Physics Letters B 730 (2014) 115–121

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

# The magnetic moment of the $\rho$-meson

D. Djukanovic [a], E. Epelbaum [b,*], J. Gegelia [b,c], U.-G. Meißner [d,e]

[a] Helmholtz Institute Mainz, University of Mainz, D-55099 Mainz, Germany
[b] Institut für Theoretische Physik II, Ruhr-Universität Bochum, D-44780 Bochum, Germany
[c] Tbilisi State University, 0186 Tbilisi, Georgia
[d] Helmholtz Institut für Strahlen- und Kernphysik and Bethe Center for Theoretical Physics, Universität Bonn, D-53115 Bonn, Germany
[e] Institute for Advanced Simulation, Institut für Kernphysik and Jülich Center for Hadron Physics, Forschungszentrum Jülich, D-52425 Jülich, Germany

A R T I C L E I N F O

*Article history:*
Received 25 September 2013
Received in revised form 29 December 2013
Accepted 2 January 2014
Available online 20 January 2014
Editor: W. Haxton

A B S T R A C T

The magnetic moment of the $\rho$-meson is calculated in the framework of a low-energy effective field theory of the strong interactions. We find that the complex-valued strong interaction corrections to the gyromagnetic ratio are small leading to a value close to the real tree level result, $g_\rho = 2$. This is in a reasonably good agreement with the available lattice QCD calculations for this quantity.



## 1. Introduction

Phenomenological low-energy chiral Lagrangians with vector mesons were considered already in the 1960s, see e.g. Refs. [1–3]. Later, the chiral symmetry of QCD has been taken into account in the framework of effective field theories describing the interaction of vector mesons with pseudoscalars and baryons, see, e.g., Refs. [4–14].

In a covariant formalism, massive vector bosons are described by Lagrangians with constraints. The self-consistency of a system with constraints imposes non-trivial conditions on the form of the Lagrangian. In Ref. [15], the effective Lagrangian of Ref. [3] describing the interaction among $\rho$-mesons, pions and nucleons was considered. Requiring perturbative renormalizability in the sense of effective field theory [16], the universality of the vector-meson couplings was derived. The crucial ingredient of any effective field theory (EFT) is power counting. It is possible to consistently include virtual (axial-) vector mesons in EFT [11,17,18] provided they appear only as internal lines in Feynman diagrams involving soft external pions and nucleons with small three-momenta. The issue becomes highly non-trivial for energies when the intermediate resonant states can be generated. The problem is that vector mesons decay in light modes and therefore large imaginary parts appear [13]. First attempts have been made to handle this problem by applying the complex mass scheme [19–26].

In this work we calculate the magnetic moment of the $\rho$-meson as a function of quark masses in the framework of a low-energy effective theory of the strong interactions. As for any unstable particle, this quantity is a complex number. For a detailed discussion on this issue, see Ref. [27]. We start with the most general chirally invariant effective Lagrangian of vector mesons interacting with pions in the presence of external fields. It contains an infinite number of terms which respect the underlying symmetries of QCD. All divergences appearing in loop integrals can be absorbed into redefinition of parameters entering the effective Lagrangian order-by-order in the expansion in terms of derivatives and quark masses. In the present work, we assume that all dimension-full coupling constants associated with vector mesons are suppressed by powers of some large scale. Based on renormalization-group arguments, this large scale can be expected to be given by masses of heavy hadrons multiplied with some numerical factors such as e.g. $4\pi M_\rho$. Due to this assumption, only a finite number of terms of the effective Lagrangian has to be considered in our calculation. We further emphasize that while the considered EFT is self-consistent and leads to systematic perturbative results for physical quantities, we can only assume that the obtained results can be matched to the corresponding QCD observables. In this sense the considered framework can be viewed as an EFT-based model. We apply the complex mass renormalization scheme [19,20] and calculate the magnetic moment of the vector meson and its pion mass dependence at one-loop order.

* Corresponding author.

http://dx.doi.org/10.1016/j.physletb.2014.01.001
0370-2693 

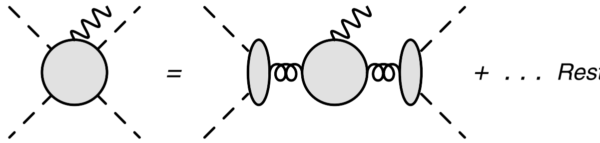


**Fig. 1.** The reaction $\pi\pi \to \gamma\pi\pi$. Dashed, wavy and wiggled lines refer to pions, photons and $\rho$-mesons, respectively. Non-resonant contributions denoted by "Rest" are not shown explicitly.

