chairman of the National Organizing Committee of SAFEPROCESS 2000 and served as the editor of the proceedings volume of the conference featuring more than 200 technical contributions (Pergamon Press, London, ISBN 0 08 043250 6, Vols. 1/2/3).

possible and only indirect measurements may be used to infer the component fault status using an analytical model of the system. A possible method to analytically detect the existence of a failure is to look for anomalies in the plant's output relative to a model-based estimate of that output by producing a residual signal that is zero in normal operation, but non-zero if a particular fault occurs.

Plant models, however, are generally incomplete and inaccurate, moreover, these fault detection and isolation algorithms often assume a particular failure mode. These plant dynamics and failure mode modeling errors can either cause a high false alarm rate, or make it difficult to detect faults. Any detection and isolation test that is designed to overcome the problems associated with these modeling errors has to be robust, *i.e.*, it must be able to distinguish between model uncertainties and failure modes and separate the effects of unmodeled dynamics or uncertain knowledge of the system parameters in order to avoid excessive false alarms or missed detections.

A possible approach to robust detection is based on the use of models, which describe the behavior of the plant more precisely. This often leads to varying structure, time dependent or nonlinear models the successful treatment of which depends on the development of new, more complex theories. The questions of robustness and sensitivity of the detection process, and the demand for new algorithms imposed by nonlinear problems are present implicitly in many approaches of the recent research. Robust FDI can be achieved *e.g.*, by assigning (decoupling) the fault effects and similarly the disturbances into disjoint subspaces in the detector output space, if possible. In many case, when this is not possible, approximate decoupling (in contrast to exact decoupling) is to be used. Approximate decoupling has to trade-off the amplification rate of fault effects and disturbances at the filter output. This problem usually leads to the theory of optimal filtering.

The author of this thesis volume has been involved in research and development of safety intensive systems for more than a decade. He took part in a number of industrial projects in the nuclear industry with activity of safety assessment, safety and reliability management, dependability analysis, dependable system installations and design. He is the author and co-author of a number of research papers in the field of fault detection and diagnosis.

The basic purpose of editing this volume together was to give a summary on the results accomplished by the author during the past ten years of research and characterize the achievements which have been done in his post-doctorial work period. Another objective of this work is to enable the reader to follow the new developments in this area. It presents some fundamentals and advances of the theory and design of FDI methods, with special emphasis on the use of detection filters. The interested reader with no prior background in the problems of filtering and detection theory has many introductory textbooks to choose from. For a historical reference the reader is directed to the succession of tutorials and survey papers in the constantly expanding literature.

In the past 15 years most of the work in FDI has been devoted to the generation of residuals in a framework referred to as analytical redundancy. While residual generation methods have been proposed in the literature in a great variety, two important tendencies of development may clearly be recognized: a certain convergence of the approaches applied to linear systems and the attempt to the extension of linear results to nonlinear problems.

In this work we attempt to provide a brief summary on the general course of this development which lead researchers from the late 70's to the most recent results of fault detection and isolation. In the framework of the theory of detection filters, we bring together two of the most exciting approaches of fault detection and diagnosis. The first involves residual generation based on state estimation, the second is the subject of inversion-based direct input reconstruction: methods, both rooted in classical control and mathematical system theory that have considerable potential in a range of future applications. This treatment may well demonstrate the methodological convergence seen in linear approaches. We also provide the connection of ideas between the two areas demonstrating the progress from state estimation techniques to direct input reconstruction methods.

The discussion of the problems develops in two surfaces: the fields of linear and nonlinear problems. Though the unified treatment of linear and nonlinear approaches would be very fortunate, the generalization of a linear approach to the nonlinear case cannot be done in every situation. While the solution of a robust estimation problem, for example, is crucial to solving the robust fault detection problem effectively, traditional fault estimation techniques may not be applied to nonlinear problems with an ease. There are methods, however, originated in traditional state estimation techniques that provide extensibility of linear concepts to the nonlinear domain much easier: the system inversion based direct input reconstruction method not only demonstrates the existing analogies in between traditional linear approaches but also links together linear and nonlinear worlds in a straightforward way.

The presentation is organized as follows. In order to provide insight into the recent ideas one always need to look at the past, review the links between the main types of algorithms and have a view of their historical origins. Therefore, the historical outlook will be present in each of the chapters.

The thesis volume consists of four parts. The first part giving *foundations and outlook* of the presented approaches familiarizes the reader with the basic definitions, properties and typical problems of fault detection and isolation. From the aforementioned reasons, in Chapter 1 a general system setup for modeling dynamical systems subject to faults and uncertainties is given. From the broad field of the theory of residual generation the parity space and the state estimation-based techniques are selected for comparison to review traditional (linear) residual generation methods. Special care is taken to provide a special view on the congruencies of these approaches.

The second part consists of three chapters. First, a geometric interpretation of the residual is given in Chapter 2, which is supported by some background knowledge of geometrical system theory quoted from the literature. Chapter 3 reviews a range of optimization-based approaches by using the standard problem formulation known from robust control, summarizing the role of state estimation approaches in robust fault detection and isolation. This chapter makes an effort to highlight distinctions as well as similarities between different well-known residual generation approaches, such as the Kalman filter, the optimal H_∞ filter and the various likelihood ratio tests. Though most of the detection approaches presented are based on a deterministic setting, this chapter reviews a range of ideas regarding a stochastic setting giving different conditions of robustness when statistical properties of the disturbances are available.

As a direct response on the previous chapter, Chapter 4 continues with the discussion of optimal game theoretic filters and illustrates the usefulness of these approaches to robust residual generation. It adds a little advance to the theory of traditional H_∞ detection filters by conditioning the optimization problem with scaling, all this in the framework of a deterministic problem formulation.

The third part deals with a completely new idea of residual generation which was given the title *direct input reconstruction* for fault detection. This part consists of three chapters again: first the original idea of the inversion-based input reconstruction method is introduced in an algebraic approximation in Chapter 5 for linear, and in Chapter 6 for nonlinear systems. Chapter 7 highlights the same idea in view of a unified geometrical approach for both linear and nonlinear systems. In the course of the discussion, special attention is devoted to the concept how these methods could be extended from linear to nonlinear systems and under what conditions.

The parallelism between the linear and nonlinear approaches and the (sometimes hidden) interdependencies between the inversion-based and other traditional residual generation approaches are always kept in front thus completing one of the basic objective of this work, *i.e.*, to show the inherent congruences and interdependencies of the often sparsely interrelated approaches. One of the most important such parallelism is revealed between the parity space and inversion-based detection approaches in Chapter 7.

To conclude the work and characterize the basic theoretical achievements given in this thesis, a case study is presented in Chapter 8 to demonstrate the effectiveness of the direct input reconstruction (inversion) method to fault detection and isolation. Quite unexpectedly, this case study serves not only for the purpose of an application example but it provides a framework of presentation upon which new results are built up. Based on various solution methods of the robust fault estimation problem represented by this real application example it is shown, how novel approaches to old problems may lead to new solution alternatives by demonstrating that advanced methods of filtering such as inversion-based residual generation and H_∞ optimal filtering and the novel combination of them may contribute to the solution of earlier not solvable problems: It is shown how the state estimate of the inverse dynamics could be obtained *on-line* by using conventional Luenberger-type state observers as well as H_∞ optimal filters, thus simplifying the filter design procedure, significantly.

We note that it *was not* (and it *could not*) be possible to include *all* the most recent developments in this area. Our story is only one of many, and other contributors all have at least as interesting, if not more interesting stories. New ideas in the field of fault detection and identification are not lacking, and we have suggested just a few during the last decade.

Budapest, September 2005 ANDRÁS EDELMAYER

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A journey is always easier when you travel together. Interdependence is certainly more valuable than independence. This volume is the result of ten years work whereby the author has been accompanied and supported by many people. It is a pleasant aspect that he has now the opportunity to express his sincere gratitude for all of them. Systems and Control Laboratory (SCL) of Computer and Automation Research Institute of the Hungarian Academy of Sciences (MTA-SZTAKI) headed by Prof. József Bokor was the first research site which, recognizing both theoretical and practical significance of fault detection and diagnosis, initiated research and development in this field in Hungary. In the course of the years, the laboratory has became one of the most acknowledged centers of excellence in the scientific world in this area. It was the author's pleasure to be able to join this activity at a very early stage and continue this work from the late 80's belonging to the scientific community of the laboratory. Special thanks to MTA-SZTAKI which provided excellent conditions of research which helped to accomplish this work in the defined quality. The author is deeply indebted to Prof. József Bokor whose help, stimulating suggestions and encouragement aided in all the time of the research. He was the first who put the idea forward and made the proposal for the realization of this project which ended with the preparation of this thesis volume. The author would like to express his sincere gratitude to Prof. Tibor Vámos and Prof. László Keviczky academicians and Prof. Péter Várlaki D.Sc. for their careful reading of the manuscript and for the many useful comments which allowed to improve the style of the presentation. He also expresses his gratitude to all members of SCL who substantially contributed to the development of this work. Special thanks to Dr. Zoltán Szabó and Dr. László Nádai for their fruitful discussions and valuable suggestions that considerably improved the content and presentation quality of this work. The author is grateful to Prof. Ferenc Szigeti for his unselfish help providing the sophisticated math background and rendering the sense and value of his unconditional friendship. It is also to be noted that the research has been supported by various grants and organizations. Special thanks to the *Hungarian Scientific Research Fund* (OTKA) and the *National Research and Development Foundation* (NKFP) which supported projects between 1996 and 2004.[2]

[2] *"Analysis and Synthesis of Robust Detection Filter Design in Uncertain Dynamical Systems"*, OTKA Grant No: *T 019448*, 1996–1998, *"Advanced Analytical Methods to Fault Detection and Isolation: Synthesis of Robustness and Sensitivity in Dependable Systems Applications"*, OTKA Grant No: *T 032408*, 2000–2002, *"COSMOS: Computerized operation support for the management, optimization and surveillance of large-scale industrial processes"*, NKFP-2/016/2001, 2001–2004.

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PART 1

FOUNDATIONS AND OUTLOOK

*With the ever increasing safety and security requirements of advanced technical systems there has been significant research activity for the enhancement of safety and reliability and, as a closely related field, in fault detection and diagnosis in the past fifteen years. Most of the efforts have been devoted to methods relying on the idea of analytical redundancy and the generation of residuals based on the knowledge of the mathematical model of the system. While residual generation methods have been proposed in the literature in great variety, two important tendencies of development may clearly be recognized in the past years: a methodological convergence of the approaches applied to linear systems and the attempt to the extension of linear results to nonlinear problems (*Gertler, 2002*). In order to provide insight into the recent ideas one need to look at the past, review the links between the main types of algorithms and have a view of their historical origins. Therefore, as an introduction, the basic definitions, properties and typical problems of model-based fault detection and isolation is presented and a general system setup for modeling dynamical systems subject to faults and modeling uncertainties is given. Special emphasis is taken on parity relations and diagnostic observers. This, from the one hand, may well demonstrate the methodological convergence could be seen in linear approaches and, from the other hand, provides the theoretical basis to further discussion.*

CHAPTER 1

Model-Based Detection and Isolation of Faults in Dynamical Systems

The basic objective of a fault detection methodology applied to dynamic systems is to provide techniques for detection and isolation of failed components. The most obvious method for automatic fault detection is the use of hardware redundancy, where measurements from multiple sensors are compared with each other and the existence of a failure is determined by implementing a voting mechanism. In many situations, however, the application of hardware redundancy may not be possible or desirable, since it imposes a penalty in terms of volume, weight and costs etc. In other situations, such as with actuators, direct access to certain variables is often not possible via physical measurements. In these cases, indirect measurements may be used to infer the component fault status using a mathematical *model* of the system. Most of the model-based methods rely on the idea of analytical redundancy in which, — in contrast to physical or hardware redundancy, — real physical measurements are completed with analytically computed redundant variables.

One method to analytically detect the existence of a failure is to look for anomalies in the plant's output relative to a model-based estimate of that output. Analytical redundancy takes two basic forms such as direct and temporal redundancy. Direct redundancy exist among relationship of instantaneous sensor measurements. Temporal redundancy is based on relationship of dissimilar sensor measurements provided at different times and relates sensor outputs and actuator inputs. For an extensive discussion of the idea see (Chow and Willsky, 1984). In the following discussions we only consider temporal redundancy relations in dynamical systems.

Plant models, however, are generally incomplete and inaccurate. Moreover, fault detection and isolation methods often assume a particular failure mode. The plant dynamics and failure mode modeling errors can either cause high false alarm rates, or make it difficult to detect the failures. Any detection and isolation test that is designed to overcome the problems associated with modeling errors must be able to distinguish between model uncertainties and failures in order to avoid excessive false alarms or missed detections. The robustness and sensitivity issue of fault detection is in the focus of the research, worldwide.

One possible approach to robustness relies on the use of models that describe the behavior of the plant more precisely. The use of nonlinear system models, however, may lead to difficulties in real life implementation. The underlying problem with these methods is that most of the established standard results of linear system theory must be relinquished, even though they comprise the basis of our understanding of dynamical systems. But, perhaps more importantly, nonlinear problems are often not tractable from a computational point of view. The challenge is therefore not only in the development of efficient failure detection methods in theory, but also in ascertaining they are computationally efficient and robust with respect to model uncertainties, unavoidable system variations and nonlinearities.

The contents of this chapter is as follows. In the introduction we briefly review the system of principles developed by the discipline of fault detection and isolation in the last two decades. We start with reviewing the different types of tasks and layers in the field of fault diagnosis and continue with a general system setup, including the basics of systems and fault modeling. The basic concepts of the modeling of nonlinear systems and the approaches used dealing with nonlinear systems is summarized.

Both direct and indirect approaches of the general concept of analytical redundancy is reviewed. The parity or consistency equations method is the *direct* implementation of the concept of analytical redundancy as it uses and manipulates the measurement variables directly. The traditional methods used for the implementation of residual generators in an *indirect* way are usually based on the error dynamics of a state observer. These approaches are used in a number of situations differing in the assumptions on noise, disturbances, robustness properties and in the specific design methods. For comparison, see representations in the literature such as (Mangoubi, 1998; Mangoubi and Edelmayer, 2000) and (Gertler, 1997).

An interesting relationship between parity space and observer-based approaches which can be revealed through the close analysis of these approaches are shown as a conclusion of this introductory part of this thesis.

1.1. Introduction

The development of model based diagnostic systems is aimed at detecting incipient faults, and following permanently the state of the supervised process on the basis of some *a priori* information of the plant dynamics. This *a priori* information is captured in the form of a mathematical representation which is called model.

Technically, a fault diagnostic system typically consists of three basic parts: a residual generator, a residual evaluation module, and a decision logic, see Fig 1.1. A usual residual generator takes the actuator commands and the measured outputs of the supervised system as inputs, and it returns a signal (or a set of signals) that is called residual. In the absence of faults in the supervised system, the nominal value of the residual is theoretically zero, after the transient due to initial conditions has vanished. However, it becomes significantly different from zero when a particular fault occurs.

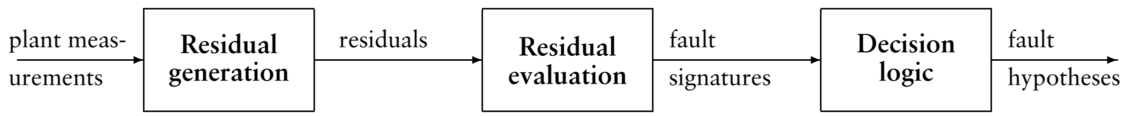

Figure 1.1. Computational stages of fault detection and diagnosis

The residual evaluation module has to detect, using adequate tests, when a given residual is indeed distinguishably different from zero. Finally the decision logic analyzes the result of the evaluation of a set of residuals, and, from the pattern of triggered and non-triggered tests, it returns a decision as to which component is faulty in the supervised system. The determination of the defective component is called fault identification or fault isolation, hence the name fault detection and isolation (FDI). In this work, we mainly focus on the design problems relating to the first part of the detection process, i.e., residual generators.

The system of principles used by the discipline of fault detection and isolation has been well categorized in the survey paper by (Willsky, 1976). It has been pointed out that there exist three different types of tasks or layers in the area of fault diagnosis, such as (1) fault detection, (2) fault identification, and (3) fault signal estimation. *Fault detection* consists of designing a residual generator that produces a residual signal enabling one to make a binary decision as to whether a fault occurred or not. *Fault identification* imposes a stronger requirement: when one or more faults occur, the residual signal must enable us not only to detect that there are faults occurring in the system, but it must also enable us to identify (isolate) which faults have occurred at which time. In certain cases providing information about the real magnitude of the fault signal is required: *fault signal estimation* is the determination of the extent of the failure. The latter is done by trying to reconstruct the fault signals. The three tasks have been considered in a large number of books and papers, see *e.g.,* the textbooks of (Basseville and Nikiforov, 1993; Chen and Patton, 1998; Gertler, 1998), and the references therein.

There are several methods used for generating residuals. Classical approaches to fault detection place emphasis on the use of a more or less accurate model of a linear time invariant (LTI) system and the whole problem of modeling uncertainty and its impact on the detection process is usually ignored. Actually, the physical parameters of a real dynamical system are rarely stay invariant as the time varies. Parameter variations and internal fluctuations are inherent dynamical phenomena of physical systems. Moreover, the systems are in contact with a complex, unpredictable environment causing them to change their behavior in time. They are subject to disturbances and our observation is always corrupted by some unpredictable measurement noise. Since most systems are inherently nonlinear by their nature, the use of linear models results in modeling uncertainties due to neglected high order terms in the Taylor series expansion of the nonlinear description of the plant.

To reflect our imprecise or partial knowledge of that unpredictable behavior it is desirable to think of it as inherently uncertain. As a result, it becomes practically impossible to detect any changes with unlimited sensitivity in the practice. Namely, the consequence of the uncertainties is that actual measurements will never match the estimated values and residuals will thus be nonzero even when there are any deviations in the system. Such kind of parasitic residual variations may cause non-desirable false alarms which may undermine the reliability and dependability of the detection system in the practice.

A fundamental requirement of FDI methods implemented in process environments is to accomplish performance objectives in the presence of modeling uncertainties and uncertain measurement data. The major concern is detection performance, *i.e.*, the ability to detect and identify faults promptly with minimal delays and false alarms even in the presence of environmental disturbances, unavoidable variations of system parameters when the mathematical model of the system is imperfectly known. It has been recognized early that feasibility of FDI methods requires satisfactory robustness with respect to the effects of model uncertainties whose impact on the detection process cannot be ignored.

Another class of problems where FDI designs might lead to false alarms or missed detection, are those that are subject to substantial *unknown* nonlinear dynamics. Even for a process with a *known* nonlinearity, most FDI design methods lead to a situation with large probabilities of false alarms, simply due to the fact that they rely on linear methods and, hence, erroneously tend to detect the nonlinear effects as faults. Nonlinear FDI detectors has until the late 90's only been considered in rather few papers in spite of the tremendous problems caused by nonlinear phenomena. Nonlinearities in connection with FDI has shortly been discussed in (Frank, 1990) and in (Patton and Chen, 1996). Lately, however, there has been increased interest in this issue, for a short summary see e.g. (Frank et al., 2000).

Ensuring robustness is one of the most exciting problems in research and development of FDI systems. The first reasonably effective results in this area – in parallel with the latest results of the new normative approaches of robust control theory – have only emerged quite lately. The earliest results concerning robustness, such as e.g., the robust diagnostic observer scheme with respect to structured LTI system perturbations appeared, see e.g., (Olin and Rizzoni, 1991). Reference (Douglas and Speyer, 1996) studied robustness issues of the FDI problem in the framework of the original detection filter idea.

There have been a number of papers on the disturbance decoupled estimation problem, or, what amounts to the same thing, the unknown input observer scheme, see e.g., (Ding and Frank, 1991), moreover, the robust eigenstructure assignment approach of (Patton and Chen, 1991), all of which taking the stand of perfect disturbance decoupling in LTI systems. Although there is extensive research in the field, this area has a substantial potential for both academic research and engineering development. Yet, there is an important literature on identification methods for faults detection and isolation (Isermann, 1993; Isermann, 1984; Basseville and Nikiforov, 1993) that tackles specifically multiplicative type faults and has been used with success on several applications.

One possible route towards improving robustness, on the one hand, consists in using models which describe the behaviour of the plant more precisely. This often leads to the area of varying structure, linear time dependent as well as bilinear and nonlinear uncertain systems whose successful treatment depends on the developments of new models and new theories for these models. On the other, the use of nonlinear system models may lead us to a dangerous area. The basic problem with these approaches is that we have to abandon most of the well elaborated standard results of linear system theory which form the basis of our understanding on the behavior of dynamical systems. But, perhaps more importantly, nonlinear problems may generate models which are computationally untractable. Therefore the challenge is not only in the development of efficient failure detection methods performing well in the theory, but also in making these methods computable and robust with respect to modeling uncertainties, unavoidable system variations and nonlinearities. Our purpose is, therefore, to study questions regarding the effects of nonlinearities from this point of view.

Perhaps needless to say, there is a personal bias in the approaches we discuss in the next chapters. First of all, we concentrate on the use of advanced algebraic methods and geometric concepts of linear and nonlinear system theory which, according to our view, may contribute in significant ways to the development of new results in this field as well as avoiding computational difficulties.

1.2. SETUP AND PROBLEM FORMULATION FOR LINEAR SYSTEMS

Consider the overall dynamical system as illustrated in Fig. 1.2 consisting of actuators, sensors and the main system components as usual. Actuators are driven by the input signals $u(t)$ while observation signals $y(t)$ are provided by the array of sensors. Malfunctions may occur either in the actuator and sensor dynamics as well as in the components of the system. The malfunctions can be treated separately and they enter the model as actuator, sensor or component failures. Here we will consider methods developed for dynamic models in which the faults appear in the system as additive terms. This assumption is not very restrictive, as various type of faults, such as parameter changes or sensor failures, can be converted into additive type faults (with some non-negligible implications), for the proof of this proposition, see (Edelmayer, 1994). One of these possible implications is that even in time invariant systems, the coefficient of such faults is time varying, and another is that in case of handling a group of parameter changes this way,

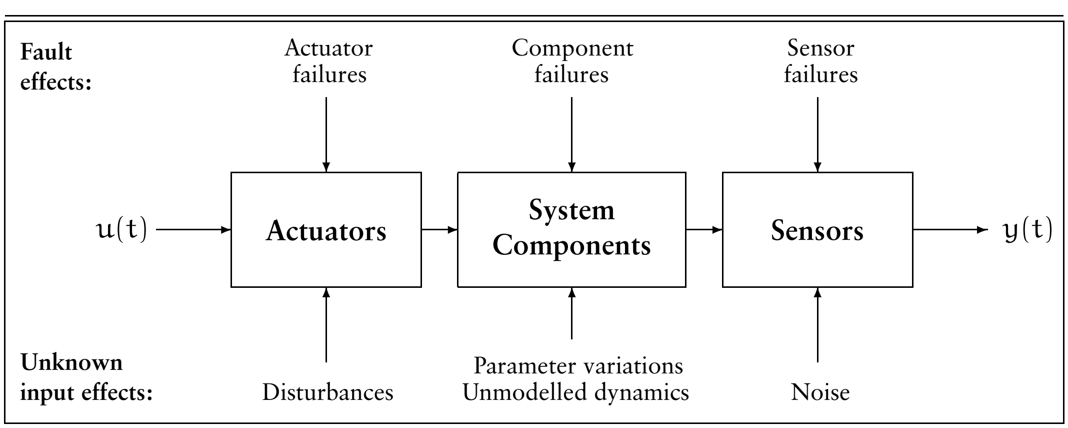

Figure 1.2. Characterization of the system in terms of faults and unknown inputs

it is possible to end up with ”equivalent” disturbances whose entry direction is colinear with most additive faults.

In normal conditions, the control literature means the term *system* which contains both the plant and the feedback controller at the same time (for a schematic representation see Fig 1.3). Fault detection problems can be solved by using both closed-loop and open-loop methods. Closed-loop methods consider the presence of the controller while open-loop methods are concerned with the problems without taking care of how the control signal is calculated.

We just note that if we work with a nominal system model in which modeling uncertainty and external disturbances are not considered, there is in principle no difference between open loop and closed loop detection methods. In the case of taking uncertainties into consideration, however, the good sensitivity performance of the fault detection method and the good performance of the closed loop control system operation is always compromised by each other.

Throughout the discussion of this paper we focus on open-loop detection and on modeling methods which does not incorporate the controller. It is always assumed that the state space description of the system is given by the nominal system matrices A, B, C, moreover, that the directions of the particular failures are known, *i.e.*, the possible distribution (structure) of faults is known in advance from fault modeling. Obviously, inevitable modeling uncertainty arises due to external disturbances, sensor noise, internal system fluctuations, parameter variations and unmodeled system dynamics.

The uncertainty factors can be characterized as *unknown inputs* acting on the system. Their effect is described by perturbation techniques in the nominal system model. The choice of characterizing uncertainty depends highly on the purpose of modeling which may be varied from application to application. In fact, this is the factor what makes distinctive differences in the sequence of modeling approaches in this work. A general system setup, which can be applied

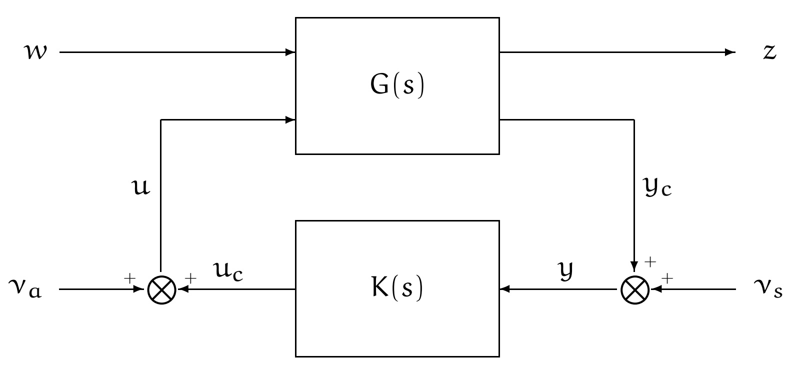

Figure 1.3. Linear control system with actuator and sensor faults, ν_a and ν_s, respectively. G(s) is the system, K(s) is the controller.

in connection with fault detection and isolation for systems with model uncertainty, fault diagnosis for systems with parametric system uncertainty and fault diagnosis for nonlinear systems is given in the following. This general setup is considered throughout the whole discussion of this volume consistently, with obvious omissions and changes in the notations depending on the specialities of the problem to be solved.

Consider the representation given in Figure 1.4, which is an extension of the setup shown in Figure 1.3, but without a feedback controller included.

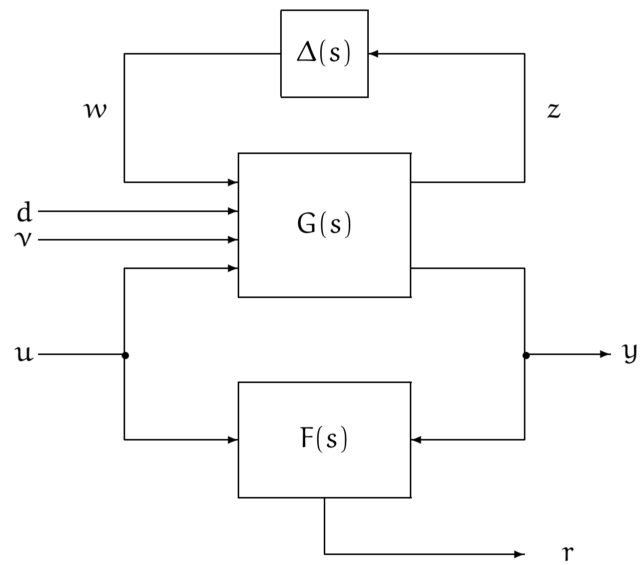

Figure 1.4. General setup for robust fault detection in open loop. $G(s)$ *is the system,* $F(s)$ *is the detector,* $\Delta(s)$ *is the uncertainty description and* r *is the residual.*

The system G in Figure 1.4 has the following state space realization:

$$\begin{pmatrix} \dot{x} \\ z \\ y \end{pmatrix} = \left(\begin{array}{c|cccc} A & B_w & B_v & B_d & B \\ \hline C_z & D_{zw} & D_{zd} & D_{zv} & D_{zu} \\ C_y & D_{yw} & D_{yd} & D_{yv} & D_{yu} \end{array}\right) \begin{pmatrix} x \\ w \\ d \\ v \\ u \end{pmatrix}, \tag{1.1}$$

where $x \in \mathbb{R}^n$ is the state vector, $z \in \mathbb{R}^q$ and $y \in \mathbb{R}^m$ are the external output signal and the measurement output signal, respectively. The maps $A : \mathcal{X} \to \mathcal{X}$, $B : \mathcal{U} \to \mathcal{X}$, are fixed throughout and will be associated with the nominal representation of the dynamical system described by the triple (A, B, C) (assuming $D = 0$ in the cases when generality is not lost). Equivalently, the system G can also be given by its transfer functions

$$\begin{pmatrix} z \\ y \end{pmatrix} = \begin{pmatrix} G_{zw} & G_{zd} & G_{zv} & G_{zu} \\ G_{yw} & G_{yd} & G_{yv} & G_{yu} \end{pmatrix} \begin{pmatrix} w \\ d \\ v \\ u \end{pmatrix}. \tag{1.2}$$

The inputs are external input $w \in \mathbb{R}^r$ from the uncertain block Δ, disturbance input $d \in \mathbb{R}^s$, fault input signal $v \in \mathbb{R}^k$ and the control input signal $u \in \mathbb{R}^p$, respectively. Further, it is assumed that all other relevant weight matrices are included in G. The connection between the external output z and the external input w is given by the relation $w = \Delta z$.

The nominal system output $y(t)$ and input $u(t)$ are always assumed to be available through measurements and will be referred to as observables of the system. The vector valued function $\nu(t)$ is an arbitrary and unknown function of the time and is called failure mode of the system. Note that by this definition of the failure mode we do not constrain $\nu(t)$ to any special function class, therefore, a wide variety of faults can be modeled by this representation.

The general system setup given in (1.1) and (1.2) above describes a large class of different fault detection and isolation problems. The different cases will be characterized by the properties of the uncertainty block Δ in Figure 1.4.

1.3. Nonlinear system models and their application in fault detection

In this work we are concerned with the continuous-time deterministic nonlinear systems described by ordinary differential equations

$$\begin{aligned}\dot{x}(t) &= f(x(t), u(t)),\\ y(t) &= h(x(t))\end{aligned} \tag{1.3}$$

in which the control appears linearly (or affine) and which can be written in state space form, by means of a set of equations of the following type

$$\dot{x} = f(x) + \sum_{i=1}^{m} g_i(x)u_i, \tag{1.4}$$

$$y_i = h_i(x), \qquad 1 \le i \le p,$$

where $x \in \mathcal{X} \subset \mathbb{R}^n$, $u \in \mathcal{U} \subset \mathbb{R}^m$, $y \in \mathbb{R}^p$ denote respectively the state, the input and the output of the system. The mappings $f, g_1 \dots, g_m$ which characterize the dynamics of the system are $\mathbb{R}^n$-valued mappings defined on the open set $\mathcal{X}$, *i.e.,* $f(x), g_1(x), \dots, g_m(x)$ correspond to the values at a specific point $x \in \mathcal{X}$ in the state space. The functions $h_1 \dots, h_p$ are real-valued functions defined on $\mathcal{X}$, and $h_1(x), \dots, h_p(x)$ correspond to the values taken at a specific point x which characterize the output of the system. These mappings may be represented in the form of n-dimensional vectors of real-valued functions of the real variables $x_1, \dots, x_n$, as

$$f(x) = \begin{bmatrix} f_1(x_1, \dots, x_n) \\ f_2(x_1, \dots, x_n) \\ \vdots \\ f_n(x_1, \dots, x_n) \end{bmatrix}, \quad g_i(x) = \begin{bmatrix} g_{1i}(x_1, \dots, x_n) \\ g_{2i}(x_1, \dots, x_n) \\ \vdots \\ g_{ni}(x_1, \dots, x_n) \end{bmatrix}, \quad h_i(x) = h_i(x_1, \dots, x_n). \tag{1.5}$$

System representation (1.4) can be extended with additional inputs which may represent faults and other unknown external excitations. One possible form of this extension can be written in the form

$$\dot{x}(t) = f(x,u) + \sum_{i=1}^{m} g_i(x,u)\nu_i \tag{1.6}$$
$$y_j(t) = h_j(x,u) + \sum_{i=1}^{m} \ell_{ij}(x,u)\nu_{ij}, \qquad 1 \le j \le p,$$

where ℓ_i are real valued functions defined on $\mathcal{X}$ and $\nu(t)$ is the fault signal $(\nu_1, \dots, \nu_m)^T$ whose elements $\nu_i : [0, +\infty) \to \mathbb{R}$ are arbitrary bounded functions of time in L_2. The fault signals ν_i can represent both actuator and sensor failures, in general.

The relationship of nonlinear system models (1.4) and (1.6) to linear systems can be established provided that $f(x), g_i(x), h_j(x), \ell_i(x)$ are linear functions of x, *i.e.*, $f(x) = Ax$, $g_i(x) = b_i$, $h_j(x) = c_j x$, $\ell_i(x) = d_i$ for some $n \times n$ matrix A and $b_i \in \mathbb{R}^{n\times 1}$, $c_j \in \mathbb{R}^{1\times n}$, $d_i \in \mathbb{R}^{p\times 1}$, $i = 1, \dots, n$, $j = 1, \dots, p$. This relationship between the classes of linear and nonlinear system models will be frequently referred to in the next sections.

The system representation (1.4) characterized above can be written in the form

$$\dot{x}(t) = A_o x(t) + \sum_{i=1}^{k} a_i(t) A_i x(t) + \sum_{i=1}^{m} u_i(t) B_i x(t), \qquad x(0) = x_o \in \mathbb{R}^n,$$
$$y(t) = Cx(t) \tag{1.7}$$

assuming $A_o, A_i \in \mathbb{R}^{n\times n}$ are linearly independent constant real matrices. We assume $f : \mathbb{R}^n \to \mathbb{R}^n$, $g : \mathbb{R}^n \to \mathbb{R}^{m\times n}$, $h : \mathbb{R}^n \to \mathbb{R}^p$ (and also $a_i(t)$) to be smooth (analytic) mappings. Note that the output $y(t)$ of the systems (1.4) and (1.7) which is affine in the inputs depends only on the state $x(t)$.

The systems written either in the form of (1.4) or (1.7) describe a large number of physical systems of interest in many engineering applications, including fault detection and isolation. In respect of the application of different modeling formulations of systems of type (1.4) and (1.7) we shall be more specific later on.

1.3.1. *Operation on vector fields*

The mappings $f, g_1 \dots, g_m$ of the system models (1.4) and (1.6) are smooth mappings in their arguments assigning to each point $x \in \mathcal{X}$ a vector of $\mathbb{R}^n$, *i.e.*, $f(x)$, $g_1(x), \dots, g_m(x)$, $h_1(x), \dots, h_p(x)$ according to (1.5). Therefore they are referred to smooth vector fields defined on $\mathcal{X}$.

DEFINITION 1.1. (Dual space of linear functional over $\mathcal{X}$). The dual space $\mathcal{X}'$ of an n-dimensional vector space $\mathcal{X}$ can be identified with an n-dimensional vector space whose elements are called covectors. If $\{x_1, \dots, x_n\}$ is a basis for $\mathcal{X}$, the dual basis for $\mathcal{X}'$ is the set $\{x'_1, \dots, x'_n\} \subset \mathcal{X}'$ such that $x'_i x_j = \delta_{ij}$ $(i, j \in \mathbf{n})$. □

Let $\mathcal{S} \subset \mathcal{X}$. The annihilator of $\mathcal{S}$ denoted by $\mathcal{S}^\perp$ is the set of all $x' \in \mathcal{X}'$ such that $x'\mathcal{S} = 0$. One can find that $\mathcal{S}^\perp$ is a subspace of the dual space $\mathcal{X}'$.

Suppose now that $\omega_1(x), \ldots, \omega_n(x)$ are smooth real-valued functions in the real variables $(x_1, \ldots, x_n)$ of $\mathcal{X} \subset \mathbb{R}^n$, and consider the row vector

$$\omega(x) = \big(\omega_1(x_1, \ldots, x_n) \ldots \omega_n(x_1, \ldots, x_n)\big).$$

The mapping ω assigning to each point $x \in \mathcal{X}$ an element $\omega(x)$ of $\mathcal{X}'$ is called a covector field. We will see that in many cases, it is more convenient to use, together with vector fields, the dual counterparts of these objects *i.e.*, covector fields assigning to each point $x \in \mathcal{X}$ an element of the dual space $\mathcal{X}'$.

A covector field that will be used more frequently in the following parts of this work is the socalled *differential* of the real-valued function λ. This covector field, denoted $d\lambda$ is defined as the $1 \times n$ row vector whose i-th element is the partial derivative of λ with respect to x_i. Its value at a point x is therefore

$$d\lambda(x) = \left[\frac{\partial\lambda}{\partial x_1}\,\frac{\partial\lambda}{\partial x_2}\cdots\frac{\partial\lambda}{\partial x_n}\right], \tag{1.8}$$

or simply $d\lambda(x) = \partial\lambda/\partial x$. Consider the real-valued function λ and a vector field f both defined on $\mathcal{X}$. A new function called the derivative of λ along f, is the inner product, written as

$$L_f\lambda = \langle d\lambda(x), f(x)\rangle = \frac{\partial\lambda}{\partial x} f_i(x) = \sum_{i=1}^{n} \frac{\partial\lambda}{\partial x_i} f_i(x). \tag{1.9}$$

Repeated use of this operation lets extending the scope of the operation. The derivative of λ first along the vector field f and then along a vector field g defines

$$L_g L_f \lambda(x) = \frac{\partial(L_f\lambda)}{\partial x} g(x). \tag{1.10}$$

Continuing a recursion by differentiating λ k-times along f satisfies

$$L_f^k \lambda(x) = \frac{\partial(L_f^{k-1}\lambda)}{\partial x} f(x), \quad \text{with} \quad L_f^0\lambda(x) = \lambda(x). \tag{1.11}$$

A standard assumption in modern control theory is that the state space (and the input and output space) are standard Euclidean vector spaces. This assumption is both valid and natural in many situations, but there is a significant class of problems for which it cannot be made. The Euclidean vector space is not a suitable state-space for a large class of nonlinear, time varying and varying structure control problems. Typical of these are the problems which arise in the control of systems type (1.7). The state space in this case is not a vector space.

It can be shown that the general description of nonlinear systems (1.4) and (1.7) which are considered throughout this paper can be properly defined on a state space which is not an Euclidean vector space $\mathbb{R}^n$, but instead is a curved n-dimensional subset of $\mathbb{R}^m$ for some m which is called a manifold.

One of the basic problems with treating system models like representations (1.4-1.7) is that they usually do not describe the system everywhere, but only on a part of the state space. For different part of the state space one may generally need another representation of the nonlinear system which can be obtained by mathematical transformations of (1.4) or (1.7). In geometrical language we say that Eqs. (1.4) and (1.7) are local coordinate descriptions of the system. In order to cover the whole system behaviour, more than one coordinate description might be needed which require tedious mathematical calculations in the analysis.

There are approaches, however, which enable us to define a nonlinear system on a curved state space independently of any choice of local coordinates. It can be shown, for example, that an alternative (global) description of (1.4) and (1.7) evolves on a state space which in fact is a Lie group of the real orthogonal matrices A_i.

Application of the theory of Lie groups, Lie algebras and their representations is a rapidly growing field of modern mathematics which occure in the solution of problems in many fields of applied mathematics and physics. The related notions will be defined later in the next section.

Recent approaches of applied Lie theory are motivated basically by control theory. During the period from the early 60's to the late 70's, for example, several research papers appeared that made use of Lie algebraic techniques to study controllability of nonlinear differential equations. These early results paved the way to a systematic use of these techniques in other system-theoretic studies.

There is another perhaps more important motivation behind the application of Lie theory to nonlinear problems. By embedding the original nonlinear problem in the framework of matrix Lie groups and associated Lie algebras, it is possible to reduce some system theoretic questions to problems which can be solved by using standard tools of linear algebra. Abstractly, a Lie algebra $\mathcal{L}$ represents a new kind of vector space to the problems which is equipped with a product $[x, y]$, which is called Lie product or Lie bracket in the sequel, satisfying certain axioms.

In order to illustrate the idea consider, for instance, the varying structure and possible nonlinear system of (1.7) where $A_o(x)$ and $A_i(x)$ are smooth vector fields defined in a neighborhood of the origin in $\mathbb{R}^n$. Recall that a function defined in the Euclidean space $\mathbb{R}^n$ is said to be smooth at a point if it can be expressed as a convergent Taylor series in some neighborhood of that point. It can be shown that the local behavior of the system is determined by the algebraic properties of the iterated Lie products of the vector fields $A_o, A_1, \ldots, A_k$'s *i.e.*, the algebraic properties of the Lie algebra they generate. This is analogous to the fact that the local behavior of smooth functions is uniquely determined by its Taylor coefficients. Because of this fact, questions of dynamical properties of the above system can be reduced to symbolic questions about the algebraic properties of the non-commutative operators $A_o, A_1, \ldots, A_k$.

This is the problem of finding representations of the system in question invariant under a given Lie group of matrices. To be more specific, a challenging question is that of finding conditions under which, *e.g.*, a linear time invariant (LTI) system is equivalent to one whose non-linear or linear time varying (LTV) dynamics is described by vector fields which generate a finite dimensional Lie algebra.

With the motivation of the use of Lie theory, a second type of operation on covector fields that is important to introduce here involves two vector fields f and g, both defined on $\mathcal{X}$. From these a new vector field can be constructed, noted $[f, g]$ and defined as

$$[f, g](x) = \frac{\partial g}{\partial x} f(x) - \frac{\partial f}{\partial x} g(x) \tag{1.12}$$

at each x of $\mathcal{X}$, where the expressions

$$\frac{\partial g}{\partial x} = \begin{bmatrix} \frac{\partial g_1}{\partial x_1} & \cdots & \frac{\partial g_1}{\partial x_n} \\ \vdots & & \vdots \\ \frac{\partial g_n}{\partial x_1} & \cdots & \frac{\partial g_n}{\partial x_n} \end{bmatrix}, \qquad \frac{\partial f}{\partial x} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \vdots & & \vdots \\ \frac{\partial f_n}{\partial x_1} & \cdots & \frac{\partial f_n}{\partial x_n} \end{bmatrix}$$

are the Jacobian matrices of the mappings g and f, respectively. The vector field defined in (1.12) is called the Lie product (or Lie bracket) of g and f. It is a fundamental property of Lie brackets that although they appear to be second order differential operators they are in fact first order because of the cancellation of the second order partial derivatives. To be more specific, the Lie bracket of two vector fields is always a vector field.

DEFINITION 1.2. (Lie algebra). A vector space V over $\mathbb{R}$ is a Lie algebra if in addition to its vector space structure it is possible to define a binary operation $V \times V \to V$, called a product and written $[\cdot, \cdot]$, which has the properties

$$[v, w] = -[w, v] \tag{1.13}$$

$$[\alpha_1 v_1 + \alpha_2 v_2, w] = \alpha_1 [v_1, w] + \alpha_2 [v_2, w] \tag{1.14}$$

and satisfies the identity

$$[v, [w, z]] + [w, [z, v]] + [z, [v, w]] = 0 \tag{1.15}$$

where α_1, α_2 are real numbers and v, w, z are real vector fields. Properties (1.13) and (1.14) are called the skew symmetric and bilinearity properties, respectively, and (1.15) is the Jacobi identity. □

The importance of the notion of Lie bracket of vector fields and Lie algebras is very much related to their applications in the study of nonlinear systems of type (1.4) and (1.7). For demonstration, we give hereafter some interesting properties.

1.3.2. *Symmetries, Poisson brackets and Hamiltonian representations*

The Lie bracket operation (1.12) can be interpreted on linear maps and matrices. If, for example, we assume that

$$f(x) = Ax, \qquad g(x) = Bx$$

then (1.12) reduces to

$$[f, g](x) = (BA - AB)x$$

where the matrix $[A, B] = (BA - AB)$ is called the commutator of A, B.

A matrix Lie algebra is an algebra of matrices with the commutator $XY - YX$ taken as the Lie bracket $[X, Y]$. One can immediately see that if X and Y are commutative matrices then their Lie product equals to zero. For example, from the skew-symmetric property $[X, Y] = -[Y, X]$ it immediately follows that

$$[X, X] = 0, \quad \text{and} \quad [X, Y] + [Y, X] = 0.$$

Thus the Lie bracket operator may be seen as a metric deciding commutativity properties of matrices, *i.e.,* the more the magnitude of the product differs from zero the more "non-commutative" the matrices are. More generally, Lie brackets generate the symmetries of commuting operators.

Lie groups and Lie algebras arise most often as symmetry groups of transformations. These symmetries are intimately related to conservation laws of dynamical systems. For example, if the state of a dynamical system is invariant under translations then its linear momentum is conserved while rotational invariance of a system implies conservation of angular momentum. The following considerations may illustrate the relationship between symmetries and conservation laws by using Hamiltonian representation of dynamical systems.

Recall that the state of a classical dynamical system is described in Hamiltonian mechanics by giving n coordinates $q = (q_1, \ldots, q_n)$ and n momenta $p = (p_1, \ldots, p_n)$. These $2n$ independent variables are called canonical variables of the system. Physically important quantities such as energy and momentum are functions $F = F(q, p)$ of the canonical variables. These functions called observables of the system.

Consider the observables F and G. They form an infinite dimensional Lie algebra with respect to the Poisson bracket

$$\{F, G\} = \sum_{i=1}^{N} \left(\frac{\partial F}{\partial q_i} \frac{\partial G}{\partial p_i} - \frac{\partial F}{\partial p_i} \frac{\partial G}{\partial q_i} \right).$$

The equations of motions are

$$\dot{q}_i = \frac{\partial H}{\partial p_i}, \qquad \dot{p}_i = -\frac{\partial H}{\partial q_i}, \tag{1.16}$$

where H is the Hamiltonian *i.e.,* the total energy of the system which in conservative systems, by definition, is constant over the time

$$\frac{dH}{dt} = \frac{\partial H}{\partial q} \frac{dq}{dt} + \frac{\partial H}{\partial p} \frac{dp}{dt} = 0.$$

Equations (1.16), called Hamilton's equations, may be written in terms of Poisson brackets as

$$\dot{q}_i = \{q_i, H\}, \qquad \dot{p}_i = \{p_i, H\}.$$

More generally, the time evolution of an observable F is given by

$$\dot{F} = \{F, H\}. \tag{1.17}$$

EXAMPLE 1.3. Consider a system of 2-bodies *i.e.,* a system consisting of two particles on a line. The canonical variables are $\{q_1, q_2, p_1, p_2\}$ with q_i representing the position of the i-th particle and p_i its momentum. Translation of the coordinate system by an amount x means that the canonical variables in the new system become $\{q_1', q_2', p_1', p_2'\} = \{q_1 - x, q_2 - x, p_1, p_2\}$. This induces an automorphism on the observable $F \to F'$ given by $F'(q_1', q_2', p_1', p_2') = F(q_1, q_2, p_1, p_2)$. Note that F is a one parameter transformation group. The infinitesimal generator of F is

$$\frac{dF'}{dx}(q_1', q_2', p_1', p_2')|_{x=0} = \frac{d}{dx} F(q_1' + x, q_2' + x, p_1', p_2')|_{x=0} = \frac{\partial F}{\partial q_1} + \frac{\partial F}{\partial q_2}.$$

Now $\partial F/\partial q_i = \{F, p_i\}$ so the infinitesimal generator may be expressed as $\{F, p_1 + p_2\}$. Then, if the Hamiltonian H is invariant under translations, we have $\{H, p_1 + p_2\} = 0$. By (1.16) the quantity $p_1 + p_2$ that generates the translation in space is therefore constant in time:

$$\frac{d}{dt}(p_1 + p_2) = \{p_1 + p_2, H\} = 0.$$

In general, if G is a function of the canonical variables such that $\{G, H\} = 0$ where H is the Hamiltonian, then G generates a symmetry of the system found by solving the equations

$$\frac{dq_i}{ds} = \{q_i, G\} = \frac{\partial G}{\partial p_i}, \qquad \frac{dp_i}{ds} = \{p_i, G\} = -\frac{\partial G}{\partial q_i},$$

for if $\{q(s), p(s)\}$ is the flow generated by these equations we have

$$\frac{dH}{ds} = \sum_i \left(\frac{\partial H}{\partial q_i}\frac{dq_i}{ds} + \frac{\partial H}{\partial p_i}\frac{dp_i}{ds} \right) = \sum_i \left(\frac{\partial H}{\partial q_i}\frac{dG}{\partial p_i} - \frac{\partial H}{\partial p_i}\frac{\partial G}{\partial q_i} \right) = \{H, G\} = 0.$$

By this way the symmetries of the Hamiltonian with respect to the set of functions G is given as $\{H, G\} = 0$. The canonical variables q_r, p_s satisfy $\{q_r, q_s\} = 0$, $\{p_r, p_s\} = 0$, $\{q_r, p_s\} = \delta_{rs}$, the corresponding operators Q_r, P_s also satisfy the commutation relations $\{Q_r, Q_s\} = 0$, $\{P_r, P_s\} = 0$, and therefore $\{Q_r, P_s\} = \delta_{rs}$.

1.4. Residual generation for fault diagnosis in linear systems

Residuals are signals that represent the inconsistency between the variables acquired from the actual plant and their ideal or expected counterparts represented by a mathematical model. From a practical point of view *detector* and *residual generator* are synonymous with each other and they are often used interchangeably in the literature. For a dynamic system a residual generator can be represented as another dynamic system as well. It may be constructed by various different techniques.

Let $y \in \mathbb{R}^m, u \in \mathbb{R}^p, d \in \mathbb{R}^s, v \in \mathbb{R}^k$ be vector valued functions of time. By following standard notations y, u, d, v may denote signals in both time and frequency domain depending on context. Based on the general system setup represented by (1.1-1.2) consider the system model

$$y = G^{\Delta}_{yu}u + G^{\Delta}_{yd}d + G^{\Delta}_{yv}v \tag{1.18}$$

where the superscript Δ indicates that the transfer functions G_{yu}, G_{yd}, G_{yv} are subjected to uncertainties.

DEFINITION 1.4. Consider Fig.1.4. A stable and proper linear single output filter $F(s)$ is an ideal residual generator for (1.18) if when $v = 0$ it holds that

$$r = F(s)\begin{pmatrix} y \\ u \end{pmatrix} = 0, \tag{1.19}$$

for all control and disturbance signals $u(t)$, $d(t)$, respectively. □

Since model uncertainty should always be taken into consideration the ideal residual generator (1.19) has no much practical significance. Considering robust detection in the presence of model uncertainty insert (1.18) into (1.19) that gives the representation

$$r = F(s)\begin{bmatrix} G^{\Delta}_{yu}(s) & G^{\Delta}_{yd}(s) \\ I & 0 \end{bmatrix}\begin{pmatrix} u \\ d \end{pmatrix} + F(s)\begin{bmatrix} G^{\Delta}_{y\nu}(s) \\ 0 \end{bmatrix}\nu. \tag{1.20}$$

To ensure ideal design, the control $u(t)$ and disturbance inputs $d(t)$ are to be decoupled from the fault effects in the residual. This means that the first term of (1.20) must be zero while the second term must be different from zero to be useful. If this design criteria is met, *i.e.*, the filter $F(s)$ can be chosen such that the first term of (1.20) can be made identically zero the robust residual generation problem was solved by perfect decoupling. By doing so, as a special case, the residual can be made single direction or structured (or directional) residual.

A single directional residual is a scalar residual signal that represents the change of the system caused by a single failure but it does not allow isolation of faults, whether those faults are simultaneous or not. Even if we use a multiple fault model in system modeling, by using a single direction residual generator, the case of multiple simultaneous failures cannot be assumed.

With uncertain models, however, in the most cases, the design requirements mentioned above are not possible to satisfy without losing the desired minimum detection sensitivity. As this decoupling can usually be made approximately, generally some compromise between sensitivity to faults and disturbance/uncertainty attenuation is needed. The basic idea of this robust residual generation problem is to find an optimal solution to the above described trade-off in the framework of a mathematical optimization problem.

1.5. The linear parity space approach

The parity or consistency equations method is the direct implementation of the concept of analytical redundancy as it uses and manipulates the measurement variables directly as it was characterized by a series of papers in the literature such as in (Chow and Willsky, 1984; Frank, 1990; Gertler and Singer, 1990; Gertler, 1997; Gertler, 1998; Chen and Patton, 1998; Ding et al., 1999). Actually, *parity space* (parity relations) and *consistency relations* (also called parity relations) are two different things, though closely related, which relationship will be explained later in this section.

