2020-07-08

An Investigation of the Role of Spatial Ability in Representing and Solving Word Problems Among Engineering Students

Gavin Duffy
Technological University Dublin, gavin.duffy@tudublin.ie

Sheryl A. Sorby
University of Cincinnati, sheryl@mtu.edu

Brian Bowe
Technological University Dublin, Brian.Bowe@TUDublin.ie

Recommended Citation

Duffy, G., Sorby, S., & Bowe, B. (2020). An investigation of the role of spatial ability in representing and solving word problems among engineering students. *Journal of Engineering Education*, 109(3),, doi:10.1002/jee.20349

Funder: National Science Foundation

An investigation of the role of spatial ability in representing and solving word problems among engineering students[1]

Gavin Duffy[a], Sheryl Sorby[b], Brian Bowe[a]

[a]Technological University Dublin, [b]University of Cincinnati

Abstract

Background

Spatial ability is significantly related to performance in engineering education. Problem solving, an activity that is highly relevant to engineering education, has been linked to spatial ability.

Purpose/Hypothesis

To what extent is spatial ability related to problem solving among engineering students and how do approaches to problem representation and solution vary with spatial ability level?

Design/Method

Three instruments – a spatial ability test, word math problems and accompanying core math competency questions – were administered to two samples of first year engineering students in two different countries. Data were analyzed at the test level to evaluate the relationship of spatial ability to problem representation and solution. A detailed item level analysis was conducted to compare approach to problem solving with spatial ability level.

Results

Spatial ability was found to be significantly related to problem solving but not to the core competency questions indicating the relationship was limited to the problem representation phase and not the solution phase. Key aspects of representation were identified for each problem to reveal a more pronounced relationship between representation and spatial ability than between problem solving (representation and solution) and spatial ability.

Conclusions

Problem solving can be considered to consist of two cognitively distinct phases: spatial ability is significantly related to problem representation but not to problem solution. Hence, this study shows that spatial ability plays a key role in engineering education that is not limited to visualization of imagery but extends to thinking during problem solving, a non-routine activity that requires mental representation.

Introduction

Long considered to be a primary factor of intelligence (e.g., Thurstone, 1938), spatial ability relates to visualization and mental processing of images and is required for everyday thinking (e.g., navigation) and to support learning (e.g., visualize the number line when

[1] This is the submitted for review version of the paper. Final version available at https://onlinelibrary.wiley.com/journal/21689830

learning math). It occupies a prominent role in several models of intelligence (Lohman, 1996) and working memory (Baddeley & Logie, 1999). The role of spatial ability in shaping education and career choice was thoroughly documented through Project TALENT, a very large scale (n ≈ 400,000) longitudinal study conducted in the US (Wai, Lubinski, & Benbow, 2009). In this study, spatial, verbal and mathematical abilities were measured in high school and participants were followed up 11 years later to find out what career choices they made. Retrieving their cognitive ability data from adolescence, those who pursued education and careers in humanities had a very different cognitive ability profile compared to those destined for engineering. Broadly similar in verbal ability, these two groups were separated by their math and spatial abilities with the biggest difference to be found in the latter. "*Spatial ability is a salient psychological characteristic among adolescents who subsequently go on to achieve advanced educational and occupational credentials in STEM* [science, technology, engineering and mathematics]" (Wai et al., 2009, p. 827). Supported by other studies, these findings have prompted the US National Science Board to call for greater attention to be paid to spatial ability in identification of STEM talent (National Science Board, 2010).

Some reasons why spatial ability is connected to success in engineering education can be found in studies that have focused on particular curriculum components, including physics, electric circuits, chemistry and math. Using concept tests as a measure of subject knowledge, a significant relationship between spatial ability and Newtonian mechanics has been measured (Kozhevnikov, Motes, & Hegarty, 2007; Kozhevnikov & Thornton, 2006). Tasks included those found on the Force and Motion Concept Inventory (Thornton & Sokoloff, 1998) such as frame of reference problems and interpreting graphs of velocity and acceleration over time. Understanding concepts associated with physical aspects of simple direct current electric circuits was found to be significantly correlated with spatial ability (Duffy, 2017; Duffy & O'Dwyer, 2015), a study that used the DIRECT concept test (Engelhardt & Beichner, 2004). Based mostly on data collected from course grades, Bodner and colleagues (Bodner, 2015; Carter, LaRussa, & Bodner, 1987; Pribyl & Bodner, 1987) have found high spatial ability students to have an advantage when mentally manipulating 2-D representations of molecules and when problem solving skills are required.

"*The relation between spatial ability and mathematics is so well established that it no longer makes sense to ask whether they are related*", according to Mix & Cheng (2012, p. 206) who list several findings from research including that visuospatial working memory (VSWM) was found to be related to counting tasks, nonverbal problem solving and reasoning tasks among school children and mental rotation was found to be related to word problem solving among 6^{th} graders. Some of this research is motivated by the widely reported gender gap in spatial ability in favor of males (Lippa, Collaer, & Peters, 2010) such as the study by Casey, Nuttall, & Pezaris (2001) which found significant correlations between spatial ability and two math subsets, one which favored males and had higher imagery ratings than the other which was more procedural and favoured females. The correlations with spatial ability were much higher for the male math subset (.44 to .55) than for the female subset (.17 to .29) indicating the strength of the relationship depends on the particular math task.

What emerges from these studies is that spatial ability is relevant when tasks require visualization of images, reasoning about questions on concept tests and non-routine activities such as problem solving. A common theme appears to be that those with high levels of spatial ability appear to perform well when reasoning about problem scenarios that often, but not always, include well-structured images. While this contributes towards explaining the overall relationship unearthed by Project Talent, two observations are worth making which are relevant to the rationale for this study. First, based on definitions of spatial ability found in the factorial models of intelligence (e.g., Carroll, 1993; Linn & Petersen, 1985; McGee, 1979), only tasks that require visualization should be related to spatial ability. Yet, some findings suggest this limitation is too narrow as performance on tasks that do not contain images can also be related to spatial ability. Second, findings related to the relationship between spatial ability and math performance have mostly emerged from studies using samples of children with few, if any, reports in the literature of the role of spatial ability in problem solving among samples of engineering students.

Another motivation for this study is that, despite the trend revealed by Project Talent, not all engineering students have high spatial ability. It has been shown that a sizeable minority of first year engineering students – in the range of 10 to 20 % (Sorby & Veurink, 2010a) – can fail a spatial test. In terms of cognitive ability profile, these students are more similar to their peers in humanities than engineering. In addition, this group is over represented by women given the gender gap in spatial ability (Lippa et al., 2010). To illustrate, a sample of 535 engineering students that was 17 % female contained a low spatial ability group that was almost 50 % female (Sorby, 2009). It is likely these low spatial students – male and female - face greater challenges in interpreting graphs, reasoning about frame of reference scenarios, completing electric circuit tasks and solving math and chemistry problems. A better understanding of the relationship of spatial ability to performance in engineering education would facilitate the development of learning, teaching and assessment activities to better support these students.

