Probability and Quantum Mechanics: A Christian Theistic Interpretation

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ABSTRACT

It has been claimed by some in creationist circles that modern physics is derived from a priori atheistic reductionistic presuppositions. On the contrary, it is claimed in this paper that it is possible to construct a Christian theistic interpretation of the experimental facts which is consistent with the Biblical doctrines of man, creation, revelation, and God. This reconciliation is derived by the correct method which subordinates "scientific" interpretation to the authority of special revelation. It is also demonstrated that some of the presuppositions and arguments of creationists against theories such as quantum mechanics and relativity actually exhibit anti-theistic philosophies (Thomism and Arminianism) or fundamental misunderstandings of these theories. A Christian theistic view of the phenomena and theories of moderr physics is presented.

INTRODUCTION

This essay is a modest attempt to advance and articulate the Christian theistic worldview, especially in regard to the philosophy of science in the areas of probability and quantum mechanics. To some extent the issues and views presented here have been, to varying degrees, already expounded by others. (See, e.g. Worthing [43]. A detailed discussion of points of disagreement with these authors would extend the current essay far beyond its intended scope.) As a result, this essay will be essentially a review of some prior results and a critique of other views. I will present some suggestions regarding the direction to take for a final solution and to advance the discussion of these issues.

Inasmuch as quantum mechanics is an explicitly probabilistic theory, the need for a preliminary development of a Christian theistic view of what might be termed the "classical" probability theory is essential. These theories, "classical" probability and quantum mechanics, will be established within the Christian ontology and epistemology. The Biblical basis of Christian ontology and epistemology is assumed throughout this essay and will be briefly described as background for the remainder of the essay. The apologetic methodology is presuppositional.

PART I: THE CHRISTIAN THEISTIC VIEW OF PROBABILITY THEORY

CHRISTIAN THEISTIC VIEW OF GOD, MAN, KNOWLEDGE AND SCIENCE

As mentioned previously, the methodology of this essay is presuppositional, and — except for a few brief introductory remarks to follow — will be used without extensive elaboration. In this section I will also give a brief foundation of the Christian theistic epistemology which will set the framework for the explanations of classical probability theory and quantum mechanics. Readers unfamiliar with the presuppositional apologetic method or the theistic foundations of human knowledge should refer to the works by Cornelius Van Til [38],[39],[40],[41], Greg Bahnsen [1], and John Frame [14],[15] for a comprehensive exposition.

Christian theistic doctrine of creation

Central to Christian theism is the revealed doctrine of God, as Creator, and the relation of this doctrine to man and epistemology, or Christian theory of knowledge. The creation account in Genesis presents God as creating the space-time continuum and the material creation *ex nihilo*. When we speak of creation ex nihilo, we are intent on denying the dualist notion of reality that God merely formed the creation out

of eternally coexisting material possessing inherent preexistent properties. God established the laws of the material universe by Divine fiat. The Genesis account also relates the *beginning of time*, as indicated by the phrase "and the evening and the morning were the *first* day." Since God has created time, He is "above" time. These special revelations are the theistic, Biblically based foundation of the Christian philosophy of reality and theory of knowledge. The Biblical doctrines of God's transcendence and immanence are opposed, respectively, to the anti-theistic philosophies of pantheism and deism. God is transcendent: He is *beyond* the creation and *other than* the creation; He is not dependent on the creation. God is also immanent: He providentially maintains and works *within* the creation; the creation is totally dependent upon God. God is the ultimate cause of all space-time factuality, and all such factuality is according to His comprehensive and *all-encompassing* plan. Herman Bavinck [2, p.55] gives a succinct summary :

> In the order of being, God is undoubtedly first. He is Creator and Preserver of all things. *His* thought and knowledge precedes the coming into existence of things. We are not to suppose that the world was first called into being, and that, as a result of this, God afterward learned to know it.

Since God is the ultimate cause of everything in creation, each event has its particular meaning and purpose because God, the root of all truth, has established it within His overarching plan; consequently, there is no ultimate contingency. This foundational theology revealed in Genesis is, of course, in stark contrast to the pagan philosophies which contain a plurality of equally ultimate gods, men, and things, ruled over by "reified" Chance.

Also, the Scripture speaks of God having created man "in His image," which includes an immaterial spiritual component; and that man was endowed with a rationality *derivative from and analogical to* God's rationality. It is in this sense that man is the *imago Dei*; man's rationality is a finite replica of God's rationality. The presuppositional approach seeks to take seriously and to remain true to the revealed doctrine of creation and God the Creator. The theistic doctrine of creation implies that limited, yet true, knowledge is available to man. Again, God is the source of and the root of all truth, and His revelation to man, both special (verbal) and general (natural), is true but not exhaustive. True knowledge is possible for man because God, as the source of all, is the ultimate subject of all predication. Man, in response to God's revelation, is to "think God's thoughts after Him."

Furthermore, God communicates to man in accordance with man's created finite capacity. Consequently, it is in the nature of the case that all of man's knowledge is thus finite, creaturely and, to varying degrees, an idealization. This must also be so in regard to the Scriptures — and this does not diminish the doctrine of the perspicuity of Scripture. The Scriptures are written in language created by God in order to communicate truth to man. When language expresses truths about God's being and His attributes in finite terms, it is necessarily anthropomorphic and, therefore, necessarily an idealization. Examples of anthropomorphic expressions in the Scriptures are well known and could be quoted extensively.

Since man's knowledge is derivative or analogical, it is necessarily an idealization, for only God himself knows all exhaustively. Man cannot — nor ever will — plumb the depths of reality (i.e. God and His creation) at any one point. There always will be inscrutable aspects of the creation. For example, man can study an electron and, due to the rationality implanted in him by his Creator, truly know various aspects of the nature and makeup of the electron. But only God himself knows exhaustively — not only the design of the electron, but also each electron's place and purpose within God's plan.

Thus, we see that the creation account provides the first articulation of the Christian ontology and epistemology, which in summary is as follows: What "exists" is God the Creator and His creation. God is independent of the creation, and the creation is totally dependent upon God as creator and sustainer (Col. 1:17). God's knowledge is *exhaustive*; man can attain to true knowledge but it is always incomplete. These facts constitute a portion of the Creator/creature distinction. This distinction is the fundamental and true dualism; it is opposed to anti-theistic dualism. We again see that the Biblical doctrine of creation is absolutely foundational to the Christian philosophy of men and things.

Christian theism and the philosophy of science

This section will conclude with a somewhat abbreviated discussion of the Christian theistic epistemology as the foundation of science.

Any philosophy of science must include, among its presuppositions, a methodology of science and a philosophy of facts. A bare, naive empiricism is untenable — it offers no intelligible explanation for the existence of rationality in the universe, or for intelligibility, in general. The methodology of science must include an epistemology and criteria by which truth claims are justified. Christian theism alone offers an *intelligible* and *coherent* framework for understanding the existence of knowledge, in general, and for justifying the hypothetical-deductive method (the latter understood in the Christian sense); the converse is that unless Christian theism is true, there is no truth or rationality. Yet within Christian theism — as revealed by special revelation — we know that man's knowledge must always be (1) limited, i.e. non-exhaustive; yet (2) true, since man's knowledge is a finite derivative (or analog) of God's exhaustive knowledge. These points — in anticipation of the later discussion on the implications of quantum mechanics — need to be underscored, since quantum mechanics demonstrates that there is, *in principle, not just in practice,* a boundary to man's knowledge beyond which lies mystery. This conclusion from the quantum mechanical formalism is consonant with the Biblical data on the nature of man. Van Til [41, p.24] summarizes these points as follows:

> Before the world was, God existed alone. In him, existence and interpretation are co-extensive. The Spirit of God searches the deep things of God. It follows from this that any human interpreters would have to be <u>derivative</u> interpreters or re-interpreters.
>
> The interpretation that man would give to anything in this world can therefore never be comprehensive and exhaustive. This much of truth there is in the recent emphasis on the part of the men of science on the mysteriousness of the facts of the universe. However, as Christians, we hold that the reason for the mysteriousness of the facts of the universe is not given by scientists today. Science today, in consonance with non-Christian thought in general, holds that the facts of the universe are surrounded by an ultimate void, that is, by an ultimate irrationality. We, on the contrary, hold that God as absolute Light is back of the facts of the universe. We hold that the atom is mysterious for us, but not for God.

Van Til goes on to correctly point out that the non-Christian scientist does, in fact, have a religion — when he sees the atom surrounded by mystery, "he worships the void." We, on the other hand, as Christians and scientists, worship God when confronted with this mystery. Again, Van Til summarizes, "As Christians, then, we believe that human knowledge of the world and of God is (a) not exhaustive and yet (b) true." This is so because we are created in God's image. Since we are *created*, our knowledge must be finite, limited, and non-exhaustive. Yet, since we are created in the image of *God*, the limited knowledge we have is true.

Special and general revelation and the philosophy of science

The central truth which the presuppositional apologetic method intends to defend is that all areas of man's knowledge are to be subordinated to and judged by the self-attesting authority of God and His revelation to man — both special and general. Presuppositionalism rejects — without qualification — the premise of unbelieving thought that there are "neutral areas of fact and knowledge" that are outside the sphere of theology. This idea has been termed the '*myth of neutrality*.' There are no "brute" facts; no "fact" can be rationally understood independently of God. Another way of stating this is that it is impossible to discuss science or any area of knowledge without bringing God into the picture[1], and then later, as a kind of afterthought, discuss the "possibility of God's existence." Thus, consistent theism rejects a neutral "naturalism" as if any fact could exist without God behind it. Consequently, there is no division in knowledge between so-called "scientific" facts and "non-scientific" facts. Such phrases as "scientific facts" and "scientifically proven" are misrepresentations. Van Til [39, p.175] summarizes as follows:

> Believers can objectively show to unbelievers that the unity of science can be attained only on the Christian theistic basis. It is the idea of God controlling whatsoever comes to pass that forms the foundation of science. And no one can or does believe that idea unless by the sovereign grace of God through Christ he has repented of his sin. Thus it is Christianity that furnishes the basis of the structure of science.

We can summarize the basics of the presuppositional apologetic as it relates to the philosophy of science in the following way. First, the foundation of all knowledge is the self-attesting God of the Scriptures, and, as such, Scripture (understood by the normative hermeneutic[2]) is the gauge of all science, never the reverse. The self-contained God, who has revealed himself in Scripture and nature, is the sole basis for

all knowledge and predication. Man, as God's finite creation, can know truly but not exhaustively. Second, in light of these observations, there is only one method to be applied in all fields of inquiry.

A BRIEF SURVEY OF ANTI-THEISTIC ONTOLOGY AND EPISTEMOLOGY: THOMISTIC AND ARMINIAN

In contrast to theistic epistemology and the presuppositional apologetic, we have the more or less anti-theistic Thomistic (as espoused by Romanism) and Arminian views of God, man, and knowledge. Again, the survey here must be very brief, and I refer the reader to the previously mentioned works by Van Til, Bahnsen, and Frame for a complete discussion.

The Thomistic view, in contradistinction to theism, is based not upon Scripture, but rather upon autonomous man. This is a natural consequence of its infection by pagan philosophies of truth, especially that of Aristotle as embraced by Scholasticism. The main ingredients of Thomistic thought are as follows: First, both God and man participate jointly in "being in general." "Being" is an abstract principle. God is thus correlative to man and creation in general, as both God and man are contained in a "greater reality." God is "open" and subject to "chance," inasmuch as it lies in the power of man to determine the future *independently of God*. As a result, God is also dependent on the creation. Thomism thus has a defective view of the ontological Trinity. Second, in the realm of epistemology, Thomism embraces a dualistic notion of knowledge. There are two "orders" of knowledge: the order of *reason* and the order of *faith*. Thomism accepts the pagan notion of exhaustive knowledge to man in the natural realm. To this extent it embraces rationalism, in that it accords to man the power to ascend to truth solely with the intellect via abstract principles — principles that exist apart from God. Yet, in the realm of "faith," Thomism denies the ability of the rationality of man; and to this extent embraces irrational fideism. In terms of methodology, this philosophy results in a false dichotomy of methods: a "scientific method" applicable to "scientific" questions, and a distinct "theological method" applicable to "theological" questions.

To a large degree the Arminian view coincides with the Thomistic view, though it does render a more consistent view of Scripture than that of Thomism. It, too, accepts the notion of autonomous man who can bring about the truly new apart from and outside of the plan of God. This is because Arminianism embraces the axiom that in order for man to be "free," he must be able to do that which is outside of the plan (or decrees) of God, i.e. man is able to thwart God. Arminianism — in the same way as Thomism and pagan philosophy — places "possibility" above God. Also, in the realm of epistemology, Arminianism consents to the notion that man can attain to exhaustive knowledge of the creation. Man is autonomous in all respects; both in action and thought, man is ultimate. Arminianism leans toward a naturalism in the material realm and appeals to abstract principles independent of Scripture.

As briefly commented upon above, contained in this "naturalism," borrowed from pagan philosophy, is the idea that the material universe can be understood *apart from God and without bringing into the picture the self-existent God*. It is ironic that on this point the atheist and the anti-theist share common ground — even though they are motivated by entirely different presuppositions. The atheist builds his worldview on the belief that there is no God, while the anti-theist builds his on the belief of autonomous man. The atheist searches for exhaustive explanation since he believes the laws of nature can be rationally understood precisely because there is no God. For the anti-theist, on the other hand, it follows that if nature can be understood apart from God, it must be understood exhaustively. Otherwise, there would be an "irrational" residue of the physical universe which would be "beyond" God; and, if this were so, this material "void" would be shrouded in ultimate Mystery beyond the knowledge of God and of man. In consequence, the "natural" realm is viewed as completely self-determining via natural laws which can be exhaustively and empirically discovered by the law of induction. So, both the atheist and the anti-theist are driven by a desire to discover exhaustive knowledge of the natural universe.

Bahnsen [1 p.229] summarizes this "naturalism" of Deism, Thomism and Arminianism, as follows:

> The religious version of this notion [the Greek philosophy of eternal impersonal laws] that there are "laws of nature" postulates a personal God as the origin of the material world and of the causal principles by which it operates, but this God (and the free or arbitrary exercise of His almighty will) is nevertheless "separated" from the ordinary and ongoing workings of the world He made. God has chosen not to directly govern every detail in the created world on a moment by moment basis, and thus "nature" has laws inherent in it which determine what things are like and how things happen. Variations on this conception of God's world

as governed by impersonal natural laws are found in a wide range of Christian professions, from Deism to Thomism (Roman Catholicism) to evangelical Arminianism.

The anti-theistic view is self-defeating. The atheist will point out that ". . . if you start with the idea of cause and purpose as intelligible to man without God when these concepts apply to relations within the universe, then you cannot consistently say that you need God for the idea of cause or purpose when these concepts apply to the universe as a whole." [39, p.252]. The ironic consequence of this position is that the anti-theist is indeed left with a "God of the gaps." Further, there are no gaps which remain in the self-determining natural realm for God to fill. God maintains a "hands off" policy until He decides to perform a miracle to correct some defect in the creation. But the atheist will respond to the anti-theist that an appeal to miracles is "cheating," for it does not conform to the "scientific method." The anti-theist, in response to this charge, retreats behind the defense of his idea of two ways of knowledge. Science is good and well in the natural realm, but it cannot apply in the realm of faith.

