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(NON-RELATIVISTIC) PHENOMENON?

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ABSTRACT

The general theory of relativity claims that the excess of precession of the planetary orbits has its origin in the curvature of space-time produced by the Sun in its near vicinity. In general relativity, gravity is thought to be a measure of the curvature of space-time when matter is present. By arguing that free falling particles follow geodesics inside gravitational fields, Schwarzschild's solution to Einstein's field equations explains that when space-time is approximately flat (weak gravitational pull of the Sun), the planetary orbits describe minute precessions which, for Mercury, agrees well with observation. This brief paper explains, first by elaborating on pure special relativity arguments, and second, by considering another solution to Newton's gravitational law, that Mercury's orbital precession does not necessarily demonstrate the unique validity of general relativity.

INTRODUCTION

History shows that the precession of Mercury's orbit has been one of the three proofs in support to Albert Einstein's general relativity theory. That theory, as its name implies, is a generalization of Newton's gravity law, and it interprets gravitation as the curvature of a four-dimensional geometry known as space-time.

The way of visualizing gravitation as a geometrical curvature rather than as a force is radically new and can be easily understood by imagining a stretched elastic sheet. The sheet's surface will appear plane when it holds no objects, and will resemble the space-time used in special relativity (commonly known as "minkowskian space"). Contrary to this, when the sheet holds a heavy body, its surface will exhibit a deformation which will increase if the weight of the body increases. This deformation can be compared in principle with the curvature of space-time in the vicinity of matter. Mass warps space and this warp is understood as gravitation.

Because the theory of general relativity explains gravity without the absurdity of instantaneous propagation at a distance, as newtonian gravitation does, it has acquired much acceptance and popularity among physicists. In the limit of weak gravitational interactions such as that of the solar system, Einstein's theory predicts phenomena which up-to-date classical mechanics has not been able to explain quantitatively. Among these predictions, the bending of light, the gravitational red shift, and the orbital precessions of the planets, have been satisfactorily confirmed by observation. This paper deals precisely with the last of these three phenomena; and particularly with the case associated to Mercury.

It was the french astronomer Jean J. Urbain LeVerrier who, during the second half of the XIX Century, discovered the strange behavior of Mercury's orbit. Apparently there existed a discrepancy between the observed motion of the planet around the Sun and its predicted displacement from Newton's law of gravity. The anomaly caused much uneasiness in the scientific community of the time.

The problem was the following. According to newtonian mechanics, the orbit of a planet shifts continuously about the Sun as a consequence of the gravitational pull between the planet and the remaining members of the solar system. For Mercury's orbit, such theory predicts a shifting rate of about 8.2×10^{-13} rad/sec, or 532''arc/Century. This value however, is less than what LeVerrier calculated from his observations. He found Mercury's orbit to precess about the Sun at a rate close to 573''arc/Century; that is, 41''arc/Century more than what should be accounted for from classical mechanics. What strange cause was responsible for such an excess of precession? A planet orbiting between the Sun and Mercury

with mass and velocity   so as to justify for the extra shift, was one of the most serious suggestions proposed by the scientists of the XIX Century; however, its existence was never confirmed.

Actually, with the advent of high technology, the observed rate of precession of Mercury's orbit is known to be ~575''arc/Century, with a rectified difference of 43''arc/Century over the results found from newtonian mechanics.

The discrepancy can be resolved elegantly by means of the general theory of relativity, without adding fudge terms. From Schwarzschild's solution to Einstein's field equations, one can obtain a relationship between Mercury's parameters and the precessional rate of its orbit. As a single condition, we require the approach to the newtonian limit; that is, we require the gravitational pull of the central body to be weak. The result thus obtained is exactly the 43''arc/Century!(1).

Consider a particular planet and replace the orbits of the other planets by rings of linear mass density, each equal to the mass of the corresponding planet divided by its circumference. Assume furthermore that the planet under consideration is located a distance (thought to be very small when compared with the radius of any of the outer mass rings) from the central gravitational force of the Sun. One can then derive an expression for force which, added vectorially to the Sun's gravitational pull, can make the orbit of the planet in question depart from fixed orientation in space. Such a departure can be either a precession in the same direction as the motion of the planet around the Sun (called a precessional advance) or a precession in the opposite direction (called a regressive precession); it all depends on the way gravity acts on a particular planet.

Following the above approach, we see that Mercury's orbit will show a precessional advance of 532''arc/Century, as LeVerrier concluded(2). Hence, of the total 575''arc/Century precessional rate, 532''arc/Century are caused by the other members of the solar system while the remaining 43''arc/Century are due to relativistic effects caused by the curvature of space-time in the vicinity of the Sun.

It has been found however, that general relativity is not required to obtain the orbital precessions of the planets. In this article we demonstrate two things. First, that only the concepts of special relativity are needed to satisfactorily solve the problem. And second, that a new solution to Newton's gravitational law can predict the observed precession of Mercury's orbit provided the existence of a gravitational solar quadrupole moment.

SPECIAL RELATIVITY AND THE ORBITAL PRECESSIONS

If gravity is somehow turned off, general relativity will reduce to special relativity. Under this condition, dynamics will in general obey the minkowskian metric $ds^2=-c^2dt^2+dx^2+dy^2+dz^2$ (where c is the speed of light in vacuum), and space, time, and matter will behave differently than in ordinary newtonian mechanics. In special relativity, time and space are taken to be functions of motion, as mass is. Time and matter will dilate, and space will contract for events moving with respect to an observer. For this to hold however, two restrictions must be imposed. These are: 1) to take the speed of light as being independent of the motion of either the source or the observer, and 2) to consider the uniform relative motion as the only valid reference.