Should curricula be reformed to cater for low spatial students or should they learn ways to cope with these challenges? Spatial ability is malleable, it can be developed through focused intervention (Uttal et al., 2013). Spatial ability training has been shown by Sorby and colleagues to lead to significant gains in spatial ability, retention rates and grades in certain subjects among first year engineering students (Sorby, 2012; Sorby, Casey, Veurink, & Dulaney, 2013; Sorby & Veurink, 2010b). Since engineering curricula are already struggling to cover everything science has discovered since Galileo made his telescope and more, adding another course can be difficult or impossible. Alternatively, students weak in spatial ability could be guided in ways to cope with the tasks that have been shown to be related to spatial ability.

To inform such decisions, a clearer understanding of the role spatial ability plays in carrying out engineering activities is needed. Without that knowledge, both evaluation of spatial ability development and ways to develop coping strategies may be misguided. In the case of the math-spatial relationship, it may not be appropriate to apply findings from samples of 6th grade children to engineering students as these populations differ in two ways. First, the age gap coincides with the teenage years, a period in life of much cognitive development – 12 year olds are cognitively different to 18 year olds. Second, as shown through Project Talent, engineering students have a different cognitive profile compared to the general population. Hence, there is a need to study the math-spatial relationship among engineering students and, if found to be present, to learn as much as possible about the nature of this relationship.

Solving word problems in mathematics

Despite achieving very high grades in mathematics courses, engineering students can struggle to solve relatively simple math problems as shown by Clement (1982) who administered the following problem and instructions to 150 freshman engineering students:

> *"Write an equation using the variables C and S to represent the following statement:*
>
> > *At Mindy's restaurant, for every four people who ordered cheesecake, there are five people who ordered strudel.*
>
> *Let C represent the number of cheesecakes and S the number of strudels."*
> (Clement, 1982, p. 17)

The success rate on this problem was 27 % - approximately two thirds of the class failed to convert the word statement into the correct equation (5C = 4S). The most common incorrect response (68 %) was 4C = 5S which can be obtained by translating one word at a time in the order of appearance without comprehending the entire statement. Several other studies have shown how rephrasing a problem without changing the mathematical properties can have a large effect on success rate (e.g., Coquin-Viennot & Moreau, 2003; Hegarty, Mayer, & Green, 1992). This literature illustrates that, for many, a problem that is simple in mathematical content can be very difficult to solve; comprehending and translating the problem statement can be very difficult even when the mathematical procedures are simple.

Solving problems is theorized in cognitive psychology to begin with a representation phase, which draws on linguistic, semantic and schematic knowledge, followed by a solution phase in which core competencies are deployed guided by strategic knowledge (Mayer, 1992). In examining the relationship between visualization and problem solving among 6^{th} grade children, both Hegarty & Kozhevnikov (1999) and Boonen, van Wesel, Jolles, & van der Schoot (2014) found a large variation in the quality of visualizations produced during problem solving. Some participants provided 'pictorial' visualizations that included images of the objects or persons contained in the problem statement but failed to include the relations between them. Others provided accurate, schematic visualizations that included both objects and relations and these participants had significantly higher success rates than those who produced pictorial imagery. To some extent, the variation in visualization quality contained within these samples was accounted for by variation in spatial ability. Mental representation, as evidenced by visualization, varied among the samples and contributed towards explaining differences in success rates but an association between quality in mental representation and high spatial ability was only partially supported among these samples of 6^{th} graders.

Research questions

The purpose of this study was to contribute to knowledge of the relationship between spatial ability and problem solving skills of engineering students. While problem solving is relevant to many discipline specific subjects in engineering education, the emphasis in this study was on word problems in mathematics because (i) it is a subject area that is common to all engineering disciplines, (ii) very basic core competencies in mathematics are needed to solve these problems and (iii) findings could be compared to the existing literature on the relationship between spatial ability and math problem solving.

The research questions addressed in this study are as follows:

1. To what extent is math word problem solving related to spatial ability among engineering students?
2. If significant, which particular phase in problem solving (representation, solution) is this relationship associated with?
3. For an individual word problem, what representation is required to solve the problem and how is ability to produce this representation related to spatial ability?

Method

A mixed methods design was employed to first expose a relationship using quantitative methods and then investigate the nature of that relationship using an interpretive approach to coding problem solutions. Once the solutions were coded, quantitative methods were employed to describe spatial ability differences between those whose solutions did and did not satisfy these codes.

Participants

A sample of 115 participants enrolled in first year engineering in two universities was recruited for this study. Sixty-two students enrolled in the general first year engineering program at a university in Ireland participated in the study along with 53 students from first year engineering at a university in the US. Ethical approval was obtained from each institution prior to data collection, each participant was fully informed about the study and each signed a consent form before participating. The US participants were recruited from those enrolled in a spatial skills development course in the Fall semester of 2016. These students were offered this course based on achieving a low score on the spatial test during orientation in August 2016, hence the low average spatial score for this group and the relatively high proportion of 'weak visualizers' when the samples are combined. The sample therefore represented engineering students from two universities in two countries and contained a wide distribution in spatial ability.

Procedure

Both samples were administered three instruments: the Purdue Spatial Visualization Test: Rotations (PSVT:R, Guay, 1976), a set of 6 math problems and a set of 6 math questions. Data from the Ireland sample were collected during a regular class session in April 2016. Spatial ability data were collected from this sample during orientation in September 2015. The US sample was administered the spatial test during orientation in July/early August 2016 and the math problems and questions were administered individually in August 2016.

Instruments

Spatial test

The PSVT:R consists of multiple choice questions designed to measure 3-D mental rotation ability. The test was timed with 20 minutes allowed so the participant's speed and accuracy are assessed. Each question shows a target figure in original and rotated positions. A second 3D shape is then presented in its original position and participants select one of five possible rotated versions to match the target rotation. One of the two practice questions from the test is shown in Figure 1. This question involves the rotation of the object by 90°

around the vertical axis. There are 30 questions on the test with rotations varying around one, two and three axes. Reliability measures for the PSVT:R are reported by (Yoon, 2011) with Cronbach's α = .81 measured using data collected from a sample of 180 education major undergraduate students enrolled in mathematics courses. Average scores on this test have been measured to be 21.32 for females and 24.62 for males among a sample of US first year engineering students (Sorby et al., 2013).

