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CONSTRUCTING THE EGG CURVES USING THE GOLDEN RATIO OF PENTAGON

Maja Petrović[117]
Marija Obradović[118]

RESUME

In the paper it is considered a construction of an egg curve obtained by continual joining of the circular arcs with three different centers. These three centers form sharp or flat triangles of the pentagon (sharp and flat triangles are the basic building shapes of Penrose tiling). They have many relationships with both the Fibonacci numbers and Phi. In such an egg curve, it is possible to inscribe two juxtaposed pentagons. We give also another construction which employs several, even infinite number of the pentagons. Those pentagons are forming a sequence related to Fibonacci sequence. The construction using the flat triangles will provide a curve which ratio between length of the perimeter and the sum of the length of its minor and major axes will give the coefficient Φ.

***Key words**: pentagon, egg curve, golden ratio, circular arc*

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1. INTRODUCTION

Forming a planar curve using the method of continuation of circular arcs does not require a complicated geometric construction, or exact mathematical formula. This construction has a practical application in the designing.

The construction of egg-shaped curve by method of ***45º*** angle (**Figure 1**): At the first semicircle of radius ***r*** with center C_1, there are attached, symmetrically on the both sides of it, the two eighth of circle twice the larger radii (***2r***) with centers C_2 and C_3 which are located at the endpoints of the semicircle. Then there is attached another quarter circle arc, with center C_4, set in the intersection of the radii of the previous arcs lined at an angle of ***45º*** respectively to the axis of the curve, with radius $a=2r-r\sqrt{2}$.

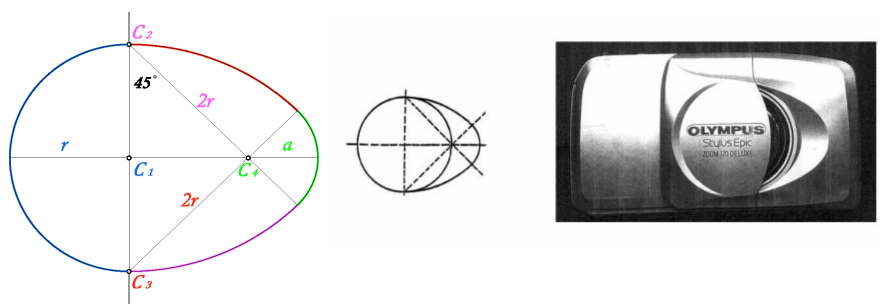
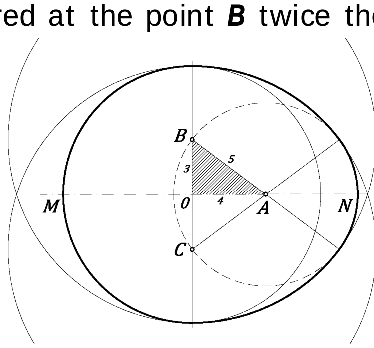


Figure 1*: Construction Of The Egg Curve By The Method Of **45º** Angle*

The egg-shaped plane figure formed with continuous circular arcs centered on the vertices of the Pythagorean triangle ***AOB*** - ***the method 345*** - is shown in the **Figure 2**: The first semi-circle with center ***A*** has the radius ***AN = 5*** (Pythagorean hypotenuse of a triangle), the other one is centered at the point ***B*** twice the larger radius, while the third circular arc with center at the point ***O*** has the radius equal to the sum of the laterals of the



Pythagorean triangle (***3 +4 = 7***).

Figure 2: *Construction Of The Egg Curve By The Method **„345“***

1.1 The Construction Of Egg-Shaped Curves With 3, 4 Or 5 Points

In the book Mathographics (*Mathographics - How to draw a real Chicken Egg*), the mathematician Robert Dixon shows the constructions of seven egg-shaped curves, using circular arcs continuing to each other, and he called such an egg - the Euclidean egg (Egg Euclidean) which is shown in the **figure 3**.

