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Analysis of Active Suspension Control Policies for Vehicle using Robust Controllers

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Abstract
Better ride comfort and controllability of vehicles are pursued by automotive industries by considering the use of suspension system which plays a very important role in handling and ride comfort characteristics. This paper presents the design of an active suspension of quarter car system using Robust H-infinity, Robust H_2, Robust Mu-synthesis controllers with passive suspension technique. Parametric uncertainties were also considered to model the non linearities associated in the system. Numerical simulation was performed to the designed controller. Results shows that inspite of introducing uncertainties, the designed active controller improves ride comfort and road holding of the car when compared to the traditional passive suspension system.

Keywords: *active suspension, suspension system, quarter car, robust H_∞control, robust H_2 control, Mu-synthesis*

1. INTRODUCTION

While in motion, vehicles experience vibration from various sources such as road roughness, wheel assembly, engine excitation, driveline excitation, transmission excitation and aerodynamic forces. Usually, disturbance from road is the major source that excites the car body while vibrations from other sources contribute less because of because of their own design that its noise transmits through longer routes until it reaches passenger [1]. To reduce this major source of vibration from road, suspension system is used which acts as a bridge between the passenger and the surface it travels. These suspension systems are categorized as passive, semi-active and active systems. Conventional shock absorber creates tradeoff between vehicle body's vertical acceleration and vehicle's tire contact with the road [2].

In passive suspension system, these parameters are coupled to each other, i.e., for better ride comfort, it is desirable to limit body acceleration by permitting high relative velocity between body and wheel with a softer shock rate. But in-turn it experience higher variation in tire and road contact thus reducing vehicle's handling performance. On the other hand, by reducing relative velocity by designing a stiffer shock absorber promise better tire and road contact with increased handling performance but compromises ride quality. [3]studied on unconstrained optimization for passive suspension systems which indicates the desirability of reduced wheel mass, low suspension stiffness and an optimum damping ratio for the best car handling. The main drawback is that the spring and damper do not provide energy to the suspension system and controls only the motion of the body and wheel by limiting the suspension velocity with respect to the rate determined by the designer. Thus, the performance of the passive suspension system varies subject to road profile.

Researcher has proposed many designs to improve semi-active suspension performance ranging from simple adjustable shock absorber to complex hydraulically actuated suspension system. Driver adjustable shock absorber is one such example of semi-active suspension system, where, the driver can adjust the amount of damping in the shock absorber with a switch depending on driver's preference and road condition. The switch determines the position of the valve inside the shock absorber which in turn provide different amount of passive damping. One setting may provide smooth ride with poor car handling, whereas, another setting may provide a firm ride with good handling. By using intelligent systems to make decision instead of the driver, these systems uses sensors that identify the steering wheel position, brake pedal position, vehicle speed and road profile. However, the amount of damping is decided by the system based on information from various sensors [4].

The semi-active device must be either conserve or dissipate energy in a suspension system. One such device is a variable resistor which dissipates energy. Constitutive laws between the system variables of force and velocity characterize these elements which can be altered using a control input. In a typical suspension system it is known as variable orifice viscous damper because damping characteristics change from soft to hard and vice versa by closing or opening the orifice. Recently, electro-rheological or magneto-rheological fluids are being used for flow control. The other popular semi-active suspension elements are variable force transformer and variable stiffness which converse and dissipate energy respectively.

Active suspension system dynamically responds to continuously changing road profile because of its ability to supply energy to produce relative motion between the sprung mass and the unsprung mass. Typically, it uses sensors to measure variables such as body acceleration, wheel acceleration, body velocity, wheel velocity, suspension deflection. The use of electronic control in automobile suspension system design has attracted much attention. Such suspension system does not only improve the performance of the vehicle but also overcome various design trade-offs intrinsic in conventional passive suspension system. Active suspensions are a kind of electronic suspension in which conventional spring damper arrangement are supplemented by actuators which can be powered by hydraulics and controlled by an algorithm. Overall performance of a car suspension is determined by two main factors. They are vehicle handling and passenger's ride comfort. The former determines the performance under the dynamic loads experienced during cornering and braking. This in-turn gives rise to low frequency dynamics. The vehicle ride comfort is its response to an uneven road profile in which high frequency dynamics arise from the wheel motion. The design of an active suspension controller should meet both this criteria without compromising either of them.

Various types controller are being used from the past few decades on active suspension system of vehicle with different degrees of freedom. Control techniques such as sliding mode control [5], sliding mode neural network interference fuzzy logic control [6], optimal controls [7], backstepping control [8], fuzzy-PID control [9], fuzzy logic control [10][11], linear quadratic controller (LQR) [12], linear quadratic Gaussian control [13], gain scheduling control [14], fuzzy sliding mode control [15], hybrid control [16-19], skyhook [16-19], groundhook[16-19], adaptive sliding mode control [20] and so on.

[21]developed robust tracking control scheme to improve ride comfort at any specified location by designing two ideal vehicles so that ride comfort becomes best at each different location. [22]designed the whole system linearly except actuator which was designed nonlinearly. Robust H_∞ control method was used to design linear part to achieve robustness and non-linear adaptive based on back-stepping control method was used to design the actuator. [23] considered the error between the desired acting force from H_∞controller and force generated by actuator as the disturbance to the linear system. [23]used pragmatic approach to select the uncertainty and performance weight. [24]considered vehicle inertial properties to design robust controller. Ride comfort is optimized by minimizing H_∞norm, while, road holding of the vehicle and suspension deflection are carried out by constraining the generalized H_2 norm. [25]presented robust control method with MR damper and evaluated the performance in a Hardware-in-the-loop HiL platform to evaluate the performance using Robust H_∞ control. [26]used an input delay approach to change the sampling measurement to continuous time signal with delay in the state. A polytopic parameter uncertainty was used to characterize the uncertain situation and lyapunov functional approach was employed to achieve the H_∞ performance. [27]designedH_∞ control with a three dimensional kinematic model of Macpherson suspension system to study the influence of the control force variation in wheel motion such as wheel performance, toe angle, camber angle and track width were simulated. However, in this study, the passenger ride comfort and car handling were not carried out and were ignored. [28] designed Robust H_∞ controller with ER suspension system subject to parametric uncertainty. Dynamic bandwidths of cylindrical ER damper

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operating with two different fluids (fast response and slow response characteristics) were identified. [29]used sampled data for H_∞ control to ensure asymptotical stability of the closed loop system with a given level of disturbance attenuation and to satisfy desirable output constraint performance, input delay approach with sampled data H_∞ control were introduced. [30]proposed robust H_∞state feedback quadratic controller from the solution of convex optimization problem. [31]investigated the issue of robust quantized H_∞ control by considering the vehicle load variation. Then required performance of the vehicle suspension such as ride comfort, car handling and suspension deflection was transformed with sampling and quantization measurements into a continuous time system with input delay and sector bound uncertainties by using input delay method.

During the last decade of the twentieth century, many researchers explored non-linearities of suspension system which requires the use of non-linear model and non-linear control scheme [32-33]. These non-linear models make active suspension control system too complex and challenging to employ in practice. In application, engineers often have to deal with complex systems having multiple parameter models with possible non-linear coupling. The linear way of modeling such a system for predicting its behavior based on analytical techniques can be proved to be inadequate even at the initial stages of mathematical modeling. It is evident that one has to deal with high degree of uncertainty in real time systems [34]. Thanks to robust control in dealing with system uncertainties which include unmodelled lags (time delay), parasitic coupling, hysteresis and other non-linearities. Such perturbation represents variation of particular system parameters over some possible range [35]. In this paper, we have designed different types of robust controllers namely, Robust H_∞, Robust H_2 and Robust Mu synthesis with system uncertainty to study the performance of a quarter car system. These active suspension techniques were further compared with passive suspension system.