In closing, it should be noted that the end result of compromising God's declaration concerning Himself and his creation results in inconsistent theology, and those who have felt the inconsistency have descended into a full return to paganism as manifested in "process theology" and the "open God" worldviews (cf. e.g. Pinnock et al. [27]). Deism, in that it denies special revelation, asserts that man resides in an "absent landlord" universe. Man is in charge of everything and ascends to "truth" by his own "enlightened" mind; God becomes a mere spectator. Thomism and Arminianism likewise compromise the Biblical doctrine of creation and make God dependent upon the contingent events in creation. However, the Biblical declaration is that God is not a mere spectator of "contingent possible events" transpiring beyond His will in an independent creation ruled by "probabilities." We thus need to turn to God's own testimony regarding "probability."

PROBABILITY: THE BIBLICAL DECLARATION

The lot is cast into the lap; but the whole disposing thereof is of the LORD. Proverbs 16:33

This oft-quoted verse is central to the topic of this essay, and is the definitive Scriptural declaration of the absence of ultimate contingency in the creation. Any consistent Christian theistic articulation of probability theory must ascribe, at the outset, to the non-existence of "random" sequences. God is the source of all "possibility." The further inference is that probability is an issue of limited creature knowledge. This verse also refutes the deistic notion that God is merely a prescient observer of the creation, for it declares God's providential activity even in the case of die rolls (the model of probability systems which introduces all students to the study of probability theory).

In anti-theistic worldviews, the result of a die toss is under the control of impersonal chance events acting under the impersonal laws of nature; the die roll is something that can transpire outside the plan of God. To the theist, nothing is outside the plan of God; there are no impersonal "chance" events in the spatio-temporal realm; rather, all events speak of the personality of God. God acting via the intermediary laws of physics is a *personal* act — the laws are what they are by the will of God. In view of the Biblical testimony, we ought then to regard any set of die rolls (even in an unimportant game of Yahtzee), not as an impersonal act of impersonal forces, but the personal act of our sovereign Lord.

Two important testimonies from Scripture show men casting lots in order to discover God's choice; the choosing of Saul in 1 Sam. 10:20-21, and the selection of Matthias as the replacement for Judas in Acts 1:26[3].

In addition to Prov. 16:33, there are numerous other passages which declare God's providential and sovereign governing of the whole created realm. Time does not allow these to be covered here in depth; however, the following passages are especially pertinent to the discussion at hand.

Job 14:16 declares that God determines (numbers) Job's steps, while in Matt. 10:29-31, even the sparrows lives are determined by God and our hairs are determined (numbered). Also, James 4:13-15 says that all we plan to do can come about only if God, who has numbered our days, sustains our lives another day. Prov. 21:1 declares that the "king's heart is as channels of water," and that God turns it as He wills. Eph. 1:11 states, "In whom also we have obtained an inheritance, being predestinated according to the purpose of him who works all things after the decree of his own desire" (Greek Text); so that all things are decreed according to God's wishes[4]. Finally, Rev. 4:11 unequivocally states that *all things* were created and exist according to God's pleasure. Whether or not we like to admit it, this verse includes,

within God's decretive will, the presence of sin in the creation, Satan and his fallen angels, and all the lost of humankind[5]. These verses give sufficient breadth and show that all of creation is ordered according to God's decree: This includes both the inanimate and animate, and the affairs of men's hearts as articulated in the doctrines of sovereign grace (cf. Rom. 9:13,16-21, for one). *No part* of creation comes about independently from God's decretive will. This doctrine is clearly summarized in the Westminster Confession of Faith, Chapter III — Of God's Eternal Decree.

PHILOSOPHIES OF PROBABILITY AND CHANCE

Having presented God's self-attesting declaration in Scripture that no part of reality is governed by "chance" or "probabilities," we now proceed to the evaluation of several philosophies of probability theory. The goal of this section is to revisit the well-known and insoluble difficulties of the anti-theistic philosophies of probability and then to articulate the Christian theistic foundations of probability theory.

What is meant by the word "random?"

The key ingredient expressed by the idea of "random" is indeterminism. Popper [28, p.150] describes randomness as events which are characterized by the perception that it would take a prophet to predict the outcomes. "We have as it were, the feeling that not a scientist but only a prophet could predict them,"[6] or, that it has a certain "lawless" character. The character of the indeterminism — whether the randomness is related to limited knowledge (of otherwise determinate events) or intrinsic to nature — is the core of the debate between subjectivist and objectivist theories of probability to be discussed later. Anticipating later results, note that the debate makes a false dichotomy between the epistemological and the objective components of "random." The issue is solved in the theistic worldview by realizing the following: (1) There is simultaneously a creaturely limit to both knowledge (subjective aspect) and control (objective aspect) of the physical universe; and (2) As Scripture attests, there is no ultimate (absolute) probability, for God is the sufficient cause of all things.

As far as technical definitions of random sequences, we can consider the following developments. Richard von Mises [23], who was a radical empiricist and a logical positivist, attempted a synthetic theory of random events. Von Mises' idea of a random sequence was based on the idea of the "collective." The collective is an infinite sequence of values

$$S = x_1, x_2, \ldots, x_n, \ldots \tag{1}$$

whose values cannot be computed as a function of n (i.e. are "anomalous"), and such that for *every* sub-sequence (such that the selection of members to retain did not depend on the value of x_n) is also "anomalous." For example, in the toss of a fair coin, heads appears with a frequency of 0.5 in the original sequence. Also, heads appears half the time in the sequence obtained by considering only the odd tosses and in the sequence obtained by considering every third toss, etc. This characterization is based on the intuitive concept of "non-existence of successful gambling systems." Such sequences have the property that they automatically reproduce the "law of large numbers," i.e. the frequency of occurrences of events approaches the probability in the limit of infinite sequences. Stated conversely, von Mises' theory excludes every "lawful" sequence from the class of random sequences. Hence, any sequence which can be generated algorithmically is "non-random." This would include, for example, the sequence of decimal digits of transcendental numbers such as π, or the output of a deterministic chaotic system. Later developments by Doob, Copeland, and Church showed that von Mises' idea of a collective was too strong, and that no such sequences could be constructed. See, for example, Popper's discussion of this point [30, p.361 sequens, and footnote (7) on p.362]. The idea which remedies the contradiction in Von Mises' original idea of the collective is the notion of the "effectively" random sequence. These are sequences which, although deterministic, cannot be predicted because there is no effectively (i.e. knowable) computable algorithm to determine the generating function.

Another attempt to give a consistent "definition" of random is Kolmogorov's "complexity" definition. In this characterization, a random sequence is one which cannot be "encoded" in fewer "bits" than the string itself. In this notion, a random string is one that can be described only by printing the string. This definition drives home the fact that in the anti-theistic worldview, everything is really only anecdotal.

These technical definitions, though they can contribute to understanding the issues, are merely descriptive; they offer no explanatory power.

Remarks on empirical and singular "probabilities"

The technical meaning of probability refers to the assignment of a numerical value which expresses the relative likelihood of an event in a "random" sequence or set. This is expressed mathematically as

$$p(x) = \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} x_i \tag{2}$$

where x_i is an indicator function which is 1 if the ith trial is a success for event "x" and 0 otherwise. In other words, $p(x)$ is the frequency of occurrence of event "x." If we examine the technical and philosophical underpinnings of assigning numerical values to likelihoods of events, and examine the technical uses of these terms — analyzed to the limit of human understanding — we can then discover the anti-theistic biases of modern science. One of the goals of anti-theistic probability theory is to analyze the concepts of probability or chance to discover whether they are an irreducible something "in the world" or whether they are reducible to something else "in the world." The question is: "How does the limit of the observed frequency approach a fixed value?" The subjectivist view — which comes closest to the correct theistic view — roots "probability" in the ignorance of the observer. In this view, "random" comes close to the idea of "uncontrollability." For example, in a "chaotic" deterministic system, "outcomes" are widely separated even though initial conditions can be arbitrarily close. In this case the absence of control in practice gives rise to the dispersion of outcomes.

Empirically, probabilities can only be computed statistically as in equation (2), and this means that only frequencies of events — all other things being equal — can be assigned a meaningful value. Singular probabilities are meaningless. How do we assign a probability to the event that a specific individual, for example a 40-year-old non-smoking male named John Doe, lives to the age of 45? On the basis of actuarial charts, we can estimate that within a group of such individuals a certain percentage will live to be 45 — but what does this say about John Doe? What is the singular probability that John Doe will live to be 45? Clearly, statistical averages cannot be applied to a single individual.

Popper points out that formally singular probability statements (such as "the probability that the next roll of a die is 6 = 1/6") can never be falsified, (see Popper [28 p.228]). To clarify this, consider the toss of the mythological fair coin. We say that the probability that heads or tails will occur on the next toss is one-half. Yet when we actually toss the coin, it must land heads or tails. So, what does that say about the so-called objective probability for that specific toss? Further, in the frequentist view, no finite string of random flips of a coin can be used to decide that a coin is biased. In the frequentist view — that probabilities are the limits of frequencies of events when the length of the sequence approaches infinity — no finite string contributes to the limit! Or, paradoxically, all of the probability always "resides in the results of the infinite tail"! This observation is also the core of the problem of induction, namely, "How can any finite sequence of observations be *infallibly generalized* to a universal statement?" Consider, for example, the "fair" coin whose first hundred tosses turn up "heads" but thereafter are "random" (i.e. the results reproduce the correct frequency limits). According to statistical tests the coin would be adjudged to be "biased" based solely upon the first hundred rolls, yet the frequentist interpretation declares this sequence "random." These remarks point to the fundamental problem of the anti-theistic philosophies to make a convincing connection between the empirical observations of frequencies and a putative "objective" probability.

When we turn to modern philosophies of probability, we find that they can generally be categorized as to whether they view probability and chance as an issue of epistemology or ontology. In the ontological view, chance is something ultimate and objectively "in the world." In the epistemological category, the theories can be further subdivided into a subjectivist and objectivist point of view. But before discussing these philosophical viewpoints I will briefly deal with the formal or axiomatic theory of probability.

Kolmogorov axiomatic theory of probability

Kolmogorov's axiomatic approach is the usual flavor of probability taught in introductory probability classes. This axiomatization, however, only defines the mathematical properties which "probability" must satisfy. *It offers no philosophical view on how or from whence the probability measure on the field of events arises; nor does it supply a formalism for constructing such spaces.* These structures are assumed as "givens." They are a priori abstract entities much the same as the concept of a point in the Euclidean axiomatization of plane geometry. The construction of the σ-field of events ("possibilia") and the measure μ ("likelihoods") is left to empirical, philosophical, and physical enquiry. It can be shown that the axiomatic

approach recovers the frequency interpretation, but only when the idea of "selection at random" of events can be performed according to the relative frequency specified by μ. This is clearly an invalid circular definition. The measure per se is supposed to provide an axiomatic idea of probability (or randomness), yet the measure μ only describes a relative frequency, which describes observed proportions of events as they occur — it does not, nor can it, provide a more precise idea of randomness. Another point is that the Kolmogorov axioms, in actuality, reduce probability theory to set theory and the theory of measures on these sets.

As to the problem of assigning measures to sets, we are confronted with the following problems: (1) specifying the universal set Ω (always within a limited context); (2) specifying predication on Ω; and (3) setting up the "a priori" probabilities or measure. In the most trivial of cases — such as die roll experiments — specifying these quantities is not difficult or usually liable to much controversy. But on larger more "universal" statements, such as "degree of confirmation" of theories or the likelihood of "spontaneous generation of life from lifeless matter," the methodology of constructing such probabilities will, *ab initio*, place one either on theistic or anti-theistic ground. This is because the very process of delineating *possibilia* is dependent upon a person's worldview. As Van Til [40, p.107] states:

> When one defines possibility he *ipso facto* defines reality. When one opens his mouth about possibility he also opens his mouth about God. God is either the source of possibility or he comes out of bare possibility, or for that matter any other term would have no significance if God were not back of it as the final subject of predication.

And elsewhere [41, p.38]:

> On the question of possibility, the same difference of opinion exists that we found on the matter of predication in general. For the theist, possibility has its source in God, while for the anti-theist, God has His source in possibility. Hence, what one will deem most possible, the other will consider altogether impossible.

> The question is similar with respect to the matter of probability. For theism God's plan is back of what is probable. For anti-theism, the probable is independent of God. Hence, what one thinks altogether probable, the other will think altogether improbable because altogether impossible.

The epistemological views

In the epistemological view, probability is a *theoretical term* which is connected to our degree of knowledge. The subjectivist version is that probabilities are *only* a property of "something in our minds," and not an objective property; while the objectivist version grants that there are objective things "outside of our minds," and thus there are criteria for assigning measures to outcomes, usually based on physical and symmetry considerations.

The extreme and, in fact, deistic viewpoint is that the material universe is deterministic and that probability relates solely to our lack of detailed knowledge of the initial conditions which enter into a specific solution of the Hamiltonian equations of motion. Laplace expresses this crypto-determinism view in his often-quoted statement [21, p.4]:

> We ought then to regard the present state of the universe as the effect of its anterior state and as the cause of the one which is to follow. Given for one instance an intelligence which could comprehend all the forces by which nature is animated and the respective situation of the beings who compose it — an intelligence sufficiently vast to submit data to analysis — it would embrace in the same formula the movements of the greatest bodies of the universe and those of the lightest atom; for it nothing would be uncertain and the future, as the past, would be present to its eyes.

According to Laplace's view of the universe, only those things decreed by Mathematics take place, and anything disallowed is forever impossible. The Laplacian view, as further developed in the causal determinism of Newtonian mechanics, is that initial conditions *plus* exact laws of propagation yield history for all time. In symbols:

$$\{X(0), \mathcal{L}\} \rightarrow X(t) \ . \tag{3}$$

The deduction proceeds both ways. Given exhaustive knowledge of the history of X, we can deduce the law (assuming that the laws are expressible as differential equations of arbitrary but finite order); or, given the law and the initial conditions (expressible in terms of initial positions and time derivatives up to order n-1, where n is the order of the differential equation), Laplace's "Great Mathematician" can reconstruct the entire history. Reality is just a differential equation.

Elaborating on this view further, it claims that all of material nature is deterministic, and there is an exhaustive immanent description *within* the physical universe, described via a differential equation

$$\frac{dX}{dt} = F(X,t) \, , \tag{4}$$

where X is a complete set of state variables, and all other quantities and qualities are reducible to functions of X. Given X at t_0 we know X for all time, and thus also all derived quantities[7]. Note the form of the equation includes the Hamiltonian formulation if $X = (q,p)$ and $F = \{X,H(q,p,t)\}$. The Hamiltonian flow is non-linear and exhibits "chaotic" behavior, which places limits on man's ability to predict the outcome; and it is in this sense that things *appear* random. Nevertheless, each trajectory in phase space can be labeled by its initial conditions. If we assign a probability density for the initial conditions, then this density evolves according to the Liouville equation. This formalism corresponds to the "probability as ignorance" view which forms the basis of Boltzmannian statistical mechanics and classical thermodynamics. The use of a probability density in this case is useful for limiting the amount of data required to model the average aggregate properties, such as temperature and pressure, without solving the n-body problem systems composed of molar collections of particles. Such an approach is also useful in the case of highly chaotic systems involving relatively small numbers of particles when measurement of the initial conditions is of limited precision.