Consider the deterministic representation of the LTI system subject to multiple faults in the state space

$$\dot{x}(t) = Ax(t) + Bu(t) + L\nu(t), \tag{1.21}$$

$$y(t) = Cx(t) + M\nu(t), \tag{1.22}$$

with $u \in \mathbb{R}^r$, $x \in \mathbb{R}^n$, $y \in \mathbb{R}^m$ and the matrices A, B, C with appropriate dimensions where the fault signals $\nu \in \mathbb{R}^q$ may represent both actuator and sensor faults as reflected in the structure of the matrices L and M.

Assuming there exist relationships between measured variables and model parameters which are directly related to faults, one can construct the generic input output relations by differentiating the observation equation (1.22) s-times and combine with Eq. (1.21) to get

$$\begin{aligned}
\dot{x} &= Ax + Bu + L\nu, \qquad (1.23)\\
y &= Cx + M\nu,\\
\dot{y} &= C\dot{x} + M\dot{\nu} = CAx + CBu + CL\nu + M\dot{\nu},\\
\ddot{y} &= CA^2x + CABu + CB\dot{u} + CAL\nu + CL\dot{\nu} + M\ddot{\nu},\\
&\vdots\\
y^{(s)} &= CA^s x + CA^{s-1}Bu + \ldots + CBu^{(s-1)} +\\
&\qquad + CA^{s-1}L\nu + \ldots + CL\nu^{s-1} + M\nu^s.
\end{aligned}$$

The set of equations obtained in the above procedure can be written as

$$\begin{bmatrix} y(t)\\ \dot{y}(t)\\ \ddot{y}(t)\\ \vdots\\ y^{(s)}(t) \end{bmatrix} = \begin{bmatrix} C\\ CA\\ CA^2\\ \vdots\\ CA^s \end{bmatrix} x(t) + \qquad (1.24)$$
$$+ \begin{bmatrix} 0 & 0 & \cdots & 0\\ CB & 0 & \cdots & 0\\ CAB & CB & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ CA^{s-1}B & \cdots & CB & 0 \end{bmatrix} \begin{bmatrix} u(t)\\ \dot{u}(t)\\ \ddot{u}(t)\\ \vdots\\ u^{(s)}(t) \end{bmatrix} +$$
$$+ \begin{bmatrix} M & 0 & \cdots & 0\\ CL & M & \cdots & 0\\ CAL & CL & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ CA^{s-1}L & \cdots & CL & M \end{bmatrix} \begin{bmatrix} \nu(t)\\ \dot{\nu}(t)\\ \ddot{\nu}(t)\\ \vdots\\ \nu^{(s)}(t) \end{bmatrix},$$

i.e., one can write

$$\nu_y = \mathcal{O}^s x + S_u \nu_u + S_\nu \nu_\nu, \qquad (1.25)$$

where $\nu_u \in \mathbb{R}^{r(s+1)}$, $\nu_\nu \in \mathbb{R}^{q(s+1)}$ and $\nu_y \in \mathbb{R}^{m(s+1)}$ contain the variables u, ν and y and their time derivatives of the appropriate order, respectively.

The system descriptions (1.24-1.25) containing a mix of input, output and state variables, characterize all the analytical redundancies of system (1.21-1.22) since it provides all the possible relationships among inputs $u(t)$ and outputs $y(t)$. Let $\omega \in \mathbb{R}^{(s+1)m}$ be a nonzero row vector satisfying

$$\omega\, \mathcal{O}^s = 0, \qquad \forall x. \qquad (1.26)$$

Then, there exist p linearly independent vectors ω_j, $j = 1, \ldots, p$, such that

$$p = m(s+1) - \operatorname{rank} \mathcal{O}^s.$$

By satisfying (1.26) the (unknown) state can be eliminated from (1.25). Now let

$$W = [\omega_j]_{j=1,\ldots,p} \subset \mathbb{R}^{p\times(s+1)m}.$$

Then, one obtains

$$Wv_y = W(S_u v_u + S_\nu v_\nu). \tag{1.27}$$

It is clearly seen that this generic equation may be used to provide information about the appearance of the faults under the assumption that the condition

$$Wv_y = WS_u v_u \tag{1.28}$$

in fault-free case (*i.e.,* when $\nu(t)$ and all its derivatives are zero) holds but there exists a bias

$$P(u, y) = W(v_y - S_u v_u) \tag{1.29}$$

for any other case. Any linear combination of the rows of (1.29) is called a parity equation or parity relation and $p > 0$ is used to indicate the order of the parity relation; the right hand side of (1.29) is called parity function; the vectors $P(u, y)$ are called parity vectors and the p dimensional space $\mathcal{P}$ of all such vectors is called the parity space.

It can be seen that the parity functions in representation (1.29) can be taken as residuals and used for the purpose of constructing a detector. Thus, based on the knowledge of (S, v_u, v_y), the traditional direct parity space methods provide solution to the implementation of the residual generator by finding $W = \{\omega_j\}$ so that the design specifications of the detector are met. Clearly, the p elements of W have to satisfy a set of s linear homogenous equations. If the system is observable these s equations are independent. Apart from this, W can be chosen freely, leading to a wide variety of parity relations. The individual approaches differ in the particular methods how the solutions for W are sought.

Consistency relations use transfer function type description. The consistency-type residual generators can be written as

$$r = W(s)\big(y - G_u(s)u\big) = G_\nu(s)\nu. \tag{1.30}$$

Note that the Chow-Willsky-type parity space residual generator given in the form (1.29) can be easily converted into representation (1.30) and turns out to be a special case, namely, a polynomial residual generator.

1.6. Observer-based residual generation in linear systems

There can be situations, when direct access to state variables of the system may not be feasible. Another class of residual implementation methods uses the anticipated values of the unknown and unmeasurable state variables: these methods can be seen as the *indirect* realization of the concept of analytical redundancy. The state of unmeasured variables are determined by using state estimation techniques and the device providing the state estimation is called state observer. State observers are auxiliary dynamic systems that are connected to the input $u(t)$ and output $y(t)$ of the observed system and provide an asymptotic estimate of its state, *i.e.*, provide an output $\hat{x}(t)$ that asymptotically approaches the observed system state, see (Luenberger, 1971). Consider the LTI system representation given by its triple (A, B, C) as

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t). \end{aligned} \tag{1.31}$$

If A is asymptotically stable a state asymptotic estimate $z(t)$ (state reconstruction in real time) can be achieved by applying the same input signal to a model of the system, (see Fig.1.5) *i.e.*, another dynamical system, built *ad hoc*, with a state $\hat{x}(t)$ whose time evolution is described by the same matrix differential equation

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t). \tag{1.32}$$

Obviously, this solution has three particular drawbacks: First, the solution is not feasible if the system is not stable, moreover the solution is sensitive to initial conditions and it does not let to establish a design strategy which might influence the performance properties of the filter.

One obvious way to avoid these difficulties is to use an error correction on the system equations. This idea results in a closed-loop state estimation method which can be generated as follows. Let $\tilde{x}(t)$ be the estimation error, defined by

$$\tilde{x}(t) \triangleq \hat{x}(t) - x(t). \tag{1.33}$$

Subtracting the system equation (1.31) from the state estimate (1.32) one gets

$$\dot{\tilde{x}}(t) = A\tilde{x}(t)$$

from which it follows that the estimation error has a time evolution depending only on system matrix A and converges to zero whatever its initial value is if and only if A is stable. This property is discussed below in the framework of the fault detection problem. Consider the state space description of the nominal LTI system subject to multiple faults

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + \sum_{i=1}^{k} L_i \nu_i(t), \\ y(t) &= Cx(t) \end{aligned} \tag{1.34}$$

with $x \in \mathcal{X} \subset \mathbb{R}^n$, where $A \in \mathbb{R}^{n\times n}$ is a stable nominal system matrix and the real constant matrices B and C are in the appropriate dimensions. Assume, moreover, that (A, C) is an observable pair. The cumulative effect of k number of faults appearing in known directions L_i of the state space are modeled by the additive linear term $\sum L_i \nu_i(t)$. Here $u(t)$ is the control, $L_i \in \mathbb{R}^{n\times s}$ and $\nu_i(t)$ are called fault signatures and failure modes, respectively. Note that $\nu_i(t)$ are arbitrary unknown time functions for $t \geq t_{j_i}$, $0 \leq t \leq T$, where t_{j_i} is the time instant when

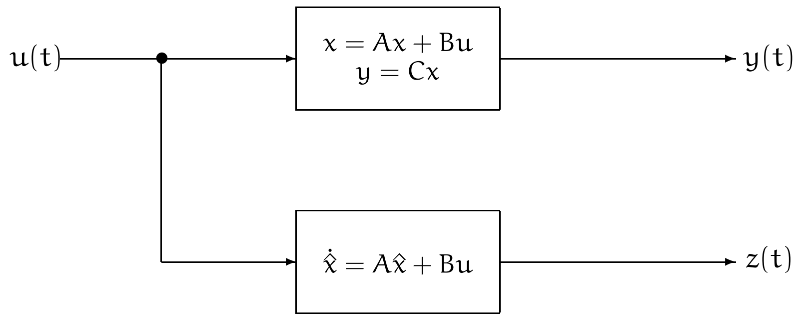

Figure 1.5. Open-loop state estimate obtained by using the linear model of the system and its input

the i-th fault appears and $\nu_i = 0$, if $t < t_{j_i}$. If $\nu_i(t) = 0$, for all i, then the plant is assumed to be fault free.

Assume, however, that there is only one failure present in the system at a time. That is to say if $\nu_i(t) \neq 0$, then $\nu_j(t) = 0$ for all $j \in k$, $j \neq i$.

Our goal is to detect and isolate failure modes $\nu_i(t)$ by applying a residual generator based on the reconstructed state of the system. A more general asymptotic observer where the drawbacks of the open-loop state estimator can be eliminated is a (usually full order) state observer relying both on the input $u(t)$ and output $y(t)$ variables of the system, see Fig. 1.6.

In the scheme of the observers of this kind there exists an arbitrary constant gain matrix $D \in \mathbb{R}^{n\times r}$ such that the estimate $\hat{x}(t)$ of $x(t)$ will be the solution to the full order observer equation

$$\begin{aligned} \dot{\hat{x}}(t) &= A\hat{x}(t) + Bu(t) + D\big(C\hat{x}(t) - y(t)\big), \\ \hat{y}(t) &= C\hat{x}(t) \end{aligned} \tag{1.35}$$

where $\hat{x}(t)$ and $\hat{y}(t)$ stand for the estimated state and observer output vectors respectively and D is the observer feedback gain matrix to be suitably chosen. Clearly, the error between the states and their estimates $\tilde{x}(t) = \hat{x}(t) - x(t)$ causes an error in the innovation series of the observer

$$r(t) \triangleq \hat{y}(t) - y(t). \tag{1.36}$$

Subtracting Eq. (1.34) from Eq. (1.35), the state error (1.33) will satisfy the differential equation

$$\begin{aligned} \dot{\tilde{x}}(t) &= (A + DC)\tilde{x}(t), \quad \tilde{x}(0) = \hat{x}(0) - x(0) = \epsilon \\ r(t) &= C\tilde{x}(t) \end{aligned} \tag{1.37}$$

with initial condition ϵ where $(A + DC)$ is the closed loop transition matrix and $r(t)$ is the output error of the observer. The solution of this system of differential equations is $\tilde{x}(t) = e^{(A+DC)t}\epsilon$, which shows that the rate at which the error vector approaches zero can be controlled by appropriate assignment of the eigenvalues of $(A + DC)$. If the matrix $(A + DC)$ is stable *i.e.*, its eigenvalues have negative real parts the effects of the error in the initial estimates

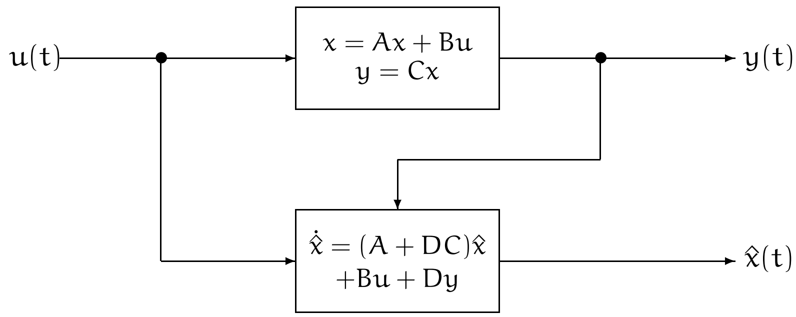

Figure 1.6. Closed-loop linear asymptotic observer which derives information from system input and output

ϵ will exponentially die out by $\lim_{t\to\infty} \tilde{x}(t) = 0$, and the observer is called exponential or asymptotic observer.

Assume that in the time $t = t_1$, due to some failure, the system changes. It is obvious that the error dynamics will reflect all the deviations with respect to the nominal system description (1.34) in steady-state. If we model the deviation of the system from its normal behavior as an additive term in the representation (1.34), such that if $t < t_1$ then $\nu_i(t) = 0$ but $\nu_i(t) \neq 0$ if $t \geq t_1$, the cumulative effect of the deviations for k failure modes will appear in the observer's error system as

$$\dot{\tilde{x}}(t) = (A + DC)\tilde{x}(t) + \sum_{i=1}^{k} L_i \nu_i(t),$$
$$r(t) = C\tilde{x}(t) \tag{1.38}$$

where L_i are the known fault direction matrices and $\nu_i(t)$ are arbitrary unknown time functions representing the faults. It can be seen that the effect of the changes in (1.34) is accentuated on the innovation of the observer ($\tilde{x}(t) \neq 0$ and $r(t) \neq 0$). Therefore, the observer's output error $r(t)$ in every respect satisfies the criteria of detection residual and the two parts of Eqs. (1.38) correspondingly, may be called state error and output error residuals of the observer.

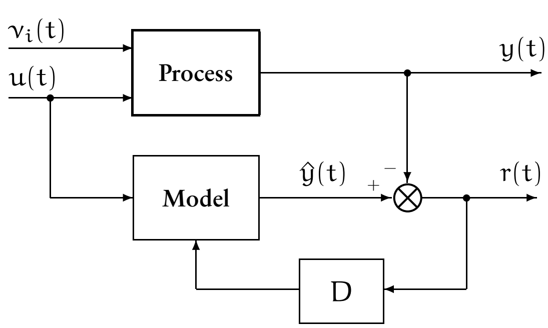

Figure 1.7. Basic scheme of the observer-based residual generator for observable linear systems operating in closed-loop

A close analysis of (1.38) shows that, in the presence of faults, the estimation error does not converge asymptotically to zero even if $A + DC$ is stable, but converges asymptotically to a *subspace* which is different from zero. This property cannot further be explained here, until the geometrical interpretation of the residual is not introduced in the next chapter. Still worth to note, however: the basic advantage of observer based residual generation lies in its closed loop nature that makes this method insensitive to unmatched initial conditions and other deterministic system variations.

We remind the reader, that by using the single observer approach we are not able to make a clear distinction between different fault effects: *i.e.*, we cannot isolate the effects of multiple failures from each other. While a single residual is sufficient to detect a single fault, a set of residuals is required for fault isolation. Multiple residuals can be generated by using a bank of observers. Using the assumption that there is only one failure present at a time, we can think of the observer-based detection and isolation as k separate observer design problem. The idea is to use a set of observers, each of which is specifically designed (*i.e.*, dedicated) to the detection of a particular failure mode. The plant measurements are processed by different observers each of which is designed on a particular hypothesis of the system behavior. To be more specific, the space of the output error residual of a particular (say the i^{th}) observer, through the appropriate choice of its observer gain D_i, is matched with the failure direction L_i to be actually monitored by the i^{th} observer in the bank.

The idea of dedicated observer scheme is in contrast to *detection filters* where the output prediction residual is a vector that can be designed so that the response of the filter is directional or structured *i.e.*, the residual vector $r(t)$ of the observer is confined to a specific direction in the output error space. The detection filter design problem for LTI systems is formulated in Section 2.6 in the next chapter. In order to use this idea in the following sections extensively, it is necessary to formalize some properties of the residual using some basic concepts of geometrical system theory in the next chapter.

1.7. On the equivalence relations of parity space methods and diagnostic observers in linear systems

Consider the nominal linear system (1.31) which, in input-output form is described as

$$y(s) = G(s)u(s), \tag{1.39}$$

where $G(s)$ is the general nominal transfer function matrix of the system. Whatever is the realization and internal structure of the observer-based residual generator, from (1.35) and (1.37) it is obvious that the residual can always be written in the general input-output form as

$$r(s) = Q(s)y(s) + P(s)u(s) \tag{1.40}$$

where $Q(s)$ and $P(s)$ are transfer function matrices. Under fault-free conditions the residual (1.40) is to be zero by definition. By using (1.39) it means that

$$Q(s)G(s)u(s) + P(s)u(s) = 0, \tag{1.41}$$

i.e.,

$$Q(s)G(s) = -P(s). \tag{1.42}$$

Using the relation (1.42) the residual (1.40) can be written as

$$r(s) = Q(s)y(s) + Q(s)G(s)u(s). \tag{1.43}$$

Now, it can be easily seen that any first order parity relations obtained in Section 1.5 in the form (1.29) is identical with (1.43) assuming the choice of the matrices

$$W(s) = Q(s).$$

The above characterized close relationship between parity equations and diagnostic observers was first known by (Gertler, 1991) based on the preliminary results of the works of (Massoumnia and Van der Velde, 1988; Frank and Wünnenberg, 1989).

1.8. SUMMARY

In this chapter the introduction to the foundations of the basic methods applied to fault detection and isolation which are based on the principle of analytical redundancy has been presented. The idea of residual generation and the main concepts behind system and fault modeling were briefly summarized. The issue of robustness in the view of modeling uncertainty and external disturbances has been addressed in a first approximation. Two traditionally fundamental model-based residual generation methods, namely parity equations and diagnostic observers have been reviewed and the equivalence of the two techniques were highlighted.

It was shown, how input-output measurement data acquired from a dynamical system can usually be used for generating residuals. The considerations we made on analytical redundancy methods such as parity relations and state observers may be readily applied to detecting and isolating faults in a number of situations differing in the assumptions on noise, disturbances, robustness properties and in the specific design methods. The simplest configuration is by using a single observer like the one in Fig. 1.7. Clearly, one can use Kalman filters, Luenberger observers or dead-beat observers in the design depending on the assumptions of the model and the nature of the disturbances.

In this part of the thesis we have focused on the basic knowledge of fixed direction residual generation, *i.e.,* on detection methods that do not consider the presence of multiple simultaneous failures. The detection of multiple failures with the dedicated observer scheme, for instance, relies on the restrictive assumption that the failure modes are always mutually detectable and they never occur in the system simultaneously.

Robustness for different kind of uncertainties will be considered in the following chapters in more details. Robustness and the problem of the detection of multiple simultaneous faults by producing structured or directional residuals require further analysis, which, in one way or in another, relies heavily on concepts of geometric system theory. These concepts — which will lead us to the geometric approach of detection filters, — will be introduced in the next chapter.

PART 2

ROBUST ESTIMATION AND FAULT DETECTION

Robustness in the face of plant modeling errors and other model uncertainties is the most fundamental problem in model-based fault detection and isolation. Residuals generated to indicate faults may also react in the presence of noise, disturbances and modeling errors. The plant dynamics and failure mode modeling errors can either cause high false alarm rates, or make it difficult to detect the failures. Therefore, in real conditions, we are always faced a decision problem whether the estimated fault condition is really due to a failure or is simply due to the poor system model. Any robust detection and isolation test that is designed to overcome the problems associated with modeling errors must be able to distinguish between model uncertainties and failures in order to avoid excessive false alarms or missed detections. To make the residual generation process insensitive to these uncertainties is a most important aspect in the design of efficient detection algorithms. In the previous part the theoretical foundations of robust fault detection based on state estimation techniques have been reviewed. This part will take further considerations upon robustness and detection sensitivity which rely heavily on geometric concepts and partly, on some assumptions on the properties of the disturbances. This discussion will lead us from the techniques of robust estimation to optimal estimation. Characterization of the relationship of Kalman filters and H_∞ *optimal filters, by relating robustness and optimality in a novel way, adds further interesting facts to the interpretation of the equivalence conditions in some classical fault detection and isolation approaches.*

CHAPTER 2

Geometric Concepts in Residual Generation for Linear Systems

Embedding the basic concepts of control in the system of geometry and the interpretation (and re-interpretation) of the results of mathematical system theory by using a geometric approach was initiated in the beginning of the 1970's by Basile, Marro and Wonham, see (Basile and Marro, 1969a; Wonham and Morse, 1970). By now, the approach has proved to be an effective means to the analysis and design of control systems and the idea gained some popularity that was followed by many authors succesfully. Good summaries of the subject can be found in the classical books of (Wonham, 1974; Wonham, 1979) and (Basile and Marro, 1991).

The term *geometric* suggests several things. On the one hand it suggests that the setting is linear state space and the mathematics behind is primarily linear algebra (with a geometric flavor). On the other hand it suggests that the underlying methodology is geometric. The theory treats many important system concepts, for example the classical Kalman controllability theory, as geometric properties of the state space or its subspaces. These are the properties that are not affected by coordinate changes and always preserved under linear transformations, for example, the so-called invariant or controlled invariant subspaces. Using these concepts the geometric approach captures the essence of many analysis and synthesis problems and treat them in a coordinate-free fashion. By characterizing the solvability of a problem as a verifiable property of some constructible subspace, calculation of the solution law becomes much easier. The computational aspects are considered independently of the theory and handled by means of the standard methods of matrix algebra, once a suitable coordinate system is defined. In many cases, the geometric approach can convert what is usually a difficult time varying or nonlinear problem into a more easier linear time invariant one.

The linear geometric systems theory was extended to nonlinear systems in the 1980's, see *e.g.,* (Isidori, 1985). In the nonlinear theory, the underlying fundamental concepts are almost the same, but the mathematics is different. For nonlinear systems the tools from differential geometry and Lie-theory are primarily used.

In the first part of this chapter some important concepts of geometric system theory are referred, basically those which will be used in the forthcoming discussions of this volume. In the rest of the chapter, we use some typical problems to illustrate the basic ideas of the geometric approach to control and filtering with special emphasis to the problem of residual generation to fault detection and isolation where the residual is given a geometric interpretation.

The concepts of input observability and the design of input estimators have evident applications in system supervision and fault detection which will be used in many times when discussing fault detectors in various contexts in the next chapters. Therefore, the problem of input reconstruction from the viewpoint of input observability is presented through the introduction of the unknown input observer problem. The unknown input observer is a very important idea as it leads out from the theory of classical state estimation methods by linking the theory of traditional observer-based residual generators to geometric ideas.

It will be shown that the solution of this problem, while firmly based on the geometric approach, has a tight relation with system inversion. This knowledge will serve useful in later parts when the concept of system inversion will be used more extensively.

2.1. INTRODUCTION

In this section we give a short summary on the basic notions and notations of linear geometric system theory. Two important concepts: the invariant subspace and the controlled invariant subspace, that will be used later on in the discussion, are introduced. For further details the interested reader is directed to the books of (Wonham, 1979; Basile and Marro, 1991).

2.1.1. *Elementary invariant subspaces*

Consider the n-dimensional linear system

$$\dot{x} = Ax \tag{2.1}$$

with $x \in \mathbb{R}^n$.

DEFINITION 2.1. The set $\Omega \subseteq \mathbb{R}^n$ is called an invariant set of (2.1) if for any initial condition $x_o \in \Omega$, we have $x(x_o, t) = e^{At}x_o \in \Omega$, for all $t \geq 0$. Trivial examples of invariant sets are $\mathbb{R}^n$ and $x = 0$. □

In this volume, only a special class of invariant sets is considered, *i.e.*, invariant subspaces which can be interpreted in the following way. Consider the conditions of a subspace $\mathcal{W}$ to be invariant: Since by the Taylor series expansion

$$x(x_o, t) = x_o + tAx_o + \frac{t^2}{2}A^2x_o + \dots,$$

it is obvious that if $A^i x_o \in \mathcal{W}$ forall $i \geq 0$, then $x(x_o, t) \in \mathcal{W}$, for all $t \geq 0$. Obviously again, this argument is true only if $\mathcal{W}$ is a linear subspace. In other words, this condition implies that if a mapping from $\mathbb{R}^n$ to $\mathbb{R}^n : w = Az$ is defined, then the image of $\mathcal{W} \subseteq \mathbb{R}^n$ is contained in $\mathcal{W}$. This property can be denoted as $A\mathcal{W} \subseteq \mathcal{W}$.

DEFINITION 2.2. Let $A : \mathcal{X} \to \mathcal{X}$. Then a subspace $\mathcal{W} \subset \mathcal{X}$ is called A-*invariant* if it has the property $A\mathcal{W} \subseteq \mathcal{W}$. □

Extensions of invariance provide means for analysis and synthesis of linear systems such as controllability and observability. Controlled invariant subspaces are associated to the controllability (reachability) properties of dynamical systems. In fact they relates input and state in the dynamics of the systems. More specifically, controlled invariant subspaces are subspaces such that, from any initial state of the linear system $\dot{x} = Ax + Bu$ belonging to these subspaces, at least one state trajectory can be maintained on them by means of a suitable control action.

DEFINITION 2.3. We say that a subspace $\mathcal{W} \subset \mathcal{X}$ is (A, B)-*controlled invariant* or simply (A, B)-*invariant*, if assuming $A : \mathcal{X} \to \mathcal{X}$ and $B : \mathcal{U} \to \mathcal{X}$ there exist a map $F : \mathcal{X} \to \mathcal{U}$ such that $(A + BF)\mathcal{W} \subseteq \mathcal{W}$. In other words, the subspace $\mathcal{W}$ is called a controlled invariant subspace of the control system $\dot{x} = Ax + Bu$ if there exists a feedback control $u = Fx$ such that $\mathcal{W}$ is an invariant subspace of $\dot{x} = (A + BF)x$. □

By duality, an analysis similar to controllability can be carried out by relating the interaction of state and output: the dual concept of (A, B)-controlled invariance is (C, A)-invariance or (C, A)-conditioned invariance.[3]

DEFINITION 2.4. Let $A : \mathcal{X} \to \mathcal{X}$ and $C : \mathcal{X} \to \mathcal{Y}$. A subspace $\mathcal{W} \subseteq \mathcal{X}$ is (C, A)-*invariant* or (C, A)-*conditioned invariant* if there exists a map $D : \mathcal{Y} \to \mathcal{X}$ such that $(A + DC)\mathcal{W} \subset \mathcal{W}$. Equivalently, $\mathcal{W} \subseteq \mathcal{X}$ is (C, A)-invariant if $A(\mathcal{W} \cap \ker C) \subseteq \mathcal{W}$. □

Note that any A-invariant is also an (A, B)-controlled invariant for any $\mathcal{B}$ and an (A, C)-conditioned invariant for any $\mathcal{C}$.

2.1.2. *Self-contained controlled and conditioned invariants*

Self-contained controlled and conditioned invariants are particular classes of invariants which have interesting properties the most important of which is to admit both a supremum and an infimum.

PROPERTY 2.5. The sum of any two (A, B)-controlled invariant subspaces is an (A, B)-controlled invariant subspace.

PROPERTY 2.6. The intersection of any two (A, C)-conditioned invariant subspaces is an (A, C)-conditioned invariant subspace.

An immediate consequence of Property 2.5 is that the set of (A, B)-controlled invariants contained in a given subspace $\mathcal{E} \subseteq \mathcal{X}$ admits a supremum, the maximal (A, B)-controlled invariant contained in $\mathcal{E}$. Similarly, Property 2.6 implies that the set of all (A, C)-conditioned invariants containing a given subspace $\mathcal{D} \subseteq \mathcal{X}$ admits an infimum, the minimal (A, C)-conditioned invariant containing $\mathcal{D}$.

Subspaces which are the supremum of all (A, B)-controlled invariants contained in a given subspace $\mathcal{E}$ and the infimum of all (A, C)-conditioned invariants containing a given subspace $\mathcal{D}$ are introduced through the following definitions, respectively.

[3] The naming conventions of these invariant subspaces are due to Basile and Marro that have been independently adopted by Wonham, see (Basile and Marro, 1969a) and (Wonham and Morse, 1970).

DEFINITION 2.7. Consider a subspace $\mathcal{L} \subseteq \mathcal{X}$. We use the notation $\langle A \,|\, \mathcal{L} \rangle$ to denote the family of all A-invariant subspaces bounded to (containing or contained in) the subspace $\mathcal{L}$ where $\mathcal{L} = \operatorname{Im} L$ and L is a linear map $\mathcal{L} : \mathcal{Y} \to \mathcal{X}$. The extremal subspaces from this set (*i.e.,* the maximal or minimal) are denoted by $\sup \langle A \,|\, \mathcal{L} \rangle$ and $\inf \langle A \,|\, \mathcal{L} \rangle$, respectively. □

DEFINITION 2.8. Given any subspace $\mathcal{E} \subseteq \mathcal{X}$ the family of (A, B)-controlled invariant subspaces contained in $\mathcal{E}$ is defined as $\mathcal{V} = \langle (A, B) \,|\, \mathcal{E} \rangle$. The supremal subspace from this set, which will be referred to frequently in this book, is denoted by $\mathcal{V}^* = \sup \mathcal{V} \langle (A, B) \,|\, \mathcal{E} \rangle$. □

DEFINITION 2.9. Analogously, if we have $\mathcal{D} \subseteq \mathcal{X}$ the set of all (C, A)-invariant subspaces containing $\mathcal{D}$ is denoted by $\mathcal{W} = \langle (A, C) \,|\, \mathcal{D} \rangle$. In the following, the infimal nonzero subspace from this set will be referred to $\mathcal{W}^* = \inf \mathcal{W} \langle (A, C) \,|\, \mathcal{D} \rangle$. The characterizing property of these subspaces is to provide the possibility to make a part of the state space unobservable, that is to place it in $\ker C$, by means of output injection. □

The most elementary class of controlled invariant subspaces are the reachability (controllability) subspaces. Controlled invariants are subspaces such that, from any initial state belonging to them, at least one state trajectory can be maintained on them by means of a suitable control action. Consider for instance the system $\dot{x} = Ax + Bu$. The reachable (controllable) subspace of this system is defined as

$$\inf \langle A \,|\, \operatorname{Im} B \rangle = \operatorname{span} \{B, AB, \ldots, A^{n-1}B\}$$

i.e., the minimal A-invariant subspace that contains $\operatorname{Im} B$.

DEFINITION 2.10. If we consider the feedback law $u = Fx + Gv$, the corresponding closed-loop system $\dot{x} = (A + BF)x + BGv$ will have the reachable (controllable) subspace

$$\mathcal{R} = \sup \langle A + BF \,|\, \operatorname{Im} (BG) \rangle. \tag{2.2}$$

Thus $\mathcal{R}$ is precisely the reachable (controllable) subspace of the pair $(A + BF, BG)$. □

The controllable subspace of the pair $((A + BF, BG)$ is called a controllability subspace of the original system (A, B). The significance of the reachability (controllability) subspace derives from the fact that by the restriction of $A + BF$ to an $(A + BF)$-controlled invariant subspace, an arbitrary spectrum can be assigned by suitable choice of the feedback F.

DEFINITION 2.11. A subspace $\mathcal{R}$ is called a reachability (controllability) subspace of $\dot{x} = Ax + Bu$ if there exist F and G with the feedback law $u = Fx + Gv$ such that (2.2) holds. The reachability subspace $\mathcal{R}$ is (A, B)-controlled invariant. □

In general, it is not possible to reach any point of a controlled invariant from any other point (in particular, from the origin) by a trajectory completely belonging to it. For this property consider the following definition:

DEFINITION 2.12. Given a subspace $\mathcal{E} \subseteq \mathcal{X}$, by leaving the origin with trajectories belonging to $\mathcal{E}$ (*i.e.,* to the maximal (A, B)-controlled invariant subspace contained in $\mathcal{E}$, which is denoted as $\mathcal{V}^*$), it is not possible to reach all the points of $\mathcal{V}^*$, but only a subspace of $\mathcal{V}^*$, which is called the reachable set on $\mathcal{E}$ (or on $\mathcal{V}^*$) and denoted by $\mathcal{R}_{\mathcal{E}}$ (or $\mathcal{R}_{\mathcal{V}^*}$). □

THEOREM 2.13. (Basile and Marro, 1991). The reachable set on $\mathcal{E}$ (or on $\mathcal{V}^*$), coincides with the maximal (A, B)-controlled invariant contained in $\mathcal{E}$, *i.e.*,

$$\mathcal{R}_{\mathcal{E}} \triangleq \mathcal{R}_{\mathcal{V}^*} = \sup \langle A + BF \,|\, \mathcal{V}^* \cap \operatorname{Im} B \rangle. \qquad \square$$

Note that the reachable set on $\mathcal{E}$ can be defined independently of the feedback matrix F. This definition is more practical because F is not unique.

DEFINITION 2.14. The reachable set on $\mathcal{E}$ is defined as

$$\mathcal{R}_{\mathcal{E}} \triangleq \mathcal{R}_{\mathcal{V}^*} = \mathcal{V}^* \cap \mathcal{S}_2^* \quad \text{with} \quad \mathcal{S}_2^* \triangleq \inf \langle \mathcal{E}, A \,|\, \operatorname{Im} B \rangle.$$

The duality relation, which defines the reachable set on $\mathcal{E}$ as the intersection of the maximal (A, B)-controlled invariant contained in $\mathcal{E}$ with the minimal $(\mathcal{E}, A)$-conditioned invariant containing $\operatorname{Im} B$ was first derived by (Morse, 1973). This observation led to the construction of important computational algorithms which will be discussed in Section 2.2. $\square$

DEFINITION 2.15. Dual to the properties of reachability is the observability. The pair of maps (C, A) is observable if

$$\bigcap_{i=1}^{n} \ker(CA^{i-1}) = 0. \tag{2.3}$$

$\square$

DEFINITION 2.16. (Unobservable subspace). Definition 2.15 with condition (2.3) suggests that the subspace $\mathcal{S} \subseteq \mathcal{X}$ defined as

$$\mathcal{S} = \bigcap_{i=1}^{n} \ker(CA^{i-1}) \tag{2.4}$$

is the unobservable subspace of the pair (C, A). Since $A\mathcal{S} \subset \mathcal{S}$, in fact, $\mathcal{S}$ is the largest in family A-invariant conditioned subspace contained in $\ker C$, *i.e.*,

$$\mathcal{S}^* = \sup \langle A \,|\, \ker C \rangle.$$

$\square$

DEFINITION 2.17. (Unobservability subspace). We say a subspace $\mathcal{S} \subseteq \mathcal{X}$ is a (C, A) unobservability subspace if $\mathcal{S} = \langle A + DC \,|\, \ker C \rangle$ for some output injection map $D : \mathcal{Y} \to \mathcal{X}$, *i.e.*, $\mathcal{S}$ is the unobservable subspace of the pair $\big(C, (A + DC)\big)$. The significance of the unobservability subspace derives from the fact that by the restriction of $A + DC$ to a $(C, A + DC)$-conditioned invariant subspace can be assigned an arbitrary spectrum by suitable choice of the output injection matrix D. $\square$

Clearly, from the orthogonality property, the annihilator of $\mathcal{S}$ is $\mathcal{S}^{\perp} = \langle A^T + C^T D^T \,|\, \operatorname{Im} C \rangle$ and $\mathcal{S}^{\perp}$ is an (A^T, C^T) controllability subspace, *cf.* (Wonham, 1979).

2.2. SOME SPECIFIC COMPUTATIONAL ALGORITHMS

Subspaces $\mathcal{S}^* = \inf \mathcal{S}\langle(C,A)\,|\,\mathcal{D}\rangle$ and $\mathcal{V}^* = \sup \mathcal{V}\langle(A,B)\,|\,\mathcal{E}\rangle$, which are respectively the infimum of the semilattice of all (C,A)-conditioned invariants containing a given subspace $\mathcal{D}$ and the supremum of the semilattice of all (A,B)-controlled invariants contained in a given subspace $\mathcal{E}$, are frequently used in our detection theory. They can be determined with computational algorithms. The basic algorithms which will be referred to in the later parts of this work are the following.

ALGORITHM 2.18. (Minimal (C,A)-conditioned invariant containing the subspace $\mathcal{D}$). The subspace $\mathcal{S}^* = \inf \mathcal{S}\langle(C,A)\,|\,\mathcal{D}\rangle$ can be obtained in a recursive procedure

$$\begin{aligned} &\mathcal{W}_o = \mathcal{D} \\ &\mathcal{W}_i = \mathcal{D} + A(\mathcal{W}_{i-1} \cap \ker C), \quad i = 1, \dots, k \end{aligned}$$

where the terminating $k \le n-1$ is determined by the condition $\mathcal{W}_{k+1} = \mathcal{W}_k$, *i.e.,* $\mathcal{W}^* = \lim \mathcal{W}_k$. We will refer to this algorithm as the (C,A)-invariant subspace algorithm (CAISA) in the following.

ALGORITHM 2.19. (Unobservability subspace). The unobservability subspace, which, by Definition 2.17 is, in fact a $(C, A+DC)$-conditioned invariant subspace can be given by the recursive procedure

$$\begin{aligned} &\mathcal{W}_o = \mathcal{X} \\ &\mathcal{W}_i = \mathcal{S}^* + (A^{-1}\mathcal{W}_{i-1}) \cap \ker C, \quad i = 1, \dots, k \end{aligned}$$

for $k = 1, 2, \dots, n-1$. The infimal element $\mathcal{W}^*$ is obtained as $\lim \mathcal{W}_k$ where the subspace $\mathcal{S}^* = \inf \mathcal{S}\langle(C,A)\,|\,\mathcal{D}\rangle$ can be precalculated by the CAISA above. The algorithm is called the unobservability subspace algorithm (UOSA) in the literature. □

From Property 2.5 and from

$$\mathcal{E} \supseteq \mathcal{V} \quad \Leftrightarrow \quad \mathcal{E}^{\perp} \subseteq \mathcal{V}^{\perp} \tag{2.5}$$

the orthogonality relation of subspaces

$$\sup \mathcal{V}\langle(A,B)\,|\,\mathcal{E}\rangle = \left(\inf \mathcal{S}\langle(A^T, B^{\perp})\,|\,\mathcal{E}^{\perp}\rangle\right)^{\perp} \tag{2.6}$$

can be easily checked which relates the determination of the subspace $\mathcal{V}^* = \sup \mathcal{V}\langle(A,B)\,|\,\mathcal{E}\rangle$ to that of $\mathcal{S}^* = \inf \mathcal{S}\langle(A,C)\,|\,\mathcal{D}\rangle$. Relation 2.6 makes the dualization of the (C,A)-conditioned invariant algorithm (Algorithm 2.18) possible in the following way:

ALGORITHM 2.20. (Maximal (A,B)-controlled invariant contained in the subspace $\mathcal{E}$). A recursive procedure which provides the subspace $\mathcal{V}^* = \sup \mathcal{V}\langle(A,B)\,|\,\mathcal{E}\rangle$ in the last recursion can be given as

$$\begin{aligned} &\mathcal{W}_o = \mathcal{E} \\ &\mathcal{W}_i = \mathcal{E} \cap A^{-1}(\mathcal{W}_{i-1} + \operatorname{Im} B), \quad i = 1, \dots, k \end{aligned}$$

where the terminating $k \leq n-1$ is determined by the condition $\mathcal{W}_{k+1} = \mathcal{W}_k$. It can be seen that the recursion converges to the orthogonal complement of $\mathcal{S}^* = \inf \mathcal{S}(A^T, B^\perp, \mathcal{E}^\perp)$ which in fact equals to $\mathcal{V}^* = \sup \mathcal{V}(A, B, \mathcal{E})$ by (2.6).

ALGORITHM 2.21. (Reachability (controllability) subspace). For an arbitrary, fixed subspace $\mathcal{R} \subset \mathcal{X}$ define the sequence of recursion

$$\begin{aligned} \mathcal{W}_o &= 0 \\ \mathcal{W}_i &= \mathcal{R} \cap (A\mathcal{W}_{i-1} + \operatorname{Im} B), \quad i = 1, \ldots, k \end{aligned}$$

for $k = 1, 2, \ldots, n-1$, where $\mathcal{W}^* = \lim \mathcal{W}_k$. The algorithm is called the controllability subspace algorithm (CSA) in the following.

ALGORITHM 2.22. (Supremal reachability (controllability) subspace). Due to Definition 2.12, every subspace $\mathcal{E} \subset \mathcal{X}$ contains a unique supremal controllability subspace $\mathcal{R}^* = \mathcal{R}_{\mathcal{V}^*}$. The computation of this subspace requires *a priori* knowledge of $\mathcal{V}^* = \mathcal{V} \sup \langle (A, B) \,|\, \mathcal{E} \rangle$. A method that can be used for precomputing the subspace $\mathcal{V}^*$ was presented by Algorithm 2.20. For the computation of this supremal subspace $\mathcal{R}_{\mathcal{V}^*}$ consider the recursive sequence

$$\begin{aligned} \mathcal{W}_o &= 0 \\ \mathcal{W}_i &= \mathcal{V}^* \cap (A\mathcal{W}_{i-1} + \operatorname{Im} B), \quad i = 1, \ldots, k \end{aligned}$$

for $k = 1, 2, \ldots, n-1$. Then, $\mathcal{R}_{\mathcal{V}^*}$ is obtained in $\lim \mathcal{W}_k$.

2.3. THE CONCEPT OF UNKNOWN INPUT OBSERVER FOR LINEAR SYSTEMS

Different types of modeling uncertainties such as nonlinearities, parameter variations and other unmeasurable external disturbances can conveniently be represented as unknown inputs which, in general, are also termed disturbances. Consider the system

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + Ld(t) \\ y(t) &= Cx(t) \end{aligned} \tag{2.7}$$

where $u(t)$ denotes the manipulable known input, $d(t)$ is the unknown input, which at the moment is assumed to be completely unaccessible for measurement, and inspect the problem of realizing, if possible, a state feedback of the type shown in Fig. 2.1 such that, starting at the known initial condition, y(t)=0 results for all admissible d(t). This problem can be identified as the unaccessible disturbance localization problem in the literature, see *e.g.,* (Basile and Marro, 1969a; Basile and Marro, 1969b). The system with state feedback is described by

$$\begin{aligned} \dot{x}(t) &= (A + BF)x(t) + Ld(t) \\ y(t) &= Cx(t), \end{aligned} \tag{2.8}$$

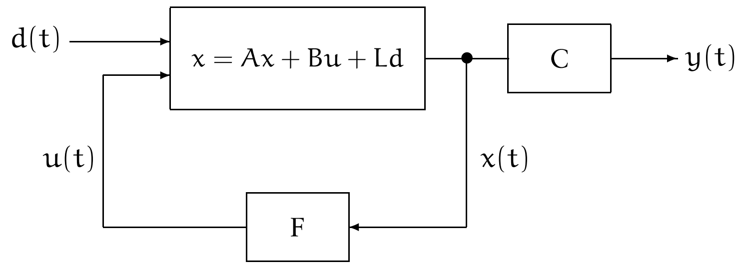

Figure 2.1. The unaccessible disturbance localization problem with state feedback.

and presents the requested behavior if and only if its reachable set by $d(t)$, *i.e.*, the minimal $(A+BF)$-invariant containing $\operatorname{Im} L$, is contained in $\ker C$. Since any $(A+BF)$-invariant subspace is an (A, B)-controlled invariant, the unaccessible disturbance localization problem admits a solution if and only if the following structural condition holds

$$\mathcal{L} \subseteq \mathcal{V}^* \tag{2.9}$$

where $\mathcal{V}^* = \sup \mathcal{V} \langle A, \mathcal{B} \,|\, \ker C \rangle$ and $\mathcal{L} = \operatorname{Im} B$.

It is worth noting two things here. First, the assumption of the possibility of the application of full state feedback shown in Fig. 2.1 is not feasible in practice since, in most cases, state is not completely accessible for measurement; second, the assumption of stability, *i.e.*, the property of the system that the matrix F, besides disturbance localization, achieves stability of the overall closed-loop system $A + BF$ is to be taken into consideration especially from practical points of view. These issues are considered in the following part of this section.

Introducing some subtle differences in the setup of the previous problem we shall now consider the asymptotic estimation of the state (or a linear function of the state, possibly the whole state) in the presence of the unaccessible disturbance input. This problem is referred to the disturbance decoupled estimation problem (DDEP) in the literature. If the problem of simultaneous observation of states and the estimation of unknown inputs is investigated the problem is also known as the unknown input observer problem (UIOP). As one of the first solutions to UIOP, Wang *et al.* proposed a minimal-order observer for the system (2.7) without making any assumptions on the properties of unknown inputs (Wang et al., 1975). This approach was followed by many authors presenting different unknown input observer design ideas (Willems and Commault, 1981; Hou and Müller, 1988; Hou and Müller, 1992). The geometric approach of the problem was first introduced by (Bhattacharyya, 1978). Different solutions of UIOP, from many obvious reasons, have a number of implications to fault detection and isolation, which have been investigated in a series of papers, see *e.g.*, (Frank and Wünnenberg, 1989; Frank, 1990; Hou and Müller, 1994).

The mathematical problem of obtaining the state from input and output measurements when some of the inputs are unknown has solvability conditions more extended than the problem of estimating the state by using a dynamic observer. For the cases when the input function is (at least partially) unknown the concept of unknown-input observability or unknown-input

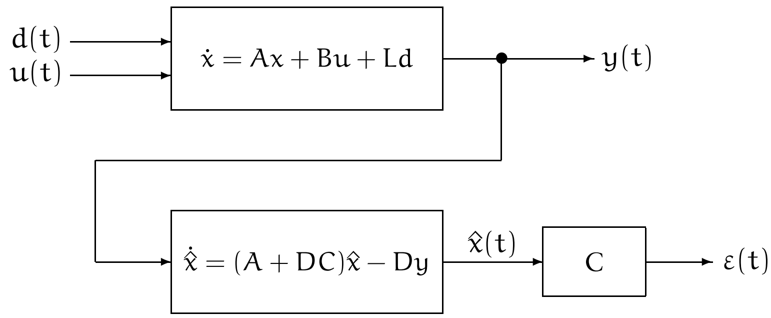

Figure 2.2. Estimation in the presence of unaccessible disturbance i.e., the concept of the unknown input observer problem.

reconstructability[4] is presented by showing that unknown-input reconstructability is closely related to invertibility.

Consider system (2.7) and the behavior of the observer designed for estimating the states (see the discussion in the previous chapter with Fig. 1.6) when, as represented in (2.7), the observed system has, besides the accessible input $u(t)$, the unaccessible input $d(t)$

$$\dot{\hat{x}}(t) = (A + DC)\hat{x}(t) + Bu(t) - Dy(t). \tag{2.10}$$

Subtracting (2.10) from (2.7) the error dynamics $\varepsilon(t) = \hat{x}(t) - x(t)$ is obtained in the form

$$\dot{\varepsilon}(t) = (A + DC)\varepsilon(t) - Ld(t), \tag{2.11}$$

which shows that the estimation error does not converge asymptotically to zero, even if $A+DC$ is stable, but converges asymptotically to a subspace $\inf \langle A+DC \,|\, \mathcal{L} \rangle$, which, in fact, equals with the reachable set $\mathcal{R}$ of the system (2.11) (cf. Definition 2.11).

It follows that, in order to obtain the state estimate in presence of unknown inputs, it is convenient to choose D to make this subspace of minimal dimension: since it is an (A, C)-conditioned invariant, the best choice of D corresponds to transforming into an $(A + DC)$-invariant subspace: *i.e.*, the minimal (A, C)-conditioned invariant which contains $\operatorname{Im} L$.

2.3.1. *System invertibility and reconstructability*

In the following part let us briefly characterize the property of unknown-input observability. From a strictly mathematical viewpoint, unknown-input observability can be introduced and characterized as follows. It is well known that, collecting the effects of the inputs u and d in a common term, the response of system (2.7) given by the triple (A, B, C) can be related to the initial state $x(0)$ and control function $u(t)$ by

$$y(t) = Ce^{At}x(0) + C\int_0^t e^{A(t-\tau)}Bu(\tau)d\tau. \tag{2.12}$$

[4] For continuous time linear systems observability and reconstructability is equivalent.

Equation (2.12) consists of two terms, the free and the forced responses of the system denoted by the operators $\gamma_1(x(0), t)$ and $\gamma_2(u, t, \tau)$, respectively. By using this notation, Eq. (2.12) for the finite time interval $[0, T]$ can be rewritten as

$$y(t)\,|_{[0,T]} = \gamma\big(x(0), u\,|_{[0,T]}\big) = \gamma_1\big(x(0)\big) + \gamma_2\big(u(t)\,|_{[0,T]}\big). \tag{2.13}$$

The initial (and the final) state of the system can be calculated from input and output observations: this requires the pair (A, C) to be observable (reconstructable). Recall that (A, C) is observable, if γ_1 is invertible, *i.e.*, $\ker \gamma_1 = 0$, see (Wonham, 1979).

For the cases when the input function is unknown the concept of reconstructability can be extended by relating it to the concept of system invertibility, see (Basile and Marro, 1991). The term system invertibility denotes the possibility of reconstructing the input from the output function. For the sake of precision it is possible to define both the unknown-state, unknown-input invertibility and the zero-state, unknown-input invertibility. Consider the following definitions.

DEFINITION 2.23. The triple (A, B, C) is said to be unknown-state, unknown-input reconstructable (or unknown-state, unknown-input invertible) if, in (2.13) γ is invertible, *i.e.*, $\ker \gamma = 0$. □

DEFINITION 2.24. The triple (A, B, C) is said to be zero-state, unknown-input reconstructable (or zero-state, unknown-input invertible) if, in (2.13) γ_2 is invertible, *i.e.*, $\ker \gamma_2 = 0$. □

When (A, C) is not observable or reconstructable

$$\ker \gamma_1 = \mathcal{S}^* \triangleq \sup \langle A \,|\, \ker C \rangle, \tag{2.14}$$

that follows from the definition of the unobservability subspace (*cf.* Definition 2.19). This means that the state canonical projection on the factor space $\mathcal{X}/\mathcal{S}^*$ can be determined from the output function $y(t)$.

Unknown-input reconstructability can be approached in a similar way: by linearity, when reconstructability is not complete, only the canonical projection of the final state on $\mathcal{X}/\mathcal{S}_1$ or $\mathcal{X}/\mathcal{S}_2$ can be determined, where $\mathcal{S}_1$ is the unknown-state, unknown-input unreconstructability subspace and $\mathcal{S}_2$ is the zero-state unknown-input unreconstructability subspace. From the mentioned properties it follows that $\mathcal{S}_2 \subseteq \mathcal{S}_1$.

The geometric characterization of these subspaces can be given by the following properties (Basile and Marro, 1991). The unknown-state, unknown-input unreconstructability subspace is

$$\mathcal{S}_1 = \mathcal{V}^* \triangleq \sup \langle (A, B) \,|\, \ker C \rangle, \tag{2.15}$$

i.e., the maximal (A, B)-controlled invariant subspace contained in $\ker C$. The zero-state, unknown-input unreconstructability subspace is

$$\mathcal{S}_2 = \mathcal{R} = \mathcal{V}^* \cap \mathcal{S}^* \quad \text{with} \quad \mathcal{S}^* = \inf \langle (A, C) \,|\, \operatorname{Im} B \rangle \tag{2.16}$$

i.e., the minimal (C, A)-conditioned invariant subspace containing in $\operatorname{Im} B$.

Although both unknown-state, unknown-input invertibility and the zero-state, unknown-input invertibility was given by Definitions (2.23) and (2.24), in the following discussions the term 'invertibility' will be referred to the latter, *i.e.,* zero-state, unknown-input invertibility which is related to the invertibility of the operator γ_2 in (2.13). Furthermore, invertibility can be associated either to controllability or reconstructability.

DEFINITION 2.25. On the one hand, by the term *functional controllability* we entitle the property of the system upon which, by applying a suitable input function u(t), starting from the zero state, any sufficiently smooth output function y(t) can be approximated. From the identity

$$y(t) = \gamma_2\big(u(t)\big) \circ \gamma_2^{-1}\big(y(t)\big)$$

this property is usually referred to *right invertibility*.

DEFINITION 2.26. On the other hand, by considering reconstructability of the input from the output one arrives to the identity

$$u(t) = \gamma_2^{-1}\big(y(t)\big) \circ \gamma_2\big(u(t)\big)$$

which is normally referred to *left invertibility*.

In the following discussions the term invertibility will always refer to the property of *left invertibility* in sense of the above definition even if it is not mentioned directly.

2.4. GEOMETRIC INTERPRETATION OF THE RESIDUAL FOR LINEAR SYSTEMS

Now we can return to the formulation of the observer-based residual generation problem presented in the previous chapters whose solvability in the presence of unmeasurable disturbance input can now be formulated in terms of the (A, C)-conditioned invariant subspaces.