2. QUARTER CAR MODELING

A quarter car model as shown in the Fig. 1 is considered in this paper. In the suspension model shown in Fig. 1, $m_b$denotes body mass of the vehicle, m_w denotes mass of the wheel, k_b is the suspension stiffness, $c_b$is the suspension damping, k_w is the tire stiffness, x_b, and x_w refers to vehicle and wheel displacement respectively, f denotes an actuator which exerts control force.

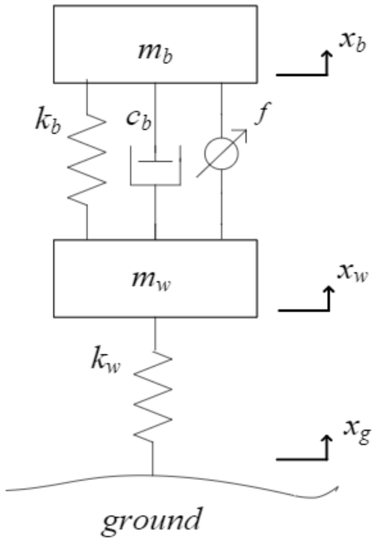

Fig. 1. Quarter car suspension model

The dynamic equation for the above vehicle model can be derived as follows,

$$m_b \ddot{x}_b + c_b(\dot{x}_b - \dot{x}_w) + k_b(x_b - x_w) = f \tag{1}$$

$$m_w \ddot{x}_w + c_b(\dot{x}_w - \dot{x}_b) + k_b(x_w - x_b) + k_w(x_w - x_g) = -f \tag{2}$$

In realistic system, mass of the body varies significantly with and without passenger which is very uncertain in nature. The uncertainty of the spring stiffness and damping coefficient should also be considered. However, it is considered that their values are within certain known intervals.

$$\begin{aligned} m_b &= \overline{m}_b(1 + p_{mb}\delta_{mb}) \\ k_b &= \overline{k}_b(1 + p_{kb}\delta_{kb}) \\ c_b &= \overline{c}_b(1 + p_{cb}\delta_{cb}) \\ k_w &= \overline{k}_w(1 + p_{kw}\delta_{kw}) \end{aligned} \tag{3}$$

Where, m_b, k_b, c_b, k_w and $\overline{m}_b, \overline{k}_b, \overline{c}_b, \overline{k}_w$ are uncertain parameters and its corresponding nominal values respectively. On the other hand, p_{mb}, p_{kb}, p_{cb}, p_{kw} and $\delta_{mb}, \delta_{kb}, \delta_{cb}, \delta_{kw}$ represents possible perturbations on the parameters listed in Table 1. In this study, we have taken p_{mb}= 0.3, p_{kb} = 0.2, p_{cb}= 0.2, p_{kw} = 0.1 and δ_{mb}, δ_{kb}, δ_{cb}, $\delta_{kw} \leq 1$. It should be noted that this represents up to 30% uncertainty in body mass, 20% uncertainty in damping coefficient, 20% uncertainty in sprung mass spring stiffness and 10% uncertainty in tire stiffness. Moreover, uncertainty of the actuator is also included.

The nominal values for the quarter model is taken from [12] and are shown in table 1,

Table 1.

Model parameter	Symbol	Values	Unit
Vehicle body mass	m_b	285	Kg
Suspension stiffness	k_b	17756	N/m
Suspension damping rate	c_b	535	N/(m/sec)
Wheel assembly mass	m_w	41	Kg
Tire stiffness	k_w	190125	N/m

3. CONTROLLER DESIGN

Robust controller design is selected due to uncertainty in the model and the difficulty to predict the disturbance.

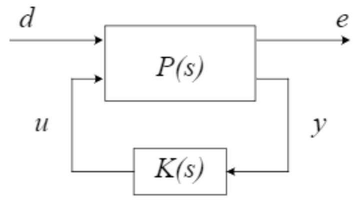

Figure 2., Robust controller K(s)

In Fig.2.*P(s)* is the generalized plant, *d* denotes all external disturbance, output signal is taken as error *e* which is to be minimized, *y* is the input to controller *K* and *u* is the vector of controlled signals which is also to be reduced to avoid the saturation of the actuator.

3.1 H-Infinity Theory

Robust H_∞technique with nominal model and modeling uncertainty is considered due changing system parameters owing.

$$\begin{aligned} \dot{x} &= Ax + B_1 d + B_2 u \\ e &= C_1 x + D_{11} d + D_{12} u \\ y &= C_2 x + D_{21} d + D_{22} u \end{aligned} \quad (4)$$

The above state space representation can be written in matrix form as,

$$P(s) = \begin{bmatrix} P_{11}(s) & P_{12}(s) \\ P_{21}(s) & P_{22}(s) \end{bmatrix} \quad (5)$$

$$= \begin{bmatrix} A & B_1 & B_2 \\ C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{bmatrix} \quad (6)$$

By taking lower LFT for the plant matrix *P(s),*

$$F_L(P,K) = P_{11} + P_{12}K(I - P_{22}K)^{-1}P_{21} \quad (7)$$

The goal of H-infinity controller is to find a stabilizing controller *K(s)* to minimize

$$\|T_{ed}\|_\infty = \|F_L(P,K)\|_\infty \tag{8}$$

Usually it is sufficient to find a stabilizing controller *K*, such that H-infinity norm of the closed loop transfer function is less than a given positive number, i.e.,

$$\|F_L(P,K)\|_\infty < \gamma \tag{9}$$

We may obtain an optimal solution by successively reducing the value of γ from a large number [36]. It should be taken care that reducing γ more than the nominal range makes the solution unreliable [37].

We could approximate

$$P(s) = \begin{bmatrix} A & B_1 & B_2 \\ C_1 & 0 & D_{12} \\ C_2 & D_{21} & 0 \end{bmatrix} \tag{10}$$

In order to simplify the computation D_{11} and D_{22} are assigned zeros in Eq. (10)

We also define two Hamiltonian matrices as given in Eq. (11) and Eq. (12)

$$H_\infty := \begin{bmatrix} A & \gamma^{-2} B_1 B_1^* - B_2 B_2^* \\ -C_1^* C_1 & -A^* \end{bmatrix} \tag{11}$$

$$J_\infty := \begin{bmatrix} A^* & \gamma^{-2} C_1^* C_1 - C_2^* C_2 \\ -B_1 B_1^* & -A \end{bmatrix} \tag{12}$$

It is assumed that

1. The pair *(A,B₁)* is stabilized and the pair *(A,C₁)* is detectable,
2. The pair *(A,B₂)* is stabilized and the pair *(A,C₂)* is detectable,
3. $D^T{}_{12}\begin{bmatrix} C_1 & D_{12} \end{bmatrix} = \begin{bmatrix} 0 & I \end{bmatrix}$
4. $\begin{bmatrix} B_1 \\ D_{21} \end{bmatrix} D^T{}_{21} = \begin{bmatrix} 0 \\ I \end{bmatrix}$

The H infinity controller further states that there exists an admissible controller such that $\|T_{ed}\| < \gamma$ if the following three conditions hold

1. $H_\infty \in dom(Ric) \quad and \quad X_\infty : Ric(H_\infty) \geq 0$
2. $J_\infty \in dom(Ric) \quad and \quad Y_\infty : Ric(J_\infty) \geq 0$
3. $\rho(X_\infty Y_\infty) < \gamma$ (the spectral radius of the product $X_\infty Y_\infty$)

When these conditions hold, one such controller is

$$K_{opt}(s) = \begin{bmatrix} \hat{A}_\infty & -Z_\infty L_\infty \\ F_\infty & 0 \end{bmatrix} \tag{13}$$

Where,

$$\hat{A}_\infty := A + \gamma^{-2} B_1 B_1^* X_\infty + B_2 F_\infty + Z_\infty L_\infty C_2$$

$$F_\infty := -B_2^* X_\infty$$

$$L_\infty := -Y_\infty C_2^*$$

$$Z_\infty := (I - \gamma^{-2} Y_\infty X_\infty)^{-1}$$

3.2 H_2 control theory

The first objective in this is to find an admissible controller K which minimizes $\|T_{ed}\|_2$.