Since each trajectory can always be labeled by its initial condition, this means that as we trace back in time along a trajectory corresponding to a coin toss of "heads," each event along that trajectory is both determined and always labeled "heads." Conversely, the event "heads" is just the terminal event in a long chain of events stretching back into the remote past. Of course the actual picture is more complicated than this simple description. The coin is actually an assemblage of atoms, and according to the material determinist view, a particular result of the coin toss was determined even before the atoms of the coin were assembled into a coin. A similar remark holds for a coin which lands "tails." Thus, the space of all trajectories is divided into disjoint equivalence classes which correspond to the mechanistically predestined outcomes. However, in this view there must be an initial asymmetric selection of the equivalence class in which the terminal result began its existence. This asymmetry is unaccounted for. As the coin toss is traced back in time, there is an initial "selection" which is "rooted in ultimate chaos and irrationality." So, as is typical of anti-theistic thought, even though this view appears to be "rational," the determinism is ultimately "irrational" since the initial conditions just "spring out of nothing." This admission of an initial indeterminism is the Achilles heel of the determinist philosophy. The empirical determinist, having allowed chaos to get one foot in the door of his house, will find it hard to refuse the admission of Chaos to his living room. If there was once indeterminism or chance, why not always? What is special about t=0?

The Christian view is that there is *no* exhaustive intra-creation determinism. Thus, anticipating Part II of this essay, Christian theism, in fact, implies some form of "quantum mechanical"-like epistemology. Man in his finiteness can not exhaustively plumb the depths of God's creation. There always will be mystery to man; it is inherent in his creatureliness. At the ultimate level, creation is governed and sustained by Christ according to His will, so that all equations of motion are of the form

$$\frac{dX}{dt} = F(X,t,''\text{will of God}'') \, . \tag{5}$$

No matter how "non-mathematical" such an expression appears, I have intentionally refrained from a notation which would reduce the "will of God" to a set of "hidden variables"! Clearly, God is not subject to a back reaction according to Newton's third law! The atheistic worldview, of course, implicitly assumes equations of the above form but with the phrase "will of God" replaced by "(irreducible) chance" or by terms which are irreducible random events. At the "macroscopic" level, we have a view where "will of God" appears absent; however, this is really not the case since at the meta-level even the equation itself is a statement of God's will[8].

In summary, the purely subjectivist theory views all probability issues as Bayesian conditionalization working from some putative absolute initial probability distribution on phase space. How the initial

condition arose is left to Chance in the atheistic worldview or to the absent-landlord god of Deism; and "randomness" is equivalent to deterministic chaotic dynamics.

Ontological probability, or "reified chance"

This view rejects the deterministic physical causalism of the epistemological view and by an appeal to "Occam's razor" substitutes a "simpler" hypothesis, indeterminism, which rather than being an initial one-time fluke, is uniformly present, everywhere and forever. In this atheistic view, "chance" is the *impersonal* purposeless "cause" or determiner of unaccountable happenings. Of course, this is an empty idea. It is not a rational explanation, for it posits a *determiner of the undetermined*.

Proponents of this view are Popper [28],[29],[30],[31] (as expounded in his "neo-classical" or "propensity theory"), David Bohm [7], and Reichenbach [33]. As Brody [9, p.108] points out, attempts to objectify "chance" as a quantitative property of the physical universe meet with difficulties from the first. As he cogently states, we can measure temperatures with thermometers and electric potential with voltmeters, but where can one find a "pitanometer" or probability meter? Probabilities are always empirically measured statistically by performing many measurements on many individuals of supposedly identically selected states (identical for the purposes of the experiment) and taking averages. This corresponds to the frequentist view, but it gets one no closer to a putative "objective" Chance.

To further demonstrate the emptiness of the belief in ontological chance, we should examine in more detail some remarks of Bohm [7] in this regard. Bohm relies upon the typical dualism of anti-theistic thought in which nature is governed by two distinct, independent, and equally ultimate principles — causal laws and the laws of chance. This is just another way of stating the anti-theistic duality principles of "rationality *and* irrationality," "closed universe *and* open universe," and "continuity *and* discontinuity." As Bohm [7, p.28] puts it: ". . . the causal laws and the laws of chance *together* are what bring about the actual development of things. . . ." (emph. his). Yet, for Bohm, "chance phenomena" — though supposedly objectified and observable in the "laws of chance" — are always "randomly fluctuating and independent contingencies *lying outside the context* under discussion," (emph. added) [7, p.28]. Bohm believes chance is ultimate, for, as he admits later [7, p.29]: "Thus we never really can eliminate contingencies. Rather, the categories of necessary causal connection and chance contingencies are seen to represent two sides of *all* processes." (emph. added). Therefore, for Bohm, a neo-Heraclitean, chance is a synonym of ultimate mystery and ultimate irrationality. Chance always lies forever beyond the horizon of an ever-expanding sequence of contexts. This "Chance of the gaps" is part of the atheist's creed.

With these few brief remarks, I proceed to the Christian foundations of probability theory. More will be said about ontological probability below in the section titled "Critique of probability arguments."

The Christian Theistic Foundations of Probability Theory

Theistically the truth is a combination of the epistemological and objective views. Probability is attributable to a *correlation* between the limited knowledge of man and the external objective state of affairs — again arranged according to the eternal decree of God. The concept of probability thus reduces to an issue of the creaturely and *necessarily finite nature* of man. To state it another way, probability characterizes the relation of the derivative and finite mind of man to the finite world of facts or states of affairs. Yet, in regard to the description or enumeration of these states of affairs (or better, *possibilia*) only insofar as these *possibilia* can be categorized can the concept of probability be applied. There will be cases where deterministic explanation fails and also when probabilistic explanation fails — both of these understood theistically — and that leaves only the will of God as the ultimate explanation. Since God is personal, the outcomes are ultimately His free choice. This view is required by the Biblical data, which as we have seen, asserts that all *possibilia* must be rooted in and come solely from God Himself; and the personal free choices of God are not subject to a higher deterministic law or to a more ultimate indeterministic principle.

This discussion shows that the Christian view of the objective component of probability should be construed as *inter-subjective*. Thus, I am suggesting that the Christian view is a combination of the subjective and objective philosophies of probability. In this regard it perfectly fits the fundamental Christian dualism of the Creator/creature distinction. By this is meant that for certain probability distributions there will be objective criteria for assigning probabilities, yet the probability is subjective in the sense that it results from creaturely ignorance of the mind of God. There are objective standards for

assigning "possibilia" because these are based upon the kindness and providence of God, who sustains the creation moment by moment in a regular fashion.

To elaborate on the relation of probability to a correlation between man's knowledge and the external possibilia, consider a sample space Ω which is the collection of (denumerable) possibilia, or events, denoted as $\omega_1...\omega_n...$[9] I qualify this statement by the remark that only in the case of very narrowly circumscribed systems can we even specify Ω. Now consider the class U of unary relations (or single argument predicates) on Ω, viz. $U = 2^\Omega$. For a given $u_i \in U$ we can construct the frequency or probability p_i as

$$p_i = \frac{o(u_i)}{o(\Omega)} , \tag{6}$$

where o(X) denotes the "order" or number of elements in set X. This definition is consistent with the Kolmogorov axioms. Thus, as this example presumed, if we speak of an ω "drawn at random" (i.e. without human foresight) from Ω, ceteris paribus, then p_i is the probability that $\omega \in U_i$.

CRITIQUE OF "PROBABILITY ARGUMENTS"

In this final section I would like to digress to submit what I believe to be a constructive criticism of Christian uses of probability arguments against evolutionary theory. The point may seem subtle, but it is a point that we need to keep in mind when testifying to the truth of Christian theism: In any presentation, we must be clear that our argument is designed to show the untenability of the purely atheistic evolutionary theory as viewed *from within their worldview*. In light of the presuppositional apologetic and Scripture's testimony regarding probability, we need to be clear that the argument is not an argument that we, as Christians, are presenting from within our worldview.

One type of probability argument considers the likelihood that the fundamental constants of physics have value to support life. The argument then proceeds along the following general lines. Denote by Ω the set, {all possible states of affairs of the fundamental constants}. Let $\mu(L)$ denote the measure of set L, where $L \subset \Omega$ is the set L = {all possible states of affairs of the fundamental constants that can sustain life}. (We ignore discussion of the impossible task of how science can assign a measure to such a space.) Then the likelihood of life is given by the probability that the value of the fundamental constants (denoted by a) is in the set L

$$p(a \in L) = \frac{\mu(L)}{\mu(\Omega)} , \tag{7}$$

which even though perhaps improbable *is not impossible*. Thus, assuming the anti-theist is willing to believe (1) in "self-existent immaterial laws inherent in matter," (2) minimal entropy initial conditions, and (3) finite time — then the material reductionist faces the *apparent* dilemma of a low likelihood of life occurring within the supposed age of the universe. It is apparent that some atheists feel the pressure of this argument on the internal inconsistency of their worldview in that they continually appeal to "laws of self organization" which supposedly accelerate the appearance of life and which mitigate the local effects of the second law of thermodynamics.

However, to the atheist there is no "law" which ultimately determines the trajectory of a(t) within Ω, except perhaps a non-deterministic Markov process. Thus, this worldview reduces to the atheistic "trinity" of Chronos, Tyche and Anagke. (Note: In this worldview space-time itself, as modeled by a space-time metric $g_{ij}(t)$, is viewed as a derived or synthetic feature of reality, cf. "The Black Box: The Reprocessing of the Universe," in Misner, Thorne and Wheeler [24, p.1209].) In summary, the womb of Tyche produces all things and everything by the insemination of father Chronos. Note the intentionally pagan cast used here in order to expose the ludicrousness of the view. In spite of this irrational approach, modern science still attempts, in general, to project a rationalist view of science to the general public.

Another current strain of irrationalism in science is that all is ultimately irrational, that there are no permanent or eternal "laws" of the physical universe (as Christians we would agree that there are no eternal laws of the physical universe unless God chooses to so maintain them). In this worldview the All is governed by ultimate Chaos, and the laws which we now observe are ultimately just a "fluke." Given enough time, the All will transmute itself into every configuration. As a result, there are no probability rebuttals to this view, except that it offers no intelligible explanation for consciousness, ethics, laws of logic, or rationality. For example, the atheist has no rational explanation of how rationality can arise from

an initially irrational universe. Literally this view is that the All is ultimately irrational and, truly, nothing but "sound and fury signifying *nothing*." As Bahnsen has pointed out, this view will even allow the historicity of Christ and His resurrection. The atheist will retort, "Well, it's a strange universe and strange things happen." He can even allow that the scriptures exist as testimonies to these events but disallow that God is behind these phenomena. After all, the All is the mother of all phenomena, including a "cycle of the All" in which such events as the person of Christ occur along with "mistaken interpretations" of those events. Our response is to reprove such an extreme view by asking the atheist why he even "goes through the motions," since in his view there is no "truth" and thus he has no intelligible basis for making the claim that the Scriptures are a "mistaken interpretation." In an attempt to escape from this dilemma, unbelieving thought always appeals to a "meta" level of eternal rationality which is actually external to the supposed All. It is at this point that the unbelieving scientist is really attacking Christianity by stealing, as Van Til said [41, p.84], the very capital of Christianity. In order to "rationally" attack Christianity using the "successes" of science, the unbelieving scientist must assume the Christian view of truth!

The point of these observations for the defense of the faith is that we must not only declare the resurrection as a historical fact (for it is not just a brute fact, as the above brand of atheists will allow), but we must also declare the resurrection to be what Christ's self-attesting testimony declares it to be — His victory over death and His finished work which redeemed the elect and further gave proof of future judgment awaiting those outside Christ's work on the cross.

Finally, as an example of how different worldviews interpret so-called brute facts, consider the "implications" of time-varying fundamental physical "constants," such as G, c, h, etc. An observation of time-varying "constants," as interpreted in a Christian theistic worldview, would be confirmatory of God's control over whatsoever comes to pass. God is the governor and sustainer of the variations. Yet, from the atheistic or anti-theistic worldview, indications of varying constants would be seen as a confirmation that the current state of affairs sprang from ultimate chaos. In rebuttal to the evidentialist approach that the constants are "tuned" to support life and thus "prove" the existence of God, the atheist would retort that we "just happened to be around" when the constants randomly drifted into the "conscious life"-supporting region of the global parameter space of constants. In other words, given enough time, chance will spawn anything and everything, and, of course, it is then trivially true that conscious life will always "observe" constants compatible with life. This view will raise its head in the "many worlds" interpretation of quantum mechanics in Part II.

CONCLUSION

The Biblical testimony is that there is no such thing as inherently "random" processes. All of creation is ruled according to God's plan. There will always be processes and events for which there are no humanly knowable or assignable causes since they are the result of God's all-inclusive providence. Thus we conclude: (1) Probability statements are always an expression of creaturely ignorance, and in this sense, "probability" is epistemological; (2) God, who is the source of all possibilia, is the absolute objective standard of predication, and He sets the bounds for all events; therefore, there is also an objective basis for creaturely estimates of probabilities; and (3) Since there is no such thing as objective chance, the concepts of probability in classical mechanics and in quantum mechanics are identical and refer to limited human knowledge. The further consequence of this theistic approach to the interpretation of quantum mechanics will be continued in Part II.

PART II: THE CHRISTIAN THEISTIC VIEW OF QUANTUM MECHANICS

INTRODUCTION

"The moon is definitely not there when nobody looks . . .." *David Mermin*

"The only law is the law that there is no law." *J.A. Wheeler*

Such views as Mermin's anti-realist interpretation of quantum mechanics or Wheeler's praise of ultimate irrationality are typical of the vast onslaught against the revealed truth of Christian theism. Such claims as these are widespread, not just in professional journals, but also in popular books on quantum mechanics which fill the shelves of science sections in local bookstores. Other popularizations tout the "affinity" of quantum mechanics and modern physics with eastern religions, and thus are a direct attack on the revealed truth of Christianity. Typical of these is the opinion expressed by Menas Kafatos [19, p.3]

in his book,"The Conscious Universe" — a revealing title which leaves no doubt as to where the book will lead — wherein he states: "Where metaphysics is the primary emphasis in books . . . the usual conclusion seems to be that the worldview of modern physics is more consistent with Taoism, Hinduism, and Buddhism."

The purpose of the second part of this essay is intended not only to refute such claims as these, but also to show that the revealed doctrines concerning the nature of God, man, knowledge, and the creation, in general, provide the presuppositional basis for the correct interpretation of certain aspects of quantum mechanics.

Since quantum mechanics (though as I will show later it is really the *Copenhagen interpretation* of quantum mechanics) has been used as what might be termed "evidentialism for false religions," the reader can understand the general aversion of Christians toward quantum mechanics. Such aversion would be correct if these anti-theistic interpretations were a necessary consequence of quantum mechanics; however, I will argue that those interpretations (based upon the Copenhagen interpretation and its derivatives) that favor eastern mysticism are not required. I will give a quick outline of the most publicized interpretations and also describe the less well-known interpretations such as the deBroglie-Bohm "pilot wave" theory, and the ensemble interpretation, as these have features which comport with Christian theism. I will also briefly discuss the issues that have contributed to what Popper aptly called the "Great Quantum Muddle." These issues are the "acausal" collapse problem, the infamous "quantum jumps," and the invalid reasoning behind interpretations of the Heisenberg uncertainty relation. My goal is to remove objections to quantum mechanics based on the faulty interpretations which have been promulgated. Finally, and perhaps most important, I will also point out some of the logical flaws in the analysis of the Aspect experiment and its violation of the Bell inequality (this experiment has been used to promote everything from pantheism and mysticism to paranormal phenomena and parapsychology). It will be shown that the Aspect experiment does not lend support to the Copenhagen interpretation (henceforth occasionally abbreviated as CI). These latter comments will be based upon the work of Brody [9]. I will conclude with a critique of some papers by Lucas and Bergman in which they argue against quantum mechanics and in favor of classical physics.

INTERPRETATIONS OF QUANTUM MECHANICS

There are, in general, four well-known interpretations of quantum mechanics which have been proposed to "explain" the quantum phenomena via an underlying "metaphysics." These views are: (1) the Copenhagen interpretation and its variants; (2) the deBroglie-Bohm pilot wave interpretation; (3) the many worlds interpretation; and (4) quantum logic. A less well-known view is the ensemble interpretation.