Events taking place in a reference frame $S'(x',y',z',t')$, but viewed from another frame $S(x,y,z,t)$ in uniform motion with respect to the first, will be described by a set of equations known as Lorentz transformations. These, in cartesian coordinates, are given by:

$$x'=\gamma(x-vt), \quad t'=\gamma(t-vx/c^2), \tag{1}$$

where x is the line of motion between both frames, v is their relative velocity, and γ is a relativistic factor equal to $1/(1-v^2/c^2)^{1/2}$. Here the coordinates y and z remain fixed for both frames. Notice that under these transformations, the minkowskian metric becomes covariant.

From Eq.(1), time dilation and length contraction are derived. The results are

$$t=\gamma t', \quad and \quad l=l'/\gamma. \tag{2}$$

On the other hand, from the energy conservation principle we know that a particle, subject only to the gravitational influence of a spherically symmetric body of mass M, will approach from infinity to a distance r from the origin of M with a velocity equal to

$$v=(2GM/r)^{1/2}; \tag{3}$$

where G is the gravitational constant. This last expression is known as the escape velocity. It is the velocity needed to actually escape from the gravitational influence of M. If r is a fixed quantity, v will always have the same value. This condition applies, for example, to objects at rest on the surface of M, or to objects orbiting around M.

To attack the problem of the orbital precessions using the above results we proceed with the following train of thought. First, we postulate a set of two reference frames small enough to confidently assure the local uniformity of the gravitational field. And second, we let one of these frames be the "free falling" frame $S(r,\theta,\phi,t)$ and the other the rest frame $S'(r',\theta',\phi',t')$ with respect to M. There is a definite distinction between these two frames of reference. Only one of them is an inertial frame. Hence, if dt' and dr' are the increments of clock reading and radial length measurement in the rest frame, in the free falling frame such increments will be given by

$$dt=\frac{dt'}{(1-2GM/rc^2)^{1/2}}, \text{ and } dr=dr'(1-2GM/rc^2)^{1/2}, \tag{4}$$

where $\sqrt{2GM/r}$ is now the relative velocity between both frames.

The validity of special relativity is assured for the case of planetary motion because: (a) the escape velocity is maintained approximately constant, (b) their motion is equivalent to a free fall, and (c) freely falling frames are equivalent to inertial frames.

Eq.(4) represents a translation of clock rate and scale length inside a homogeneous gravitational field. Thus, conversion from free fall to rest is achieved by adapting Eq.(4) to the minkowskian metric, yielding

$$ds^2=-c^2(1-2\phi/c^2)dt^2+(1-2\phi/c^2)^{-1}dr^2+r^2(d\theta^2+\sin^2\theta d\phi^2), \tag{5}$$

in spherical coordinates. Here $\phi(=GM/r)$ represents the newtonian potential. Observe that when gravitation vanishes, as in free fall for example, ϕ disappears and Eq.(5) reduces to the minkowskian metric. This is equivalent to a transformation from rest to free fall.

With Eq.(5) now established, the problem of the planetary orbits can be solved by choosing a reference point fixed with respect to the Sun. For all practical purposes we take the orbital plane of Mercury to coincide with the solar equatorial plane. This being the case we let the polar angle $\theta=\pi/2$, and Eq.(5) reduces to

$$ds^2=-c^2(1-2\phi/c^2)dt^2+(1-2\phi/c^2)^{-1}dr^2+r^2d\phi^2. \tag{6}$$

The metric coefficients of this equation are:

$$g_{tt}=-(1-2\phi/c^2), \quad g_{rr}=(1-2\phi/c^2)^{-1}, \quad \text{and} \quad g_{\phi\phi}=r^2. \tag{7}$$

Tensor analysis shows that if the metric coefficients constituting ds^2 are not functions of their respective coordinates then, if it applies, the covariant component of four-velocity for that particular coordinate is a constant(3).

Because g_{tt}, and $g_{\phi\phi}$ are independent of t and ϕ respectively, the only covariant components of four-velocity that remain constant are

$$U_t=g_{tt}cdt/d\tau=-(1-2\phi/c^2)cdt/d\tau,$$

and

$$U_\phi=g_{\phi\phi}d\phi/d\tau=r^2d\phi/d\tau,$$

where $d\tau$ is called the "proper time". These covariant components, multiplied by the planet's mass m represent the time component of four-momentum (-E/c) and angular momentum L of the orbit respectively. Therefore we write

$$U_t=-(1-2\phi/c^2)cdt/d\tau=-E/mc, \quad \text{and} \quad U_\phi=r^2d\phi/d\tau=L/m. \tag{8}$$

Knowing that in special relativity $(U^i)^2=(U_i)^2$, and $g_{ij}U^iU^j=g^{ij}U_iU_j$, where $i=j(=t,r,\theta,\phi)$, then from the inner product of four-velocity and Eq.(8), we see that

$$-(E/mc)^2/(1-2\phi/c^2)+(dr/d\tau)^2/(1-2\phi/c^2)+(1/r^2)(L/m)^2=-c^2. \tag{9}$$

Our purpose is to find the equation of motion of m around M. For that matter we use $u=1/r$

in the second expression of Eq.(8) to get $dr/d\tau=-Ldu/md\phi$. With this, and letting $k=2GM/c^2$, Eq.(9) becomes

$$c^2(1-ku)=(E/mc)^2-(L/m)^2(du/d\phi)^2-(Lu/m)^2(1-ku). \qquad (10)$$

Dividing Eq.(10) by $(L/m)^2$ and operating both sides by $d/d\phi$ we obtain

$$d^2u/d\phi^2+u=3GMu^2/c^2+GMm^2/L^2. \qquad (11)$$

The first term on the right hand side of Eq.(11) is the post-newtonian factor introduced by general relativity for all orbital motions under spherically symmetric gravitational fields. Actually, Newton predicts that $d^2u/d\phi^2=GMm^2/L^2$.