We can define the two Hamiltonian matrices belong to dom(Ric)

$$H_2 := \begin{bmatrix} A & -B_2 B_2^* \\ -C_1^* C1 & -A^* \end{bmatrix} \tag{14}$$

$$J_2 := \begin{bmatrix} A^* & -C_2^* C_2 \\ -B_1 B_1^* & -A \end{bmatrix} \tag{15}$$

Moreover, $X_2 := Ric(H_2)$ and $Y_2 := Ric(J_2)$ are positive semi definite.

$A_{F2} := A + B_2 F_2$ $\qquad C_{1F2} := C_1 + D_{12} F_2$

$A_{L2} := A + L_2 C_2$ $\qquad B_{1L2} := B_1 + L_2 D_{21}$

$\hat{A}_2 := A + B_2 F_2 + L_2 C_2$

Where, $F_2 = -B_2^* X_2$, $\quad L_2 := -Y_2 C^*{}_2$

The unique optimal controller is $K_{opt}(s)$ is given in Eq. (16),

$$K_{opt}(s) = \begin{bmatrix} \hat{A} & -L_2 \\ F_2 & 0 \end{bmatrix} \tag{16}$$

3.3 Mu-Synthesis

Mu synthesis is an optimization problem which relies on structured singular value Mu rather than larger maximum singular value resulting in a less conservative design approach.

$$\min_{K(s)\in S} \sup_{\omega\in R} \mu_\Omega \{F_L[G(j\omega), K(j\omega)]\} \tag{17}$$

To avoid complexity in computing Mu synthesis, lower and upper bounds are computed. Lower bound is the spectral radius of M given in Eq. (18).

$$\rho(M) \leq \mu_{\Gamma}(M) \leq \overline{\sigma}(M) \qquad (18)$$

Scaling matrix D and its inverse is one of the ways to reduce upper bound on Mu without affecting the value of Mu.

$$D := \{diag\,\{D_1,...,Ds,\, d_1 I_{m1},...,\, d_{F-1} I_{mF-1},\, I_{mF}\}: \\ D_i \in C^{Ti \times Ti},\, D_i = D_i^{*} > 0,\, d_j \in R,\, d_j > 0\}$$

… (19)

The upper bound on Mu can be written as

$$\inf_{D \in D} \overline{\sigma}(DMD^{-1}) \qquad (20)$$

Our goal is to minimize both *K(s)* and *D(s)*.

$$\min_{K(s) \in S} \inf_{D, D^{-1} \in RH\infty} \| D(s) F_L(P(s), K(s)) D^{-1}(s) \|_{\infty} \qquad (21)$$

It should be noted that, *D-K* iterations is solved through iterations by alternating between a minimization over *K* and over *D*.

Step 1: Minimizing over *K* while *D* is fixed. Usually *D* is set to *I*.

$$\min_{K(s) \in S} \| DF_L(G,K) D^{-1} \|_{\infty} \qquad (22)$$

Step 2: Minimizing over *D* while *K* is fixed.

$$\inf_{D, D^{-1} \in RH\infty} \| DF_L(G,K) D^{-1} \|_{\infty} \qquad (23)$$

The above equation is solved first by minimizing the below equation over each frequency from the range of frequency of interest and then followed by fitting the frequency dependent Dw with a transfer function DERHin

$$\min_{D\omega \in D} \overline{\sigma}[D_{\omega} F_L(G,K)\,(j\omega)\, D_{\omega}^{-1}] \qquad (24)$$

Reducing the upper bound allows increased performance while still being robust.

3.4 Weighting function

Choosing the appropriate weighting function is an important step in designing robust controller. The weighting functions are useful to avoid actuator saturation, closed loop performance specification and to represent the frequency content of sensor noise and road disturbance [38]. The weighting function *We* and *Wu* represents error and actuator weights respectively. Whereas, *Wn* = 0.01 and *Wd*= 0.06 represents sensor noise and road disturbance respectively.

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$$W_e = \frac{1}{0.02s+1} \tag{25}$$