I will begin with brief descriptions of these views, and then proceed to the positive task of developing the Christian realist interpretation of QM. In order to set the framework for the subsequent discussion, I will first review the "one mystery" of quantum mechanics, namely the well-known double-slit experiment.

DOUBLE-SLIT EXPERIMENT

The double-slit experiment first performed by Young in 1801 provided firm experimental evidence of the wave properties of light. Before Young's experiment, Newton had postulated that light consisted of particles (corpuscles); however, corpuscular light moving according to the laws of Newton's mechanics would instead produce a superposition of two bell-shaped curves, I_1 and I_2, as shown to the right of S in Figure 1. The wave nature of light remained unchallenged for nearly a century until the quantitative measurements by Millikan in the early 1900s on the photoelectric effect. Millikan's experiments added impetus to the developing quantum theory and its concept of the dual nature of light eventually became canonized in the principle of wave-particle duality. It is this duality which gives rise to some of the most baffling concepts central to the theory and which are used as the prime supports
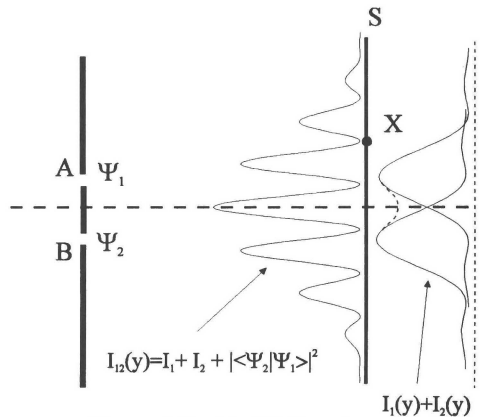

$$I_{12}(y) = I_1 + I_2 + |\langle\Psi_2|\Psi_1\rangle|^2$$

$$I_1(y) + I_2(y)$$

Figure 1 Double Slit Experiment

of the CI of quantum mechanics. To elucidate the source of these mysteries, we now turn to a detailed discussion of the double-slit experiment with electrons.

When the experiment is performed with particles, such as electrons, the mystery begins. Electrons are incident from the left on a screen in which are two slits labeled A and B, and finally strike a screen or photographic plate S, where they are detected. The impacts of the electrons result in a series of alternating bright and dark strips, or interference fringes. The observed interference fringes could be easily explained by the phenomena of wave motion and the related effects of constructive and destructive addition of wave motion as detailed in introductory physics texts. At first one might think that the interference is due to a "crowding wave" effect as a result of the presence of vast numbers of electrons, which as a collective effect act somewhat similarly to gas molecules which collectively produce pressure waves or "sound." However, this is not the case.

If the experiment is run at low intensity such that *only one electron at a time* is incident on the slits, what is observed is a sequence of scintillations which slowly build up until, after a long running time, an interference pattern emerges (I_{12} in Figure 1). When the experiment is run with only one slit open, no interference pattern appears. But how can the particle be *influenced by the other slit through which it did not pass*, assuming the electron has a well-defined trajectory and can only travel through one slit at a time? When only one slit is open, we obtain a bell curve (I_1 or I_2 to the right of "S" in Figure 1). But when both slits are open, allowing two possible paths for the electron, what results is that certain points on the screen (e.g. point X in Figure 1), which detected electrons when one slit was open, now detect none! This redistribution of impacts is baffling since in a particle view, opening the other slit should not cause a decrease in the number of electrons detected at point "X"[10]. This effect has all the prima facie earmarks of either: (1) action-at-a-distance (non-locality), or (2) that there is no distinct trajectory for the electron and yet it does, in some sense, pass through both slits at once like a wave. (I will later discuss and refute the contention that the electron has no distinct trajectory.)

THE COPENHAGEN INTERPRETATION(S)

It was the attempt to interpret such empirical data as the interference of single electrons that gave rise to the Copenhagen interpretation and its notorious "acausal" collapse of the wave function. It should be noted here that the phrase "acausal" really means "indeterminate" and not "acausal" in the sense of surd events (though in the atheist interpretation even though the collapse is caused, the final outcome is ultimately undetermined and, hence, ultimately irrational).

The Copenhagen interpretation and its variants can be considered as an outgrowth of either a confusion between ontology and epistemology, or as an indecision between the two. This confusion is manifest in the canonized principle of "wave-particle duality." Though the CI is the "orthodoxy" presented in most modern quantum textbooks, the list of dissenters includes such notable physicists as Einstein, deBroglie, Schrödinger, Landé, and Bohm. In the CI the Ψ function represents a probability wave for a *single particle* (the potentia of Heisenberg), and actually describes a "*matter of fact*" pertaining to a *single particle*. As long as no measurement is made (for example by removing the screen S in Figure 1), the wave function propagates away from the slits and represents *possible locations of the electron*. Only when the screen is inserted (a measurement) does the electron reveal its location. Since the electron wave is viewed as "real," this means that at the moment the plate was inserted the electron had to be prepared to "collect itself" from all points in space and "collapse" to a single point on the screen. Where it will appear is unpredictable, but it will appear somewhere and then be describable by a Ψ function whose distribution is sharply defined. We note at this point that this "collapse" has all the features of probabilistic conditionalization as stated in Bayes' theorem. This has been pointed out by Popper [29, pp.78-9,124-5] and has been further investigated by Brody [9] in the ensemble interpretation to be discussed later.

At this point, let me summarize the axioms of the Copenhagen interpretation, which are added to the mathematical framework of quantum mechanics. It is these which will require modification or rejection to bring the interpretation of quantum mechanics into a Christian theistic worldview. These assumptions, which will be referred to as (CI-1), (CI-2) and (CI-3), are:

(CI-1) ontological probability assumption;
(CI-2) "acausal" collapse assumption; and
(CI-3) non-disturbance (or repeatability) of measurements.

According to CI-1, the wave function Ψ, *qua* probability, has ontological significance; it describes an actual "matter of fact" or "state of affairs" of (a subsystem of) the universe. Since the wave function is a solution of a linear equation, it can thus represent a superposition of many physically contrary states of affairs. Second, according to CI-2, the wave function Ψ develops "causally" or deterministically, according to the linear Schrödinger equation, until a measurement is performed when the Ψ-function undergoes a non-unitary "acausal" or "undetermined" collapse to an eigenstate of the measured quantity. This collapse is not governed by the Schrödinger equation. An act of measurement results in a suspension of the normal evolution of the probabilities as determined by the Schrödinger equation. The actual state which will result from a measurement is undetermined except that the results of many such experiments will exhibit statistical frequencies consistent with a "singular" probability of transition for each state n, according to the *Born rule*

$$p_n - |\langle \Psi | a_n \rangle|^2 , \tag{8}$$

where $|a_n\rangle$ is an eigenstate of the quantity $\hat{A}$ measured by the experiment. Finally, CI-3 states that if a system is in a pure eigenstate of an observable, then subsequent measurements of that observable yield (with probability = 1) the same value.

It is clear that CI-1, the ontological probability assumption, is the radical anti-theistic claim, and that all of the difficulties of the CI center on the insistence that the wave function *pertains to a single particle* quantum system. The consequences of this assumption are the measurement problem and the notorious "acausal" collapse, CI-2, of the wave function. To illustrate the difficulties, let us explore, once more, the two-slit interference experiment. If, as CI-1 states, the electron wave function really is a potentiality wave spreading from both slits in all directions, and the measurement of the wave at the plate S in Figure 1 is a discrete localized scintillation, then the consequence is that the wave function has to collapse in a blink from a wave dispersed over the whole spatial extent of the interference fringe (essentially $y = \pm \infty$) by gathering itself instantaneously from "hither and yon" to condense at the observed location. This is clearly a non-local phenomenon and also non-deterministic in the sense that there is no law which can predict where the particle appears and how the wave function evolves. These observations lead to the measurement problem in the CI, and all of the failed attempts to construct a theory of *physical measurements* within the CI. The battle among the anti-theists is between the Cartesian dualists ("mind and matter" is all that exists), on the one hand, and the material monists and the idealist monists, on the other. In particular it is here that one sees the anti-realists and phenomenologists (who subordinate everything including measurements to the consciousness of the observer) making statements that ultimately lead to the ridiculous conclusion that the "state of the universe" (which to a potential observer is nothing but a bundle of "sense perceptions") depends on the mental health of the observer.

As stated earlier, CI-1 and CI-2 exhibit the confusion between ontology and epistemology. CI-1 blends an ontology with probability (which, as we have seen, is theistically always epistemological); while CI-2, as a consequence of CI-1, must blend ontology with epistemology, by making Bayesian conditionalization a purely physical phenomenon. This suggests that (1) if we exorcise the ontological probability assumption, that is, if the wave function is not a potentia but a physical field, and (2) the measurement problem as expressed by (CI-2) is not just physical but also *epistemological*, then QM can be given a consistent Christian theistic interpretation.

The Heisenberg uncertainty relation

I now come to the interpretation of the well-known Heisenberg uncertainty relation. This relation has a long history of debated interpretations, which, as Popper has pointed out, have contributed to what he appropriately called the "great quantum muddle." [29, p.5].

The subjectivist and idealist trends in quantum mechanics have led to the anti-realist tendency to refer to the uncertainty relation as the "indeterminacy relation." The intent of this change in terminology is to emphasize the anti-realist denial that quantum particles have simultaneous positions and momenta — that is, the quantum particle cannot even be said to have a "classical trajectory." Or, in a less anti-realist (though still irrational) vein, we have the sentiment of Hans Reichenbach [33] that the indeterminacy is rooted in "causal anomalies." The subsequent analysis presented here, based in part on Brody and Popper, will describe the extent of the "muddle" and the solution.

The uncertainty relation states that, in principle, it is impossible to simultaneously determine the *post-measurement* position and momentum of a particle. This destroys mechanistic determinism, for no initial

data can be found to predict the future (since we cannot collect the initial data for a Cauchy surface). We note that this is different from the limits imposed by chaotic dynamics which places a practical limit on predicting the future of mechanical systems, i.e. we can predict the future development of a sufficiently small system provided we are willing to pay the cost of obtaining the required precision. QM, on the other hand, says that there is a limit beyond which no expenditure of additional effort can produce any increase in knowledge.

Disturbance interpretation

Heisenberg explicitly bases his interpretation of the uncertainty relation upon the disturbance of the physical system by the act of measurement. In his remarks on "Illustrations of the Uncertainty Relations" [18, p.20], he states:

> This may be expressed in the concise and general terms by saying that every experiment destroys some of the knowledge of the system which was obtained by previous experiments. This formulation makes it clear that the uncertainty relation *does not refer to the past*; if the velocity of the electron is at first known and the position measured, the position for time previous to the measurement may be calculated. *Then for these past times* $\Delta p \Delta q$ *is smaller than the usual limiting value* . . .." (emph. added).

However, we also note Heisenberg's concession to anti-realism when he immediately remarks [18, p.20], "It is a matter of personal belief whether such a calculation concerning the past history of the electron can be assigned any physical reality or not." It seems ironic that Heisenberg would make such a statement when his whole argument has just hinged on the basis of the disturbance interpretation. The question in this author's mind is: How can one physically disturb something that is not real? How could any science be done when all measurement must necessarily be based upon past histories, and yet these past histories are "not real?" This lapse again shows how even high-caliber physicists like Heisenberg can succumb to the quantum "muddle"!

Central to the interpretation of the uncertainty relation is the meaning of the variance or "deltas" in the formula

$$\sqrt{\langle \Delta Q^2 \rangle}\sqrt{\langle \Delta P^2 \rangle} \geq \frac{\hbar}{2} \ . \tag{9}$$

Any empirical test of this relation would require that frequencies of precise simultaneous position and momentum values be computed. Thus, it is clear that this is an ensemble verification and not a singular case. Every verification of the Heisenberg relation requires statistics over an ensemble *in which both P and Q have been measured*, so where does it require the relation to hold for the singular object? When we review the proofs of this relation we note that it applies to any quantum state, and that means "unreduced." Yet the empirical proof of this relation requires reduction of the wave function by a measurement. The conclusion is inescapable — as pointed out by Popper and Brody [9, p.162] — this relation *pertains to an ensemble* of particles, *not to a single particle*. The variances that appear cannot apply *per se* to a limitation on simultaneous measurement of position and momentum. If the formula means that the momentum and position cannot be simultaneously measured to sufficient precision, then the relation would not be experimentally testable.
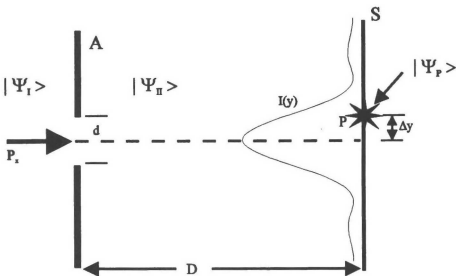

Figure 2 Quantum Diffraction Experiment

We should note that even measurements of momentum rely upon *measurements of position* and retrodiction of momentum which requires an assumption of continuous trajectories. In the single-slit diffraction experiment, illustrated in Figure 2, the momentum is related to the deflection distance Δy via

$$P_y = m\frac{\Delta y}{\Delta t} \ . \tag{10}$$

Of course, this measurement of momentum is a retrodiction, yet nonetheless, it ascribes *a simultaneous position and momentum* to the particle that strikes the screen at point P in

Figure 2, for at least some time interval shortly prior to the impact. These measurements satisfy the relation

$$\Delta p \Delta q \ll h , \qquad (11)$$

which is contrary to the Heisenberg uncertainty relation. Of course, to determine the momentum of the electron *after the impact* would require a second screen, and that screen would also exhibit a scatter relation in agreement with the Heisenberg uncertainty relation as interpreted by the ensemble interpretation (to be described below). As Popper [29, p.63] states, "But once we ascribe physical reality to measurements for which, as Heisenberg admits, $\Delta p \Delta q \ll h$, the whole situation changes completely: for now there can be no question whether according to the quantum theory, *an electron can 'have' a precise position and momentum. It can.*"

What about those "quantum jumps"?

"If we have to go on with these damned quantum jumps, then I'm sorry that I ever got involved." *E. Schrödinger*

The collapse of the wave function under a measurement is sometimes called the non-existence of "classical" trajectories. The statement is often made that QM implies that particles have no trajectories in the classical sense of the term. Further, this statement is made in the context of the Copenhagen interpretation with its acausal collapse assumption and the false interpretations of the Heisenberg Uncertainty relation. At this point, I will show that postulate CI-3 of "non-disturbance" of states (or "repeatability of measurements) as applied to a dynamical variable is logically inconsistent with the acausal collapse assumption. In other words, non-disturbance of measured states implies that there are no "quantum jumps" or "acausal collapses" in the real (i.e. mechanical) sense asserted by CI in the assumption (CI-2) above. Quantum mechanics does deny *mechanistic determinism*, and thus there is no exhaustive equation of motion which we can integrate forward in time to obtain a future mechanical state — yet, this does not imply that the trajectories are *discontinuous* — rather, QM implies a sort of Brownian-like motion of particles, i.e. the trajectories are continuous, but the first derivatives are not continuous[11]. It may be further remarked that this continuity is explicitly employed in the path integral formulations of quantum mechanics, inasmuch as the "sum over histories" is always performed over a space of *continuous trajectories*.