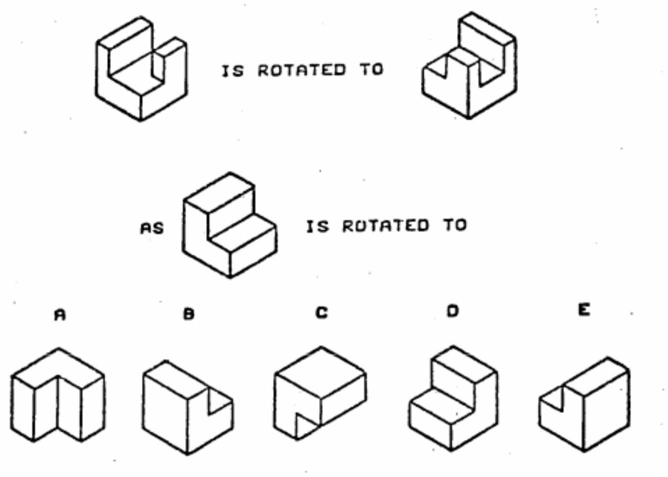


Figure 1. Sample question from the PSVT:R (Guay, 1976)

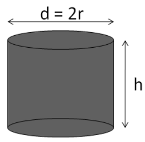
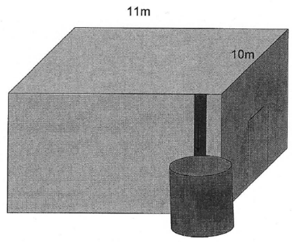
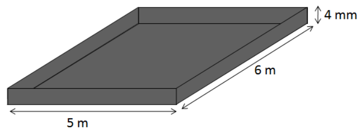
Word Problems

A pilot study was conducted with a sample of 13 participants from the engineering undergraduate teaching assistant pool at the US university. Based on the results from the pilot study, 6 problems were selected based on: a) each taking approximately 5 minutes to solve, and b) ability to discriminate by spatial ability. The data collected by administering the set of 6 problems to the 115 engineering students were found to be normally distributed. The internal consistency of the set of problems was found to be medium (Cronbach's α = .49, 6 items) which reflects how each problem is distinct to some extent. The 6 problems are provided in Table 1.

Core Competency Questions

A set of 6 questions was developed to assess the math competencies needed to solve the problems. For example, the Lawn problem required the creation and solution of a quadratic equation. Hence, a core competency was identified - obtain the factors of a quadratic equation – and a question was created that presented a quadratic in x, with the same structure but different constants as the problem quadratic, and asked the participant to find the factors. Internal reliability of this test was found to be medium (Cronbach's α = .61, 6 items) but there was significant skew in the distribution to the upper end of the range as the average score on this test was high. The 6 questions are provided in Table 1.

Table 1. Set of problems and accompanying set of questions used in the study

Problem	Question
1. A square lawn was extended in width by 2 m and in length by 3 m. The area of the new lawn is twice as big as the area of the old lawn. What are the dimensions of the old lawn?	1. Find the roots of $2x^2 + 6x - 8 = 0$ using factoring.
2. Stainless steel cylindrical jugs are made to hold a volume of 2 litres (2000 cm^3). If the 1 litre mark is at 8.84 cm what is the radius of the jug to the nearest centimetre?	2. What is the volume of this cylinder?  (Also needed for Problems 4 and 6)
3. Drink cans are made by stamping out circular discs from a sheet of metal. The rectangular sheet from which the discs are stamped out measures 1 m by 2 m. If the cans have a radius of 10 cm, how many discs can be made from this sheet of metal?	3. How many centimetres are in a metre?
 4. The diagram above shows the dimensions of a flat roofed commercial shed. During one week 5 mm of rain fell on the roof of the shed. The rain was collected by gutters that flowed into a cylindrical water barrel with a diameter of 1 m. By how much did the depth of the water in the barrel increase as a result of this rain?	4. What is the equation for the area of a circle? (Also needed for Problems 2 and 6)
5. I have some pencils and some jars. If I put 4 pencils into each jar I will have one jar left over. If I put 3 pencils into each jar I will have one pencil left over. How many pencils and how many jars are there?	5. What is the volume of this tank? 
6. When blood samples are centrifuged the blood separates into two distinct layers – one made up mainly of plasma and the other made up of red blood cells. A sample of blood was put in a flat bottomed test tube with a diameter of 3 cm. When the blood sample was added to the tube it filled the tube to a depth of 7.5 cm. After centrifuging, the red blood layer was 1.5 cm deep. What volume of plasma was in the sample?	6. Determine the value of x and y by solving these two equations $x + y = 6$ $-3x + y = 2$

Data analysis

Relationships at the Test Level

In order to address the first two research questions, answers to both the set of 6 problems and the set of 6 questions were scored as either correct or incorrect. Answers to the PSVT:R questions were also scored as correct/incorrect. These data were examined using frequency plots and checked for normality. Depending on the normality of the distributions, correlations between the measures were calculated using either the Pearson or the Spearman coefficient. Then, for each problem and question, the sample was grouped as being correct or incorrect and the spatial test scores of each group were compared using an independent samples t-test. Finally, cases were excluded if the answer to the core competency question was incorrect in order isolate procedural mathematical knowledge as a determining factor in problem solving. The sample was again grouped as correct or incorrect on each problem and the spatial test scores of each group were compared using an independent samples t-test.

Identifying a key aspect of representation for each problem

To answer the third research question, a second round of analysis focused on the solutions to the problems only. No cases were excluded in this analysis based on the assumption that a problem can be represented even if a participant lacks procedural knowledge required in the solution phase.

All solutions to all problems were analysed based on a coding scheme developed from Mayer's framework for problem solving (Mayer, 1992) which consists of several types of knowledge that are required to solve math problems. This was tailored for each problem as each problem was different and had a different solution path. An illustration of how this was tailored for the Lawn problem is outlined in Table 2.

Table 2. Mayer framework (Mayer, 1992) for math problem solving applied to the Lawn problem

Type of knowledge	Application to the Lawn problem
Linguistic knowledge	ability to understand the words used in the problem, e.g. the lawn is square, area of new lawn is twice the old area, new width equals old width plus 2, new length equals old length plus 3
Semantic knowledge	ability to draw on common sense or knowledge that is taken for granted, e.g. a square has four equal sides, a lawn is an area of grass beside a house (this definition was provided in the problem statement) that is 2 dimensional (not provided)
Schematic knowledge	ability to draw on knowledge of schema, i.e. that have been previously learnt; in this case, a schema that is required is that area = length x width
Strategic knowledge	ability to set subgoals in the problem solving process and to monitor progress, in this case one should develop an equation for one unknown – the side of the original square – and then solve for this unknown using algebra
Procedural knowledge	ability to perform standard mathematical procedures; in this case, an important procedure is to factorise a quadratic, (hence the core competency question testing this skill)

Each transcript was studied in turn while checking for evidence of these actions in each participant's written solutions to the Lawn problem. In practice, the above table was broken up and simplified to allow the coding to proceed more efficiently with some modification of codes required. For example, every participant that showed evidence of the lawn being extended in width by 2 m also showed evidence of it being extended in length by 3 m, hence these codes were combined into one. In the case of the Lawn problem, this process led to the final set of codes shown in Table 3. The same process was followed for each problem so that a unique table of codes was prepared and used to score each solution. Problem representation was based on problem solving actions and, for each problem, the full set of actions evident in the data set was identified and used as a checklist for each problem solution.