## 2. Lagrangian

We start with the most general chiral effective Lagrangian for $\rho$- and $\omega$-mesons, pions and external sources in the parametrization of the model III of Ref. [7][1]

$$\mathcal{L} = \mathcal{L}_\pi + \mathcal{L}_{\rho\pi} + \mathcal{L}_\omega + \mathcal{L}_{\omega\rho\pi} + \cdots. \tag{1}$$

Below we specify only those terms of the Lagrangian which are relevant for the calculation of the magnetic moment of the $\rho$-meson presented in this work:

$$\begin{aligned}
\mathcal{L}_\pi &= \frac{F^2}{4}\operatorname{Tr}\left[D_\mu U\left(D^\mu U\right)^\dagger\right] + \frac{F^2 M^2}{4}\operatorname{Tr}\left(U^\dagger + U\right),\\
\mathcal{L}_{\rho\pi} &= -\frac{1}{2}\operatorname{Tr}\left(\rho_{\mu\nu}\rho^{\mu\nu}\right) + \left[M_\rho^2 + \frac{c_x M^2 \operatorname{Tr}(U^\dagger + U)}{4}\right]\operatorname{Tr}\left[\left(\rho^\mu - \frac{i\Gamma^\mu}{g}\right)\left(\rho_\mu - \frac{i\Gamma_\mu}{g}\right)\right],\\
\mathcal{L}_\omega &= -\frac{1}{4}\left(\partial_\mu\omega_\nu - \partial_\nu\omega_\mu\right)\left(\partial^\mu\omega^\nu - \partial^\nu\omega^\mu\right) + \frac{M_\omega^2\omega_\mu\omega^\mu}{2},\\
\mathcal{L}_{\omega\rho\pi} &= \frac{1}{2} g_{\omega\rho\pi}\epsilon_{\mu\nu\alpha\beta}\omega^\nu \operatorname{Tr}\left(\rho^{\alpha\beta}u^\mu\right),
\end{aligned} \tag{2}$$

where

$$\begin{aligned}
U &= u^2 = \exp\left(\frac{i\vec{\tau}\cdot\vec{\pi}}{F}\right),\\
\rho^\mu &= \frac{\vec{\tau}\cdot\vec{\rho}^\mu}{2},\\
\rho^{\mu\nu} &= \partial^\mu\rho^\nu - \partial^\nu\rho^\mu - ig\left[\rho^\mu, \rho^\nu\right],\\
u_\mu &= i\left[u^\dagger\partial_\mu u - u\partial_\mu u^\dagger - i\left(u^\dagger v_\mu u - u v_\mu u^\dagger\right)\right],\\
\Gamma_\mu &= \frac{1}{2}\left[u^\dagger\partial_\mu u + u\partial_\mu u^\dagger - i\left(u^\dagger v_\mu u + u v_\mu u^\dagger\right)\right],\\
\Gamma_{\mu\nu} &= \partial_\mu\Gamma_\nu - \partial_\nu\Gamma_\mu + [\Gamma_\mu, \Gamma_\nu],\\
f_+^{\mu\nu} &= u F_L^{\mu\nu} u^\dagger + u^\dagger F_R^{\mu\nu} u,\\
D_\mu A &= \partial_\mu A - i v_\mu A + i A v_\mu.
\end{aligned} \tag{3}$$

Here, $F$ denotes the pion decay constant in the chiral limit, $M^2$ is the lowest order expression for the squared pion mass, $M_\rho$ and $M_\omega$ refer to the $\rho$ and $\omega$ masses in the chiral limit, respectively. Further, $g$, $c_x$ and $g_{\omega\rho\pi}$ are coupling constants and $v_\mu$ is the external vector field. Notice that we do not show the counterterms explicitly. For the electromagnetic interaction we have $v_\mu = -e\tau^3 A_\mu/2$. Demanding that couplings with different mass dimensions are independent, the consistency condition for the $\rho\pi\pi$ coupling [15] leads to the KSFR relation [28,29]

$$M_\rho^2 = 2g^2F^2. \tag{4}$$

The Lagrangian of Eq. (2) results from Eq. (1) by switching off all coupling constants with negative mass dimensions of the interactions involving the vector mesons.

## 3. Magnetic moment of the vector meson

As the $\rho$-meson is an unstable particle it does not appear as an asymptotic state in the effective field theory. Therefore, to define the magnetic moment of the $\rho$-meson, we follow the strategy of Ref. [27] and consider an amplitude of a process in which the $\gamma\rho\rho$ vertex contributes as a sub-diagram. For the sake of definiteness, we take the process $\pi\pi \to \gamma\pi\pi$ shown in Fig. 1. We parameterize the amplitude of this process as

[1] The $\rho$-vector fields transform inhomogeneously under chiral transformations in this parametrization. This coincides with the parametrization adopted by Weinberg for the vector fields in Ref. [3].