Repeatability of measurements (CI-3) means that if one measures the value of a conserved quantum quantity, then a subsequent measurement will reveal the same value. Or, if one measures a dynamic observable such as the position Q, then the following limiting condition must obtain:

$$\lim_{\delta t \to 0} (\hat{\Pi}_Q \hat{U}(t + \delta t, t) \hat{\Pi}_Q - \hat{\Pi}_Q) |\psi(t)\rangle = 0 , \qquad (12)$$

where U is the unitary time evolution operator and

$$\hat{\Pi}_Q = |Q\rangle \langle Q| \qquad (13)$$

is the non-unitary (idempotent) projection operator corresponding to the result of a position measurement with value Q. Equation (12) states that if we measure the position of a particle at a given time and compare it with the position measured a short time later, then the difference approaches zero as the time interval approaches zero. This is equivalent to the standard "epsilon-delta" proof of continuity. The limit is clearly zero since the unitary operator U is continuous in time. This proof can be repeated by applying the same steps in the momentum representation, the conclusion then being that the momentum also changes in a continuous (though unpredictable) manner as a result of measurement. This empirically verified "recipe" is accepted by the CI without realizing the logical consequence: *the "quantum object" has continuous trajectories*. We should also point out that the continuity of trajectories is also empirically verified repeatedly by cloud chamber and spark chamber tracks. Based on these observations, we conclude, as expected, that CI-1 must be rejected or modified.

These observations thus demonstrate that the commonly held view of "quantum jumps" as applied to canonical position and momentum is incorrect. But what of "quantum jumps" between stationary states, as characterized by energy? It was in this context that some of the original debates concerning jumps were argued; in particular, the energy transitions of atoms in thermal equilibrium with external electromagnetic fields. The question was raised whether the electrons "jumped" instantaneously from one orbital to another as it either absorbed or emitted quanta of photons.

Suppose that an electron is in a pure energy eigenstate prior to time t_1:

$$|\psi(t)\rangle \sim e^{\frac{-iE_n t}{\hbar}} |E_n\rangle. \tag{14}$$

Suppose now that the atom is coupled to the electromagnetic field by a time-dependent interaction

$$\hat{H} \sim \hat{H}_0(p,q) + \hat{V}(p,q,t), \tag{15}$$

where the time-dependent potential, V, is non-zero only for $t_1 \le t \le t_2$. In this case the issue of which stationary energy states apply during the interaction interval is not so clear cut. The usual prescription for dealing with such issues, for example in quantum scattering theory, is to ignore the question of the state during the interaction. What is asked instead is what are the probabilities of transitions between the asymptotic non-interacting stationary states, at $t=-\infty$ and $t=+\infty$. These stationary states can be clearly defined; however, ignoring the issue of states during the transition is not to deny the essential continuity of the trajectories during the interaction. That this transition is continuous again follows from the fact that the state evolves via the unitary Lie transformation (necessarily continuous in the group parameter t)

$$|\psi(t'')\rangle = U(t'',t')|\psi(t')\rangle = \exp\left(-\frac{i}{\hbar} \int_{t'}^{t''} H(t)\,dt\right)|\psi(t')\rangle. \tag{16}$$

DEBROGLIE-BOHM "PILOT WAVE" THEORY

This interpretation (for a detailed exposition, see [8]) avoids most of the conundrums of the Copenhagen class of interpretations, and if it is divorced from Bohm's pagan conceptions — which are not intrinsic to the interpretation — it seems to merit further investigation and a study of its features which can be given a consistent Christian theistic construction. It posits both particle *and* wave. The wave is a "pilot" wave that exists *in addition* to the particle. In the case of the double-slit experiment, the wave given by

$$\Psi(x,t) \sim R(x,t) \exp\left(\frac{iS(x,t)}{\hbar}\right) \tag{17}$$

passes through both slits but the particle only goes through one. The particle experiences a "quantum" force due to the presence of the wave which acts as a "quantum potential" given by the equation

$$Q = -\frac{\hbar^2}{2m} \frac{\nabla^2 R}{R}. \tag{18}$$

The net result is that the force acts to move the particle into regions of higher $\rho = |\psi|^2$ and away from regions of lower ρ. This interpretation can thus account for the interference fringes and the discrete particle nature of the electron itself. In this theory the actual trajectory is deterministic and causal, the final position being predetermined by the electron's initial conditions. The theory then requires that

$$\rho(0) \sim |\Psi(t=0,x)|^2 \tag{19}$$

also represent the likelihood of the electron being prepared in the specified initial state. In this regard the interpretation shares the features of the ensemble interpretation to be discussed later. The theory is incomplete. We need not worry about this; whether there is a lower level accessible to man is irrelevant to the debate between Christians and unbelievers.

As for the supposed objection that the "pilot wave" is of an unusual sort in that it does not carry energy in the usual sense, we need only note that when we go to the relativistic version of quantum field theory (QFT), the field there plays the same theoretical role as the deBroglie wave. To clarify this, recall that in QFT of a scalar field, the Hamiltonian of the field is given by

$$\hat{H} = \frac{1}{2}\int[\dot{\varphi}^2 + (\nabla\varphi)^2 + m^2\varphi^2]d^3x = \sum_k \hat{N}(k)\omega(k). \tag{20}$$

Thus, the energy is solely "carried by" the particles; yet, the field represents the "amplitude" for "creation" of particles in the various momentum states.

This concept of "particle *and* wave" removes the objections due to the paradoxical conclusions of "wave-particle" duality. Further, it can account for particle "interference" phenomena. These features are compatible with Christian theism and provide a basis for a Christian theistic interpretation of QM, to be presented below.

EVERETT-WHEELER MANY WORLDS INTERPRETATION

This interpretation need not detain us long. It is clearly anti-theistic and reductionistic. This view — entertained as an alternative to the difficulties of the CI collapse assumption and measurement problem — is the conclusion one would arrive at in order to maintain linearity of the wave equation and to exorcise the interpretation of the irrationality of the wave function collapse, while maintaining the ontological status of Ψ as a probability function. So, rather than abandoning the ontological assumption of the Ψ function, this interpretation elevates Ψ to "all there is." According to this view, Ψ never collapses but evolves linearly according to the universal Hamiltonian which includes all measurement interactions. As Ψ evolves, all possible worlds which could result from measurements actually do occur. The universe bifurcates continuously and infinitely into "alternate" history universes which have no causal connection with each other.

In brief, any solution of the Schrödinger equation *always describes the state of affairs.* Reality actually is the ensemble or totality of "all possible worlds." In this worldview, all things which can happen (provided they are consistent with the Hamiltonian and the initial wave function Ψ(t=0)) in fact do happen.

It is clear that this is an irrational view for rational beings to hold. For in this view, there are worlds in which "beings" irrationally ascribe to the many worlds view — not because it is "logical" — but rather, because the bifurcation of all quantum states has placed the "believers" of the many worlds view in a quantum state that corresponds to this "belief." In this particular "possible" world, they must, by necessity, believe the many worlds view. This is so because it is one of all possible worlds. In another "parallel universe" there are "beings" who do not believe this to be the case. This is another worldview where rationality is just an illusion. With these few remarks we will proceed to the next aberrant view.

QUANTUM LOGIC INTERPRETATION

One attempt to circumvent the problems of the Copenhagen interpretation is the "quantum logic" interpretation. Fortunately, this view has not attracted many adherents, and our discussion of this interpretation, as with the many-worlds view, need not detain us long.

Some of its proponents will make the outlandish claim that the "true" logic of reality is non-Boolean, and that we should beat a hasty retreat from "dated, man-made" Aristotelian (or "classical") logic. We can dispense with quantum logic by observing first that quantum "logic" and other such deviant logics are not logics at all. This fact has been commented on by Quine and Haack. The first point to make is that the deviant logics confuse the issue of predication with the laws of logic, per se. Issues of what can be predicated of reality belong to the realm of godly philosophy. As Quine puts it [32, p.85], they confuse knowledge of the single fact of knowing "something to be true or false" with the entirely different issue of two separate statements: (a) "knowing something to be true" *or* (b) "knowing something to be false." As to the claim that deviant logics (of which quantum logic is an example) are the "real" laws of logic which govern *all* processes in the Universe — including our "minds" — and which should replace "classical logic," we merely need to point out that, at the meta-level, (i.e. when discoursing about many-valued logics), we use classical 2-valued logic. This point has been cogently argued by many philosophers of logic. Haack, for instance states "(most) many-valued logics are contained in 2-valued logic (i.e. all of their theorems are theorems in 2-valued logic, and not vice versa)." [17, p.2]. Brody [9, p.251] makes a similar point when he states (emph. added):

> If quantum logic is the only acceptable logic for quantum mechanics (indeed, if any but Boolean logic is required), *the fact that the entire development of quantum theory could occur without it needs explaining*; but to the author's knowledge no such explanation is yet forthcoming. Moreover, the Boolean-based derivation of quantum results should sometimes be wrong, yet no such misleading derivations appear to be known.

Once this is admitted, it follows that the so-called deviant "logics" are not really addressing logic (i.e. modes of valid inference), but rather, they are merely discussing *what can be predicated.* As I pointed out previously, from a theistic point of view, all predication, i.e. statements about reality, has its source in God.

ENSEMBLE INTERPRETATION

This interpretation, presented by Brody [9, pp.159-184], has a lot to say for it. Since it avoids the extreme forms of anti-theistic metaphysics, I believe that it should be investigated and elaborated on within the Christian theistic worldview. The main feature of the CI which the ensemble interpretation is intent on exorcising is the belief that the wave function describes a single quantum system. According to the ensemble interpretation, the wave function is like the probability density in classical probability theory. It does not describe individuals, but rather a population of "identically" prepared systems. Consequently, the ensemble view grants the incompleteness of the quantum mechanical description. As Brody [9, p.178] states: "To sum up, we have seen that, from the point of view of the ensemble interpretation, quantum mechanics is incomplete, that any completion should be stochastic in nature, and that the resulting theory will likely be of a very different character."

From the point of view of the ensemble interpretation, we can immediately dismiss the paradox of that other "macabre measurement problem," viz. Schrödinger's cat. Recall that the Copenhagen school ascribes a "smoky" reality to the state

$$\psi(t) = e^{-t/\tau} |''live \; cat''\rangle + (1 - e^{-t/\tau}) |''dead \; cat''\rangle. \tag{21}$$

However, in the ensemble view, $\psi(t)$ represents the ensemble of "identically prepared" experiments. In half of the experiments the atom has decayed, killing *different* cats, and in the other half the cats are still living.

These remarks should be sufficient to at least show — even if they do not decisively prove — that the ensemble interpretation has enough explanatory power to warrant further investigation and elaboration by Christian scientists. From this summary, note that, as pointed out previously, incompleteness of all humanly stated truths is a necessary consequence of the Biblical revelation concerning man and God — man is finite and his knowledge is true but non-exhaustive (i.e. incomplete). Second, as Christians we would note that any theory which appears stochastic from the human perspective has really touched the limits of human knowledge, and apparently random data is not random in the anti-theistic sense, but really a testimony to the providence of God. Thus, the stochastic aspect of the ensemble interpretation can be given a Christian theistic interpretation, for, as I have pointed out, God is still behind all apparently "random" events, as Prov. 16:33 explicitly states.

THE EPR PARADOX, BELL'S INEQUALITIES AND ASPECT EXPERIMENT

I now examine several quantum mechanical systems which have been widely publicized via popularizations in the press and on television. Such systems have been used to promulgate anti-theistic worldviews to the man in the street. Inasmuch as these systems and the descriptions employed have a prima facie plausibility, they would seem to require that a Christian theistic worldview reject quantum mechanics. However, it has been known for some time — though not widely publicized — that the popularized interpretations which have been used to promote mysticism and parapsychology (such as mental telepathy), based on supposed "spooky action at a distance," are erroneous and that, in fact, *there are classical systems which violate Bell's inequality*. This last observation thus shows the *irrelevance of the Bell inequality* to the debate over the metaphysical foundations of quantum mechanics. Since these discoveries have not been widely disseminated, I will spend some time giving a brief explanation of the issues involved; the reader is referred to the original literature for more details, [9],[25],[36].

Figure 3 is a schematic of the model system investigated by Bell and which forms the theoretical basis that motivated the Aspect experiment. It consists of an atom (at S) in an excited spin-zero state which decays
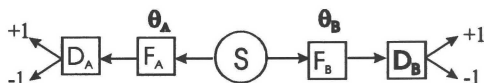

Figure 3 Bohm's Version of EPR Experiment

by the emission of two spin-correlated electrons. The two electrons are emitted in opposite directions toward two filters F_A and F_B which consist of Stern-Gerlach magnets with orientation angles θ_A and θ_B relative to an arbitrary reference direction. Quantum mechanics predicts a correlation between the electron spin components when measured along different spin directions at detectors D_A and D_B. The detectors count particle incidence at the positions indicated by +1 for particles deflected "up" and -1 for particles deflected "down." Denote by (a,b) the paired values of particle counts at (A,B). The theoretical QM value of the correlation, C, of particle counts at detector A and at detector B is given by

$$C_{ab} = \langle \psi | (\sigma_A \cdot \hat{a})(\sigma_B \cdot \hat{b}) | \psi \rangle = -\cos(\theta_A - \theta_B). \tag{22}$$

The Bell inequality (in the form given by Clauser, Holt, Horne and Shimony [11], and derived from assumptions to be stated later) expresses limits on the range of the correlations when the experiment is performed at different values of the angles θ_A and θ_B, indicated by the presence of the primes, thus

$$V = |C_{AB} - C_{AB'}| + |C_{A'B} + C_{A'B'}| \leq 2. \tag{23}$$

The choice of angles: $\theta_A = 0°$, $\theta_B = 45°$, $\theta_{A'} = 90°$ and $\theta_{B'} = 135°$, along with the theoretical QM prediction of eq. (22), leads to the value $V=2\sqrt{2}$, which violates eq. (23).

Assumptions of the Bell inequality theorems

The explicitly mentioned issues and assumptions that typically enter into the derivations of Bell inequalities are the following (cf. Jarrett in [13]): (1) conservation (correlation); (2) locality (or "factorizability"); and (3) completeness. Sometimes these last two are called "separability" and "hidden variables." The argument goes as follows: Since theory and experiment violate the Bell inequalities, one or more of the assumptions must be false. The conclusion is then drawn that reality is either not local, or quantum mechanics is not complete. (Note that rejection of completeness is a perfectly acceptable theistic conclusion. Conservation is not a candidate for rejection since it is part of the "description" of the type of experiments under consideration.) However, Brody and others have noted that there is one "hidden" assumption not listed above. This is the assumption that the quantities being measured (assumed to be "random") *possess a joint probability distribution*, or what is equivalent, *that the quantities under consideration are capable of being jointly measured* — this, following Brody, will be referred to as the *joint measurability assumption*, or JMA, for short.

Brody [9, p.232], in particular, outlines four derivations of the Bell inequality: (1) Bell's original proof, (2) Wigner and Holt, (3) Eberhard (1977) and Stapp (1971), and, finally, (4) Suppes and Zanotti [37]. I will briefly outline and discuss these four derivations, following the discussion given by Brody [9, p.231]. In place of Bell's original proof, I will give my own "rotational symmetry" derivation of Bell's inequality, assuming the spin itself is the "random" input variable for the random outcomes at detectors A and B. In this form the proof is more general; a deterministic model can be obtained by setting the outcome probabilities to 1. I will also give a longer discussion of the Suppes-Zanotti probability theoretic derivation since it more clearly demonstrates the minimal core of assumptions necessary for the Bell inequality to be satisfied, and is, thus, worth repeating in detail. The reader is referred to Brody [9] and the references therein for a fuller discussion of the original derivations by Bell, Wigner-Holt, and Eberhard-Stapp.

