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Worldsheet Instantons and Torsion Curves

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Abstract. We study aspects of worldsheet instantons relevant to a heterotic standard model. The non-simply connected Calabi-Yau threefold used admits $\mathbb{Z}_3 \times \mathbb{Z}_3$ Wilson lines, and a more detailed investigation shows that the homology classes of curves are $H_2(X, \mathbb{Z}) = \mathbb{Z}^3 \oplus (\mathbb{Z}_3 \oplus \mathbb{Z}_3)$. We compute the genus-0 prepotential, this is the first explicit calculation of the Gromov-Witten invariants of homology classes with torsion (finite subgroups)[**1, 2, 3**]. In particular, some curve classes contain only a single instanton. This ensures that the Beasley-Witten cancellation of instanton contributions cannot happen on this (non-toric) Calabi-Yau threefold.

1. Heterotic Standard Models

1.1. Heterotic M-theory. Probably the most promising corner of string theory to construct models with realistic particle spectra is heterotic M-theory, also known as the Horava Witten setup [**4, 5**]. In it, the spacetime is taken to be Minkowski space $\mathbb{R}^{3,1}$ times a Calabi-Yau threefold X times an interval I in the eleventh direction. The two 10-dimensional boundaries each support a E_8 gauge theory, one of which should be broken by instantons and/or Wilson lines to the standard model gauge group. The other E_8^{hid} is then hidden and only couples gravitationally to the visible sector. In

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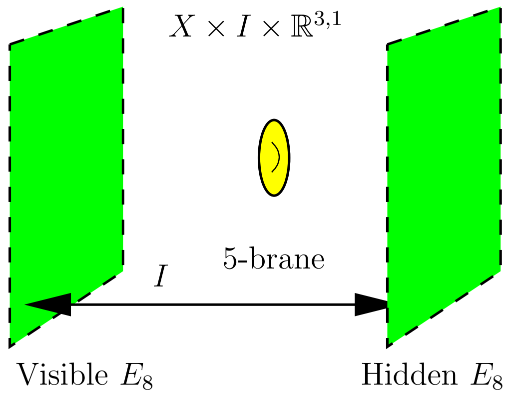

FIGURE 1. Horava-Witten setup.

addition, there can be a number of 5-branes wrapping a curve of the Calabi-Yau threefold in the interior of the interval. In order to realize weak-scale supersymmetry breaking either the 5-brane or the hidden E_8^{hid} gauge sector should break supersymmetry and gravitationally mediate the effect to the ($N = 1$ supersymmetric) visible E_8^{vis} sector.

1.2. A Heterotic Standard Model. In order to find a compactification of heterotic M-theory one thus has to specify a Calabi-Yau threefold X and two E_8 gauge bundles on it. For our purposes it will be convenient to pick an $SU(n)$ subgroup in E_8, and use this group embedding to promote $SU(n)$ bundles (that is, vector bundles V_{vis} and V_{hid}) to E_8 bundles. The commutant of its holonomy $\text{Hol}(V_{\text{vis}})$ inside the visible E_8^{vis} is then the unbroken low-energy gauge group in 4 dimensions.

Here, we are going to be interested in the case where V_{vis} is a rank 4 holomorphic, slope stable (hence preserving $N = 1$ by itself) vector bundle together with a $\mathbb{Z}_3 \times \mathbb{Z}_3$ Wilson line. In other words,

$$\text{Hol}(V_{\text{vis}}) = SU(4) \times \mathbb{Z}_3 \times \mathbb{Z}_3 \quad \subset E_8^{\text{vis}}. \tag{1.1}$$

The unbroken gauge group is then

$$SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}, \tag{1.2}$$

the standard model gauge group times a $U(1)_{B-L}$ which constrains proton decay.

The (visible) matter fields of the effective 4-dimensional theory correspond to the zero modes of the Dirac operator $\not{D}_{E_8}$ on the Calabi-Yau threefold X. They can be computed as the H^1 cohomology groups of the vector bundle V_{vis} and associated bundles $V_{\text{vis}}^\vee$, $\wedge^2 V_{\text{vis}}$, ... In [**6, 7**], we constructed a suitable vector bundle yielding precisely the MSSM matter spectrum of

- three families of quarks and leptons,
- one (up, down) pair of Higgs, and

- no other charged fields.