Consider the single-fault single-residual approach, *i.e.,* consider the case when $k = 1$ in the state-space representation (1.34). Our objective is to design an observer for this nominal system in the form (1.35) in an attempt to detect this fault. One can see that the closed-loop eigenvalues λ_j and eigenvectors ν_j of the error system (1.38) are determined by

$$\big(\lambda_j I - (A + DC)\big)\nu_j = 0 \tag{2.17}$$

for $j = 1, 2, \ldots, n$, where $C\nu_j \neq 0$ since observability is assumed. The eigenvectors ν_j are independent and span the error variable state-space. The fault vector L can therefore be expressed in this basis and written as a linear combination of ν_j as

$$L = \sum_{j=1}^{n} \alpha_j \nu_j. \tag{2.18}$$

It follows from (2.17) and (2.18) that

$$(A + DC)L = \sum_{j=1}^{n} \alpha_j \lambda_j \nu_j. \tag{2.19}$$

DEFINITION 2.27. The observable space with respect to $(A + DC, C)$ of (1.38) is spanned by the vectors of the Kalman observability matrix

$$\mathcal{O} = \{C^T \quad (C(A + DC))^T \ldots (C(A + DC)^{n-1})^T\}.$$

□

DEFINITION 2.28. The subspace of the error variable state-space which represent that part of the state-space affected by L is the subspace $\mathcal{W} \subseteq \mathcal{O}$ containing $\operatorname{Im} L$ and is called the detection space of the observer. □

Since the summation term in (2.19) represents some vector in the detection space of L it can be seen that the detection space $\mathcal{W}$ is an $(A + DC)$ invariant subspace *i.e.*, in the sense of Definition 2.4, it is a (C, A)-invariant subspace of the error variable space containing $\operatorname{Im} L$

$$\mathcal{W} = \langle A + DC \,|\, \mathcal{L} \rangle. \tag{2.20}$$

2.5. THE FUNDAMENTAL PROBLEM OF RESIDUAL GENERATION IN LINEAR SYSTEMS

Consider the system model given in (1.34) subject to k simultaneous fault effects. Let our objective be to design a residual generator providing output which is sensitive to only one of the failure modes but insensitive to all of the others. This problem is identified as the fundamental problem of residual generation (FPRG) in the literature which was first given by (Massoumnia et al., 1989). The problem can be considered an early approach to robust fault detection filter design because — as it will be shown in this section, — it gives the conditions of the detections of a single fault independently from another faults or, what makes the same, independently from any unmeasurable disturbances acting as unknown inputs.

2.5.1. *FPRG for two failure modes*

Consider $k = 2$ in the system model (1.34), *i.e.*, we assume two failure events may affect the system. Let our objective be to design a residual generator providing output sensitive to one of the failures but insensitive to the other. For convenience (1.34) is then written

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + L_1\nu_1(t) + L_2\nu_2(t) \\ y(t) &= Cx(t). \end{aligned} \tag{2.21}$$

Based on the discussion in the previous chapter the most general form of a residual generator, that was given in the form of (1.35), takes the observables $y(t)$ and $u(t)$ and provides the residual (1.36) which can be represented in the more general form

$$r(t) = E\hat{x}(t) - Fy(t) + Gu(t) \tag{2.22}$$

where $\hat{x}(t) \in \hat{\mathcal{X}}$. From representations (1.35), (2.21) and (2.22) one can construct the extended state-space

$$\begin{bmatrix} \dot{x}(t) \\ \dot{\hat{x}}(t) \end{bmatrix} = \begin{bmatrix} A & 0 \\ -DC & A+DC \end{bmatrix} \begin{bmatrix} x(t) \\ \hat{x}(t) \end{bmatrix} + \begin{bmatrix} B & L_2 \\ B & 0 \end{bmatrix} \begin{bmatrix} u(t) \\ \nu_2(t) \end{bmatrix} + \begin{bmatrix} L_1 \\ 0 \end{bmatrix} \nu_1(t),$$

$$\varepsilon(t) = [-FC \quad E] \begin{bmatrix} x(t) \\ \hat{x}(t) \end{bmatrix} + [G \quad 0] \begin{bmatrix} u(t) \\ \nu_2(t) \end{bmatrix}. \tag{2.23}$$

With the definition of the extended state-space $\mathcal{X}_e = \mathcal{X} \oplus \hat{\mathcal{X}}$ and, by similarly constructing the space $\mathcal{U}_e$, the representation (2.23) can be rewritten as

$$\begin{aligned} \dot{x}_e(t) &= A_e x_e(t) + B_e u_e(t) + L_e \nu_1(t) \\ r(t) &= E_e x_e(t) + G_e u_e(t) \end{aligned} \tag{2.24}$$

where $x_e(t) \in \mathcal{X}_e$ and $u_e(t) \in \mathcal{U}_e$ and the matrices A_e, B_e, L_e, E_e, G_e are in a clear correspondence with (2.23).

Reviewing (2.24) closely, the relationship between FPRG and the disturbance decoupled estimation problem (DDEP) presented in Chapter 2.3 can be readily recognized. The difference between the two problems is that while in DDEP, the state to be estimated is always given as part of the problem statement, in FPRG this is usually the part of the problem to find that part of the state space that can be estimated even in the presence of the unknown input. In FPRG, incomplete state estimate may be fully satisfactory if, for instance, it is not necessary to know the whole state, but only a given linear function of it.

The residual system (2.24) can be written in its transfer function form as

$$\varepsilon(s) = G_{\varepsilon u}(s)u(s) + G_{\varepsilon\nu_1}(s)\nu_1(s) + G_{\varepsilon\nu_2}(s)\nu_2(s). \tag{2.25}$$

Based on representations (2.24-2.25) the design criteria of the filter producing residual in which the failure mode $\nu_1(t)$ shows up while $\nu_2(t)$ remains hidden can be given in various conditions. On the one hand, the most natural approach is to require that the transfer function from $\nu_1(t)$ to $\varepsilon(t)$, (*i.e.*, $G_{\varepsilon\nu_1}(s)$ in (2.25)), be left invertible by ensuring that $G_{\varepsilon\nu_2}(s)$ and $G_{\varepsilon u}(s)$ are identically zero (*cf.* with the problem presented in (2.12-2.13). If this invertibility condition is satisfied, then any nonzero $\nu_1(t)$ will result in a residual $\varepsilon(t)$ different from zero. On the other hand, it is clear from Definitions 2.23-2.24 that, instead of imposing invertibility (which is a stronger condition), it is enough to request that the system relating $\nu_1(t)$ to $\varepsilon(t)$ be input observable. Note that in the vast majority of the cases the failure signatures, and so the transfer matrix $G_{\varepsilon\nu_1}(s)$, are column vectors for which the properties of left invertibility and input observability are equivalent.

We remind the reader that the system (2.24) is input observable if B_e is monic (*i.e.*, it has maximal column rank) and $\operatorname{Im} B_e$ does not intersect the unobservable subspace $\mathcal{S}_e^*$ of the pair (A_e, E_e), *i.e.*,

$$\operatorname{Im} B_e \cap \mathcal{S}_e^* = 0.$$

The idea of the residual generator design satisfying FPRG is, therefore, to place the failure signature not wishing to represent in the residual (*i.e.*,$\nu_2(t)$ in our case) in the unobservable subspace in the output space of (2.24). Using the geometric language this problem has a solution if

$$\mathcal{S}_1^* \cap \mathcal{L}_1 = 0 \tag{2.26}$$

where $\mathcal{S}_1^* = \inf \langle \mathcal{S} \,|\, \mathcal{L}_2 \rangle$ and $\mathcal{S} = \sup \langle A_e \,|\, \ker E_e \rangle$. Based on the above idea, the design procedure of the filter which enforces $\nu_1 \to \varepsilon_1$ input observable and $\nu_2 \to \varepsilon_2 = 0$ can be summarized in the following two steps procedure.

STEP 1. Determine the effect of ν_2 in the state space by establishing the minimal (C, A)-conditioned invariant containing the subspace $\mathcal{L}_2$ *i.e.*, $\mathcal{W}_2^* = \inf \mathcal{W}_2 \langle (C, A) \,|\, \mathcal{L}_2 \rangle$ using the (C, A)-controlled invariant subspace algorithm (Algorithm 2.18).

STEP 2. Design an observer for the detection of ν_1 such that its unobservability subspace contains this (C, A)-conditioned invariant $\mathcal{W}^*$, that is to say, we attempt to hide ν_2 at the output of the detector. For the solution of this problem the unobservability subspace algorithm (Algorithm 2.19) can be used as

$$\begin{aligned} \mathcal{S}_1^o &= \mathcal{W}_2^* + \ker C \\ \mathcal{S}_1^i &= \mathcal{W}_1^* + (A^{-1}\mathcal{S}_{i-1}) \cap \ker C, \quad i = 1, \dots, k \end{aligned}$$

for $k = 1, 2, \dots, n-1$, where $\mathcal{S}_1^*$ is obtained as $\lim \mathcal{S}_1^k$.

2.5.2. *Extension of FPRG to multiple faults and the relation to system invertibility*

FPRG can be extended to multiple (*i.e.*, $k > 2$) fault events. In this case we have the system representation

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + L_i\nu_i(t) + \sum_{j \neq i} L_j \nu_j(t) \quad i = 1, \dots, k \\ y(t) &= Cx(t). \end{aligned} \tag{2.27}$$

By making such an assumption, we want to design a residual generator that generates k residuals, $\varepsilon_i(t)$, $i \in k$, such that the i^{th} failure mode $\nu_i(t)$ affects the i^{th} residual but only this residual and not showing up in any other. For system (2.27) the extended state space can be given in the same form as (2.24), but now the term $B_e u_e(t)$ will contain the multiple fault effects too.

Considering the conditions of detectability of the single fault in presence of another simultaneous fault presented in the preceding section, now the system relating $\nu_i(s)$ to $\varepsilon_i(s)$ is required to be input observable, while the transfer functions from $\nu_i(s)$ to all other residuals $\varepsilon_j(s)$, $j \neq i$, should be zero. Note that the identification of k simultaneaous failure modes necessitates the production of at least k residuals, one residual for each of the fault effects. The solvability condition of FPRG for multiple faults can be given analogously to (2.26) as

$$\mathcal{S}_i^* \cap \mathcal{L}_i = 0, \quad i \in k \tag{2.28}$$

where $\mathcal{S}_i^* = \inf \langle \mathcal{S} \,|\, \sum_{j \neq i} \mathcal{L}_j \rangle$ and $\mathcal{S} = \sup \langle A_e \,|\, \ker E_e \rangle$.

It is interesting to relate this general solvability condition of FPRG to system invertibility. Consider therefore the system model (2.27) in the frequency domain as

$$y(s) = Gu(s)u(s) + G_\nu(s)\nu(s) \tag{2.29}$$

with $\nu(s) = (\nu_1(s), \dots, \nu_k(s))^T$ and the transfer functions

$$G_u(s) = C(sI - A)^{-1}B, \tag{2.30}$$

$$G_\nu(s) = C(sI - A)^{-1}(L_1, \dots, L_k). \tag{2.31}$$

Again, from Definitions (2.23) and (2.24) it follows that the FPRG has a solution if and only if the transfer matrix $G_\nu(s)$ is left invertible. In this case the fault identifiability conditions (2.26) and (2.28) are equivalent to the left invertibility of (2.31) assuming $\nu(s)$ is a column vector.

The reader may find different design methods satisfying the failure identifiability conditions (2.26) and (2.28) of FPRG in the literature which are not discussed here. Most of them are relied on the appropriate assignment of the spectrum (*i.e.*, relocation of the eigenvalues) of the residual generator, see *e.g.*, (Massoumnia et al., 1989), (White and Speyer, 1987).

One thing, however, should be important to note. By solving the FPRG problem the detection of the failure signature $\nu_i(t)$ could be achieved *independently* of any other $\nu_j(t)$, $i \neq j$, which appear simultaneously in the system. It is said, therefore, that the failure effect $\nu_i(t)$ could be exactly decoupled from other faults in the output space of the residual generator.

Obviously, if looking at the problem of FPRG in view of the DDEP, a single failure $\nu(t)$ could be detected in the presence of unmeasurable disturbance input by using the very same design methodology. This indicates the potential of the idea in robust fault detection filter design.

2.6. Detection and isolation by means of exact geometric decoupling in linear systems

Detection filters are classical tools which rely on the use of particular type of state observers producing residuals with directional characteristics. The basic idea of detection filters is that when using a single observer, the observer gain can be chosen such that the direction of the innovation vector in the space of the output error residual can uniquely identifies the location of the failure. Different state space directions can be associated with prespecified failure types thus ensuring both *detection* and *isolation* of failures. The formulation of this type of detection problem together with the geometric approach to the analysis of this particular type of filters were originally devised by (Beard, 1971) and (Jones, 1973), hence the literature knows about this filtering problem as the Beard-Jones detection filter problem (BJDFP). The idea was further refined in a series of papers by a long line of other researchers such as *e.g.*, (Frank and Keller, 1980; White and Speyer, 1987; Massoumnia, 1986; Massoumnia et al., 1989), just to mention a few.

Consider for instance the LTI system (1.34). Let our objective be to detect and isolate failure modes $\nu_i(t)$ by applying a single state observer (1.35) shown in Fig. 1.7. By using the geometric language, the traditional detection filter design problem can be summarized as follows.

DEFINITION 2.29. A detection filter, capable to detect and isolate multiple faults, is a state observer of form (1.35) whose static gain matrix D is chosen such that the effects of failure modes $\nu_i(t)$ are assigned to independent subspaces $\mathcal{W}_i \subseteq \mathbb{R}^n$, i.e.,

$$\operatorname{Im} L_i \subseteq \mathcal{W}_i, \quad (A + DC)\mathcal{W}_i \subseteq \mathcal{W}_i, \quad i = 1, \ldots, k \tag{2.32}$$

such that

$$\operatorname{Im} L_i \cap \ker C = 0, \tag{2.33}$$

moreover, the output image of $\mathcal{W}_i$ in the output error space is decoupled, i.e.,

$$C\mathcal{W}_i \cap \sum_{i \neq j}^{k} C\mathcal{W}_j = 0 \quad i, j = 1, \ldots, k. \tag{2.34}$$

□

As a further design consideration, the closed loop transition matrix $(A + DC)$ is required to be stable, *i.e.*, its eigenvalues λ have negative real parts assuming its spectrum σ is arbitrarily assignable with only conjugate symmetry constraints, $\max\{\operatorname{Re}\lambda : \lambda \in \sigma(A + DC)\} < 0$.

Relations (2.32), (2.33) and (2.34) are the *detectability*, *input observability* and *output separability* principle of the design, respectively. The subspaces $\mathcal{W}_i$ are called detection spaces of the filter. Relation (2.32) shows that $\mathcal{W}_i$ are (C, A)-invariant subspaces of the pairs $(A + DC, L_i)$. For basic practical reasons it is important to use extremal (C, A)-invariant subspaces in the design. We want to find the family of the smallest possible subspaces $\mathcal{W}_i$ satisfying Principles (2.32–2.34).

The (C, A)-invariance property of $\mathcal{W}_i$ implies that the *controllable* space of L_i with respect to the closed-loop transition matrix $(A + DC)$ is the infimal (C, A)-invariant subspace containing $\operatorname{Im} L_i$, *i.e.*, $\inf\langle A + DC \,|\, \mathcal{L}_i\rangle$. This will be denoted by $\mathcal{W}_i{}^*(\mathcal{L}_i)$ in the sequel. That is to say, the family of the controllable subspaces of $(A + DC, L_i)$ is a subfamily of the (C, A)-invariant subspaces of the filter.

Another family of minimal subspaces is the socalled supremal unobservability subspace $\mathcal{T}_i^* = \inf \mathcal{S}_i^*(\mathcal{W}_i^*)$ of the controllability subspace $\mathcal{W}_i{}^*$, $i = 1, \ldots, k$. We focus on the controllability subspace methods in the sequel.

Recall that the controllability subspace is the set $\mathcal{W}^* \subseteq \mathbb{R}^n$ of initial points $x(0)$ that can be controlled by appropriate state feedback K to the origin of the state-space in finite time. $\mathcal{W}^*$ is always a linear subspace of $\mathbb{R}^n$. When $\mathcal{W}^* = \mathbb{R}^n$ the system is said to be controllable. More precisely, $\mathcal{W}_i^*$ is the controllable subspace of the pair $(A + DC, L_i)$, *i.e.*,

$$\mathcal{W}_i^* = \langle A + DC | \operatorname{Im} L_i \rangle.$$

For the computation of these minimal (C, A)-invariant subspace constructions the recursive algorithm

$$\mathcal{W}_i^{\ell+1} = \quad \operatorname{Im} L_i + A(\mathcal{W}_i^{\ell} \cap \ker C), \qquad \mathcal{W}_i^{o} = 0, \tag{2.35}$$

is available due to (Wonham, 1979), and the infimal subspace $\mathcal{W}_i^*$ is given as $\lim_{\ell\to\infty} \mathcal{W}_i^{\ell}$.

The geometrical filter design problem was reformulated as an eigenvalue assignment problem in (White and Speyer, 1987) that is useful for filter synthesis in LTI systems. This theory is based on an additional design condition. It requires that the output directions CL_i associated by the state-space directions L_i of the particular faults maintain the fixed output directions Cv_j^i in the output space, i.e., the residual directions $\varepsilon_i(t)$ be unidirectional with the eigenvectors v_j for all k. It immediately follows that the maximum number of faults k which can be detected and isolated with a single filter is always less than or equal to the number of eigenvectors, *i.e.*, the dimension of the error variable state space.

2.7. Generalized state observer for LTV systems

For an easier transition from the linear time invariant world to the area of nonlinear systems, let us mention some interesting geometric properties based on our paper (Edelmayer et al., 1999). The idea presented in this section will prove useful for the introduction of the system inversion-based filter design methods discussed in the later chapters of this thesis.

Consider the representation of the linear system subjected to multiple faults and linear time varying (LTV) perturbations

$$\begin{aligned} \dot{x}(t) &= A(t)x(t) + Bu(t) + \sum_{i=1}^{k} L_i \nu_i(t), \\ y(t) &= Cx(t) \end{aligned} \tag{2.36}$$

where $x \in \mathcal{X} \subset \mathbb{R}^n$, and the real constant matrices B and C are in the appropriate dimensions. Assume that the parametric perturbations can be represented by the LTV perturbation structure

$$A(t) = A + \Delta A(t) = A + \sum_{i=1}^{m} a_i(t) A_i, \tag{2.37}$$

where the nominal system matrix A is stable. A_i are assumed to be non-destabilizing[5] known constant matrices and $a_i(t) \in L_2$ are arbitrary bounded smooth perturbation functions of time. For non-LTI systems, the eigenvector assignment approach mentioned in the previous section, is not useful because of the possible stability problems. In order to show this property, consider, for example the system (2.36) with asymptotically stable nominal A matrix and time varying perturbations

$$A(t) = \begin{pmatrix} \cos t & \sin t \\ -\sin t & \cos t \end{pmatrix} \begin{pmatrix} -1 & 5 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \cos t & -\sin t \\ \sin t & \cos t \end{pmatrix}$$

that has a (non-stable) branching solution at te^t, as it was presented *e.g.*, in (Cronin, 1994). This suggests that the solution of the disturbance robust DFP requires the application of new

[5] A system is said *stabilizable* if all its unstable modes are reachable, which implies that they can be stabilized. In this sense, the term non-destabilizing denotes a particular A_i that does not make any modes of the system unreachable.

design considerations, especially with respect to the description of (C, A)-invariant subspaces of the error variable system of the detector.

2.7.1. *Controllability and observability of LTV systems*

The significance of controllability of the LTV systems (2.36) was first recognized in the late sixties. Modern studies of controllability were initiated by (Mohler, 1973), (Kuĉera, 1970) which adopted Lie algebraic approaches and differential algebraic techniques to this problem. On the basis of the results of (Chow, 1939) it was shown by (Brockett, 1972) that the set of matrices reachable from the identity forms a Lie group whose algebra is generated by the structure matrices (A, A_i) of (2.36). The following proposition is based on earlier results of (Szigeti, 1992) and (Szigeti et al., 1995).

PROPOSITION 2.30. The controllability and observability subspaces of systems (2.36), are contained in the generalized Kalman subspaces,

$$\mathcal{K}_c = \sum_{n_o=0}^{n-1} \cdots \sum_{n_\ell=0}^{n-1} \mathrm{Im}\,(A_o^{n_o} \ldots A_\ell^{n_\ell} B) \tag{2.38}$$

$$\mathcal{K}_o = \sum_{n_o=0}^{n-1} \cdots \sum_{n_\ell=0}^{n-1} \mathrm{Im}\,({A'_o}^{n_o} \ldots {A'_\ell}^{n_\ell} C'), \tag{2.39}$$

correspondingly, assuming $A \triangleq A_o$, where $\{\bar{A}_o, \bar{A}_1, \ldots, \bar{A}_\ell\}$ forms a basis of the Lie algebra $\mathfrak{L}$ generated by $A_o, A_1, \ldots, A_m$ under the commutator product $[A_i, A_j] = A_iA_j - A_jA_i$. Equivalently, the unobservability subspace is contained in

$$\mathcal{K}_{uo} = \bigcap_{n_o=0}^{n-1} \cdots \bigcap_{n_\ell=0}^{n-1} \ker(CA_o^{n_o} A_1^{n_1} \ldots A_\ell^{n_\ell}). \tag{2.40}$$

□

PROPOSITION 2.31. If the perturbation functions $a(t) = \{a_1, \ldots, a_m\}$ in (2.37) are persistently exciting, *i.e.*, if they are differential algebraically independent, for definition see (Szigeti et al., 1995), then the controllability and observability subspaces are exactly the image spaces of the generalized Kalman matrices (2.38) and (2.39). Moreover, the unobservability subspace of (2.36) is the largest $A(t)$-invariant subspace of (2.40) contained in $\ker C$. □

2.7.2. *Detectability of LTV systems*

The notion of detectability is an extension of observability in systems where state observers are designed in an attempt to produce residuals for detecting and isolating faults. In order to generalize the concept of detectability traditionally used in LTI systems and increase design freedom, consider the generalization of the classical Luenberger observer (for $\nu_i = 0, \forall i$) by using the successive derivatives of the observation vector as

$$\begin{aligned}
\dot{y}(t) &= \sum_i a_i(t) C A_i x(t) + C B u(t) \\
&\vdots \\
y^{(k)}(t) &= \sum_i P(a_i, \dot{a}_i, \ldots, a_i^{(k-1)}) x(t) + \\
&\qquad + \sum_i Q(a_i, \dot{a}_i, \ldots, a_i^{(k-2)}, u, \dot{u}, \ldots, u^{(k-1)}),
\end{aligned}$$

which can be written in the more general form

$$y^k(t) = P_k(A, C)x(t) + \sum_{i=1}^{k-1} Q_k^i(A, B, C) u_i(t),$$

where $P_k(A(t), C)$ and $Q_k^i(A(t), B, C)$ are differential polynomials. Then, the state observer yields

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + \textstyle\sum_{j=0}^{J} D_j\big(-y^j(t) + P_j(A, C)\hat{x}(t) + \sum_i Q_k^i(A, B, C) u^i(t)\big) \tag{2.41}$$

which can be called the *prolongation* of the Luenberger observer of order j, where $D_j = \{d_{k\ell}\}^j$ is the *j*-th element of the observer gain. In the following part one needs to prove that (2.41), indeed, is a useful observer structure. Moreover, it is shown that the closed loop observer system obtained by the injection of the state and its derivatives is assignable over the observable subspace, is decoupled from the unobservable subspace, and is invariant over the unobservable subspace. From (2.41) for the error variable system $e = \hat{x} - x$ we get

$$\dot{e}(t) = (A + D_o C + D_1 \sum_i a_i(t) C A_i + \ldots)\ e(t)$$

i.e.,

$$\dot{e}(t) = \Big(A + \sum_{j=0}^{J} D_j P_j\big(A(t), C\big)\Big) e(t). \tag{2.42}$$

DEFINITION 2.32. The LTV system (2.36) is detectable if there exists a prolongated (often called as P-feedback) state observer of form (2.41) whose corresponding error equation (2.42) is asymptotically stable.

PROPOSITION 2.33. If the non-observable part of the LTV representation is asymptotically stable, the LTV system is still detectable.

PROPOSITION 2.34. The LTV system is assignable, *i.e.,* its spectrum σ can be assigned arbitrarily, if for all G there exists a state observer (2.41) such that its error equation can be written in the form

$$\dot{e}(t) = Ge(t). \tag{2.43}$$

Assume the derivative of the highest order term which is to be injected to the observer equation (2.41) is s. Let the matrix

$$\begin{aligned} G(t) &= A + \sum_{j=0}^{s} D_j P(j)\big(A(t), C\big) \\ \text{with} \qquad G &= \left[\begin{array}{c|c} G_{11} & 0 \\ \hline G_{21} & G_{22} \end{array}\right] \end{aligned} \tag{2.44}$$

be the desired error dynamics of the filter, where $G_{22} \triangleq A_{22}$ is the unobservable dynamics of the original system. Eq. (2.44) represents a set of linear equations for the coefficients $\{d_{k\ell}\}^j$. For a particular selection of the filter gains $D_j(t)$, namely, which satisfy the linear equations for $G_{21}(t) = 0$ and an asymptotically stable $G_{11}(t)$ the error equation is decoupling to

$$\begin{bmatrix} \dot{e}_o(t) \\ \dot{e}_{uo}(t) \end{bmatrix} = \begin{bmatrix} G_{11} & 0 \\ 0 & G_{22} \end{bmatrix} \begin{bmatrix} e_o(t) \\ e_{uo}(t) \end{bmatrix}. \tag{2.45}$$

□

PROPOSITION 2.35. Suppose there exists a subinterval $[0, T_o] \subset [0, T]$ where the functions $a_i(t)$ are persistently exciting. Then the system (2.36) is detectable and there exists a generalized Luenberger observer with asymptotically stable error equation iff the unobservable subsystem

$$\dot{e}_{uo}(t) = G_{22} e_{uo}(t)$$

is asymptotically stable, moreover the observable part

$$\dot{e}_o(t) = G_{11} e_o(t)$$

is assignable. □

PROPOSITION 2.36. Let $\mathcal{P}_o : \mathbb{R} \to \mathbb{R}$ and $\mathcal{P}_{uo} : \mathbb{R} \to \mathbb{R}$ be the orthogonal projectors onto the observable and unobservable subspaces, respectively. Then, the DFP for detection and isolation of faults in (2.36) can be solved by using generalized Luenberger observers if the detectability rank condition

$$\ker(\mathcal{P}_o L_i) = 0, \quad i = 1, \ldots, k, \tag{2.46}$$

moreover, the fault separability condition (not sufficient)

$$\operatorname{Im} CL_i \cap \sum_{i \neq j}^{k} \operatorname{Im} CL_j = 0, \quad i, j = 1, \ldots, k \tag{2.47}$$

hold. □

In sense of the above proposition the ultimate goal is to find a stable G for which the sufficient condition

$$C(G, L_i) \cap \sum_{j \neq i} C(G, L_j) \tag{2.48}$$

holds. Assuming observability of the system, let us give the matrix G on the basis of L such that the separability condition (2.48) is satisfied

$$\mathcal{W}_i = (G, L_i) = \langle G \,|\, \operatorname{Im} L_i \rangle \supset \operatorname{Im} L_i,$$

with $L_i = (\ell_{i_1}, \dots, \ell_{i_{\mu_i}})$, and $\ell_i = \sum_{i=1}^{\mu_i} \alpha_i \nu_i$, where ν_i are the eigenvectors of the desired closed loop observer gain G. Therefore, $G\ell_{i_j} = \lambda_{i_j} \ell_{i_j}$, and the basis of G can be given as

$$\{A'^{i_1} C' e_{j_1}, A'^{i_2} C' e_{j_2}, \dots, A'^{i_k} C' e_{j_k}\}$$

i.e.,

$$D_{ij} C A^i = \left(\sum d_{ij_\ell} A'^{i_\ell} C' e_{j\ell} \right) C A^i.$$

Observe that

$$\operatorname{Im} CA_i \subset \operatorname{Im} C[A_1^{n_1} \dots A_\ell^{n_k}],$$

where the column space of the matrix $[A_1^{n_1} \dots A_\ell^{n_k}]$ is spanned by the set $\{\bar{A}_1 \dots \bar{A}_\ell\}$ which forms the basis of the Lie algebra $\mathfrak{L}$ generated by $A_o, A_1, \dots, A_m$ under the commutator product $[A_i, A_j] = A_i A_j - A_j A_i$. In fact, there is a basis of the column vectors of form

$$\{C', A_1' C', A_2' C', \dots, A_\ell' C', {A_1'}^2 C', A_1' A_2' C', \dots\}$$

for $\sum n_i \le k$, such that k is minimal, and G can be expressed over $\mathbb{R}^n$ in this minimal basis construction selected from the Lie algebra representation $\mathfrak{L}$.

PROPOSITION 2.37. Because the observability subspace of the system is contained in the generalized Kalman subspace (2.39), the generalized observer gain can be given by

$$G = (A + D_o C + D_1 \sum_i a_i C A_i + \dots + D_k(-y^{(k)} + P_k + Q_k),$$

where the coefficients D_j can be given in the basis of $\mathfrak{L}$ generated by $(A_1 \dots A_m)$. □

EXAMPLE 2.38. Assume that only y and $\dot{y}$ are used for output feedback in the observer. Let us expect, therefore, the observer gain in the second order form

$$G = A + D_o C + D_1 \sum_i a_i(t) C A_i.$$

Then, it can be reconstructed in the basis $\{C, CA_i\}$, and the system of linear equations for the coefficients of the gain matrix is obtained as

$$D_o(t) = d_{oo}(t) C' + \sum_i d_{oi} A_i' C',$$

$$D_1(t) = d_{1o}(t) C' + \sum_i d_{1i} A_i' C'.$$

2.8. SUMMARY

In this chapter some elementary facts of linear geometric system theory together with some specific subspace computation algorithms were quoted from the literature, namely those, which will play a distinctive role in the following discussions. On this basis, the traditional concept of residual generation (neglecting sensor faults) was given a geometric interpretation. For a direct consequence of this new geometric inspection, a special formulation of the disturbance decoupled estimation problem, *i.e.,* the unknown input observer, was discussed. This approach simplifies the detection filter problem by dividing the entire set of faults into two sets: the faults which are to be detected and the faults which are to be neglected (*e.g.*, disturbances). Unknown input observers work by making the second set unobservable (if possible).

As the generalization of this idea to multiple faults the formulation of the so called fundamental problem of residual generation has been introduced giving some basics of the theory of detection filters providing exact geometric decoupling of failure modes and disturbance effects. As a related fact, it was shown that the concepts of invertibility, reconstructability and functional controllability are closely related concepts which link a number of seemingly different residual generation ideas together.

The cleanest realization of the geometric detection filter idea the Beard-Jones filter works by imparting a special invariant subspace structure into the observer. This subspace structure is ideal for isolating the effects of different faults because it makes use of the fact that failures drive a system like unexpected inputs. As such, they bias observer residuals and can be associated with reachable subspaces. The detection filter restricts each of these reachable subspaces to lie within an invariant subspace and fixes the set of invariant subspaces containing the faults to be nonoverlapping. The end result is that the effect of a failure is wholly contained within a single invariant subspace.

With this structure, simultaneous detection and identification can be achieved by projecting the biased residual onto each of the invariant subspaces. A nonzero projection (or one that exceeds a threshold) indicates a fault; the subspace associated with the projection identifies the fault. Unknown input observers are clearly less capable than Beard-Jones filters, since they can identify only one fault out of the complete set of faults, which is due to a simpler invariant subspace structure unless further considerations are not made to make the residual structured.

The relevance of the application of the geometric approach to the solution of FDI problems in LTV systems based on the concepts of controllability and observability subspace methods were discussed. The idea of the generalized Luenberger observer and the application of Lie algebra have proved to be convenient mechanisms for finding conditions under which solvability of the non-LTI problem could be established.

It was shown how the concept of (C, A)-invariant subspaces, generally used for detection filter design in LTI systems, could be applied to the LTV case by generalizing the subspace construction procedure. For comparison, recall that in LTI systems the solution of the DFP could be found using the (C, A)-invariant subspaces

$$\langle A + DC \,|\, \operatorname{Im} L \rangle$$

with closed loop filter gain $A - DC$. For LTV systems, the generalized subspace

$$\left\langle A + \bar{D} \begin{bmatrix} C \\ CA_i \\ \vdots \\ CA_i^k \end{bmatrix} \mid \operatorname{Im} L \right\rangle \tag{2.49}$$

with $\bar{D} = [D_o\ D_1\ \ldots]$ can be used in the design. The generalized subspace (2.49) has many interesting features which, in the further discussions, will be referred to in many times. In fact, the generalized subspace (2.49) can be considered an immediate antecedent of the results related to inversion-based direct input reconstruction that will be presented in the later part of this work.

Overall in this chapter we assumed $u = Fx + Gv$ for state feedback and the general observer state and error equations $\dot{\hat{x}} = A\hat{x} + Bu + D(-y + C\hat{x})$, $\tilde{x} = \hat{x} - x$, respectively, for output injection causing $A + BF$ and $A + DC$ in the discussion, respectively. In later chapters, for convenience, the notations $A - B\hat{F}$ and $A - \hat{D}C$ will often be used, according to $u = -\hat{F}x + Gv$ and $\dot{\hat{x}} = A\hat{x} + Bu - \hat{D}(y - C\hat{x})$, and $\tilde{x} = x - \hat{x}$.

CHAPTER 3

Robust Estimation and Fault Detection

In this part of this work the development of robust estimators for detection and isolation of faults in uncertain dynamical systems is considered when knowledge about the statistical properties of the disturbance is assumed available. In the case of noise corrupted measurements an estimator has to be used for the purpose of residual generation. It is well known that the Kalman filter is a minimum variance optimal estimator for a linear process with zero-mean Gaussian white noise if its design is based on the accurate dynamic system model, which includes also the (deterministic) input. The treatment of fault detection includes algorithms that make use of this Kalman filter, such as the generalized likelihood ratio test that rely on robust state estimation.

It was stated earlier very shortly that the innovation (*i.e.,* the prediction error) of the Kalman filter can be applied for the purpose of a detection residual. This means that the statistical mean of the filter innovation is zero if there is no fault that becomes nonzero if any fault effect appears. Since the innovation sequence is white, it is relatively easy to apply statistical tests to recognize the changes.

The idea of the application of Kalman filters to fault detection and diagnosis can be traced back to the early 70's, see (Mehra and Peschon, 1971). Other important contributions to this subject can be found in (Willsky and Jones, 1976; Willsky, 1986; Friedland, 1979; Basseville, 1986), followed more recently by (Nikoukhah, 1994; Mangoubi et al., 1993; Mangoubi et al., 1994; Mangoubi, 1995).

In this chapter a formulation for a class of robust fault detection problems and its corresponding solution approach is presented. In order to provide insight into the robust algorithm, one needs to consider the past and understand its historical origins in optimal fault detection, where it is assumed that both plant and noise models are accurate. The deleterious effect of plant model uncertainties on the performance of the optimal tests is illustrated, thus motivating the idea of robust fault detection, which makes use of robust game theoretic or risk sensitive estimators. These robust estimators provide test residuals or statistics that are insensitive to uncertainties, but still sensitive to failure modes.

3.1. INTRODUCTION

In general, work in robust fault detection deals with one or more of the following three categories of uncertainties: — (i) failure mode, (ii) noise, and (iii) plant modeling errors. Traditionally these uncertainties are regarded either in a deterministic or a stochastic setting, depending on the available knowledge on the statistical properties of the noise to be dealt with.

The classical detection filter of (Beard, 1971) and (Jones, 1973), for example, is robust to failure mode uncertainty, but assumes perfect knowledge of the plant dynamics and noise characteristics, while the Generalized Likelihood Ratio Test (GLRT) of (Willsky and Jones, 1976) assumes accurate knowledge of the failure mode, noise statistics, and plant dynamics. Likelihood ratio tests can also be generalized to a large class of failure models, see *e.g.*, (Grenander, 1981). Some versions are also made robust to noise uncertainties (see below).

Previous work on robustness to plant model uncertainty includes that of (Lou et al., 1986), where a geometric interpretation of the concept of analytical redundancy leads to a procedure involving singular value decompositions for determining redundancy relations that are maximally insensitive to model uncertainties. (Gertler and Singer, 1990) uses an alternative approach, where the authors assume that model errors may be deduced from the uncertainties of a set of underlying parameters. The partial derivatives of the residuals with respect to these parameters are then computed and the residual generator with lowest partial sensitivity is selected.

Another approach is that of (Horak, 1988; Emami-Naeini et al., 1988), and (Tsui, 1994), where a bound on the effect of model uncertainties on the residual is estimated. This bound is then used to set the detection threshold accordingly. The unknown input observer attempts to mask disturbance (noise) from certain input channels and model uncertainties from the output residual. The authors of (Chen and Patton, 1998) are prolific contributors to the subject. The use of inputs to systems in order to robustly detect and isolate failures has been studied *e.g.*, by (Ribbens and Riggins, 1991).

Generally speaking, faults are detectable if their spectral characteristics is well distinguished from those of the uncertainties or, alternatively by using the geometric language, if failure modes and uncertainty effects enter the plant in different directions of the state space. Faults having very similar frequency characteristics as those of uncertainties, and lying in the same subspace of the state space, might not be detectable.

Detection filters are devices that make use of the directional characteristics of the faults in the state space. Previous results presented in the previous chapter have shown that if the effects of faults and system uncertainties show up in independent subspaces of the state space, then geometrical methods for decoupling their effects at the residual space can be used in enhancing robustness of the detection process. Classical detection filters producing directional residuals, *i.e.*, filters that map failure modes and unknown inputs to orthogonal (C, A)-invariant output subspaces denoted by $C\mathcal{W}_{\nu_i}, (i = 1, \dots, k)$ and $C\mathcal{W}_d$ of the filter respectively, separate the effects of the failure modes from disturbances thus making the detection robust, as shown, for instance in (Massoumnia, 1986; White and Speyer, 1987; Edelmayer et al., 1997d).

Unfortunately, however, in most real engineering applications, the effects of faults and model uncertainties cannot be separated from each other either because the structure of the uncertainty is not known or, from other structural reasons, the subspace separability condition (2.34) for the failure modes and unknown inputs cannot be satisfied, *i.e.*, the condition $C\mathcal{W}_{\nu_i} \cap C\mathcal{W}_d \neq 0$, for any i holds. In these cases an enhancement of detection performance can only be achieved if the improvement of the disturbance attenuation capability of the filter with respect to a particular fault direction has a feasible solution. As disturbance attenuation and detection sensitivity are two contradictious requirements the above problem tends to suggest the use of optimization techniques in the state estimation procedure.

In this chapter, an alternative optimization approach is presented ensuring sensitivity to failure modes while remaining robust to noise, plant model and failure mode uncertainties. For robustness to failure mode uncertainty, a broad-band Gauss-Markov model is used that embraces a large class of failures. The failure model is appended to the plant's dynamic model, giving an augmented linear system with plant and failure states. For robustness to noise and plant model uncertainty, the algorithm relies on a robust risk sensitive (exponential Gaussian) or a robust H_∞ filter in order to synthesize a robust failure estimate that is used in detection and/or isolation tests. These robust filters were first derived in (Mangoubi, 1995) and appeared also in (Mangoubi and Edelmayer, 2000). The close relationship between H_∞ and risk sensitive optimization makes it possible to apply the algorithm in either a deterministic or a stochastic setting.

The algorithm is a generalization of the well known Likelihood Ratio Tests (LRT) to failure detection. For this reason, we first derive such an algorithm, and discuss the effect of plant model uncertainties on its decision function. Nevertheless, the likelihood ratio test we derive, which relies on the optimal Kalman smoother for residual generation, has some robustness properties. By generalizing the problem formulation in order to extend these properties, we motivate the use of robust filters (Mangoubi, 1998; Mangoubi et al., 1995) that are insensitive to plant modeling errors, but at the same time sensitive to failures.

It is perhaps appropriate to state one principle that was adopted while developing this approach to robust detection: — The motivation for robust FDI methods lies in the limitation of conventional optimal algorithms, specifically their sensitivity to modeling errors. As such, our robust algorithms are, as mentioned earlier, extensions or generalizations of optimal tests. More precisely, in the absence of modeling errors, the robust algorithm reduce to an optimal one. It therefore behoves us to first take a look in this chapter at the past, or the pre-robust era, and to discuss the optimal algorithm from which the robust algorithms originate.

3.2. System model, fault model

A robust failure detection test is a hypothesis test between a set of unfailed plants and a set of failed plants. Fig. 3.1 illustrates the two hypotheses. The figure shows a general input/output representation of a nominal plant P with modeling uncertainty $\Delta \in \boldsymbol{\Delta}$. The vector u represents the known input to the plant, r represents the combined process and measurement noise, x_0

represents the initial state vector of the nominal plant, and y the measurement vector. The signals ϵ and η represent the interaction between the nominal plant and the perturbation Δ. Finally, f represents the failure signal.

Our concern is for failures in the actuators and sensors of the plant. This class of failure modes may have the additive representation shown in Fig. 3.1. Consider now an interval of interest, say $[k_0, ..., K]$. The notation $r = [r_{k_0}, ..., r_K]$ is used to denote the input disturbance signal. The vectors η, ϵ, y, and f are similarly defined, *i.e.*, $r = [r_{k_0}, ..., r_K]$, $\eta = [\eta_{k_0}, ..., \eta_K]$, $\epsilon = [\epsilon_{k_0}, ..., \epsilon_K]$, $y = [y_{k_0}, ..., y_K]$ and $f = [f_{k_0}, ..., f_K]$.

Note that each of the above signals is a matrix, whose k^{th} column represents the value of all the signal components at time step k, and whose rows are the time history of various components of the signal. Thus, f_k, for $k = k_0, ..., K$, denotes the vector of all the failures at time k, or

$$f_k = \left[f'_{1k}, \ldots, f'_{\mathcal{M}k}\right]'.$$

By contrast, we use the notation $f_{i,\cdot}$ to denote the i^{th} row of f (or the time history of the i^{th} failure element), for $i = 1, \ldots, \mathcal{M}$ as:

$$f_{i,\cdot} = \left[\ f_{ik_0}, \ldots, f_{iK}\ \right]. \tag{3.1}$$

Note that the direction of the failure vector f_k for all k depends on the failed channel, a fact that makes isolation a simple task. The ℓ_2 norm of the input disturbance r is given by

$$\|r\| = \left(\sum_{k=k_0}^{K} r'_k r_k\right)^{\frac{1}{2}}. \tag{3.2}$$

The ℓ_2 norms of η, ϵ, and y are similarly defined. The norm of the failure signal f can be slightly more general, since it can include weights. In addition, $\|x_0 - \hat{x}_0\|_{P_0^{-1}}$ represents the weighted Euclidean norm of the initial estimation error $x_0 - \hat{x}_0$.

The disturbances can be viewed as either deterministic or stochastic. For a deterministic model, r has a bounded ℓ_2 norm, while $x_0 - \hat{x}_0$ is assumed to have a bounded weighted Euclidean norm. For a stochastic model, r is a white noise sequence with unit variance and $x_0 - \hat{x}_0$ is Gaussian with zero mean and covariance P_0.

Note that because of the existing relationship between risk sensitivity and H_∞, which will be discussed in Section 3.5 in more details, the algorithm developed here is applicable to either

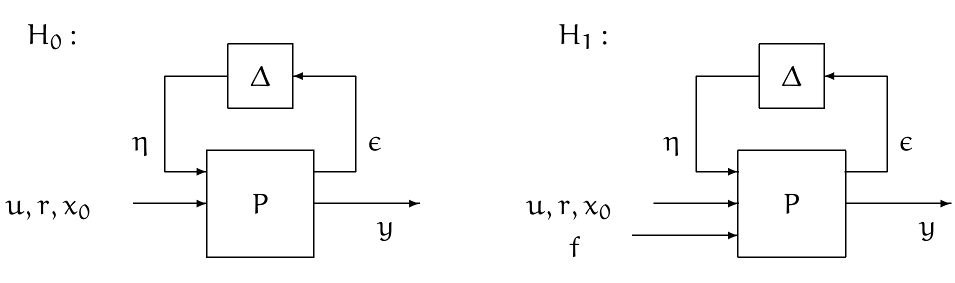

Figure 3.1. Hypothesis test for additive failures in the presence of model uncertainty

a stochastic or deterministic setting. These two interpretations provide insight into the design and analysis of the algorithm. Furthermore, the model uncertainty is characterized by the set of *scaled* perturbations whose induced 2-norm is given by

$$\Delta = \left\{ \Delta \mid \|\Delta\|_{i2} = \sup_{\epsilon \neq 0} \frac{\|\eta\|}{\|\epsilon\|} < \gamma \right\}, \tag{3.3}$$

where γ is a positive constant and the index i stands for induced norm.

The developments that follow are for linear plants, time-varying (LTV) and time-invariant (LTI). If the stochastic setting is assumed, then the set Δ will be restricted to linear perturbations only. For the deterministic view, no such assumption on the perturbation is needed.

Let x_k be the state of the dynamic plant P at time k, along with states associated with frequency weights on the uncertainty, and let y_k be the observations. The no-failure (H_0) and failure (H_1) hypotheses for the detection test over an interval $[k_0, K]$ can now be formally introduced as

$$\mathbf{H_0} \quad : \quad \begin{array}{rcl} x_{k+1} &=& Ax_k + Q\eta_k + Br_k + Uu_k \\ \epsilon_k &=& Sx_k + T\eta_k \\ y_k &=& Cx_k + R\eta_k + Dr_k + Wu_k \end{array} \tag{3.4}$$

$$\mathbf{H_1} \quad : \quad \begin{array}{rcl} x_{k+1} &=& Ax_k + Q\eta_k + Br_k + Uu_k + Ff_k \\ \epsilon_k &=& Sx_k + T\eta_k \\ y_k &=& Cx_k + R\eta_k + Dr_k + Wu_k + Lf_k \end{array} \tag{3.5}$$

with initial condition $x_{k_0} = \hat{x}_{k_0}$. In Eqs. (3.4-3.5), the perturbation's output signal η enters the plant through the matrices Q and R, while S and T represent the plant's input into the uncertainty. This formulation can represent a large class of uncertainties, including parametric, as well as nonparametric uncertainties, such as unmodeled dynamics. In (Mangoubi, 1998), examples are shown of how parametric and nonparametric uncertainties can be represented.

Each f_{ik} represents the failure mode of a control or measurement channel i at time k. The matrices F and L describe the way the control input and measurement failures enter into the system. Note that the time at which the failure occurs does not figure in the hypothesis test. That is, the failure is assumed either to exist or not to exist for the entire interval of observations. For the sake of notational compactness, the problem statement, and the subsequent development is presented for LTI systems, although both are applicable to LTV systems as well, with A_k replacing A, etc. For the hypothesis test we will assume the Gauss-Markov failure model of the form

$$\varphi_{k+1} = A_f \varphi_k + B_f \vartheta_k, \tag{3.6}$$

$$f_k = C_f \varphi_k. \tag{3.7}$$

The above model can also be LTI or LTV. It can be viewed as deterministic or stochastic. For a deterministic model, ϑ has a bounded ℓ_2 norm, and φ_0 is assumed to have a bounded Euclidean norm. For a stochastic model, Eq. (3.6) is a Gauss-Markov model, where ϑ is a white noise sequence with unit variance and φ_0 is Gaussian with zero mean and covariance P_{φ_0}. We discuss the issue of parameter selection at a later point.

The failure model of (Willsky and Jones, 1976) can be seen as a special case of the shaping filter (3.6-3.7). In this case, $A_f = I_{\mathcal{M}}$, where $I_{\mathcal{M}}$ is the identity matrix of dimension $\mathcal{M}$, $B_f = 0$, $C_f = I$, and $P_{\varphi_0} = \infty$. The choice of an infinite initial covariance is dictated by the fact that, in (Willsky and Jones, 1976), no prior information on the failure is assumed. The more general model of (Grenander, 1981) is also a special case of the model (3.6-3.7). Other special cases are the failure models of (Hall, 1985). In this work a first-order model for each failure is assumed.

The state dynamic equation of the failed hypothesis (3.5) can be augmented with the failure model of Eqs. (3.6-3.7) to give

$$\begin{aligned}\begin{bmatrix} x_{k+1} \\ \varphi_{k+1} \end{bmatrix} &= \begin{bmatrix} A & FC_f \\ 0 & A_f \end{bmatrix}\begin{bmatrix} x_k \\ \varphi_k \end{bmatrix} + \begin{bmatrix} Q \\ 0 \end{bmatrix}\eta_k + \\ &\quad + \begin{bmatrix} B & 0 \\ 0 & B_f \end{bmatrix}\begin{bmatrix} r_k \\ \vartheta_k \end{bmatrix} + \begin{bmatrix} U \\ 0 \end{bmatrix} u_k \end{aligned} \tag{3.8}$$

$$\epsilon_k = \begin{bmatrix} S & 0 & T \end{bmatrix}\begin{bmatrix} x_k \\ \varphi_k \\ \eta_k \end{bmatrix}, \tag{3.9}$$

with initial estimate

$$\begin{bmatrix} x_0 \\ f_0 \end{bmatrix} = \begin{bmatrix} \hat{x}_0 \\ 0 \end{bmatrix}. \tag{3.10}$$

If a stochastic setting is assumed, then the initial error has a mean of zero and a covariance given by

$$P_0 = \begin{bmatrix} \check{P}_0 & 0 \\ 0 & P_{\varphi_0} \end{bmatrix}. \tag{3.11}$$

For a deterministic setting, the initial estimation error has a weighted Euclidean norm, with weight given by P_0^{-1}. A joint bound is assumed on the norm of the disturbances and initial error $\|r\|^2 + \|x_0 - \hat{x}_0\|^2_{\bar{P}_0^{-1}} + \|\vartheta\|^2 < \mathcal{B}$. Then, the associated observation equation can be written as

$$y_k = \begin{bmatrix} C & LC_f \end{bmatrix}\begin{bmatrix} x_k \\ \varphi_k \end{bmatrix} + \begin{bmatrix} R & D & W & 0 \end{bmatrix}\begin{bmatrix} r_k \\ \eta_k \\ u_k \\ \vartheta_k \end{bmatrix}. \tag{3.12}$$

Note that the direction of the vector f can be used to isolate the failed component. Finally, from the above, it is clear that the failure is detectable only if the pair

$$\left(\begin{bmatrix} A & FC_f \\ 0 & A_f \end{bmatrix}, \begin{bmatrix} C & LC_f \end{bmatrix}\right)$$

is observable. Obviously, the speed of detection depends on the bandwidth of the filter's response.

3.3. LRT WITH ROBUSTNESS TO FAILURE MODE AND NOISE MODEL NEGLECTING PLANT UNCERTAINTIES

The objective in this section is to derive the likelihood ratio test for the detection and isolation tests of the previous section, assuming an accurate knowledge of the plant and model statistics and the feasibility of the Gauss-Markov failure model given in (3.6-3.7). It follows that the matrices Q, R, S, T in the hypotheses (3.4) and (3.5) are assumed to be zero. By showing that a version of this test can rely on an optimal (risk sensitive or game theoretic) estimator, we demonstrate robustness to failure mode and noise model uncertainties only. The robust algorithm presented in the next section is an extension of this test to the case where plant uncertainties are present, and reduces to it in the absence of such uncertainties.