Assuming u to have a minute displacement u' about an equilibrium distance $u_o(=1/r_o)$, then $u=u_o+u'$. Substituting this in Eq.(11) and keeping only first order terms in u' yields

$$\frac{d^2u'/d\phi^2}{1-6GM/r_oc^2} + u' = \frac{3GM/r_o{}^2c^2+GMm^2/L^2}{1-6GM/r_oc^2} ,$$

and by making $1-6GM/r_oc^2$ part of the azimuthal angle ϕ we see that

$$d^2u'/dq^2+u' = \frac{3GM/r_o{}^2c^2+GMm^2/L^2}{1-6GM/r_oc^2} ,$$

where $q=\phi\sqrt{1-6GM/r_oc^2}$.

The general solution to the above differential equation represents an almost circular motion, with the quantity $\sqrt{1-6GM/r_oc^2}$ having a very particular characteristic: it causes the orbit to continuously precess about the Sun M, as shown in Figure 1.
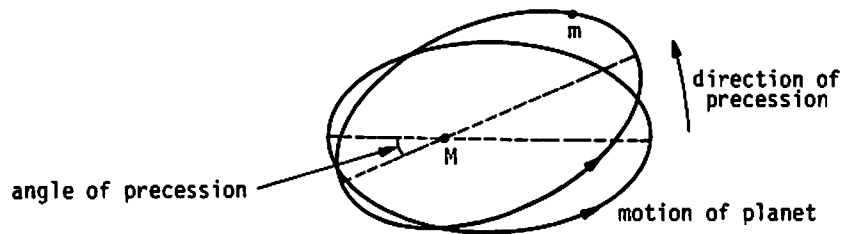

Figure 1 - Shifting of an almost circular orbit. Every time the planet completes a revolution around the Sun, the point of closest approach becomes shifted in the same direction as the orbital motion of the planet.

Choosing as our reference point the perihelion of the orbit, we see that the angular distance covered from one perihelion to the next is given by

$$q=\phi\sqrt{1-6GM/r_oc^2}. \qquad (12)$$

For any planetary orbit in the solar system the quantity $GM/c^2<< r_o$, thus expanding Eq.(12) by the binomial theorem, and maintaining only terms up to first order in $1/r_o$ yields: $q=\phi(1-3GM/r_oc^2)$. But q is just 2π rad. Therefore, the shifting between two successive perihelia, in units of rad/orbit, is found to be

$$\phi-2\pi=6\pi GM/r_oc^2. \qquad (13)$$

The fact that Eq.(13) is positive means that the sense of precession of the orbit is equal to the sense of motion of the planet around M. Substituting numbers, this equation gives a value which is in good agreement with the observed shift of $\sim 43.1''$ arc/Century for Mercury's orbit.

It is interesting to know that the same theoretical results for orbital precessions, derived from general relativity, can be obtained from the sole arguments of the special theory of relativity.

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WHAT IS THE SHAPE OF THE SUN?

Can the actual conformation of the Sun be more complicated than that of a simple sphere? Back in 1967, Robert H. Dicke and H. Mark Goldenberg[4] studied several observations of the Sun's shape and concluded, based on the information at hand, that it is conformally oblate. Their conclusions give an estimated oblateness value of $5\pm0.7\times10^{-5}$; although, a year before other scientists found a $4.51\pm0.34\times10^{-5}$ oblateness[5]. Such values imply directly the existence of a solar quadrupole moment, and accordingly, a more elaborated gravitational potential, now given by the expression[6]

$$\phi = \frac{GM}{r}(1+R^2\Delta/3r^2) \tag{14}$$

for points lying on the equatorial plane of the Sun. Here R represents the solar radius, Δ the solar oblateness, and $r(>R)$ the distance from the origin of M to the point in question. Note additionally that when $\Delta=0$, the potential reduces to the familiar spherically symmetric form.

Adapting Eq.(14) to Eq.(6) provides a much different metric. From it, and following the procedure of the previous section, we obtain the equivalent of Eq.(10)

$$c^2\{1-ku(1+Nu^2)\}=(E/mc)^2-(L/m)^2(du/d\phi)^2-(Lu/m)^2\{1-ku(1+Nu^2)\},$$

where $u=1/r$, $k=2GM/c^2$, and $N=R^2\Delta/3$.

The differential equation for the small displacement u', derived from the above expression, contains the following terms:

$$(d^2u'/d\phi^2)/A+u'=\{\frac{GMm^2}{L^2} + \frac{3GM}{c^2r_o^2}(\frac{m^2c^2R^2\Delta}{3L^2} + 1)+ \frac{5GMR^2\Delta}{3c^2r_o^4}\}/A,$$

with $A=\{1-(6GM/r_oc^2)(m^2c^2R^2\Delta/3L^2+1)-20GMR^2\Delta/3c^2r_o^3\}$. This particular equation is analogous to that succeeding Eq.(11); consequently, making A part of the angle ϕ, yields the desired expression

$$\frac{d^2u'}{dq^2} + u'=\{\frac{GMm^2}{L^2} + \frac{3GM}{c^2r_o^2}(\frac{m^2c^2R^2\Delta}{3L^2} + 1) + \frac{5GMR^2\Delta}{3c^2r_o^4}\}/A$$

from which, by the same token as in the last section, $q=\phi\sqrt{A}$.

Considering that angular momentum for a circular, or nearly circular, motion is given by $L^2=GMm^2r_o$ the orbital precession obtained from q will be

$$\phi-2\pi= \frac{6\pi}{r_oc^2}(\frac{c^2R^2\Delta}{3r_o} + GM)+ \frac{20\pi GMR^2\Delta}{3r_o^3c^2} . \tag{15}$$

Notice that when oblateness vanishes, this expression reduces to Eq.(13).