Table 3. Codes for the Lawn Problem

No.	Code	1 if participant	0 if	Type of knowledge
1	Square Lawn	Discerns lawn is a square	Does not, e.g. rectangle	Linguistic Assignment
2	Area change	Discerns A_{New}=2xA_{Old}	Does not	Linguistic Relational
3	Size change	Discerns correct change in width and length	Does not	Linguistic Relational
4	Apply 1, 2 and 3	Gets all three linguistic ingredients	Gets less than 3	Linguistic
5	A=WxL	Includes Area = width x length	Does not	Schematic
6	Combine 4 & 5	Get all 4 ingredients needed to write the equation	Does not	Representation – schematic & linguistic
7	Correct equation	Write the correct equation	Makes error	Procedural
8	Solve equation	Correctly solve the equation through factoring	Makes error	Procedural

In order to measure the relationship between spatial ability and problem solving actions the sample was split into two groups – low (PSVT:R ≤ 18) and high (PSVT:R ≥ 19) spatial ability – and the number in each group who did and did not show evidence of each action was summed. This low/high categorisation was used previously by Sorby & Baartmans (1996). Rather than having a psychological basis, this approach is consistent with a commonly used pass/fail threshold of 60 % on the PSVT:R. Dividing the sample at this point was very close to having a median split as the median score was 19 with four participants having this score, 55 below and 56 above the median. Kozhevnikov & Thornton (2006) classified the sample in their study as either low or high in spatial ability on the basis of a median split. In addition, for each problem solving code, the sample was divided into those who did and didn't show evidence of this action and the difference in spatial level of each group was tested for significance.

The outcome of this analysis was an aspect of representation that was unique to each problem but was essential for the problem to be solved successfully, i.e. the problem could not be solved without this aspect of representation.

Results and analysis

Descriptive statistics and correlations between measures

Descriptive statistics are presented separately for each and combined samples in Table 4.

Table 4. Descriptive statistics for the variables measured

Sample	n Total	n Male	n Female	Age	PSVT:R Mean (S.D.)	Problems Mean (S.D.)	Questions Mean (S.D.)	n Low-spatial	n High-spatial
Ireland	62	53	9	20.4 (4.1)	22.37 (5.53)	2.85 (1.48)	5.00 (1.22)	10	52
US	53	25	28		16.58 (3.86)	1.91 (1.32)	5.68 (1.47)	45	8
Combined	115	78	37		19.70 (5.61)	2.42 (1.48)	5.31 (1.37)	55	60

The lower spatial test score for the US university reflects the sample being recruited from those enrolled in the spatial skills development course. The combined samples contained almost equal numbers of low and high-spatial participants. Performance on the math questions was very high – very few made errors on these questions – whereas performance on the math problems was more varied. The extent to which variation in each variable was shared with the other was measured by determining a Pearson correlation coefficient for each pairing of the variables. Given the non-normal distribution of the math question data, correlations related to this variable were also determined using the Spearman correlation coefficient. As shown in Table 5, a significant amount of variation in the spatial test data is shared with the problem solving measure but not with the math question data.

Table 5. Correlation matrix for scores on the PSVT:R, math problems and math questions.

	Math Problems r (Pearson)	Math Questions r (Pearson)	Math Questions r_S (Spearman)
PSVT:R	.544***	.131	.153
Math Problems		.418***	.459***

** significant at p < .01
*** significant at p < .001

Further statistical analysis was conducted to examine how spatial ability was related to performance on each individual problem. This was first examined by determining point bi-serial correlations between individual problem score (0 or 1) and the spatial test data and these results are presented in Table 6. For problems 2, 3, 4 and 6, the correlation was found to be significant at the p < .01 level, for problem 1, it was significant at the p < .05 level and for problem 5 the correlation was small and insignificant.

Table 6. Point bi-serial correlation between PSVT:R and each math problem.

Problem	1	2	3	4	5	6
PSVT:R	.203*	.414**	.362**	.336**	-.072	.395**

*. Correlation is significant at the 0.05 level (2-tailed).
**. Correlation is significant at the 0.01 level (2-tailed).

Another way to present the relationship between performance on each problem and spatial ability is to group the sample into correct and incorrect response and compare the PSVT:R scores of these two groups using an independent samples t-test. These results, shown in Table 7, exhibit the same pattern as the point bi-serial correlations, as expected.

Table 7. Comparison of means of PSVT:R scores for those correct and incorrect on each math problem.

	Correct			Incorrect					
Problem/ Question	n	M	SD	n	M	SD	t-test	Sig (2-tailed)	Cohen's d (Size)
P1 Lawn	30	21.53	5.53	85	19.06	5.53	-2.107	.037*	0.45 (Medium)
P2 Jug	69	21.59	4.93	46	16.87	5.43	-4.837	.000**	0.92 (Large)
P3 Cans	45	22.16	4.73	70	18.13	5.60	-3.993	.000**	0.78 (Large)
P4 Rain	33	22.67	4.35	80	18.44	5.79	-3.776	.000**	0.83 (Large)
P5 Jars	43	19.19	5.24	72	20.01	5.84	.764	.447	0.15 (Small)
P6 Blood	62	21.76	4.71	53	17.30	5.67	-4.604	.000**	0.86 (Large)

*. Correlation is significant at the 0.05 level (2-tailed).
**. Correlation is significant at the 0.01 level (2-tailed).

Cases were next excluded from the problem data set if the answer to the corresponding math question was incorrect. For example, if a participant was incorrect in the question on factoring a quadratic equation, s/he was removed from the Lawn problem data as this problem required this math competency. Since this was done separately for each problem the number of cases now varied per problem and it was not possible to determine a second correlation matrix. However, it was possible to redo independent samples t-tests for each problem, grouping the sample as before into correct and incorrect groups. As shown in Table 8, the pattern found earlier still holds – large, significant differences in spatial test score between those correct and incorrect on problems 2, 3, 4 and 6, medium to large, significant difference on problem 1 and a small, insignificant difference on problem 5. Excluding cases based on errors in math competencies did not alter the previous findings.

Table 8. Comparison of means of PSVT:R scores for those correct and incorrect on each math problem with cases excluded if the answer to the corresponding question is incorrect (n varies by problem)

	Correct			Incorrect					
Problem	**n**	**M**	**SD**	**n**	**M**	**SD**	**t-test**	**Sig (2-tailed)**	**Cohen's d (Size)**
P1 Lawn (Q1=1)	27	22.30	5.74	62	19.48	5.29	-2.247	.027*	0.52 (Large)
P2 Jug (Q2=1)	64	21.88	4.85	22	16.41	5.24	-4.470	.000**	1.09 (Large)
P3 Cans (No Q)	44	22.27	4.72	71	18.11	5.56	-4.125	.000**	0.81 (Large)
P4 Rain (Q2 & 5=1)	27	22.41	4.45	41	18.27	5.48	-3.277	.002**	0.83 (Large)
P5 Jars (Q6=1)	36	19.03	5.33	60	20.30	5.73	1.080	.283	0.23 (Small)
P6 Blood (Q2=1)	59	21.81	4.80	27	17.30	5.97	-3.744	.000**	0.84 (Large)

*. Correlation is significant at the 0.05 level (2-tailed).