$$W_u = \frac{100(s+1000\times 2\pi)}{(s+1000\times 2\pi\times 1000)} \tag{26}$$

4. RESULTS AND DISCUSSION

Frequency domain response

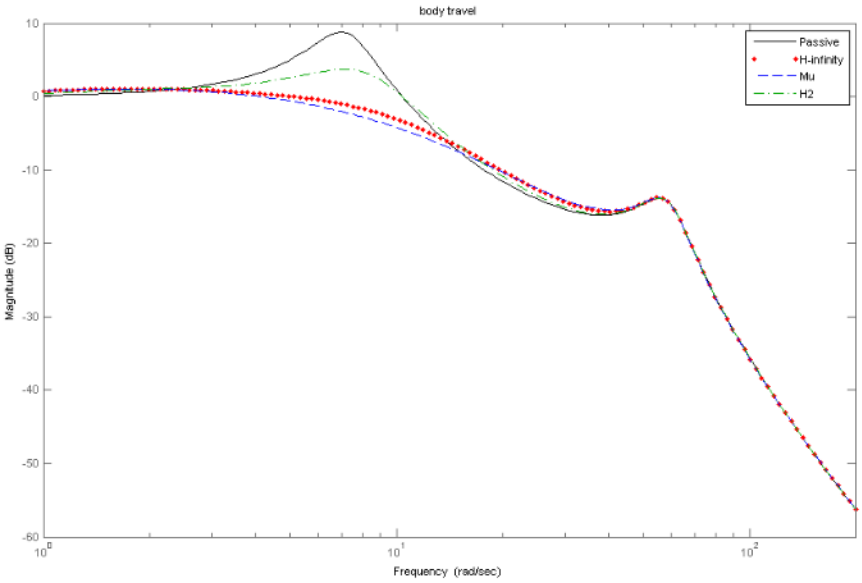

Figure 3. Body travel

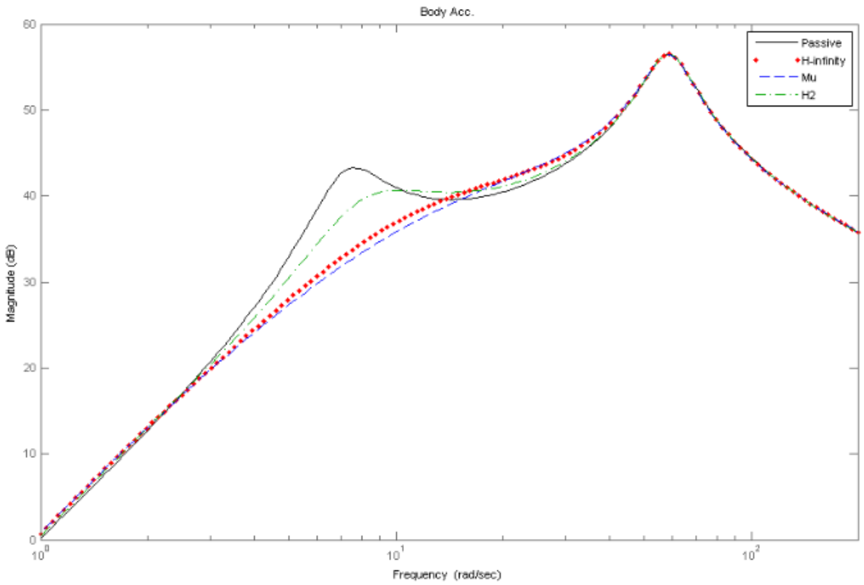

Figure 4. Body acceleration

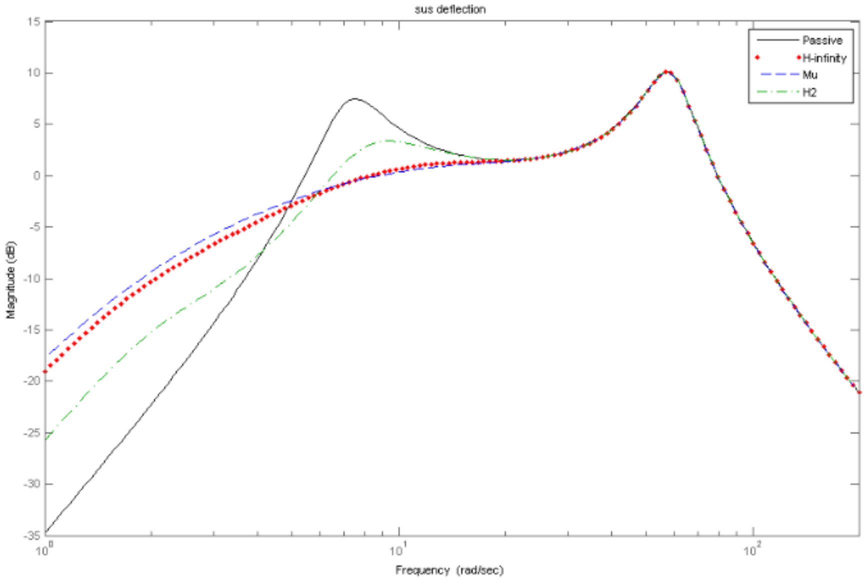

Figure 5. Suspension deflection

While in motion, human body is more prone to the effects of vibration in the frequency range of 10 – 20 Hz [39]. Figure 3 – 5 show the frequency response plot of body travel, body acceleration and suspension deflection of Robust H-infinity, Robust Mu-synthesis, Robust H_2 controllers and passive suspension technique. From the result, it is evident that there exist two natural frequency which can be classified as lower frequency and higher frequency. Higher frequency which is of wheel mass and cannot be controlled, all controller show similar response, whereas, at lower frequency which is of vehicle body, active controller technique is more effective is suppressing the vibration than passive suspension technique. However, robust H_2 controller does have the effect of vehicle's body natural frequency which is not completely suppressed unlike in Robust H-infinity and Robust Mu-synthesis. Comparing all the results, it is clear that Robust H-infinity and Mu-synthesis controllers is better than H_2 and passive in controlling the vibration at low frequency region.

Time domain response

To study the effect of disturbance on vehicle, two types of input signal, namely bump and pothole disturbance with 0.05 m maximal value is considered as shown in the figure 6 (a) and 6(b) respectively.

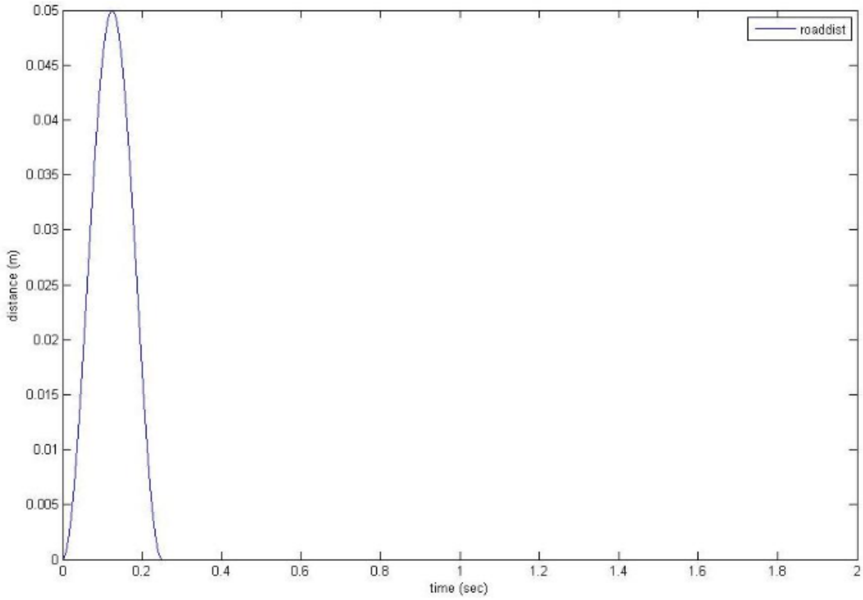

Figure 6(a). Bump disturbance

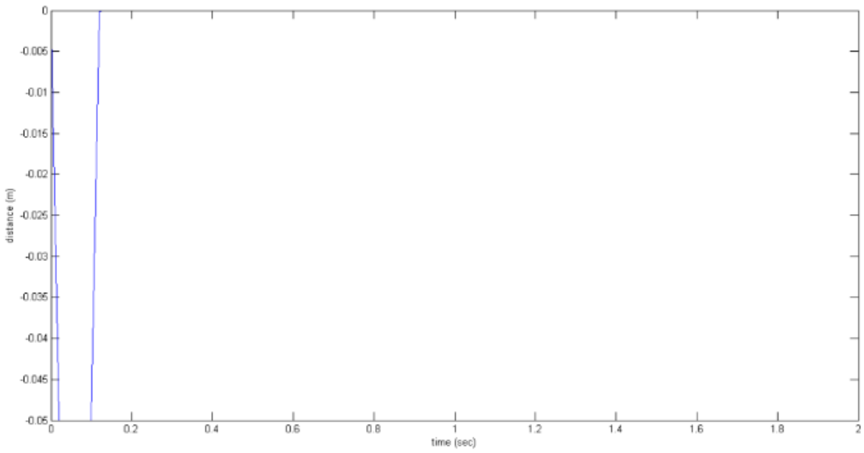

Figure 6 (b).pothole disturbance

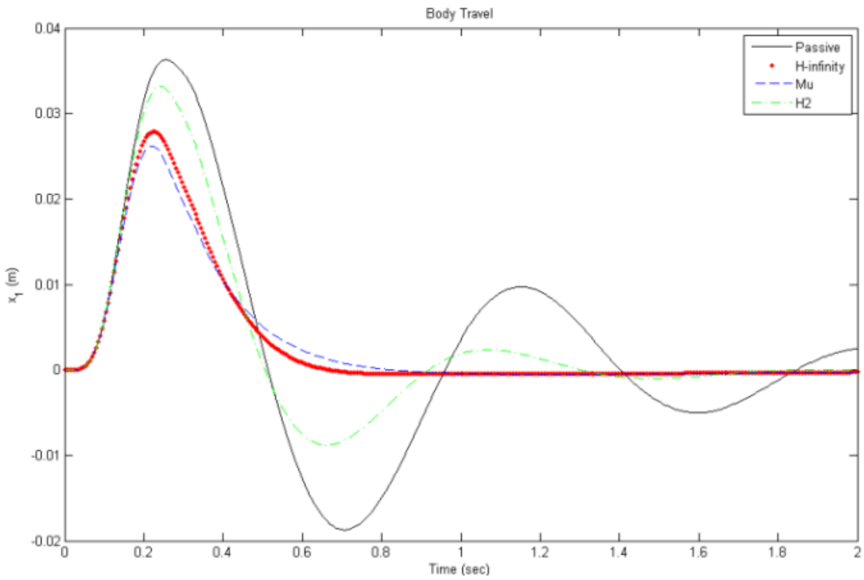

Figure 7(a). Body travel (bump)