For Mercury's orbit ($r_o=5.971\times10^{10}$ meters), and with the solar data ($R=7\times10^8$ meters, $M=1.97\times10^{30}$ kg, $\Delta=5\times10^{-5}$), we will get a perihelion shift equal to 5.2113×10^{-7} rad/orbit, which in more conventional units is

$$\phi-2\pi=44.63''arc/Century. \tag{16}$$

Similarly, a solar oblateness of 4.51×10^{-5} will contribute with a perihelion shift equivalent to $44.24''arc/Century$. Since both of these precessional rates go beyond the observed shift of Mercury's orbit, we deduce that any measurable solar oblateness will render Einstein's relativity theory untenable for planetary motions.

PERTURBATION THEORY AND MERCURY'S PERIHELION PRECESSION

Based on classical perturbation theory, and taking into account the existence of a solar quadrupole moment, the theory developed in this section clearly shows that Mercury's excessive orbital precession is caused by the attraction of an oblate Sun. Hence, starting with the potential

$$\phi= \frac{GM}{r}(1+ \frac{R^2\Delta}{3r^2} - \frac{R^2z^2\Delta}{r^4})$$

(which reduces to that of Eq.(14) when z=0), and letting each planet be represented by a subscript i or j, we see that the gravitational interaction between M and a planet of mass m_i located at a distance r_i from the origin of M is

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$$\vec{F}_i = \frac{-GMm_i\vec{r}_i}{r_i^3} - \frac{GMm_iB}{r_i^5}(1-\frac{5z_i^2}{r_i^2})\vec{r}_i - \frac{2GMm_iB\vec{z}_i}{r_i^5} \; , \tag{17}$$

where $B=R^2\Delta$, and the cap arrow ($\rightarrow$) denotes a vectorial quantity.

Similarly, to an excellent approximation, the gravitational force between two planets is described by

$$\vec{F}_{ij} = -(Gm_im_j/r_{ij}^3)\vec{r}_{ij} \; , \tag{18}$$

where r_{ij} is the distance from m_i to m_j, and $i,j=1,2,3,\ldots,9$ ($i{\neq}j$). Both forces are calculated from the reference frame (x,y,z) of the Sun.

Since the motion of the planets is small when compared with the speed of light, they obey Newton's law $F=dp/dt$ if $p(=mv)$ is the momentum of each planet. From this law, and Eqns.(17) and (18), and the aid of Figure 2, we obtain the equation of motion of Mercury (m_1) relative to the outer planets and to the Sun. This is

$$\frac{d^2\vec{r}_1}{Gdt^2} = \sum_j^9 \{\frac{m_j\vec{r}_{1j}}{r_{1j}^3} - \frac{m_j\vec{r}_j}{r_j^3} - \frac{Bm_j}{r_j^5}(1-\frac{5z_j^2}{r_j^2})\vec{r}_j$$

$$- \frac{2Bm_j\vec{z}_j}{r_j^5}\} - \frac{\vec{r}_1\mu}{r_1^3} - \frac{\mu B}{r_1^5}(1-\frac{5z_1^2}{r_1^2})\vec{r}_1 - \frac{2\mu B\vec{z}_1}{r_1^5} \; , \tag{19}$$
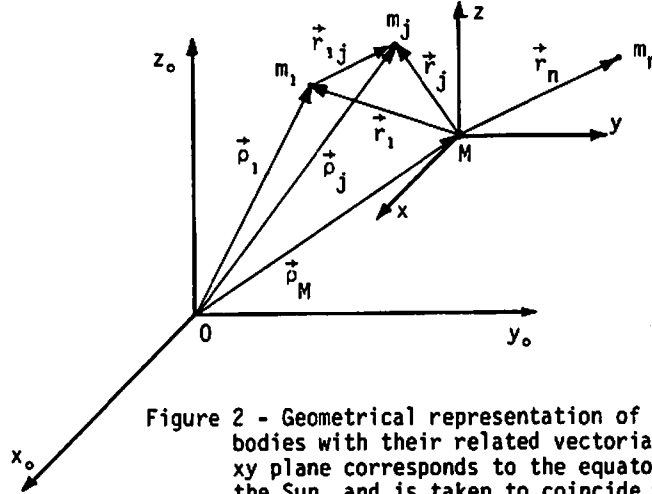
where $\mu=M+m_1$.

Figure 2 - Geometrical representation of a system of n bodies with their related vectorial distances. The xy plane corresponds to the equatorial plane of the Sun, and is taken to coincide with the ecliptic. All members constituting the solar system move with respect to a fixed reference frame (x_0,y_0,z_0).

Figure 2 also shows that the magnitude of the vector $\vec{r}_{1j}$ is equal to $\{(x_j-x_1)^2 + (y_j-y_1)^2 + (z_j-z_1)^2\}^{1/2}$, from where the following relations are deduced:

$$\frac{\partial}{\partial x_1}(\frac{1}{r_{1j}}) = \frac{x_j-x_1}{r_{1j}^3} \; ; \quad \frac{\partial}{\partial y_1}(\frac{1}{r_{1j}}) = \frac{y_j-y_1}{r_{1j}^3} \; ; \quad \frac{\partial}{\partial z_1}(\frac{1}{r_{1j}}) = \frac{z_j-z_1}{r_{1j}^3} \; ; \tag{20}$$

and

$$\frac{\partial}{\partial x}(\vec{r}_1\cdot\vec{r}_j)=x_j; \quad \frac{\partial}{\partial y}(\vec{r}_1\cdot\vec{r}_j)=y_j; \quad \text{and} \quad \frac{\partial}{\partial z}(\vec{r}_1\cdot\vec{r}_j)=z_j. \tag{21}$$