One interesting possibility for the hidden sector (cancelling the heterotic anomaly) consists of an unbroken E_8^{hid} plus anti-five branes wrapping a rigid curve in the bulk [**8**]. See also [**9, 10**] for other heterotic constructions.

1.3. Bundle Superpotential. Mathematically, deformations of holomorphic vector bundles are unobstructed (at a smooth point in the bundle moduli space). Therefore, there cannot be any perturbative superpotential for the bundle moduli fields.

However, world-sheet instantons can, in principle, generate a superpotential. The action for a worldsheet instanton wrapped on a holomorphic curve C is

$$S[C \to X] = \int_C \omega, \tag{1.3}$$

where ω is the complexified Kähler class

$$\omega = iJ + B = it^a J_a + B. \tag{1.4}$$

The instanton-generated superpotential W_{bundle} is then the sum over all holomorphic curves and weighted with the Pfaffian (a function of those moduli which determine the bundle restricted to C):

$$W_{\text{bundle}}\big(\phi, c, t\big) = \sum_{C \subset X} \mathrm{Pf}(\phi, c) e^{i \int_C \omega}, \tag{1.5}$$

where

- ϕ are the $\dim H^1(X, V \otimes V^\vee)$ vector bundle moduli,
- c are the $h^{21}(X)$ complex structure moduli, and
- $t = \{t^a\}$ are the $h^{11}(X)$ Kähler moduli.

Ignoring any torsion parts in homology and cohomology for the moment, we can generate $H^2(X, \mathbb{Z})$ by the basis two-forms J_a of eq. (1.4) and, therefore, write the homology class of the curve C as

$$[C] = (n_1, \ldots, n_{h^{11}}) = \Big(\int_C J_1, \ldots, \int_C J_{h^{11}} \Big) \in \mathbb{Z}^{h^{11}} \simeq H_2(X) \tag{1.6}$$

Usually, we use the Fourier-transformed Kähler moduli $q_a = e^{2\pi i t^a}$ and write the instanton action part as

$$e^{i \int_C t^a J_a} = \prod q_a^{n_a}. \tag{1.7}$$

Unlikely as this may seem, in common constructions, like gauged linear sigma models and monad constructions, the bundle moduli superpotential eq. (1.5) vanishes. One possible explanation was offered by Beasley and Witten, who have argued [**11**] that the sum over all instantons within a given homology class vanishes under certain circumstances. That is, rewrite eq. (1.5) as

$$W_{\text{bundle}} = \sum_{C\subset X} \mathrm{Pf}(\phi, c) \prod q_a^{n_a} = \sum_{\vec{n}\in H_2(X)} \underbrace{\Bigg(\sum_{[C]=\vec{n}} \mathrm{Pf}(\phi, c)\Bigg)}_{=0\ ?} \prod q_a^{n_a} \tag{1.8}$$

Then, for bundles coming from the toric ambient space, the inner sum actually does vanish. As we will see later in Subsection 3.3, this argument does not apply to our vector bundle. Since our underlying Calabi-Yau threefold X is not a toric complete intersection, this should not be too surprising.

2. The $\mathbb{Z}_3 \times \mathbb{Z}_3$ Calabi-Yau Threefold

2.1. The Manifold. We now construct the Calabi-Yau threefold X as a free $\mathbb{Z}_3 \times \mathbb{Z}_3$-quotient of a simply connected Calabi-Yau threefold $\widetilde{X}$. Clearly, then, the fundamental group $\pi_1(X) = \mathbb{Z}_3 \times \mathbb{Z}_3$ allows for suitable Wilson lines, see Subsection 1.2.