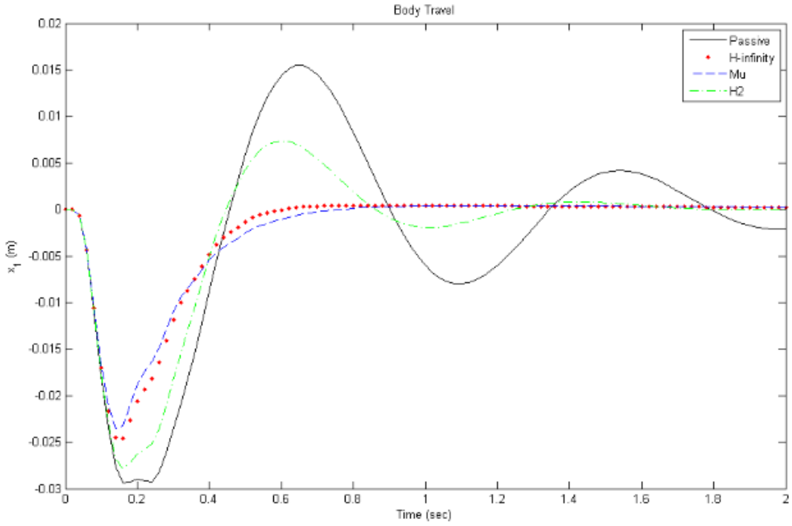

Figure 7(b). Body travel (pothole)

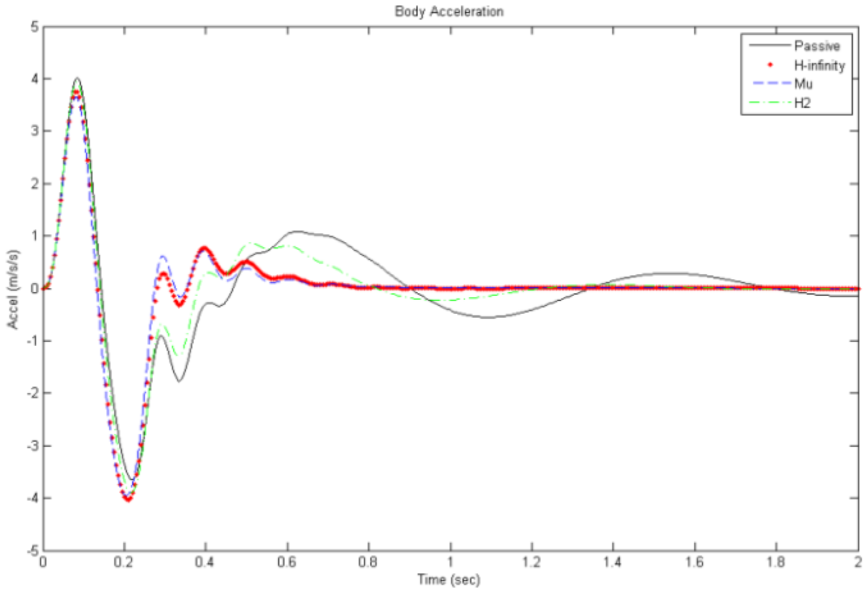

Figure 8(a). Body Acceleration (bump)

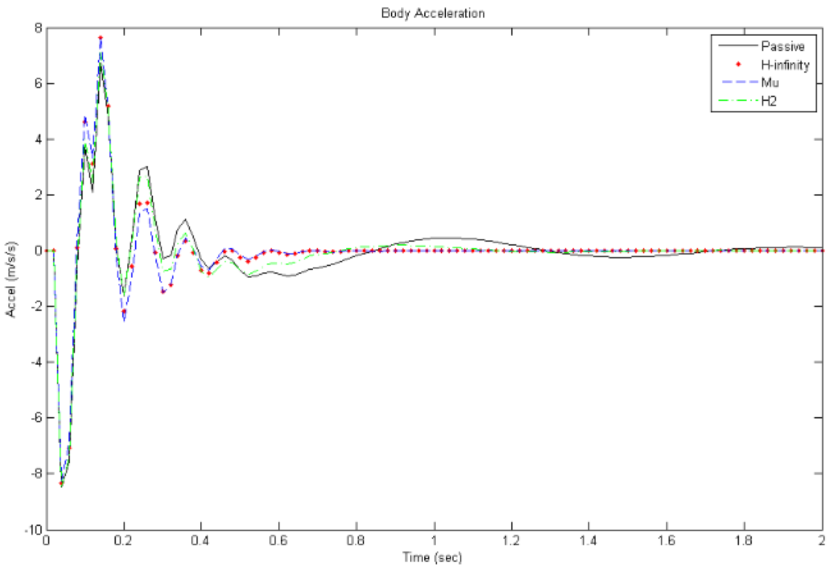

Figure 8(b). Body acceleration (pothole)

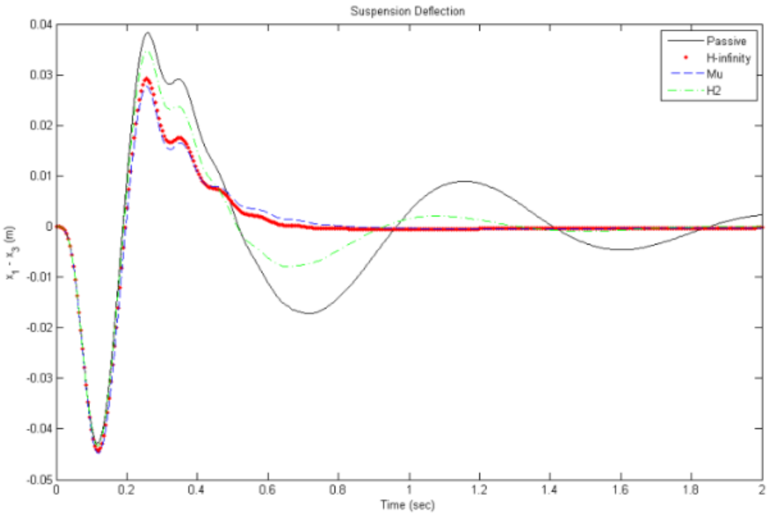

Figure 9(a). Suspension deflection (bump)

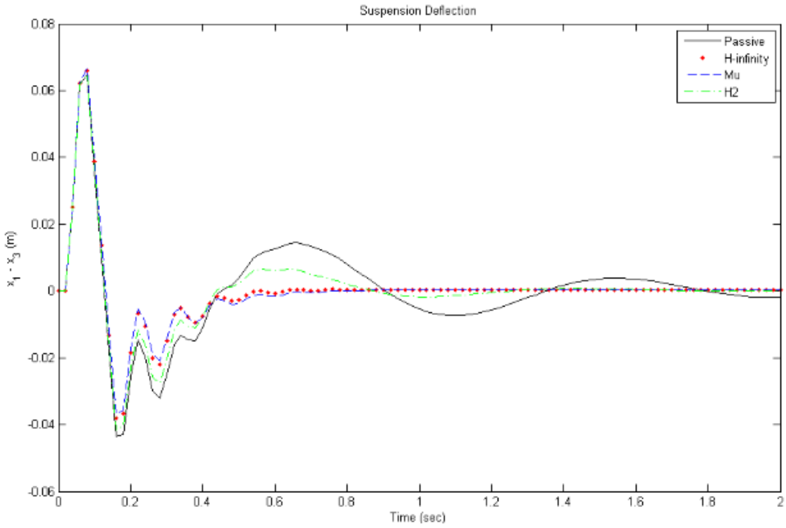

Figure 9(b). Suspension deflection (pothole)