For a nominal model, the *weighted* likelihood ratio over an interval $[k_0, ..., k_0 + N]$ for the failure detection test is

$$\Lambda^s = \frac{E_{\underline{f}}[p(Y \mid H_1)]}{p(Y \mid H_0)} = \frac{\int_{-\infty}^{+\infty} p(Y \mid H_1, \underline{f}) p(\underline{f} \mid H_1) d\underline{f}}{p(Y \mid H_0)}. \tag{3.13}$$

The superscript s will be explained shortly. The vector Y represents the observations over the entire interval $[k_0, k_0 + N]$ stacked into one column:

$$Y = \left[y'_{k_0}, ..., y'_{k_0+N}\right]'.$$

Similarly, $\underline{f}$ is simply the signal f with its columns f_k, $(k = k_0, ..., k_0 + N)$ stacked into one column:

$$\underline{f} = \left[f'_{k_0}, ..., f'_{k_0+N}\right]'.$$

The vector $\underline{f}$ has a mean of zero and a covariance $\Sigma_{\underline{f}}$ that can be derived from Eqs. (3.6-3.7), seen as a Gauss-Markov model. For the entire interval, this test is given as

$$H_0 \quad : \quad Y \quad = \quad Y_0 \tag{3.14}$$

$$H_1 \quad : \quad Y \quad = \quad Y_0 + \mathcal{G}\underline{f} \tag{3.15}$$

where Y_0 is the vector of observations in the absence of failure, and the matrix $\mathcal{G}$ represents the projection of the failure process onto the observations. Specifically,

$$\mathcal{G} = \begin{bmatrix} L & & & & & \\ CF & L & & & & \\ \cdots & \cdots & \cdots & & & \\ CA^{k-k_0-1}F & CA^{k-k_0-2}F & \cdots & L & & \\ \cdots & \cdots & \cdots & & & \\ CA^{N-1}F & CA^{N-2}F & \cdots & \cdots & CF & L \end{bmatrix} \tag{3.16}$$

The test therefore reduces to the classical problem of the detection of the stochastic signal, f or $\underline{f}$, in colored noise, Y_0. The density function of the observation under each of the two hypotheses is given by

$$H_0 \quad : \quad Y \sim \mathcal{N}(0, \Sigma_0), \tag{3.17}$$

$$H_1 \quad : \quad Y \sim \mathcal{N}\left(0, \Sigma_0 + \mathcal{G}\Sigma_{\underline{f}}\mathcal{G}'\right), \tag{3.18}$$

where Σ_0 is the covariance of the observation under the null hypothesis, computed from the model of Eqs. (3.8-3.12), together with the observation equation in (3.4). Note that $\underline{f}$ and Y_0 are independent, which allows us to add the covariances in Eq. (3.18). In terms of Y, the log-likelihood ratio of Eq. (3.13) is given by :

$$\mathcal{D}^s = -Y' \left[\left(\mathcal{G}\Sigma_{\underline{f}}\mathcal{G}' + \Sigma_0 \right)^{-1} - \Sigma_0^{-1} \right] Y,$$

see (Mangoubi, 1998) for details. The above ratio can be expressed in terms of the maximum a posteriori (MAP) estimate of $\underline{f}$, given the observation sequence Y. In the linear Gaussian context, the MAP estimate is also the *smoothed* minimum variance estimate, denoted by $\hat{\underline{f}}^s$. The superscript s on the ratios Λ^s and $\mathcal{D}^s$ have been added to emphasize the fact that they depend on a smoothed estimate of the failure. To see this, first express this estimate as a function of the observations, or

$$\hat{\underline{f}}^s \quad = \quad E\left(\hat{\underline{f}} \mid Y\right) = \Sigma_{\underline{f}\mid Y}\mathcal{G}'\Sigma_0^{-1}Y \tag{3.19}$$

where

$$\Sigma_{\underline{f}\mid Y} = \left(\mathcal{G}'\Sigma_0^{-1}\mathcal{G} + \Sigma_{\underline{f}}^{-1} \right)^{-1} \tag{3.20}$$

is the a posteriori covariance of the failure given the observation, again, see (Mangoubi, 1998) for details. Comparing the above equation with Eq. (3.3), we can see that the log-likelihood ratio test can be expressed as

$$\mathcal{D}^s = \hat{\underline{f}}^{s'} \Sigma_{\underline{f}|Y}^{-1} \hat{\underline{f}}^s \quad \gtrless \quad \text{Threshold}. \tag{3.21}$$

The above expression shows explicitly that likelihood ratio detection tests over an interval make use of a smoothed rather than a filtered estimate. This fact is not made obvious in (Willsky and Jones, 1976) because the failure is assumed to be a jump whose magnitude does not vary with time. This is because the filtered estimate of a constant failure at the end of the interval $[k_0, k_0 + N]$, which can be obtained using a Kalman filter, is also a smoothed estimate.

Notice that $\hat{\underline{f}}^s$ is a fixed-interval smoothed estimate, and it can be obtained using a backward and forward filter, as explained in Chapter 2 of (Mangoubi, 1998), where it has also been shown that the minimum variance fixed-interval smoothing is equivalent to game theoretic or minimax fixed-interval smoothing. For this reason, one can consider the estimate $\hat{\underline{f}}^s$ robust to noise model uncertainty.

For convenience, it is often desirable to use a causal version of Eq. (3.21). To do so, we replace the smoothed estimate of each element of the failure vector by the filtered estimate. That is, instead of using a smoother based on Eqs. (3.8-3.12), we use the estimate given by the forward game theoretic or minmax filter based on the same equation. Recall that the minmax filter is parametrized by a parameter γ (or $\theta = \gamma^{-2}$, as is customary in the literature on risk sensitive optimization). If we set γ to infinity (or θ to zero), we get the Kalman filter, see Section 3.5.2.

On the other hand, if we set γ (θ) to its minimum (maximum) possible value, we have the H_∞ filter. Intermediate values of these parameters trade off average and worst case noise performance. Define

$$\hat{f}^s = \left[\hat{f}^s_{k_0}, ..., \hat{f}^s_k, ..., \hat{f}^s_{k_0+N} \right].$$

That is, $\hat{f}^s$ is simply $\underline{\hat{f}}^s$ rearranged in the same way as f itself. Then,

$$\hat{f}^s_k = E\left(f \mid y_{k_0}, ..., y_{k_0+N}\right), \quad \forall k \in [k_0, \dots, k_0+N]$$

is replaced by the causal estimate $\hat{f}^c_k$, or simply $\hat{f}_k$, the solution to the following risk sensitive estimation problem

$$\min_{\hat{f}} \theta^{-1} \log\ E\left(e^{\theta J}\right), \tag{3.22}$$

where

$$J = \frac{1}{2}\sum_{k=1}^{k_0+N} e_k' e_k, \tag{3.23}$$

and, with some known weighting M_k, the estimation error is given

$$e_k = M_k(f_k - \hat{f}_k) = M_k C_f\left(\varphi_k - \hat{\varphi}_k\right) \tag{3.24}$$

subject to the dynamic constraints of Eqs. (3.8)-(3.11) with the plant uncertainty matrices *all* set to zero. The recursive filter equations that are a solution to the above estimation problem will not be given here, as they are a special case of the robust filter equations given in Section 3.5.2.

Now, if we denote the error covariance of the filtered estimate Σ_k, we have a recursive decision function for the interval window $[k_0, k_0+N]$,

$$\mathcal{D}^c_{k_0+N} = \sum_{k=k_0}^{k=k_0+N} \|\hat{f}^c_k\|^2_{\Sigma_k^{-1}} \ \gtrless\ \text{Threshold.} \tag{3.25}$$

Finally, if the failure model of Eq. (3.6-3.7) is used, then the parameters A_f and B_f can be chosen so as to obtain rapid and accurate tracking of the failure. This can be done by choosing the steady-state gain of the transfer function $\mathcal{T}_{\hat{f}^c f}$ between the failure input and the failure estimate to be close to unity for a large bandwidth. That is,

$$\mid \mathcal{T}_{\hat{f}^c f}(\omega) \mid \simeq 1, \qquad \forall \omega < \omega^*. \tag{3.26}$$

The goal is to design a filter that is as fast as possible, without causing a large false alarm probability by having the disturbances pass as a failure. The frequency ω^*, as well as the elements of A_f and B_f, are therefore determined by trial and error with Eq. (3.26) in mind.

Note that we have derived a family of failure detection and isolation tests applicable to a large family of failures, parametrized by the parameter θ. Due to the application of this parameter, these tests are robust to noise model uncertainty. In fact, the alteration of θ allows us to make a trade-off between average and worst-case noise performance. But these tests still assume perfect knowledge of the plant model. Before we relax this assumption, we first analyze the effect of the violation of this condition in the next section.

3.4. LRT and plant modeling uncertainties

Given Gaussian noise, and no plant dynamic modeling uncertainty, the failure estimate is Gaussian with zero mean and covariance Σ_0. The decision function

$$\mathcal{D} = \underline{\hat{f}}'\Sigma_0^{-1}\underline{\hat{f}}$$

therefore has a χ^2 distribution with degrees of freedom equaling the length of the smoothed or filtered estimate $\hat{\underline{f}}$. Plant dynamic uncertainty will degrade the Kalman filter (smoother) estimate producing a covariance Σ different, and usually larger than the nominal value Σ_0. In addition, with a known plant model, known deterministic inputs (such as actuator deflections) are propagated through the plant model and nominally have no effect on the estimation error. With modeling uncertainty, these deterministic inputs can produce large biases in the failure estimate, significantly degrading performance.

The effects of modeling uncertainty are illustrated in Fig. 3.2 for a two element Gaussian vector. The covariance can be represented by an ellipse with major and minor axes formed

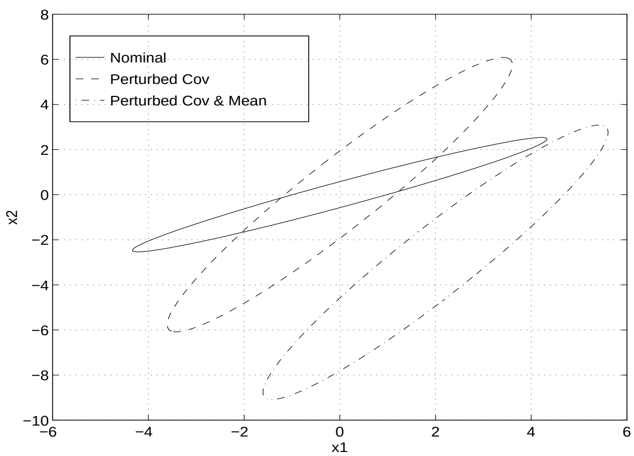

Figure 3.2. Effect of model uncertainty on the covariance of two elements

by the eigenvectors of the covariance matrix with lengths given by the eigenvalues. Modeling uncertainty will tend to increase and distort the covariance and non-zero mean inputs will produce non-zero mean outputs. Fig. 3.3 shows the effect of these distortions on the decision function for a hypothetical example. In general, the decision function can be represented as a

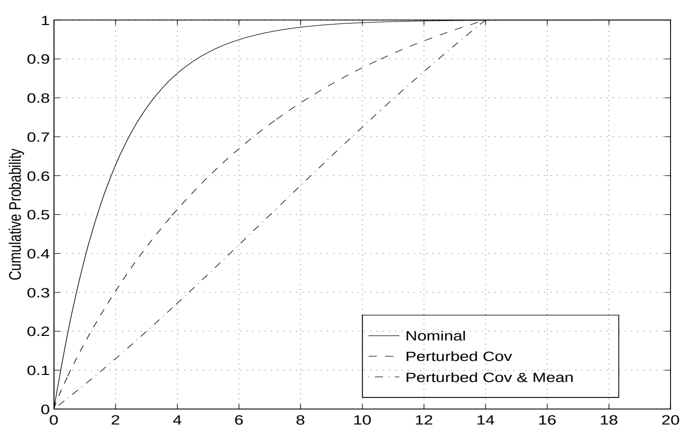

Figure 3.3. Effect of model uncertainty on decision function

weighted sum of χ^2 variables. Given $\underline{f}$ with a Gaussian density with mean m_f and covariance

Σ, the decision function can be written as

$$\mathcal{D} = Z'\Sigma^{1/2}\Sigma_0^{-1}\Sigma^{1/2}Z$$

where

$$Z = \Sigma^{-1/2}\underline{\hat{f}}$$

The decision function can be expressed as

$$\mathcal{D} \overset{d}{=} \sum_{j=1}^{\mathcal{M}} \lambda_j Z_j^2,$$

where $\overset{d}{=}$ stands for equal in distribution, λ_j are the eigenvalues of $\Sigma_0^{-1/2}\Sigma^{1/2}$, and the Z_j's are independent Gaussian random variables with unity variance. The decision function is therefore the weighted sum of non-central χ^2 random variables with one degree of freedom.

Modeling uncertainty in the plant dynamics can produce a large degradation in performance. Simply ignoring the uncertainty can lead to excessive false alarms. Accounting for the uncertainty by just increasing the threshold can lead to missed detection. Ideally, the mean and covariance of the failure estimates should be made as small as possible (with no failures) over the range of modeling uncertainty. In addition, the decision function $\mathcal{D}$ would ideally be formed using the actual covariance Σ. Unfortunately, there are as many possible covariances as there are possible plant models given the range of modeling perturbations; the selection of the normalization matrix that produces the best performance remains an open research question.

3.5. FDI with robustness to failure mode, noise and plant model uncertainties

In the last section, we have shown how model uncertainties can seriously degrade the performance of the likelihood ratio test, which assumes accurate plant models. In this section, we describe a robust failure detection and isolation algorithm that is insensitive to failure mode, noise *and* plant model uncertainties.

Fig. 3.4 describes the logic of the algorithm design. The inputs to the plant P are the perturbation's output η, the known inputs u, the disturbance r, the initial condition x_0, and the failure f, which is assumed to be the output of a shaping filter P_f. As discussed in Section 3.2, this failure model embraces a large class of failure modes, making the algorithm robust to failure mode uncertainties. The measurement y is to be fed to the robust estimator F, along with the initial state estimate $\hat{x}_0$. The outputs are state and failure estimates that are robust to failure mode, plant and noise modeling errors. The test's detection and isolation functions are based on that estimate.

Finally, Fig. 3.5 is a general description of failure detection and isolation algorithms. For the GLRT described in (Willsky and Jones, 1976), P_f is simply a unit step function, $\Delta = 0$, F represents the Kalman filter, and the block 'Test' compares the square of the step magnitude estimate to a threshold. For the likelihood ratio test of Section 3.3, P_f is a shaping filter, while

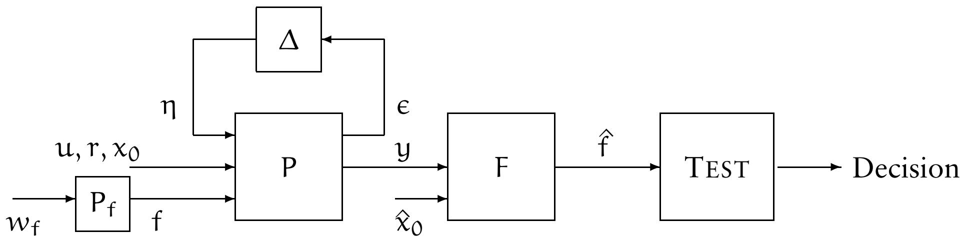

Figure 3.4. The scheme of the robust FDI algorithm

$\Delta = 0$, F is game theoretic filter or smoother, and the block TEST represents the comparison of the square of a failure estimate to a threshold. The block TEST also uses the failure estimate for isolation. The decision function is described in Section 3.5.1, and the robust estimator is described and discussed in Section 3.5.2.

3.5.1. *The decision function*

In Section 3.3, we have shown that the likelihood ratio test is based on a quadratic function of the observation signal. This decision function can also be described as a function of a failure estimate obtained by minimizing the energy in the failure estimation error. That is, the detection function of Eq. (3.21), rewritten below, is

$$\mathcal{D}^s = \underline{\hat{f}}^{s'} \Sigma_{\underline{f}|Y}^{-1} \underline{\hat{f}}^s \tag{3.27}$$

where $\hat{f}^s$ is obtained by solving the optimization problem

$$\min_{\hat{f}^s} E\left(\|\underline{f} - \underline{\hat{f}}^s\| \mid Y \right) \tag{3.28}$$

subject to the nominal plant and failure model constraints of Eqs. (3.8-3.9). Again, this smoothed estimate is robust to noise model uncertainty since it is the solution to the H_∞ smoothing problem as shown *e.g.*, in (Mangoubi, 1998). For the filtered version, the decision function

$$\mathcal{D}^c_{k_0+N} = \sum_{k=k_0}^{k_0+N} \|\hat{f}^c_k\|^2_{\Sigma_k^{-1}} \tag{3.29}$$

is obtained by minimizing

$$\min_{\hat{f}} \theta^{-1} \log \, E\left(e^{\theta J} \right) \tag{3.30}$$

subject to the same constraints. Here J is given by Eqs. (3.23-3.24).

The game theoretic smoother and filter are, respectively, the solutions to the above two optimization problems. Again as $\theta \to 0$, we have the traditional Kalman filter based test.

In the context of our algorithm, an exact model is not assumed. Instead, a bound on the induced 2-norm of the plant perturbation can be assumed, and likewise for the ℓ_2 norm of the disturbance over an interval. A reasonable detection function is one which, in the absence of noise and plant model uncertainties, can reduce to that of the likelihood ratio test, *i.e.*, Eq. (3.21) or (3.25). A smoothed version would be

$$\mathcal{D}^{rs}_{k_0+N} = \sum_{k=k_0}^{k_0+N} \hat{f}^{rs\prime} \mathcal{S}_k \hat{f}^{rs} \tag{3.31}$$

where $\hat{f}^{rs} = \hat{f}^{rs}(Y)$, the robust smoothed estimate, minimizes the worst case error energy, or ℓ_2 norm

$$\min_{\hat{f}^{rs}(Y)} \max_{r,\vartheta,\Delta,\hat{x}_0} \sum_{k=k_0}^{k_0+N} \left(\|f_k - \hat{f}^{rs}_k(Y)\|^2_{\mathcal{S}_k} \right) \tag{3.32}$$

subject to the plant and failure models, as well as the constraints on the noise and the perturbations, rewritten below

$$\|u\|^2 + \|r\|^2 + \|\vartheta\|^2 + \|x_0 - \hat{x}_0\|^2_{P_0^{-1}} \quad < \quad 1 \tag{3.33}$$

$$\|\Delta\|_{i2} \quad < \quad 1. \tag{3.34}$$

Again, due to rescaling, there is no loss of generality implied when using a unity bound. The argument Y in Eq. (3.32) is added in order to emphasize the fact that, at each time step k, $\hat{f}^{rs}_k$ is a function of the entire set of observations on $[k_0, k_0 + N]$. The elements of the matrix $\mathcal{S}_k$ are free parameters.

The filtered version of Eq. (3.31) is

$$\mathcal{D}_{k_0+N} \quad \equiv \quad \mathcal{D}^{rc}_{k_0+N} \tag{3.35}$$

i.e.,

$$\sum_{k=k_0}^{k_0+N} \hat{f}^{rc\prime}_k \mathcal{S}_k \hat{f}^{rc}_k = \sum_{k=k_0}^{k_0+N} \hat{f}^{\prime}_k \mathcal{S}_k \hat{f}_k \tag{3.36}$$

subject to the same constraints as the smoothed version. The superscript rc, for robust causal filtering, will be dropped from now on for the sake of simpler notation. The failure estimate is obtained by replacing the objective function (3.32) with

$$\min_{\hat{f}} \max_{r,\vartheta,x_0,\Delta} \sum_{k=k_0}^{k_0+N} \left(\|f_k - \hat{f}_k\|^2_{\mathcal{S}_k} \right). \tag{3.37}$$

The above decision function has a stochastic interpretation. As explained in (Mangoubi, 1998), if we assume that the noise is Gaussian, then $\hat{f}$ is the estimate that minimizes the risk sensitive objective function. For the filtered estimate, we have

$$\hat{f} = \arg\min_{\hat{f}} E \left(e^{\theta \sum_{k=k_0}^{k_0+N} \|f_k - \hat{f}_k\|^2_{\mathcal{S}_k}} \right) \tag{3.38}$$

subject to plant and failure models, as well as the induced-norm bound constraint on the perturbation of Eq. (3.34). If a stochastic interpretation is assumed, then a constant diagonal matrix $\mathcal{S}$ can be used, where the diagonal entries are equal to the reciprocals of the variances of the failures, based on the Gauss-Markov model of Eqs. (3.6-3.7). As mentioned at the end of Section 3.4, the best choice of $\mathcal{S}$ is an open research question. Note that if the failure is a scalar, *i.e.*, only one control command or measurement is of interest, then the question of choosing $\mathcal{S}$ does not arise.

It is obvious from the above discussion that, in order to obtain the desired decision function, we must use a robust filter or smoother, which would give us $\hat{f}^{rc}$ or $\hat{f}^{rs}$. The robust filter's equations are given in Section 3.5.2.

Finally, note that in the absence of model uncertainties and norm bounds, the decision functions of the robust algorithm reduce to the corresponding likelihood ratio test functions, provided the appropriate weighting matrix is used, *i.e.*, $\mathcal{S}_k = \Sigma_k^{-1}$. This is by virtue of the fact that robust H_∞ or risk sensitive optimization reduces to linear quadratic optimization in the absence of model uncertainties, as shown in earlier chapters. The robust FDI algorithm is therefore an extension of the likelihood ratio test of Section 3.3, just as the robust H_∞ or risk sensitive estimator is an extension of the maximum likelihood, or more accurately the maximum a posteriori estimator, which is given by the fixed-interval smoother.

3.5.2. *The robust estimator*

One can give the recursive equations of the robust filter just described. Define the matrices

$$\breve{A} = \begin{bmatrix} A & FC_f \\ 0 & A_f \end{bmatrix}, \quad \breve{B} = \begin{bmatrix} B & 0 & U \\ 0 & B_f & 0 \end{bmatrix}, \quad \breve{C} = \begin{bmatrix} C & LC_f \\ 0 & 0 \end{bmatrix}, \quad \breve{D} = \begin{bmatrix} D & 0 & W \\ 0 & 0 & I \end{bmatrix}.$$

Then, as derived in (Mangoubi, 1998),

$$\begin{bmatrix} \hat{x}_{k+1} \\ \hat{\varphi}_{k+1} \end{bmatrix} = \left(\overline{A}_k - K_k\overline{C}_k\right) \begin{bmatrix} \hat{x}_k \\ \hat{\varphi}_k \end{bmatrix} + K_k y_k \tag{3.39}$$

$$K_k = \left(\overline{B}_k\overline{D}_k' + \overline{A}_k H_k \overline{C}_k'\right)\left(\overline{D}_k\overline{D}_k' + \overline{C}_k H_k \overline{C}_k'\right)^{-1} \tag{3.40}$$

where

$$\begin{aligned}
\overline{A}_k &= \breve{A} + \gamma^{-2}\breve{B}Z_k^{-1}F_k' \\
\overline{B}_k &= \breve{B}Z_k^{-1/2} \\
\overline{C}_k &= \breve{B} + \gamma^{-2}\breve{D}Z_k^{-1}F_k' \\
\overline{D}_k &= \breve{D}Z_k^{-1/2} \\
F_k &= S'T + \breve{A}'X_{k+1}\breve{B} \\
H_k &= \left(P_k^{-1} - \gamma^{-2}M_k'M_k\right)^{-1} \\
Z_k &= I - \gamma^{-2}\left(T'T + \breve{B}'X_{k+1}\breve{B}\right)
\end{aligned}$$

where M_k is the known weighting factor of the state estimation and the matrices X_k and P_k are, respectively, positive definite solutions to the two Riccati equations:

$$\begin{aligned} X_k &= \breve{A}' X_{k+1} \breve{A} + S'S + \gamma^{-2} F_k Z_k^{-1} F_k' \\ X_{k_0+N} &= 0 \quad \text{(backward recursion)} \end{aligned} \tag{3.41}$$

$$\begin{aligned} P_{k+1} &= \left(\overline{A}_k - K_k \overline{C}_k\right) H_k \left(\overline{A}_k - K_k \overline{C}_k\right)' + \left(\overline{B}_k - K_k \overline{D}_k\right) \left(\overline{B}_k - K_k \overline{D}_k\right)' \\ P_0 &= \text{given} \quad \text{(forward recursion)}. \end{aligned} \tag{3.42}$$

The solution to this problem is an extension of both the game theoretic or H_∞ optimal estimator and the Kalman filter for nominal systems. If there are no model perturbations, then $S_k = 0,\ T_k = 0$ in all the above equations, so that the Riccati equation (3.41) for X_k is superfluous, *i.e.*, $X_k = 0$. The estimator is then reduced to solving one Riccati equation based on the nominal plant dynamics. The Riccati equation (3.42) and the gain (3.40) are then parametrized by $\gamma = \theta^{-1/2}$ which, once more, gives us a whole family of estimators, with the two extremes being the Kalman filter, (when $\gamma \to \infty$), and the H_∞ filter, (when γ attains a minimum value with P_k singular).

The robust estimator can therefore be viewed as a further extension of the Kalman filter that permits the designer to make a trade-off of nominal performance in the minimum error variance sense to provide robustness to disturbance *and* plant modeling errors. For further reference and numerical results the reader is referred to (Mangoubi, 1998) and (Agustin et al., 1999).

3.6. Risk seeking estimation

The approach to robust detection presented in the previous section is not without its limitations. One potential drawback is the need to add at least one state for each failure under consideration. Despite the advantage it offers, such as the ability of the estimator to provide a reliable estimate for the failure, this increase in the size of the state space may lead to numerical difficulties in the realization. In the following we investigate a formulation of the problem very shortly that may make it unnecessary to model the failure as an additional state.

Let us first consider the case where the plant is known accurately, and where no noise is present. With failures as inputs, our system would then be

$$x_{k+1} = Ax_k + Bu_k + Ff_k, \tag{3.43}$$

$$y_k = Cx_k + Du_k + Lf_k. \tag{3.44}$$

If we treat the failure as a disturbance, then one possible approach is to attempt to design an estimator that would provide an output residual $y_k - \hat{y}_k$ that is sensitive, rather than insensitive, to the disturbance. A possible formulation of this problem is

$$\min_{\hat{x}} \theta^{-1} \log\ E\left(e^{\theta J}\right) \tag{3.45}$$

subject to the dynamic constraints of Eqs. (3.43-3.44), with

$$J = \frac{1}{2} \sum_{k=1}^{k_0+N} e_k' e_k, \tag{3.46}$$

$$e_k = M_k(y_k - \hat{y}_k), \tag{3.47}$$

where the parameter θ is *negative*, rather than positive. This is called a *risk seeking* estimation problem. Because θ is negative, the resulting residual e_k would be sensitive to f_k, as opposed to the case we presented earlier, where a positive θ would give an estimate that is insensitive to the disturbance. The direction of the residual in the presence of failure would then determine the failed component.

3.7. Summary

As the main conclusion of this chapter it is notable that *estimation* and *detection* can be as closely linked in the H_∞ context as they are in the least-square Gaussian context. For the robust failure detection and isolation algorithm, the detection and isolation functions are given in terms of a failure estimate obtained using a robust game theoretic (H_∞) or risk sensitive estimator, just as the likelihood ratio decision functions are given in terms of a failure estimate obtained from a Kalman filter. In fact, the robust estimation and detection methods developed reduce, in the absence of model uncertainties, to the classical linear Gaussian estimation and detection methods. Specifically, the robust estimation algorithm reduces to the Kalman filter, and the robust detection test reduces to the likelihood ratio test.

Another interesting conclusion is to compare the classical geometric detection filter relied on the deterministic system model presented in the previous chapters with the estimation techniques discussed in this chapter. While in the idea of the geometric detection filter, pursued by the model following approach by both R. V. Beard and H. L. Jones in (Beard, 1971; Jones, 1973), respectively, requires that the failure detector possess the same mathematical structure as a Kalman filter (incorporating the system model). However, the filter gains are chosen not to minimize the mean square error of the state estimate, as done in an optimal Kalman filter; but are chosen instead to emphasize or enchance the estimates of the failure mode states and not to necessarily satisfy any other objectives such as acceptable tracking the other important system states that necessitated the use of a Kalman filter in the first place. This construction is amenable to purely deterministic systems subject to failures, but some questions relating to extent of applicability are raised when the approach is to be used for failure detection in the presence of plant and measurement noises (as are frequently encountered in applications). Consequently, to be feasible for use, a second Kalman filter would be needed so that one could be used for the usual tracking and estimation functions while the other (the geometric filter) is used to detect the presence of prespecified or previously characterized failures.

Another general observation is the fact that detection (and isolation) based on observations over an interval requires a smoothing, rather than a filtering operation. Filters are used for convenience. This fact is obscured if the failure is assumed to be a constant scalar or vector since, in this case, the filtered and smoothed estimates are the same.

An interesting feature of the approach presented in this chapter is that the robust filter it uses can be designed so as to provide an estimate of the plant's state immediately after a failure occurs. This, however, requires increasing of the dimension of the state space. An alternative formulation of the problem called risk seeking estimation that does not suffer from this drawback was also briefly mentioned and the idea introduced.

CHAPTER 4

Game Theoretic Robust Optimal Estimation in Linear Systems

THIS CHAPTER DEALS WITH THE SENSITIVITY OPTIMIZATION of detection filters in linear time invariant (LTI) as well as linear time varying (LTV) systems which are subject to multiple simultaneous faults and disturbances. The game theoretic robust optimal estimation problem and its solution approach is presented which is based on a deterministic problem formulation. Game theoretic formulation means that the disturbance and modeling errors act as opponents of the state estimator in the optimization process. The goal of the state estimator is to find the state estimate for the worst possible combination of initial condition, disturbances and model errors.

Beyond the idea of classical H_∞ design, a scaling optimization approach is presented. It means that the robust fault detection filter design problem is cast as a *scaled* H_∞ filtering problem. The effect of two different input scaling approaches to the optimization process is investigated. The objective is to provide the smallest scaled L_2 gain of the unknown input of the system (minimizing the maximum energy of the disturbances in the estimation error) that is guaranteed to be less than a prespecified level. That is to say, the goal is to produce an optimal filter with the possible best disturbance suppression capability in such a way that sufficient sensitivity to failure modes should still be maintained.

It is shown how to obtain bounds on the scaled L_2 gain by transforming the standard H_∞ filtering problem into a convex feasibility problem, specifically, a structured, linear matrix inequality (LMI). Numerical examples demonstrating the effect of the scaled optimization with respect to conventional H_∞ filtering is presented.

4.1. Introduction

Consider the deterministic description of the dynamical system subjected to structured linear time varying uncertainties, system and observation noise and multiple simultaneous faults, which can be represented in state space form by the time varying dynamics

$$\dot{x}(t) = A(t)x(t) + B_u u(t) + B_w w(t) + \sum_{i=1}^{k} L_i \nu_i(t),$$
$$y(t) = Cx(t) + v(t), \tag{4.1}$$

where the vectors x, y and u belong to real linear vector spaces $\mathcal{X}^{(n)}$, $\mathcal{Y}^{(m)}$, $\mathcal{U}^{(p)}$ and B_u, B_w and C are appropriate constant matrices. Assume that the structured parametric uncertainties of system (4.1) can be represented by the LTV perturbation structure

$$A(t) = A + \Delta A(t) = A + \sum_{i=1}^{m} a_i(t) A_i, \tag{4.2}$$

where the nominal system matrix A is considered stable and A_i are assumed to be non-destabilizing known constant matrices. $a_i(t) \in \mathcal{L}_2$ are arbitrary bounded smooth perturbation functions of time and the input functions $w(t) \in \mathbb{R}^p$ and $v(t) \in \mathbb{R}^m$ are the process and observation noise, respectively. These functions are referred to as *unknown inputs* in the sequel. The unknown time functions $\nu_i(t)$ are the *failure modes* which affect the system in the directions $L_i \in \mathbb{R}^n$. L_i are considered to be known and pre-determined by fault modeling.

Our objective is the detection and isolation of failure modes $\nu_i(t)$ in the presence of the modeling uncertainties and disturbances. In this chapter we consider the situation when separation (decoupling) of the fault effects from unknown inputs at the residual space of the filter is not possible, and therefore, the detection performance of the filter will always be compromised by the effect of uncertainties. In order to avoid excessive false alarm rates or missed detections the improvement of the disturbance attenuation capability of the filter with respect to unknown inputs is the only viable solution to the problem.

Studies on the use of H_∞ optimal state estimation methods to FDI have shown that these filters can produce output residuals that are insensitive to disturbances to certain limits. In (Mangoubi et al., 1993), a preliminary study on the use of H_∞/μ filters with robustness to noise and plant model uncertainties for fault detection, shows that these filters can generate output residuals that are highly insensitive to uncertainties. Other works based on the use of H_∞ filtering techniques with focus on the application to FDI include that of (Mangoubi, 1995; Qiu and Gertler, 1993; Edelmayer et al., 1994; Mangoubi et al., 1994; Frank, 1994; Frank and Ding, 1994) as well as (Patton and Hou, 1997; Chung and Speyer, 1998; Douglas and Speyer, 1995; Douglas and Speyer, 1999).

A general design goal of H_∞ filtering is to provide the optimal estimate of the state vector of the system by taking assumptions about the bound of the cumulative effects of the uncertainties ensuring that the magnitude of the transfer function computed from unknown inputs to the output error of the filter is always less than a prespecified level $\gamma > 0$. This kind of H_∞ filtering problem was first considered by (Grimble, 1987), then followed by (Doyle et al., 1989) and (Bernstein and Haddad, 1989). Different interpretations of the problem can be found *e.g.*, in (Nagpal and Khargonekar, 1991; Yaesh and Shaked, 1992; Limebeer et al., 1992).

A notable incident reported by research papers regarding the application of traditional H_∞ filtering techniques to FDI was that, in certain cases, these filters may provide poor detection performance. Although, improving disturbance suppression is a primordial design goal in filtering, the solution of the detection filter design problem requires maintaining adequate sensitivity to failure modes either. Obviously, these conflicting objectives lead to unavoidable design tradeoff: capturing all robustness and performance objectives in a single H_∞ norm cost function is not possible. As a result, traditional H_∞ optimization approaches cannot guarantee any desired level of sensitivity.

One could postulate this estimation problem as a mixed H_2/H_∞ filtering problem which amounts to finding a filter gain which minimizes the H_∞ norm of the transfer function from unknown inputs to the residual of the filter subject to the H_2 norm of the transfer function from failure modes to the filter error. Unfortunately, the design freedom which is usually available in practice doesn't make the realization of this idea possible. Early results obtained by mixed-criterion optimization were inevitably compromised by inferior disturbance suppression capability – these filters tend to indicate insufficient sensitivity to failures.

A more realistic interpretation of the problem, utilizing the standard H_∞ filtering solution, minimizes the effect of unknown inputs on the filter error by simultaneously guaranteeing an *expected minimal* amplification rate of failure modes or, what amounts to the same thing, optimize filtering sensitivity.

Our earlier results of the application of H_∞ filtering to robust FDI, see *e.g.*, (Edelmayer et al., 1997a) have shown that the proper selection of a free design parameter, namely, the output map of the state estimation, may have significant influence on detection performance as well as on direction dependent sensitivity of the filter. In most practical situations, however, the heuristic choice of the estimation weighting guaranteed neither a definite sensitivity increase nor improved separability for the particular failure signals.

Continuing the original concept of (Edelmayer et al., 1997b) in this chapter the ideas of sensitivity optimization is discussed in more details. In our view, a well-conditioned optimization problem is a prerequisite for obtaining accurate results. It will be shown that an alternative approach, still utilizing the H_∞ norm cost function, involves similarity scaling of certain closed loop transfer functions. The robust FDI problem as a scaled H_∞ filtering problem is presented where the effect of estimation weighting on filtering performance is optimized through two different input similarity scaling approaches, namely, *diagonal scaling* and *rotation*. The objective is to provide the smallest scaled L_2 gain of the disturbance input of the system that is guaranteed to be less than a pre-specified positive constant γ, and, at the same time to increase filtering sensitivity as much as possible. The solution is considered to be an extension to the standard solutions of the H_∞ detection filter design problem.

The layout of the chapter is as follows. In Section 4.2 the basic concepts of the conventional H_∞ detection filter problem for LTV systems are formulated. The general solution to the filtering problem is briefly given. In Section 4.3 the idea of input similarity scaling and its role in the optimization process is discussed. It is shown how to obtain bounds on the scaled L_2 gain by transforming the original problem into a convex feasibility problem, specifically, a structured, linear matrix inequality (LMI). Numerical results demonstrating the effect of the scaled optimization on the filter performance conclude the paper.

4.2. FILTERING IN AN H_∞ SETTING

Consider the LTV system model (4.1). In standard H_∞ (or min-max) optimal filtering problem the objective is to design a state observer for the nominal LTI representation of the system (4.1) which gives an estimate $\hat{z}(t)$ of the weighted state vector

$$z(t) = C_z x(t). \tag{4.3}$$

The estimate $\hat{z}(t)$ is then used in generating a residual for the detection of the failure mode $\nu(t)$. Assuming that the observer design is based on the nominal pair (A, C) of representation (4.1), the observer can be given in the form

$$\begin{aligned} \dot{\hat{x}}(t) &= A\hat{x} + D(y - C\hat{x}) + B_u u(t), \qquad (4.4)\\ \hat{y}(t) &= C\hat{x}(t),\\ \hat{z}(t) &= C_z\hat{x}(t) \end{aligned}$$

with the observer state $\hat{x} \in \mathbb{R}^n$ and weighted output estimation $\hat{z} \in \mathbb{R}^p$ where D is the static observer gain matrix and C_z is the constant estimation weighting. Obviously, the filter error system can be derived as

$$\begin{aligned} \dot{\tilde{x}}(t) &= (A - DC)\tilde{x}(t) + \Delta A(t)x(t) + B_w w(t) + \sum_{i=1}^{k} L_i \nu_i(t),\\ \varepsilon(t) &= C\tilde{x}(t), \qquad (4.5) \end{aligned}$$

where the state error $\tilde{x}(t)$ and weighted output error $\varepsilon(t)$ of the filter are defined, respectively, as

$$\tilde{x}(t) = x(t) - \hat{x}(t), \qquad \varepsilon(t) = z(t) - \hat{z}(t).$$

It is important to notice that the state and state-error equations become coupled through the time varying perturbation term $\Delta A(t)$. It is assumed that the inputs and perturbations are norm bounded. Therefore, the approaches are concerned with obtaining an estimate $\hat{z}(t)$ of $z(t)$ over the finite horizon $[0, T]$ providing a uniformly small estimation error $\varepsilon(t)$ for any $w(t), \nu(t) \in \mathcal{L}_2[0, T]$ and all admissible uncertainties assuming $\nu_i(t) = 0$.

4.2.1. *The classical solution to* H_∞ *detection filters*

Mathematically, the performance measure considered is defined as

$$J(w, \nu, \hat{z}) \triangleq \frac{1}{2}\left[\| z - \hat{z} \|_2{}^2 - \gamma^2 \left(\| w \|_2{}^2 + \| \nu \|_2{}^2\right)\right].$$

Using this criterion, the robust H_∞ filtering problem can be defined as follows: find an estimate $\hat{z}(t)$ which minimizes

$$\sup_{w, \nu, a_i} J(w, \nu, \hat{z}). \tag{4.6}$$

The basic idea of the solution of this linear-quadratic optimization problem is that the estimation error $z - \hat{z}$ is minimized *w.r.t.* the worst-case effect of unknown inputs using a min-max optimization.

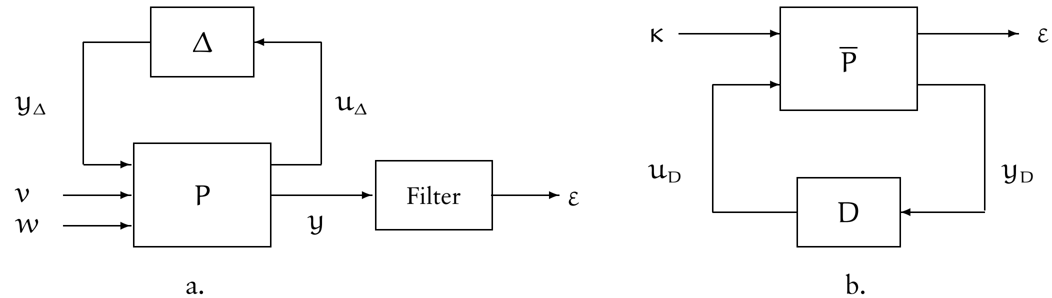

Figure 4.1. Diagram for the input-output specification of the H_∞ *filtering problems for nominal plant* P *in terms of modelling uncertainties* Δ *and disturbances* $w(t), v(t)$ *on the one hand, and worst-case input* $\kappa(t)$ *on the other.* $\overline{P}$ *is the generalized plant.*

Several alternative solutions to this H_∞ estimation problem exist depending on the ways the worst-case inputs are derived and included in the filtering problem. The most general solution can be achieved by using a two-stage design procedure which requires the handling of two coupled Riccati equations, see *e.g.*, (Mangoubi, 1995). In (Edelmayer et al., 1994) a different approach was presented which, with certain assumptions, requires the solution of a single Riccati equation. (Edelmayer et al., 1996) gives the comparison of the two ideas by showing that the different approaches share the common concept of using an auxiliary representation of the original system (4.1). The auxiliary representation is based on a generalized model of the plant. Technically, this amounts to the transformation of the original system to a representation which does not contain modelling uncertainties but is affected by auxiliary inputs which are treated as worst-case disturbances. Then, one needs to solve the H_∞ optimization for the auxiliary system.

The methodology of solving the robust H_∞ filtering problem in the presence of parametric uncertainty via an auxiliary H_∞ filtering problem was first proposed in (Xie et al., 1991) and (de Souza et al., 1992) for discrete and continuous-time systems, respectively. The aim is to construct the generalized filtering scheme as it can be seen in Fig. 4.1/a and Fig. 4.1/b, respectively, on the basis of the original LTV representation (4.1). The auxiliary system representation can be obtained as

$$\begin{aligned}\dot{x}(t) &= Ax(t) + B_u u(t) + B_\kappa \kappa(t) + \sum_{i=1}^{k} L_i \nu_i(t)\\ y(t) &= Cx(t),\end{aligned} \tag{4.7}$$

which does not involve parametric uncertainty and is equivalent to the original uncertain system in the sense that both (4.1) and (4.7) have equivalent (C, A)-invariant subspaces. $B_\kappa = [B_w, L_\Delta]$ is the worst-case input direction and $\kappa(t) \in \mathcal{L}_2[0, T]$ is the input function for all $t \in \mathbb{R}_+$ representing the worst-case effects of modelling uncertainties and external disturbances propagating into any of the nominal system matrices A, B or C. Note that sensor fault is not considered in this setting.

Consider the system with B_κ and arbitrary unknown perturbation function $\kappa(t)$. Based on the results of (Limebeer et al., 1992) and (Nagpal and Khargonekar, 1991), an optimal detection filter which makes a tradeoff between worst-case disturbance κ and $\mathcal{L}_2$ norm of the filter error $\tilde{z} = (z - \hat{z})$ on $\mathcal{L}_2$ which minimizes the worst-case performance measure

$$J(D, \kappa) \triangleq \sup_{\kappa \in \mathcal{L}_2} \frac{\| z - \hat{z} \|_2}{\| \kappa \|_2} = \| H_{\varepsilon\kappa}(s) \|_\infty \tag{4.8}$$

can be derived as,

$$\begin{aligned} \dot{\hat{x}}(t) &= (A - QC^T C)\hat{x} + B_u u(t) + QC^T y(t), \\ \hat{z}(t) &= C_z \hat{x}(t) \end{aligned} \tag{4.9}$$

in which, corresponding to a given estimation weighting matrix C_z, one optimal detection filter gain $D = QC^T$ could be obtained through gamma iteration *i.e.*, by solving the modified filter algebraic Riccati equation (MFARE)

$$AQ + QA^T - Q\left(C^T C - \frac{1}{\gamma^2} C_z^T C_z\right) Q + B_\kappa B_\kappa^T = 0 \tag{4.10}$$

for Q starting from a sufficiently large γ_o recursively, until $\gamma_{min} \in \mathbb{R}_+$ is found for which the constant $\gamma_{min} - \epsilon$ with an arbitrary small $\epsilon > 0$ no longer produces a positive definite solution, see (Edelmayer et al., 1994). One can see that the solution is defined over the set of allowable weighting matrices C_z, and as such, is dependent on its proper choice. Our latest results on the application of H_∞ filtering to robust FDI (see *e.g.*, (Edelmayer et al., 1994) and (Edelmayer et al., 1997a)) have shown that the proper selection of the map C_z may have significant influence on detection performance as well as on direction dependent sensitivity of the filter. The observation was confirmed by the results of (Chen and Patton., 2000), as well.

4.2.2. *Characterization of filtering sensitivities*

Based on the previous results, one may give standard quantities which can be used for the characterization of sensitivity as well as an overall performance of detection filters as follows. The detection threshold with respect to a particular failure mode can be given as

$$\tau(C_z) = \gamma_{min} \| \kappa \|_2, \tag{4.11}$$

which is exactly the magnitude of the effects of worst-case inputs at the output error of the filter. Obviously, the failure modes which produce ingredients in the residual smaller than that of this limit, cannot be detected by the filter. Notice that τ is a function of the estimation weight C_z.

Similarly, the amplification rate of failure modes relative to the amplification of worst-case inputs can be given by the dimensionless quantity

$$\mu_i = \frac{\| H_{\varepsilon\nu_i} \|_\infty \ \| \nu_i \|_2}{\gamma_{min} \| \kappa \|_2}. \tag{4.12}$$

By substitution of γ_{min}, the ratios

$$S_i = \frac{\| H_{\varepsilon\nu_i} \|_\infty}{\| H_{\varepsilon\kappa} \|_\infty}, \tag{4.13}$$

which can be regarded as the measures of filtering sensitivity, represent the magnitude of the frequency response of the particular failure modes relative to the effects of worst-case input where the matrices

$$H_{\varepsilon\nu_i}(s) = C_z(sI - A + DC)^{-1}L_i \tag{4.14}$$

$$H_{\varepsilon\kappa}(s) = C_z(sI - A + DC)^{-1}B_\kappa, \tag{4.15}$$

are the transfer functions calculated from failure modes $\nu_i(t)$ and unknown inputs $\kappa(t)$ to the weighted error residual $\varepsilon(t)$ of the filter, respectively. Obviously, S_i may characterize sensitivity only locally at a particular frequency. It is desirable to keep S_i as high as possible over the whole frequency range where the detection of $\nu_i(t)$ is to be considered.

4.3. SCALED H_∞ DETECTION FILTERS

It was shown in the previous sections that filtering sensitivity is subject to the proper selection of the estimation weight. The choice of the set of applicable weights is always problem dependent, which reflects assumptions about the disturbance and fault characteristics as well as the desired performance requirements of the filter. Simple-minded approaches of the selection of C_z may not provide optimal results. In this chapter we propose a new approach namely scaled optimization. This makes the inclusion of an optimality seeking algorithm into the problem possible which helps finding the optimal value of the free parameters like C_z.

4.3.1. *Scaling and the idea of scaled* H_∞ *optimization*

In order to be able to introduce the concept of scaled optimization we need to review the historical origins of the idea. Numerical aspects of optimization algorithms require the use of well conditioned problems in terms of numerical conditioning of matrices, namely those that contribute to the process of optimization. Traditionally, one should scale the (A, B, C) matrices of a system to improve their conditioning. In this traditional sense the condition number of a matrix is the ratio of the largest to the smallest singular values. This number should ideally be close to unity. The importance of this property arises from the fact that each time we execute matrix multiplications (and these are ubiquitous in numerical optimization utilizing iterative methods) the resulting quantities are more sensitive than the original ones (*e.g.*, with respect to the solution of equations arising from the latter) thus accumulating computational (rounding) errors.

There are differing options as to how scaling should be done. A common practice is to divide each variable by its maximum expected or allowed change. For the system $\dot{x} = Ax + Bu$, $y = Cx$, for instance, this is achieved by scaling each component input as $\hat{u}_i = u_i/u_i^{max}$ and similarly for outputs and states. The overall effect of scaling is that of multiplying inputs, outputs and states by positive definite diagonal matrices D_1, D_2 and D_3 resulting in the system

$$\begin{aligned}\dot{\hat{x}} &= D_1AD_1^{-1}\hat{x} + D_1BD_2^{-1}\hat{u}\\ \hat{y} &= D_3CD_1^{-1}\hat{x}\end{aligned}$$

where $\hat{x} = D_1 x$, $\hat{u} = D_2 u$, $\hat{y} = D_3 y$.

In light of the above introduction let us return to our original problem and investigate the effect of weighting by considering the similarity scaling of the worst-case disturbance/error closed loop transfer function

$$H_{\varepsilon\kappa}(s) = C_z(sI - A + DC)^{-1} B_\kappa. \tag{4.16}$$

PROPOSITION 4.1. Let our objective be to solve

$$\inf_{C_z, T>0} \| T(H_{\varepsilon\kappa})T^{-1} \|_\infty. \tag{4.17}$$

Namely, by modifying Eq. (4.7) and introducing the scalar non-singular *diagonal* matrix T which has the interpretation of scaling, one can consider the system

$$\dot{x}(t) = Ax(t) + B_u u(t) + B_\kappa T^{-1}\kappa(t) + \sum_{i=1}^{k} L_i \nu_i(t)$$

$$z(t) = TC_z x(t), \tag{4.18}$$

assuming (4.18) has as many disturbance inputs as state estimates. Analogously, the scaled estimation

$$\hat{z}(t) = TC_z \hat{x}(t) \tag{4.19}$$

and, respectively, the scaled L_2 gain can be given

$$\inf_{\substack{T>0 \\ T\text{diagonal}}} \sup_{\|\kappa\|_2} \frac{\| z - \hat{z} \|_2}{\| \kappa \|_2}, \tag{4.20}$$

which has the interpretation

$$\max_i \sup_{\| \kappa_i \|_2} \frac{\| (z - \hat{z})_i \|_2}{\| \kappa_i \|_2} \tag{4.21}$$

for $i = 1, \ldots, n_z$ and every fixed scaling $T = \text{diag}(t_1, \ldots, t_{n_z})$. □

As a matter of fact, by using the idea of Proposition 4.1 the conditioning of the worst-case input $\kappa(t)$ and its effect on the estimation process is investigated as it will be shown in the following section. The usefulness of such optimizing solutions is known in the control literature and can also be found *e.g.*, in (Safonov, 1986), (Boyd and Yang, 1989) and (Packard et al., 1992).

For the application of the idea to H_∞ detection filter design, consider the result of (Edelmayer et al., 1997b) which first proposed the application of a diagonal input scaling method by giving the solution of problem (4.17) for T diagonal. In the following part, the solution method developed in the series of papers (Edelmayer et al., 1997b; Edelmayer and Bokor, 2000; Edelmayer and Bokor, 2002) is detailed.

4.3.2. *LMI solution of diagonally scaled optimization*

Unfortunately, the problem described by Eq. (4.17) is not convex in general, see for instance (Safonov, 1986) and, previously, no simple optimization methods were available for the solution of this kind of problem. In the following part an alternative solution method is presented.

This solution is based on the transformation of the original non-convex problem into a convex feasibility problem in the framework of structured linear matrix inequality (LMI).

PROPOSITION 4.2. The L_2 gain scaled by T is guaranteed to be less than $\gamma > 0$ if there exists $R > 0$ which satisfy

$$\begin{bmatrix} RA + A^T R - C^T C & RB_\kappa & C_z^T \\ B_\kappa^T R & -S & 0 \\ C_z & 0 & -\gamma^2 S^{-1} \end{bmatrix} < 0, \tag{4.22}$$

with $S \triangleq T^T T$. □

Proof. From representation (4.18) the scaled Riccati equation is obtained

$$AQ + QA^T - Q\left(C^T C - \frac{1}{\gamma^2} C_z^T T^T T C_z\right) Q + B_\kappa T^{-1} T^{-T} B_\kappa^T = 0. \tag{4.23}$$

Alternatively, (4.23) is equivalent to the Riccati inequality in $R \triangleq Q^{-1}$,

$$RA + A^T R - C^T C + RB_\kappa T^{-1} T^{-T} B_\kappa^T R + \frac{1}{\gamma^2} C_z^T T^T T C_z < 0 \tag{4.24}$$

if there exists a real $R = R^T$ satisfying (4.24). Note that the inequality (4.24) which is a quadratic matrix inequality in the variable R can be obtained by applying the positive-real lemma to (4.23) and substituting the equality for inequality, see *e.g.*, (Boyd et al., 1994).

The nonlinear inequality (4.24) can be converted to LMI form

$$\begin{bmatrix} RA + A^T R - C^T C & -RB_\kappa \\ B_\kappa^T R & -S \end{bmatrix} + \frac{1}{\gamma^2} \begin{bmatrix} C_z \\ 0 \end{bmatrix} S \begin{bmatrix} C_z & 0 \end{bmatrix} < 0, \tag{4.25}$$

by using Schur complements of block matrices. Inequality (4.25) is also equivalent to

$$\begin{bmatrix} C_z^T S C_z & 0 \\ 0 & 0 \end{bmatrix} < \gamma^2 \begin{bmatrix} -RA - A^T R + C^T C & -RB_\kappa \\ -B_\kappa^T R & S \end{bmatrix}, \tag{4.26}$$

which gives the LMI formulation (4.22) and proves the result of Proposition 4.2. ■

The optimal scaled L_2 gain γ is, therefore, obtained by minimizing γ over (γ, R, S) subject to (4.22). It can be deduced from (4.25) that the problem leads to the convex Generalized Eigenvalue Minimization Problem (GEVP), *i.e.*, to the minimization of the maximum generalized eigenvalue of a pair of matrices that depend affinely on a variable subject to an LMI constraint. The general form of this GEVP is:

$$\begin{array}{ll} \text{minimize} & \lambda \\ \text{subject to} & \lambda B(x) - A(x) > 0 \\ & B(x) > 0 \\ & C(x) > 0 \end{array} \tag{4.27}$$

where A, B, C are affine functions of the variable x, see again (Boyd et al., 1994). Note that GEVP is a quasi-convex problem, which, for the variable γ can be solved by using appropriate optimization algorithms (LMI toolbox in Matlab) by applying the convex constraints

$$\begin{bmatrix} RA + A^T R - C^T C & RB_\kappa \\ B_\kappa^T R & -S \end{bmatrix} < 0 \tag{4.28}$$

and

$$\begin{bmatrix} -C & -RB_\kappa \\ -B_\kappa^T R & S \end{bmatrix} > 0. \tag{4.29}$$

The optimization algorithm of GEVP returns the optimal γ and also the parameters R and S which are necessary for filter implementation.

4.3.3. *Scaling and rotation of the worst-case input*

Nondiagonal invertible scaling matrices lead to general coordinate transformations. By choosing the scaling $T > 0$ to be a full matrix, one can introduce rotation in the input space beyond simple diagonal scaling.