Dividing each of the terms in Eq.(21) by r_j^3, and combining them their corresponding terms of Eq.(20) yields

$$\frac{\partial}{\partial x_1}(\frac{1}{r_{1j}}) - \frac{\partial}{\partial x_1}\frac{\vec{r}_1\cdot\vec{r}_j}{r_j^3}) = \frac{\partial}{\partial x_1}H_j, \quad \frac{\partial}{\partial y_1}(\frac{1}{r_{1j}}) - \frac{\partial}{\partial y_1}(\frac{\vec{r}_1\cdot\vec{r}_j}{r_j^3}) = \frac{\partial}{\partial y_1}H_j, \text{ and } \frac{\partial}{\partial z_1}(\frac{1}{r_{1j}}) - \frac{\partial}{\partial z_1}(\frac{\vec{r}_1\cdot\vec{r}_j}{r_j^3}) = \frac{\partial}{\partial z_1}H_j.$$

For Mercury's trajectory H_j plays the role of a perturbative function and it is given by $H_j = 1/r_{1j} - (\vec{r}_1 \cdot \vec{r}_j)/r_j^3$. The use of this function in Eq.(19) gives

$$\frac{d^2\vec{r}_1}{Gdt^2} = \sum_j^9 m_j \{\vec{\nabla}_1 H_j - \frac{B}{r_j^2}(1 - \frac{5z_j^2}{r_j^2})\vec{\nabla}_1(\frac{1}{r_{1j}} - H_j) - \frac{2B\vec{z}_j}{r_j^5}\} - \mu\{\frac{\vec{r}_1}{r_1^3} + \frac{B\vec{r}_1}{r_1^5}(1 - \frac{5z_1^2}{r_1^2}) + \frac{2B\vec{z}_1}{r_1^5}\} ,$$

where $\vec{\nabla}_1 = (\partial/\partial x_1)\hat{i} + (\partial/\partial y_1)\hat{j} + (\partial/\partial z_1)\hat{k}$ is known as the del operator; $\hat{i}$, $\hat{j}$, and $\hat{k}$ are unit vectors in the x_1, y_1, and z_1 directions respectively. The summation term of the right hand side of the above equation is the perturbation contribution of the outer planets on the motion of Mercury. Hence, by ignoring this contribution, the equation reduces to

$$\frac{d^2\vec{r}_1}{dt^2} = -G\mu\{\frac{\vec{r}_1}{r_1^3} + \frac{B\vec{r}_1}{r_1^5}(1 - \frac{5z_1^2}{r_1^2}) + \frac{2B\vec{z}_1}{r_1^5}\} .$$

This expression will describe the motion of m_1 about an oblate mass M. Thus, calling

$$L_a = \frac{G\mu B}{r_1^4}(1 - \frac{5z_1^2}{r_1^2}), \quad \text{and} \quad L_b = \frac{2G\mu B z_1}{r_1^5} , \tag{22}$$

the above equation becomes

$$\frac{d^2\vec{r}_1}{dt^2} = -\frac{G\mu\vec{r}_1}{r_1^3} - L_a\hat{r} - L_b\hat{z} , \tag{23}$$

where $\hat{r}$ and $\hat{z}$ are unit vectors in the r_1 and z_1 directions respectively.

Suppose we want to express the coordinates of m_1 in terms of Mercury's orbital elements[***] and time. If $\alpha_1, \alpha_2, ..., \alpha_6$ represent those elements, the coordinates in vector notation may then be written as

$$\vec{r}_1 = \vec{r}_1(\alpha_1, \alpha_2, ..., \alpha_6, t). \tag{24}$$

For the ideal case in which the Sun is spherical, the orbital motion of m_1 about M is said to be keplerian, and the elements describing the orbit will remain constant in time. In reality however, the solar oblateness must be considered. Thus, the motion of m_1 gradually deviates from the simple keplerian path. This being the case, the orbital elements are actually functions of time. The problem then is to find the time rates of change of the α's when the perturbative effects of Eq.(23) are included.

Differentiating Eq.(24) with respect to time in the true motion of m_1 gives

$$\frac{d\vec{r}_1}{dt} = \sum_k^6 \frac{\partial\vec{r}_1}{\partial\alpha_k}(\frac{d\alpha_k}{dt}) + \frac{\partial\vec{r}_1}{\partial t} ,$$

where partial differentiation refers to the keplerian orbit, and total differentiation refers to the perturbed orbit.

Since the orbital elements slowly change with time, we can assume the change to be uniform and consider their accelerations as being practically zero. Hence, a second differentiation of Eq.(24) yields

$$\frac{d^2\vec{r}_1}{dt^2} = \sum_k^6 \frac{\partial^2\vec{r}_1}{\partial t\partial\alpha_k}(\frac{d\alpha_k}{dt}) + \frac{\partial^2\vec{r}_1}{\partial t^2} .$$

Substituting this expression in Eq.(23) gives

$$\sum_k^6 \frac{d^2\vec{r}_1}{\partial t\partial\alpha_k}(\frac{d\alpha_k}{dt}) + \frac{\partial^2\vec{r}_1}{\partial t^2} + \frac{G\mu\vec{r}_1}{r_1^3} + L_a\hat{r} + L_b\hat{z} = 0 . \tag{25}$$

If B=0, the Sun will show no oblateness and the relative motion of Mercury will obey the equation $\partial^2\vec{r}_1/\partial t^2 = -G\mu\vec{r}_1/r_1^3$. This is the well-known newtonian law of motion under gravi-

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[***] The orbital elements of a planet are: the semi-major axis a, the eccentricity e, the inclination angle i, the argument of the perihelion ω, the argument of the ascending node Ω, and the time of perihelion passage T. Shown graphically in Figure 3 are the elements denoted by i, ω, Ω, and T.