The covering space $\widetilde{X}$ is the complete intersection Calabi-Yau threefold defined by a degree-$(3,1,0)$ and $(0,1,3)$ polynomial in $\mathbb{P}^2 \times \mathbb{P}^1 \times \mathbb{P}^2$. Discrete symmetries appear for special values of the complex structure moduli. In fact, there are *two different* loci in the complex structure moduli space with free $\mathbb{Z}_3 \times \mathbb{Z}_3$ group actions, leading to two different $\mathbb{Z}_3 \times \mathbb{Z}_3$ quotients. The two quotients have different complex structures, but cannot be distinguished at the level of cohomology groups. One of the quotients was used in [**6, 7**] to construct a heterotic MSSM, and the explicit group action and polynomial equations can be found in [**12**]. However, we are now going to consider the other quotient, where one can use toric mirror symmetry. The covering space of the second quotient

$$\widetilde{X} \subset \mathbb{P}^2_{[x_0:x_1:x_2]} \times \mathbb{P}^1_{[t_0:t_1]} \times \mathbb{P}^2_{[y_0:y_1:y_2]} \tag{2.1}$$

is the complete intersection (see also [**13**])

$$\begin{gathered} t_0\Big(x_0^3 + x_1^3 + x_2^3\Big) + t_1\Big(x_0 x_1 x_2\Big) = 0 \\ \Big(\lambda_1 t_0 + t_1\Big)\Big(y_0^3 + y_1^3 + y_2^3\Big) + \Big(\lambda_2 t_0 + \lambda_3 t_1\Big)\Big(y_0 y_1 y_2\Big) = 0 \end{gathered}, \tag{2.2}$$

where we labelled three particular complex structure moduli $\lambda_1, \lambda_2, \lambda_3 \in \mathbb{C}$. The significance of these three moduli is that they parametrize $\mathbb{Z}_3 \times \mathbb{Z}_3$ symmetric complete intersections. Explicitly, the group action is ($\zeta = e^{\frac{2\pi i}{3}}$)

$$\begin{aligned} g_1 &: \begin{cases} [x_0 : x_1 : x_2] \mapsto [x_0 : \zeta x_1 : \zeta^2 x_2] \\ [t_0 : t_1] \mapsto [t_0 : t_1] \text{ (no action)} \\ [y_0 : y_1 : y_2] \mapsto [y_0 : \zeta y_1 : \zeta^2 y_2] \end{cases} \\ g_2 &: \begin{cases} [x_0 : x_1 : x_2] \mapsto [x_1 : x_2 : x_0] \\ [t_0 : t_1] \mapsto [t_0 : t_1] \text{ (no action)} \\ [y_0 : y_1 : y_2] \mapsto [y_1 : y_2 : y_0]. \end{cases} \end{aligned} \tag{2.3}$$

Since the two equations are each cubic in one $\mathbb{P}^2$, one can easily see that $\widetilde{X}$ is a double elliptic fibration. Hence its Euler number must be $\chi(\widetilde{X}) = \chi(X) = 0$. A straightforward geometric calculation yields the Hodge diamond, which turns out to be

$$h^{p,q}\big(\widetilde{X}\big) = \begin{matrix} & & & 1 & & & \\ & & 0 & & 0 & & \\ & 0 & & 19 & & 0 & \\ 1 & & 19 & & 19 & & 1 \\ & 0 & & 19 & & 0 & \\ & & 0 & & 0 & & \\ & & & 1 & & & \end{matrix}, \quad h^{p,q}\big(X\big) = \begin{matrix} & & & 1 & & & \\ & & 0 & & 0 & & \\ & 0 & & 3 & & 0 & \\ 1 & & 3 & & 3 & & 1 \\ & 0 & & 3 & & 0 & \\ & & 0 & & 0 & & \\ & & & 1 & & & \end{matrix} \tag{2.4}$$

Note that, as mentioned above, X has $h^{2,1}(X) = 3$ complex structure parameters.