PROPOSITION 4.3. The LMI solution of (4.17) for nondiagonal invertible scaling matrices T can be given by solving the GEVP subject to the LMI (4.22) and the constraints (4.28) and (4.29) in the same way as for T diagonal.

EXAMPLE 4.4. For illustrating the effect of the scaling and rotation, consider the following simple example. Let the system (4.1) in Jordan canonical form be given by the matrices

$$A = \begin{bmatrix} -0.8 & 0 & 0 \\ 0 & -0.5 & 0.6 \\ 0 & -0.6 & -0.5 \end{bmatrix}, \quad B_u = \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix},$$

i.e., one may consider the representation where the real and imaginary parts of the complex eigenvalues of the nominal system matrix A are affected by unknown LTV perturbations $a_1(t), a_2(t) \in \mathcal{L}_2$. Assume for simplicity that $w(t) = 0$. Let the objective of the design be the detection of the actuator faults $\nu_1(t)$ and $\nu_2(t)$ appearing in the directions $L_1 = [1\ 1\ 0]^T$ and $L_2 = [1\ 0\ 1]^T$, respectively. Let us select

$$B_\kappa = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

for worst-case disturbance direction and the estimation weight be defined as

$$C_z = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}.$$

(i) First, the filter is calculated by using conventional H_∞ optimization methods. The filter solution can be derived by performing gamma iteration on the Riccati equation (4.10). This solution ensures that the magnitude of the disturbance/error transfer function is always lower than $\gamma_o = 1.351$, see Fig 4.2. It is important to note that one of the failure modes (namely,

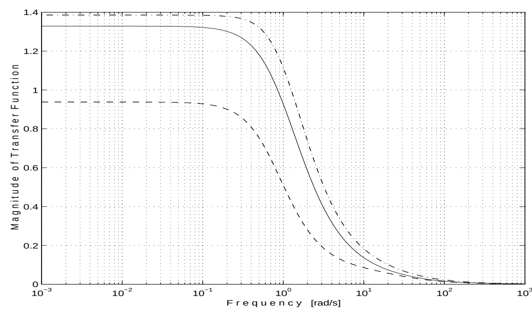

Figure 4.2. Maximal singular values of the transfer functions $H_{\varepsilon\nu_i}$ (dotted lines) and $H_{\varepsilon\kappa}$ (solid line) of the detection filter derived by using conventional H_∞ optimization. Solid line is the detection threshold.

the one which is not sufficiently amplified in the output error residual to exceed the detection threshold γ_o) cannot be detected by this filter.

(ii) Subsequently, the diagonally scaled H_∞ filter with the same C_z is computed by using the LMI-based design. As a solution to the GEVP represented by the set of constraints (4.22) and the standard LMI constraints (4.28) and (4.29), it is obtained that the diagonally scaled L_2 gain is guaranteed to be less than $\gamma_o = 1.283$, see Fig. 4.3.

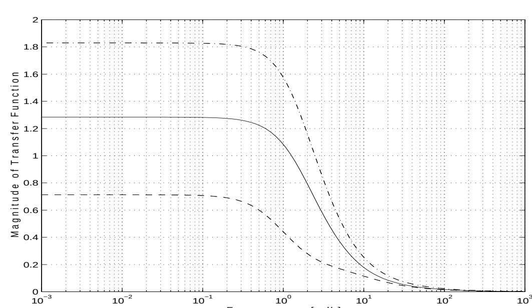

Figure 4.3. Magnitudes of $H_{\varepsilon\nu_i}$ (dotted lines) and $H_{\varepsilon\kappa}$ (solid line) derived by using optimal diagonal input scaling for H_∞ optimization.

(iii) The result of the solution to the full scaling problem is illustrated in Fig. 4.4. Here, the L_2 gain is guaranteed to be less than $\gamma_o = 1.303$. Though the detection threshold γ_o is not any better than in the previous case, however, the sensitivity of the filter with respect to one of the failure modes has increased somewhat.

The above results indicate clearly that the real effect of the proposed method lies in the construction of optimal input directions for the optimization algorithm by scaling (rotation) of the worst-case input. In the design process one need to choose the coordinate axes (*i.e.*, do coordinate transformations) and/or units (*i.e.*, do diagonal scaling) so that the mathematical

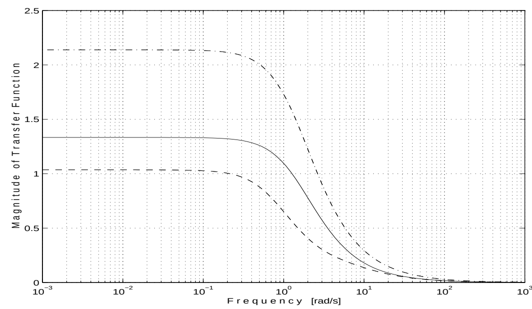

Figure 4.4. Magnitudes of $H_{\varepsilon\nu_i}$ *(dotted lines) and* $H_{\varepsilon\kappa}$ *(solid line) derived by using optimal full input scaling and rotation for* H_∞ *optimization.*

optimization problem accurately reflects the sensitivity of the physical problem. As a result, the overall sensitivity of the detection filter can be improved significantly over the results provided by the traditional H_∞ optimization approaches.

4.4. SUMMARY

This chapter was concerned with the robustness and sensitivity issues of the design of detection filters. The basic objective was to attenuate the worst-case effects of time varying parametric system perturbations and external disturbances on the filter error by maintaining sensitivity to failure modes by using game theoretic (min-max or H_∞) optimization.

It was shown that in robust FDI problems, capturing all robustness and performance objectives in a single H_∞ norm cost function is not possible. An alternative approach, still utilizing the H_∞ norm, involves input similarity scaling of certain transfer functions of the filter. H_∞ filters were designed by using the ideas of two different input scaling approaches, namely, diagonal scaling and rotation of the worst-case input.

The results can be considered as direct extensions to the standard solutions of conventional H_∞ filtering. The solution methods which are based on convex optimization algorithms and LMI representation of the system enhance the disturbance suppression capability of the filter by scaling the effect of the estimation weight C_z. Simulation results confirm that this scaled optimization may provide improved filter performance, lower detection thresholds and improved directional sensitivity with respect to conventional H_∞ design methods.

Further studies are required to investigate the mechanism by which the directional sensitivity of the filter may have been influenced. The clarification of this idea could help in solving the task of fault isolation by using H_∞ filters.

PART 3

DIRECT INPUT RECONSTRUCTION TO FAULT DETECTION

The problem of reconstructing inputs which may represent control actions, unknown external disturbances, faults and other uncertainty effects in a dynamic system is, in certain sense, the subject of all fault detection and isolation approaches. In fact, input reconstruction was of great interest as long as the problem of reconstructing the faults in the presence of other (known and unknown) inputs has been considered. This indirect approach seldom aims to reconstruct the input of the system in its entirety since, in the majority of cases, the problem of fault signal reconstruction is in focus by trying to ignore all the others. In this approach, the characteristics of the fault is interesting only in scope of its detectability w.r.t a well-established threshold. This interpretation of the problem was the topic of discussion in the approaches presented in the previous parts. In contrast to this indirect approach, direct input reconstruction corresponds to finding a reconstructor, whose inputs are the observables obtained from the original system and whose outputs correspond to the original inputs: control signals, faults and disturbances in a well-structured way. In the previous parts the conceptual relationship of input reconstruction and system inversion has been shown. It is shown, how input reconstruction methods, based on the representation of the inverse system can be considered a useful means in obtaining the estimate of the driving functions exerted on the input of a dynamical system.

CHAPTER 5

Input Reconstruction by System Inversion: an Algebraic Approach to Linear Systems

In this chapter the idea of system inversion to the design of detection filters to fault detection and isolation in dynamic systems is addressed. This approach is an application of dynamic inversion to filtering which is dual to the concept of dynamic inversion for control. The difference between these inversion approaches is that control uses a right inverse whereas estimation uses a left inverse of the system (see the explanation of these concepts in Section 2.3.1). The method arrives at detector architectures whose outputs are the fault signals while the inputs are the measured system inputs and outputs and possible their time derivatives. This approach will make not only the detection and isolation but also the estimation of the fault signals possible. The idea is basically relies on the concept studied for example by (Sain and Massey, 1968; Silverman, 1969) for LTI systems and considered by (Hirschorn, 1979a; Hirschorn, 1979b; Fliess, 1986) and also (Isidori, 1985) for nonlinear systems.

Though inverse problems became particularly important in control and system theory in the last 50 years, and the close relation of input reconstruction to system inversion was emphasized by many authors the idea of the application of this concept to solve various detection problems first appeared in the works of Szigeti quite lately, see *e.g.*, (Szigeti et al., 2000a; Szigeti et al., 2000b).

Very briefly, in inversion-based detection filter design the goal is to find the left inverse of the fault-to-output residual transfer function such that the fault estimation error transfer function is diagonal.

The analysis of the interaction between input and state, on the one hand, and between state and output, on the other hand, is of a fundamental importance in solving the input reconstruction problem. Key tools for the analysis of such interactions are the notions of reconstructability and invertibility and the relative degree and zero dynamics of the representation of a dynamical system. We review in this chapter some relevant aspects of this algebraic theory, namely those which are used in the algebraic approach presented in the last part of this chapter.

This chapter beyond presenting a general introduction to the ideas deals with linear systems only. The linear structure allows the results to be carried forward in a simpler form and easier computational procedure to be developed for the derivation of the inverse system. In fact, while in the nonlinear case it will be necessary to use structural properties of the system such as relative degree and zero dynamics, for linear time invariant systems it is possible to relate the inverse to some structure independent problems such as the purely algebraic approach presented in this chapter. The notion of relative degree and zero dynamics will be introduced in the next chapter where the nonlinear problems are introduced.

The main contribution of this chapter is, by using an algebraic state space approach, the elaboration of an inversion algorithm for LTI systems that can be used for detector design represented as minimum order stable linear dynamic systems.

5.1. INTRODUCTION

In this chapter the classical detection filter design problem as a standard input reconstruction problem is considered. The residual generation problem is viewed as an inverse problem and is aimed at being solved by dynamic system inversion. Input reconstruction by means of system inversion is a relatively new idea to construct residual generators for robust detection and isolation of faults. This approach is based on the existence of the left inverse and arrives at detector architectures whose outputs are the fault signals while the inputs are the measured system inputs and outputs and possibly their time derivatives. This will make not only the detection and isolation but also the estimation of the fault signals possible. The theory is developed for linear time invariant and nonlinear systems having vector relative degree. The nonlinear problem is presented in the next chapter. This chapter presents a view on the properties of the inverse for linear multivariable systems from the aspect of the fault detection and isolation. The applicability of the inversion process to fault reconstruction is demonstrated by examples.

Residual generation both for linear and nonlinear systems can be viewed as an input reconstruction process and solved by using the idea of system inversion. In order to introduce this idea consider the following disturbance free linear control system subject to faults given in state space form as

$$\dot{x} = Ax + Bu + Lv, \qquad (5.1)$$

$$y = Cx + Du + Mv, \qquad (5.2)$$

where $x \in \mathbb{R}^n$, is the state vector, $u \in \mathbb{R}^r$, $y \in \mathbb{R}^p$ are the inputs and the measured outputs, respectively. It is noted again that the fault signal $v \in \mathbb{R}^q$ can represent both actuator and sensor failures, in general, as reflected in the structure of the matrices L, M. The goal is to detect the presence of the components of the fault signal independently from each other.

Properties of unknown input reconstructability discussed in Section 2.3.1 suggest the application of a novel solution approach to this problem. Recall that unknown input reconstruction addresses the problem of designing a filter or detector which, on the basis of the input and output measurements, returns the unknown inputs of the original system. The idea is to construct an input reconstructor or detector, *i.e.*, another dynamic system with outputs $v(t)$ and with inputs $u(t), y(t)$ and possible their time derivatives or integrals (Szigeti et al., 2001) which we expect to have in the general form

$$\dot{\bar{x}} = \bar{A}\bar{x} + B_u\xi_u + B_y\xi_y, \tag{5.3}$$

$$\nu = \bar{C}\bar{x} + D_u\xi_u + D_y\xi_y, \tag{5.4}$$

where the elements of the vectors ξ_u, ξ_y consist of the input and output signals and also their time derivatives of the appropriate order as

$$\xi_y = [y, \dot{y}, \ddot{y}, \ldots]', \quad \xi_u = [u, \dot{u}, \ddot{u}, \ldots]'.$$

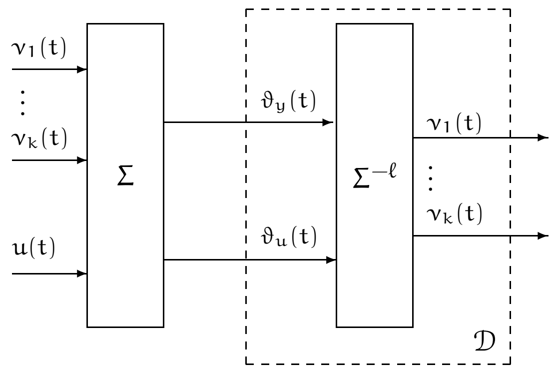

Figure 5.1. Input reconstruction and the idea of system inversion: Σ is the plant, $\mathcal{D}$ is the reconstructor or detector which can be obtained as the (left) inverse $\Sigma^{-\ell}$ of the original system.

The analogy between input reconstructability and system invertibility presented in Section 2.3.1, suggest that one possible way to obtain a dynamical system (5.3-5.4) is through the construction of the left inverse of (5.1-5.2) *w.r.t.* the failure signal $\nu(t)$. For the schematic interpretation of the idea see Fig. 5.1.

The solution of various types of inverse problems became particularly important in control and filtering which received a considerable attention already in the classical age of control sciences. The feasibility of system inversion for solving detection problems, however, was first demonstrated in (Szigeti et al., 2001). Additional issues of inverse computation for the FDI problem can be found *e.g.*, in (Szigeti et al., 2002) and (Varga, 2002). More recently, on-line dynamic inversion methods were successfully applied to many interesting problems in aerospace and aviation, such as *e.g.*, (Krupadanam et al., 2002). A summarizing study on related ideas was published in (Goodwin, 2002). Still, however, there remained a number of open problems in this area especially regarding the properties and calculation of the inverse in problems of fault detection and isolation.

Silverman's left inversion algorithm was proposed for the linear dynamic systems $\dot{x} = Ax + Bu$, $y = Cx + Du$ in the form

$$\dot{z} = (A - BD_\alpha^{-1}C_\alpha)z + BD_\alpha^{-1}y_\alpha$$

$$u = -D_\alpha^{-1}C_\alpha x + D_\alpha^{-1}y_\alpha,$$

where D_α, C_α were obtained in a recursion ensuring the invertibility of D_α in the α-*th* step, see (Silverman, 1969). This procedure, however, guaranteed neither minimality (or observability, detectability) nor stability properties of the resulting inverse system, causing difficulties when using this idea in detector (residual generator) design or applied to various kind of signal estimation problems.

In Section 5.3 a recursive procedure, that generates minimal state space representation of a left inverse system if the original system was given in minimal left invertible state space form is proposed. This algorithm is basically a constructive one and, therefore, only the first step of the procedure will be discussed in details.

5.2. INPUT (FAULT) OBSERVABILITY OF LTI SYSTEMS

The input or fault observability of linear dynamical systems were closely related to invertibility in Section 2.3.1. In order to show this property as well as for the properties of fault observability, let us summarize some important results from the literature. For the sake of simplicity, we discuss input observability in the sequel. Fault observability can be interpreted in an analogous way. Let the minimal state space representation of the LTI systems be given

$$\dot{x} = Ax + Bu, \quad y = Cx + Du \tag{5.5}$$

and consider the following proposition:

DEFINITION 5.1. (Hou and Patton, 1998). The input $u(t)$ is said to be observable if $y(t) = 0$ for $t \geq 0$ implies $u(t) = 0$ for $t > 0$ provided that $x(0) = 0$. □

DEFINITION 5.2. (Basile and Marro, 1969a). A linear system is called left invertible if the input $u(t)$ can be recovered from the knowledge of output function $y(t)$ and the initial state $x(0)$. □

REMARK 5.3. For any known initial condition $x(0)$ input observability implies left invertibility.

Let us denote the set of all possible inputs of (5.5) by Ω and assume they are at least n-times differentiable.

PROPOSITION 5.4. By taking the restriction of the input set

$$\Omega_o = \{u \in \Omega : u(0) = 0,\, \dot{u}(0) = 0, \ldots, u^{(n-1)}(0) = 0\}$$

and considering system (5.5) over Ω_o, left invertibility and input observability are equivalent. □

Proof. Consider the output $y(t)$ and of its derivatives with respect to time $y^{(k)}(t)$, for $k = 1, 2, \ldots,$ and $t = 0$ on Ω_o. We obtain the equations

$$\begin{aligned} y &= Cx + Du, \\ \dot{y} &= CAx + CBu + D\dot{u}, \\ &\vdots \\ y^{n-1} &= CA^{n-1}x + CA^{n-2}Bu + \ldots + CBu^{(n-2)} + Du^{(n-1)}, \end{aligned}$$

$$\begin{aligned} y &= Cx(0), \\ \dot{y} &= CAx(0), \\ &\vdots \\ y^{n-1} &= CA^{n-1}x(0). \end{aligned}$$

Since the system is minimal by assumption hence it follows that the output function $y(t)$ determines the initial state $x(0)$, uniquely, which, according to Remark 5.3, means that left invertibility and input observability are equivalent on Ω_o. ■

REMARK 5.5. (Fault observability and invertibility). In case we work with fault detection problems, *i.e.*, we consider systems of type (5.1-5.2) where the fault signals $\nu \in \mathbb{R}^q$ may represent both actuator and sensor faults as reflected in the structure of the matrices L, M, all derivatives of the fault signals in the diagnostic system models will be zero for $t = 0$, since it is always supposed that $\nu(t) = 0$ if $t \leq t_o > 0$. It follows that the residual system is invertible iff it is input observable. Clearly, if M in (5.2) is a full rank matrix the inverse can be obtained by simple algebraic calculation. For treating more general cases, however, we need to consider the properties of invertibility in more details in the next sections.

5.3. A CONSTRUCTIVE ALGORITHM FOR INVERSION OF LINEAR SYSTEMS

Consider the state space representation (5.1-5.2) of the linear dynamical system subject to faults. The algorithm for the calculation of the left inverse of (5.1-5.2) by using pure algebraic considerations can be presented as follows. Initiate the procedure by

$$\begin{aligned} A_o &= A, & B_o &= B, & L_o &= L, \\ C_o &= C, & D_o &= D, & M_o &= M, \\ x_o &= x, & y_o &= y, & \nu_o &= \nu, \end{aligned}$$

and follow the algorithmic steps as below:

STEP 1. Denote the projection to $\operatorname{Im} M_o$ by P_o, then $\ker(I - P_o) = \operatorname{Im} M_o$. Applying P_o to Eq. (5.2) one arrives at

$$(I - P_o)y_o = (I - P_o)C_o x_o + (I - P_o)D_o u \tag{5.6}$$

$$P_o y_o = P_o C_o x_o + P_o D_o u + P_o M_o \nu_o. \tag{5.7}$$

It is also possible to decompose ν as $\nu_o = \nu_1^* + \nu_1$, where $\nu_1^* \in M_o^*$ and $\nu_1 \in \ker M_o$. Denote the pseudoinverse of M by M^+, then by using Eq. (5.7) one obtains:

$$\nu_1^* = M_o^+ P_o (y_o - C_o x_o - D_o u). \tag{5.8}$$

STEP 2. Consider now Eq. (5.6) and assume that $(I - P_o)C_o$ is of full rank. Decompose the state vector as $x_o = x_1^* + x_1$, where $x_1^* = Q_o x \in \operatorname{Im} C_o^*(I - P_o)$ and $x_1 = (I - Q_o)x \in \ker(I - P_o)C_o$ where Q_o denotes the orthogonal projection onto $\operatorname{Im} C_o^*(I - P_o)$. Then,

$$\begin{aligned} x_1^* &= T_o(y_o - D_o u), \\ T_o &= \big(C_o^*(I - P_o)C_o\big)^+ C_o^*(I - P_o). \end{aligned} \tag{5.9}$$

Substituting Eq. (5.9) into Eq. (5.8) one obtains

$$\nu_1^* = M_o^+ P_o (I - C_o T_o)(y_o - D_o u) - M_o^+ P_o C_o x_1. \tag{5.10}$$

STEP 3. Generate new dynamics and outputs as follows.

$$\dot{x}_1 = (I - Q_o)\dot{x} = (I - Q_o)\big(A(x_1^* + x_1) + Bu + L(\nu_1^* + \nu_1)\big).$$

Substitute x_1^* from Eq. (5.9) and ν_1^* from Eq. (5.10) and introduce the notations:

$$\begin{aligned} \tilde{A}_o &= A_o - L_o M_o^+ P_o C_o, \\ \tilde{B}_{y,o} &= A_o T_o + L_o M_o^+ P_o (I - C_o T_o) \\ \tilde{B}_o &= B_o - \tilde{B}_{y,o} D_o \\ A_1 &= (I - Q_o)\tilde{A}_o, \\ B_{u,1} &= (I - Q_o)\tilde{B}_o \\ B_{y,1} &= (I - Q_o)\tilde{B}_{y,o}, \\ L_1 &= (I - Q_o)L_o, \\ C_1 &= Q_o \tilde{A}_o, \\ D_{u,1} &= Q_o \tilde{B}_o, \\ D_{y,1} &= Q_o \tilde{B}_{y,o}, \\ M_1 &= Q_o L_o. \end{aligned}$$

The new system generated in this first step is obtained in the form:

$$\begin{aligned} \dot{x}_1 &= A_1 x_1 + B_{u,1} u + B_{y,1} y_o + L_1 \nu_1, \\ y_1 &= C_1 x_1 + D_{u,1} u + D_{y,1} y_o + M_1 \nu_1, \end{aligned}$$

where $y_1 \triangleq T_o \dot{y}_o$. In general, the last state space form can be obtained in the k-th step as:

$$\begin{aligned} \dot{x}_k &= A_k x_k + B_{u,k} \xi_{u,k-1} + B_{y,k} \xi_{y,k-1} + L_k \nu_k, \\ y_k &= C_k x_k + D_{u,k} \xi_{u,k} + D_{y,k} \xi_{y,k-1} + M_k \nu_k, \end{aligned}$$

where

$$\xi_{u,k} = [u, \dot{u}, \ldots, u^{(k)}]', \quad \text{and} \quad \xi_{y,k-1} = [y, \dot{y}, \ldots, y^{(k-1)}]'.$$

Thus, the failure output ν is constructed recursively in k-steps as:

$$\nu = \nu_1^* + \nu_2^* + \dots \nu_{k+1}^*, \tag{5.11}$$

$$\nu_1^* \in M_o^*, \quad \nu_2^* \in M_1^*, \quad \dots, \quad \nu_{k+1}^* \in M_k^*. \tag{5.12}$$

The recursion continues by defining P_k, the orthogonal projection to $\operatorname{Im} M_k$. The procedure ends up if M_k is invertible.

PROPOSITION 5.6. Assume that the system in (5.1-5.2) is minimal and left invertible *w.r.t.* the failure signal ν. Then the recursion detailed from Step 1 to Step 3 above ends up with a minimal state space left inverse system. The state dimension of the inverse system will be given by

$$\bar{n} = n - (k+1)p + (k+1)\operatorname{rank} M_o + \operatorname{rank} M_1 + \dots + \operatorname{rank} M_k,$$

where

$$\operatorname{rank} M_o + \operatorname{rank} M_1 + \dots + \operatorname{rank} M_k = q = \dim \nu.$$

□

Proof. The proof is based on the constructive procedure presented above. Considering the algorithmic step 1, in Eq. (5.2) the map $P_o M_o$ is isomorphism between $\operatorname{Im} M_o$ and $\operatorname{Im} M_o^*$ ensuring the invertibility *w.r.t.* ν and $P_o y$. This ensures that the inverse is minimal if the original system was minimal, too. Also in step 2, if $(I - P_o)C_o$ was full rank, then Eq. (5.1) provides an isomorphism between $\operatorname{Im} C_o^*(I - P_o)$ and $\ker M_o^*$ implying that x_1^* can be substituted by $(I - P_o)y_o$ without effecting minimality. ■

REMARK 5.7. The inverse system, obtained in the above procedure, is not necessarily stable. If, however, there are more sensor outputs available than failure signals (*i.e.,* $y \in \mathbb{R}^q$, $\nu \in \mathbb{R}^p$, $q > p$), then it is possible to define a parameter matrix $\theta \in \mathbb{R}^{p \times q}$ and a new $y_\theta = \theta y$ output. In many cases this new output can be used to obtain a stable inverse.

REMARK 5.8. The input and output derivatives can be removed from the state equation of the inverse system by applying the method of state translation, for this technique see Example 5.9 for illustration. The output equation for $\nu(t)$ will contain these derivatives after state translation, however.

EXAMPLE 5.9. The following examples are to demonstrate this inversion procedure. Consider the dynamical system which is subject to failures and given in the state space as:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0.5 & 2 & 3 \\ 1 & 1 & 2 \\ -1 & -1 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} u + \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} \nu_1 \\ \nu_2 \end{bmatrix},$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \nu_1 \\ \nu_2 \end{bmatrix},$$

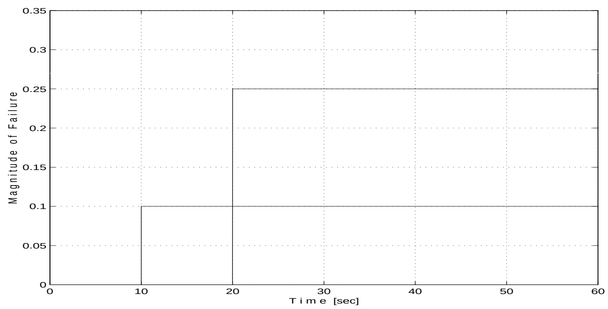

Figure 5.2. Illustration to Example 5.9: Fault signals $\nu_1(t), \nu_2(t)$ affecting the system independently at time 10 and 20.

where $\nu_1(t)$ and $\nu_2(t)$ are the failure modes. The subspace generated by $(1 \quad -1)^T$ is orthogonal to $(1 \quad 1)^T$, therefore

$$y_1 - y_2 = x_2 - x_3. \tag{5.13}$$

Thus

$$x_3 = x_2 - y_1 + y_2. \tag{5.14}$$

On the other hand

$$\nu_1 = y_1 - x_2. \tag{5.15}$$

Deriving (5.13), then substituting (5.14) and (5.15), the new output is obtained

$$\dot{y}_1 - \dot{y}_2 = \dot{x}_2 - \dot{x}_3 = x_1 + x_2 + 2(x_2 - y_1 + y_2) + u + (y_1 - x_2) + \nu_2 - \\ - \big(-x_1 - x_2 - 2(x_2 - y_1 + y_2) + y_1 - x_2 - \nu_2\big) = 2x_1 + 6x_2 + u - 4y_1 + 4y_2 + 2\nu_2$$

i.e.,

$$\dot{y}_1 - \dot{y}_2 + 4y_1 - 4y_2 = 2x_1 + 6x_2 + u + 2\nu_2.$$

This imposes the equation for the second component of the fault signal as

$$\nu_2 = -x_1 - 3x_2 + 0.5(\dot{y}_1 - \dot{y}_2 - u) + 2y_1 - 2y_2.$$

The new dynamics of the states x_1 and x_2 is then given as follows:

$$\begin{aligned} \dot{x}_1 &= 0.5x_1 + 2x_2 + 3(x_2 - y_1 + y_2) + u + (y_1 - x_2) + \\ &\quad (-x_1 - 3x_2 + 0.5(\dot{y}_1 - \dot{y}_2 - u) + 2y_1 - 2y_2) = \\ &\quad -0.5x_1 + x_2 + 0.5u + 0.5\dot{y}_1 - 0.5\dot{y}_2 + y_2, \\ \dot{x}_2 &= x_1 + x_2 + 2(x_2 - y_1 + y_2) + u + (y_1 - x_2) + \\ &\quad (-x_1 - 3x_2 + 0.5(\dot{y}_1 - \dot{y}_2 - u) + 2y_1 - 2y_2) = \\ &\quad -x_2 + 0.5u + 0.5\dot{y}_1 - 0.5\dot{y}_2 + y_1. \end{aligned}$$

Using state translation

$$\begin{aligned} \bar{x}_1 &= x_1 - 0.5y_1 + 0.5y_2, & x_1 &= \bar{x}_1 + 0.5y_1 - 0.5y_2, \\ \bar{x}_2 &= x_2 - 0.5y_1 + 0.5y_2, & x_2 &= \bar{x}_2 + 0.5y_1 - 0.5y_2, \end{aligned}$$

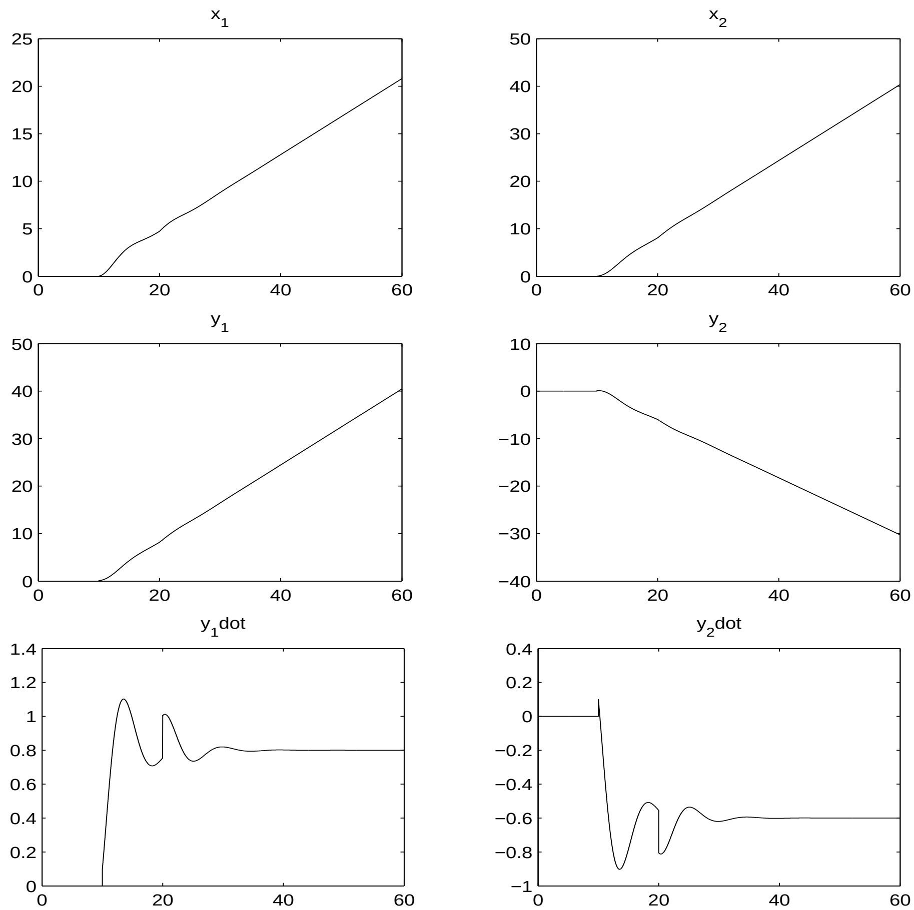

Figure 5.3. Illustration to Example 5.9: Simulation of the state and output variables and the derivative of the output.

one can eliminate the derivatives of y_1 and y_2 as

$$\begin{aligned}
\dot{\overline{x}}_1 &= -0.5(\overline{x}_1 + 0.5y_1 - 0.5y_2) + (\overline{x}_2 + 0.5y_1 - 0.5y_2) + 0.5u + y_2 = \\
&\qquad -0.5\overline{x}_1 + \overline{x}_2 + 0.25y_1 + 0.75y_2 + 0.5u, \\
\dot{\overline{x}}_2 &= -(\overline{x}_2 + 0.5y_1 - 0.5y_2) + 0.5u + y_1, \\
\nu_1 &= y_1 - (\overline{x}_2 + 0.5y_1 - 0.5y_2) = -\overline{x}_2 + 0.5y_1 + 0.5y_2, \\
\nu_2 &= -(\overline{x}_1 + 0.5y_1 - 0.5y_2) - 3(\overline{x}_2 + 0.5y_1 - 0.5y_2) + 0.5(\dot{y}_1 - \dot{y}_2 - u) + 2y_1 - 2y_2.
\end{aligned}$$

We can see that the inverse system is obtained in the following form

$$\begin{aligned}
\dot{\overline{x}}_1 &= -0.5\overline{x}_1 + \overline{x}_2 + 0.25y_1 + 0.75y_2 + 0.5u, \\
\dot{\overline{x}}_2 &= -\overline{x}_2 + 0.5y_1 + 0.5y_2 + 0.5u, \\
\nu_1 &= -\overline{x}_2 + 0.5y_1 + 0.5y_2 \\
\nu_2 &= -\overline{x}_1 - 3\overline{x}_2 + 0.5(\dot{y}_1 - \dot{y}_2 - u).
\end{aligned}$$

This inverse state space is asymptotically stable. Therefore, one can use it to generate the states and the components of the failure signal. The initial conditions for generating the states can be obtained by applying the transformation steps on the original initial conditions. Denoting the

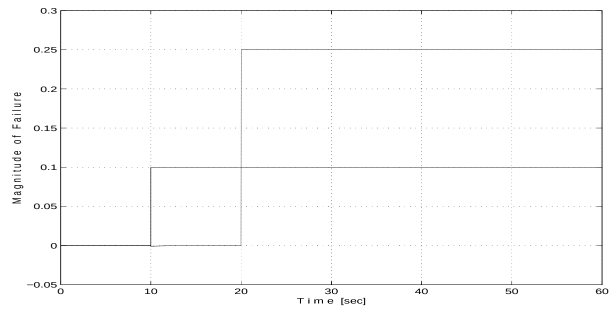

Figure 5.4. Illustration to Example 5.9: The calculated failure signals at the output of the unknown input reconstructor.

initial conditions for the original system in Eq. (5.1-5.2) by

$$x(0) = [\xi_1, \xi_2, \xi_3]',$$

the initial conditions for the inverse system become

$$\begin{aligned}
\bar{x}(0) &= [(\bar{x}_1(0), \bar{x}_2(0)]' = [\xi_1 - 0.5\xi_2 + 0.5\xi_3, 0.5\xi_2 + 0.5\xi_3]' \\
y(0) &= [y_1(0), y_2(0)]' = [\xi_2, \xi_3]', \\
\dot{y}(0) &= [\xi_1 + \xi_2 + 2\xi_3 + u(0), -\xi_1 - \xi_2 - 2\xi_3]'.
\end{aligned}$$

EXAMPLE 5.10. The following example is to show that in cases when the system can not be decoupled using an asymptotically stable filter, there is still an opportunity to handle the problem with inversion. Consider for instance the representation:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix},$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}.$$

Then,

$$\begin{aligned}
\dot{y}_1 &= \dot{x}_3 = x_2 - y_2 + u_2 + v_1 + v_2, \\
\dot{y}_2 &= \dot{x}_4 = 2x_3 + v_1 - v_2 = 2y_1 + v_1 - v_2.
\end{aligned}$$

Now the system of equations of the new outputs

$$\begin{aligned}
\dot{y}_1 + y_2 &= x_2 + u_2 + v_1 + v_2 \qquad (5.16) \\
\dot{y}_2 - 2y_1 &= v_1 - v_2,
\end{aligned}$$

can be solved for ν_1 and ν_2 from the inverse output equations as follows:

$$\nu_1 = \frac{1}{2}\left[\dot{y}_1 + \dot{y}_2 + y_2 - 2y_1 - x_2 - u_2\right], \tag{5.17}$$

$$\nu_2 = \frac{1}{2}\left[\dot{y}_1 - \dot{y}_2 + y_2 + 2y_1 - x_2 - u_2\right]. \tag{5.18}$$

Substituting $x_3 = y_1$ and $x_4 = y_2$ into the original dynamics (the dynamics of x_3 and x_4 are expressed by Eqs. (5.17) and (5.18), respectively) one arrives at the inverse state equations

$$\dot{x}_1 = y_2 - y_1,$$
$$\dot{x}_2 = x_1 + u_1,$$

that is

$$x_1(t) = \int_0^t y_2(t) - \int_0^t y_1(t), \qquad x_2(t) = \int_0^t\int_0^s y_2(t) - \int_0^t\int_0^s y_1(t) + \int_0^t u_1(t).$$

The failure modes $\nu_1(t)$ and $\nu_2(t)$ can be determined from (5.17-5.18), which, therefore, can be viewed as residual.

5.4. SUMMARY

In this chapter an inversion procedure for LTI systems that can be used to construct residual generators for fault detection and isolation has been proposed. The input of this fault detector are composed from the derivatives of the input and output signals of the original system and its outputs are the components of the failure modes.

In some situations the derivatives of certain output signals of the system are directly measured, and these can be utilized in this approach. This procedure can be used in some cases when other approaches like the (C, A)-invariant subspace based detection filter design method fails to provide a stable filter. The cost at which it can be obtained is that one needs to use the integrals of certain output signals in the residual generators as artificial inputs. This was illustrated by Example 2.

One of the advantages of the inversion approach discussed in this chapter is that the extension of the idea to some classes of nonlinear systems (bilinear and input affine) is possible. It will be shown that, by using this concept, linear and nonlinear problems can be treated in the same theoretical framework and the methodology presented can be easily generalized to nonlinear systems. As soon as the results for linear systems were obtained, the corresponding results for nonlinear systems can be regarded as natural generalizations of the linear case. In most of the fault detection and residual generation methods developed for LTI systems, this generalization cannot be made.

CHAPTER 6

Inversion-Based Input Reconstruction for Nonlinear Systems

In this chapter the idea of inversion-based input reconstruction in systems with nonlinear dynamics is considered. We shall see, how the theory of system inversion applied to the solution of the detection filter design problem presented in the previous chapter can be extended to the nonlinear platform. In finding the left inverse of a nonlinear system, the idea is always to solve first the output zeroing problem, *i.e.*, to find initial conditions and inputs consistent with the constraint that the output function $y(t)$ is identically zero for all times in a neighborhood of $t = 0$, and to analyze the corresponding internal dynamics. This will provide an appropriate extension of the notion of zero dynamics to a system having relative degree. The analysis can be made either in algebraic and geometric way. In this chapter we focus on the algebraic approach while the geometric concepts will be discussed in the next chapter.

6.1. Introduction

The main objective addressed in this chapter is the design and analysis of a residual generator for the classes of nonlinear input affine systems described in the general form of (1.6) which is subject to multiple, possible simultaneous faults. Recall that this class of systems was written in the general form

$$\begin{aligned}\dot{x}(t) &= f(x) + \sum_{i=1}^{m} g_i(x,u)\nu_i \\ y(t) &= h(x) + \sum_{i=1}^{m} \ell_i(x,u)\nu_i, \end{aligned} \tag{6.1}$$

where f, g, h, ℓ are functions smooth in their arguments and $x(t) \in \mathcal{X} \subset \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$, $y(t) \in \mathbb{R}^p$ being the vector valued state, input and output variables of the system, respectively, $\nu(t)$ is the fault signal $(\nu_1, \dots, \nu_m)^T$ whose elements $\nu_i : [0, +\infty) \to \mathbb{R}$ are arbitrary functions of time. The fault signals ν_i can represent both actuator and sensor failures, in general. The goal is to detect the occurrence of the components ν_i of the fault signal independently of each other and identify which fault component specifically occurred.

For certain classes of nonlinear state space systems one can find algorithms (and also sufficient or necessary conditions) of invertibility, see *e.g.*, (Isidori, 1985). The main result of this paper is an algorithm that provides a (left) inverse system $\Sigma^{-\ell}$ in some finite $k < m$ steps. The procedure, that can be viewed as a generalization of the procedure described in (Isidori, 1985), is discussed in the next sections.

6.2. INVERTIBILITY AND THE RELATIVE DEGREE OF LINEAR SYSTEMS

The existence of the left inverse determines the feasibility of the inversion-based approach to detector design. Therefore, we will study a series of problems concerned with the analysis of the properties of invertibility of dynamical systems. We will discuss first the linear case. It will be seen that the point of departure of the invertibility analysis is the notion of relative degree of dynamical systems. Consider the LTI system Σ given in (5.5) and the construction of its inverse

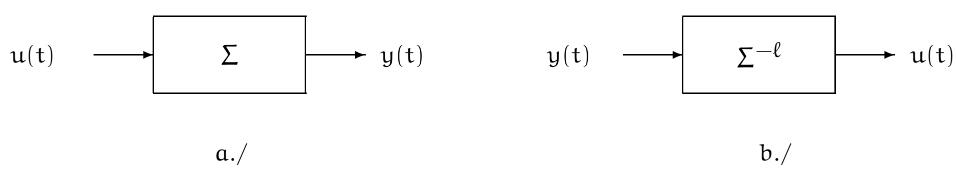

Figure 6.1. The system Σ and its inverse representation $\Sigma^{-\ell}$.

representation. In the first approximation Σ in Fig. 6.1/a is said to be left invertible (*i.e.*, it has a left inverse) if there exists a corresponding system representation in Fig. 6.1/b such that the composition, shown in Fig. 6.2, results in the identity for each input-output pair (u, y), (*cf.* Definition 2.26 and the discussion in Section 2.3.1).

More specifically, if one considers the input-output representation of Σ in the form of

$$
\begin{aligned}
G(s) &= C(sI - A)^{-1}B \\
y(s) &= G(s)u(s),
\end{aligned}
\tag{6.2}
$$

the (left) inverse $G^{-\ell}$ satisfies the identity

$$
G^{-\ell}(s)y(s) = G^{-\ell}(s)G(s)u(s) \tag{6.3}
$$

with $G^{-\ell}G = I$, (see Fig. 6.2). It means that the input $u(t)$ can be uniquely identified by the output function $y(t)$ and the left inverse $G^{-\ell}$.

For treating more general cases, and considering the inversion problem in the time domain the notion of relative degree will be the point of departure of the whole analysis. Let us introduce therefore the notion of relative degree of the state space representation of a left invertible linear

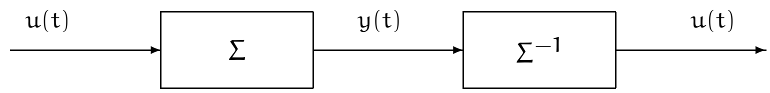

Figure 6.2. The composition of systems Σ and $\Sigma^{-\ell}$ resulting in the identity.

dynamical system

$$\dot{x} = Ax + Bu, \quad y = Cx. \tag{6.4}$$

Suppose we wish to calculate the value of the output function $y(t)$ and of its derivatives with respect to time $y^{(k)}(t)$, consecutively, for $k = 1, 2, \ldots$.

Then, the resulting equations can be written as

$$\begin{aligned}
\dot{x} &= Ax + Bu && (6.5)\\
y &= Cx && (6.6)\\
\dot{y} &= C\dot{x} = CAx + CBu && (6.7)\\
\ddot{y} &= CA\dot{x} + CB\dot{u} = CA(Ax + Bu) + CB\dot{u} = CA^2x + CABu + CB\dot{u} && (6.8)\\
&\vdots\\
y^{(k)} &= CA^kx + CA^{k-1}Bu + CA^{k-2}B\dot{u} + CA^{k-3}B\ddot{u} + \ldots + CBu^{(k-1)}. && (6.9)
\end{aligned}$$

By elaborating the equations component-by-component, we obtain, (see also the system (1.24))

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_p \\ y_1^{(1)} \\ y_2^{(1)} \\ \vdots \\ y_p^{(1)} \\ \vdots \\ y_1^{(k)} \\ y_2^{(k)} \\ \vdots \\ y_p^{(k)} \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_p \\ c_1A \\ c_2A \\ \vdots \\ c_pA \\ \vdots \\ c_1A^kB \\ c_2A^kB \\ \vdots \\ c_pA^kB \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ c_1B \\ c_2B \\ \vdots \\ c_pB \\ \vdots \\ c_1A^{k-1}B \\ c_2A^{k-1}B \\ \vdots \\ c_pA^{k-1}B \end{bmatrix} u + \cdots + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ \vdots \\ c_1B \\ c_2B \\ \vdots \\ c_pB \end{bmatrix} u^{(k-1)}, \tag{6.10}$$

where $k \geq 0$.

DEFINITION 6.1. (Relative degree of linear systems). Consider the procedure completing in the equation (6.9) and the system representation (6.10) where c_i, $i = 1, \ldots, p$ denote the rows of the matrix C. If there exists integers $r_i > 0$, such that

$$c_iA^kB = 0 \quad \text{and} \quad c_iA^{r_i-1}B \neq 0, \quad \text{for all } \; k < r_i - 1, \tag{6.11}$$

moreover,

$$\operatorname{rank}\begin{bmatrix} c_1 A^{r_1-1}B \\ \vdots \\ c_p A^{r_p-1}B \end{bmatrix} = m, \tag{6.12}$$

then r_i is called a relative degree of the system. □

Obviously, in single-input single-output systems the representation (A, b, c) may have only one relative degree. For the generalization of the notion of relative degree for multivariable systems consider the following definition.

DEFINITION 6.2. (Vector relative degree of linear systems). Based on the individual components r_i the vector relative degree r of a multivariable linear system is defined as $r = [\,r_1, \dots, r_p\,]$. □

For the illustration of the role of relative degree in the analysis of systems we present two simple interpretations of it. First, it is known that the integer satisfying the conditions (7.70) is exactly equal to the difference between the degree of the denominator and the numerator polynomials of the transfer function $G(s)$ of the system (6.2), (*cf.* Section 7.2).

For the second, suppose we wish to calculate the value of the output function $y(t)$ and of its derivatives with respect to time $y^{(k)}(t)$, for $k = 1, 2, \dots$, *i.e.*, follow the same procedure as resulted in (6.10). It is easy to see that if the relative degree r is larger than 1, then $CBu = 0$ in (6.7) and therefore $\dot{y} = CAx$. This yields $\ddot{y} = CA\dot{x} = CA(Ax + Bu) = CA^2x + CBu$ in (6.8). Similarly, if the relative degree is larger than 2, we have $CABu = 0$ in (6.8) and we get $\dddot{y} = CA^2x$. Continuing in this way, we can arrive at Eq. (6.9) with the assumption $CA^kBu = 0$ for all $k \geq 0$, *i.e.*, if the model has relative degree equal to or possibly larger than r, we have that

$$CB = CAB = \dots = CA^{r-2} = 0$$

which means that the first $r - 1$ derivatives of $y(t)$ do not depend explicitly on $u(t)$, and the r-th one depends explicitly on $u(t)$ but not on its derivatives. That is to say, the output of the system is not affected by the input: the output function of the system depends only on the initial state x_o.

Thus, in this interpretation, the relative degree r is exactly the number of times the output function $y(t)$ is to be differentiated in order to have the value $u(t)$ of the input explicitly appearing in the equations. The above interpretation of relative degree suggests that the matrix products $Cx, CAx, \dots, CA^{r-1}x$ have special importance in the analysis. It will be seen in the next sections that they can be used to define a coordinate transformation to obtain a representation of the inverse system in a very convenient way.

By definition of relative degree from the state space representation of the linear system (5.5) one can construct the equations

$$\begin{bmatrix} y_1^{(r_1)} \\ \vdots \\ y_p^{(r_p)} \end{bmatrix} = \begin{bmatrix} c_1 A^{r_1} \\ \vdots \\ c_p A^{r_p} \end{bmatrix} x + \begin{bmatrix} c_1 A^{r_1-1}B \\ \vdots \\ c_p A^{r_p-1}B \end{bmatrix} u, \tag{6.13}$$

from which, the input variable $u(t)$ can be obtained by inversion. The inverse system of (5.5) can be represented in the possible non-minimal state space form

$$\dot{\eta} = A_{inv}\eta + B_{inv}\nu_{inv} \tag{6.14}$$

$$u = C_{inv}\eta + D_{inv}\nu_{inv}, \tag{6.15}$$

where Eq. (6.14) describes the inverse dynamics and the vector ν_{inv} contains the measurements and its derivatives in the respective orders as

$$\nu_{inv} = \begin{bmatrix} y_1 & \dots & y_1^{(r_1)} & \dots & y_p & \dots & y_p^{(r_p)} \end{bmatrix}^T. \tag{6.16}$$

If the realization of the inverse system is minimal, then A_{inv} gives the so-called zero dynamics of (A, B, C). Throughout this paper it will be assumed that the zero dynamics of the system is asymptotically stable, *i.e.*, the residual system is minimum phase. If this condition does not hold, the system inversion-based method presented here does not give a feasible solution to the detection problem.

6.3. RELATIVE DEGREE OF NONLINEAR SYSTEMS

For the formal description of the relative degree of nonlinear systems recall the representation of the multivariable input affine system of the form

$$\begin{aligned} \dot{x} &= f(x) + \sum_{i=1}^{m} g_i(x)u_i, \quad u \in \mathbb{R}^m, \quad y \in \mathbb{R}^p \\ y_j &= h_j(x), \quad j = 1, \dots, p \end{aligned} \tag{6.17}$$

and consider the following definition.

DEFINITION 6.3. (Relative degree of nonlinear systems). The relative degree of the nonlinear system (6.17) is the integer r_j derivatives of $y_j = h_j(x)$, such that

$$\begin{aligned} &L_{g_i} L_f{}^k h_j(x) = 0 \quad \text{for} \quad 0 \le k < r_j - 1 \\ &\exists_j \quad L_{g_i} L^{r_j - 1} h_j(x) \neq 0. \end{aligned} \tag{6.18}$$

□

If the relative degree r_j does not exist *i.e.*,

$$\forall\, i, k \quad L_{g_i} L_f{}^k h_j(x) = 0,$$

then r_j equals to $+\infty$ by definition. It can be seen, that the j^{th} output derivatives have the forms

$$y^{(j)} = L_f^k h_j(x), \quad k = 0, 1, \dots, r_j - 1,$$

$$y^{(r_j)} = L_f^{r_j} h_j(x) + \sum_{i=1}^{m} L_{g_i} L_f^{(r_j - 1)} h_j(x) u_i.$$

DEFINITION 6.4. (Vector relative degree of multivariable nonlinear systems). Let the vector relative degree of (6.17) be defined as

$$r = (r_1, \ldots, r_p). \tag{6.19}$$

The multivariable nonlinear system (6.17) is said to have a vector relative degree r at a point x_o if

$$L_{g_i} L_f^k h_j(x) = 0, \quad j = 1, \ldots, p, \;\; i = 1, \ldots, m, \quad \text{for all} \quad k < r_j - 1, \tag{6.20}$$

for all x in a neighborhood of x_o assuming the matrix

$$A(x) \triangleq \begin{bmatrix} L_{g_1} L_f^{r_1-1} h_1(x) & \cdots & L_{g_m} L_f^{r_1-1} h_1(x) \\ \vdots & & \vdots \\ L_{g_1} L_f^{r_p-1} h_p(x) & \cdots & L_{g_m} L_f^{r_p-1} h_p(x) \end{bmatrix}$$

is nonsingular at $x = x_o$, or equivalently

$$\operatorname{rank} A(x_o) = m. \tag{6.21}$$

□

DEFINITION 6.5. (Relative order of nonlinear systems). If the rank condition (6.21) does not hold but there exist numbers r_j satisfying property (6.20) then r_j are called *pseudo relative degree* (or in other sources *relative orders*) of the system (6.17). □

REMARK 6.6. It is easily seen that for linear systems represented in the form

$$\dot{x} = Ax + Bu, \quad y = Cx$$

the conditions (6.20) and (6.21) include condition (7.70), inherently, since, in this case we write $f(x) = Ax$, $g(x) = B$, $h(x) = Cx$, which implies that $L_f^k h(x) = CA^k x$ and therefore $L_g L_f^k h(x) = CA^k B$. Thus the relative degree r is characterized by the conditions (6.9) with $CA^k B = 0$ for all $k < r - 1$ and $CA^{r-1} B \neq 0$.

If the matrix

$$A(x) = \begin{bmatrix} L_f^{(r_1-1)} h_i(x) \\ \vdots \\ L_f^{(r_m-1)} h_i(x) \end{bmatrix} [g_1(x), \ldots, g_m(x)] \tag{6.22}$$

generated on the analogy of (6.12) is nonsingular, then the inverse of the system can be computed from

$$\begin{bmatrix} {y_1}^{(r_1)} \\ \vdots \\ {y_p}^{(r_m)} \end{bmatrix} = \begin{bmatrix} L_f^{r_1^1} h_1(x) \\ \vdots \\ L_f^{r_m^1} h_p(x) \end{bmatrix} + A(x) \begin{bmatrix} u_1 \\ \vdots \\ u_m \end{bmatrix} \tag{6.23}$$

see, (Isidori, 1985). This can be referred to as a *1-step* algorithm to obtain an inverse. The non-singularity of $A(x)$, however is a strong requirement that restricts the possible use of this algorithm. In the next section of this chapter an extension of this algorithm will be presented and demonstrated. The idea is to construct new output functions and use their derivatives leading to a procedure that generates the inverse in some finite steps. This idea appeared first in (Szigeti et al., 2001).