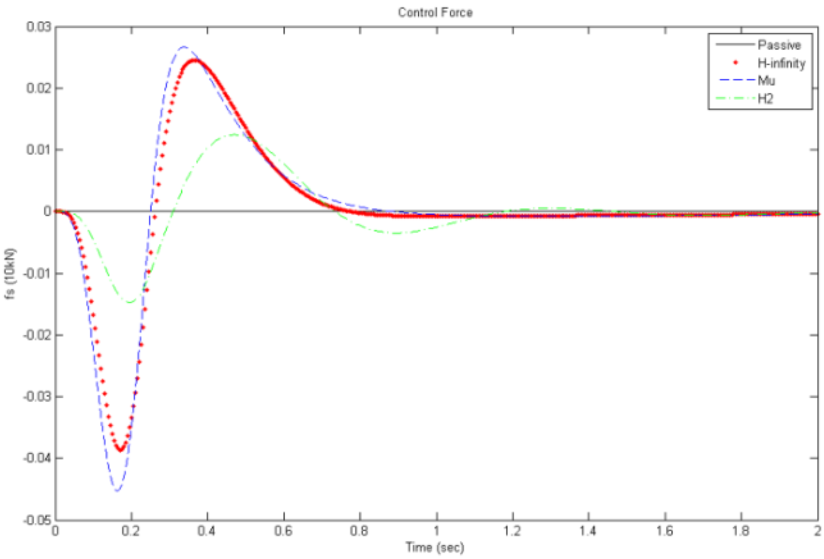

Figure 10(a). Actuator Force (bump)

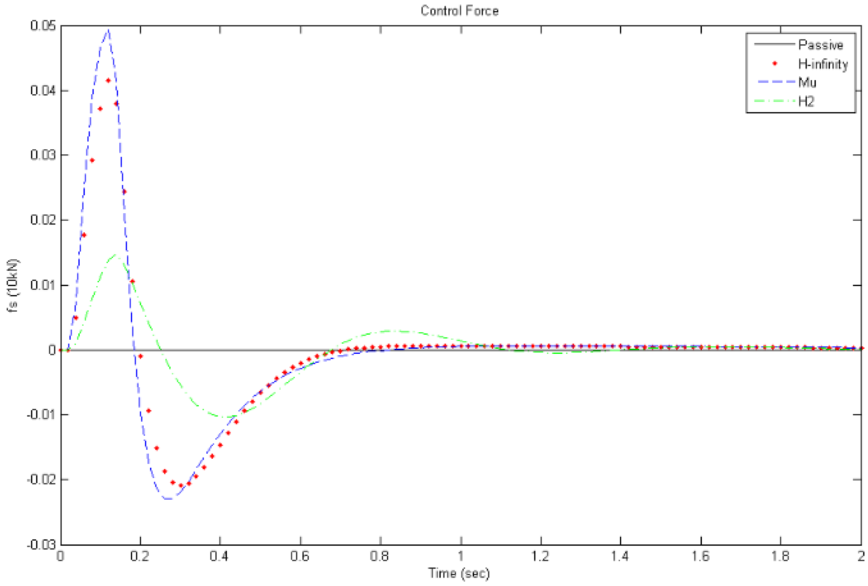

Figure 10(b). Actuator force (pothole)