2.2. Coinvariant Homology. The Hurewicz isomorphism tells us that

$$H_1(X,\mathbb{Z}) = \pi_1(X)_{\mathrm{ab}} = \mathbb{Z}_3 \oplus \mathbb{Z}_3. \tag{2.5}$$

Together with the Hodge diamond eq. (2.4), the Universal Coefficient Theorem and Poincaré duality, this determines the integral homology up to a finite, Abelian group T ("torsion"). The resulting integral homology groups are given in Table 2.2. The hard part is to compute the torsion

i	0	1	2	3	4	5	6
$H_i(X,\mathbb{Z})$	$\mathbb{Z}$	$\mathbb{Z}_3 \oplus \mathbb{Z}_3$	$\mathbb{Z}^3 \oplus T$	$\mathbb{Z}^8 \oplus T$	$\mathbb{Z}^3 \oplus \mathbb{Z}_3 \oplus \mathbb{Z}_3$	0	$\mathbb{Z}$
$H^i(X,\mathbb{Z})$	$\mathbb{Z}$	0	$\mathbb{Z}^3 \oplus \mathbb{Z}_3 \oplus \mathbb{Z}_3$	$\mathbb{Z}^8 \oplus T$	$\mathbb{Z}^3 \oplus T$	$\mathbb{Z}_3 \oplus \mathbb{Z}_3$	$\mathbb{Z}$

TABLE 1. Integral homology and cohomology of X.

part $T = H_2(X,\mathbb{Z})_{\mathrm{tors}}$, that is, the torsion homology classes of curves in X. Clearly, such a torsion homology class cannot be represented by a holomorphic curve. However, as we will see, torsion homology classes can be represented by formal differences of holomorphic curves. It turns out that

$$H_2(X,\mathbb{Z}) = \mathbb{Z}^3 \oplus \underbrace{\mathbb{Z}_3 \oplus \mathbb{Z}_3}_{=T} \tag{2.6}$$

How did we obtain this? First, note that by definition we have a quotient map q which induces the push-forward in homology,

$$\begin{array}{ccc} \widetilde{X} & & H_2(\widetilde{X},\mathbb{Z}) = \mathbb{Z}^{19} \\ {\scriptstyle q}\big\downarrow & & \big\downarrow{\scriptstyle q_*} \\ X & & H_2(X,\mathbb{Z}) = \mathbb{Z}^3 \oplus \mathbb{Z}_3 \oplus \mathbb{Z}_3 \end{array} \tag{2.7}$$

The $\mathbb{Z}_3 \times \mathbb{Z}_3$ group action on $\widetilde{X}$ identifies curves

$$\widetilde{C} = g\widetilde{C} \qquad \forall g \in \mathbb{Z}_3 \times \mathbb{Z}_3,\ \widetilde{C} \in H_2(\widetilde{X}, \mathbb{Z}). \tag{2.8}$$

The quotient under this equivalence relation is called "coinvariant Homology"[1], and denoted as $H_2\big(\widetilde{X}, \mathbb{Z}\big)_{\mathbb{Z}_3\times\mathbb{Z}_3}$. By explicitly constructing a basis of curves on X and identifying the group action on them, one can explicitly determine the $\mathbb{Z}_3 \times \mathbb{Z}_3$-group action on $H_2\big(\widetilde{X}, \mathbb{Z}\big) \simeq \mathbb{Z}^{19}$ in terms of two commuting 19×19 matrices. We found that [**2**]

$$\begin{aligned} H_2\big(\widetilde{X}, \mathbb{Z}\big)_{\mathbb{Z}_3\times\mathbb{Z}_3} &= H_2\big(\widetilde{X}, \mathbb{Z}\big) \Big/ \operatorname{Span}\big\{\widetilde{C} - g\widetilde{C}\big\} = \\ &= \mathbb{Z}^3 \oplus \mathbb{Z}_3 \oplus \mathbb{Z}_3. \end{aligned} \tag{2.9}$$

In general, the relationship between coinvariant homology and the homology of the quotient $X = \widetilde{X}/(\mathbb{Z}_3 \times \mathbb{Z}_3)$ is determined by the Cartan-Leray spectral sequence

$$E^2_{p,q} = H_p\Big(\mathbb{Z}_3 \times \mathbb{Z}_3, H_q\big(\widetilde{X}, \mathbb{Z}\big)\Big) \quad \Rightarrow \quad H_{p+q}\big(X, \mathbb{Z}\big). \tag{2.10}$$