Under the assumption of completeness, the joint probability of the result "a" at A and "b" at B is obtained by using the conditional probabilities averaged over the density of the random hidden variable λ. Thus,

$$p(a,b|\theta_A,\theta_B) = \int p(a,b|\theta_A,\theta_B,\lambda)\rho(\lambda)d\lambda. \tag{24}$$

Here, λ represents the hidden common cause of the correlations, and is the variable which "completes" (and exhausts) the physical description.

The assumptions of locality and separability allow the integrand to be expressed by the relation

$$p(a,b|\theta_A,\theta_B,\lambda) = p(a|\theta_A,\lambda)p(b|\theta_B,\lambda). \tag{25}$$

It expresses the condition that the result at each measuring device is dependent only upon its local setting and the common cause denoted by the "hidden random" variable λ. In other words, the measured value "a" at A is independent of the setting of the filter at B, and vice versa. Thus, the correlation function is computed via the expression

$$C_{ab}(\theta_A,\theta_B) = \langle ab \rangle = \sum_{a,b} \int ab\, p(a|\theta_A,\lambda)p(b|\theta_B,\lambda)\rho(\lambda)d\lambda. \tag{26}$$

Finally, correlation is expressed by the relation

$$p(a,b|\theta_A,\theta_A) = \begin{cases} 1 & \text{if } ab = -1 \\ 0 & \text{otherwise}, \end{cases} \tag{27}$$

which states that when the magnets are positioned at the same angle, the results are always perfectly anti-correlated.

In the next section these formulae are applied to a proof of the incompatibility of these assumptions with the experimental results which agree with eq. (22).

A rotational symmetry proof

Again, we assume that the system decays into two spin-correlated pairs, and that the spins are the completely specifiable "hidden variable." The probability can then be written as

$$p(a|\hat{a},\hat{s}) - \sum_l f_l(a) P_l(\hat{a}\cdot\hat{s}). \tag{28}$$

This is the probability that the particle will be deflected in the $a = \pm 1$ direction when the particle's spin is in the direction of unit vector s and the magnet is aligned in direction of unit vector a. Here P_l is the lth Legendre polynomial.

The correlation is then given by

$$C_{ab} - \langle \sum_{ab} abP(a|\hat{a},\hat{S}_a) P(b|\hat{b},\hat{S}_b) \rangle_{\hat{S}_a, \hat{S}_b}, \tag{29}$$

where the separability and locality assumptions have been used to factor the probabilities. By the "correlation" assumption, the spins incident at detectors A and B are related as

$$S_a - - S_b - S. \tag{30}$$

Equation (29) can then be simplified to

$$C_{ab} - - \langle \sum_{ab} abP(a|\hat{a},\hat{S}) P(b|\hat{b},\hat{S}) \rangle_{\hat{S}}. \tag{31}$$

Again, assuming that the spins are emitted isotropically so that the probability density of a given spin direction on the unit sphere is uniform, we obtain the following:

$$C_{ab} - - \frac{1}{4\pi} \int \sum_{ab} abp(a|\hat{a},\hat{s}) p(b|\hat{b},\hat{s}) d\hat{s}$$

$$- - \frac{1}{4\pi} \int [p(+1|\hat{a},\hat{s}) - p(-1|\hat{a},\hat{s})][p(+1|\hat{b},\hat{s}) - p(-1|\hat{b},\hat{s})] d\hat{s}. \tag{32}$$

Integrating over $\hat{s}$ and using the orthogonality properties of the Legendre polynomials straightforwardly leads to the following:

$$C_{ab} - - \sum_l \frac{[f_l(+1) - f_l(-1)]^2}{2l+1} P_l(\hat{a}\cdot\hat{b}). \tag{33}$$

QM theoretically predicts and the experimental results (eq. (22)) imply

$$f_l(+1) - f_l(-1) - \begin{cases} 0 & \text{if } l \neq 1 \\ \sqrt{3} & \text{if } l - 1, \end{cases} \tag{34}$$

but the requirement that

$$p(+1|\cdot) + p(-1|\cdot) - 1 \tag{35}$$

for all values of a magnet's orientation implies

$$f_l(+1) + f_l(-1) - \begin{cases} 1 & \text{if } l - 0 \\ 0 & \text{if } l \neq 0. \end{cases} \tag{36}$$

Equations (34) and (36) together imply

$$p(+1|\hat{a},\hat{s}) - \frac{1}{2}(\sqrt{3}\hat{a}\cdot\hat{s} + 1) \tag{37}$$

and this clearly violates the probability bound $0 \leq p \leq 1$ whenever

$$|\hat{a}\cdot\hat{s}| > \frac{1}{\sqrt{3}}. \tag{38}$$

In this form of the proof, it is clear that not only are locality and separability assumed, *but also that there is a joint probability function for all three cartesian components of the spin vectors.* It is this last feature of the proof which is responsible for the violation of the Bell inequality, and this assumption is a feature of QM which is present at the outset, for QM requires that probabilities such as

$$p(\hat{S} \cdot \hat{a} - a \& \hat{S} \cdot \hat{b} - b), \qquad (39)$$

when the vectors a and b are not parallel, are incompatible. Thus, the proof cannot proceed even beyond the first step. Therefore, it is clear that the Bell inequalities have nothing to do with non-locality or "spooky" action at distance. All that QM specifies are "transition" probabilities of the form

$$p(\hat{S} \cdot \hat{a} - x \mid \hat{S} \cdot \hat{b} - y), \qquad (40)$$

and these are specifiable regardless of the unit vectors a and b. Finally, we should note that equation (40) is consistent with the disturbance model for measurements. When an electron's spin direction is measured in a sequence of measurements along linearly independent directions, the spin is unavoidably altered to align along the measured directions.

Suppes-Zanotti and Wigner-Holt derivations

The Suppes-Zanotti proof is a pure probabilistic theoretical derivation. It assumes nothing about physical processes which give rise to the random variables. In particular, it mentions nothing about whether the processes are local, separated in space and time, etc. Its only assumption is the existence of joint probabilities for the random variables. Therefore, it is clear that locality plays no essential part in the derivation of the inequality. I now proceed to give an expanded outline of the proof as given in [37].

Denote by X,Y and Z three random variables with zero means and outcomes in the set {-1,+1}. Denote by p(x,y,z) the probability of the specific outcome X=x, Y=y and Z=z. Further, assume that the values of the three expectations, E(XY), E(XZ) and E(YZ) are given. It should be noted that the values for these expectations cannot be arbitrarily assigned. First, it is clear that they must all be bounded by constraints such as: $-1 \le E(XY) \le 1$. Second, it is impossible that E(XY) = E(XZ) = E(YZ) = -1. There is no assignment of probabilities which can realize this. The first equality requires both Y and Z to be the negative of X and thus to be equal, and this contradicts the last equality that Y and Z must be anti-correlated.

There are eight independent values of the probabilities p(x,y,z) to be specified for the values of X= $\pm$ 1, Y= $\pm$ 1, Z= $\pm$ 1. The three correlations plus the three means and the requirement that the probabilities sum to one then yield a system of seven equations for the eight unknown probabilities, namely

$$\sum_{xyz} p(x,y,z) - 1,$$

$$\sum_{xyz} x p(x,y,z) - E(X) - 0 , \quad \sum_{xyz} y p(x,y,z) - E(y) - 0 , \quad \sum_{xyz} z p(x,y,z) - E(z) - 0 , \qquad (41)$$

$$\sum_{xyz} xz p(x,y,z) - E(XZ) , \quad \sum_{xyz} yz p(x,y,z) - E(YZ) , \quad \sum_{xyz} xy p(x,y,z) - E(XY) .$$

The requirement that the arbitrarily selected eighth probability, say p(+1,+1,+1), satisfies the probability bound

$$0 \le p(+1,+1,+1) \le 1, \qquad (42)$$

then yields a constraint on the correlations

$$-1 \le E(XY) + E(YZ) + E(XZ) \le 1 + 2\min(E(XY), E(YZ), E(XZ)). \qquad (43)$$

This equation can be rewritten as four separate, independent equations obtained by considering three out of a larger set of four random variables, say w,x,y, and z. One equation involves {x,y,z}, another {w,y,z}, etc. Combining these four equations then yields eq.(23). It should be noted that in this form, the proof requires a joint probability function for measurements performed on the same particle.

The Wigner-Holt proof is essentially the same as the Suppes-Zanotti proof. Rather than considering the eight values of the joint probability function of three random variables, it looks at the sixteen values of the joint probabilities of the four results a,a',b,b'; results a and a' at detector A using the settings θ_A and $\theta_{A'}$, and results b and b' at detector B using the settings θ_B and $\theta_{B'}$. The proof then proceeds along similar lines, assuming only a joint probability function; but it assumes a joint probability function for four counterfactual events. For example, a given probability value

$$p(a=+1, a'=+1, b=+1, b'=+1) \qquad (44)$$

gives a putative probability that we will measure all four values as +1. But it is impossible to empirically obtain a result at detector A with a given setting and empirically pair it with results at detector B, *supposing the run had been made with different settings.*

To reiterate and summarize: Each of these proofs of the Bell inequalities assumes progressively less, and it thus becomes clear that the common ingredient of all the proofs is the existence of a joint probability function. This is emphasized by the Suppes-Zanotti derivation whose only assumption is the existence of a joint probability distribution. It makes no assumptions regarding the physics such as locality, finite speed of light, or separability. The Suppes-Zanotti theorem states that the sole necessary and sufficient condition for the existence of a joint probability density is the existence of Bell-like inequalities. Both the Aspect experiment and the theoretical prediction of quantum mechanics violate the Bell inequalities. We thus deduce that there is no joint probability function which can generate the putative four correlations

$$C_{AB} \quad C_{A'B} \quad C_{AB'} \quad C_{A'B'} . \qquad (45)$$

But, *we already knew this!* Since the components of spin are non-commuting observables, they are also *not simultaneously measurable.* The empirical results of the Aspect experiment say nothing more than that the experiment is measuring incommensurate quantities. This is all we can conclude — it does not imply non-locality, action-at-a-distance, mental telepathy, solipsism, or any of the other claims of expositors of the "mysteries of quantum mechanics" as revealed by the Aspect experiments[12].

As for incommensurate quantities, they are not limited to quantum mechanical systems. There are many examples of pairs (x,y) of classical or macroscopic "random" variables for which a joint probability p(x,y) does not exist. Recall that p(x,y) represents the probability that both events x and y are *realized* and are *pairable in an unambiguous way.* One example of a non-quantum system for which joint probabilities do not exist is the singular probability for lifetimes of light bulbs which are run at different specified voltages. For instance, let the variable x represent the event, "the light bulb lifetime ≤ x when operated at a constant voltage of 60 volts," and y represent the event, "the light bulb lifetime ≤ y when operated at 120 volts." It is clear that there is no joint probability for this set of events. A light bulb, under the specification of the experiment, cannot have two lifetimes at two different voltages. The measurement of the lifetime at 60 volts precludes the measurement of its lifetime at 120 volts — and this type of situation is included in the more general case where the measurement of one quantity disturbs the value of the other quantity for which a joint probability is to be determined. So, any pair of events (x,y) which are incompatible in the sense that they cannot coexist (such as the lifetimes of light bulbs obeying classical physics, or conjugate variables such as independent spin components of an electron in quantum mechanics) will result in the impossibility of a joint probability function — and this, in turn, is a necessary condition for a possible violation of Bell's inequalities.

This observation — that the central feature leading to the violation of the Bell inequalities was not any of the traditional assumptions of locality or causality, but rather the absence of joint measurability — leads to the possibility of classical systems also exhibiting violations of the inequality, and indeed *classical systems have been found which violate Bell's inequalities.* These are mentioned by Brody in several of his articles (cf. [9, pp.217,236]), and I refer the reader to the original literature [25], [36] where some of these classical systems are described.

A Christian realist interpretation

Based upon the prior discussion, we can begin to construct a Christian realist interpretation. This interpretation is built upon the features of the deBroglie-Bohm interpretation, the ensemble interpretation and relativistic quantum field theory (QFT) which are consistent with Christian theism. In summary, the features of a Christian theistic interpretation are as follows: First, CI-1 is rejected. Instead, along the lines of the deBroglie-Bohm and QFT, the wave function is a *real physical* wave. This wave *never collapses*. Thus, we can account for the observed interference patterns in a rational but incomplete manner. Second, QM is incomplete and there never will be a complete description. (There may be replacements for QM which are richer and are covering theories for QM, but those theories will also be incomplete.) Third, measurements always result in a disturbance of a physical system in an inscrutable manner, such that incommensurate (i.e. conjugate observables) cannot be simultaneously predicted or controlled. The wave function of QM represents the limits of humanly attainable knowledge and control of physical systems. Preparation of a physical system in a state, followed by measurements of incompatible observables, A, B, etc., always results in unpredictable outcomes determined by God. The state is not

a *sufficient* causal predecessor of the observed effects. The "acausal" collapse assumption (CI-2) is not acausal, but rather unpredictable. In this sense, CI-2 is really an epistemological construct, similar to Bayesian conditionalization. Remember that "unpredictable" is not a synonym for "uncaused." Consequently, particle trajectories are continuous but cannot be exhaustively determined by man, and are thus unpredictable.

SUMMARY

The conclusions of these discussions are the following: First, the usual interpretations of the CI in terms of anti-theistic metaphysical assumptions are not a necessary consequence of the QM formalism, but are the result of the additional assumptions of CI added to QM. The outlandish claims of paranormal phenomena, due to misinterpretations of Bell inequalities, are unwarranted. Second, when purged of the anti-theistic assumptions, the experimental results and theoretical predictions are consistent with Christian theism. Third, a Biblically based Christian theism actually reveals a QM-like limitation on man's knowledge. The necessary incompleteness of QM, in particular, and all physical theories, in general, is known from special revelation. Further, the theory of QM implies absence of *physical* causalism (not causation in general) and epistemological limits on man — the exact presuppositions of Christian theism. QM does not deny ultimate causation by God. Therefore, QM is, in fact, consistent with the revealed truth of God controlling all things! Rather than revealing a "God of the gaps," quantum mechanics is consistent with the fact that there are no gaps for God to fill, for, in fact, *God fills all in all*.

A CRITIQUE OF "COMMON SENSE SCIENCE"

Before I begin my critique of "Common Sense Science," (hereafter CSS) I wish to state that the men in this organization say much with which I agree, in principle. The points of agreement are:

1. The foundational authority of Scripture;
2. The criterion of the superiority of "covering theories." That is, first, if theory A, using fewer primitive notions, can explain some unexplained feature of (an accepted) theory B; and, second, if theory A explains all other features of theory B, then theory A is the superior and preferred theory. (This principle can be established from Scripture, but to do so is beyond the scope of this essay. It should also be stated that this criterion says nothing about the possibility of, or requirement that, such covering theories exist.)

However, CSS rather than grounding their critique and rejection of QM and relativity solely upon Biblical principles, goes on to invoke other abstract principles as reasons to reject quantum mechanics and the theories of relativity. These principles are, primarily: (1) "common sense," (2) "Mach's criterion," and (3) rejection of "idealizations." In regard to idealizations, there is the further criticism of the "point particle approximation," and a distinction between "mathematical" models as opposed to "physical" models. Also implicit in some of the argumentation is an appeal to *physical* causalism — though this appeal seems to be invoked more as an objection to the irrational acausalism expounded by the radical proponents of the Copenhagen interpretation. Before I proceed, let me say that I am not suggesting that the members of CSS are self-consciously appealing to principles that are Thomistic or Arminian. However, to the extent that they do adopt the premises and the consequences of those schools of thought, they are not using a *consistently* scripturally based theistic apologetic. It is the approach of appealing to these non-theistic "principles" with which I disagree.