ty. Therefore, substitution of this in Eq.(25) gives

$$\sum_{k}\frac{\partial^2\vec{r}_1}{\partial t\partial\alpha_k}(\frac{d\alpha_k}{dt}) = -L_a\hat{r} - L_b\hat{z} .$$ (26)

Since the mathematical representation for the time rates of change of the orbital elements in a keplerian orbit is

$$\sum_{k}\frac{\partial\vec{r}_1}{\partial\alpha_k}(\frac{d\alpha_k}{dt}) = 0,$$ (27)

dotting Eq.(26) with $\partial\vec{r}_1/\partial\alpha_\ell$, and Eq.(27) with $\partial^2\vec{r}_1/\partial t\partial\alpha_\ell$, and substracting one from the other, yields the total perturbation due to the solar oblateness. This operation gives

$$\sum_{k}(\frac{\partial\vec{r}_1}{\partial\alpha_\ell}\cdot\frac{\partial^2\vec{r}_1}{\partial t\partial\alpha_k}-\frac{\partial\vec{r}_1}{\partial\alpha_k}\cdot\frac{\partial^2\vec{r}_1}{\partial t\partial\alpha_\ell})(\frac{d\alpha_k}{dt}) = -L_a\partial r_1/\partial\alpha_\ell - L_b\partial z_1/\partial\alpha_\ell ,$$ (28)

where $\ell=1,2,3,4,5,6$ ($\ell\neq k$).

In classical mechanics, the left hand side of Eq.(28) is commonly known as Lagrange brackets. These are generally expressed as

$$\sum_{k}\{(\partial q_k/\partial u)(\partial p_k/\partial v)-(\partial p_k/\partial u)(\partial q_k/\partial v) = {}_k\{u,v\}_{q,p} .$$
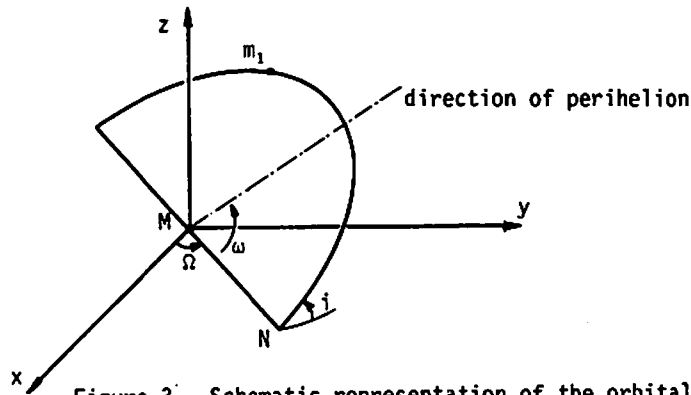

Figure 3 - Schematic representation of the orbital
elements i, ω, Ω, and T. Ω is the angle made by
the x axis (oriented towards the vernal equinox)
and the line of ascending node MN of the orbit.
ω is the angle between the line MN and the direc-
tion of the perihelion. Finally, i is the angle
of inclination of the orbit relative to the eclip-
tic (plane xy).

Comparing these brackets with the quantities $\vec{r}_1$, $\partial\vec{r}_1/\partial t$, α_ℓ, and α_k, Eq.(28) simplifies more so that

$$\sum_{k}\{\alpha_\ell,\alpha_k\}(\frac{d\alpha_k}{dt})= L_a\partial r_1/\partial\alpha_\ell - L_b\partial z_1/\partial\alpha_\ell; \quad k,\ell=1.2,...,6.$$ (29)

Independently of the relation that might exist between the right hand side of Eq.(29) and the Lagrange brackets, the latter have already been evaluated (in terms of orbital ele-ments) by several authors; see for example W.M. Smart(7) and F.R. Moulton(8). Quoting the results we have

$$\alpha_\ell = \Omega, \quad \alpha_k = a; \quad \{\Omega,a\} = (\frac{1}{2})na\cos i\sqrt{1-e^2} ,$$ (30a)

$$\alpha_\ell = \omega, \quad \alpha_k = a; \quad \{\omega,a\} = (\frac{1}{2})na\sqrt{1-e^2} ,$$ (30b)

$$\alpha_\ell = e, \quad \alpha_k = \Omega; \quad \{e,\Omega\} = \frac{na^2e\cos i}{(1-e^2)^{1/2}} ,$$ (30c)

$$\alpha_{\ell} = e, \quad \alpha_k = \omega; \quad \{e,\omega\} = \frac{na^2e}{(1-e^2)^{1/2}}, \tag{30d}$$

$$\alpha_{\ell} = i, \quad \alpha_k = \Omega; \quad \{i,\Omega\} = na^2Sini\sqrt{1-e^2}, \tag{30e}$$

and

$$\alpha_{\ell} = a, \quad \alpha_k = T; \quad \{a,T\} = \frac{an^2}{2}, \tag{30f}$$

where $n=\sqrt{G(M+m_1)}/a^{3/2}$ is the orbital angular speed.