$$\mathcal{M} = \mathcal{M}_1^\alpha(-i)D_{\alpha\mu}(p_i)V^{\lambda\mu\nu}(q,p_f,p_i)(-i)D_{\nu\beta}(p_f)\mathcal{M}_2^\beta\epsilon_\lambda + Rest, \tag{5}$$

where "*Rest*" denotes the non-resonant contributions. Here, $\mathcal{M}_1^\alpha$ and $\mathcal{M}_2^\beta$ are the $\rho\pi\pi$ vertex functions, $\epsilon_\lambda$ is the photon polarization, and $-iD^{\mu\nu}(p)$ with

$$D^{\mu\nu}(p) = Z_V \frac{g^{\mu\nu} - p^\mu p^\nu/z}{p^2 - z} + Rest \tag{6}$$

is the dressed propagator of the vector meson. Further, $Z_V$ is the (complex) residue at the pole $z$ and *Rest* denotes the non-pole part. The $\gamma\rho\rho$ vertex function can be written as

$$V^{\lambda\mu\nu}(q,p_f,p_i) = \sum_j t_j^{\lambda\mu\nu} V_j\big(q^2,p_f^2,p_i^2\big), \tag{7}$$

where $t_j^{\lambda\mu\nu}$ denote the possible tensor structures which depend on the momenta $p_i$, $p_f$ and the metric tensor, and $V_i(q^2,p_f^2,p_i^2)$ are the corresponding scalar functions. Here and in what follows, we do not show the isospin indices (unless stated otherwise) for the sake of compactness. Expanding the $V_j$ about the $z$-pole and substituting, together with the expression for the propagator in Eq. (6), into Eq. (5) we obtain for the leading double-pole contribution

$$\mathcal{M}_{\rm dp} = -\mathcal{M}_1^\alpha \frac{g^{\alpha\mu} - p_i^\mu p_i^\nu/z}{p_i^2 - z} Z_V \sum_j t_j^{\lambda\mu\nu} V_j\big(q^2,z,z\big) Z_V \frac{g^{\nu\beta} - p_f^\nu p_f^\beta/z}{p_f^2 - z}\mathcal{M}_2^\beta. \tag{8}$$

In order to properly renormalize the $\gamma\rho\rho$ vertex function we rewrite Eq. (8) in the form

$$\mathcal{M}_{\rm dp} = -\mathcal{M}_1^\alpha \sqrt{Z_V}\frac{g^{\alpha\mu} - p_i^\mu p_i^\nu/z}{p_i^2 - z} \sqrt{Z_V}\sum_j t_j^{\lambda\mu\nu} V_j\big(q^2,z,z\big)\sqrt{Z_V}\frac{g^{\nu\beta} - p_f^\nu p_f^\beta/z}{p_f^2 - z}\sqrt{Z_V}\mathcal{M}_2^\beta \tag{9}$$

and define

$$i\Gamma^{\lambda\mu\nu}(q,p_i,p_f) := \sqrt{Z_V}\sum_j t_j^{\lambda\mu\nu} V_j\big(q^2,z,z\big)\sqrt{Z_V}. \tag{10}$$

Noting that $p^\mu D_{\mu\nu}$ does not have a pole, we drop structures containing $p_f^\nu$ and $p_i^\mu$ and parameterize the "on-mass-shell" $\Gamma$ as follows

$$\Gamma^{\lambda\mu\nu}(q,p_i,p_f) = f_1\big(q^2\big)\big(p_i^\lambda + p_f^\lambda\big)g^{\mu\nu} + f_2\big(q^2\big)\big(q^\nu g^{\lambda\mu} - q^\mu g^{\lambda\nu}\big) + \cdots, \tag{11}$$

where the ellipsis refer to structures which do not involve the metric tensor.

The charge and magnetic moment $e$ and $\mu_\rho$ of the $\rho$-meson are defined in terms of the corresponding form factors $f_1(0)$ and $f_2(0)$ as

$$\begin{aligned} f_1(0) &= -e, \\ f_2(0) &= -2M_\rho\mu_\rho. \end{aligned} \tag{12}$$

There are both tree-level and loop contributions to these quantities. Loop diagrams are suppressed by powers of $\xi = g_i^2/(16\pi^2)$, where $g_i$ stands for coupling constants in general. Even for a sizeable coupling like $g_{\rho\pi\pi}$, this expansion parameter is small, $\xi \simeq 0.2$. Vertices generated by the $c_x$-term of the Lagrangian are only included at tree order in our calculations as their contributions are suppressed at the one-loop order by two additional powers of the pion mass. At tree order, we obtain

$$\begin{aligned} f_1^{\rm tree}(0) &= -e, \\ f_2^{\rm tree}(0) &= -2e, \end{aligned} \tag{13}$$

in agreement with the findings of Refs. [30,31]. One-loop diagrams contributing to the $\gamma\rho\rho$ vertex function are shown in Fig. 2. Their full contributions to $f_2(0)$ are given in Appendix A. Taking into account the wave function renormalization we obtain