In addition to disagreement on these foundational approaches, there are a number of technical areas of disagreement which space does not allow to be covered here. But one technical issue which I will discuss is the question of stability of Bergman's classical electromagnetic ring model of elementary particles (cf. e.g. Bergman [6], Bergman and Wesley [5]).

Appeals to "common sense" and Mach's criterion

To begin, we will first consider the appeals to "common sense" and Mach's criterion (cf. [4]). These criteria are appeals to authority. To state the question succinctly: To whom or what can Common Sense Science appeal to justify the authority of "common sense" and "Mach's criterion?"

Consider first the appeal to "common sense." Though this appeal is primarily to "establish" the existence of "objective reality," CSS admits this is an *assumption* and *unprovable*, yet *reasonable*. As CSS states in their web pages [12]: "Common Sense Science is based on three unprovable but reasonable

assumptions: "Reality . . . Causality . . . Unity . . .." This admission is the result of abandoning a thoroughly theistic presuppositional approach to all knowledge. CSS — rather than building a foundation from the start on the presupposition of God — instead starts by building a foundation upon abstract principles. To admit that the assumptions are unprovable is a concession which will make the resolute skeptic smile, as he could ask for nothing more than this! It concedes the central presupposition of skepticism, which states that in the realm of knowledge, nothing is certain. Also, the statement that these assumptions are "reasonable" is to beg the question — whether they are "reasonable" is the precise issue which the resolute skeptic will want verified by "laws of logic" which he will accept.

"Common sense" is not the answer to David Hume's devastating arguments against realism (which are unanswerable on anything but Biblically based Christian theism, and this defended presuppositionally). Further, though I grant that an *initially* unproven "assumption" is permissible as a proximate starting point in any argumentation, yet, the fact that there is an objective reality is *not unprovable* since there is an ultimate and absolute starting point, namely, God. The proof of the existence of objective reality is as certain as the existence of God — for there is an absolutely certain proof of God's existence. In a nut shell, the "transcendental" proof of God's existence is: "God exists because of the impossibility of the contrary," (cf. Van Til [39, pp.100-1],[40, pp.205-6,222-3]). The approach of CSS is an example of the Thomistic approach of appealing to abstract principles *first*, then, after the fact, discussing the "possibility" or "probability" of God's existence — as if, without God, there could be a reality, causality, and unity, and, we should mention, rationality and the immaterial laws of logic. (The point is: Reality, causality, and unity in the created realm are derivative from God, and they are intelligible only on the presupposition of the God of Christian theism.) A God that just "possibly" exists is not the God of Scripture. Such a "god" is one whose "possible" existence rests upon neutral, independent laws of thought and abstract principles which are apparently rooted "in a vacuum." This is an unacceptable approach to defending the faith. Appeal to principles that are rooted "in a vacuum" is unintelligible and places one on anti-theistic ground. The laws of logic do not rest "in a vacuum" or in "neutral territory" between the anti-theist and theist "camps." There is no "vacuum." All of reality is in the theist "camp," and this includes the abstract laws of logic. The laws of logic are constituted in man's rationality in accordance with his being created in imago Dei. The laws of logic as placed in man are rooted in God and are thus reflective of God's rationality. Therefore, we conclude that the "reasonableness" to which CSS appeals is provable — but only on the presupposition of the God of Christian theism.

We should also note that the appeal to "common sense" in any argument is a double-edged sword, and one risks being thrown by his own petard when employing it. To the extent that "common sense" is intended to mean "rational" (in the Christian theistic sense that man can attain to a rational but non-exhaustive understanding of God and creation), the view can be supported by Scripture, as presented in Part I. But in this case, the appeal then becomes an appeal to the correct foundation, namely, the Scriptures, and not to "common sense." On the negative side, once one has appealed to "common sense," the question arises as to when to limit its "authority." To what higher principle do we appeal to limit common sense only to the question of objective reality and not other issues? In fact, an unlimited appeal to common sense was the universal principle of the era of Enlightenment — "enlightened common sense" was the ultimate judge of truth. The fruit of this philosophy was the promulgation of deistic philosophy, such as that espoused, for example, by Thomas Jefferson[13]. In the case of Jefferson, he believed that Reason was sufficient to judge that the Hebrew scriptures were not God's special revelation, and he thereby rejected the Hebrew scriptures. Similarly, he rejected the deity of Christ, miracles, and the Trinity. Jefferson called his religious beliefs "rational Christianity." We thus see, in this regard, that if one is intent on retaining a belief in inspired special revelation but also an appeal to common sense, one must adopt the Thomistic notion of two realms and methods of knowledge — reason which adjudicates truth in the natural sphere, and faith (not subject to "common sense") which adjudicates truth where reason must be curbed.

Next, consider the appeal to "Mach's criterion," which as quoted in [4], is: "Only those propositions should be employed in physical theory from which statements about observable phenomena can be deduced." Mach's criterion is clearly the first principle of *logical positivism*. E. Mach, generally considered to be the father of the seminal idea of logical positivism, held to a bare phenomenalism. As a radical phenomenologist, Mach attempted to exorcise all "metaphysics" from science. On one hand, Mach's extreme phenomenology led him to embrace a *form of idealism*[14] and a form of a static Parmenidean world in which nothing really happens, in fact, a world in which there is no cause and effect. To quote Mach himself [20]: "There is no cause and effect in nature; nature has but an individual existence; nature simply *is*. Recurrences of like cases exist only in the abstraction which we perform for the purpose of mentally reproducing the facts."

Inherent in Mach's view, as embodied in the "criterion," is the idea that all knowledge is "scientific" and can only be discovered and judged through "sense phenomena" via the "operationalism" of the "scientific method." As a result of this outlook, theological statements were not even considered to be false, just meaningless, (i.e. theology is "metaphysical" in the pejorative sense). As Gordon H. Clark [10, p.199] rightly observes in referring to Laplace, Haeckel and Mach: ". . . all . . . in one way or another construct a worldview that makes theism impossible." Mach, "the scientific skeptic," is not the "authority" to appeal to when building a consistently theistic worldview.

The appeal of CSS to Mach's criterion is also a relapse into the appeal to an abstract principle in accordance with the rationalism of Thomistic and Arminian thought which accords autonomy to man. It is important to note that Lucas repeatedly uses Mach's criterion as the standard to judge that various modern theories are "non-scientific," (cf. [22, e.g. pp.21,27, etc.]). First, we should note that this is an acceptance of the idea that there is a compartment of knowledge disjoint from theology and other knowledge, in general, that can be termed "scientific." As pointed out in Part I, this distinction is rooted in Thomism and is not theistic. Second, for the sake of argument, if we suppose that there is a class of "scientific" facts and knowledge, we can ask the question: Is Mach's criterion itself "scientific?" The answer, according to the criteria of its own standard, is "no!" Mach's criterion cannot be found written on a rock or seen through a microscope. This is the Achilles heel of Mach's worldview and of logical positivism — *by its own declaration it is meaningless*. This brings us once again to the conclusion that there is only one method for judging the truth of physical theories, namely: Are they consistent with Christian theism as revealed in Scripture?

Mechanism and reductionism

Other parts of the CSS approach are direct appeals to mechanistic explanation. To see this strain in some of the writings of Lucas, consider the following quote [22, p.7]:

> ". . . to allow identifications of *mechanisms* in agreement with scripture by which God *could* create the universe, daily sustain it in the most minute detail, and perform miracles." (emph. added)

I certainly agree with Lucas in his desire to develop a Christian philosophy of science, and again to subordinate science to special revelation so that it is "in agreement with Scripture." However, the desire to identify a mechanism is reductionistic and mechanistic. For example, a similar argument could be made in the case of the nature of man, with the aim to identify mechanisms of "will" and "mind." But such a program would place one dangerously close to "behaviorism." This desire to find mechanisms is an appeal to the very same principle which undergirds the atheistic worldview which Lucas rightly condemns. And that principle is the search for naturalistic and exhaustive knowledge within creation. In the defense of Christian theism no such explanation is wanted. We would, of course, point out that the theistic doctrine of creation explicitly denies the possibility of a *mechanism* by which God *could* create. The creation is a miracle performed by divine fiat; the secret of the creation lies within God. Scripture itself declares that, by Christ "all things consist" (Col. 1:17); and that He upholds "all things by the word of his power," (Heb. 1:3). Scripture provides no more than this and does not include descriptions of "mechanisms" as to "how" God does this. As God manifests Himself and interacts with the creation — whether He does so within the bounds of the physical laws which He has created, or suspends them — all is within His power. Even assuming we could trace back in time all of the physical mechanisms and make infinitely precise measurements of all events which contribute to a coin landing "heads," we know from Scripture that this is still an incomplete description — mechanistic causalism alone does not determine the result! It is in the nature of the case that no mechanism will ever be found. Scripture states that the result is "of the Lord." The continual and omnipresent providence of God, acting both directly and indirectly through the physical laws which He has established, determines the result.

Retreat to mechanistic causality

Lucas and Bergman, rather than rejecting the *confusion* of modern atheistic physicists of causality with predictability, accept this claim at face value and then on this basis reject quantum mechanics. Lucas states [22, p.34]: "Within the context of the wave equation one has a purely statistical interpretation of nature in which the notions of causality are essentially lost." Bergman [3] states, "In the classical theory of science, there is an underlying assumption that events follow the law of cause and effect. Events do not occur at random or spontaneously, but are the result of some cause." I agree with this assessment, that all physical effects have a cause, *but the choice is not between physical causation and no causation*

at all. Rather, as argued in Part I, *not all causes are open to the scrutiny of man* — God is the ultimate cause of all things, and the whole being of God is not communicable unto man. The Copenhagen interpretation in its radical embracing of *ultimate* indeterminism is at fault, *but this is not a necessary requirement of quantum mechanics*. Thus, these arguments for rejecting QM are unpersuasive.

Rejection of certain idealizations

Lucas [22, p.14] states, "Many of the problems with modern science stem from the use of idealizations. . .," of which he identifies and singles out the point particle idealization as the most significant. It is important to emphasize that the point particle idealization is really only an approximation, and, as such, it is routinely employed not only in modern physics but also classical applications, relativistic applications, and quantum mechanical applications. We should also add that relativity is not founded on the point particle approximation, as Lucas states. Extended and composite bodies are and were routinely employed in the development of classical charged particle models, as a literature search will readily reveal. The same comments apply to QM and QFT. Models of many-particle bound states are not antithetical to QM, and the discovery of neutron and proton structure did not come as a "shock" to the physics community. Nor would the discovery of "lepto-quarks" and a discovery of internal electron structure. Such a discovery would only move the question of "point" particles and their stability to another level.

It is true that a point electron model is afflicted with the problem of infinite self-energy, and that it is unstable *if* it is viewed and modeled as an extended body of divisible electric fluid. But the question is why must it be an extended body of divisible electric fluid? Could it not be an irreducible (i.e. atomos) object, held together by Christ? The Biblical answer is: Yes, it could, for by Christ *all things hold together*. On the other hand, the atheist would embrace with relish the discovery of a consistent and *purely physical* stable model of the electron. (It should be added, that such a putative discovery is, of course, an impossibility as Col. 1:17 attests.) Such a putative model would be viewed as a confirmation of the atheist worldview — as for God, the atheist has no need of that hypothesis, for then the electron consists *in itself*.

Lucas identifies certain idealizations as ultimately false. He states [22, p.9], "From the Christian perspective it is important to know . . . what sort of conclusions science comes to as a result of using these idealizations." The anti-theistic worldview does not come at the end of a syllogism starting with an antecedent assumption of the point particle idealization. Rather, the worldview *is* the presupposition. And, as pointed out in Part I, man's knowledge is always analogical and idealized due to his finite creatureliness. The arguments Lucas espouses for rejecting certain idealizations are not persuasive, since the conclusions he draws are not necessary inferences of the specifically identified idealizations.

Distinction between "mathematical" and "physical" models

Another point which is puzzling is the appeal to a distinction between *physical* models as opposed to *mathematical* models. Yet, it is unclear in what sense Bergman's models are less "mathematical." Even Bergman's "physical" models are expressed in mathematical language and are thus idealized descriptions of physically measurable quantities. Bergman's ring model consists of an idealized torus consisting of what is essentially a distribution of "electric fluid." This leads one to the familiar dilemma of reductionism: What is this electric fluid? Is it a continuum? (This too is a mathematical idealization). Or, is it really an aggregate of more fundamental particles? And if so, are they points? This is an infinite regress.

I agree that the Copenhagen interpretation of the wave function as an abstract probability wave approaches the sense of "mathematical" (abstract with no connection to physical reality), to which Bergman objects. But when examined closer, we find that Bergman's argument — though I believe valid in intent — is not as strong as it may seem at first examination. As related above, the problem, and, in fact, the nonsensical portion of the CI, is the raising of an abstract wave function (in a Hilbert space[15]) to the level of physical ontological probability or ultimate chance. To see that the argument based on abstraction alone is not valid, we need only one counterexample. As a counterexample, we need only note that classical probability theory uses abstract mathematics to describe epistemological states or degrees of knowledge. Such mathematical expressions for degrees of belief, statistics, etc., are used in essentially the same sense as the wave function of QM. Both are mathematical expressions of states of mind. Theistically, mind is both real and immaterial. Thus, probabilistic descriptions (theistically understood) — inasmuch as they are descriptive of our mental states of knowledge — are necessarily abstract, for they do not correspond exactly to the outcomes determined by God. A more concrete counterexample from classical physics is the distribution function, $f(x,v)$, of classical statistical mechanics. It is purely mathematical, and it bears a similar relation to the wave function in the ensemble interpretation.

Further, f(x,v) is an epistemological construct, and to the extent that it describes ontology, it describes idealized group behaviors and degrees of ignorance.

We should also note that the electric and magnetic fields of classical electrodynamics provide an example of a similar metamorphosis from *abstract concept* to *ontological existence*. At one time the physical existence of such "field" concepts was not part of "common sense." It has since become part of the standard scientific "orthodox" doctrine to "reify" the electric and magnetic fields, though being vector fields they are to be pictured mathematically as arrays of numbers attached at each point of space. These numbers then came to be pictured as tiny "vector fields" which indicate "directions" at each point in space[16]. But the question is: "A direction in what?" A direction is a *displacement in space*, while an electric field, on the other hand, is not a distance in space since it has units of "volts/m"; and when multiplied by charge ("coulombs") yields the direction of a *force*, which also does not "reside" in space. The field and the force are not the same thing. *Where* does this force reside? It certainly is not a vector attached to the particle and extending so many meters in physical space. This force can, in turn, be converted to acceleration and be integrated to obtain velocities and, finally, positions in space. Thus, we ultimately come to the visualizable *kinematics*. Through this conceptual manipulation of mathematical concepts we come to the *illusion* of explanation — but closer scrutiny reveals mere description. The question remains: *What* are the electric and magnetic fields?[17] Though the well-known arrangement of iron filings under the influence of magnets does lead one to acquiesce to this concept of a field, yet even in this circumstance we can ask — as Mach asked Boltzmann concerning the atom — "Have you seen one?" What one sees in actuality is only the action of the magnet on the filings. So if we are really pressed to describe this field — for example, if we ask the question, "Of what 'stuff' is it composed"[18] — we find that further explanation (or "reduction") is hard to find. In this case, it is well to remember the initial resistance to Newton's gravitational action at a distance, which was called an "occult force" since it was not "action by contact."

A TECHNICAL REMARK ON BERGMAN'S SPINNING RING MODEL

This model is proposed as a replacement to quantum mechanics, in part motivated by a desire to eliminate the model of the electron as a point particle. Though I believe that the prior portions of this paper have demonstrated that the methodological foundations of CSS's rejection of quantum mechanics are unjustified, I do want to address a technical problem in the spinning ring model. This difficulty centers on the claim that the ring model is stable.