**. Correlation is significant at the 0.01 level (2-tailed).

Problem Representation

Results from the analysis of approach to problem solving are presented for the Lawn problem in Table 9. Errors were made by both low and high-spatial participants at each step in solving this problem but the percentage of the low-spatial category failing at each step was always higher as shown in Table 10. In order to write the correct equation, a participant had to extract four key ingredients from the problem statement – (i) the original lawn is square (an assignment statement), (ii) the change in width and length (a relational statement), (iii) change in area (a relational statement) and (iv) area = width x length. According to Mayer's framework, the first three relate to linguistic knowledge and the last to schematic knowledge. This latter piece of schematic knowledge was in evidence in the solution of all but 8 participants, presumably because the problem statement contains many cues for this schema through words such as 'square', 'width', 'length' and 'new area', for example. There was little difference between the number who correctly translated all three linguistic ingredients but failed to apply the correct schema and the number who were correct in all of these aspects. It was therefore decided that the key aspect of representation of this problem that presented a challenge to this sample was the sum of square lawn, area change and size change.

Table 9. Success rates and differences in spatial ability for each code in the Lawn Problem

No.	Code	n Low-spatial 1	0	n High-spatial		PSVT:R M (SD) 1	0	t (p)	Cohen's d
1	Square lawn	31	24	42	18	20.27 (5.38)	18.71 (5.93)	-1.441	0.28 (Medium)
2	Area change	43	12	51	9	20.34 (5.49)	16.86 (5.41)	-2.637*	0.64 (Large)
3	Size change	41	14	53	7	20.43 (5.36)	16.48 (5.72)	-3.017**	0.72 (Large)
4	Apply 1, 2 and 3	21	34	33	27	21.22 (5.24)	18.36 (5.63)	-2.810**	0.53 (Large)
5	Apply A=WxL	49	6	58	2	20.05 (5.52)	15.13 (5.06)	-2.444*	0.93 (Large)
6	Combine 4 & 5	20	35	32	28	21.29 (5.29)	18.40 (5.58)	-2.833**	0.54 (Large)
7	Correct Equation	9	46	22	38	22.13 (5.26)	18.81 (5.50)	-2.904**	0.62 (Large)
8	Solve	6	49	17	43	22.43 (5.52)	19.02 (5.46)	-2.678**	0.63 (Large)

* Significant at the p < .05 level
** Significant at the p < .01 level

The largest contrast in representation error between low and high-spatial participants was to be found not in each individual linguistic item but in the combination of all three, i.e. high-spatial participants were significantly more likely to correctly identify all three linguistic items than low-spatial. Hence, the finding from this round of analysis of the Lawn problem was the key aspect of representation that was discriminating and revealed a significant difference in spatial ability related to extracting and translating assignment and relational statements from word to mathematical form.

Table 10. Numbers of all, weak and strong visualizers who failed to show evidence of Lawn problem solving codes

Action	Description	Error rate (%) All	Low-spatial	High-spatial	Cohen's d
1	Treat lawn as square	37	44	30	.28
2	Size change	18	25	12	.72**
3	Apply $A_{new}=2xA_{old}$	18	22	15	.64*
4	Apply 1, 2 and 3	53	62	45	.53**
5	Apply A = W x L	7	11	3	.93*
6	Combine 4 & 5, get all 4 ingredients	55	64	47	.54**
7	Write correct equation	73	84	63	.62**
8	Solve equation	80	89	72	.63**

* Significant at the p < .05 level
** Significant at the p < .01 level

This approach to analyzing the data was followed for each problem in turn so that a key code for each problem could be identified that (i) was essential for solving the problem and (ii) revealed individual differences in performance. Presented in Table 11 are the results of the second round of analysis – a key representation code from each problem, the percentage of low and high-spatial participants that failed to show evidence of this code in their solutions, the effect size (Cohen's d) for the difference between the spatial test scores of those who did and didn't show this code and the type of knowledge required for this aspect of representation to be possible (Mayer, 1992).

Table 11. Key codes extracted from each problem.

Problem	Key code	Error rate (%) Low-spatial	High-spatial	Cohen's d	Knowledge type
Lawn	Combination of applying lawn is square, width + 2 and A_{new} = 2 x $A_{old.}$	62	45	.53**	Linguistic
Jug	Correctly match height with volume	29	12	.93**	Linguistic
Cans	Apply any grid layout schema	60	29	.81**	Schematic
Rain	Adopt volume is conserved schema	59	27	.89**	Schematic
Pencils & Jars	Algebraic model attempted	62	45	.50**	Schematic
Blood	Cylinder volume schema	31	10	.99**	Schematic

** Significant at the p < .01 level

Discussion

Spatial ability is a primary factor of intelligence that has been shown to be significantly related to success in STEM education (Wai et al., 2009). The purpose of this study was to both quantify the relationship between spatial ability and word problem solving among engineering students and, by examining differences in approach to problem solving that varied with spatial ability, provide reasons to explain this relationship. In this section we discuss themes that emerged from answering the research questions: (i) the role of spatial ability in problem representation, (ii) what this suggests more broadly about the relevance of spatial ability to engineering education, and (iii) the implications of transferring findings from this research into engineering education practice. Each theme is discussed below.

The role of spatial ability in problem representation

We believe that most engineering faculty would describe the math word problems used in this study as basic, simple and easy to solve. However, they presented a difficulty to the some of the sample recruited for this study. As shown by the core competency data, that difficulty was not a mathematical one as the mean score on this test was 5.31 out of 6 or 89%. These students had the mathematical knowledge to perform well on these problems yet many failed to do so. The first research question asked to what extent spatial ability is related to the ability to represent and solve word problems in mathematics among this sample of first year engineering students. Problem solving was found to be significantly related to spatial ability ($r(113) = .544$, $p < .001$) but for reasons other than math core competency as it was found to have a small and insignificant relationship with spatial ability ($r_s(113) = .153$, N.S.). The challenge faced by participants was in problem representation and it was this phase in the problem solving process that contained the relationship with spatial ability.

Analyzing the data at the item level revealed the individual steps – both correct and incorrect – taken by all participants in the sample when solving each problem. This analysis began with a set of codes based on Mayer's (1992) knowledge framework for problem solving but this was updated, with scripts scored retrospectively, as the data were examined. It emerged that spatial ability was relevant to the application of linguistic knowledge to translating assignment and relational statements and also to selecting schemata but this varied across problems. For two – Lawn and Jug - schema selection appeared to be trivial as almost all participants selected the correct schema in each case. For these problems, the difficulty lay in correctly and consistently translating statements that assigned values to variables and related variables to each other. While these statements were clearly written and not ambiguous, high spatial ability students performed significantly better than low spatial students at this level.