$$\begin{aligned} f_1^{\rm loop}(0) &= 0, \\ f_2^{\rm loop}(0) &= \frac{e}{384\pi^2F^2}\big\{24M^2B_0\big(M_\rho^2,M^2,M^2\big) + 30M_\rho^2B_0\big(M_\rho^2,M_\rho^2,M_\rho^2\big) - 30A_0\big(M_\rho^2\big) - 24A_0\big(M^2\big) + 17M_\rho^2 + 24M^2\big\} \\ &\quad + \frac{eg_{\omega\rho\pi}^2}{576\pi^2F^2M_\rho^2}\big\{12A_0\big(M_\omega^2\big)\big(2M_\omega^2 - M_\rho^2 - 2M^2\big) - 6A_0\big(M^2\big)\big(4M_\omega^2 + 5M_\rho^2 - 4M^2\big) \\ &\quad - 9M_\rho^2\big(M_\omega^2 + M^2\big) + M_\rho^2\big(21M_\omega^2 - 19M_\rho^2 + 21M^2\big) \\ &\quad - 6\big[4M_\omega^4 - 2M^2\big(4M_\omega^2 + M_\rho^2\big) + M_\omega^2M_\rho^2 - 2M_\rho^4 + 4M^4\big]B_0\big(M_\rho^2,M_\omega^2,M^2\big)\big\} \\ &\quad + \frac{eg_{\omega\rho\pi}^2[M_\omega^2A_0(M_\omega^2) - M^2A_0(M^2)]}{32\pi^2F^2(M_\omega^2 - M^2)}, \end{aligned} \tag{14}$$

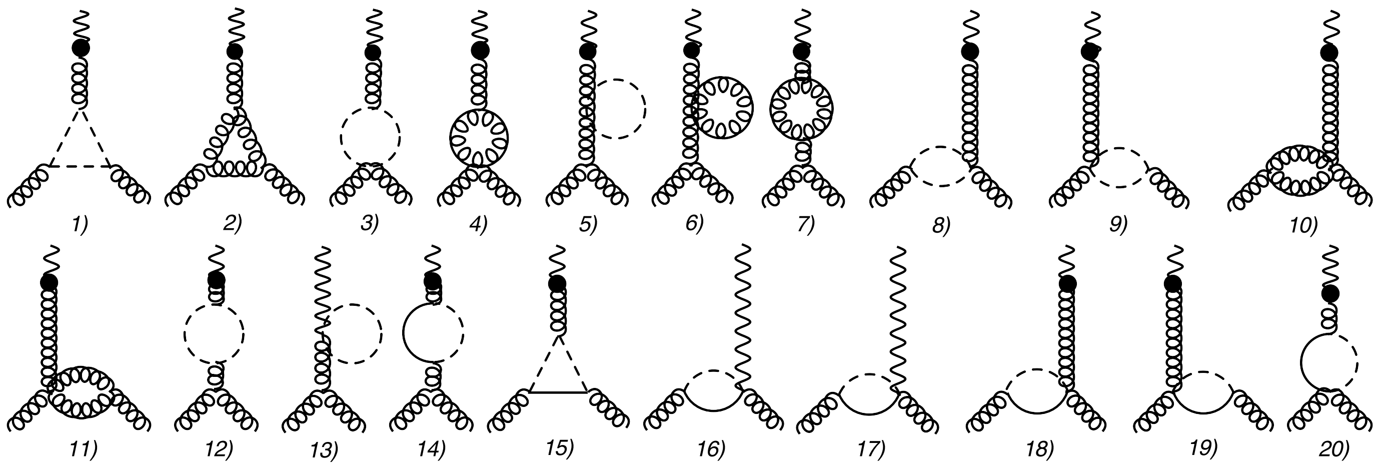


**Fig. 2.** Leading one loop diagrams contributing to the magnetic moment of the $\rho$-meson. Wavy, wiggly, dashed and solid lines correspond to the photon and $\rho$, $\pi$ and $\omega$-mesons, respectively. The solid circle corresponds to the photon-$\rho$-meson mixing.

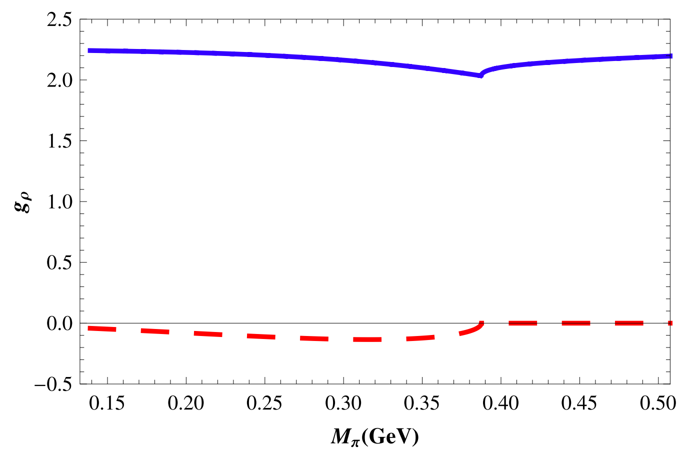


**Fig. 3.** The factor $g_\rho$ as a function of the pion mass. Solid (blue) and dashed (red) lines correspond to the real and imaginary parts, respectively.

where the loop functions $A_0(m^2)$ and $B_0(p^2, m_1^2, m_2^2)$ are also defined in Appendix A. Note that in Eq. (14), the quantity $f_1^{\text{loop}}(0)$ only vanishes when the complex residue of the dressed propagator at the pole is used as the wave function renormalization constant for the vector field. A similar result has been obtained in Ref. [32].