Is the Bergman ring model stable?

Bergman and Wesley [5] claim their physical model of the electron (and other elementary particles) is "completely stable under electromagnetic forces alone." I will show that this claim is incorrect, using Bergman's own stated formula for the total energy of the ring.

Recall that if we have a configuration of matter for which we can write its energy, U, as a function of generalized coordinates q^i as:

$$U = U(q^1, ..., q^n),$$ (46)

then the generalized force acting to change coordinate q^i is given by

$$F_{q^i} = -\frac{\partial U}{\partial q^i},$$ (47)

and the configuration is stable only if there is a point such that the force vanishes, and such that U is a minimum, i.e.

$$\frac{\partial^2 U}{\partial q^2} > 0.$$ (48)

For the Bergman model (cf. Equation (6) of [6]), the energy is given by (q is the particle's charge)

$$E_T = E_e + E_m = \frac{q^2}{4\pi\epsilon_0 R}\left[\ln\left(\frac{8R}{r}\right) - 1\right] \equiv E(R, r).$$ (49)

Computing the generalized forces gives

$$F_R = -\frac{\partial E}{\partial R} = \frac{q^2}{4\pi^2 \epsilon_0 R^2}\left[\ln\left(\frac{8R}{r}\right) - 2\right].\tag{50}$$

This equation does admit a zero at (note here e is the base of natural logarithms)

$$R = \frac{e^2 r}{8},\tag{51}$$

but this value is outside the range of Bergman's approximation (R>>r). For R>>r the force is repulsive and causes an increase in R.

The force for the r coordinate is

$$F_r = -\frac{\partial E}{\partial r} = \frac{q^2}{4\pi^2 \epsilon_0 R r}.\tag{52}$$

This force is repulsive and has no zero except at r=+∞, and thus, the ring explodes, since the forces act to increase both r and R.

CONCLUSION

In summary, the Christian theistic philosophy of science, which is Biblically based, includes the following truths. First, man is finite. Man is a created being, and he cannot exhaustively penetrate God's creation. Second, though man is finite, he is nevertheless created in the image of God, and what he can know he can know truly. *QM is consistent with this Biblically based epistemology*. Thus, the claims of modern science — when they *describe* the discovery of a world in which there are limits to man's knowledge (such as those found in quantum mechanics and relativity) — pose no threat to Christianity and the doctrines of God and of creation and of man. However, when the anti-theistic philosophies of some modern physicists are invalidly added to *explain* the phenomenon modeled by quantum mechanics (or when theories such as relativity are irresponsibly and falsely applied to sociology to argue for ethical relativism), we must be prepared to rebut. Our rebuttal needs to carefully separate observations and theory from the presupposed anti-theistic worldview and must concentrate only upon the anti-theistic claims. I hope this essay has provided some intellectual ammunition for the apologetic warfare in which we as Christians and scientists are engaged.

END NOTES:

1. This last statement usually elicits the question: "Are you suggesting that unbelievers cannot discover truth or do science using their methods?" The answer is simply no. First, the noetic effect of sin on unbelievers is a moral and ethical alienation from God; sin has not reduced man to an irrational beast; the unbeliever is still the imago Dei. Unbelievers plow their fields similarly to believers, but, "even the plowing of the wicked is sin," (Prov. 21:4, KJV). Second, the unbelieving scientist can discover truth *in spite of* his worldview, because the methods of science are actually based upon and presuppose the Christian view of truth. Therefore, the unbeliever can successfully engage in science because he is not consistent. His methodology, stolen from Christianity, is not consistent with and cannot be established by his avowed worldview. See Van Til [39, p.103].

2. The normative hermeneutic is characterized by the following rules for assigning the meaning of a passage: "Unless the context of the passage or parallel passage requires otherwise, the standard usage of a word, phrase, grammatical construction, figure of speech, etc. prevails." For further details cf. R.E. Walsh [42].

3. Incidentally, the selection of Matthias by lot (Acts 1:26) was not a mistake on the part of the eleven. The twelfth apostle had to be one who had been present throughout the Lord's earthly ministry (Acts 1:21-26), as he was required to be one of the rulers of the twelve tribes of Israel (Matt. 19:28). Moreover, the casting of lots was the correct, God-ordained, priestly procedure for the eleven to use. The conclusion is that the names of the twelve apostles of the Lamb written on the foundations of the New Jerusalem include Matthias, not Paul! The apostle Paul, on the other hand, is the special channel through whom Christ directly revealed (Gal. 1:11-12) the dispensation of the mystery (Eph. 3:3-5; Rom. 16:25-26; and Col. 1:26) or Body of Christ program. Paul is not a member of the Jewish dispensation.

4. We should note that this verse includes both Greek words which are rendered "will" in English translations. Boulomai - the word which expresses deliberative will (to decree or execute counsel) and

thelo - the word which expresses the desire. This verse emphatically teaches the doctrine of the unity of God's will of decree and desire. God decrees (boulomai) exactly that which He desires (thelo), and He also accomplishes (energeo) it. Further, since Scripture centers "desire" in the person of the Father, and "decree" in the person of the Son (who, as the Logos, is the "word" or articulator of the Father's desire), and the working of the desire as decreed in the Spirit, this verse includes all three persons of the ontological Trinity in their relation to the creation. This verse thus also shows that those who hold to a modified Calvinism (Amyraldianism), by rejecting particular redemption by the Son while paying lip service to particular election by the Father, have a defective view of God. They are not truly trinitarian, since they make a division between the Father and the Son. Scripture clearly teaches that the Father and the Son are one, in essence and purpose, and that Christ came to do the Father's will (desire), that is to die in the place of (an actual substitutionary atonement) and to save only the elect (cf. e.g. John 10:11 with 10:26; and Rom. 8:32). The "age old" arguments against particular redemption have been answered ably by John Owen in his work *The Death of Death in the Death of Christ* [26], to which I refer the interested reader.

5. We should point out in connection with these Biblical testimonies, that the presence of sin in the universe is inconsistent with the Arminian's non-Scriptural axiom of "free" will and the plan of God. To see this, consider the following argument from Van Til [40, p.83]. Either the sin of Adam was or was not within the plan of God. If it was, then there is an inconsistency with the Arminian's "axiom" of free will; for Adam's choice was *free* (i.e. not necessary), yet *certain* according to the plan of God. The alternative is that sin was not within the plan of God, and thus it was beyond the control of God. And this puts one on anti-theistic ground. The correct Biblical view is that man is a free agent; man is responsible and does choose according to his nature, yet every choice is within God's plan (determinate counsel).

6. Along this line, as Christian theists, we insist that the prophets of God could, Deus volens, predict the results of a long string of die rolls. (This hypothetical example is meant to be illustrative of a principle and is not meant to trivialize the prophetic office by discussing what might appear at first inspection to be a "inconsequential" activity — though in the broader scheme of God's overarching plan, the occurrence of such an experiment would serve the purpose to corroborate a prophet as from God.) But we must not say that God intervenes to "alter" die rolls in an *otherwise* contingent universe (such is a deistic view), or that God just communicates his prescience of otherwise contingent die rolls to his prophets; but rather, God communicates the results of the die rolls to his prophets because the die rolls are determined by God according to His complete and all-encompassing plan for creation. That is, God controls all sequences of events which creatureward appear as contingent events. God controls all things, *whatsoever comes to pass*. There is no ultimate contingency; all possibilities are rooted in God who is behind all of creation.

7. According to the extreme reductionist — for whom there is no such thing as "mind" but only "brains" (composed in turn of matter in motion) — there is a given state X which corresponds to Beethoven "thinking" and "writing" the first note of his first symphony. For the determinist, the second note was a necessary consequence of the first note; thus, Beethoven is a mechanical automaton. For the indeterministic reductionist, the second note is not a necessary consequence, but is the result of reified Chance. So in this case, though the anti-theistic philosophies of modern physics may have freed man from the tyranny of eternal impersonal necessity of Laplace's clockwork universe, the "freedom" it has to offer is thralldom to ultimate eternal irrational chaos. In either case, there is no "Beethoven" who is a real individual personality and a responsible free agent, as Christian theism asserts. Instead, "Beethoven" is a "zombie," a complex mass of (determined or undetermined) "behaviors" which are the mechanical reactions to other "behaviors." Thus, there are no such things as rationality, laws of logic, intelligibility, ethics, etc. Again, we see that in the atheist worldview everything is "sound and fury signifying nothing."

8. *Excursus on mind-body problem*. Equation (5) is similar to that which we must write when considering the nature of man. Man has volition, and as a result of our mental activity and wills, we exercise control over the material realm — in particular our brains and bodies. Yet, we are aware of the fact that our material bodies really are not "us." In fact, we know that the actual matter of our bodies has been replaced many times over in the course of our lives, so that there is no particular assemblage of matter which is "us." The biblical doctrine is that the real self is the immaterial soul and spirit. We are also aware that our material bodies also obey the laws of physics; for example, our involuntary response to the laws of inertia as we fall, or involuntary reflexes when we are acutely aware that our leg moves though we did not "will it." At any rate, it is the exercise of our wills over our bodies which is one aspect of the mind-body problem. Similar to equation (5) above we have for our case:

$$\frac{dX}{dt} = F(X,t) + v(''mind'')$$

where v("mind") is a term which expresses the physical forces manifested in our bodies as a result of our mental agency. The term v cannot be reduced to physical law, and it is apparent that Newton's third law of motion cannot be invoked as a back reaction of v on "mind." It is interesting that this equation is analogous to God's providential term in eq. (5) — I would suggest that this is another example of our being the imago Dei.

9. The limitation to a denumerable set of events is arbitrary and is selected merely for notational convenience. The argument can be easily generalized to included continuous events.

10. The purely additive effect which should be the case for a particle can be made more exact by considering the following: Suppose the particle's path is random as to which hole it passes through. Let $P(A)$ and $P(B)$ be the probabilities that the particle passes through slit A and B, respectively. Let $P(y|X)$ be the probability that the particle strikes the screen at point y given it passes through slit $X \in \{A,B\}$. Then, based on the standard formula for conditional probability, the probability, $P(S)$, that the particle arrives at S, if both slits are open, is given by:

$$P(S) = P(S|A)P(A) + P(S|B)P(B)$$

This formula is clearly the sum of the number of particles arriving from either slit and, thus, does not exhibit the observed interference. Note this expression is denoted as $I_1 + I_2$ in Figure 1.

11. Mathematically QM implies that the coordinates of trajectories, considered as functions of time, are in the class of C^{1-} functions.

12. It should be explicitly stated and emphasized that Christian theism need not be committed to the axiom of locality. Since God is omnipresent, He can act nonlocally, simultaneously and instantaneously; and it is conceivable that He has arranged the natural order such that even the conservation laws explicitly (through non-locality) manifest His providential upholding of creation. However, I want to emphasize again that the logical structure of the proofs of Bell's inequalities do not require non-locality. Thus, those forms of the discussion which introduce spatially separated correlated systems, which seem to imply super-luminal action-at-a-distance, actually obscure a more fundamental mystery. This fundamental mystery is that a *single localized* electron cannot be described as having a spin vector in R^3 (consisting of three well-defined projections on the Cartesian axes)! This is the case *even before* the particles of a spin-correlated system have been spatially separated. We can say that an electron has a probability distribution for a single spin component S_z, but *not* one for S_x and S_y and S_z. This, in itself, is baffling, since it means we cannot visualize the electron as a spinning geometrical object *embedded* in R^3. On the other hand, we can think of the spin *operator* — which represents *measurements* on the spin object — as three dimensional (since this is how we construct Stern-Gerlach magnets, for example). I would like to thank the anonymous referee whose comment on non-locality suggested this additional endnote.

13. Two good references on the religious views of Thomas Jefferson are Sanford [35] and Gaustad [16].

14. I have intentionally avoided calling Mach an "idealist." It is a characterization which Mach would have denied. Nevertheless, Mach's view is, in essence, similar to Hume's idealism. When the consequences of Mach's philosophy are explored, it must be admitted that Mach embraced a *form of idealism*. This has been pointed out by Popper (cf. [30, Part I, Sect. 11, pp.88-91]). Mach's philosophy has been called "neutral monism." The word 'neutral' means a (skeptical) indifference between realism and idealism, while 'monism' means that there is only one 'stuff' of the universe: sensations. The indifference between realism and idealism is a result of the positivist's or phenomenologist's view that such labels are "metaphysical," not subject to test, and thus are meaningless. Whether the world is "real" or "ideal" is irrelevant in science. To a "neutral monist" such concepts are not needed to construct operational theories. This underscores why Mach is a strange ally for a Christian theist who must necessarily be a realist.

15. We should note the fact that though QM uses a Hilbert space to describe the physical phenomena, that alone does not thereby make QM abstract — no more so than a Fourier transform of classical wave motion into an ideal superposition of "elementary vibrations of infinite duration" makes classical electrodynamics abstract.

16. The idea of *tangent vectors* is also an abstract (idealized but real and true!) concept as has been clearly emphasized by modern developments in differential geometry. In rigorous developments of curved

spaces, it is shown that tangent vectors *do not* "reside" in the manifold M (the "base space"), but actually reside (especially as illustrated in the case of a curved manifold such as the two-dimensional surface of the earth) in another space, T(M), called the *tangent bundle* of M. The usual concept of vectors in *flat spaces* — such as three-dimensional Euclidean space E^3 with its concept of "sliding vectors" — obscures this fact by identifying a copy of each tangent space $T_p(E^3)$ with E^3 itself. As pointed out in the main text, on the basis of dimensional considerations, a velocity vector having dimensions of [L/T] cannot be added to points which have dimensions [L]. As a result, a velocity vector cannot "reside" in E^3 but rather in a *copy* of E^3. As a practical application of these observations, note that in a (computer) graphics representation of a fluid flow, velocity tangent vectors must be converted to a spatial displacement by multiplying by a convenient time scale. The tip of the vector then represents where the flow would be if unaccelerated during that time scale. But this is a *counterfactual* representation and an idealization which only aids in *understanding* and *visualization*. Such a visualization is not false. To actually represent the flow, one should view a movie of the flow. This underscores the fact that the very idea of derivatives is an idealized concept expressed as abstract limits of differences of tangent vectors.

17. It is interesting to note that many nineteenth-century physicists spent a lot of energy trying to construct fluid and mechanical analogies of Maxwell's equations. The goal was to reduce electrodynamics to the familiar fluid and mechanical systems of the day. The mechanical versions used fantastic assemblages of tiny wheels and elastic bands, while the fluid analogies, though prima facie more credible, were likewise unsuccessful. In face of these insurmountable difficulties and failures of this reductionism, physicists finally accepted the physical existence of Maxwellian fields as irreducible "stuff" of the physical universe. The lesson to be drawn from this is that the "common sense" of one age can easily become the "myth" of the next. Once again, this highlights the futility of basing the Christian apologetic on the "scientific fads" of the age. Science is not a *crutch* which supports Christianity; rather, Christian theism is the *foundation* on which science rests.

18. It was this realization that the electromagnetic tensor could be viewed as a convenient mathematical description which lead to the Feynman-Wheeler "action at a distance" formulation of electrodynamics. The E and B fields can be totally eliminated and the dynamics expressed entirely in terms of equations for the dynamics of charge distributions under the influence of forces acting at a distance. If I am allowed to speak in the point particle approximation, all we need are the initial data for the collection of charged particles of charges q_n located at points $x_n(t)$, cf. e.g. Rohrlich [34, p.194].

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