Without going into details, we point out that the $p = 0$ column of the E^2 tableau is the coinvariant homology,

$$E^2_{0,q} = H_0\Big(\mathbb{Z}_3 \times \mathbb{Z}_3, H_q\big(\widetilde{X}, \mathbb{Z}\big)\Big) = H_q\big(\widetilde{X}, \mathbb{Z}\big)_{\mathbb{Z}_3\times\mathbb{Z}_3}. \tag{2.11}$$

But, as usual, there are more entries in the spectral sequence, as well as higher differentials and extension ambiguities. However, one can show [**2**] that these ultimately do not contribute to $H_2(X, \mathbb{Z})$.

To summarize, the homology classes of curves are given in eq. (2.6). The torsion curve classes can be understood as coinvariant curve classes on the covering space $\widetilde{X}$.

3. Counting Curves

3.1. Prepotential. Having constructed the Calabi-Yau threefold X, we would like to identify the holomorphic curves on it. The well-known trick to count the curves is via the instanton corrections to the Yukawa couplings in the standard embedding,

$$W^{\text{np}}_{\text{Yuk}}\big(t, \underline{\mathbf{27}}\big) = \sum_{\vec{n}\in H_2(X)} \underline{\mathbf{27}}_i \underline{\mathbf{27}}_j \underline{\mathbf{27}}_k \underbrace{C_{ijk}(\vec{n})}_{\in\mathbb{Q}} \prod q_a^{n_a} \tag{3.1}$$

The instanton numbers are encoded in the power series $\sum_{\vec{n}} C_{ijk}(\vec{n}) \prod q_a^{n_a}$, multi-indexed by $i, j, k = 1, \dots, h^{1,1}$. Fortunately, the $(h^{1,1})^3$ power series come from a single prepotential

$$\mathcal{F}^{\text{np}}_{X,0}(t) = \sum_{\vec{n}\in H_2(X)} \underbrace{n_{\vec{n}}}_{\in\mathbb{Z}} \operatorname{Li}_3\Big(\prod q_a^{n_a}\Big). \tag{3.2}$$

[1] Since this is the dual of invariant Cohomology.

The trilogarithm in the prepotential takes care of multi-covers of a single instanton. Written as in eq. (3.2), the expansion coefficients $n_{\vec{n}}$ are the instanton numbers, and they count[2] the holomorphic curves in the homology class $\vec{n} \in H_2(X, \mathbb{Z})$.

3.2. Torsion Curves. In our case, the curve homology classes are

$$H_2\big(X, \mathbb{Z}\big) = H_2\big(\widetilde{X}, \mathbb{Z}\big)_{\mathbb{Z}_3 \times \mathbb{Z}_3} = \mathbb{Z}^3 \oplus \mathbb{Z}_3 \oplus \mathbb{Z}_3, \tag{3.3}$$

and we have to take into account the finite subgroup. Therefore, the prepotential $\mathcal{F}^{\mathrm{np}}_{X,0}$ must be

- a power series in the $h^{11}(X)$ variables p, q, r, and
- a polynomial in b_1, b_2 with $b_i^3 = 1$.

After properly distinguishing the torsion curves, the prepotential must be of the form

$$\mathcal{F}^{\mathrm{np}}_{X,0} = \sum_{\substack{(n_1,n_2,n_3,m_1,m_2)\\ \in H_2(X,\mathbb{Z})}} n_{(n_1,n_2,n_3,m_1,m_2)} \operatorname{Li}_3\left(p^{n_1} q^{n_2} r^{n_3} b_1^{m_1} b_2^{m_2}\right). \tag{3.4}$$

Knowing the general form of the series expansion, how do we go about and compute it? We compared three different approaches [**2, 3**]