We now estimate numerically the derived one-loop contributions. Using $g_{\omega\rho\pi} = 1.478$ from Ref. [33] and adopting the physical values for the various meson masses and the pion decay constant instead of the corresponding chiral-limit values as appropriate at the order we are working, namely $M_\rho = 0.775$ GeV, $M_\omega = 0.782$ GeV, $M \sim M_\pi = 0.1395$ GeV, $F \sim F_\pi = 0.0924$ GeV, we obtain

$$f_2^{\text{loop}}(0) = (0.2416 - 0.0423i)e. \tag{15}$$

Notice that using the complex values $M_\rho^2 = (0.775^2 - i0.775 \times 0.149)$ GeV$^2$, $M_\omega^2 = (0.782^2 - i0.782 \times 0.0085)$ GeV$^2$ for the renormalized masses of the $\rho$- and $\omega$-mesons corresponding to the pole positions leads to a very similar numerical result of

$$f_2^{\text{loop}}(0) = (0.2124 - 0.0415i)e. \tag{16}$$

Comparing Eqs. (15) and (16) with Eq. (13) we see that the loop contributions are clearly suppressed in comparison with the tree-level result and also that the imaginary part of the vector meson masses has a little impact on the value of the loop correction. We thus conclude that the leading quantum corrections to the classical value of the $g$-factor, $g_\rho = 2$, are suppressed. This gives a strong indication that the strong corrections to this observable are small. This conjecture is further supported by the value $g_\rho = 2.1 \pm 0.5$ extracted in Ref. [34] from the $e^+e^- \to \pi^+\pi^-2\pi^0$ BaBar data and the available lattice QCD calculations for this quantity, namely $(g_\rho)_{\text{quenched}} \sim 2.3$ of Ref. [35] and $(g_\rho)_{\text{unquenched}} = 1.6(1)$ of Ref. [36] (see also Ref. [37] for an early study).

Finally, we also plot in Fig. 3 the real and imaginary parts of the $g$-factor $g_\rho$ as functions of the pion mass. Both real and imaginary part show very little pion mass dependence, the cusp appears at the value of the pion mass, at which the $\rho$ pole moves from the second to the first Riemann sheet.

## 4. Summary

In this Letter, we have calculated the complex-valued magnetic moment of the $\rho$-meson in a chirally invariant EFT utilizing the complex-mass renormalization scheme. Assuming that the interaction terms with a higher number of derivatives and/or more fields

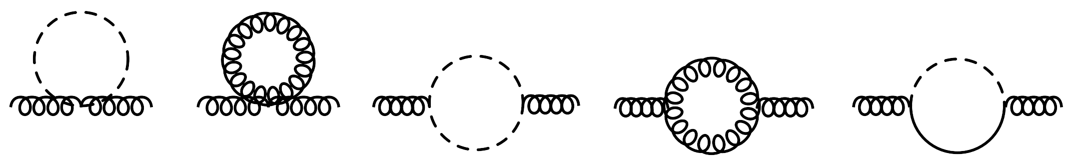

**Fig. 4.** Vector meson self-energy diagrams at the one-loop level. Wiggly, dashed and solid lines correspond to $\rho$, $\pi$ and $\omega$ mesons, respectively.

are suppressed by powers of some large hadronic scale, we perform a one-loop calculation in terms of the expansion parameter $\xi = (g_{\rho\pi\pi}/4\pi)^2 \simeq 0.2$. The pertinent results of our investigation can be summarized as follows:

- At tree level (leading order), the magnetic moment of the $\rho$ is real and its gyromagnetic ratio is $g_\rho = 2$.
- At one-loop order, the magnetic moment picks up an imaginary part. We find that the one-loop corrections to $g_\rho$ are of the order of 10%, cf. Eqs. (15), (16), and the imaginary part is about 0.04 (in units of the charge). The results are in agreement with recent lattice QCD determinations.
- We find that the pion mass dependence of the gyromagnetic ratio is very weak. This could be tested on the lattice for sufficiently small pion masses, say $M_\pi \lesssim 0.3$ GeV, that also allow for the $\rho$-meson to decay.