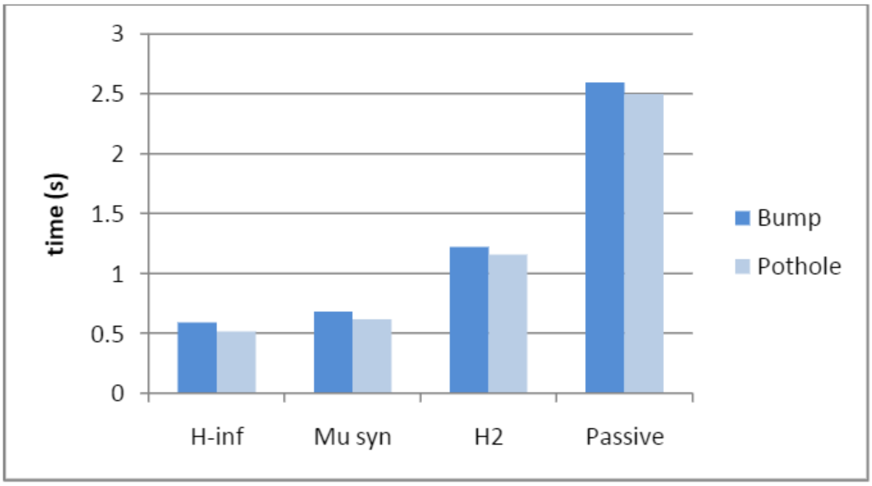

Figure 11. Body travel settling time comparison

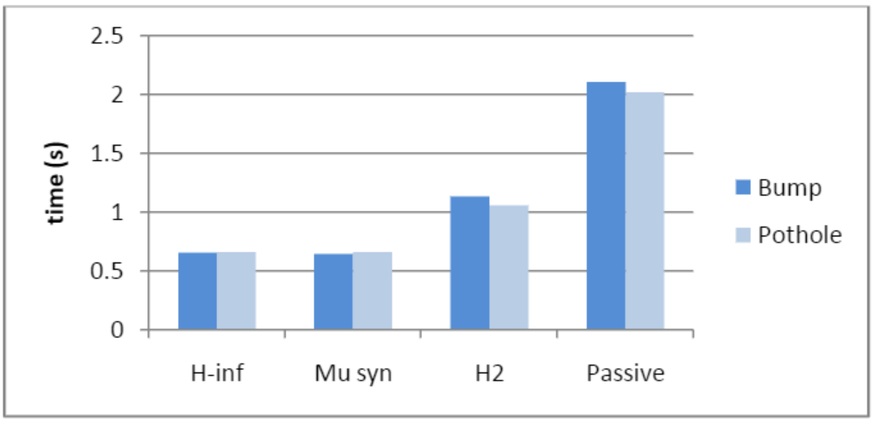

Figure 12. Body acceleration settling time comparison

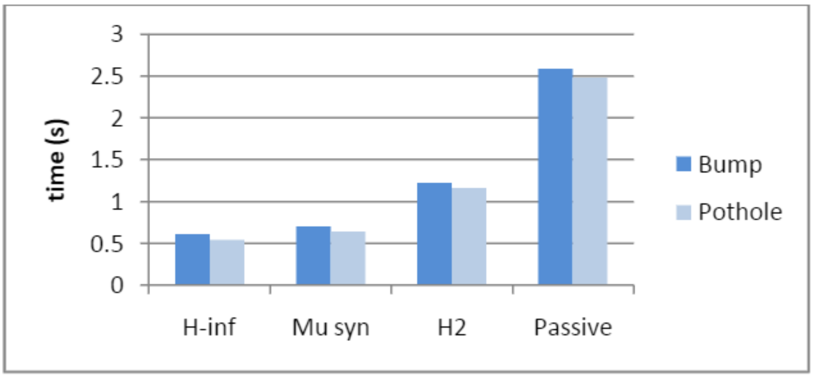

Figure 13. Suspension deflection settling time comparison

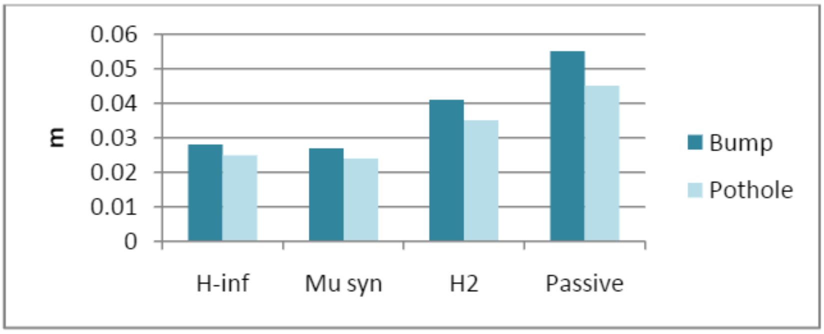

Figure 14. Body travel peak to peak comparison

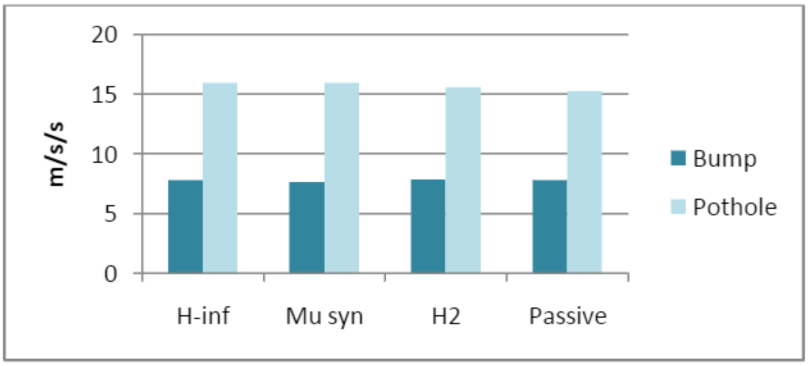

Figure 15. Body acceleration peak to peak comparison

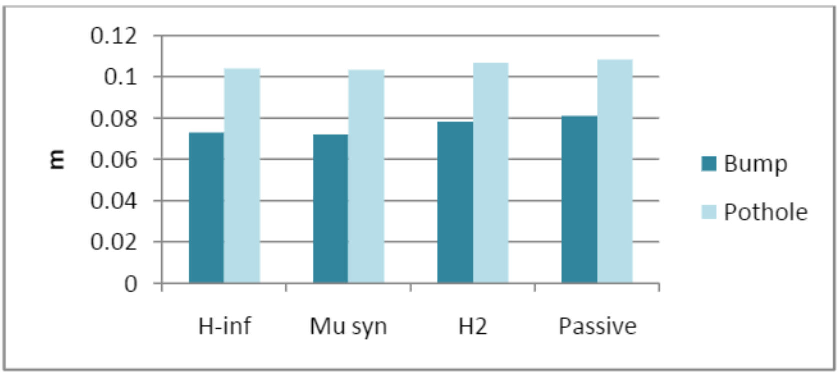

Figure 16. Suspension deflection peak to peak comparison

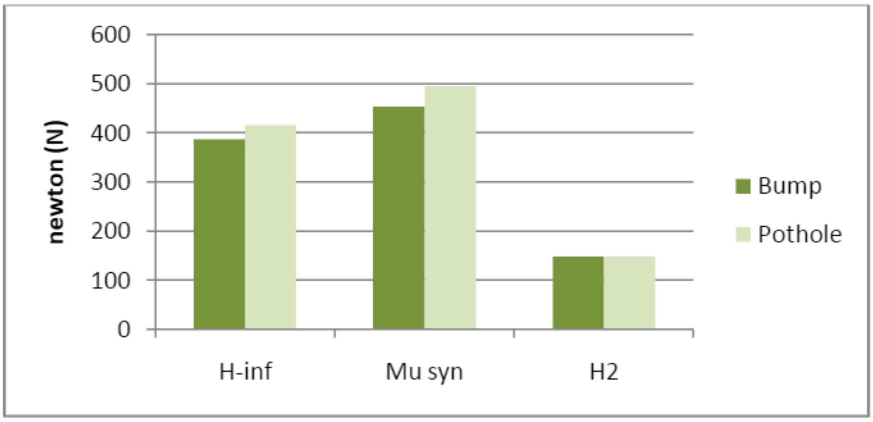

Figure 17. Actuator force comparison

Figure 7 – 10 show the plot of body travel, body acceleration, suspension deflection and actuator force of all the controllers used namely, robust H-infinity, robust Mu-synthesis, robust H_2 and passive controller with bump input disturbance and pothole disturbance. Fig 11 – 17 shows the comparison graph of settling time and peak to peak performance of controllers with different disturbance.

It can be seen in figure 11 that H-infinity controller has faster settling time in all results except in body acceleration with pothole input. On the other hand, Mu-synthesis controller performs closely with H-infinity controller as far as settling time is concerned. Passive suspension technique proves to be worst from the active suspension controllers.

Mu-synthesis has the best peak to peak response from other controllers except in body acceleration with pothole disturbance, as shown in fig 14 – 16. However, in this case, H-infinity controller closely follows the performance of My-synthesis. H_2 controller performance remains to be worst among active controller technique.

External enerygy required by the actuator depends on required control force, so that an acceptable trade-off between controller effectiveness and energy consumption can be achieved. As shown in figure 17, Robust H_2 performs better than H-infinity controller and Mu-synthesis controller but it is at the expense of ride comfort. Control force for passive is zero as shown in figure 10(a) and 10(b) since passive controller do not have actuator, whereas, Mu-synthesis requires highest external engery for actuator force generation. Robust. It should be noted that the better performance of Mu-synthesis in peak to peak response is with the expense of more external energy required by the actuator.

CONCLUSION

This paper presents the design of active suspension of quarter car system using Robust H-infinity, Robust H_2, Robust Mu-synthesis controllers and passive suspension technique. Parametric uncertainties were also considered to model the non-linearities associated in the system. Numerical simulation was performed to simulate the designed controller. Results show that inspite of introducing uncertainties, the designed active controller improves ride comfort and road holding of the car when compared to the traditional passive suspension system. However, Robust H-infinity and Robust Mu synthesis controller showed promising performance from other active suspension technique used in this study. Overall performance of passive suspension technique proved to be worse due to the limitation of actuator force.

REFERENCES

[1] W. F. Faris, *et al.*, "Ride quality of passenger cars: an overview on the research trends," *International Journal of Vehicle Noise and Vibration,* vol. 8, pp. 185-199, 2012.

[2] W. F. Faris, *et al.*, "Assessment of different semi-active control strategies on the performance of off-road vehicle suspension systems," *International Journal of Vehicle Systems Modelling and Testing,* vol. 5, pp. 254-271, 2010.

[3] A. G. Thompson, "Design of Active Suspension," *Proc. Institute of Mechanical Engineers,* vol. 185, pp. 553-563, 1970.

[4] Z. BenLahcene, *et al.*, "Analysis and simulation of semi-active suspension control policies for two-axle off-road vehicle using full model," *International Journal of Vehicle Systems Modelling and Testing,* vol. 6, pp. 219-231, 2011.