- Start on $\widetilde{X}$ and mod out $\mathbb{Z}_3 \times \mathbb{Z}_3$.
  The prepotential on $\widetilde{X}$ is partially known [**14**]. The quotienting by the $\mathbb{Z}_3 \times \mathbb{Z}_3$ action essentially amounts to a variable substitution [**2**], and one obtains

$$\begin{aligned} &\mathcal{F}^{\mathrm{np}}_{X,0}(p, q, r, b_1, b_2) = \frac{1}{9} p \times \\ &\qquad \Theta_{E_8}\big(q^3;\, q^2 b_1^2 b_2^2,\, q b_2,\, 1,\, b_1^2 b_2,\, b_2^2,\, 1,\, b_1^2,\, q^{-1}\big) \times \\ &\qquad \Theta_{E_8}\big(r^3;\, r^2 b_1 b_2,\, r b_2^2,\, 1,\, b_1 b_2^2,\, b_2,\, 1,\, b_1,\, r^{-1}\big) \times \\ &\qquad P\big(q^3\big)^{12} P\big(r^3\big)^{12} + O(p^2). \end{aligned} \tag{3.5}$$

- Identify and count curves directly on X.
  By counting sections and singular Kodaira fibers in the double torus fibration X, one can directly count the curves up to linear order in p. The result is

$$\mathcal{F}^{\mathrm{np}}_{X,0}(p, q, r, b_1, b_2) = p \left(\sum_{i,j=0}^{2} b_1^i b_2^j \right) P(q)^4 P(r)^4 + O(p^2). \tag{3.6}$$

  Setting $b_1 = b_2 = 1$, the equality of eq. (3.5) and eq. (3.6) is a known identity of E_8 theta functions [**15**], but we do not know a mathematical proof in general.

[2] If all curves are rigid, then the instanton number is the naive number of curves in the given homology class. Otherwise, the expansion coefficients $n_{\vec{n}}$ are still integers, but may be negative.

- Toric mirror symmetry.
  This approach yields the full power series for the whole prepotential, but only numerical with degree bounded by computer resources. In practice, it is useful to look at (toric) Z_3-quotients of the toric complete intersection $\widetilde{X}$ [**3**].

3.3. Instanton Numbers. The instanton numbers $n_{(1,n_2,n_3,m_1,m_2)}$ turn out to be independent of m_1 and m_2, that is, the same for all $3 \times 3 = 9$

$n_2 \backslash n_3$	0	1	2	3	4	5	6	7
0	1	4	14	40	105	252	574	1240
1	4	16	56	160	420	1008	2296	4960
2	14	56	196	560	1470	3528	8036	17360
3	40	160	560	1600	4200	10080	22960	49600
4	105	420	1470	4200	11025	26460	60270	130200
5	252	1008	3528	10080	26460	63504	144648	312480
6	574	2296	8036	22960	60270	144648	329476	711760
7	1240	4960	17360	49600	130200	312480	711760	1537600

TABLE 2. The instanton numbers $n_{(1,n_2,n_3,*,*)}$.

choices of the torsion part. We list them in Table 2. In particular, observe that

$$n_{(1,0,0,*,*)} = 1, \tag{3.7}$$

that is, the 9 smallest volume curves are alone in their homology class. Hence, their contribution to the bundle moduli superpotential cannot cancel within their curve class and Beasley-Witten's cancellation cannot happen on the non-toric Calabi-Yau threefold X.

Although it is a priori surprising that the instanton numbers in Table 2 do not depend on the torsion part of the homology class, this can be argued form a remaining discrete symmetry. However, this phenomenon is special to $n_1 = 1$ and does not hold in general. For example, the instanton numbers

$n_{(3,n_2,n_3,0,0)}$

$n_2 \backslash n_3$	0	1	2
0	0	**3**	**36**
1	**3**	**108**	
2	**36**		

$n_{(3,n_2,n_3,m_1,m_2)}$, $(m_1, m_2) \neq (0,0)$

$n_2 \backslash n_3$	0	1	2
0	0	**0**	**27**
1	**0**	**81**	
2	**27**		

TABLE 3. The instanton numbers $n_{(3,n_2,n_3,*,*)}$.

$n_{(3,n_2,n_3,m_1,m_2)}$ listed in Table 3 do explicitly depend on m_1 and m_2.

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