While comparing our results with those of lattice QCD tempts us to conclude that our assumption about the contributions from terms with more derivatives and/or fields being suppressed is justified, we cannot derive it from QCD due to the lack of separation of scales in the spectrum of hadrons of non-Goldstone boson type.

## Acknowledgements

We are grateful to Genaro Toledo Sánchez for drawing our attention to related recent work on this topic. This work was supported in part by Georgian Shota Rustaveli National Science Foundation (grant 11/31), DFG (SFB/TR 16, "Subnuclear Structure of Matter"), by the European Community-Research Infrastructure Integrating Activity "Study of Strongly Interacting Matter" (acronym HadronPhysics3, Grant Agreement no. 283286) under the Seventh Framework Programme of EU, and ERC project 259218 NUCLEAREFT.

## Appendix A. Loop functions and explicit expressions

The loop functions $A_0$ and $B_0$ are defined as follows:

$$A_0\left(m^2\right) = \frac{(2\pi)^{4-n}}{i\pi^2}\int \frac{d^n k}{k^2-m^2},$$
$$B_0\left(p^2, m_1^2, m_2^2\right) = \frac{(2\pi)^{4-n}}{i\pi^2}\int \frac{d^n k}{[k^2-m_1^2][(p+k)^2-m_2^2]}, \tag{A.1}$$

where $n$ is the space–time dimension.

The sum of all one-particle-irreducible diagrams contributing in the vector meson two-point function can be parameterized as

$$i\Pi^{ab}_{\mu\nu}(p) = i\delta^{ab}\left[g_{\mu\nu}\Pi_1 + \left(g_{\mu\nu}p^2 - p_\mu p_\nu\right)\Pi_2\left(p^2\right)\right], \tag{A.2}$$

where $\Pi_1$ is momentum-independent and $\Pi_2(p^2)$ is regular at $p^2 = 0$. Further, $a$ and $b$ are the isospin indices. The wave function renormalization of the vector meson is defined as the (complex) residue at the (complex) pole of the dressed propagator. In terms of Eq. (A.2) it reads

$$Z_V = \frac{1}{1-\Pi_2(z) - z\Pi_2'(z)} = 1 + \delta Z_\rho + \cdots, \tag{A.3}$$

where $\delta Z_\rho$ is the LO one-loop contribution and the ellipses stand for higher order corrections.

Calculating the vector-meson self-energy diagrams, cf. Fig. 4, we obtain

$$\begin{aligned}
\delta Z_\rho = & -\frac{1}{1152\pi^2 F^2}\left\{4\left[3\left(M_\rho^2+2M^2\right)B_0\left(M_\rho^2, M^2, M^2\right) - 6A_0\left(M^2\right) - M_\rho^2 + 6M^2\right]\right. \\
& \left. + M_\rho^2\left[99B_0\left(M_\rho^2, M_\rho^2, M_\rho^2\right) + 113\right] - 258A_0\left(M_\rho^2\right)\right\} \\
& + \frac{g^2_{\omega\rho\pi}}{288\pi^2 F^2 M_\rho^2}\left\{3M_\omega^2 M_\rho^2 B_0\left(M_\rho^2, M_\omega^2, M^2\right) - 6M_\omega^2 M^2 B_0\left(M_\rho^2, M_\omega^2, M^2\right)\right. \\
& + 3M_\rho^2 M^2 B_0\left(M_\rho^2, M_\omega^2, M^2\right) + 3M^4 B_0\left(M_\rho^2, M_\omega^2, M^2\right) - 6M_\rho^4 B_0\left(M_\rho^2, M_\omega^2, M^2\right) \\
& + 3M_\omega^4 B_0\left(M_\rho^2, M_\omega^2, M^2\right) + A_0\left(M_\omega^2\right)\left(-3M_\omega^2 + 6M_\rho^2 + 3M^2\right) \\
& \left. + 3A_0\left(M^2\right)\left(M_\omega^2 + 2M_\rho^2 - M^2\right) - 6M_\omega^2 M_\rho^2 + 5M_\rho^4 - 6M_\rho^2 M^2\right\}.
\end{aligned} \tag{A.4}$$

The contributions of the one-loop diagrams to $f_1(0)$ are:

