PHYSICAL REVIEW C **83**, 065502 (2011)

Dressing the electromagnetic nucleon current

H. Haberzettl,[1,*] F. Huang,[2,†] and K. Nakayama[2,3,‡]

[1]*Center for Nuclear Studies, Department of Physics, The George Washington University, Washington, DC 20052, USA*

[2]*Department of Physics and Astronomy, University of Georgia, Athens, Georgia 30602, USA*

[3]*Institut für Kernphysik and Jülich Center for Hadron Physics, Forschungszentrum Jülich, 52425 Jülich, Germany*

(Received 10 March 2011; revised manuscript received 5 May 2011; published 16 June 2011)

A field-theory-based approach to pion photoproduction off the nucleon is used to derive a microscopically consistent formulation of the fully dressed electromagnetic nucleon current in an effective Lagrangian formalism. It is shown how the rigorous implementation of local gauge invariance at all levels of the reaction dynamics provides equations that lend themselves to practically manageable truncations of the underlying nonlinearities of the problem. The requirement of consistency also suggests a novel way of treating the pion photoproduction problem. Guided by a phenomenological implementation of gauge invariance for the truncated equations that has proved successful for pion photoproduction, an expression for the fully dressed nucleon current is given that satisfies the Ward-Takahashi identity for a fully dressed nucleon propagator as a matter of course. Possible applications include meson photo- and electroproduction processes, bremsstrahlung, Compton scattering, and ee' processes off nucleons.

DOI: 10.1103/PhysRevC.83.065502 PACS number(s): 13.40.−f, 13.60.Le, 25.20.Lj, 13.60.Fz

I. INTRODUCTION

The electromagnetic interaction provides the cleanest probe of hadronic systems available to experimentalists. Many experimental facilities, such as JLab, MAMI, ELSA, SPring-8, GRAAL, and others around the world, therefore, use reactions employing real or virtual photons to gain information about the internal dynamics of hadronic systems. (For a recent review, see Ref. [1].) However, while we understand the electromagnetic interaction perfectly well at the elementary level, its applications in actual experiments do not concern elementary particles but rather composite systems of elementary particles that describe the internal structures of the baryonic or mesonic systems that take part in the experiments. At intermediate energies, for baryons in particular, there usually is no need to invoke quark degrees of freedom to understand their internal structures, since the internal dynamics of baryons can be described very well in terms of baryonic and mesonic degrees of freedom.

One very successful, quite fundamental way of dealing with these degrees of freedom is the effective-field-theory framework of chiral perturbation theory [2]. However, in view of its perturbative nature, this cannot be easily extended to energy regions too far away from threshold. At higher energies, one usually must rely on effective Lagrangian formulations that offer a more direct avenue to the actual meson and baryon degrees of freedom that manifest themselves in the experiments.

It is important, therefore, to understand the nature of the electromagnetic interaction with mesons and baryons in a more detailed picture. One of the most important and most basic systems in this respect is the nucleon itself. The matrix element of the electromagnetic current operator J^μ of the nucleon between on-shell nucleon spinors is given by

$$\bar{u}J^\mu u = \bar{u}(p')\left[e\delta_N\,\gamma^\mu F_1(k^2) + e\frac{i\sigma^{\mu\nu}k_\nu}{2m}\kappa_N F_2(k^2)\right]u(p), \tag{1}$$

where e is the fundamental charge unit, δ_N is 1 or 0 for proton or neutron, respectively, and κ_N is the nucleon's anomalous magnetic moment; m is the physical nucleon mass (which here is related to the incoming and outgoing nucleon four-momenta by $p^2 = p'^2 = m^2$). The (scalar) Dirac and Pauli form factors, F_1 and F_2, respectively, are functions of the squared photon four-momentum $k = p' - p$, normalized here such that $F_1(0) = F_2(0) = 1$. The expression appearing within the square brackets, with two independent coefficient functions, F_1 and F_2, is the most general expression for the current J^μ for on-shell nucleons. A large number of works exist that investigate the possible physical mechanisms that lead to the observed functional behavior of the on-shell form factors $F_1(k^2)$ and $F_2(k^2)$ either in terms of mesonic or quark degrees of freedom, or as hybrid approaches that link both particle regimes (see, e.g., Refs. [3–8] and references therein).

The current in the form (1) appears only in physical processes involving virtual photons, such as electron scattering off the nucleon. While it is well known [9] that any physical mechanism involving off-shell nucleons, in general, requires an expansion of the current operator in terms of six independent form factors, the simple expression inside the square brackets of Eq. (1) nevertheless remains the parametrization of choice for the nucleon current operator J^μ in many if not most descriptions of photoprocesses within effective Lagrangian approaches, irrespective of whether the photon is real or the

*helmut.haberzettl@gwu.edu

†huang@physast.uga.edu

‡nakayama@uga.edu

incoming and outgoing nucleons are on-shell.[1] *A priori*, of course, it is not clear how much of the dynamics of the full electromagnetic coupling to the nucleon is ignored by such a simplified approach.