6.4. ALGEBRAIC CONSTRUCTION OF THE INVERSE FOR NONLINEAR SYSTEMS

Suppose now that the matrix $A_1(x) = A(x)$ constructed in this first step is well defined, *i.e.*, each pseudo relative degree is finite, but $A_1(x)$ is singular.

Denote the vector relative degree associated to A_1 by $\rho^1 = (\rho_1^1, \rho_2^1, \ldots, \rho_m^1) = r$. Suppose that $\max_x \operatorname{rank} A_1(x) = d_1$ and assume the first d_1 rows are linearly independent. Then, there exist a matrix $F_1(x) \in \mathbb{R}^{(m-d_1)xm}$, rank $F_1(x) = (m - d_1)$, with entries $F_{ij}(x)$, $i = 1, 2, \ldots, m - d_1$, $j = 1, 2, \ldots, m$, that are polynomial functions in $L_{g_j} L_f^{r_i - 1} h_i(x)$ such that

$$F_1(x) A_1(x) = 0. \tag{6.24}$$

Using the following vectorial notations

$$y^{(r)} = (y_1^{(r_1)}, y_2^{(r_2)}, \ldots, y_m^{(r_m)})^T,$$

$$L_f^r h(x) = (L_f^{r_1} h_1(x), \ldots, L_f^{r_m} h_m(x))^T,$$

one can write

$$F_1(x)(y^{(r)} - L_f^{(r)} h(x)) = 0.$$

These equations will be considered later as additional new output relations. Then the new output relations will be defined as

$$\begin{bmatrix} y - h(x) \\ F_1(x)(y^{(r)} - L_f^{(r)} h(x)) \end{bmatrix} = 0. \tag{6.25}$$

Next calculate the derivatives of all components of these new output relations up to the inputs appear. In this way one can define a second set of relative degrees, *i.e.*, a new *pseudo* vector relative degree denoted by

$$\rho^2 = (\rho_1^2, \ldots, \rho_{d_1}^2, \rho_{d_1+1}^2, \ldots, \rho_m^2).$$

It is clear that the first d_1 elements of ρ^2 are identical to those of ρ^1, since the first d_1 rows of (6.25) are identical to the original ones in (6.22).

Define now the matrix $A_2(x)$ such that its first d_1 rows are the same as those rows of $A_1(x)$, but the remaining $m - d_1$ rows are selected from the derivatives of the new output relations. These will have the form:

$$A_2(x, y)_{d_1+k,j} = \sum_{i=1}^{m} (L_{g_j} L_f^{r_k^2 - 1} F_{ki}(x)(y_i^{(r_i^1)} h_i(x))) - F_{ki} L_{g_j}(x) L_f^{r_i^1 + r_k^2 - 1} h_i(x)$$

where

$$d_2 = \operatorname{rank} A_2(x) \geq \operatorname{rank} A_1(x) = d_1.$$

If $d_1 = d_2 < m$ holds then the system is not invertible. If $d_2 = m$ then the input functions can be obtained in this step from the equation analogous to (7.73) as

$$\sum_{l=0}^{r^2} \binom{r^2}{l} L_f^l F(x) \otimes (y^{(r^1+r^2-l)} - L_f^{r^1+r^2-l} h(x)) + A_2(x, y^r)u = 0 \qquad (6.26)$$

where

$$\binom{r^2}{l} = \left[\binom{r_1^2}{l_1}, \ldots, \binom{r_m^2}{l_m}\right], \quad l = (l_1, \ldots, l_m),$$

and $\otimes$ is the Kronecker product, and the procedure stops. The vector relative degree can be written as

$$r^2 = (r_1^2, \ldots, r_{d_1}^2, r_{d_1+1}^2, \ldots, r_m^2),$$

where, for $i = 1, \ldots, m$

$$r_i^2 = \rho_i^2, i = 1, \ldots, d_1; \quad r_{d_1+i}^2 = \rho_{d_1+i}^1 + \rho_{d_1+i}^2.$$

REMARK 6.7. Assuming the special technical hypothesis that for a given k and r_k

$$F_{ki} L_{g_j}(x) L_f^{r_i^1 + r_k^2 - 1} h_i(x) \neq 0,$$

$$L_{g_j} L_f^{r_k^2 - 1} F_{ki}(x) = 0, \quad \forall i, j,$$

then the definition of A_2 will be replaced by

$$A_2(x)_{d_1+k,j} = -F_{ki} L_{g_j}(x) L_f^{r_i^1 + r_k^2 - 1} h_i(x).$$

If $A_2(x, y^{(r_1)})$ (or $A_2(x)$, respectively) is not invertible but $\operatorname{rank} A_2 = d_2 < m$, then it is possible to select its linearly independent rows. Assume that the first d_2 rows are linearly independent (if not, one can permute the rows) and it is possible to define an $(m - d_2) \times m$-dimensional matrix $F_2(x, y^{(r_1)})$ (or $F_2(x)$, respectively) analogously to F_1 in (6.24). The algorithm continues by defining new output equations similarly to (6.25). Suppose that the above algorithm terminates in k steps, *i.e.,* when $d_k = m$. Then the relative degree will be defined as follows.

DEFINITION 6.8. The (vector) relative degree of the extended system computed by the above algorithm is the ordered set of integers:

$$r = (r_1^1, \ldots, r_{d_1}^1; r_{d_1+1}^2, \ldots, r_{d_2}^2; \ldots; r_{d_{k-1}+1}^k, \ldots, r_m^k).$$

where for $k \geq 2$,

$$r_i^1 = r_1, i = 1, \ldots, d_1, \qquad r_i^j = \sum_{l=1}^{j} \rho_i^j,$$

$$d_j \leq i \leq d_{j+1}, \quad 2 \leq j \leq k.$$

□

It is to be noticed that the relative degree defined in this way is not unique since the extended system depends on the order of selection of the independent original and new output relations. It satisfies, however,

$$r_1^1 + \dots r_m^k \leq n. \tag{6.27}$$

REMARK 6.9. The vector relative degree specified in Definition 6.8 plays the same role as the one defined in Eq. (6.18) in constructing canonical (or normal) forms for the inverse dynamics. The basic difference in the structure of normal forms described *e.g.*, in Chapter 5 of (Isidori, 1985), when using the coordinates

$$\Phi(x) = (dh_1, \dots, L_f^{r_1-1}h_1; \dots; h_m, \dots, L_f^{r_m-1}h_m; \dots, \phi_n)$$

is that in our case the output components and their derivatives appear in the state transform. This implies that the normal equations are not explicit, they can, however, be transformed into the matrix pencil form

$$Q(\Phi, y, \dot{y}, \dots)\dot{\Phi} = CF(\Phi),$$

where $CF(\Phi)$ is a symbol for the usual nonlinear canonic forms consisting of the m blocks $(\Phi_2^i, \dots, \Phi_{rj-1+i^j}, \dots)$, see (Isidori, 1985).

In case the above algorithm generates matrices $A_1(x), A_2(x), \dots, A_k(x)$, depending only on x, then the matrix pencil Q will also depend only on x, *i.e.*, $Q = Q(x)$. If Eq. (6.27) is satisfied with equality, then the system has no zero dynamics as expected.

6.4.1. *A recursive algorithm for calculation of the inverse*

In order to formalize the theoretical result presented in the previous section the following proposition is provided. For convenience, introduce the notation and consider the system representation (6.17) in the form

$$\begin{aligned} \dot{x} &= f(x) + g(\lambda)u \qquad u \in \mathbb{R}^m, \quad \lambda \in \mathbb{R}^p \\ \lambda_i &= H_i(x). \end{aligned} \tag{6.28}$$

PROPOSITION 6.10. (A recursive algorithm for calculating the inverse in nonlinear systems). Consider the generation of a recursive procedure according to the following algorithmic steps:

STEP 1. Initiate the recursion by initializing the values $\lambda_o = y$, $p_o = p$, $H_o = h(x)$ and $i = 1$.
STEP 2. Compute $A_i(x)$ according to (6.22)

$$\begin{bmatrix} \lambda_1^{(r_1^i)} \\ \vdots \\ \lambda_p^{(r_{p_i}^i)} \end{bmatrix} = \begin{bmatrix} L_f^{r_1^i} H_i^1(x) \\ \vdots \\ L_f^{r_{p_i}^i} H_i^{p_i}(x) \end{bmatrix} + A_i(x) \begin{bmatrix} u_1 \\ \vdots \\ u_m \end{bmatrix}. \tag{6.29}$$

STEP 3. Check rank condition. If $\operatorname{rank} A_i(x) < m$, then continue and go to Step 4, else the procedure is terminated and go to Step 5.

STEP 4. By introducing the notation from (6.29)

$$\tilde{\lambda}_i = \begin{bmatrix} \lambda_1^{(r_1^i)} \\ \vdots \\ \lambda_p^{(r_{p_i}^i)} \end{bmatrix}, \tag{6.30}$$

according to (6.24) calculate

$$\lambda_{i+1} = H_{i+1}(x) \triangleq \begin{bmatrix} \lambda_i \\ F_1(x)\tilde{\lambda}_i \end{bmatrix}. \tag{6.31}$$

By incrementing the index of recursion ($i = i+1, p_{i+1} = \dim \lambda_{i+1}$), go to Step 2 and continue.
STEP 5. Procedure ended. □

6.4.2. *Examples*

EXAMPLE 6.11. For illustration of the idea, consider the following system representation:

$$\dot{x}_1 = x_1 + (x_2 - 1)v_1, \tag{6.32}$$
$$\dot{x}_2 = x_3 + (x_1 + 1)v_1, \tag{6.33}$$
$$\dot{x}_3 = x_2 + (1 + x_1 x_3)v_2, \tag{6.34}$$
$$y_1 = x_1, \;\; y_2 = x_2, \tag{6.35}$$
$$f(x) = (x_1,\, x_3,\, x_2)^T,$$
$$g_1(x) = (x_2 - 1,\, x_1 + 1,\, 0)^T,$$
$$g_2(x) = (0,\, 0,\, 1 + x_1 x_3)^T.$$

where, in this case, let the variable v denote the input. Differentiating the output in the first step ($k = 1$), we get

$$\dot{y}_1 = x_1 + (x_2 - 1)v_1,$$
$$\dot{y}_2 = x_3 + (x_1 + 1)v_1.$$

It can be seen that the pseudo relative degree is $\rho^1 = (1, 1)$, and the matrix

$$A_1(x) = \begin{bmatrix} x_2 - 1 & 0 \\ x_1 + 1 & 0 \end{bmatrix}$$

is singular. The matrix $F_1(x)$ in (6.24) can be chosen as

$$F_1(x) = \begin{bmatrix} x_1 + 1 & -(x_2 - 1) \end{bmatrix}.$$

In the second step ($k = 2$), from (6.32-6.34), eliminate v_1 and define the new output in the form

$$y_3 = (x_1 + 1)\dot{y}_1 - (x_2 - 1)\dot{y}_2 = (x_1 + 1)\dot{x}_1 - (x_2 - 1)\dot{x}_2 = (x_1 + 1)x_1 - (x_2 - 1)x_3. \tag{6.36}$$

Applying (6.35) to (6.36) one obtains

$$(y_1+1)\dot{y}_1-(y_2-1)\dot{y}_2=(y_1+1)y_1-(y_2-1)x_3. \tag{6.37}$$

Calculating the derivatives we get

$$\begin{aligned}
\dot{y}_1 &= x_1+(x_2-1)v_1,\\
\dot{y}_3 &= (2x_1+1)(x_1+(x_2-1)v_1)-\\
&\quad (x_3+(x_1+1)v_1)x_3+\\
&\quad (1-x_2)(x_2+(1+x_1x_3)v_2)=\\
&\quad (2x_1+1)x_1+(1-x_2)x_2-{x_3}^2+\\
&\quad ((2x_1+1)(x_2-1)-(x_1+1)x_3)v_1+\\
&\quad (1-x_2)(1+x_1x_3)v_2.
\end{aligned}$$

It follows that the pseudo relative degree is $\rho^2=(1,1)$, and the matrix

$$A_2(x)=\begin{bmatrix} x_2-1 & (2x_1+1)(x_2-1)-(x_1+1)x_3\\ 0 & (1-x_2)(1+x_1x_3)\end{bmatrix}^T$$

is nonsingular. The relative degree is $r^2=(1,2)$. Since the sum of the relative degrees is equal to the state dimension, the inverse has no zero dynamics and the unknown inputs can be obtained by measurements

$$v_1=\frac{\dot{y}_1-y_1}{y_2-1},\quad v_2=\frac{\dot{y}_3-2y_1\dot{y}_1-\dot{y}_1+\dot{y}_2x_3+(y_2-1)y_2}{(y_2-1)(1+y_1x_3)},$$

where, from (6.37)

$$x_3=\dot{y}_2-\frac{(y_1+1)(\dot{y}_1-y_1)}{(y_2-1)}.$$

EXAMPLE 6.12. The following example is to show the effect of the choice of the new outputs on the inversion process. To this end consider the system represented by the equations

$$\begin{aligned}
\dot{x}_1 &= (1+x_1)x_3,\\
\dot{x}_2 &= x_2+(x_3-1)v_1,\\
\dot{x}_3 &= x_4+(x_2+1)v_1,\\
\dot{x}_4 &= x_3+(1+x_2x_3)v_2,
\end{aligned} \tag{6.38}$$

assuming the state variables x_1 and x_2 are directly measurable *i.e.*,

$$y_1=x_1,\quad y_2=x_2. \tag{6.39}$$

Now let us consider the new outputs which are selected in two different ways.

6.4.3. *Output selection scheme No.1 to Example 6.12*

The outputs considered in the natural order $y = (y_1, y_2)^T$ have pseudo relative degree $\rho_1 = (2, 1)$. Indeed,

$$\begin{aligned} \dot{y}_1 &= (1 + x_1)x_3, \\ \ddot{y}_1 &= (1 + x_1)(x_3^2 + x_4) + (1 + x_1)(1 + x_2)v_1, \\ \dot{y}_2 &= x_2 + (x_3 - 1)v_1. \end{aligned}$$

Hence the matrix

$$A_1(x) = \begin{bmatrix} (1 + x_1)(1 + x_2) & 0 \\ (x_3 - 1) & 0 \end{bmatrix}$$

is not invertible. Since

$$F_1(x)\,A_1(x) = 0 \tag{6.40}$$

with the matrix

$$F_1(x) = [(x_3 - 1) \quad -(1 + x_1)(1 + x_2)]$$

the redefined outputs become $[y_1, y_3]^T$ with

$$y_3 = \ddot{y}_1(x_3 - 1) - \dot{y}_2(1 + x_1)(1 + x_2).$$

The pseudo relative degree is $\rho_2 = (2, 1)$.

$$\begin{aligned} \dot{y}_3 \quad = \quad & (x_1 + 1)(x_3^2 + x_4) + (2x_4 + 1)(x_3^2 - x_3) \\ & (x_1 + 1) + (x_1 + 1)(x_3^2 + x_4)x_4 - (x_1 + 1) \\ & (x_2^2 + x_2)x_3 - (x_1 + 1)(2x_2^2 + x_2) + (2(x_1 + 1) \\ & (x_2 + 1)(x_3^2 - x_3) + (x_1 + 1)(x_3^2 + x_4) \\ & (x_2 + 1) - (x_1 + 1)(2x_2 + 1)(x_3 - 1))\,v_1 + \\ & +(x_1 + 1)(x_3 - 1)(1 + x_2x_3)v_2. \end{aligned}$$

Hence the matrix

$$A_2(x) = \begin{bmatrix} (1 + x_1)(1 + x_2) & 0 \\ {A_2}^{21} & {A_2}^{22} \end{bmatrix} \tag{6.41}$$

with entries

$$\begin{aligned} {A_2}^{21} &= 2(x_1 + 1)(x_2 + 1)(x_3^2 - x_3) + (x_1 + 1)(x_3^2 + x_4)(x_2 + 1) - (x_1 + 1)(2x_2 + 1)(x_3 - 1) \\ {A_2}^{22} &= (x_1 + 1)(x_3 - 1)(1 + x_2x_3), \end{aligned}$$

is invertible. Then the relative degree r of system (6.38-6.39) can be computed by using ρ_1 and ρ_2:

$$r^2 = (\rho_1^1,\ \rho_2^1 + \rho_2^2) = (2,\ 1 + 1) = (2, 2).$$

The inverse can be calculated by the inversion of (6.41) from the equations given for $\ddot{y}_1$ and $\dot{y}_3$.

6.4.4. *Output selection scheme No.2 to Example 6.12*

The outputs $y = (y_2, y_1)^T$ given by permutation have the pseudo relative degree $\rho_1 = (1, 2)$. Considering the same mixed output (6.40) the redefined outputs become y_2 and $y_3 = \ddot{y}_1(x_3 - 1) - \dot{y}_2(1 + x_1)(1 + x_2)$. The pseudo relative degree of the output $(y_2, y_3)^T$ is $\rho_2 = (1, 1)$. The $A_2(x)$ modified

$$A_2(x) = \begin{bmatrix} (x_3 - 1) & 0 \\ {A_2}^{21} & {A_2}^{22} \end{bmatrix} \tag{6.42}$$

where the entries ${A_2}^{21}$ and ${A_2}^{22}$ are the same as in (6.41). It means that (6.42) is also invertible. The corresponding relative degree r of system (6.38-6.39) can be computed by using ρ_1 and ρ_2 as

$$r^2 = (\rho_1^1,\ \rho_2^1 + \rho_2^2) = (1,\ 2 + 1) = (1, 3).$$

The inverse can be calculated by the inversion of $A_2(x)$ in (6.41) by using the equations given for $\dot{y}_2$ and $\dot{y}_3$.

6.5. SUMMARY

In this chapter the fault detection and isolation problem for nonlinear systems in view of the fault reconstruction process by means of dynamic system inversion has been discussed when sensor noise and sensor faults were neglected. It was shown that a detector relying on the inverse representation of the original system fully reconstructs the failure modes at its output on the basis of standard input and output (sometimes state variable) measurements.

The main contribution of the work presented in this chapter is an algorithm which can be used for the calculation of the inverse. The procedure can be viewed as a generalization of the 1-*step* algorithm proposed by (Isidori, 1985) for systems represented in canonical normal form. The method proposed by this chapter resolves the strong requirement included in this 1-*step* algorithm by providing the inverse in some $k > 1$ finite steps thus making the applicability of the method less restrictive in the practice.

CHAPTER 7

A Geometric View of Inversion-Based Detection Filter Design

In this chapter a view of the inversion-based input reconstruction with special emphasis to the aspects of fault detection and isolation by using invariant subspaces and the results of classical geometrical system theory is provided. The applicability of the idea to fault reconstruction is demonstrated by examples.

In the past years geometric approaches have proved to be particularly useful and successful means for the design and analysis of FDI methods. They provided fundamental tools for the design of residual generators aimed at providing structured and directional residuals, *i.e.,* detection filters. Most of the results obtained for the classical detection filter theory were made available on the geometric platform, see *e.g.,* the results of (Massoumnia, 1986; White and Speyer, 1987; Massoumnia et al., 1989) for LTI, (Edelmayer et al., 1997d) for linear time varying (LTV), and (Hammouri et al., 1999) for bilinear systems based on the geometric theory originated by (Basile and Marro, 1969a) and (Wonham, 1979).

Efforts to extend geometric concepts to nonlinear problems have been made *e.g.,* by the work (De Persis and Isidori, 2001). The generalization of the geometric ideas to nonlinear systems, such as invariant subspaces used for LTI systems in a standard way, may prove to be cumbersome from several points of view in the practice. Our approach attempts to avoid difficulties deriving from nonlinear invariant subspace theory and invariant distributions. It will be shown that the inverse problem for nonlinear systems can be dealt with relative ease on the basis of standard geometric concepts introduced by (Wonham, 1979) and partly by (Isidori, 1985).

The power of this kind of geometric approach is due to its direct treatment of the fundamental structural questions at the root of many important synthesis problems in control and systems theory such as *e.g.,* the properties of inverse generation. As the reader will probably notice in the following sections, the main results will always be expressed in terms of the maximal (A, B)-controlled invariant subspaces, contained in the kernel of some other transformation.

7.1. Introduction

The main objective addressed in this chapter is the design and analysis of a residual generator for nonlinear input affine systems subject to multiple, possible simultaneous faults given in the most general form in the state space as

$$\begin{aligned} \dot{x}(t) &= f(x,u) + \sum_{i=1}^{m} g_i(x,u)\nu_i \\ y(t) &= h(x,u) + \sum_{i=1}^{m} \ell_i(x,u)\nu_i, \end{aligned} \tag{7.1}$$

where f, g, h, ℓ are analytic functions and $x(t) \in \mathcal{X} \subset \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$, $y(t) \in \mathbb{R}^p$ being the vector valued state, input and output variables of the system, respectively, $\nu(t)$ is the fault signal $(\nu_1, \dots, \nu_m)^T$ whose elements $\nu_i : [0, +\infty) \to \mathbb{R}$ are arbitrary functions of time. The fault signals ν_i can represent both actuator and sensor failures, in general. The goal is to detect the occurrence of the components ν_i of the fault signal independently of each other and identify which fault component specifically occurred.

Along the discussion of this chapter linear and nonlinear problems will be treated in parallel to each other. Results for linear time invariant (LTI) systems will always be viewed as special cases of the results obtained for the nonlinear problems specified by the general system model (7.1).

In our approach a detector, *i.e.,* another dynamic system, is constructed with outputs ν and with inputs u, y and possibly their time derivatives which, in the most general form, can be thought of

$$\begin{aligned} \dot{\zeta}(t) &= \varphi(\zeta, y, \dot{y}, \dots, u, \dot{u}, \dots), \\ \nu(t) &= \omega(\zeta, y, \dot{y}, \dots, u, \dot{u}, \dots) \end{aligned} \tag{7.2}$$

with the state variable $\zeta(t)$ assuming φ, ω are arbitrary analytic time functions. The filter reproduces the fault signal at its output that is zero in normal system operation, while it differs from zero if a particular fault occurs.

This detector should satisfy a number of requirements. It should distinguish among different failure modes ν_i, *e.g.,* between two independent faults in two particular actuators. Moreover, it is aimed to completely decouple the faults from the effect of disturbances and also from the input signals. Note that for LTI systems the filter (7.2) traditionally serves as a residual generator which assigns the fault effects and the disturbances into disjoint subspaces in the detector output space.

Therefore, it makes sense to relate the inversion problem to the classical results of geometric detection filter theory. Section 7.3 gives the geometric interpretation of the inverse problem in LTI systems. Then, we continue with input observability properties in the nonlinear framework. The generalization of the concepts obtained in the previous sections to nonlinear problems is discussed and the geometric interpretation of inversion-based fault reconstruction in nonlinear systems is given in Section 7.4. This geometric approach proved to be useful not only from the point of view of a better understanding of the idea, but it creates a theoretical basis for constructing efficient inversion algorithms. The technique is applied to simple demonstrative examples both for LTI and nonlinear input affine systems.

7.2. ZEROS AND ZERO DYNAMICS

Zeros and zero dynamics of dynamical systems are of fundamental important notions in the analysis and inverse representation of the systems. In order to be able to proceed to the next section we need to introduce these notions. Zero dynamics, which describes the internal behavior of the system when the output is forced to be zero will be important concept in the analysis and interpretation of the results in the rest of this work. The concept of zero dynamics was introduced by (Byrnes and Isidori, 1984), then applied in a series of papers, see *e.g.* (Byrnes and Isidori, 1988; Byrnes et al., 1991).

For the brief characterization of this principle consider the following problem and the corresponding definition. Consider a state space system of the form

$$\begin{aligned} \dot{x} &= f(x, u), \\ y &= h(x). \end{aligned} \tag{7.3}$$

It is assumed that the origin $x = 0$, $u = 0$ is an equilibrium point for this system ($\dot{x} = 0$) and that $h(0) = 0$. Let, furthermore, the point x_o in the state space of (7.3) such that $f(x_o, 0) = 0$ and $h(x_o) = 0$. Thus, if the initial state of (7.3) at time $t = 0$ is equal to x_o, moreover, the input $u(t)$ is zero for all $t \geq 0$, then also the output $y(t)$ is zero for all $t \geq 0$.

DEFINITION 7.1. The system dynamics described by (7.3) restricted to the set of initial conditions described above is called the zero-output constrained dynamics or shortly, the zero dynamics. To be more specific, the zero dynamics identifies the set of all pairs consisting of an initial state x_o and an output function $h(x)$ which produce an identically zero output. □

It can be easily seen that Definition 7.1 is applicable to nonlinear systems written in the form

$$\begin{aligned} \dot{x} &= f(x) + \sum_{i=1}^{m} g_i(x) u_i, \quad i = 1, \dots m \\ y_j &= h_j(x), \quad j = 1, \dots p \end{aligned} \tag{7.4}$$

and, with the assumptions $f(x) = Ax$, $g(x) = B$, $h(x) = Cx$, for linear systems too. As it will be found, the name *zero dynamics* is due to its relation to output zeroing and its relation to transmission zeros. This relationship can be characterized for linear systems in the following sections.

7.2.1. *Zero dynamics of SISO linear systems*

Recall that the transmission zeros of the linear SISO system defined by the strictly proper scalar transfer function

$$g(s) = \alpha \frac{p(s)}{d(s)} = k \frac{s^p + p_1 s^{p-1} + \cdots + p_{p-1} s + p_p}{s^n + d_1 s^{n-1} + \cdots + d_{n-1} s + d_n} \tag{7.5}$$

are the roots of the numerator polynomial $p(s)$ of the system (7.5). It is easy to check that in this case the relative degree is $n-p$.

Obviously, in order to achieve $y=0$, one just need to find initial conditions and a feedback control such that

$$y^{(i)}=0, \quad i=0,1,\dots$$

Assume that system (7.5) has a minimal state space realization (A,b,c). When we compute $y^{(i)}$ as an implication of the relative degree as discussed in the previous sections, we see that

$$cA^{i}b=0, \quad i=0,1,\dots,n-p-2, \quad cA^{n-p-1}b\neq 0. \tag{7.6}$$

In other words, we have

$$\begin{aligned} y^{(i-1)} &= cA^{i-1}x, \quad i=1,\dots,n-p \\ y^{(n-p)} &= cA^{n-p}x+cA^{n-p-1}bu. \end{aligned}$$

The implication of (7.6) leads to the fact that the rows

$$cA^{i-1}, \quad i=1,\dots,n-p \tag{7.7}$$

are linearly independent. We now do a coordinate change by letting

$$\begin{aligned} \xi_i &\triangleq cA^{i-1}x, \quad i=1,\dots,n-p \\ z_i &\triangleq x_i, \quad i=1,\dots,p, \end{aligned}$$

where one can easily verify that the z_i's are linearly independent from the ξ_i's. Then the new system can be written as

$$\begin{aligned} \dot{z} &= Nz+P\xi_1 \\ \dot{\xi}_1 &= \xi_2 \\ &\vdots \\ \dot{\xi}_{n-p-1} &= \xi_{n-p} \\ \dot{\xi}_{n-p} &= Rz+S\xi+\alpha u \\ y &= \xi_1 \end{aligned} \tag{7.8}$$

where

$$N=\begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \vdots & 1 \\ -p_p & -p_{p-1} & \cdots & \cdots & -p_1 \end{bmatrix}, \quad P=\begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \tag{7.9}$$

and

$$\xi=(\xi_1,\dots,\xi_{n-p})^T.$$

Note that Eqs. (7.8) represent the normal form of (7.5). In order to keep $y(t) = 0$, we need to have $\xi = 0$ and

$$u = \frac{1}{\alpha}(-Rz - S\xi).$$

So the zero dynamics is defined on the subspace

$$Z^* = \{x : cA^i x = 0, \quad i = 0, \dots, n-p-1\}$$

and represented by

$$\dot{z} = Nz.$$

The eigenvalues of N are the zeros of (7.5).

7.2.2. *Zero dynamics of MIMO linear systems*

Assume we have the minimal representation of the n-dimensional multivariable state space system given by the triple (A, B, C), which have the frequency domain representation $G(s) = C(sI - A)^{-1}B$.

DEFINITION 7.2. (Invariant zeros of MIMO systems). Invariant zeros of the linear multivariable system (A, B, C) are the complex numbers λ which cause the matrix

$$P(\lambda) = \begin{bmatrix} A - \lambda I & B \\ C & 0 \end{bmatrix}$$

to lose column rank. □

Associated with each zero is an invariant zero direction z such that

$$P(\lambda) = \begin{bmatrix} A - \lambda I & B \\ C & 0 \end{bmatrix} \begin{bmatrix} z \\ \xi \end{bmatrix} = 0. \tag{7.10}$$

When (7.10) holds, the vector ξ is such that $\xi = Kz$ for some matrix K. Following an analogous procedure as made for SISO systems the normal form can be derived (on the basis of the relative degree formulation, see Definition 6.2) by using a coordinate transformation analogously to (7.8) as

$$\begin{aligned} \dot{z} &= Nz + P\xi \\ \dot{\xi}_1^i &= \xi_2^i \\ &\vdots \\ \dot{\xi}_{r_i-1}^i &= \xi_{r_i}^i \\ \dot{\xi}_{r_i}^i &= R_i z + S_i \xi + c_i A^{r_i - 1} B u \\ y_i &= \xi_1^i, \quad i = 1, \dots, p \end{aligned} \tag{7.11}$$

where

$$\xi = (\xi_1^1, \dots, \xi_{r_1}^1, \dots, \xi_1^p, \dots, \xi_{r_p}^p)^T.$$

Again, in order to keep $y(t) = 0$, we must have $\xi = 0$ and $u = L^{-1}(-Rz - S\xi)$, where

$$L = \begin{bmatrix} c_1 A^{r_1 - 1} B \\ \vdots \\ c_p A^{r_p - 1} B \end{bmatrix}, \quad R = \begin{bmatrix} R_1 \\ \vdots \\ R_p \end{bmatrix}, \quad S = \begin{bmatrix} S_1 \\ \vdots \\ S_p \end{bmatrix}.$$

If the system has the vector relative degree $r = (r_1, \ldots, r_p)$, then the zero dynamics is defined on the subspace

$$Z^* = \{x : c_i A^{j-1} x = 0, \quad i = 0, \ldots, p \text{ and } j = 1, \ldots, r_i\}.$$

DEFINITION 7.3. (Transmission zeros). The eigenvalues of N are called transmission zeros of the system. □

Invariant zeros and transmission zeros are system invariants, in the sense that coordinate transformations or feedback transformations do not alter their location. Invariant zeros behave like multivariable analogs to the transfer function zeros of classical control theory. They are essential to defining special (C, A)-invariant subspaces for the design.

It will be seen that the invariance property in some cases can have detrimental effect on the invertibility property of a system. Zeros in the right half plane are especially unpleasant from this point of view. Systems with this type of zeros (*i.e.*, non-minimum phase systems), even if are invertible, would produce instable inverse that, in most of the cases, is practically useless for residual generation.

7.3. GEOMETRIC THEORY OF INVERSION-BASED INPUT RECONSTRUCTION IN LTI SYSTEMS

We now summarize the discussion in the previous section. In order to show the existence of a left inverse for an LTI dynamical system consider the following propositions.

PROPOSITION 7.4. The system $\Sigma : (A, B, C)$ given in state space form is left invertible iff B is monic (it has full column rank) and

$$\mathcal{V}^* \cap \operatorname{Im} B = 0,$$

where $\mathcal{V}^*$ is the supremal (A, B)-invariant subspace in $\ker C$ and F is the feedback, such that $(A + BF)\mathcal{V}^* \subseteq \mathcal{V}^*$ *i.e.*, $(A + BF)$ is maximally unobservable, see (Wonham, 1979). □

This condition, in particular, is equivalent to the condition that the largest controllability subspace, noted $\mathcal{R}^*$ of $\ker C$ is zero. Therefore, an equivalent description of the invertibility can also be given by the following proposition.

PROPOSITION 7.5. The system Σ is invertible iff for the maximal controllability subspace $\mathcal{R}^*$ contained in $\ker C$, the condition $\mathcal{R}^* = 0$ holds, see (Morse and Wonham, 1971). □

REMARK 7.6. The subspace $\mathcal{V}^*$ can be calculated by using the (A, B)-invariant subspace algorithm (see Algorithm 2.20) without explicitly constructing F.

PROPOSITION 7.7. Consider the left invertible system $\Sigma : (A, B, C)$. The dynamics of the (left) inverse can be given as the restriction of $(A + BF)$ on $\mathcal{V}^*$,

$$A_{inv} = (A + BF) \,|\, \mathcal{V}^*.$$

□

COROLLARY 7.8. The dimension of the state space for the inverse system is $n_{inv} = \dim \mathcal{V}^* = n - \rho(r)$, where n is the state dimension of Σ, r is its (vector) relative degree and $\rho(r) = \sum_{i=1}^{p} r_i$. □

Proof. Proposition 7.4 implies

$$(\mathcal{V}^*)^{\perp} + (\operatorname{Im} B)^{\perp} = \mathcal{X}. \tag{7.12}$$

Let us denote the insertion map of $\mathcal{V}^*$ by V^*. Then, from the subspace identity (7.12) it follows that

$$\ker \begin{bmatrix} V^{*\perp} \\ (\operatorname{Im} B)^{\perp} \end{bmatrix} = 0.$$

By using this property in the construction of a state transformation consider the mapping T as

$$z = Tx = \begin{bmatrix} \xi \\ \eta \end{bmatrix}, \quad \xi \in \mathcal{V}^{*\perp}, \quad \eta \in \mathcal{V}^*,$$

where

$$T^{-1} = \begin{bmatrix} B & \Lambda & V^* \end{bmatrix}, \quad \text{and} \quad \operatorname{Im} \Lambda \subset \mathcal{V}^{*\perp}. \tag{7.13}$$

Applying the state transformation on the linear dynamical system the state space representation is obtained

$$\begin{aligned} \dot{z} &= \bar{A}z + \bar{B}u \\ y &= \bar{C}z. \end{aligned} \tag{7.14}$$

From the invertibility condition $\mathcal{V}^* \cap \operatorname{Im} B = 0$ it follows that $\mathcal{V}^* \subset (\operatorname{Im} B)^{\perp}$, *i.e.*, the transformation T is well defined. In the new coordinate system the state matrices will take the form

$$\begin{aligned} &\bar{A} = \begin{bmatrix} \bar{A}_{11} & \bar{A}_{12} \\ \underbrace{\bar{A}_{21}}_{\rho} & \underbrace{\bar{A}_{22}}_{n-\rho} \end{bmatrix} \begin{matrix} \}\,\rho \\ \}\,n-\rho \end{matrix}, \quad \bar{B} = \begin{bmatrix} \bar{B}_1 \\ 0 \end{bmatrix} \begin{matrix} \}\,\rho \\ \}\,n-\rho \end{matrix}, \\ &\bar{C} = \begin{bmatrix} \bar{C}_1 & \bar{C}_2 \end{bmatrix}, \quad \text{where} \quad \rho = \dim(\mathcal{V}^{*\perp}). \end{aligned} \tag{7.15}$$

Since $\mathcal{V}^* \subset \ker C$, the matrix $\bar{C}_2$ should equal to zero *i.e.*,

$$\bar{C} = \begin{bmatrix} \underbrace{\bar{C}_1}_{\rho} & \underbrace{0}_{n-\rho} \end{bmatrix}.$$

Also, since $\bar{A}\mathcal{V}^* \subseteq \mathcal{V}^* + \operatorname{Im}\bar{B}$, it follows that

$$\begin{bmatrix} \bar{A}_{11} & \bar{A}_{12} \\ \bar{A}_{21} & \bar{A}_{22} \end{bmatrix} \begin{bmatrix} 0 \\ \mathcal{V}^* \end{bmatrix} = \begin{bmatrix} \bar{A}_{12}\bar{\mathcal{V}}^* \\ \bar{A}_{22}\bar{\mathcal{V}}^* \end{bmatrix} \subseteq \begin{bmatrix} \operatorname{Im} B \\ \mathcal{V}^* \end{bmatrix}.$$

Since $\bar{B}_1$ is monic there exists a unique matrix F_2 such that

$$\bar{B}_1 F_2 = -\bar{A}_{12}.$$

By choosing

$$F = \begin{bmatrix} 0 & F_2 \end{bmatrix},$$

we get

$$\bar{A} + \bar{B}F = \begin{bmatrix} \bar{A}_{11} & \bar{A}_{12} \\ \bar{A}_{21} & \bar{A}_{22} \end{bmatrix} + \begin{bmatrix} \bar{B}_1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 & F_2 \end{bmatrix} = \begin{bmatrix} \bar{A}_{11} & 0 \\ \bar{A}_{21} & \bar{A}_{22} \end{bmatrix}.$$

In order to simplify the notation the matrices $\bar{B}_1$ and $\bar{C}_1$ will be denoted as $\bar{B}$ and $\bar{C}$ in the following. By the proper selection of T, one has $\bar{B} = [\, I_m \quad 0 \,]^T$. Applying the state feedback

$$u = F_2\eta + \nu \tag{7.16}$$

to the transformed system (7.14), one obtains the equations

$$\begin{aligned} \dot{\xi} &= \bar{A}_{11}\xi + \bar{B}\nu \\ y &= \bar{C}\xi. \end{aligned} \tag{7.17}$$

One can prove by induction that from $c_i A^k B = 0$, it follows that

$$\bar{c}_i \bar{A}_{11}^k \bar{B} = 0 \quad \text{and} \quad \bar{c}_i \bar{A}_{11}^{r_i - 1} \bar{B} \neq 0,$$

for $k < r_i - 1$. Since $\dim(\mathcal{V}^{*\perp}) = \sum_{i=1}^{p} r_i$, see (Wonham, 1979), one can define a state transform S for (7.17) such that

$$w = \begin{bmatrix} y_1 \\ \vdots \\ y_1^{(r_1-1)} \\ \vdots \\ y_p \\ \vdots \\ y_p^{(r_p-1)} \end{bmatrix} = S\xi, \qquad \text{where} \qquad S = \begin{bmatrix} \bar{c}_1 \\ \vdots \\ \bar{c}_1 \bar{A}_{11}^{r_1-1} \\ \vdots \\ \bar{c}_p \\ \vdots \\ \bar{c}_p \bar{A}_{11}^{r_p-1} \end{bmatrix}. \tag{7.18}$$

It follows that the new input function is

$$\nu = \bar{B}^{-1} S^{-1} (\dot{w} - S\bar{A}_{11} S^{-1} w), \tag{7.19}$$

where $S\bar{A}_{11}S^{-1}$ is exactly the observer canonical form of $\bar{A}_{11}$. From

$$\begin{aligned} \dot{\eta} &= \bar{A}_{22}\eta + \bar{A}_{21} S^{-1} w \\ u &= F_2\eta + \nu, \end{aligned} \tag{7.20}$$

one may get the matrix in the basis represented by T

$$\bar{A}_{22} = (A + BF)|\mathcal{V}^* = A_{inv}, \tag{7.21}$$

which proves Proposition 7.7. ■

The following proposition is a corollary of the proof presented above.

PROPOSITION 7.9. The inverse dynamics of the system (A, B, C) can be obtained by following the algorithmic procedure as follows.

STEP 1. Calculate $\mathcal{V}^*$ by using the (A, B)-invariant subspace algorithm (see Algorithm 2.20).
STEP 2. Choose a basis for $\mathcal{V}^*$ and compute the state transformation matrix T as it is defined by (7.13).
STEP 3. Calculate the state transformation matrix S according to (7.18).
STEP 4. Introduce the vector of derivatives

$$v_{inv} = \begin{bmatrix} w \\ y_1^{(r_1)} \\ \vdots \\ y_p^{(r_p)} \end{bmatrix}, \tag{7.22}$$

as the input of the inverse system where w is according to (7.18). Then, the dynamics of the inverse is obtained from

$$\dot{\eta} = A_{inv}\eta + B_{inv}v_{inv} \tag{7.23}$$

using the definitions

$$A_{inv} = \bar{A}_{22}, \qquad B_{inv} = \begin{bmatrix} \bar{A}_{21}S^{-1} \\ 0 \end{bmatrix}. \tag{7.24}$$

The input function $u(t)$ can be obtained from the equations

$$u = C_{inv}\eta + D_{inv}v_{inv}, \tag{7.25}$$

where $C_{inv} = F_2$, moreover,

$$D_{inv} = Z - \begin{bmatrix} S\bar{A}_{11}S^{-1} & 0 \\ 0 & 0 \end{bmatrix}. \tag{7.26}$$

The matrix Z is given as

$$Z = \begin{bmatrix} Z_1 & 0 & \cdots & 0 & E_1 \\ 0 & Z_1 & \cdots & 0 & E_p \\ \vdots & & & & \\ 0 & 0 & \cdots & Z_p & E_p \end{bmatrix}, \tag{7.27}$$

where

$$Z_i = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 \\ \vdots & & & & & \\ 0 & 0 & 0 & \cdots & 1 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}, \quad \text{and} \quad E_i = \begin{bmatrix} 0 \\ e_i^T \end{bmatrix}, \tag{7.28}$$

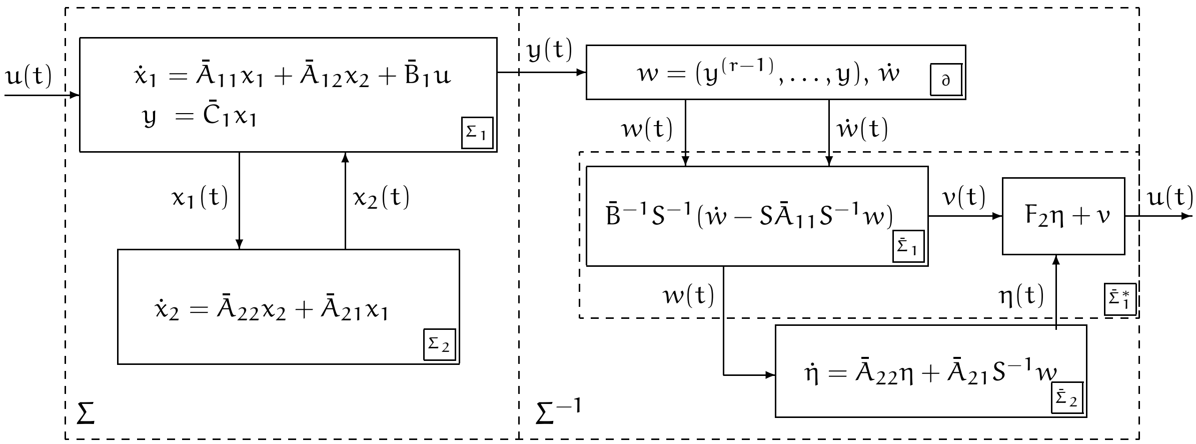

Figure 7.1. Logical structure of the direct input reconstruction method based on dynamic system inversion for linear systems. Σ is the system, Σ^{-1} is its inverse representation.

with e_i being the i^{th} unit vector in $\mathbb{R}^p$. □

Fig. 7.1 shows the logical structure of the direct input reconstruction method based on dynamic system inversion for linear multivariable systems. One can see how the system Σ is conveniently split into subsystems Σ_1 and Σ_2 which are coupled through the state variables x_1, x_2 according to (7.15). The same splitting can be identified in the Σ^{-1} inverse structure as well. The subsystems Σ_2 and $\bar{\Sigma}_2$ corresponds to the separated zero dynamics of the original system and its corresponding representation in the dynamics of the inverse system, respectively. $\bar{\Sigma}_1$ is in accord with (7.19) and, by adding the state feedback (7.20) $\bar{\Sigma}_1^*$ represents the read-out map of the inverse system. The ∂ block is the differentiator providing the derivatives of the measurement vector $y(t)$.

To conclude this section an immediate property of the inverse dynamics *i.e.,* the transmission zeros of the transfer function matrix of the system is characterized by the following proposition.

PROPOSITION 7.10. (Transmission zeros). The transmission zeros of (A, B, C) are the poles of the inverse dynamics, *i.e.,*

$$\sigma\left((\bar{A} + \bar{B}F)|\mathcal{V}^*\right) = \sigma\left(\bar{A}|\mathcal{V}^*\right).$$

□

Proof. The maximality of $\mathcal{V}^*$ implies that for all s the system matrix

$$\begin{bmatrix} sI - A_{11} & B_1 \\ C_1 & 0 \end{bmatrix} \tag{7.29}$$

is nonsingular therefore it has no transmission zero. If (7.29) were singular, then there would exist x_o, u_o, $x_o \in \mathcal{V}^*$, such that $(s_o I - A_{11})x_{1_o} + B_1 u = 0$ and $C_1 x_{1_o} = 0$. But for this case

$V = \mathcal{V}^* + \operatorname{span} x_o$ satisfies $AV \subset V + \operatorname{Im} B$, $V \subset \ker C$. It implies that $\mathcal{V}^*$ is not maximal which is a contradiction. Since it is known that

$$\begin{bmatrix} sI - A - BF & B \\ C & 0 \end{bmatrix} = \begin{bmatrix} sI - A & B \\ C & 0 \end{bmatrix} \begin{bmatrix} I & 0 \\ -F & I \end{bmatrix}, \tag{7.30}$$

the invariant polynomials of the open-loop and closed-loop system matrices are identical. Rearranging rows and columns we get

$$\left[\begin{array}{cc|c} sI - A_{11} & B_1 & 0 \\ C_1 & 0 & 0 \\ \hline -A_{21} & 0 & sI - A_{22} \end{array}\right]. \tag{7.31}$$

It follows that the invariant polynomials (or transmission polynomials) can only be associated to $sI - A_{22}$, that is to say, to $A + BF|\mathcal{V}^*$. ■

EXAMPLE 7.11. In order to demonstrate the inverse calculation in LTI systems based on the geometric characterization of the procedure presented in the previous section consider the system representation given by the matrices

$$A = \begin{bmatrix} -1 & 0 & -1 & 1 \\ 1 & -2 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 2 & 0 \end{bmatrix}, \; B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}, \; L = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 1 \\ 1 & -1 \end{bmatrix}, \; C = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

assuming D and M are zero. It is simple to arrive at $\mathcal{V}^* = \ker C$ *i.e.*, $\dim \mathcal{V}^* = 2$. Since $\dim \mathcal{V}^* = (n - \rho)$, it follows that the relative degree of the system is $\rho = 4 - 2 = 2$. Indeed, a simple calculation reveals that the relative degree is $r = [\,1 \quad 1\,]$, that is to say $r_1 = 1$ and $r_2 = 1$ and, therefore, $\rho = 1 + 1 = 2$. Since $\mathcal{V}^* \cap \operatorname{Im} L = 0$, (A, L, C) is left invertible. The calculation of $\mathcal{V}^{*\perp}$ can be carried out from the span of the rows of C, *i.e.*,

$$\mathcal{V}^{*\perp} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = C$$

and one can choose

$$L^{\perp} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}.$$

The state transform can be written as a simple change of coordinates x_i

$$T = \begin{bmatrix} \mathcal{V}^{*\perp} \\ L^{\perp} \end{bmatrix} = \left[\begin{array}{cccc} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \hline 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{array}\right].$$

Then the coordinate transforms $z = Tx$, $\bar{B} = TB$ and $\bar{L} = TL$ are written as

$$z = \begin{bmatrix} x_3 \\ x_4 \\ x_1 \\ x_2 \end{bmatrix}, \; \bar{B} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{bmatrix}, \; \bar{L} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

Moreover, with

$$\bar{C} = CT^{-1} = CT^{T} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \tag{7.32}$$

and $\bar{A} = TAT^{-1} = TAT^{T}$

$$\bar{A} = \begin{bmatrix} \bar{A}_{11} & \bar{A}_{12} \\ \bar{A}_{21} & \bar{A}_{22} \end{bmatrix} = \left[\begin{array}{cc|cc} 0 & -1 & 0 & 1 \\ 2 & 0 & 0 & 0 \\ \hline -1 & 1 & -1 & 0 \\ 0 & 0 & 0 & -2 \end{array}\right]$$

one arrives at $A_{inv} = A|\mathcal{V}^* = A_{22}$. Then, the transformed state space system can be written in the form of

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \\ \dot{z}_4 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & 1 \\ 2 & 0 & 0 & 0 \\ -1 & 1 & -1 & 0 \\ 0 & 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{bmatrix} u - \begin{bmatrix} 1 & 1 \\ 1 & -1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \nu. \tag{7.33}$$

Since the zero dynamics has the form $\eta = [z_3, z_4]^T$ the inverse system can be represented as

$$\dot{\eta} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} \eta + \begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} u.$$

When $r_1 = 1$, $r_2 = 1$ and, according to (7.18), S is the left block identity matrix of (7.32)

$$S = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

by (7.19), the unknown inputs ν_1 and ν_2 can be derived from the first two equations of the system (7.33) as

$$\begin{bmatrix} \nu_1 \\ \nu_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}^{-1} \left(\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} - \begin{bmatrix} 0 & -1 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \eta - \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \right). \tag{7.34}$$

Since

$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}^{-1} = \frac{1}{2} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix},$$

eqs. (7.34) can be expressed by using the identities: $y_1 = z_1$, $y_2 = z_2$, $z_3 = x_1$, and $z_4 = x_2$ as

$$\begin{aligned} \nu_1 &= -\frac{1}{2}(-\dot{y}_1 - \dot{y}_2 + 2y_1 - y_2 + x_2 + u_2) \\ \nu_2 &= -\frac{1}{2}(-\dot{y}_1 + \dot{y}_2 - 2y_1 - y_2 + x_2 + u_2). \end{aligned}$$

7.4. GEOMETRIC THEORY OF INVERSION-BASED INPUT RECONSTRUCTION IN NONLINEAR SYSTEMS

Consider the nonlinear input affine system written in the form

$$\begin{aligned} \dot{x} &= f(x) + g(x)u, \qquad g(x) = \sum_{i=1}^{m} g_i(x)u_i, \quad u \in \mathbb{R}^m, \quad y \in \mathbb{R}^p, \\ y_j &= h_j(x), \quad j = 1, \dots, p, \end{aligned} \tag{7.35}$$

It was shown in the previous section that the zero dynamics of the linear dynamical system gives the dynamics of the left inverse (see Proposition 7.7). Similarly to the linear case, the concept of zero dynamics will, therefore, be used extensively in the following parts of this work in which the idea of the construction of the inverse for nonlinear systems is presented. Let us begin, therefore, with the problem of how the output of the system (7.35) can be set to zero by means of a proper choice of the initial state and input (*cf.* Sections 7.2.1 and 7.2.2) and, in order to create a basis for further discussions, let us recall some elementary facts and definitions from nonlinear system theory as found *e.g.*, in (Isidori, 1985) and (Nijmeijer and Van der Schaft, 1991).

7.4.1. *Nonlinear analog of transmission zeros and zero dynamics*

Select the point x_o in the state space of (7.35) and assume $f(x_o) = 0$ and $h(x_o) = 0$. If the initial state at time $t = 0$ is equal to x_o and the input $u(t)$ is zero for all $t \geq 0$, then so is the output $y(t)$. Our purpose is to identify the set of all pairs consisting of an initial state and an input function which produce zero output of the system.

A smooth connected submanifold M of $\mathcal{X}$ which contains the point x_o is said to be locally controlled invariant at x_o if there exists a smooth feedback $\alpha(x)$ and a neighborhood U_o of x_o such that the vector field $\tilde{f}(x) = f(x) + g(x)\alpha(x)$ is tangent to M for all $x \in M \cap U_o$, *i.e.,* M is locally invariant under $\tilde{f}$.

A smooth connected submanifold M that is locally controlled invariant at x_o and with the property that $M \subset h^{-1}(0)$ is called an output-zeroing submanifold of Σ. This means that for some choice of the feedback control $\alpha(x)$ the trajectories of the system Σ which start in M stay in M for all t in a neighborhood of $t_o = 0$ while the corresponding output is identically zero.

If M and M' are two connected submanifolds of $\mathcal{X}$ which both contain x_o, we say that M locally contains M' (or, more practically, coincides with M') if for some neighborhood U of the origin, $(M \cap U) \supset (M' \cap U)$. An output zeroing submanifold M is locally maximal if, for some neighborhood U of the origin, any other output zeroing submanifold M' satisfies $(M \cap U) \supset (M' \cap U)$.

A submanifold M is said to be an integral submanifold of a distribution Δ if for every $x \in M$ and the tangent space T_xM to M at x, one has $T_xM = \Delta(x)$.

The construction of the maximal locally controlled invariant output-zeroing submanifold for a system Σ can be illustrated by the following algorithm.