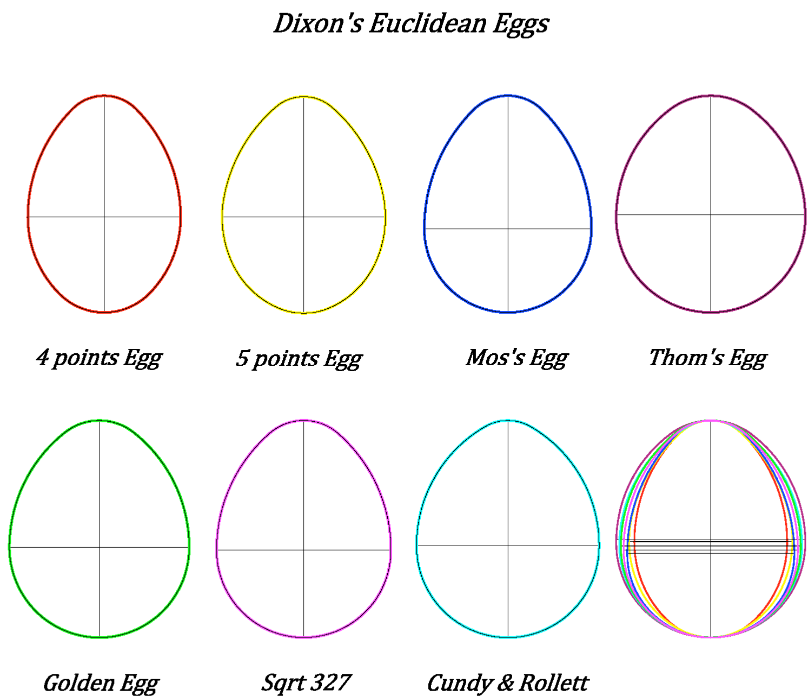


Figure 3: *The Egg Curves Obtained By Robert Dixon Construction*

This paper deals with one of the possible construction of the golden egg using the described methods.

1.2 Connection Of The Golden Section And The Construction Of Egg-Shaped Curve - The Method Of 45º Angle

The **figure 4** presents the previously described method of **45º** angle, associated with the golden section. For the specific value of the initial semicircle radius, we may observe more easily the construction of the golden egg.

If the first arc, semicircle, would have the radius **r = Φ** with the center at the point **O**, the second (and the third) arc, centered at the point **H (**and **G)** of the radius twice as large - would value **2Φ**,then the fourth arc, quarter of the circle centered at the point ***I*** would have the radius ***p***.

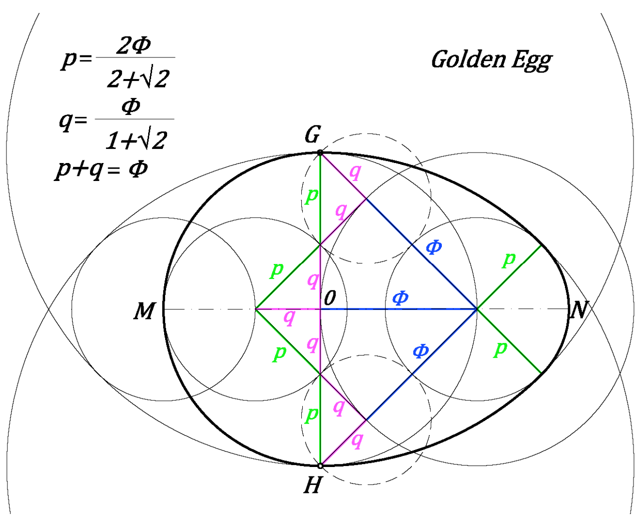


Figure 4: *The Golden Egg Curve Construction By The Method Of 45º Angle*

2. THE EGG CURVE AND THE GOLDEN SECTION

2.1. Constructing The Egg Shaped Curve Using The Regular Pentagon

Forming the egg-shaped curve using two inserted pentagons, by the method "***345***"of continuous circular arcs can produce the golden egg (**Fig. 5**).

The circular arc $\boldsymbol{l_1}$**,** with the center at the point $\boldsymbol{C_1}$ has the radius of ***1***, the circular arc $\boldsymbol{l_2}$ centered at the point $\boldsymbol{C_2}$ has the radius $\boldsymbol{1+\Phi}$, while the circular arc $\boldsymbol{l_3}$ with the center at the point $\boldsymbol{C_3}$ has the radius of $\boldsymbol{\Phi}$**.**