[5] E. Akbari, *et al.*, "Observer design for active suspension system using sliding mode control," in *Research and Development (SCOReD), 2010 IEEE Student Conference on*, 2010, pp. 207-212.

[6] N. Al-Holou, *et al.*, "Sliding mode neural network inference fuzzy logic control for active suspension systems," *Fuzzy Systems, IEEE Transactions on,* vol. 10, pp. 234-246, 2002.

[7] Z. Bao-Lin, *et al.*, "Optimal Control of Vehicle Active Suspension Systems with Actuator Delay," in *Control and Automation, 2007. ICCA 2007. IEEE International Conference on*, 2007, pp. 2257-2261.

[8] A. A. Basari, *et al.*, "Nonlinear Active Suspension System with Backstepping Control Strategy," in *Industrial Electronics and Applications, 2007. ICIEA 2007. 2nd IEEE Conference on*, 2007, pp. 554-558.

[9] S.-y. Bei, *et al.*, "On Fuzzy-PID Integrated Control of Automotive Electric Power Steering and Semi-Active Suspension," in *Intelligent Information Technology Application, 2008. IITA '08. Second International Symposium on*, 2008, pp. 847-851.

[10] M. Biglarbegian, *et al.*, "Design of a Novel Fuzzy Controller to Enhance Stability of Vehicles," in *Fuzzy Information Processing Society, 2007. NAFIPS '07. Annual Meeting of the North American*, 2007, pp. 410-414.

[11] M. Kaleemullah, *et al.*, "Design of robust H, fuzzy and LQR controller for active suspension of a quarter car model," in *2011 4th International Conference on Mechatronics: Integrated Engineering for Industrial and Societal Development, ICOM'11, May 17, 2011 - May 19, 2011*, Kuala Lumpur, Malaysia, 2011.

[12] M. Kaleemullah, *et al.*, "Comparative analysis of LQR and robust controller for active suspension," *International Journal of Vehicle Noise and Vibration,* vol. 8, pp. 367-386, 2012.

[13] R. Kashani and S. Kiriczi, "Robust stability analysis of LQG-controlled active suspension with model uncertainty using structured singular value, , method," *Vehicle System Dynamics,* vol. 21, pp. 361-384, 1992.

ISSN: 2005-4238 IJAST

[14] M.-S. Kim, *et al.*, "Gain scheduling control of levitation system in electromagnetic suspension vehicle," *WSEAS Transactions on Circuits and Systems,* vol. 5, pp. 1706-1712, 2006.

[15] J. Lin, *et al.*, "Enhanced fuzzy sliding mode controller for active suspension systems," *Mechatronics,* vol. 19, pp. 1178-1190, 2009.

[16] Z. B. Waleed F. Faris, S.I. Ihsan, "Ride Comfort Assessment in Off Road Vehicles using passive and semi-active suspension," *Asia pacific Conference on Defence & Security Technology (DSTC 2009), Kuala Lumpur, Malaysia,* 2009.

[17] S. Ihsan, *et al.*, "Dynamics and control policies analysis of semi-active suspension systems using a full-car model," *International Journal of Vehicle Noise and Vibration,* vol. 3, pp. 370-405, 2007.

[18] W. F. Faris, *et al.*, "Transient and steady state dynamic analysis of passive and semi-active suspension systems using half-car model," *International Journal of Modelling, Identification and Control,* vol. 6, pp. 62-71, 2009.

[19] S. I. Ihsan, *et al.*, "Analysis of control policies and dynamic response of a Q-car 2-DOF semi active system," *Shock and Vibration,* vol. 15, pp. 573-582, 2008.

[20] C. Hung-Yi and H. Shiuh-Jer, "Adaptive sliding controller for active suspension system," in *Control and Automation, 2005. ICCA '05. International Conference on*, 2005, pp. 282-287 Vol. 1.

[21] H. Okuda, *et al.*, "Robust active suspension controller achieving good ride comfort," in *SICE(Society of Instrument and Control Engineers)Annual Conference, SICE 2007, September 17, 2007 - September 20, 2007*, Takamatsu, Japan, 2007, pp. 1459-1464.

[22] T. T. Nguyen, *et al.*, "A hybrid control of active suspension system using H< sub>∞</sub> and nonlinear adaptive controls," in *Industrial Electronics, 2001. Proceedings. ISIE 2001. IEEE International Symposium on*, 2001, pp. 839-844.

[23] A. Yousefi, *et al.*, "Low order robust controllers for active vehicle suspensions," in *Computer Aided Control System Design, 2006 IEEE International Conference on Control Applications, 2006 IEEE International Symposium on Intelligent Control, 2006 IEEE*, 2006, pp. 693-698.

[24] J.-S. Lin and J. Kanellakopoulos, "Road-adaptive nonlinear design of active suspensions," in *American Control Conference, 1997. Proceedings of the 1997*, 1997, pp. 714-718.

[25] J. Wang, *et al.*, "Robust modelling and control of vehicle active suspension with MR damper," *Vehicle System Dynamics,* vol. 46, pp. 509-520, 2008.

[26] H. Gao, *et al.*, "Robust sampled-data H∞ control for vehicle active suspension systems," *IEEE Transactions on Control Systems Technology,* vol. 18, pp. 238-245, 2010.

[27] M. S. Fallah, *et al.*, "H∞ robust control of active suspensions: A practical point of view," in *American Control Conference, 2009. ACC'09.*, 2009, pp. 1385-1390.

[28] S.-S. H. Seung-Bok Choi, "Hinf control of electrorheological suspension system subjected to parameter uncertainties," *Mechatronics 13,* pp. 639-657, 2003.

[29] L. Hongyi, *et al.*, "A study on half-vehicle active suspension control using sampled-data control," in *Control and Decision Conference (CCDC), 2011 Chinese*, 2011, pp. 2635-2640.

[30] V. F. Montagner, *et al.*, "Robust H control for an active suspension system," in *2010 9th IEEE/IAS International Conference on Industry Applications, INDUSCON 2010, November 8, 2010 - November 10, 2010*, Sao Paulo, Brazil, 2010.

[31] H. Li, *et al.*, "Robust quantised control for active suspension systems," *IET Control Theory and Applications,* vol. 5, pp. 1955-1969, 2011.

[32] A. Alleyne, *et al.*, "Application of nonlinear control theory to electronically controlled suspensions," *Vehicle System Dynamics,* vol. 22, pp. 309-320, 1993.

[33] A. Alleyne and J. K. Hedrick, "Nonlinear adaptive control of active suspensions," *Control Systems Technology, IEEE Transactions on,* vol. 3, pp. 94-101, 1995.

[34] J. Cao*, et al.*, "State of the art in vehicle active suspension adaptive control systems based on intelligent methodologies," *Intelligent Transportation Systems, IEEE Transactions on,* vol. 9, pp. 392-405, 2008.

[35] P. Gu, Konstantinov, "Robust Control Design with Matlab," *Springer*.

[36] Z. Aghaie and R. Amirifar, "H2 and H controllers design for an active suspension system via riccati equations and LMIs," in *2nd International Conference on Innovative Computing, Information and Control, ICICIC 2007, September 5, 2007 - September 7, 2007*, Kumamoto, Japan, 2008.

[37] P. Gu, Konstantinov, "Robuts control design with Matlab," 2005.

[38] X. D. Li, "Active Vibration Control of Vehicle Suspension," *Master's Dissertation, McGill University, Montreal, Canada,* 2009.

[39] S. I. Ihsan, "Analysis of semiactive control policies for passenger vehicles," *PhD dissertation, International Islamic University Malaysia, Kuala Lumpur, Malaysia,* 2008.