ALGORITHM 7.12. (Zero dynamics algorithm). Define a nested sequence of subsets $M_o \supset M_1 \supset \cdots$ of $\mathcal{X}$ in the following way. Let U_o be a neighborhood of x_o and

$$M_o = \{x \in \mathcal{X} \,:\, h(x) = 0\}.$$

At each $k > 0$, suppose that, $M_k \cap U_k$ is a smooth manifold and let M_k^c denote the connected component of $M_k \cap U_k$ which contain x_o. Assume that M_k is a submanifold through x_o and define M_{k+1} as:

$$M_{k+1} = \{\, x \in M_k^c : \; f(x) \in \text{span}\{g_i(x)\} + T_x M_k^c\}.$$

□

If there is a U_o such that M_k is a smooth submanifold through x_o for each $k \geq 0$, then x_o is called a regular point of the algorithm and there is a k^* such that $M_{k^*+l} = M_{k^*}$ for all $l \geq 0$. Let, in addition,

$$\dim \text{span}\{\, g_i(x_o) \,|\, i = 1, \dots, m\} = m, \tag{7.36}$$

and the dimension of the subspace

$$\dim \text{span}\{\, g_i(x) \,|\, i = 1, \dots, m\} \cap \; T_x M_{k^*}^c$$

is constant for all $x \in M_{k^*}^c$. Then the maximal connected component of M_{k^*}, is the locally maximal output-zeroing submanifold of Σ which will be denoted by Z^*. Moreover, if

$$\text{span}\{\, g_i(x) \,|\, i = 1, \dots, m\} \cap T_x M_{k^*}^c = 0, \tag{7.37}$$

then there exists a unique smooth mapping (feedback control) $\alpha^* : Z^* \to \mathbb{R}^m$ such that the vector field

$$f^*(x) = f(x) + g(x)\alpha^*(x)$$

is tangent to Z^*, (Isidori, 1985) and (Nijmeijer and Van der Schaft, 1991).

Suppose the hypotheses (7.36) and (7.37) are satisfied. Since $f^*(x)$ is tangent to Z^*, $f^*(x)|_{Z^*}$ (the restriction of $f^*(x)$ to Z^*) is well defined vector field on Z^*. The submanifold Z^* is then called the *(local) zero dynamics submanifold* and the vector field $f^*(x)$ of Z^* is the *zero dynamics vector field*. The pair (f^*, Z^*) is called the *zero dynamics* of the system. By construction, the dynamical system

$$\dot{x} = f^*(x), \quad x \in Z^* \tag{7.38}$$

identifies the internal dynamics of the system when the output has been forced, by proper choice of initial state and input, to zero for some interval of time.

An algorithm for computing Z^* in general cases can be found in (Isidori, 1985) and (Nijmeijer and Van der Schaft, 1991). However, in some cases Z^* can be determined easily by relating it to the maximal controlled invariant distribution Δ^* contained in $\ker(dh)$, given by the following algorithm.

ALGORITHM 7.13. (Controlled Invariant Codistribution Algorithm - CIcDA).

$$\Omega_1 = \operatorname{span}\{dh_i \,|\, i = 1, \ldots, p\}$$
$$\Omega_{k+1} = \Omega_k + L_f(\Omega_k \cap g^\perp) + \sum_{i=1}^{m} L_{g_i}(\Omega_k \cap g^\perp),$$

moreover, $\Delta^* = \Omega_*^\perp$. □

THEOREM 7.14. (Isidori, 1985). Suppose x_o is a regular point regarding the controlled invariant codistribution algorithm and dim $g(x_o) = m$. Suppose also that

$$L_{g_i}(\Omega_k \cap g^\perp) \subset \Omega_k,$$

for all $k \geq 0$. Then, for all x in a neighborhood of x_o, one has

$$\Delta^*(x) = T_x Z^*.$$

□

REMARK 7.15. It is an easy analogy to relate the zero dynamics algorithm (Algorithm 7.12) to linear systems. One can realize the equivalence

$$M_o = \ker C$$

and

$$M_{k+1} = \{x \in M_k : Ax \in \operatorname{Im} B + M_k\}.$$

It shows that all M_k's are subspaces of the state space and the subspace $M_{k^*} = \mathcal{V}^*$ is by construction the maximal subspace in ker C satisfying

$$A\mathcal{V}^* \subset \mathcal{V}^* + \operatorname{Im} B, \tag{7.39}$$

which shows that conditions of Theorem 7.14 is trivially satisfied for linear systems. The hypotheses (7.36) and (7.37), therefore, with the identity $Z^* = \mathcal{V}^*$ reduce to

$$\dim(\operatorname{Im} B) = m, \quad \text{and} \quad \mathcal{V}^* \cap \operatorname{Im} B = 0, \tag{7.40}$$

which are exactly the conditions under which the transfer function matrix of the system is left invertible, (*cf.* Proposition 7.4). Conditions (7.39) imply the existence of the state feedback $\alpha^*(x)$ which, in this case is a linear function of the state, namely, it can be written that $\alpha^*(x) = Fx$ such that $f^*(x) = Ax + B\alpha^*(x)$ is tangent to $\mathcal{V}^*$. By construction the subspace $\mathcal{V}^*$ is invariant under the linear mapping $(A + BF)$. That is to say $(A + BF)x$ is included in $\mathcal{V}^*$ for all $x \in \mathcal{V}^*$, namely $(A+BF)\mathcal{V}^* \subset \mathcal{V}^*$ which provides the result of Proposition 7.4 in a straightforward way.

In case (7.40) holds, the restriction $F|_{\mathcal{V}^*}$ is unique and, if the triple (A, B, C) is minimal, the invariant factors of $(A + BF)|_{\mathcal{V}^*}$ are the transmission polynomials and their roots are the transmission zeros of the system.

7.4.2. *Nonlinear systems with vector relative degree*

The conditions of Theorem 7.14 are satisfied for nonlinear systems having vector relative degree.

LEMMA 7.16. Let us suppose that the system (7.35) has a relative degree. Then the row vectors

$$dh_1(x_o), \cdots, dL_f^{r_1-1}h_1(x_o), \cdots, dh_p(x_o), \cdots, dL_f^{r_p-1}h_p(x_o)$$

are linearly independent. □

REMARK 7.17. From the proof of the lemma, see (Isidori, 1985), it is clear that condition (6.21) is a necessary condition, *i.e.,* the existence of the finite relative orders alone does not ensure the linear independency of the whole system.

REMARK 7.18. Since for any real valued function λ $dL_f\lambda(x) = L_f d\lambda(x)$ and, by the algorithm CIcDA presented above, one has that all the codistributions $dL_f^k h_i(x)$, satisfying the property $L_{g_j}L_f^k h_i(x) = 0$, are contained in Ω_*, *i.e.,* in $\Delta^{*\perp}$. It follows that

$$\Delta^* \subset \text{span}\{dL_f^k h_i \,|\, k = 0, \ldots, r_i - 1, \; i = 1, \ldots, p\}^\perp.$$

The significance of the determination of the output-zeroing manifold is motivated by these issues in the principle of invertibility and the construction of the reduced order inverse for linear and nonlinear controlled systems.

Conditions (7.36) and (7.37), for example, can be interpreted as a special property of invertibility of system (7.35). As it was already stated, if rank $A(x) = m$, then

$$Z^* = \{x \,|\, L_f^k h_i = 0, \; i = 1, \cdots, p \quad k = 0, \cdots, r_i - 1\} \tag{7.41}$$

and

$$\Delta^{*\perp} = \ker \text{span}\{\, dL_f^k h_i, \; i = 1, \cdots p, \quad k = 0, \cdots, r_i - 1\}, \tag{7.42}$$

see also (Nijmeijer and Van der Schaft, 1991). Moreover, the control feedback $u^*(x) = \alpha(x)$ is the solution of the equation

$$A(x)\alpha(x) + B(x) = 0 \tag{7.43}$$

by using the notation

$$B(x) \triangleq \begin{bmatrix} L_f^{r_1} h_1(x) \\ \vdots \\ L_f^{r_p} h_p(x) \end{bmatrix}.$$

Let $\xi = \Theta(x)$ denote the diffeomorphism formed by $(\xi^i)_{i=1,\ldots,p}$, and defined by

$$\xi^i = (L_f^k h_i(x))_{k=0,\ldots,r_i-1}.$$

It is a standard computation that

$$\dot{\xi}^i = A^i \xi^i + B^i y_i^{(r_i)}, \tag{7.44}$$

where A^i, B^i are in the Brunowsky canonical form, see (Isidori, 1985) (p.231). Note that $\xi_1^i = y_i$. Let us complete $\Theta(x)$ to be a diffeomorphism (that is Θ and also Φ^{-1} be continuously differentiable) on X as

$$\begin{bmatrix} \xi \\ \eta \end{bmatrix} = \Phi(x) \triangleq \begin{bmatrix} \Theta(x) \\ \Lambda(x) \end{bmatrix}. \tag{7.45}$$

Since $\partial_x \Theta = [dL_f^k h_i]$, one has

$$\dot{\xi} = [dL_f^k h_i] f|_{\Phi^{-1}} + [dL_f^k h_i] g|_{\Phi^{-1}} u. \tag{7.46}$$

By maintaining the nonzero rows one gets the splitting

$$\dot{\xi}_{r_i}^i = B|_{\Phi^{-1}} + A|_{\Phi^{-1}} u, \tag{7.47}$$
$$\dot{\eta} = \partial_x \Lambda f|_{\Phi^{-1}} + \partial_x \Lambda g|_{\Phi^{-1}} u. \tag{7.48}$$

Then the zero dynamics can be obtained by inserting for $\xi = 0$

$$\dot{\eta} = \partial_x \Lambda f|_{\Phi^{-1}} + \partial_x \Lambda g \alpha|_{\Phi^{-1}}. \tag{7.49}$$

If g is involutive, then one can choose $d\Lambda \subset g^{\perp}$, and then

$$\dot{\eta} = \partial_x \Lambda f|_{\Phi^{-1}}. \tag{7.50}$$

Based on the above considerations the following proposition summarizes the inverse construction procedure for nonlinear systems.

PROPOSITION 7.19. The inverse dynamics of systems (7.35) can be obtained by using the algorithmic procedure as follows.

STEP 1. Calculate $\Delta^{*\perp}$ given in the form (7.42) by using the Controlled Invariant Codistribution Algorithm - CIcDA (see, Algorithm 7.13).

STEP 2. Construct the state transformation matrix $\Phi(x)$ according to (7.45), where $\Theta(x) \triangleq \Delta^{*\perp}$. Select a basis to $\Lambda(x)$ to be orthogonal to $\Theta(x)$ *i.e.,* define $\Lambda(x) \triangleq \Delta^{*}$.

STEP 3. By knowing $\Phi(x)$, get the splitting

$$\begin{bmatrix} \xi \\ \eta \end{bmatrix} = \Phi(x)$$

and obtain the system representation in the form of (7.47)-(7.48).

STEP 4. Introduce the vector of derivatives

$$v_{inv} = [\, w^T \quad y_1^{(r_1)} \ \cdots \ y_p^{(r_p)} \,]^T,$$

as the input of inverse system. Then, the dynamics of the inverse is obtained from (7.49) and the read-out map of the inverse system will be given by (7.47). □

Fig. 7.2 shows the logical structure of the direct input reconstruction method based on dynamic system inversion for nonlinear systems (for sake of notational simplicity, only SISO systems are considered in the figure). One can see how the system Σ is conveniently split into subsystems

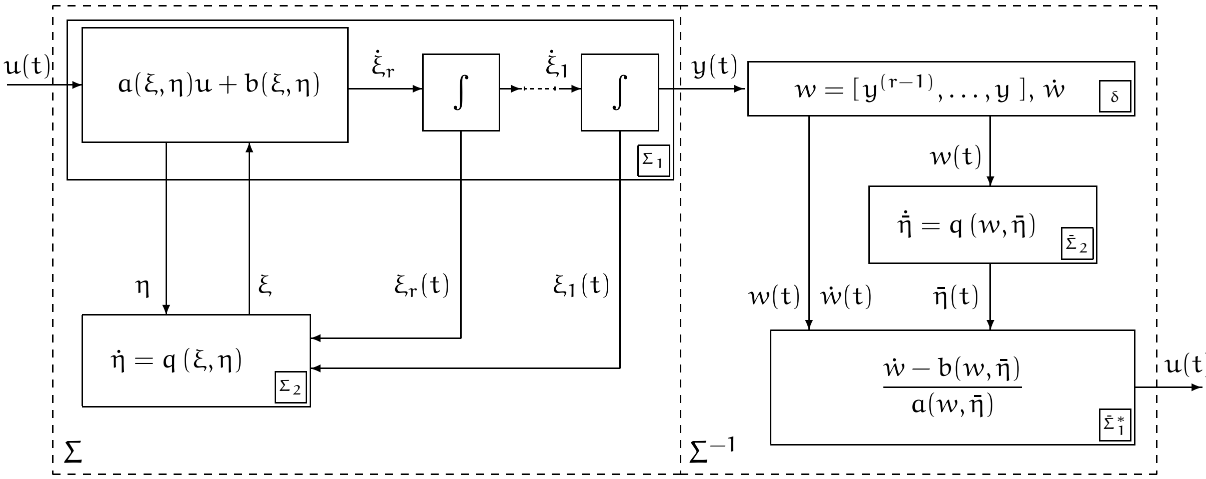

Figure 7.2. Logical structure of the direct input reconstruction method based on dynamic system inversion for SISO nonlinear systems. Σ is the system, Σ^{-1} is its inverse representation.

Σ_1 and Σ_2, (7.46) and (7.49), respectively, which are coupled through the state variables ξ and η. The same splitting can be identified in the inverse structure Σ^{-1}, though not so directly as in the linear case because $\bar{\Sigma}_1^*$ merges the dynamics and the inverse with the feedback (*cf.* $\bar{\Sigma}_1^*$ in Fig. 7.1 for linear systems).

The subsystems Σ_2 and $\bar{\Sigma}_2$ corresponds to the separated zero dynamics of the original system and its corresponding representation in the dynamics of the inverse system, respectively, in a completely similar way as in the linear case. The ∂ block is the differentiator providing the derivatives of the measurement vector $y(t)$.

REMARK 7.20. Let us consider the following controlled nonlinear input affine system, subject to multiple faults, as

$$\begin{aligned} \dot{x} &= f(x, u) + \sum_{l=1}^{q} g_\ell(x)\nu_\ell, \\ y_j &= h_j(x), \quad j = 1, \ldots, p, \end{aligned} \tag{7.51}$$

with $f(x, u) = f_o(x) + \sum_{i=1}^{m} f_i(x)u_i$. Then, by introducing the time t as an auxiliary state, one may apply the results of the previous section to the augmented system.

The decoupling matrix A will also depend on the control inputs $u(t)$ and similarly on its derivatives, *i.e.,* the condition (6.21) for having vector relative degree will also be dependent on the inputs. This is in contrast with the LTI case where the inputs $u(t)$ do not play any role in the solvability of the problem.

EXAMPLE 7.21. Consider system (7.51) determined by the functions

$$f_o(x) = f(x) = \begin{bmatrix} x_2 \\ 0 \\ x_1 x_4 \\ -1.2x_3 \\ x_1 \end{bmatrix}, \quad g_1(x) = \begin{bmatrix} 1 \\ -x_2 \\ 0 \\ -x_4 \\ 1 \end{bmatrix}, \quad g_2(x) = \begin{bmatrix} 0 \\ 0 \\ 1 \\ x_1 \\ -x_5 \end{bmatrix} \tag{7.52}$$

$$h_1(x) = x_1, \quad h_2(x) = x_3, \tag{7.53}$$

i.e., for the sake of the greatest possible simplicity, we consider an autonomous system subject to the effect of the failure modes ν_1 and ν_2. Then,

$$A(x) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad B(x) = \begin{bmatrix} x_2 \\ x_1 x_4 \end{bmatrix}. \tag{7.54}$$

Let us define the diffeomorphism

$$\Phi(x) = \begin{bmatrix} x_1 \\ x_3 \\ x_2 \\ x_4 \\ x_5 \end{bmatrix}, \quad \text{with} \quad \partial_x \Phi = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}. \tag{7.55}$$

It follows that

$$\begin{bmatrix} \dot{\xi} \\ \dot{\eta} \end{bmatrix} = \partial_x \Phi (f + g\nu)|_{\Phi^{-1}} = \begin{bmatrix} \eta_1 \\ \xi_1 \eta_2 \\ 0 \\ -1.2\xi_2 \\ \xi_1 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ -\eta_1 \\ -\eta_2 \\ 1 \end{bmatrix} \nu_1 + \begin{bmatrix} 0 \\ 1 \\ 0 \\ \xi_1 \\ -\eta_3 \end{bmatrix} \nu_2$$

and $y = \xi$. One can obtain the inverse system by using the relation

$$\nu = -\begin{bmatrix} \eta_1 \\ \xi_1 \eta_2 \end{bmatrix} + \dot{y}. \tag{7.56}$$

From $\xi = y$, one get the failure modes

$$\nu = \begin{bmatrix} \dot{y}_1 - \eta_1 \\ \dot{y}_2 - y_1 \eta_2 \end{bmatrix} \tag{7.57}$$

with

$$\dot{\eta} = \begin{bmatrix} 0 \\ -1.2y_2 \\ y_1 \end{bmatrix} + \begin{bmatrix} -\eta_1 \\ -\eta_2 \\ 1 \end{bmatrix} (\dot{y}_1 - \eta_1) + \begin{bmatrix} 0 \\ y_1 \\ -\eta_3 \end{bmatrix} (\dot{y}_2 - y_1 \eta_2).$$

7.4.3. *Feedback linearization and dynamic inversion*

Recall that, feedback linearizing a scalar system, as originated by the concept of (Brockett, 1978; Brockett, 1981),

$$\dot{x} = f(x) + g(x)u, \tag{7.58}$$

amounts to finding an output map and feedback

$$\begin{aligned} y &= h(x) \\ \nu &= \alpha(x) + \beta(x)u, \end{aligned} \tag{7.59}$$

that transforms the system into a string of integrators (*cf.* Fig. 7.2)

$$\frac{d^n y}{dt^n} = \nu.$$

Assuming, the value of the output $y(t)$ is known by measurements, and it is smooth then the computation of the input $u(t)$ from (7.59) is straightforward,

$$u(t) = \frac{\frac{d^n y}{dt^n}(t) - \alpha\big(x(t)\big)}{\beta\big(x(t)\big)}. \tag{7.60}$$

For this reason, feedback linearization is sometimes called dynamic inversion as it was more extensively studied in (Enns et al., 1994). This relationship also gives the explanation of the notion of *dynamic inversion*. Similarly, for systems with m inputs one seeks m outputs and the feedback of form (7.59) that transforms the system into m strings of integrators

$$\frac{d^{k_i} y_i}{dt^{k_i}} = \nu_i, \quad i = 1, \ldots, m,$$

where $\{k_1, \ldots, k_m\}$, are the (Kronecker) controllability indices of the system, upon which, the computation of the input $u(t)$ can be based. That is to say, when the output function of the dynamical system h can be made known (by measurements) then the problem is one of input-output linearization via state feedback. As a general assumption, the system must have well defined relative degree to be invertible, and the zero dynamics must be stable.

7.4.4. *Extension to LPV systems*

For a general nonlinear system, which cannot be represented in the form of (7.35), the question of the existence and the computation of the codistribution Δ^* is far to be trivial. Moreover, computation of the state transformation map that is necessary to determine the zero dynamics involves, in general, integration of partial differential equations. Therefore the general treatment of the problem in the framework of geometric nonlinear system theory is often not computationally tractable and some useful progress requires an intermediate level of complexity.

Linear parameter varying (LPV) modeling techniques have been proven to be useful in this application domain. The idea is that a lot of nonlinear system can be converted into a *quasi* linear form, obtaining so called quasi linear parameter varying (*q*LPV) system models in which the state matrix depends affinely on a parameter vector. These classes of systems subjected to faults can be described as

$$\dot{x}(t) = A(\rho)x(t) + B(\rho)u(t) + \sum_{j=1}^{m} L_j(\rho)\nu_j(t) \tag{7.61}$$
$$y(t) = Cx(t),$$

with

$$A(\rho) = A_o + \rho_1 A_1 + \ldots + \rho_N A_N,$$
$$B(\rho) = B_o + \rho_1 B_1 + \ldots + \rho_N B_N,$$
$$L_j(\rho) = L_{j,o} + \rho_1 L_{j,1} + \ldots + \rho_N L_{j,N},$$

where ρ_i are time-varying parameters for the LPV case and parameters that depends on measurable outputs for the *q*LPV case, respectively (Bokor and Balas, 2004). It is assumed that each parameter ρ_i and its derivatives ranges between known extremal values. Let us denote this parameter set by $\mathcal{P}$.

To apply the ideas presented in the previous sections to systems (7.61), it is necessary to introduce the parameter varying counterpart of the invariant subspace $\mathcal{V}^*$.

DEFINITION 7.22. Let $\mathcal{B}(\rho)$ denote Im $B(\rho)$. Then a subspace $\mathcal{V}$ is called a parameter-varying (A, B)-invariant subspace (or shortly (A, B)-invariant subspace) if for all $\rho \in \mathcal{P}$ one has

$$A(\rho)\mathcal{V} \subset \mathcal{V} + \mathcal{B}(\rho).$$

□

The set of all parameter varying (A, B)-invariant subspaces containing a given subspace $\mathcal{C}$, is an upper semilattice with respect to the intersection of subspaces. This semilattice admits a maximum, denoted by

$$\mathcal{V}^* = \max \mathcal{V}(A(\rho), B(\rho), \mathcal{C}).$$

This subspace can be computed by a finite algorithm for systems of type (7.61), for details see *e.g.,* (Balas et al., 2003) and (Szabó et al., 2003). Using this subspace the computation of the inverse system can be performed following the same algorithmic steps as in the LTI case.

7.5. RELATIONSHIP OF PARITY SPACE AND SYSTEM INVERSION-BASED RESIDUAL GENERATION IN LINEAR SYSTEMS

In view of the results presented in the previous sections we are now able to discuss some interesting properties which relate parity space and the system inversion-based residual generation methods.

Recall that the linear dynamical system (1.21) and its corresponding analytical redundancy equations, up to order s was given in the general form (1.25) in Section 1.5 as

$$\nu_y = \mathcal{O}^s x + S_u \nu_u + S_\nu \nu_\nu, \tag{7.62}$$

where $\nu_u \in \mathbb{R}^{r(s+1)}$, $\nu_v \in \mathbb{R}^{q(s+1)}$ and $\nu_y \in \mathbb{R}^{m(s+1)}$ contain the variables u, v and y and their time derivatives of the appropriate order, respectively. The vector variables ν_u and ν_y contain the input and output measurements and their respective time derivatives and are written in the form $(u, \dot{u}, \dots, u^{(s)})^T$, $(y^T, \dot{y}^T, \dots, y^{(s)^T})^T$ and $\mathcal{O}^s = [C^T \ (CA)^T \dots (CA^s)^T]^T$. Analogously, the unknown input is written as $\nu_v = (v, \dot{v}, \dots, v^{(s)})^T$.

It was also shown, how the unknown state x can be decoupled and eliminated from (7.62) by using p linearly independent vectors ω_j, $j = 1, \dots, p$, with $p = m(s+1) - \operatorname{rank} \mathcal{O}^s$, satisfying

$$\omega \, \mathcal{O}^s x = 0. \tag{7.63}$$

Letting $W = [\omega_j]_{j=1,\dots,p} \subset \mathbb{R}^{p \times (s+1)m}$, and following the state elimination procedure described in Section 1.5 one obtains

$$W\nu_y = W(S_u \nu_u + S_v \nu_v), \tag{7.64}$$

i.e.,

$$WS_v \nu_v = W\nu_y - WS_u \nu_u. \tag{7.65}$$

Let us introduce now the matrix-valued multivariable differential polynomials

$$q(v) = WS_v \nu_v, \tag{7.66}$$

and

$$p(u, y) = W(\nu_y - S_u \nu_u). \tag{7.67}$$

In fault free case (*i.e.*, when $v(t)$ and all its derivatives are zero) the relation

$$W\nu_y = WS_u \nu_u \tag{7.68}$$

holds but

$$p(u, y) \neq 0 \tag{7.69}$$

for any other case. By this property, $p = (u, y)$ was proposed to use as detection residual in many different ways; two of the many were shown in (Chow and Willsky, 1984) and (Gertler, 1998). Concerning the linear dynamical system (1.21) and its corresponding analytical redundancy equations, which combine the dynamics with output derivatives (1.23) up to fault input insertion, letting the integer $s = k$ denote the number of differentiation when the unknown input (fault) appears in the output. If there exists the integer $r_i > 0$, such that

$$c_i A^k L = 0 \quad \text{and} \quad c_i A^{r_i - 1} L \neq 0, \tag{7.70}$$

for $\forall \, k < r_i - 1$, assuming c_i are the row vectors of C, then r_i is called a relative degree of the LTI system (1.21), see Definition 6.2. Based on the individual components r_i the construction $r = [\, r_1, \dots, r_p \,]$ is defined as the vector relative degree of the representation (1.21). Then, by a proper selection of the basis, one can construct the equations

$$\begin{bmatrix} y_1^{(r_1)} \\ \vdots \\ y_p^{(r_p)} \end{bmatrix} = \begin{bmatrix} c_1A^{r_1} \\ \vdots \\ c_pA^{r_p} \end{bmatrix} x + \Sigma \begin{bmatrix} 0 & 0 & \cdots & 0 \\ CB & 0 & \cdots & 0 \\ CAB & CB & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ CA^{s-1}B & \cdots & CB & 0 \end{bmatrix} \nu_u + \begin{bmatrix} c_1A^{r_1-1}L \\ \vdots \\ c_pA^{r_p-1}L \end{bmatrix} \nu \tag{7.71}$$

where Σ is a selection matrix, or, in a more compact form

$$\nu_y = \tilde{T}x + \tilde{S}_u\nu_u + \tilde{R}_\nu \nu, \tag{7.72}$$

where $\nu_u \in \mathbb{R}^{r(s+1)}$ contains the input variable u and its derivatives in the appropriate order. It can be seen that, in the construction of the detection residual, the faults can be derived from (7.72) by calculating the left inverse *w.r.t.* $\nu(t)$ obtaining

$$\nu = \tilde{R}_\nu^{-1}(\nu_y - \tilde{T}x + \tilde{S}_u\nu_u). \tag{7.73}$$

Relating (7.73) with eqs. (6.13-6.15) reveals that the dynamics of (7.73) is based on the state $\eta(t)$ defined by the zero dynamics of the system.

The laplace transform of the input-output relationship for the residual system (7.73) results in the representation, analogous to (6.2-6.3)

$$G^{-\ell}(s)G_\nu(s)\nu(s) = G^{-\ell}(s)y(s) + G^{-\ell}(s)G_u(s)u(s). \tag{7.74}$$

By taking the Laplace transform of (7.65) one gets

$$Q(s)\mathcal{V}(s) = P_1(s)\mathcal{Y}(s) + P_2(s)\mathcal{U}(s), \tag{7.75}$$

which is an input-output relationship between the faults and the available data and, with the conditions $G^{-\ell}(s)G_\nu(s) = Q(s)$, $G^{-\ell}(s)G_u(s) = P_2(s)$ and $G^{-\ell}(s) = P_1(s)$, is equivalent with the representation (7.74). Based on the formalism of relation (7.75), the residual generation can now be characterized in a unified framework applying the following considerations.

The problem of residual generation in the parity space approach is to define residuals $\mathcal{R}$ to satisfy

$$\tilde{Q}(s)\mathcal{R}(s) = Q(s)\mathcal{V}(s) \tag{7.76}$$

so that the system defined by

$$\tilde{Q}(s)\mathcal{R}(s) = P_1(s)\mathcal{Y}(s) + P_2(s)\mathcal{U}(s) \tag{7.77}$$

is realizable and stable. The main concept of parity relation-based residual generator design is to select the matrix $\tilde{Q}(s)$, appropriately, leading to a variety of parity relations. Depending on the choice of $\tilde{Q}(s)$ the following basic cases can be identified.

1. The specific choice of
$$\mathcal{R}(s) = Q(s)\mathcal{V}(s)$$
($\tilde{Q}(s) = I$) leads to the classical analytical redundancy approach originated in (Chow and Willsky, 1984).

2. Other choices for $\tilde{Q}(s)$ relates the idea to the parity equation-based residual generation presented in (Gertler, 1998) which was considered in a discrete time setting.
3. If the system (1.21-1.22) is assumed left invertible and minimum phase *w.r.t.* the faults $\nu(t)$, then (7.75) defines the input-output relationship of the inverse system which corresponds to the choice $\mathcal{R}(s) = \mathcal{V}(s)$. The state space realization of this inverse is given by (7.20).

7.6. SUMMARY

In this chapter the fault detection and isolation problem in view of the fault reconstruction process by means of dynamic system inversion has been studied. In the presented setting, sensor noise and sensor fault detection problems were not considered. Along the discussion of this problem linear time invariant as well as input affine nonlinear systems with stable zero dynamics were considered. It was shown that the detector relying on the inverse representation of the original system reconstructs the failure modes at its output on the basis of standard input/output (and often also state variable) measurements.

The chapter was devoted to the exposition of the geometrical properties of the inverse and attempted to provide a better understanding of the conditions of the inversion procedure with special focus on the aspects of fault detection and isolation. A procedure for the construction of the inverse system based on the concept of invariant subspaces and, on the related coordinate transformations was given. It was shown that the solution methods obtained for nonlinear problems can be directly applied to the linear framework and the linear solutions can be viewed as special cases of the nonlinear ones.

The procedure resulted in a minimal dimensional inverse system supposed: (i) it is given in state space form, (ii) the representation has a relative degree and (iii) the representation is left invertible. The availability of state variable measurements (in certain cases the direct access to derivatives) is assumed. Considering the recent progress of advanced measurements technology and the wide availability of sensors capable to provide derivative of a measured variable — see *e.g.*, some applications in aviation technology, — this condition is not difficult to satisfy for small relative degrees. Unfortunately, this condition is seldom valid in the process control industry. Another problem is that the zero dynamics of the system should be stable (non-minimum phase).

In the last sections of the chapter the idea of dynamic inversion in light of the concept of feedback linearization of nonlinear systems has been shown, and an unexpected interesting relationship between parity space and system inversion-based methods has been revealed. The comparison of the two ideas, based on the most general concept of analytical redundancy *i.e.*, parity equations, was made in a rough technical approach. Though the close relation of the two approaches could be easily deduced, some further properties of the parity spaces generated by the two different ways, especially with respect to their geometric properties, is subject of future research.

CHAPTER 8

CASE-STUDY

TO DEMONSTRATE THE EFFECTIVENESS AND DISTINCTIVE CAPABILITIES of the direct input reconstruction (inversion) method to fault detection and isolation with relation to other residual generation approaches, in this chapter an application example is presented as a case-study. In this example, the F16XL aircraft monitoring problem is revisited that was originally considered in the papers (Douglas and Speyer, 1995) and (Chung and Speyer, 1998).

This application includes the detection and isolation of multiple simultaneous faults in the presence of external disturbances. An interesting feature of the presented problem, is that, because of some structural properties of the system, the decoupling of the faults from the disturbance effects is not possible by using conventional geometric design methods.

By relaxing the design requirements, however, posed by traditional detection filter ideas, we can determine new filter structures which admit to apply new solution ideas to a wider class of systems.

It is shown in the following sections that novel approaches to the problem may lead to new solution alternatives. It is demonstrated how the advanced methods of filtering such as inversion-based residual generation and H_∞ optimal filtering and the novel combination of them may contribute to the solution of earlier not solvable problems.

8.1. INTRODUCTION

For a brief introduction to the filter design problem that will be presented in the next sections consider the linear dynamical system which is subject to faults and also to external disturbances which is given in the state space form

$$\begin{aligned} \dot{x} &= Ax + B_u u + B_d d + Lf \\ y &= Cx + Du + Mf, \end{aligned} \tag{8.1}$$

where the system matrices A, B_u, C, D are in the appropriate dimensions. The arbitrary bounded time function $f(t)$ can represent both actuator and sensor faults which affect the system in the known directions in the state space as described by the matrices L, M, and $d(t) \in \mathcal{L}_2$ is the unknown but bounded disturbance function which affects the system in the state space direction B_d. The goal is to construct a possible *stable* and *minimal* representation of a residual generator in the state space, that generates $f(t)$ at its output in the presence of the persistent disturbance $d(t)$.

To conclude the work presented in this thesis, let us focus on the direct input reconstruction approach and consider this residual generation problem from view of the application of the idea of system inversion. From this perspective, the residual generator can be viewed as another dynamical system, which is expected in the state space form, (*cf.* with the formulation (5.3-5.4) in Chapter 5), as

$$\dot{\bar{x}} = \bar{A}\bar{x} + B_\mu \xi_u + B_y \xi_y,$$
$$f = \bar{C}\bar{x} + D_u \xi_u + D_y \xi_y,$$

where

$$\xi_y = [\dot{y}, \ddot{y}, \dots, y^{(k-1)}]^T, \qquad \xi_u = [u, \dot{u}, \dots, u^{(k)}]^T.$$

According to the idea of the procedure introduced for reconstructing the (unknown) inputs of the system by using the direct input reconstruction idea, we want to find the left inverse of the fault-to-output transfer function of the system such that the fault estimation error is diagonal.

Because of the presence of the disturbance in system (8.1), the solvability of the robust detection problem will now take a little different form (in contrast with the problem setup presented in Chapter 5 where the effects of disturbance were not considered), and the robust detection filter design problem can be formulated by using the solution alternatives that can be characterized by the following propositions.

PROPOSITION 8.1. (Exact decoupling of faults and disturbances by means of direct input reconstruction). Assume that the system subject to faults and disturbances is given as (8.1). Let our objective be the robust detection and estimation of the fault signals in the presence of the disturbances. The idea of the solution of this problem was presented in the previous chapters in various different ways. According to this method one has to invert the system for the fault $f(t)$ and also for the disturbance signal $d(t)$, thus attempting to achieve an *exact decoupling* of the faults from the disturbance effects. □

In case exact decoupling of the disturbance is not possible (*i.e.*, one cannot construct stable inverse for both $f(t)$ and $d(t)$), one can attempt to apply a filtering scheme which attenuates the effect of the disturbance on the output residual of the filter. This disturbance attenuation can be achieved in combination with the inversion method. For this solution approach consider the following proposition.

PROPOSITION 8.2. (Fault detection and isolation by means of exact fault decoupling with disturbance attenuation). Assume the system subject to faults and disturbances is given as (8.1). Assume, moreover, that there are more sensors than failures available, *i.e.*,

$$\dim y > \dim f. \tag{8.2}$$

In this case it is always possible to select and use a minimal set of output functions

$$\tilde{y} = \{ y_j \}|_{j \in \{1,\dots,m\}}, \quad \dim \tilde{y} < \dim y,$$

to obtain the inverse *w.r.t.* the elements of $f(t)$ which, from system (8.1), is obtained in the respective forms

$$f_i = C_{f_i}\bar{x} + B_{yf_i}\xi_y + B_{uf_i}\xi_u \tag{8.3}$$

for $i = 1, \ldots, k$. Substituting (8.3) to the state equations of (8.1) one obtains the inverse dynamics

$$\dot{\bar{x}} = \bar{A}\bar{x} + B_y\xi_y + B_u\xi_u + B_d d \tag{8.4}$$

that provides a useful structure for constructing a filter and generating detection residuals. Indeed, the fault detection residual of the filter can be calculated due to the output relations (8.3) assuming the state $\bar{x}$ of (8.4) is known.

Then, the following proposition is a corollary of the problem formulation above. Assume the system (8.1) is left invertible *w.r.t.* $f(t)$ and construct the inverse system resulting in the form (8.3-8.4), moreover, define the new output functions

$$\bar{y} = \bar{C}\bar{x}, \qquad \bar{y} = \{y_\ell\}|_{\ell\in\{1,\ldots,m\}, j\neq\ell}, \qquad \dim\bar{y} < \dim y \tag{8.5}$$

which, in accord with condition (8.2), are not utilized in the calculation of the inverse in (8.3), *i.e.,* the new observations $\bar{C}$ in (8.5) are composed of selected rows of C.

Consider the representation (8.3-8.4) equipped with the new output functions (8.5). Assume the pair $(\bar{A}, \bar{C})$ is observable. Then, a reduced order state observer can be designed to get an estimate of the unknown state $\bar{x}(t)$ in (8.4) by either: (i) designing an unknown input observer to get rid of the effect of $d(t)$ while estimating $\bar{x}(t)$ by $\hat{\bar{x}}(t)$ or (ii) applying an H_∞ filter to attenuate the effects of $d(t)$ on the output residual. Alternatively, if the pair $(\bar{A}, \bar{C})$ is found non-observable $\bar{y}(t)$ can be extended with one or more of the original measurements from the set $\{y_j\}$, attempting to construct an observable representation. □

PROPOSITION 8.3. (Optimal filtering). The classical solution of this filtering problem can be approached with using H_∞ optimal filtering making use of the design methodology presented in Chapter 4. □

In the next section the comparison of the solution approaches associated with Proposition 8.1, 8.2 and 8.3 is given and their effectiveness of the individual solution methods are demonstrated on the basis of a real application example.

8.2. THE F16XL AIRCRAFT MONITORING PROBLEM REVISITED

In this example, patterned after the filtering problem presented by (Douglas and Speyer, 1995) and (Chung and Speyer, 1998), we show a design example where the disturbance could not be decoupled from the sensor and actuator faults, using traditional (C, A)-invariant subspace design, and could not be attenuated neither while keeping fault effects decoupled. This problem considers the design of an aircraft fault detection filter that monitors an elevon actuator and a normal accelerometer sensor in the presence of a persistent wind gust disturbance.

Table 8.1. State variables of the system

$x_1 = u(t)$	longitudinal body axis velocity	ft/s
$x_2 = w(t)$	normal body axis velocity	ft/s
$x_3 = q(t)$	pitch rate	deg/s
$x_4 = \theta(t)$	pitch angle	deg
$x_5 = w_g(t)$	wind gust	ft/s

This is a common practice in aerospace technology that linearized models of the lateral and longitudinal dynamics for selected points along the nominal flight trajectory at both subsonic and supersonic speeds are used for controller design. These models include the linearized rigid body dynamics, the so called Dryden wind gust model and linear actuator response models with command rate limiting.

The Dryden gust model is a wind turbulence model recommended for study of vehicle response to winds for horizontally flying aircrafts with flight path angles less than 30 deg. To describe aircraft dynamics we use a model which is linearized about trimmed level flight at 10.000 ft altitude[6] (3048 m) and relative Mach speed 0.9 as presented in (Douglas and Speyer, 1995) and (Chung and Speyer, 1998). The reduced-order five-state model of the aircraft includes longitudinal dynamics only including a first-order wind gust model. The port and starboard elevons are modeled as a slaved system, no lateral dynamics and no elevon actuator dynamics are taken into consideration. This simplified aircraft model can be considered in the state space as

$$\begin{aligned} \dot{x} &= Ax + B_\omega \omega + B_\delta \delta, \\ y &= Cx + Dv, \end{aligned} \tag{8.6}$$

where the elevon deflection angle $\delta(t)$ is considered as input function and $\omega(t)$ and $\nu(t)$ are the wind gust disturbance and the sensor noise, respectively. The observables and state variables of the system contained in the system model (8.6) are summarized in Table 8.1 and 8.2.

[6] For the sake of technical faithfulness and, in order to be able to present the results in a more contrastable form, the units of measurement used by the original model data were kept and not converted to SI from the native U.S. system of measurement and, for the same reason, the notation system was retained without revision.

Table 8.2. Input/output variables of the system

$\delta(t)$	elevon deflection angle	deg
$c_1 = q(t)$	pitch rate	deg/s
$c_2 = \alpha(t)$	angle of attack	deg
$c_3 = A_z(t)$	normal acceleration	ft/s^2
$c_4 = A_x(t)$	longitudinal acceleration	ft/s^2

The parameters of system (8.6) are given by the matrices

$$A = \begin{bmatrix} -0.0674 & 0.0430 & -0.8886 & -0.5587 & 0.0430 \\ 0.0205 & -1.4666 & 16.5800 & -0.0299 & -1.4666 \\ 0.1377 & -1.6788 & -0.6819 & 0 & -1.6788 \\ 0 & 0 & 1.0000 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1.1948 \end{bmatrix},$$

$$B_\omega = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1.57 \end{bmatrix}, \quad B_\delta = \begin{bmatrix} -0.1672 \\ -1.5179 \\ -9.7842 \\ 0 \\ 0 \end{bmatrix},$$

$$C = \begin{bmatrix} 0 & 0 & 1.0000 & 0 & 0 \\ 0 & 0.0591 & 0 & 0 & 0.0591 \\ 0.0139 & 1.0517 & 0.1485 & -0.0299 & 0 \\ -0.0677 & 0.0431 & 0.0171 & 0 & 0 \end{bmatrix},$$

$$D = \begin{bmatrix} 0.01 & 0 & 0 & 0 \\ 0 & 0.143 & 0 & 0 \\ 0 & 0 & 0.245 & 0 \\ 0 & 0 & 0 & 0.245 \end{bmatrix}.$$

For notational convenience, let us introduce the matrix representation

$$A = \begin{bmatrix} A_1^T \\ A_2^T \\ \vdots \\ A_5^T \end{bmatrix}, \qquad C = \begin{bmatrix} c_1^T \\ c_2^T \\ c_3^T \\ c_4^T \end{bmatrix}$$

where A_i and c_i correspond to the i-th rows of the system matrices A and C, respectively.

Further on, carrying more practical considerations into the problem, the Dryden model (8.6) including wind gust disturbance is extended to include faults: a normal accelerometer sensor fault and an elevon fault as an actuator fault, denoted by $\mu_{A_z}(t)$ and $\nu_\delta(t)$, respectively. Let our objective be to design a filter for the detection and isolation of the faults in the presence of the $\omega(t)$ wind gust disturbance.

The accelerometer sensor fault $\mu_{A_z}(t)$ and the elevon fault $\nu_\delta(t)$ can be modeled as additive terms in the state and measurement equations as

$$\begin{aligned} \dot{x} &= Ax + B_\omega \omega + B_\delta(\delta + \nu_\delta), \\ y &= Cx + D\nu + E_{A_z}\mu_{A_z} \end{aligned} \tag{8.7}$$

Table 8.3. Faults and disturbances affecting the system

$\nu_\delta(t)$	elevon actuator fault	deg
$\mu_{A_z}(t)$	normal accelerator sensor fault	ft/sec^2
$\omega(t)$	wind gust disturbance	ft/sec

i.e., the elevon fault enter the system in the same direction of the state space as the input does, where $\mu_{A_z}(t), \nu_\delta(t)$ are arbitrary time-varying real scalars and, from the normal acceleration measurement

$$E_{A_z} = [0 \quad 0 \quad 1 \quad 0]^T.$$

It was (Beard, 1971) followed by (White and Speyer, 1987) and others, who showed that sensor faults appearing in the measurement equations can be modeled as two dimensional additive signals entering in the state dynamics of the system. The method is based on finding the input to the plant which drives the error state of the observer in the same way that μ_{A_z} will in (8.7). This is accomplished by a Goh transformation on the error space (Jacobson, 1971). Based on this idea, system (8.7) can be converted to the state space representation

$$\begin{aligned} \dot{x} &= Ax + F_{A_z} m_{A_z} + B_\delta(\delta + \nu_\delta) + B_\omega \omega, \\ y &= Cx \end{aligned} \tag{8.8}$$

where m_{A_z} is a fictitious signal representing the sensor fault effect assuming sensor noise is zero and the two dimensional fault $F_{A_z} = [F_{A_z^1} \;\; F_{A_z^2}]$ in (8.8) is equivalent to the fault E_{A_z} in (8.7) so that $F_{A_z^1}$ is the solution to $E_{A_z} = CF_{A_z^1}$ and $F_{A_z^2} = AF_{A_z^1} - \dot{F}_{A_z^1}$ (for details, see (Chung and Speyer, 1998)). When the system is time invariant, $\dot{F}_{A_z^1} = 0$ and thus

$$F_{A_z} = \begin{bmatrix} 0 & 0.9986 \\ 0 & 0.0534 \\ 0 & 0 \\ -33.444 & 0 \\ 0 & 0 \end{bmatrix}.$$

This kind of detection problem was discussed in the articles of (Douglas and Speyer, 1995) and (Chung and Speyer, 1998) in details. It has been shown how an unobservability subspace with respect to the wind gust is formed in this application, thereby decoupling the wind gust disturbance from the fault isolation residuals, happens to be nonmutually detectable with respect to the faults by using traditional geometric decoupling methods. Moreover, the faults and the wind direction combine to place a system transmission zero at 0.0002, forcing any fault detection filter design with these fault directions to have an unstable closed-loop pole. As a consequence, the application of traditional detection approaches such as the unknown input observer and other geometric decoupling methods are not possible.

For the possible solution of this robust detection and estimation problem consider the following solution alternatives.

8.2.1. *Disturbance attenuation with* H_∞ *filtering*

In the first approximation of the problem let us begin with the least ambitious assumption and consider the problem when we do not want to find a decoupling solution but only to achieve an optimal disturbance attenuation with respect to the fault signals in the filter residual.

PROBLEM 8.4. (Detection filter solution with optimal filtering). Assume that the system subject to faults and disturbances is given as (8.8). Let our objective be the robust detection and isolation of the fault signals in the presence of the disturbances with acceptable performance. Acceptable, in this example, means that the filter transmits the target faults and attenuates the disturbance so that the separation between the respective transmission levels is maintained making the detection and isolation of the faults robustly possible. The classical solution of this problem, that does not necessitate to make any consideration about separability of the faults and disturbances was given by Proposition 8.3.

The authors of the above references presented a game theoretic approach for the attenuation of the disturbance effects in (8.7) in an H_∞ sense. This solution approach closely follows the traditional H_∞ detection filter solution idea which is interpreted and the corresponding solution method to Problem 8.4 is reconstructed in the following.

It is not our intention, however, to reproduce the same algorithmic solution as that of the mentioned references. As an alternative, the raw H_∞ filter presented in Chapter 4 will be adopted for the solution of Problem 8.4 demonstrating that the two optimization approaches provide the very same results.

According to Proposition 8.3 we seek a residual generator with state $\hat{x}$ and state estimate $\hat{z}$ of the form

$$\begin{aligned} \dot{\hat{x}} &= A\hat{x} + K\big(C(x-\hat{x})\big) + F_{A_z}m_{A_z} + B_\delta(\delta+\nu_\delta) + B_\omega\omega, \\ \hat{y} &= C\hat{x}, \\ \hat{z} &= C_z\hat{x} \end{aligned} \tag{8.9}$$

where K is the feedback gain such that the effect of $\omega(t)$ on the filter innovation $C(x-\hat{x})$ is attenuated in sense of L_2-norm over a finite time interval by a fixed factor γ, (*cf.* Chapter 4). In effect, we want to minimize the transmission of the disturbance $\omega(t)$ *i.e.*, the magnification of the disturbance transfer function

$$T_{\varepsilon\omega}(s) = C_z(sI - A + KC)^{-1}B_\omega,$$

against the transmission of the other faults characterized by

$$\begin{aligned} T_{\varepsilon\nu_\delta}(s) &= C_z(sI - A + KC)^{-1}B_\delta \\ T_{\varepsilon m_{A_z}}(s) &= C_z(sI - A + KC)^{-1}F_{A_z}. \end{aligned}$$

The classical H_∞ filtering solution characterized by the transmission levels (fault and disturbance signal magnifications) of the filter for unity estimation weighing C_z is shown in Fig. 8.1/a.

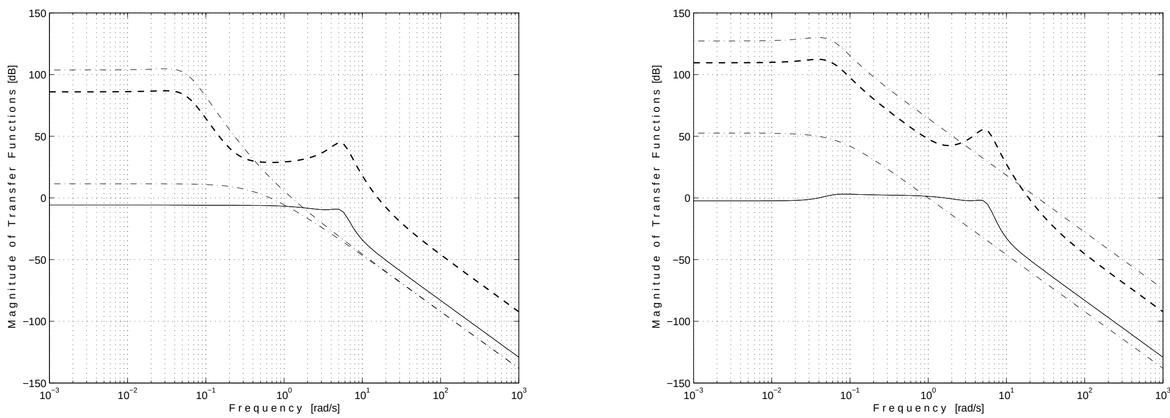

Figure 8.1. a) The magnitude (maximal singular values) of transfer functions $T_{\varepsilon\omega}$ (solid line), $T_{\varepsilon m_{A_z}}$ (dash-dot lines), and $T_{\varepsilon v}$ (bold dashed line) for unity estimation weighting C_z and b) special weighting C_z^*, respectively.

Following the same estimation technique as proposed in (Chung and Speyer, 1998) and selecting the estimation weight in the form

$$C_z^* = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 25 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

the maximal singular value plots of the transfer functions of the resulting filter, implemented by the filter gain

$$K = \begin{bmatrix} -0.0012 & -0.1289 & -0.0158 & 0.0037 & -0.0001 \\ 0.0222 & 2.1606 & 0.3447 & -0.0612 & 0.0023 \\ 0.0005 & 0.0549 & 0.0825 & -0.0016 & -0.0000 \\ 0.0013 & 0.1235 & 0.0158 & -0.0035 & 0.0001 \\ -0.0146 & -1.4276 & -0.2545 & 0.0405 & 0.0004 \end{bmatrix},$$

which evidences a minimum of the disturbance transmission level at $\gamma = 1.8669$ (-5 dB correspondingly), are shown in Fig. 8.1/b.

The results indicate that the H_∞ filter grants at least 60 dB of separation (signal-to-noise ratio, SNR) between the sensor fault and the disturbance effect and indicate almost SNR 120 dB for the elevon fault. It can be concluded that this sensitivity is usually enough to robustly detect both the elevon and the accelerometer faults in the practice, even in the presence of the disturbance. We note, however, that separation and identification of the two faults with using a single filter, by means of this solution, could be problematic. For the enhancement of fault selectivity of the filter, consider the following problem formulation.

8.2.2. *Inversion based fault decoupling with disturbance attenuation*

PROBLEM 8.5. (Fault decoupling with disturbance attenuation). We can attempt to decouple the effects of the faults μ_{A_z} and $\nu_\delta(t)$ from each other irrespectively of the presence of the disturbance signal $\omega(t)$ by using the idea of inversion-based direct input reconstruction. Note that in this case the residual, though is decoupled for the faults, is still corrupted by the disturbance. If this decoupling is possible disturbance attenuation can be used to suppress the effect of the disturbance on the fault residuals. For a possible solution of this problem, the idea presented in Proposition 8.2 can be used as it will be detailed in the next part.

For the realization of Proposition 8.2 consider the following concept. In the first step of this approach we want to invert the system for the fault signals $\mu_{A_z}(t)$ and $\nu_\delta(t)$ irrespectively of the disturbance $\omega(t)$.

STEP 1. (Fault decoupling with inversion *w.r.t* the faults signals). First, one need to select the measurements that can be used for the calculation of the particular fault signals by means of inversion. It is easily seen that the only measurement available for the determination of the accelerometer fault μ_{A_z} is the third output equation $y_3(t)$. Let us write, therefore,

$$y_3 = c_3^T x + \mu_{A_z} \tag{8.10}$$

from which the sensor fault can be expressed as the inverse *w.r.t.* μ_{A_z}, as

$$\mu_{A_z} = y_3 - c_3^T x. \tag{8.11}$$

As we *do not* want to use the derivatives of the disturbance function, let the inversion *w.r.t.* ν_δ use the derivative of the fourth equation (ω does not enter into this equation), letting

$$\dot{y}_4 = c_4^T \dot{x} = c_4^T A x + c_4^T B_\omega \omega + c_4^T B_\delta(\delta + \nu_\delta). \tag{8.12}$$

Since, it can be easily checked that, for this example $c_4^T B_\omega = 0$, for the actuator fault one obtains

$$\nu_\delta = \frac{1}{c_4^T B_\delta}\left(\dot{y}_4 - c_4^T A x\right) - \delta. \tag{8.13}$$

By substituting the new output functions (8.11) and (8.13) into the state equation (8.8) one obtains

$$\dot{x} = Ax - \frac{1}{c_4^T B_\delta} B_\delta c_4^T A x + \frac{1}{c_4^T B_\delta} B_\delta \dot{y}_4 + B_\omega \omega, \tag{8.14}$$

and, by using the definitions

$$\bar{A} = \left(I - \frac{1}{c_4^T B_\delta} B_\delta c_4^T\right) A, \quad \text{and} \quad \bar{B} = \frac{1}{c_4^T B_\delta},$$

one has the representation of the inverse system from (8.14) as

$$\begin{aligned} \dot{\bar{x}} &= \bar{A}\bar{x} + \bar{B} B_\delta \dot{y}_4 + B_\omega \omega \\ \bar{y} &= \bar{C}\bar{x} \end{aligned} \tag{8.15}$$

with

$$\bar{y} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}, \quad \text{and} \quad \bar{C} = \begin{bmatrix} c_1^T \\ c_2^T \end{bmatrix}$$

where the fault signals can be estimated in the form

$$f = \begin{bmatrix} \mu_{A_z} \\ \nu_\delta \end{bmatrix} = \begin{bmatrix} -c_3^T \\ \frac{-c_4^T A}{c_4^T B_\delta} \end{bmatrix} \bar{x} + \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{c_4^T B_\delta} \end{bmatrix} \begin{bmatrix} y_3 \\ \dot{y}_4 \end{bmatrix} \tag{8.16}$$

solely based on the measurement $y_3(t)$ (normal acceleration measurement) and the derivative of $y_4(t)$ (longitudinal acceleration measurement). It can be seen that the wind gust (state x_5) is effectively masked out in (8.16), however, it is coupled with the measurements (*e.g.*, with $y_4(t)$, *cf.* eq. (8.12)) which has the consequence that the disturbance effect will inevitably show up in (8.16).