For three problems – Cans, Rain and Blood – schema selection was a discriminating factor and, again, high spatial students were significantly more successful than low spatial students in selecting the correct schema during problem representation. Perhaps visualizing the shapes referred to in the problem statements facilitated schema selection, hence the correlation with spatial ability. It is worth comparing the Jug and the Rain problems as both statements explicitly referred to cylindrical shapes but the Rain problem also included a prism of water on the shed roof. In the Jug problem, the challenge was not to match a height and volume but to do so correctly in one of two ways. In the Rain problem, the challenge was to include the transfer of volume from one container to another but 27 participants, 19 with low spatial ability, failed to include the roof in the problem solution, assuming instead that the barrel height increased by the rainfall amount of 5 mm.

Of the six problems used in this study, Pencils & Jars is the only problem for which success was not related to spatial ability. A difference in spatial ability did emerge in terms of approach: strong visualizers favoured an algebraic approach ($d = .50$, $p<.01$) whereas weak visualizers favoured solving by guess and check ($d = .89$, $p<.001$). Participants were not instructed to use algebra to solve this problem although it had been incorrectly assumed in the research design phase that they would. This suggests that when direction is not provided, the decision to use modelling is related to spatial ability. It is possible that guess and check facilitated an approach that placed a lower demand on working memory so variation in guess and check performance was not shared with spatial ability. However, further work is needed to investigate these possibilities.

This finding is also supported by the literature as other studies have highlighted this relationship even where a separate assessment of procedural knowledge was not included. For example, Boonen et al. (2014) and Hegarty & Kozhevnikov (1999) found qualitative differences in the nature of visualizing word problems during the problem representation phase with accurate and schematic visualizations more likely to lead to a correct answer and a significant relationship between frequency of using such visualizations and spatial ability. However, visualization quality did not account for all of the data as approximately two thirds of the solutions did not appear to be accompanied by any visualization yet the success rate for these was close to 50 % (Boonen et al., 2014). As shown by this study, the application of linguistic and schematic knowledge during problem representation provides a more fundamental account for the spatial- problem solving relationship. Visualization may be embedded in this process or may be a product of representation. While visualization quality is associated with success and problem solvers are well-advised to practice creating visualizations, it does not fully account for the spatial-problem solving relationship.

Relevance of these findings to the role of spatial ability in engineering education

Based on the findings of this study, the role of spatial ability in engineering education is not limited to tasks related to visualizing and transforming well-structured images, as factorial models of intelligence would suggest, but extends to representing word problem statements. Key to these tasks is that they are problems, i.e. prior knowledge does not contain a solution path, interpretation of and reasoning about the problem scenario is required, a task that is non-routine to the problem solver. Low spatial ability students are, therefore, likely to be challenged in the engineering curriculum for two reasons – spatial visualization and problem representation. While tasks related to processing well-structured images may be easy to identify, a more intimate knowledge of both the engineering curriculum and the students' prior experiences are required to judge where the tasks are in the curriculum that require thinking about non-routine problem scenarios.

Identifying non-routine problem solving activities may be difficult for a number of reasons. First, the term 'problem' can be loosely defined among engineering educators. For example, textbooks often contain questions that have little or no ambiguity with regard to solution path and are essentially tests of routine procedure yet are labelled problems. Second, prior knowledge of the student determines whether a task is routine or non-routine. If the problem set used in this study were to be administered to another sample that had extensively practiced very similar problems then a correlation with spatial ability may not emerge. Finally, in an engineering class with a small minority of low-spatial ability students a high average performance may mask the relationship; a keen eye may be needed to notice the few that are being challenged to form appropriate representations of non-routine scenarios.

As mastery is developed, spatial ability becomes less important, as observed in fields such as geology, chemistry, geometry and chess playing (Uttal & Cohen, 2012). For example, a correlation between spatial ability and FMCE performance observed among novices (Kozhevnikov et al., 2007) may disappear in a sample that has reached a high level of conceptual understanding of Newtonian mechanics. Statistically, both variables must have variation in order for some of that to be shared; if there is little variation in FMCE scores, for example, there is little to share. At a psychological level, this is explained by the FMCE tasks changing from non-routine to routine – the items on the FMCE were initially problems but have now become procedural. Among those that persist and achieve mastery, spatial ability is unlikely to be related to performance.

To those who would like to identify curriculum components that are likely to reveal a significant relationship with spatial ability we advise they search for tasks that require the student to:

- Create a visualization of a well-structured image and mentally manipulate this visualization
- Create a visualization of a scenario from a word statement
- Produce a mental representation of a word problem statement that contains many pieces of information that must interpret at a linguistic level and is ambiguous with regard to schema
- Reason and think about non-routine scenarios that have not been seen before and where the challenge lies in the novelty rather than the level of discipline knowledge

It is possible that tasks such as these are not easily isolated but are embedded in larger assessments or hidden in the course and yet must be overcome for the novice to progress to mastery.

Implications for teaching practice

A motivation for this research was to improve the experience of a large minority - 10 to 20% - of students that join engineering programs with low levels of spatial ability, a group that is over represented by female students (Sorby & Veurink, 2010a). What can we learn from this study that might help guide interventions to better prepare low spatial ability students for non-routine problem solving in the engineering curriculum? Two strategies are suggested – one is to learn how to cope with low spatial ability and the other to improve spatial ability. These options, including the assumption of causality implied in the latter, are discussed below.

Strategies to help these students should be designed with the knowledge that spatial ability is related to problem representation but not solution. This cognitive distinction between problem representation and solution implies that problem solving contains two very different learning outcome areas and the cognitive gap between them is so large that each may need very different learning, teaching and assessment methods. As a learning outcome, problem solution relates to knowing and being able to apply procedures and low spatial students are not necessarily challenged at this level. This can arguably be well addressed using traditional or behaviourist learning and teaching methods as procedures, such as factoring a quadratic equation, for example, tend to consist of relatively well defined rules and methods.

Problem representation, in contrast, is a very different learning outcome as this process not only draws on and integrates several different types of knowledge but also requires more judgment. As shown by Boonen et al. (2014) and Hegarty & Kozhevnikov (1999), success rates in problem solving are highest when the visualization is both accurate and schematic. Hence, to improve representation ability, students should become aware of visualization as an important tool in problem solving, practice creating accurate, schematic visualizations of word statements and get feedback on these visualizations. However, in the study by Boonen et al. (2014), two thirds of the solutions provided by the sample did not contain any evidence of visualization yet there was still a correlation between spatial ability and problem solving. As our findings have shown, the ability to apply linguistic and schematic knowledge is related to spatial ability. Which comes first – visualization or representation – or do they support each other? Is visualization essential to the correct translation of all assignment and relational statements in the problem? Is it more relevant to schema selection? Perhaps visualization is more relevant to some aspects of representation than others.