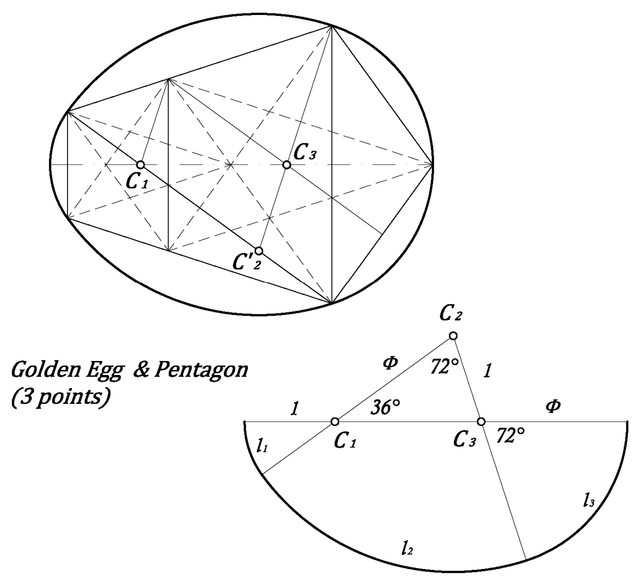


Figure 5*: The Golden Egg And The Pentagon*

For the previously described method, it is possible to obtain, by iteration, a series of egg shaped curves, which will be described in three, four, five, ... pentagons (**Fig. 6**).

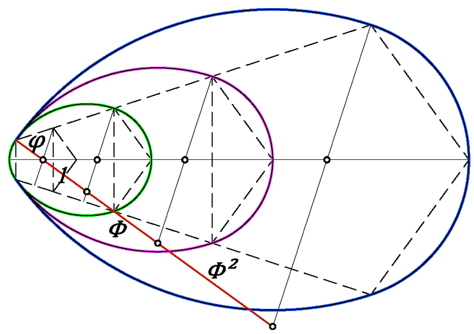


Figure 6*: The Golden Egg and the Series of Pentagons*

These curves may create interesting rosettes (**Fig.10**) which can be used in designing and arts.

2.2 The Construction Of The Egg Curve Using The Penrose Tiling Triangle

The golden Egg curve is primary obtained using the triangle formed by ***Penrose tiling***[119], so that the vertices of the triangle are the centers of circular arcs $\mathbf{C_1}$, $\mathbf{C_2}$ and $\mathbf{C_3}$ **(Fig. 7)**.

[119] Kepler (Johannes Kepler, 1571-1630) was the one of the first scientists who investigated the paving (tiling) and symmetry of the pentagon (5-fold symmetry, Harmonices Mundi, Book II), and who also inspired Roger Penrose (Roger Penrose), to discover in 1973. and 1974. the three new sets of aperiodic pentagonal tiling with symmetry, that are used as models in geometry of quasicrystal structures.

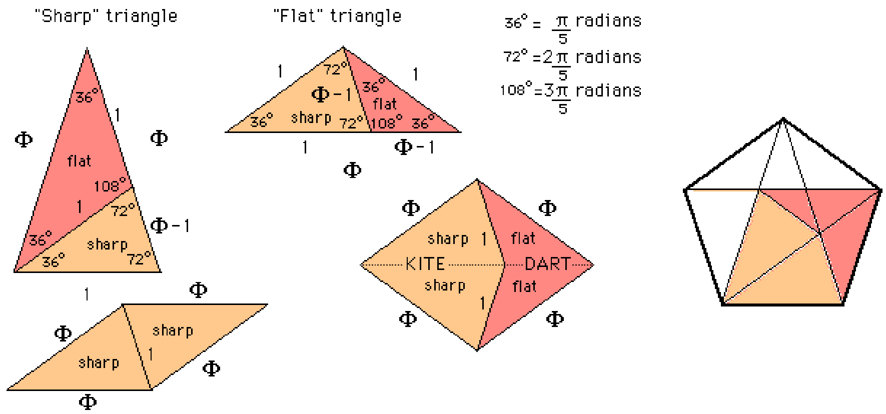


Figure 7: *Sharp And Flat Triangles Of The Pentagon*[120]

When the egg-shaped curve is formed by the sharp triangle of the Penrose tiling - the triangle with angles of ***36 º - 72 º - 72 º*** (**Figure 8**) then we will get the same curve as in the **Figure 5.**

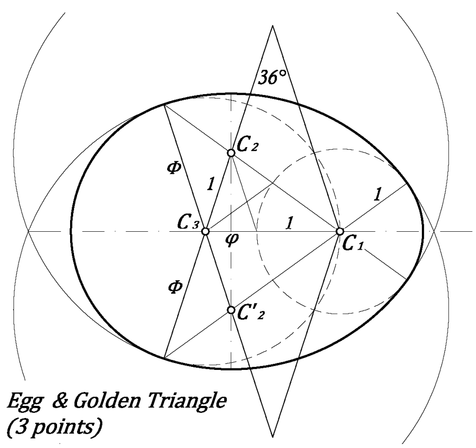


Figure 8: *The Golden Egg And The Sharp Triangle* (**36º - 72º - 72º**)

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120 The picture is taken from: http://www.maths.surrey.ac.uk/hosted-sites/R.Knott/Fibonacci/phi2DGeomTrig.html

For a blunt (flat) triangle of the Penrose tiling, with the angles of ***36º - 36º - 108º*** (**Figure 9**) there will be obtained the golden egg, ie. the curve which ratio between its volume and the sum of major and minor axis equals **Φ**.