$$
\begin{aligned}
f_1[1] &= \frac{e(-3(M_\rho^2+2M^2)B_0(M_\rho^2,M^2,M^2)+6A_0(M^2)+M_\rho^2-6M^2)}{288\pi^2F^2},\\
f_1[2] &= -\frac{e(M_\rho^2(495B_0(M_\rho^2,M_\rho^2,M_\rho^2)+196)-534A_0(M_\rho^2))}{2304\pi^2F^2},\\
f_1[3] &= 0,\\
f_1[4] &= 0,\\
f_1[5] &= 0,\\
f_1[6]+f_1[7] &= 0,\\
f_1[18]+f_1[9] &= 0,\\
f_1[10]+f_1[11] &= -\frac{e(M_\rho^2(10-99B_0(M_\rho^2,M_\rho^2,M_\rho^2))+6A_0(M_\rho^2))}{768\pi^2F^2},\\
f_1[12] &= \frac{eA_0(M^2)}{16\pi^2F^2},\\
f_1[13] &= -\frac{eA_0(M^2)}{16\pi^2F^2},\\
f_1[14] &= 0,\\
f_1[15] &= \frac{eg_{\omega\rho\pi}^2}{288\pi^2F^2M_\rho^2}\big\{3A_0\big(M_\omega^2\big)\big(-2M_\omega^2+M_\rho^2+2M^2\big)\\
&\quad+3A_0\big(M^2\big)\big(2M_\omega^2+M_\rho^2-2M^2\big)+M_\rho^2\big(-3M_\omega^2+4M_\rho^2-3M^2\big)\\
&\quad-3\big[M^2\big(4M_\omega^2+M_\rho^2\big)+M_\omega^2M_\rho^2+M_\rho^4-2M^4-2M_\omega^4\big]B_0\big(M_\rho^2,M_\omega^2,M^2\big)\big\},\\
f_1[16]+f_1[17] &= 0,\\
f_1[18]+f_1[19] &= \frac{eg_{\omega\rho\pi}^2}{288\pi^2F^2M_\rho^2}\big\{3A_0\big(M_\omega^2\big)\big(M_\omega^2+M_\rho^2-M^2\big)\\
&\quad+3A_0\big(M^2\big)\big(-M_\omega^2+M_\rho^2+M^2\big)+M_\rho^2\big(-3M_\omega^2+M_\rho^2-3M^2\big)\\
&\quad-3\big(M_\omega^4-2M_\omega^2\big(M_\rho^2+M^2\big)+\big(M_\rho^2-M^2\big)^2\big)B_0\big(M_\rho^2,M_\omega^2,M^2\big)\big\},\\
f_2[20] &= 0.
\end{aligned}
\tag{A.5}
$$

The contributions of the one-loop diagrams to $f_2(0)$ are:

$$
\begin{aligned}
f_2[1] &= -\frac{e}{288F^2\pi^2}\big\{M_\rho^2-6M^2+6A_0\big(M^2\big)+6\big(M_\rho^2-M^2\big)B_0\big(M_\rho^2,M^2,M^2\big)\big\},\\
f_2[2] &= \frac{e}{2304F^2\pi^2}\big\{\big[136-513B_0\big(M_\rho^2,M_\rho^2,M_\rho^2\big)\big]M_\rho^2+546A_0\big(M_\rho^2\big)\big\},\\
f_2[3] &= 0,\\
f_2[4] &= -\frac{3e}{64F^2\pi^2}\big\{4M_\rho^2-3A_0\big(M_\rho^2\big)\big\},\\
f_2[5] &= 0,\\
f_2[6]+f_2[7] &= 0,\\
f_2[8]+f_2[9] &= 0,\\
f_2[10]+f_2[11] &= \frac{e}{768F^2\pi^2}\big\{M_\rho^2\big[99B_0\big(M_\rho^2,M_\rho^2,M_\rho^2\big)-10\big]-6A_0\big(M_\rho^2\big)\big\},\\
f_2[12] &= \frac{e}{8F^2\pi^2}A_0\big(M^2\big),\\
f_2[13] &= -\frac{e}{8F^2\pi^2}A_0\big(M^2\big),\\
f_2[14] &= 0,\\
f_2[15] &= -\frac{eg_{\omega\rho\pi}^2}{576\pi^2F^2M_\rho^2}\big\{-6A_0\big(M_\omega^2\big)\big(M_\omega^2+M_\rho^2-M^2\big)\\
&\quad+6A_0\big(M^2\big)\big(M_\omega^2+2M_\rho^2-M^2\big)+M_\rho^2\big(-3M_\omega^2+M_\rho^2-3M^2\big)\\
&\quad+6\big(M_\omega^4-2M^2\big(M_\omega^2+M_\rho^2\big)+M_\omega^2M_\rho^2+M_\rho^4+M^4\big)B_0\big(M_\rho^2,M_\omega^2,M^2\big)\big\},
\end{aligned}
$$

$$
\begin{aligned}
& f_2[16]+f_2[17]=0, \\
& f_2[18]+f_2[19]=\frac{e g_{\omega\rho\pi}^2}{288\pi^2 F^2 M_\rho^2}\big\{3A_0\big(M_\omega^2\big)\big(M_\omega^2+M_\rho^2-M^2\big) \\
& \qquad +3A_0\big(M^2\big)\big(-M_\omega^2+M_\rho^2+M^2\big)+M_\rho^2\big(-3M_\omega^2+M_\rho^2-3M^2\big) \\
& \qquad -3\big(M_\omega^4-2M_\omega^2\big(M_\rho^2+M^2\big)+\big(M_\rho^2-M^2\big)^2\big)B_0\big(M_\rho^2,M_\omega^2,M^2\big)\big\}, \\
& f_2[20]=-\frac{e g_{\omega\rho\pi}^2}{64\pi^2 F^2(M_\omega^2-M^2)}\big[M_\omega^4-2A_0\big(M_\omega^2\big)M_\omega^2-M^4+2M^2A_0\big(M^2\big)\big].
\end{aligned}
\tag{A.6}
$$