Memory models of cognition offer an alternative explanation of the role of spatial ability in problem representation. Translating problem statements is a non-routine activity for which there is no readily available procedure stored in long term memory. Such tasks are, therefore, heavily dependent on working memory. If the process is sufficiently demanding to cause working memory overload, key information is dropped leading to inaccurate representation. As shown by Miller (1956), working memory capacity is notoriously limited and, according to more recent work (Baddeley & Logie, 1999), visuospatial ability is a central component of working memory. Assuming the PSVT:R provides a measure of visuospatial working memory (Kyttälä & Lehto, 2008), high visualizers in this study were more successful at problem representation because they had higher working memory capacity which led to fewer errors being made in problem representation.

Hence, two approaches to coping are possible – one is to employ problem-solving heuristics that emphasise the creation of accurate, schematic visualizations and the other is to avoid working memory overload. Choosing between them depends on the ontology of spatial ability and human thinking. Factorial models of intelligence are consistent with the creation of visualizations whereas the literature on working memory would advocate the latter approach. Working memory overload could be avoided by methodically searching for individual components in the problem statement. Rather than attempt to grasp the statement as a whole and risk dropping key information, the problem solver would identify each assignment and relational statement and translate them one at a time before putting them aside and devoting all attention to schema selection. Creating visualizations may then support the integration of these translations to form a complete representation.

Rather than develop coping strategies, low-spatial students could take courses on spatial skills development in the hope such development will transfer to performance in other areas. Based on a meta-analysis of training studies Uttal et al. (2013) found evidence to support the claim that spatial ability can be improved through tailored learning interventions. Spatial ability training has been shown by Sorby and colleagues at Michigan Technological University to lead to significant gains in spatial ability, retention rates and grades in certain subjects among first year engineering students (Sorby, 2012; Sorby, Casey, Veurink, & Dulaney, 2013; Sorby & Veurink, 2010). Spanning a couple of decades, this research has shown the positive impact spatial skills training can have in terms of broad measures of performance in engineering education. In a recent intervention study in Australia, a 10 week spatial training course developed collaboratively with teachers and delivered to 6th grade students was found to result in significant gains in both spatial ability and mathematics (Lowrie, Logan, & Ramful, 2017). Cheng & Mix (2014) observed transfer of spatial training to a particular, well-defined math task but in a similar study using a randomized control trial this finding was contradicted (Hawes, Moss, Caswell, & Poliszczuk, 2015). While research has shown spatial training can transfer to performance in STEM education, more studies are needed to examine causal aspects of the relationship (Stieff & Uttal, 2015).

Conclusions

Spatial ability is significantly related to the process of creating mental representations of mathematically simple word problems among first year engineering students. High spatial students demonstrate much greater proficiency and consistency in translating assignment and relational statements and in selecting appropriate schemata during problem representation than low spatial students. Our study has shown that this relationship, revealed by others in much younger samples of 6th grade students (e.g., Boonen et al., 2014), is also present at a later stage in development indicating the relationship persists throughout adolescence. Hence, the findings of this study are relevant to those involved in all stages of adolescent education.

Problem solving consists of two cognitively distinct phases with spatial ability relevant to problem representation but not to problem solution. When problem representation can be simplified through guess and check, its relationship with spatial ability becomes insignificant. From a cognitive point of view, problem solving is a highly nuanced process that requires certain conditions for it to have a relationship with spatial ability which include ambiguity with regard to solution path, translation of assignment and relational statements and schema selection and representation which cannot be simplified through guess and check. We suggest that those who teach problem solving examine how these conditions apply in their subject area and consider how well learning, teaching and assessment activities are aligned with the cognitive features of problem representation and solution.

In engineering education, spatial ability is not limited to visualization and transformation of well-structured images as factorial models of intelligence might suggest. It is manifest in the ability to mentally organise information from sources that vary from well-structured images to word descriptions of quantities, that are sufficiently novel or non-routine to place a high demand on working memory, and rearrange this information into a new format that is consistent with the original form and to do so promptly.

It is claimed that low spatial students face their greatest challenge in early stages of STEM higher education and, as mastery is developed, the relationship between spatial ability and performance in STEM fades away (Uttal & Cohen, 2012). Our findings may contribute to explaining why: during this phase of education topics are often novel and non-routine, those with high-spatial ability are better matched at a cognitive level to these demands and, therefore, more likely to succeed and persist.

Acknowledgements

This material is based upon work supported by the National Science Foundation in the US. under grants number DRL-1535307 and DRL-1818758. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the National Science Foundation.

References

Baddeley, A. D., & Logie, R. H. (1999). Working memory: The multiple-component model. In A. Miyake & P. Shah (Eds.), *Models of working memory: Mechanisms of active maintenance and executive control.* (pp. 28–61). New York, NY, US: Cambridge University Press. https://doi.org/10.1017/CBO9781139174909.005

Bodner, G. M. (2015). Research on Problem Solving in Chemistry. *Chemistry Education: Best Practices, Opportunities and Trends*, 181–202.

Boonen, A. J. ., van Wesel, F., Jolles, J., & van der Schoot, M. (2014). The role of visual representation type, spatial ability, and reading comprehension in word problem solving: An item-level analysis in elementary school children. *International Journal of Educational Research*, *68*, 15–26.

Carroll, J. B. (1993). *Human cognitive abilities: a survey of factor-analytic studies*. Cambridge: Cambridge University Press.

Carter, C. S., LaRussa, M. A., & Bodner, G. M. (1987). A study of two measures of spatial ability as predictors of success in different levels of general chemistry. *Journal of Research in Science Teaching*, *24*, 645–657.

Casey, M. B., Nuttall, R. L., & Pezaris, E. (2001). Spatial-mechanical reasoning skills versus mathematics self-confidence as mediators of gender differences on mathematics subtests using cross-national gender-based items. *Journal for Research in Mathematics Education*, 28–57.

Cheng, Y.-L., & Mix, K. S. (2014). *Spatial Training Improves Children's Mathematics Ability*. Grantee Submission.

Clement, J. J. (1982). Algebra Word Problem Solutions: Thought Processes Underlying a Common Misconception. *Journal for Research in Mathematics Education*, *13*(1), 16–30.

Coquin-Viennot, D., & Moreau, S. (2003). Highlighting the role of the episodic situation model in the solving of arithmetical problems. *European Journal of Psychology of Education*, *18*(3), 267–279.

Duffy, G. (2017). *Spatial thinking in the engineering curriculum: an investigation of the relationship between problem solving and spatial skills among engineering students* (Unpublished doctoral dissertation). Dublin Institute of Technology.