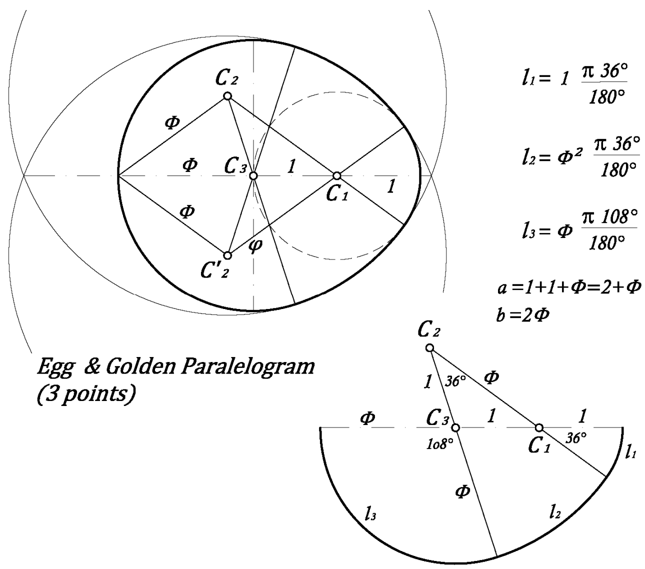

Figure 9: *The Golden Egg And The Flat Triangle* (**36º - 36º - 108º**)

$$a+b = 1+1+\Phi+2\Phi = 2+3\Phi = \Phi^4$$

$$\begin{aligned} O &= 2\,(l_1 + l_2 + l_3) \\ &= 2\,(1+\Phi^2+3\Phi)\ \frac{\pi\, 36°}{180°} \\ &= (1+1+\Phi+3\Phi)\ \frac{\pi\, 36°}{90°} \\ &= (1+2\Phi)\ \frac{\pi\, 36°}{45°} \\ &= \Phi^3\ \frac{4}{5}\ \Phi^2\ \frac{6}{5} \\ &= \frac{24}{25}\Phi^5 \end{aligned}$$

3. CONCLUSIONS

Construction of the egg shaped curves produced by the method of continuous circular arcs, so that the centers of these circles are vertices of the sharp or the flat triangles, and the construction of the curve using the pentagon were not yet presented. Thus, the resulting curve can be inspiring and can be used for further research:

• to create interesting rosettes (**Fig. 10**)

• to create aperiodic tiling, and so on.

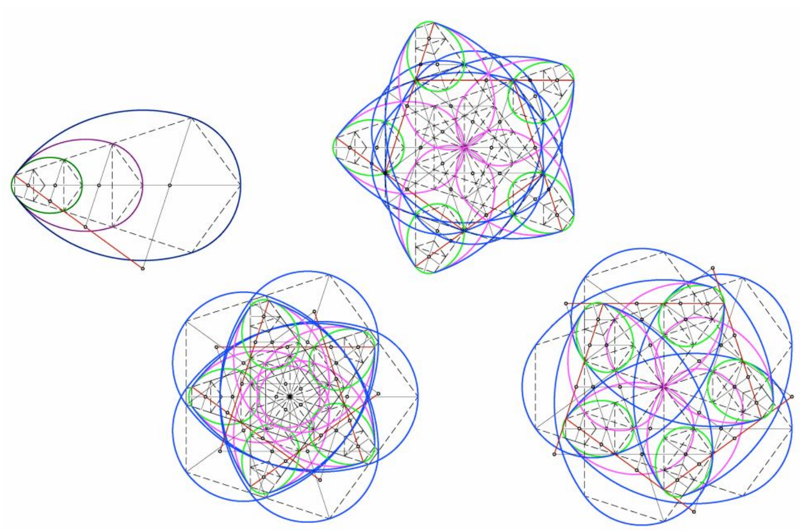

Figure 10*: Rosettes formed by rotation of golden egg shaped curves*

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