# Reply to comment on "Improvements for drift-diffusion plasma fluid models with explicit time integration"

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**Abstract.** This is a reply to the comment of Jiayong Zou on the paper "Improvements for drift-diffusion plasma fluid models with explicit time integration". The criticism in the comment, namely that the current-limited approach is inconsistent with the underlying partial different equations, seems to be invalid. However, this criticism raises an interesting question about the behavior of the current-limited scheme for a given time step, which is discussed in this reply.

## 1. Reply to comment

The comment by J. Zou [1] raises concerns about the consistency of the current-limited scheme described in [2]. A numerical scheme is *consistent* when numerical solutions converge to the true solution of the underlying partial differential equation (PDE) when the mesh spacing $\Delta x$ and time step $\Delta t$ go to zero. A similar definition is also given in the comment.

The main criticism of the comment is that the scheme proposed in [2] is not consistent. However, there seems to be a flaw in the reasoning: the case $\Delta x \to 0$ is considered without taking into account $\Delta t \to 0$. According to equation (18) of [2], the electron flux is only limited when its amplitude exceeds

$$f_{\max} = \varepsilon_0 E^* / (e\Delta t), \tag{1}$$

where $\varepsilon_0$ is the permittivity of vacuum, $e$ the elementary charge and $E^*$ the amplitude of the electric field with a correction for diffusive terms, see equation (19) of [2]. For $\Delta t \to 0$, we have $f_{\max} \to \infty$, so the electron flux will not be limited or otherwise modified. The proposed scheme is therefore consistent with the underlying PDE.

## 2. Behavior of current-limited scheme

Regardless of whether the proposed scheme is consistent, the comment does raise an interesting question: how does the current-limited scheme behave for a given time step $\Delta t$? To analyze the scheme, let us consider a simple homogeneous plasma, with the following assumptions:

- The electron density $n_e$ is homogeneous in space and time
- The electron mobility $\mu_e$ is constant
- The conductivity only comes from electrons
- There is no space charge, and the electric field $E$ in the plasma is homogeneous

Physically, the electron flux will then be given by

$$f = -n_e \mu_e E, \tag{2}$$

and the dielectric relaxation time is given by

$$\tau = \frac{\varepsilon_0}{e \mu_e n_e}. \tag{3}$$

A standard numerical discretization will give the same flux as equation (2). A semi-implicit scheme [3, 4] will also reproduce this flux, since the 'correction' term will be zero, see equation (15) in [2]. Let us now consider the current-limited approach, for which $f_{\max}$ is here given by

$$f_{\max} = \varepsilon_0 |E| / (e\Delta t). \tag{4}$$

With a time step $\Delta t \leq \tau$ we will have $|f| \leq f_{\max}$, so no limiting will take place and the correct flux is obtained. For $\Delta t > \tau$, the current-limited scheme will reduce $|f|$ so that it does not exceed $f_{\max}$, effectively reducing the conductivity of the plasma to prevent instabilities. Depending on the applied boundary condition, this can lead to two effects:

- When a constant voltage is applied over the plasma, the current through the plasma will reduce by a factor $\tau/\Delta t$.
- When a constant current is imposed through the plasma, the electric field in the plasma will increase by a factor $\Delta t/\tau$.

These unphysical effects are drawbacks of the current-limited scheme, but there seems to be no way to overcome them without making the scheme at least partially implicit.

The example above highlights the drawbacks of current-limited scheme because it contains a 'short-circuit' and therefore a potentially large current. The scheme is more suitable for discharges that do not carry large currents, like the examples shown in [2], in which the electric field is screened inside the plasma. Increasing such a screened electric field by a factor $\Delta t/\tau$, as described above, will not significantly affect the obtained solution.

The scheme can sometimes also be used when a current flows through the plasma. If a small region has a much higher conductivity than the rest of the plasma, the conductivity inside the small region will be reduced, and the local electric field will increase correspondingly. However, the total conductivity of the plasma will remain almost the same.

In summary, the current-limited scheme artificially reduces the conductivity of the plasma when $\Delta t > \tau$. How severely this will impact solutions depends on the type of system that is simulated and on the ratio $\Delta t/\tau$. The examples in [2] demonstrate that in some regimes, the errors introduced by the current-limited scheme are significantly smaller than those resulting from a first-order semi-implicit scheme.

## References

[1] J. Zou, Comment on "Improvements for drift-diffusion plasma fluid models with explicit time integration", Plasma Sources Science and Technology (Jul 2020). doi:10.1088/1361-6595/aba984.

[2] J. Teunissen, Improvements for drift-diffusion plasma fluid models with explicit time integration, Plasma Sources Science and Technology 29 (1) (2020) 015010 (Jan. 2020). doi:10.1088/1361-6595/ab6757.

[3] P. L. G. Ventzek, Two-dimensional modeling of high plasma density inductively coupled sources for materials processing, Journal of Vacuum Science & Technology B: Microelectronics and Nanometer Structures 12 (1) (1994) 461 (Jan. 1994). doi:10.1116/1.587101.

[4] G. Lapenta, F. Iinoya, J. Brackbill, Particle-in-cell simulation of glow discharges in complex geometries, IEEE Transactions on Plasma Science 23 (4) (Aug./1995) 769–779 (Aug./1995). doi:10.1109/27.467999.