Making use of the property $\{\alpha_{\ell},\alpha_k\}=-\{\alpha_k,\alpha_{\ell}\}$, and substituting from Eq.(30) into Eq.(29) gives the following set of equations:

$$\frac{1}{2}na(\frac{da}{dt})\sqrt{1-e^2}Cosi - \frac{na^2eCosi}{\sqrt{1-e^2}}(\frac{de}{dt}) - na^2(\frac{di}{dt})\sqrt{1-e^2}Sini = -L_a\frac{\partial r_1}{\partial \Omega} - L_b\frac{\partial z_1}{\partial \Omega}, \tag{31a}$$

$$\frac{1}{2}n^2a(\frac{dT}{dt}) - \frac{1}{2}na(\frac{d\Omega}{dt})\sqrt{1-e^2}Cosi - \frac{1}{2}na(\frac{d\omega}{dt})\sqrt{1-e^2} = -L_a\frac{\partial r_1}{\partial a} - L_b\frac{\partial z_1}{\partial a}, \tag{31b}$$

$$\frac{1}{2}na(\frac{da}{dt})\sqrt{1-e^2} - \frac{na^2e}{\sqrt{1-e^2}}\frac{de}{dt} = -L_a\frac{\partial r_1}{\partial \omega} - L_b\frac{\partial z_1}{\partial \omega}, \tag{31c}$$

$$\frac{na^2eCosi}{\sqrt{1-e^2}}(\frac{d\Omega}{dt}) + \frac{na^2e}{\sqrt{1-e^2}}(\frac{d\omega}{dt}) = -L_a\frac{\partial r_1}{\partial e} - L_b\frac{\partial z_1}{\partial e}, \tag{31d}$$

$$-\frac{1}{2}n^2a(\frac{da}{dt}) = -L_a\frac{\partial r_1}{\partial T} - L_b\frac{\partial z_1}{\partial T}, \tag{31e}$$

and

$$na^2(\frac{d\Omega}{dt})Sini\sqrt{1-e^2} = -L_a\frac{\partial r_1}{\partial T} - L_b\frac{\partial z_1}{\partial T}. \tag{31f}$$

These are six equations with the six unknowns da/dt, de/dt, di/dt, dω/dt, dΩ/dt, and dT/dt. To solve for each of these time rates of change, we must treat all six members of Eq.(31) simultaneously. Hence, after a few simple algebraic manipulations wee see that the time rate of change of ω is given by

$$\frac{d\omega}{dt} = \frac{\sqrt{1-e^2}}{na^2e}(-L_a\frac{\partial r_1}{\partial e} - L_b\frac{\partial z_1}{\partial e}) - \frac{(\sqrt{1-e^2})Cosi}{na^2Sini}(-L_a\frac{\partial r_1}{\partial T} - L_b\frac{\partial z_1}{\partial T})^{-1}. \tag{32}$$

This simple equation gives all the necessary information regarding the precession of Mercury's perihelion provided we define r_1 and z_1. To do this we notice that at perihelion point $r_1=a(1-e)$, so by inspecting Figure 3, the z_1 component of r_1 at the same point must be equal to $a(1-e)Sin\omega Sini$. Therefore, partial differentiation of both expressions with respect to e and i will give the following results

$$\frac{\partial r}{\partial e} = -a; \quad \frac{\partial r}{\partial T} = 0; \quad \frac{\partial z}{\partial e} = -aSin\omega Sini; \quad \text{and} \quad \frac{\partial z}{\partial T} = a(1-e)Sin\omega Cosi.$$

Replacing n, B, and these results, along with Eq.(22), in Eq.(32) we obtain as our main answer

$$\frac{d\omega}{dt} = \frac{\sqrt{G(M+m_1)(1+e)}R^2\Delta}{e\{a(1-e)\}^{7/2}}(1-3Sin^2\omega Sin^2i) + 2\left[\frac{G(M+m_1)}{1+e}\right]^{1/2}\frac{R^2\Delta}{\{a(1-e)\}^{7/2}}Sin^2\omega Cos^2i. \tag{33}$$

By the very definition of ω, Eq.(33) will predict the total shift per unit time of Mercury's perihelion, and the amount of precession will depend strictly on the oblateness of the Sun. Thus, as stated by Eq.(33), if the Sun is spherical, the perihelion will remain fixed in space.

With Mercury's data (a=5.791X10^{10} meters, e=0.205628, i=7.004167°, ω=28.7526°, Ω=47.1458°, and m_1=0.3332X10^{24} kg), and the solar oblateness values discussed in the previous section, Eq.(33) provides us with the following range of precessional rates

$$\frac{d\omega}{dt} = 49.57\pm6.94''arc/Century \quad \text{if} \quad \Delta = 5\pm.7X10^{-5},$$

and

$$\frac{d\omega}{dt} = 44.71\pm3.37''arc/Century \quad \text{if} \quad \Delta = 4.51\pm.34X10^{-5}.$$

The observed precession of 43.1''arc/Century for Mercury's orbit actually lies in the range of possible shifts predicted by Eq.(33) for either solar oblateness. Hence, from pure classical mechanics, we see that Mercury's orbital precession has a natural cause and is not a relativistic effect, as it is thought to be. The alleged shift is not a decisive proof of the uniqueness of general relativity.

FROM THE STANDPOINT OF A YOUNG EARTH

The mathematical development of the last section does not prove directly that the Earth is young. It shows rather that general relativity is not the only mathematical theory able to explain the actual gravitational fields governing the motion of light and particles in the vicinity of the Sun. As it is known, Einstein's theory also predicts the expansion of the Universe and its creation, and indirectly the creation of the Earth; by extrapolating backwards in time the information we have up to date about the behavior of the Cosmos at large, that theory predicts the Universe to have originated about 18 billion years ago, with the Earth's age not much younger than 1/4 of that value. Scientists actually believe that the Universe, as a closed system, must obey Einstein's equations, since these are known to hold for the majority of its predictions.

The fact that newtonian gravitation can also predict the correct amount of precession of Mercury's perihelion, opens the possibility for viewing the Universe (and the Earth) without the need for general relativity. This is very important because it gives ways for proposing other cosmological theories that can go much in agreement with the observations and with the conception of a young Earth.

NOTES AND REFERENCES

1. For a derivation of the precessional rates of the planetary orbits from general relativity consult for example: Landau. L. D., and Lifshitz, E. M., THE CLASSICAL THEORY OF FIELDS, Addison-Wesley Press, New York, 1951, p. 206.