## References


[1] J.S. Schwinger, Phys. Lett. B 24 (1967) 473.
[2] J. Wess, B. Zumino, Phys. Rev. 163 (1967) 1727.
[3] S. Weinberg, Phys. Rev. 166 (1968) 1568.
[4] J. Gasser, H. Leutwyler, Ann. Phys. (N. Y.) 158 (1984) 142.
[5] U.-G. Meißner, Phys. Rep. 161 (1988) 213.
[6] M. Bando, T. Kugo, K. Yamawaki, Phys. Rep. 164 (1988) 217.
[7] G. Ecker, J. Gasser, H. Leutwyler, A. Pich, E. de Rafael, Phys. Lett. B 223 (1989) 425.
[8] G. Ecker, J. Gasser, A. Pich, E. de Rafael, Nucl. Phys. B 321 (1989) 311.
[9] B. Borasoy, U.-G. Meißner, Int. J. Mod. Phys. A 11 (1996) 5183.
[10] M.C. Birse, Z. Phys. A 355 (1996) 231.
[11] B. Kubis, U.-G. Meißner, Nucl. Phys. A 679 (2001) 698.
[12] M. Harada, K. Yamawaki, Phys. Rep. 381 (2003) 1.
[13] P.C. Bruns, U.-G. Meißner, Eur. Phys. J. C 40 (2005) 97.
[14] P.C. Bruns, U.-G. Meißner, Eur. Phys. J. C 58 (2008) 407.
[15] D. Djukanovic, M.R. Schindler, J. Gegelia, G. Japaridze, S. Scherer, Phys. Rev. Lett. 93 (2004) 122002.
[16] S. Weinberg, The Quantum Theory Of Fields. Vol. 1: Foundations, Univ. Pr., Cambridge, UK, 1995.
[17] T. Fuchs, M.R. Schindler, J. Gegelia, S. Scherer, Phys. Lett. B 575 (2003) 11.
[18] M.R. Schindler, J. Gegelia, S. Scherer, Phys. Lett. B 586 (2004) 258.
[19] R.G. Stuart, in: J. Tran Thanh Van (Ed.), $Z^0$ Physics, Editions Frontieres, Gif-sur-Yvette, 1990, p. 41.
[20] A. Denner, S. Dittmaier, M. Roth, D. Wackeroth, Nucl. Phys. B 560 (1999) 33.
[21] D. Djukanovic, J. Gegelia, A. Keller, S. Scherer, Phys. Lett. B 680 (2009) 235.
[22] D. Djukanovic, J. Gegelia, S. Scherer, Phys. Lett. B 690 (2010) 123.
[23] D. Djukanovic, J. Gegelia, A. Keller, S. Scherer, PoS CD 09 (2009) 050.
[24] T. Bauer, D. Djukanovic, J. Gegelia, S. Scherer, L. Tiator, AIP Conf. Proc. 1432 (2012) 269.
[25] T. Bauer, J. Gegelia, S. Scherer, Phys. Lett. B 715 (2012) 234.
[26] T. Bauer, J. Gegelia, G. Japaridze, S. Scherer, Int. J. Mod. Phys. A 27 (2012) 1250178.
[27] J. Gegelia, S. Scherer, Eur. Phys. J. A 44 (2010) 425.
[28] K. Kawarabayashi, M. Suzuki, Phys. Rev. Lett. 16 (1966) 255.
[29] Riazuddin, Fayyazuddin, Phys. Rev. 147 (1966) 1071.
[30] D. Djukanovic, M.R. Schindler, J. Gegelia, S. Scherer, Phys. Rev. Lett. 95 (2005) 012001.
[31] C. Lorce, Phys. Rev. D 79 (2009) 113011.
[32] D. García Gudiño, G. Toledo Sánchez, Phys. Rev. D 81 (2010) 073006.
[33] M. Lublinsky, Phys. Rev. D 55 (1997) 249.
[34] D. García Gudiño, G. Toledo Sánchez, arXiv:1305.6345 [hep-ph].
[35] J.N. Hedditch, W. Kamleh, B.G. Lasscock, D.B. Leinweber, A.G. Williams, J.M. Zanotti, Phys. Rev. D 75 (2007) 094504.
[36] M. Gurtler, et al., QCDSF Collaboration, PoS LATTICE 2008 (2008) 051.
[37] W. Andersen, W. Wilcox, Ann. Phys. 255 (1997) 34.