It can be easily checked that the observability matrix of the pair $(\bar{C}, \bar{A})$ is full rank, *i.e.*, the system (8.15) is observable.

Now, one has basically two solution approaches to follow. As it was mentioned in the introduction, (i) one possibility is to design a classical *unknown input observer* for the system (8.15) for decoupling the effect of $\omega(t)$. If this decoupling is not possible or not desirable (ii) one can use H_∞ *optimal filtering* for attenuating the effect of $\omega(t)$. As the application of H_∞ filtering is potentially more flexible and more robust than other approximate detection filter design techniques, which tend to be based on geometric theory like unknown input observers, we show here how the H_∞ disturbance attenuation approach may contribute in finding the solution.

STEP 2. (H_∞ filter design to attenuate the effect of the disturbance on the fault decoupled residual). Consider the inverse system (8.15), with input directions $B_\Delta \triangleq \bar{B} B_\delta$ and $B_\kappa \triangleq B_\omega$, to be equivalent with the generalized representation (*cf.* system (4.7) in Chapter 4)

$$\begin{aligned} \dot{\bar{x}} &= \bar{A}\bar{x} + B_\Delta \nu_\Delta + B_\kappa \kappa \\ \bar{y} &= \bar{C}\bar{x}, \end{aligned} \tag{8.17}$$

in an attempt to design a state observer which gives an estimate $\hat{z}(t)$ of the weighted state vector

$$z = C_z \bar{x}. \tag{8.18}$$

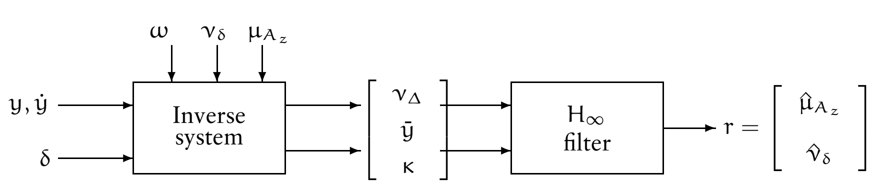

Figure 8.2. Inversion-based fault separation combined with the idea of optimal disturbance attenuation. The H_∞ filter, in fact, provides the estimation of the inverse dynamics.

The equivalence of systems (8.15) and (8.17) can be seen from the facts that (i) generating (8.17) from (8.15) by substituting the inverse equations (8.11) and (8.13), respectively, we effectively combined the faults $\mu_{A_z}(t)$ and $\nu_\delta(t)$ and control input $\delta(t)$ into a new input function $\nu_\Delta(t)$ while separating the disturbance input $\omega(t)$ in the same time, and (ii) $\omega(t)$ is the only disturbance affecting the system in the predetermined direction B_ω, therefore it can be viewed as worst-case disturbance $\kappa(t)$.

The concept of inversion-based fault separation combined with the idea of H_∞ disturbance attenuation is shown in Fig. 8.2. The inverse system, driven by the measurements $y(t), \dot{y}(t)$, and control input $\delta(t)$ provides the generalized inputs $\nu_\Delta(t), \bar{y}(t)$ and worst-case disturbance $\kappa(t) \triangleq \omega(t)$ for the H_∞ filter. The residual of the filter reconstructs (decouples and estimates) the faults in such a way that the disturbance effect is suppressed in the residual signal in H_∞ sense.

It can be interesting to realize that the resulting H_∞ filter which is given in the form

$$\begin{aligned} \dot{\hat{x}} &= (\bar{A} - K)\hat{x} + B_\delta \delta + Q\bar{C}^T \bar{y} \\ \hat{z} &= C_z \hat{x}, \end{aligned} \tag{8.19}$$

in fact, provides the estimation of the dynamics of the inverse system (8.17) where Q is the solution of the corresponding modified algebraic filter Riccati equation presented in Chapter 4 and the filter gain is calculated as $K = Q\bar{C}^T\bar{C}$.
Solving the filter optimization problem for system (8.17) one obtains

$$Q = \begin{bmatrix} 0.2974 & 0.4547 & 0.0306 & -0.0222 & -0.0267 \\ 0.4547 & 0.6974 & 0.0424 & -0.0337 & -0.0817 \\ 0.0306 & 0.0424 & 0.0142 & -0.0029 & 0.1004 \\ -0.0222 & -0.0337 & -0.0029 & 0.0635 & 0.0337 \\ -0.0267 & -0.0817 & 0.1004 & 0.0337 & 1.4688 \end{bmatrix}.$$

The transmission levels of the H_∞ filter (8.19) designed for the generalized system (8.17) are given in Fig. 8.3. In this plot, the magnitudes of the transfer functions $T_{\varepsilon\nu_\Delta}$ and $T_{\varepsilon\kappa}$ are displayed along the interested frequency range.

Note that the sensor fault appears in the measurements directly (see Eq. (8.10)) thus it has a direct feedthrough to the residual signal. The effect of this feedthrough can be seen in the figure: the ragged line at zero dB is the transmission of the sensor fault. This direct feedthrough of the accelerometer fault prevents the gradually reduced response at the upper ends of the working frequency range.

As it can be seen, the typical low frequency or steady-state transmission of the combined target fault exceeds the transmission of the disturbance by more than 115 dB in the DC range and still maintains a minimum of SNR 65 dB in the frequencies over 10 rad/s. This is an excellent sensitivity for the detection of the target faults even they are continuously corrupted by the wind gust disturbance.

This result is not characteristically better than that of obtained by the approach presented in Section 8.2.1 and in (Douglas and Speyer, 1995; Chung and Speyer, 1998), however, with

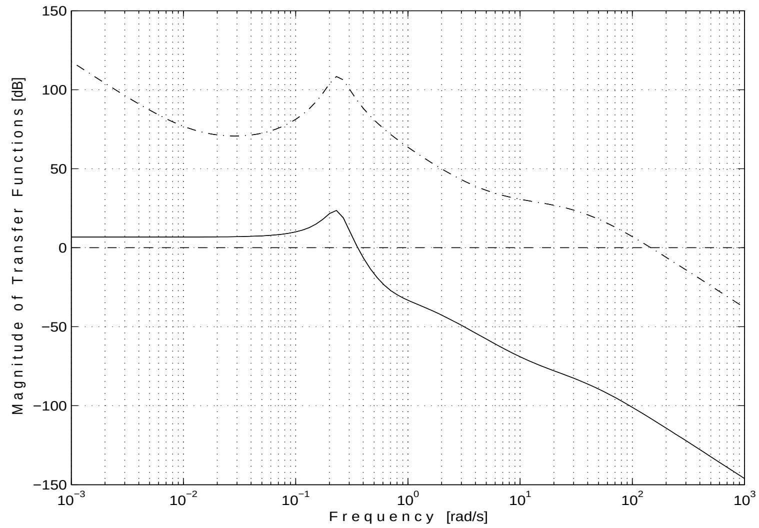

Figure 8.3. Singular value plots of the combined fault to output residual transfer function $T_{\varepsilon\nu_\Delta}$ *(dotted line) and wind gust disturbance to output residual transfer function* $T_{\varepsilon\kappa}$ *(solid line) of the detection filter derived by using standard* H_∞ *optimization technique with unity estimation weighting* C_z.

including a big difference. Namely, the faults to be detected now are subject to exact decoupling which show up in a single filter residual, separately from each other. Realizing the filter (8.19) with the filter gain

$$K = Q\bar{C}^T\bar{C} = \begin{bmatrix} 0 & 0.0015 & 0.0306 & 0 & 0.0015 \\ 0 & 0.0022 & 0.0424 & 0 & 0.0022 \\ 0 & 0.0005 & 0.0142 & 0 & 0.0005 \\ 0 & -0.0000 & -0.0029 & 0 & -0.0000 \\ 0 & 0.0048 & 0.1004 & 0 & 0.0048 \end{bmatrix},$$

a process simulation, constructed using the MATLAB 6 and Simulink 5 software tools, was used to evaluate the design and validate the fulfillment of the requirement specifications. The elevon and normal accelerometer faults applied to the system, modeled as step functions, are shown in Fig. 8.4.

The values for moderate and severe wind gust composed by a variable energy (smaller and higher, relative to the anticipated faults magnitude) 1 Hz quasi-sinusoid signal and a small energy 25 Hz random signal were used for this analysis to generate the winds referred to as *severe* and *moderate* gusts in the illustrations Fig. 8.5 and 8.7. The response of the filter to the elevon and normal accelerometer faults in the presence of the two different wind gust disturbance is shown in Fig. 8.6 and 8.8, respectively.

8.2.3. *Detection and estimation by means of direct input reconstruction*

PROBLEM 8.6. (Exact fault and disturbance decoupling with direct input reconstruction). We can attempt to decouple the effects of the faults μ_{A_z} and $\nu_\delta(t)$ and also $\omega(t)$ from each other.

For a possible solution of this problem, the idea presented in Proposition 8.1 can be used as it will be detailed in the next section.

By applying the same design considerations as in (8.10-8.13) in the previous section, the fault signals $\mu_{A_z}(t)$ and $\nu_\delta(t)$ can be calculated from the output functions $y_3(t)$ and $y_4(t)$, respectively, as

$$\mu_{A_z} = -c_3^T x + y_3, \tag{8.20}$$

$$\nu_\delta = -\frac{c_4^T A}{c_4^T B_\delta} x + \frac{1}{c_4^T B_\delta} \dot{y}_4 - \delta. \tag{8.21}$$

Now, in the construction of the disturbance function $\omega(t)$ one may choose between two solution opportunities: The representation can either be done based on the measurements $y_1(t)$ or $y_2(t)$ (and also on their derivative). Let us select the second measurement equation $y_2(t)$ and write

$$\begin{aligned} y_2 &= c_2^T x \\ \dot{y}_2 &= c_2^T \dot{x} = c_2^T \big(Ax + B_\omega \omega + B_\delta(\delta + \nu)\big). \end{aligned} \tag{8.22}$$

By knowing the identity relation from (8.21)

$$\delta + \nu_\delta \triangleq \left(\dot{y}_4 - c_4^T A x\right) \frac{1}{c_4^T B_\delta} \tag{8.23}$$

one can write for the disturbance by substituting (8.23) into (8.22)

$$\omega = -\left(\frac{c_2^T}{c_2^T B_w} - \frac{c_2^T B_\delta}{c_4^T B_\delta c_2^T B_w} c_4^T\right) Ax + \frac{1}{c_2^T B_w} \dot{y}_2 - \frac{c_2^T B_\delta}{c_4^T B_\delta c_2^T B_w} \dot{y}_4. \tag{8.24}$$

For a more compact system of notation let us introduce the identities

$$\vartheta_1 \triangleq \frac{1}{c_2^T B_\omega}, \quad \vartheta_2 \triangleq \frac{c_2^T B_\delta}{c_4^T B_\delta} \frac{1}{c_2^T B_\omega}, \quad \vartheta_3 \triangleq \frac{c_4^T B_\omega}{c_4^T B_\delta}, \quad \vartheta_4 \triangleq \frac{1}{c_4^T B_\delta} + \vartheta_3 \vartheta_2. \tag{8.25}$$

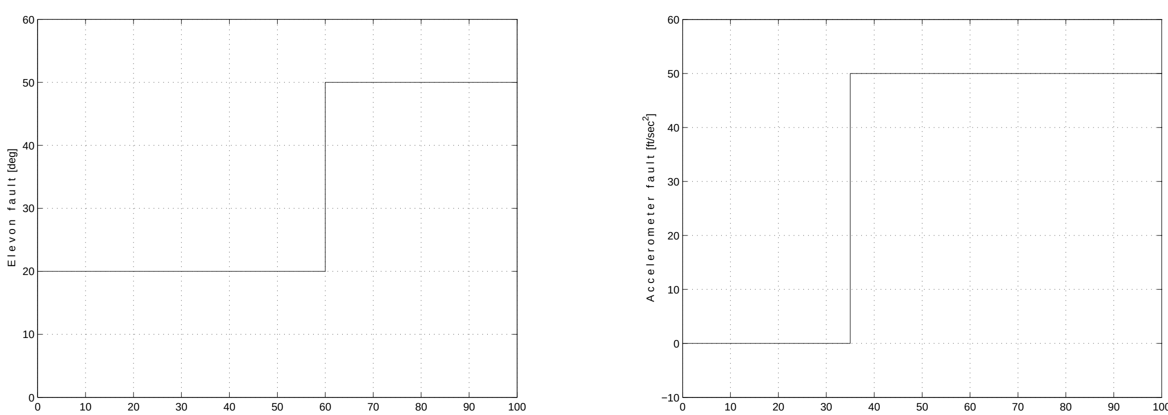

Figure 8.4. Elevon and normal accelerometer faults occurring at time $t = 60$ s *and* $t = 35$ s.

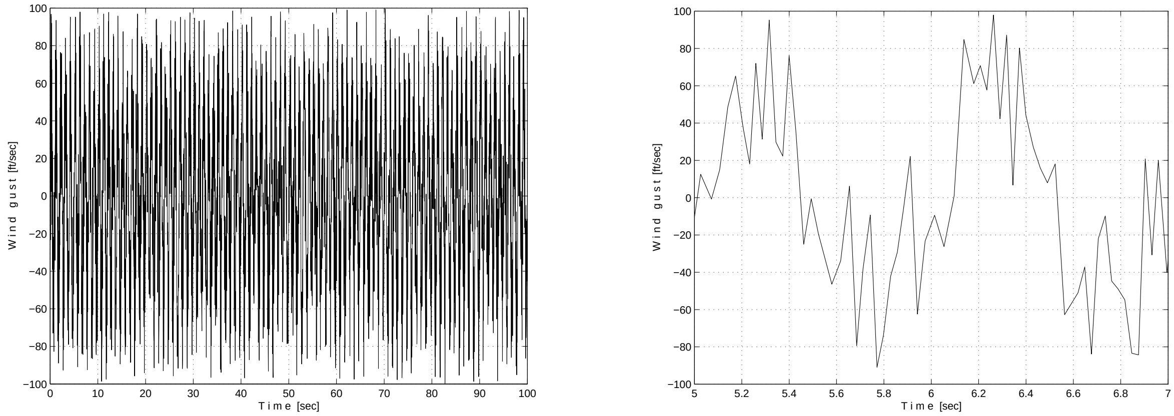

Figure 8.5. Moderate energy wind gust disturbance (see the zoomed time-scale for greater resolution).

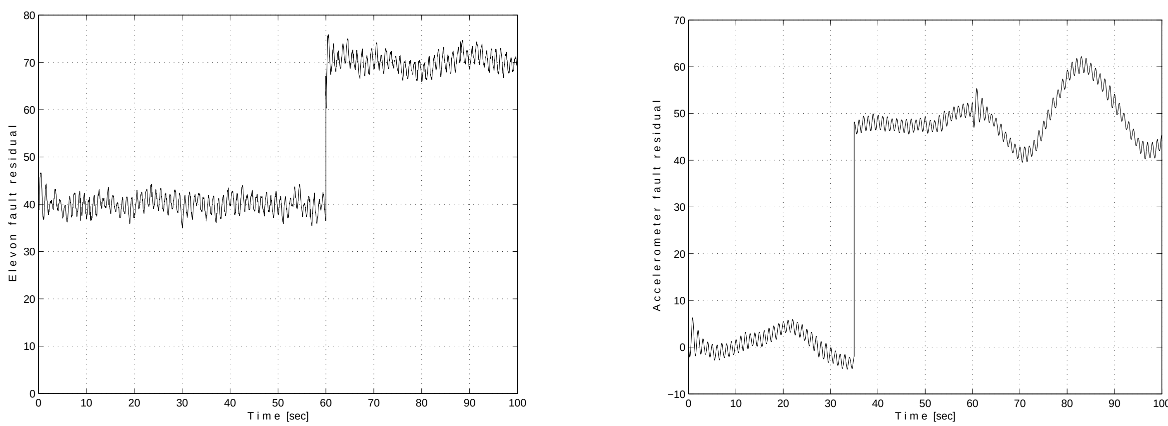

Figure 8.6. Elevon and normal accelerometer fault residuals in the presence of moderate energy wind gust disturbance. The fault effects are decoupled and the disturbance is attenuated in H_∞ *sense.*

By using (8.25) and substituting the new output functions (8.20), (8.21) and (8.24) into the state equation (8.7) one obtains the dynamics of the inverse system as

$$\dot{\bar{x}} = \bar{A}\bar{x} + \bar{B}\bar{u}, \tag{8.26}$$

where

$$\bar{A} = \Big(I + B_\omega\big(\vartheta_2 c_4^T - \vartheta_1 c_2^T\big) + B_\delta\big(\vartheta_3\vartheta_1 c_2^T - \vartheta_4 c_4^T\big)\Big)A, \tag{8.27}$$

$$\bar{B} = \begin{bmatrix} 0 & 0 & 0 & \vartheta_1 B_\omega - \vartheta_3\vartheta_1 B_\delta & -\vartheta_2 B_\omega + \vartheta_4 B_\delta \end{bmatrix}, \tag{8.28}$$

$$\bar{u} = \begin{bmatrix} \delta & y_3 & \dot{y}_2 & \dot{y}_4 \end{bmatrix}^T. \tag{8.29}$$

Since, for this case, the product $c_4^T B_\omega$ is obtained zero the identities (8.25) and the matrices (8.27-8.28) reduce to

$$\vartheta_1 \triangleq \frac{1}{c_2^T B_\omega}, \quad \vartheta_2 \triangleq \frac{c_2^T B_\delta}{c_4^T B_\delta}\frac{1}{c_2^T B_\omega}, \quad \vartheta_3 = 0, \quad \vartheta_4 \triangleq \frac{1}{c_4^T B_\delta}, \tag{8.30}$$

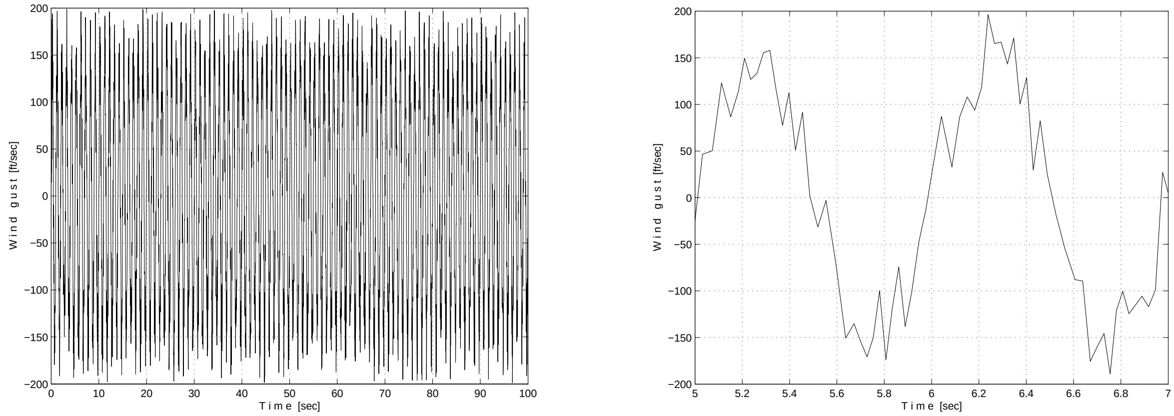

Figure 8.7. Severe wind gust disturbance (see the zoomed time-scale for greater resolution).

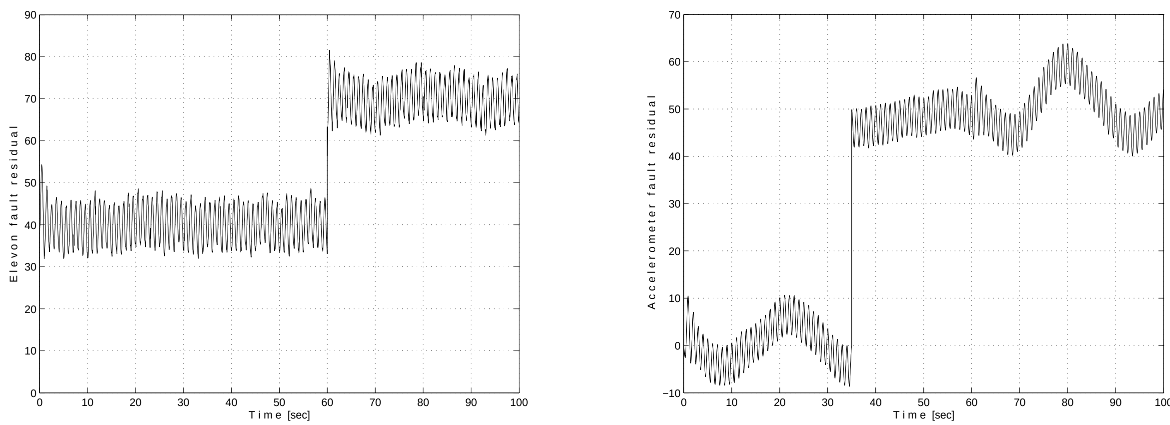

Figure 8.8. Elevon and normal accelerometer fault residuals in the presence of severe wind gust disturbance. The fault effects are decoupled and the disturbance is attenuated in H_∞ *sense.*

$$\bar{A} = \left(I + B_\omega \vartheta_2 c_4^T - B_\omega \vartheta_1 c_2^T - B_\delta \vartheta_4 c_4^T\right) A,$$

$$\bar{B} = \begin{bmatrix} 0 & 0 & 0 & \vartheta_1 B_\omega & -\vartheta_2 B_\omega + \vartheta_4 B_\delta \end{bmatrix}.$$

Using the original idea of Proposition 8.2, consider the inverse system attached with the measurement equation that was not utilized for inversion in the determination of the output functions $\nu_\delta(t), \mu_{A_z}(t), \omega(t)$, given by (8.20), (8.21) and (8.24), respectively, and write

$$\begin{aligned} \dot{\bar{x}} &= \bar{A}\bar{x} + \bar{B}\bar{u} \\ \bar{y} &= \bar{C}\bar{x} \end{aligned} \tag{8.31}$$

providing the residuals

$$\begin{bmatrix} \nu_\delta \\ \omega \\ \mu_{A_z} \end{bmatrix} = \begin{bmatrix} -\dfrac{c_4^T}{c_4^T B_\delta} \\ -\dfrac{c_2^T}{c_2^T B_w} - \dfrac{c_2^T B_\delta}{c_4^T B_\delta c_2^T B_w} c_4^T \\ -c_3^T \end{bmatrix} A\bar{x} + \begin{bmatrix} -1 & 0 & 0 & \dfrac{1}{c_4^T B_\delta} \\ 0 & 0 & \dfrac{1}{c_2^T B_\delta} & -\dfrac{c_2^T B_\delta}{c_4^T B_\delta c_2^T B_w} \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \delta \\ y_3 \\ \dot{y}_2 \\ \dot{y}_4 \end{bmatrix} \qquad (8.32)$$

where, $\bar{y} \triangleq y_1$ and $\bar{C} = c_1^T$ by definition. Based on the previous considerations, the following proposition is the extension of Proposition 8.2.

PROPOSITION 8.7. (Fault and disturbance decoupling by means of direct input reconstruction and estimation of the inverse dynamics). Consider the system subject to faults and disturbances as given in (8.1). Assume, as previously, that there are more measurements than failures available. In this case one can use a subset of measurements for the calculation of the output functions of the inverse system using the concept of Proposition 8.2. If the system (8.31), obtained for the inverse dynamics equipped with the measurements which were not included in the inversion, is found observable, then a reduced order state observer can be designed to get an estimate $\hat{\bar{x}}(t)$ of the state $\bar{x}(t)$ of the zero dynamics. Optionally, if the pair $(\bar{A}, \bar{C})$ is found non-observable, $\bar{y}(t)$ can be extended with one or more of the original measurements, attempting to construct an observable representation. Rendering $\hat{\bar{x}}(t)$ available, it can be used for the calculation of the inverse outputs $\nu_\delta(t), \mu_{A_z}(t)$ and $\omega(t)$ from (8.32). □

It can be easily checked that, for our example, the pair $(\bar{C}, \bar{A}) = (c_1^T, \bar{A})$ is non-observable. By appending the original measurements to $\bar{y}$ (all but not y_3), by constructing the output matrix $\bar{C} = [c_1^T\ c_2^T\ c_4^T]^T$, this newly constructed system is found observable that makes the observer design problem viable.

In the next step of the solution, let us construct this state observer for obtaining an estimate $\hat{\bar{x}}$ of the inverse state $\bar{x}$ in (8.31). The problem of finding a feedback matrix K of the observer, such that the closed loop filter gain $\bar{A} + K\bar{C}$ is stable and the observer satisfy some performance requirements (*i.e.,* it has the desired set of eigenvalues) is a standard problem of modern control theory which has a number of solution methods available.

With the application of the eigenvalue assignment approach, a stable filter with an acceptable transient behavior by posing the design requirement that the filter eigenvalues along the real axis are smaller than −0.5, the observer gain K (and its spectrum σ) is obtained as

$$K = \begin{bmatrix} 20840 & -162670 & -70140 \\ 32730 & -255510 & -110200 \\ 30 & 20 & 10 \\ -2670 & 20790 & 8970 \\ -32730 & 255490 & 110210 \end{bmatrix}, \qquad \sigma(K) = \begin{bmatrix} -1.3615 + 6.3015\,i \\ -1.3615 - 6.3015\,i \\ -1.0879 + 0.4244\,i \\ -1.0879 - 0.4244\,i \\ -0.7733 \end{bmatrix}.$$

The schematic diagram of the solution approach is depicted in Fig. 8.9. The results of a continuous time process simulation assuming the simultaneous occurrence of an elevon fault and a normal accelerometer fault in the presence of severe wind gust disturbance (with the same timing and disturbance signal characteristics as shown in Figs. 8.4 and 8.7, respectively), are presented in Fig. 8.10. The corresponding derivative measurements are shown in Fig. 8.11.

The result evidences that the filter gives the estimates of the two fault signals ν_δ, μ_{A_z}, and represents the faults in the residual signal *separately*, disregarding the disturbance effect ω which, in this case, shows up in an independent residual direction. Our preliminary investigations revealed that this kind of decoupling solution which uses a standard (non-robust) state observer is particularly sensitive to the noise affecting the derivative measurements. Even a small energy derivative noise may corrupt the state estimation resulting in a useless residual signal as it is clearly shown on Fig. 8.12. Further research on making the idea robust *w.r.t.* uncertainty of the derivative measurements is required.

8.3. SUMMARY

This chapter contains reproducible numerical data and application related process simulations for supporting the theoretical results presented in this thesis and depicts the work that was performed to assess and demonstrate the applicability and effectiveness (both strengths and weaknesses) of our achievements. For this purpose, a real application example taken from the literature has been investigated as a case-study.

This application (the F16XL airplane failure monitoring problem originally published by (Douglas and Speyer, 1995)) included the detection and isolation of two simultaneous faults in the presence of persistent wind gust disturbances when the decoupling of the faults from the disturbance effects was not possible by using conventional filter design methods.

It was effectively demonstrated that, by relaxing the design requirements posed by traditional detection filter ideas, new filter structures could be determined admitting to apply new solution ideas to a wider class of systems. It was shown how novel approaches of the detection problems may lead to new solution alternatives demonstrating that advanced methods of filtering such as the idea of system inversion-based residual generation and H_∞ optimal filter-

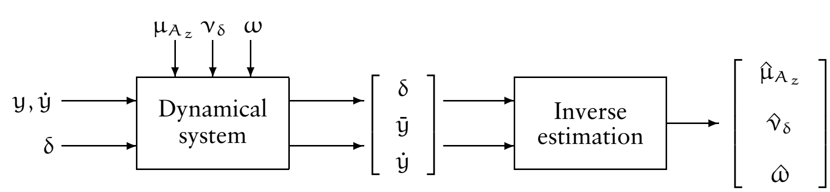

Figure 8.9. Robust detection and estimation by means of exact decoupling of faults and disturbance effects with on-line estimation of the inverse dynamics. The state observer provides the estimate of the inverse based on the knowledge of the control input δ and the selected set of measurements y, $\dot{y}$.

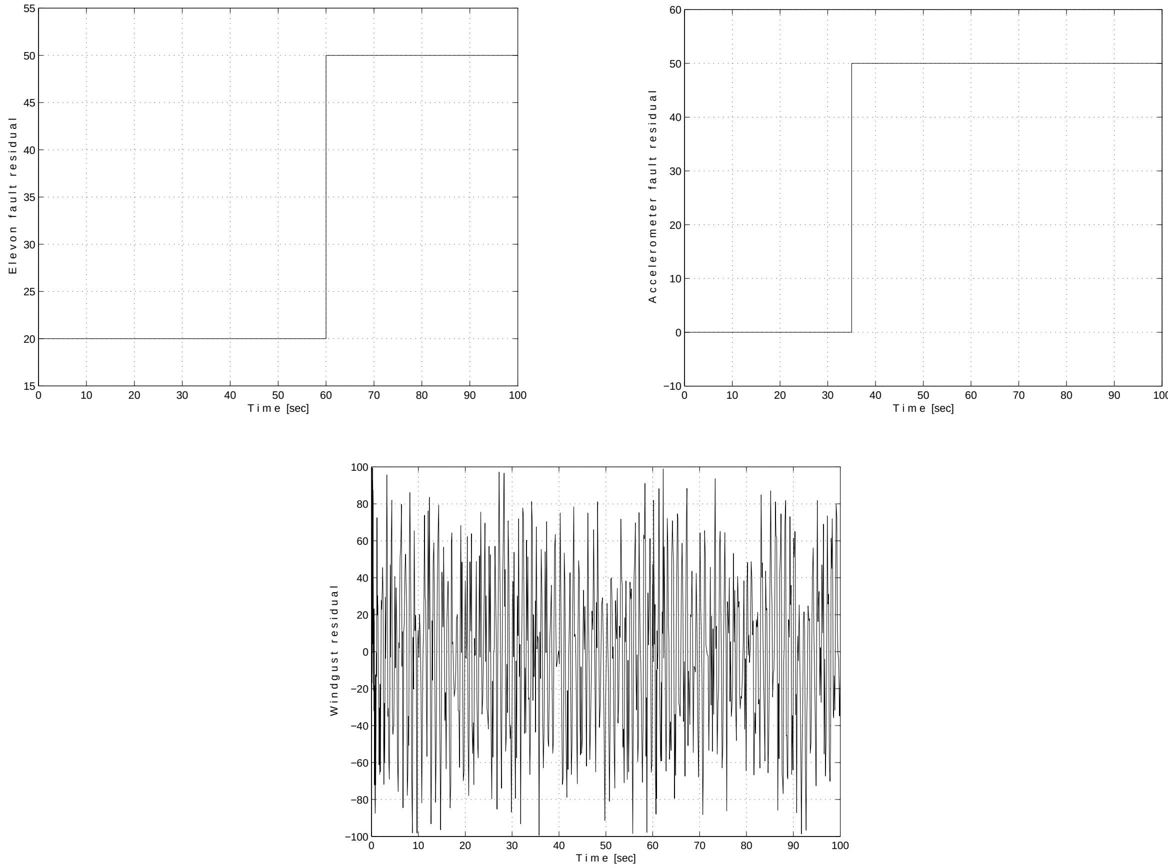

Figure 8.10. Residual components of the filter driven by the elevon faults $\nu_\delta(t)$, *normal accelerometer faults* $\mu_{A_z}(t)$, *and windgust* $\omega(t)$ *disturbance.*

ing, (moreover, the novel combination of them), may contribute to the solution of *earlier not solvable* problems.

The robust estimation problem was approached in a series of different solution concepts. These concepts included the use of the classical H_∞ optimal filtering (whose applicability was demonstrated just for reference purposes here), as well as the new idea of exact fault decoupling with H_∞ disturbance attenuation; a key part of which has been the application of the inversion-based direct input reconstruction method.

Emerging from this novel idea, as one of the most important advancements that can be made with this approach, a new concept for the determination of the inverse dynamics by using state estimation techniques has been established. It was shown that, under some specific conditions, the state estimate of the inverse could be obtained by using conventional Luenberger type state observers as well as H_∞ optimal filters. Process simulations, constructed using the MATLAB 6 and Simulink 5 software tools, were used to evaluate the design and validate the results.

The solution method is viewed a useful extension of the theoretical methods presented in the previous chapters for the calculation of the zero dynamics. It confirms viability and practical

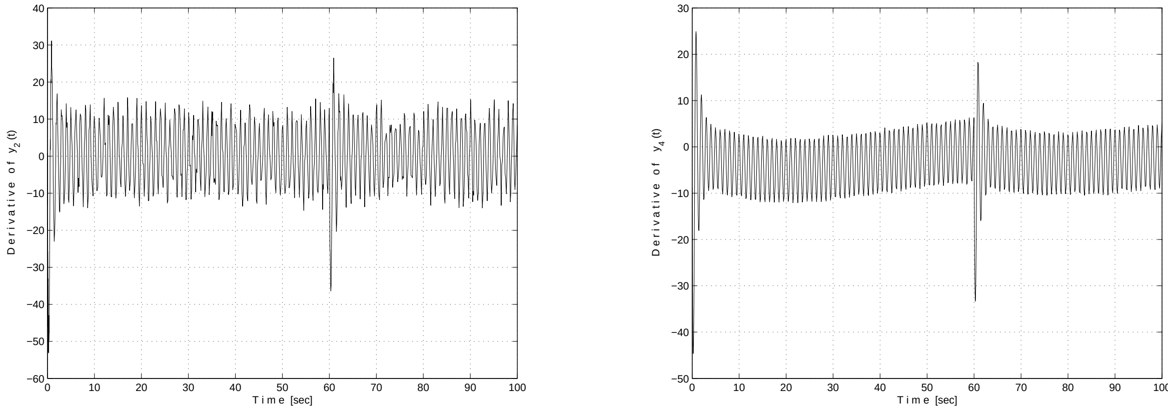

Figure 8.11. Derivative measurements of the output signals $y_2(t)$ *and* $y_4(t)$ *disregarding any measurement uncertainty.*

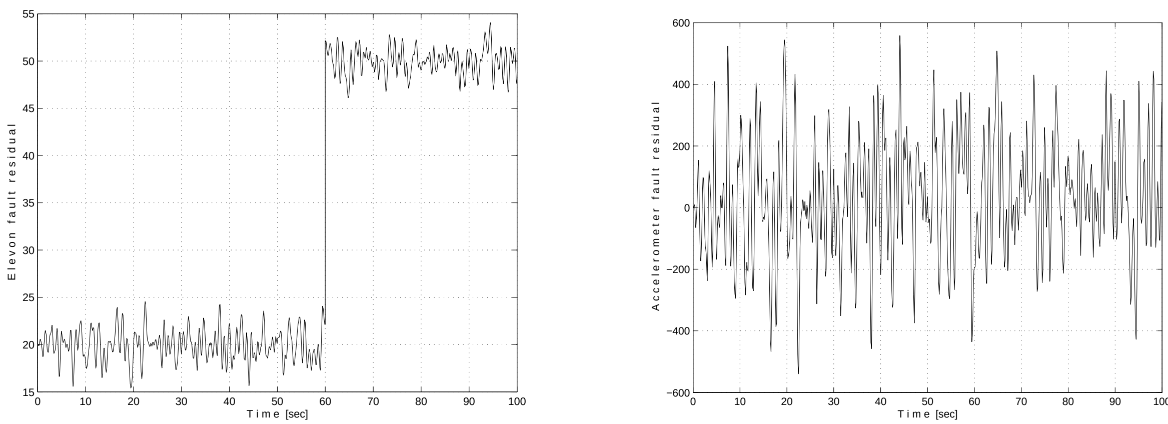

Figure 8.12. Detection residuals of the filter triggered by the elevon fault $\nu_\delta(t)$ *and normal accelerometer fault* $\mu_{A_z}(t)$*, assuming the derivative measurements* $\dot{y}_2(t)$ *and* $\dot{y}_4(t)$ *are corrupted by random noise with* $1/100$ *maximum signal amplitude relative to the expected magnitude of failure signatures.*

applicability of the novel idea for exact decoupling of faults and disturbances by means of direct input reconstruction.

The proposition gives new perspectives on the application of inversion-based fault detection filter design which, in certain applications, nicely complements the existing approaches and compares favorably with other decoupling solution methods.

PART 4

CONCLUSIONS AND REFERENCES

By first intention, the work presented in this thesis is a short overview on the analytical model-based methods applied to fault detection and isolation in contemporary engineering practice. A comparative summary of methods which led researchers from traditional state estimation to direct input reconstruction techniques is given showing the interesting analogies and congruences between the individual approaches. By second intention, it provides a brief summary of the results achieved by the author in this research area in the past ten years demonstrating the usefulness of some novel ideas. The thesis was attempted to present an engineering style of a 'design-based' research methodology of detection filters to guide the reader towards a better understanding of the types of mechanisms that render the concerns of the inversion-based (or by its new moniker, the direct input reconstruction) detection and estimation methods. Although each chapter have written as a more or less self-contained module, the earlier chapters do provide useful background material for the concepts presented later. We would therefore recommend that at least some of this earlier material is read or at least reviewed before launching into the heart of the thesis described in Part 3: 'Direct Input Reconstruction for Fault Detection'. However, it is also recognized that the volume is not a homogenous entity, though it was intended to be so - (the paper confronts geometric and algebraic approaches, stochastic and deterministic ideas which might seem completely different for some readers) different readers will decide on the relevance of each chapter according to their own particular interests. In this last chapter, we attempt to make some integrating, concluding comments with respect to the background and applicability of the presented ideas. We also describe some of the possible directions in which we hope our work will be extended in the future. The list of references and a list of collected articles published by the author in the subjected area of research conclude the volume.

CHAPTER 9

CONCLUSIONS

RELIABILITY, SECURITY AND SAFETY ARE SYNONYMOUS QUALITIES which have emerged as high priority in many countries, with significant technological, social and economic implications. It has been a subject of high relevance to research, with responsibility and probable relation to many industrial projects and applications creating the category of high reliability and dependable systems. Nevertheless, the position of the application field is a bit contradictory, particularly if one considers the huge research efforts invested world-wide in the area during the past twenty years, in view of the current spread of technology in the industrial practice.

Active and passive methods are commonly available for enhancing the measures of reliability, security and safety. While passive methods are basically related to the specification, design and implementation periods of product life-cycle (such as application of the concepts of fail-safe design, ensuring compliance with standards and product recommendations etc.), active methods are associated with the detection, elimination and removal of faults and directly related to concepts referred to as fault tolerance.

A common characteristics of the utilization of any kind of safety enhancement methods is that safety requires expenditures in a highly nonlinear manner: safety costs a lot, and a little more safety requires even more expenses, see Fig. 9.1 for illustration. Specification of the safety levels of a system is a very cost sensitive economic category which necessitates considerable engineering care. Wrong product specification, with under- or over-specified safety features, may risk product success.

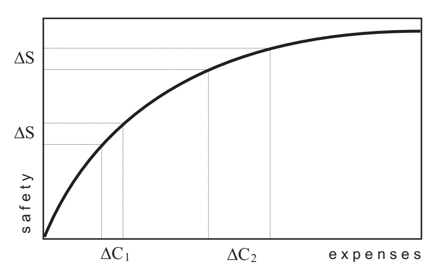

Figure 9.1. Typical characteristics of safety enhancement technologies in terms of total product costs.

In a traditional view, fault detection and diagnostics are considered as separate system functions, which can be implemented and incorporated in product functionality, independently of the original product specifications at any time during the active life-cycle of the product as an *'add-on'* function. This approach has become increasingly inconsistent with the price/value/function-oriented view of modern marketing and product management.

As long as the inclusion of fault detection and diagnostic methods in product functionality has visible costs, the direct economic advantage derived from the application of these methods is not so apparent. Detection and diagnostics is not part of primary system functions; obviously, a well-engineered system remains functional without any detection or diagnostics module included in the operation (at least for the duration of a never defined period of time). Therefore, on the one hand, the customer (consumer) is not well-motivated in the investment (purchase) of product functions by which no direct material advantage can be obtained, and, on the other hand, the supplier is not interested in the implementation of functions which necessarily lead to price increase without effectively motivating the customer for purchase: Selling safety as an *'add-on'* function in economic structures attempting to maximize profit is not an easy task.

Many sources attempt to explain this contradictory situation by bringing up statistical data showing that most of fatal system failures are due to human nonperformance and not to technical or technological cause.[7] As a consequence of this, most efforts made to safety enhancement lately have been connected to the application of passive methods; companies placed a bet on the training of operators and on improving the quality of the production and conformity of the design etc.

9.1. From stand-alone methods to embedded applications

Changing product definition. The recent alteration of the notion of the *product*, however, has created a completely new situation quite lately. There appeared an entire new class of products (both in consumer and industrial categories) sharing the common characteristics which is principally based on the application of high performance computers. In an ever increasing scale, compliance with requirement specifications (price, performance criterions) can be satisfied only by means of the use of *silicium-based* solutions.

The functionality of these products relies heavily on algorithmic computations and data processing which activity is embedded in the implementation. The literature refers to these systems as Software Enabled (Control) Systems in which the functionality implemented in the software is no longer an added feature but, literally speaking, the software (firmware, middleware) *makes* the functionality of the system. The methods applied to data processing, the way how sensor data is distributed and shared, the local *intelligence* of the components and also their interoperation, moreover, the extensive communication infrastructure characterize this technology.

Because of the very characteristics of this technology, these systems necessarily contain components of relatively low reliability: Software-based implementations are liable to sudden

[7] According to the result of air accident case analysis reported by Boeing, more than 80 % of airplane accidents are attributable to the erroneous behavior of the pilots or the air control personnel but not the malfunctioning of the system.

malfunctions. As full-scale testability of computer programs is a debated subject, the application of computers and software-based solutions in safety critical systems such as in nuclear applications, vehicle and aviation technology is still subject to intense dispute in various engineering forums. Very often, the only possibility that makes the fulfillment of safety and security requirements possible and may guarantee an acceptably long duration of reliable operation is the integration (embedding) of active safety enhancement methods in normal systems functionality. Fault detection and diagnostics will be no longer an *'add-on'* but an *'in-design'* product feature. Adopting this idea, one of the main objectives of the design is to create engineering structures in which failures can be detected and removed from the systems architecture quickly and reliably in such a way that system functionality should be continuously maintained over time.

As a result, dependable system technology on the one hand, is continuously widening (solution alternatives will be provided for new, formerly not viable applications). On the other hand, technology is drifting from the traditional fields of industry (nuclear, chemical and aviation technology etc.) to new application domains (consumer products, small applications) and the former *'add-on'* character is visibly being replaced with the *'embedded'* one.

As another new trend, system developments are pushed by consumer needs in a much more characteristic way than anytime earlier. The new products are designed to meet consumer needs, increasing product efficiency, reliability, comfort and safety demands. A typical example of this may be brought from the field of advanced automotive systems: with the advent of X-By-Wire, a concept that replaces most hydraulic and mechanical systems in an automobile with software and electronics, the realizability threshold of design ideas in terms of cost, weight and other traditional engineering considerations is shrinking rapidly resulting in the increase of overall system complexity.

This development is placing dependable system applications to another view and turning fault detection technology into another application – a cheaper and more widely available one putting the concept of high dependability to mass production. Some of the main features of this transition is characterized in Table 9.4.

Table 9.4. The drift of the development of dependable technologies

Add-on character	⟶	Embedded character
Safety view-point	⟶	Reliability view-point
Large-scale applications	⟶	Small-scale domains
Industrial systems	⟶	Consumer products
Individual production	⟶	Mass production

Along this development, the way how fault detection and diagnostic methods are built in to large-scale systems is transforming too. For reasons of tractability and modular design, it is a common practice to partition systems into components based on the functionality they provide. Typically, the components are individually designed and optimized and system interactions may be beneficially exploited to improve overall safety and reliability measures in a plant-wide manner. The emerging MEMS[8] technology is a representative example of this general tendency. Simply put, large-scale systems are increasingly partitioned into smaller applications doing extensive communications over redundant networks which the system relies on for data exchange; the plant-wide networks that link local controllers, MEMS sensors and actuators *etc.* together.

A demonstrative example of the above can be cited from the car making industry again. The structure of advanced automotive electronic systems architectures can be related to large-scale systems in many sense. In a structure like this, one can blend the system together electronically, so the steering system, brake, suspension and engine control all communicate with each other as shown in Fig. 9.2, using the emerging new communications standard FlexRay.

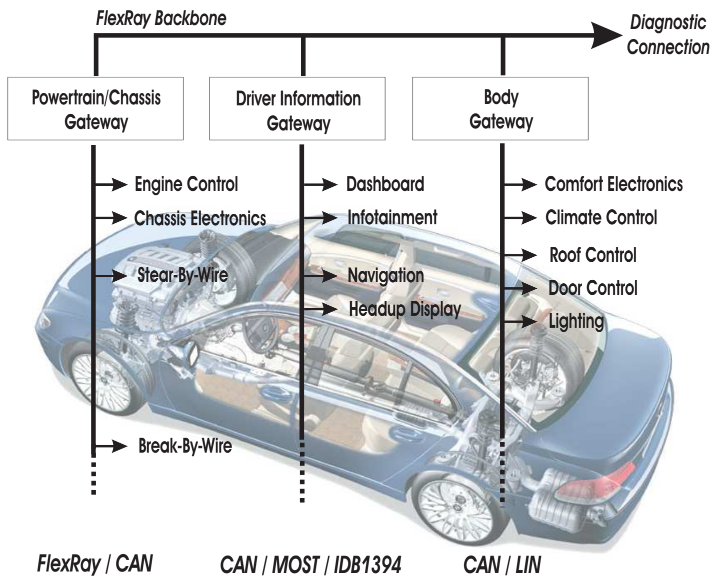

Figure 9.2. FlexRay, the next generation of fault tolerant network concept utilizing redundant CAN subnetworks in advanced X-By-Wire vehicle architectures. FlexRay is a scalable, flexible high-speed communication system that meets the challenges of growing safety-relevant requirements in the automobile by providing both single and multichannel redundancy. The system operation is organized around the distributed structure of ECUs (Electronic Control Units) or MCUs (Multiple Microcontroller Units) which keep track with the control and local supervision of the main functional units of the car such as the engine, brake, steering etc. The fault tolerant network utilizing redundant CAN subnetworks offers significantly enhanced features related to traditional solutions. These include a higher data transfer rate (10-100 Mbit/s as opposed to conventional 1 Mbit/s) plus redundant communication channels and predictable latency. This makes the concept suitable for vehicle functions where extremely high levels of performance, real-time response, and exceptional reliability are required. Inclusion of fault detection and diagnostic functions is inherent in the structure.

[8] Micro-Electro-Mechanical Systems (MEMS) is the integration of mechanical elements, sensors, actuators, and electronics on a common silicon substrate through microfabrication technology. While the electronics are fabricated using integrated circuit (IC) process sequences (e.g., CMOS, Bipolar, or BICMOS processes), the micromechanical components are fabricated by using compatible *micromachining* processes that selectively etch away parts of the silicon wafer or add new structural layers to form the mechanical and electromechanical devices. MEMS is an enabling technology allowing the development of smart products, augmenting the computational ability of microelectronics with the perception and control capabilities of (micro)sensors and (micro)actuators.

In this extensive communication structure the operation of the main functional components of the vehicle is closely linked, components interact with each other (breaking may affect steering as well as power and chassis control and the other ways around), they influence the operation of one-another, as it was *e.g.*, a typical assumption in the case of traditional large-scale systems.

Realizability of fault detection and isolation on embedded platforms. The fault detection and diagnosis methods these system architectures aim to integrate must be designed in accord with the principles of partitioning of the system: elementary fault detection jobs are implemented as parts of the device functionality of the main functional units in the above characterized embedded manner. Then, global fault-tolerance and diagnostics are realized by collecting all the relevant information, provided by these elementary jobs, over the communication networks.

In view of the above, embedded software faces some unique constraints not found in conventional systems such as limited resources (limitations in memory and processing power) and real-time requirements (interaction with the environment is to happen within specific time constraints, the computation is to be performed cyclically within a pre-determined duration of time) just to mention a few of the most severe criterions. There are numerous other variables that must be clearly understood and mastered by the designer in order to achieve the best results and satisfy the design requirements. This necessitates the use of clever modeling and algorithm design ideas to fault detection.

One of the challenges faced by researchers of advanced methods in fault detection is, therefore, in the construction of computationally efficient theories and detection algorithms which can be ported to embedded platforms. All this favors solutions supporting modularity: partitioning became a key concept which affect not only on the implementation but on modeling and algorithm design as well.

The theory of design and implementation of well-structured and efficiency-optimized software for dependable embedded use is broad and increasing and the concept of component-based technology (in common term *componentware*[9]) is generally applied in the design and realization of such systems.

The journey across methods spanning many different fields from state estimation to direct input reconstruction in this thesis presented a broad methodology of the design of system components providing analytical algorithmic methods for fault detection and fault signal estimation. The review tried to outline a common theoretical framework within which the similarities and differences, some congruences and parallelism of the individual approaches could be identified. Though, (embedded) implementation was not a central point in the discussion, this helped to build a common understanding of the different attributes of various representations (linear and nonlinear, deterministic and stochastic), and also their roles, and their relationships upon which the selection of the right solution alternative as well as the implementation of the

[9] Software designed to work as a (embedded) component of a larger application considering partitioning of tasks using standard communication interfaces between components that makes the mixing of inhomogeneous hardware and software components from different manufacturers in a single system possible.

specific methods should be based. It was always indicated, however, how particular filtering methods can be part of global solutions, such as *e.g.*,, when it was shown how specific filter banks can be constructed, involving a set of detection filters of limited scope, in an attempt to eliminate the narrowness of the solutions.

The solution and presentation of the discussed detection problems have always been inspired (sometimes constrained), by real engineering considerations. The filter design methods proposed in this work are universal, their usability is not confined to particular application fields, as much as the model (usually a deterministic state space representation) of the system is always thought to be readily available. In the vast majority of the problems, this condition is not considered restrictive; the state space representation of the system can be constructed through system identification. We enforced a few design considerations, however, which pose some restrictions particularly in the case of the proposed, new, direct fault reconstruction methods. These design restrictions necessitate a careful use of the ideas and represent a clear set of problems, subject to further research. These are summarized very shortly in the following section.

9.2. KNOWN ISSUES

STABILITY OF THE INVERSE. The stability of the system (alternatively the stability of the zero dynamics) has always been assumed. Our inversion methods, in their present forms, could not guarantee the construction of a stable inverse for nonminimal-phase systems. In the next stages of research we will be interested in the stable inversion process and its dependence on parameters of the system. This problem is of particular importance in view of uncertainty handling in nonlinear systems. Assuming that the relative degree of the system does not change as parameters vary, the continuous dependence of the stable inversion process can be studied as it was already demonstrated *e.g.*, in (Hunt et al., 1997) under appropriate assumptions.

NOISE SENSITIVITY OF THE APPLICATION OF DERIVATIVES. The pros and cons of exploitation of derivatives and some related issues of noise sensitivity have been discussed in a case-study in Chapter 8. Two fundamental types of noise can be considered: noise produced during the calculation of the derivative and noise produced during the sensing and transmission of signals. In light of the rapid instrumental and sensor development witnessed in the past decade, the application of direct derivative measurements, one the one hand, became a realistic engineering concept: the computationally costly and noise sensitive calculations of the derivatives can be accomplished more efficiently if these derivatives are determined (measured) directly. These new types of sensors letting direct access to derivatives of a signal alleviate the first type of noise but the excessive sensitivity to disturbances when using derivative signals corrupted by measurement and transmission noise still exist (see the related investigation in Chapter 8), which requires extensive further study. H_∞-filtering seems to be a viable solution alternative in the enhancement of robustness with noise in derivative measurements.

To evaluate the usefulness of the proposed new methods MATLAB and Simulink-based computer simulations were constructed and the results analyzed. The benefits and practical potential of these ideas were best illustrated in the case-study in Chapter 8.

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