2. The reader interested in the mathematical details of such a conclusion will find the following paper quite informative: Price, M. P., and Rush, W. F., "Non-Relativistic Contribution to Mercury's Perihelion Precession", AMERICAN JOURNAL OF PHYSICS. Vol. 47, No. 6, June, 1979, p. 531.

3. Price, R. R., "General Relativity Primer", AMERICAN JOURNAL OF PHYSICS, Vol. 50, No. 4, April, 1982, p. 322.

4. Dicke, R. H., and Goldenberg, H. M., "Solar Oblateness and General Relativity", PHYSICAL REVIEW LETTERS, Vol. 18, No. 9, February, 1967, p. 313.

5. Dicke, R. H., "Oblateness of the Sun and Relativity", SCIENCE, Vol. 184, No. 4135, April, 1974, p. 419.

6. Ibid., p. 420.

7. Smart, W. M., CLASSICAL MECHANICS, Longmans, Green and Co., Inc., New York, 1953, p. 63.

8. Moulton, F. R., AN INTRODUCTION TO CELESTIAL MECHANICS, Longmans and Co., New York, 1914, p. 391.

DISCUSSION

This paper by Mr. Francisco Ramirez Avila is a masterpiece. He gives a clear explanation of Einstein's General Theory of Relativity. He rigorously deduces two alternatives to General Relativity for the case of the anomalous precession of the orbit of Mercury. The first deduction shows that General Relativity can be replaced by Special Relativity. The second deduction shows that the Special Relativity can be replaced by ordinary Newtonian physics on this problem. In all three cases the solution gives the observed value of the precession of the orbit of Mercury.

Mr. Ramirez's work is revolutionary. It displaces the need for Einstein's relativity or his postulates in one of the most acclaimed areas of Einstein's relativity. This is a blow to evolutionary cosmology, which is dependent upon General Theory of Relativity. It opens the way to a cosmology that is in accord with creation.

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Mr. Ramirez shows that if one accepts the data for the oblateness of the sun as reported by Dicke and others, then it is possible to obtain a rate of precession of the perihelion of Mercury comparable to the accepted relativistic result. This is accomplished by taking into account the solar gravitational quadrupole moment due to the sun's oblateness.

This result seems very plausible and in agreement with many other research papers that have shown relativity theory's logical fallacies, internal inconsistencies, and failure to agree with data from most phenomena.(1) Also this work can be shown to be in agreement with Poincare's logical arguments showing that all forces and phenomena in nature are purely electromagnetic in nature(2), if one notices that the oblateness of the sun is an electro- magnetic phenomenon due to the sun moving about the center of the Milky Way with velocity 156 ± 23 mi./sec. From (3) the oblateness of the sun is given by

$$r(\theta) = r_0(1-v^2/c^2)1/2/(1-v^2\sin^2\theta/c^2)1/2 \qquad (1)$$

$$r = r_0(1 - 1/2\ v^2/c^2 + \ldots\ ) = r_0(1 - D) \qquad (2)$$

$$D = 1/2\ v^2/c^2 = .5(156\pm23/186000)^2 = 3.5\pm1.1\ x\ 10^{-5} \quad (3)$$

References

1. Lucas, C. W., Jr., "A Call for Reformation in Modern Science," *Proceedings of the First International Conference on Creationism*, Pittsburgh, Pa., August 1986.

2. Lucas, C. W., Jr., "A New Unified Theory of Modern Science," *Proceedings of the First International Conference on Creationism*, Pittsburgh, Pa., August 1986.

3. Lucas, C. W., Jr., "An Electrodynamic Model for Charged Elementary Particles," *SOLI DEO GLORIA*, Church Computer Services, 4511 Poppe Place, Temple Hills, MD 20748, 1979, p. 16.

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CLOSURE

I want to thank Dr. Thomas G. Barnes and Dr. Charles W. Lucas, Jr. for their very kind comments.

Indeed, the contents of my article show other possibilities to general relativity for explaining the anomalous shift of Mercury's orbit. However, let me emphasize one thing here. The equations covered apply solely to the case of Mercury. If we want to know the extra precession of Venus' orbit, or the orbit of any of the outer planets for example, using the method of the last section of my article, we must take into account that Mercury will now play the role of an inner planet. Because of this, the equations of motion will differ significantly from those contained in that last section. But still, the method followed is exactly the same.

Some physicists have even claimed that the procedure of the last section of my article is actually an alternative way of solving Newton's differential equations. These assertions are interesting and will yield terminal conclusions on their validity when we apply the same

method to the case of the outer planets. Then we will know if the approach is general or not.

The results of my work are embodied in (and support the need for) the much more significant and revolutionary task of re-deriving quantum and relativistic mechanics using pure classical physics; all this from the viewpoint that every natural phenomenon is electromagnetic in nature. Such major objective is being successfully attacked by Dr. Barnes and his students, and independently by Dr. Lucas, Jr. Their approaches are somewhat different, but their aim is the same. Dr. Barnes has compiled most of his work in two well-known books, Physics of the Future, edited by the Institute for Creation Research of Santee, California, and Space Medium, published by the Geo-Space Research Foundation of El Paso, Texas, while Dr. Lucas' achievements are contained in the three references of his review of my article.

As a final remark, let me point out that the oblateness of the Sun, as derived from Lucas' electromagnetic arguments, should be equal to $3.5 \pm 1.1 \times 10^{-7}$ instead of what it is shown in equation three of his review. The corrected value is still quite minute and it has no serious consequences against, nor in favor of, my conclusions. Nevertheless, I want to thank Dr. Lucas for spending part of his time on trying to find significant electromagnetic contributions to the solar oblateness. Thank you very much.

<div align="right">Francisco S. Ramirez Avila, IV</div>