Duffy, G., & O'Dwyer, A. (2015). Measurement of first year engineering students cognitive activities using a spatial skills test and an electrical concepts test: implications for curriculum design. Presented at the Research in Engineering Education Symposium.

Engelhardt, P. V., & Beichner, R. J. (2004). Students' understanding of direct current resistive electrical circuits. *American Journal of Physics*, *72*(1), 98–115.

Guay, R. (1976). *Purdue spatial visualization test*. Purdue University.

Hawes, Z., Moss, J., Caswell, B., & Poliszczuk, D. (2015). Effects of mental rotation training on children's spatial and mathematics performance: A randomized controlled study. *Trends in Neuroscience and Education*, *4*(3), 60–68. https://doi.org/10.1016/j.tine.2015.05.001

Hegarty, M., & Kozhevnikov, M. (1999). Types of visual–spatial representations and mathematical problem solving. *Journal of Educational Psychology*, *91*(4), 684–689. https://doi.org/10.1037/0022-0663.91.4.684

Hegarty, M., Mayer, R. E., & Green, C. E. (1992). Comprehension of arithmetic word problems: Evidence from students' eye fixations. *Journal of Educational Psychology*, *84*(1), 76–84. https://doi.org/10.1037/0022-0663.84.1.76

Kozhevnikov, M., Motes, M. A., & Hegarty, M. (2007). Spatial Visualization in Physics Problem Solving. *Cognitive Science*, *31*(4), 549–579. https://doi.org/10.1080/15326900701399897

Kozhevnikov, M., & Thornton, R. (2006). Real-Time Data Display, Spatial Visualization Ability, and Learning Force and Motion Concepts. *Journal of Science Education & Technology*, *15*(1), 111–132. https://doi.org/10.1007/s10955-006-10361-0

Kyttälä, M., & Lehto, J. E. (2008). Some factors underlying mathematical performance: The role of visuospatial working memory and non-verbal intelligence. *European Journal* *of Psychology of Education - EJPE (Instituto Superior de Psicologia Aplicada)*, *23*(1), 77–94.

Linn, M. C., & Petersen, A. C. (1985). Emergence and characterization of sex differences in spatial ability: A meta-analysis. *Child Development*, 1479–1498.

Lippa, R. A., Collaer, M. L., & Peters, M. (2010). Sex differences in mental rotation and line angle judgments are positively associated with gender equality and economic development across 53 nations. *Archives of Sexual Behavior*, *39*(4), 990–997.

Lohman, D. F. (1996). Spatial ability and g. In I. Dennis & P. Tapsfield (Eds.), *Human abilities: Their nature and measurement.* (pp. 97–116). Hillsdale, NJ: Lawrence Erlbaum Associates, Inc.

Lowrie, T., Logan, T., & Ramful, A. (2017). Visuospatial Training Improves Elementary Students' Mathematics Performance. *British Journal of Educational Psychology*, *87*(2), 170–186.

Mayer, R. E. (1992). *Thinking, problem solving, cognition*. New York: W.H. Freeman.

McGee, M. G. (1979). Human spatial abilities: psychometric studies and environmental, genetic, hormonal, and neurological influences. *Psychological Bulletin*, *86*(5), 889–918.

Miller, G. A. (1956). The magical number seven, plus or minus two: Some limits on our capacity for processing information. *The Psychological Review*, *63*, 81–97.

Mix, K. S., & Cheng, Y.-L. (2012). Chapter 6 - The Relation Between Space and Math: Developmental and Educational Implications. In J. B. Benson (Ed.), *Advances in Child*

Development and Behavior (Vol. 42, pp. 197–243). JAI. https://doi.org/10.1016/B978-0-12-394388-0.00006-X

National Science Board. (2010). *Preparing the next generation of stem innovators: Identifying and developing our nation's human capital*. Arlington, Virginia: National Science Foundation.

Pribyl, J. R., & Bodner, G. M. (1987). Spatial ability and its role in organic chemistry: A study of four organic courses. *Journal of Research in Science Teaching*, *24*(3), 229–240. https://doi.org/10.1002/tea.3660240304

Sorby, S. A. (2009). Educational Research in Developing 3-D Spatial Skills for Engineering Students. *International Journal of Science Education*, *31*(3), 459–480. https://doi.org/10.1080/09500690802595839

Sorby, S. A. (2012). Spatial Skills Training to Improve Student Success in Engineering. *Spatial Thinking Across the College Curriculum*, (2012 Specialist Meeting).

Sorby, S. A., & Baartmans, B. J. (1996). A Course for the Development of 3-D Spatial Visualization Skills. *Engineering Design Graphics Journal*, *60*(1), 13–20.

Sorby, S. A., Casey, B., Veurink, N., & Dulaney, A. (2013). The role of spatial training in improving spatial and calculus performance in engineering students. *Learning & Individual Differences*, *26*, 20–29. https://doi.org/10.1016/j.lindif.2013.03.010

Sorby, S. A., & Veurink, N. (2010a). Are The Visualization Skills Of First Year Engineering Students Changing? ASEE Conferences. Retrieved from https://peer.asee.org/16026

Sorby, S. A., & Veurink, N. (2010b). Long-term Results from Spatial Skills Intervention among First-Year Engineering Students. Presented at the Proceedings of the 65th Midyear Meeting of the Engineering Design Graphics Division of ASEE.

Thornton, R., & Sokoloff, D. (1998). Assessing student learning of Newton's laws: The Force and Motion Conceptual Evaluation and the Evaluation of Active Learning Laboratory and Lecture Curricula. *AMERICAN JOURNAL OF PHYSICS*, *66*(4), 338–352. https://doi.org/10.1119/1.18863

Thurstone, L. L. (1938). *Primary mental abilities*. Chicago: University of Chicago Press. Retrieved from http://search.ebscohost.com/login.aspx?direct=true&db=psyh&AN=1951-00635-003&site=ehost-live

Uttal, D. H., & Cohen, C. A. (2012). *Spatial Thinking and STEM Education. When, Why, and How?* (Vol. 57). Retrieved from http://www.scopus.com/inward/record.url?eid=2-s2.0-84860615431&partnerID=40&md5=9e7bafd47ed3bb637fcef4311e09dfd7

Uttal, D. H., Meadow, N. G., Tipton, E., Hand, L. L., Alden, A. R., Warren, C., & Newcombe, N. S. (2013). The Malleability of Spatial Skills: A Meta-Analysis of Training Studies. *Psychological Bulletin*, *139*(2), 352–402.

Wai, J., Lubinski, D., & Benbow, C. P. (2009). Spatial ability for STEM domains: Aligning over 50 years of cumulative psychological knowledge solidifies its importance. *Journal of Educational Psychology*, *101*(4), 817.

Yoon, S. Y. (2011). *Psychometric Properties of the Revised Purdue Spatial Visualization Tests: Visualization of Rotations (The Revised PSVT-R).* ERIC. Retrieved from http://eric.ed.gov/?id=ED534824