Based on the Lorentz structure of the spin-$1/2$ case alone, the generic structure of the electromagnetic nucleon current requires 12 independent form factors [9]. Using gauge invariance reduces this to eight, and time-reversal invariance further reduces this to six independent functions, as alluded to above. In general, the (scalar) off-shell form factors are functions of the squared incoming and outgoing nucleon four-momenta, p^2 and p'^2, respectively, and of the squared photon four-momentum k^2. However, only the two on-shell form factors $F_1(k^2)$ and $F_2(k^2)$ that appear in Eq. (1) are accessible experimentally. This mismatch between what is required for a complete Lorentz-covariant description of the current and what can be checked experimentally presents a formidable challenge for theoretical formulations of the dynamical features of the nucleon current. This problem is bypassed completely in non-Lorentz-covariant formulations such as chiral perturbation theory [2] that only require on-shell currents of the form (1) as a matter of course. However, in the resonance region, farther away from threshold, where chiral perturbation theory faces increasing difficulties in describing the mesonic and baryonic degrees of freedom of reaction processes, one must resort to effective Lagrangian approaches that require the description of the off-shell properties of the nucleon current. Most of the corresponding investigations are restricted to the off-shell p'^2 and p^2 behavior of the form factors F_1 and F_2 of Eq. (1) [10–12]. The full off-shell behavior in terms of all six form factors of the nucleon current, albeit in an *ad hoc* phenomenological manner, is considered by Surya and Gross [13] in their model description of pion photoproduction. A useful, but generic description of the constraints on the structure of hadron currents in general and of the nucleon current in particular is given in Ref. [14]; however, no practical scheme is offered for the calculation of the full nucleon current.

It is the purpose of the present work to provide a more detailed description of the electromagnetic nucleon current J^μ. We will start from a comprehensive field theory [15] that utilizes baryon and meson degrees of freedom to describe pion-nucleon scattering and that also provides—via its description of the dressed nucleon propagator—an avenue to the detailed dynamics of the nucleon's electromagnetic interaction. The electromagnetic currents of this approach are derived employing the gauge-derivative method of Haberzettl [15] that provides a general tool for coupling the photon to hadronic systems. (This procedure is also referred to as "gauging of equations" by others [16].) The full formalism is a very complex and nonlinear Dyson-Schwinger-type approach and, as such, therefore, not easily implemented in practical applications. We will show here how this formalism can be reformulated equivalently in a manner that makes it directly amenable to physically motivated approximation schemes, thus rendering the approach practically manageable. Of decisive importance in this respect will be the fact that the internal dressing effects of the nucleon current and the dynamics of pion photoproduction are very closely related.

The paper is organized as follows. In Sec. II, concentrating on contributions due to pions, nucleons, and photons only, we introduce some basic facts needed for the description of the dressed nucleon current J^μ. In doing so, we follow the corresponding field-theory formulation of Haberzettl [15]. In particular, we discuss the structure of the unique minimal Ball-Chiu current [17] that provides the current's Ward-Takahashi identity [18,19]. It is argued that the internal structure of J^μ is very closely related to pion photoproduction and we therefore revisit this problem in Sec. III, where we extend the approach of Haberzettl, Nakayama, and Krewald [20] to make the truncated formalism gauge invariant in a manner that is microscopically consistent with the dressing mechanisms of the nucleon current, which are provided in the subsequent Sec. IV. Up to this stage, the derivations of both dressed nucleon current and pion photoproduction current provide exact expressions. In Sec. V, we then discuss possible approximations to render the complex nonlinearity of the resulting equations manageable in practice. Finally, Sec. VI provides a summarizing assessment, including a discussion of possible applications.

II. NUCLEON CURRENT: BASIC CONSIDERATIONS

The generic structure of the electromagnetic current J^μ of the nucleon can be determined in a formulation that involves only pions, nucleons, and photons. Any additional hadronic degrees of freedom will only complicate the situation, but will not add any qualitatively new structure to J^μ; in other words, they will not add anything of substance to the discussion.[2] Using these degrees of freedom, the field-theory approach of Haberzettl [15] provides an expression for the current based on a Lorentz-covariant effective Lagrangian formalism. Rather than recapitulating all of the details of Ref. [15], we summarize the result given there in several diagrams.

To define the dressed nucleon current J^μ, we need the dressed nucleon propagator S which is obtained from the T matrix for πN scattering. Figure 1 shows the structure of this T matrix; Fig. 1(a), in particular, depicts the splitting of the

[1]One should mention in this context that if one uses this simplified expression with two degrees of freedom for virtual photons, at the very least one should replace the Dirac part according to

$$\gamma^\mu F_1(k^2) \to \gamma^\mu + (\gamma^\mu k^2 - k^\mu \not{k})\frac{F_1(k^2)-1}{k^2},$$

because, on general grounds, the k^2 dependence for virtual photons can only occur in manifestly transverse contributions. On-shell, this reduces to the Dirac term of Eq. (1), of course. In this respect, see also the general discussion on the structure of the nucleon current in Ref. [14].

[2]Of course, in an actual application of the present formalism, the *internal* "pion" and "nucleon" appearing here must be expanded to incorporate all relevant meson and baryon degrees of freedom, respectively.

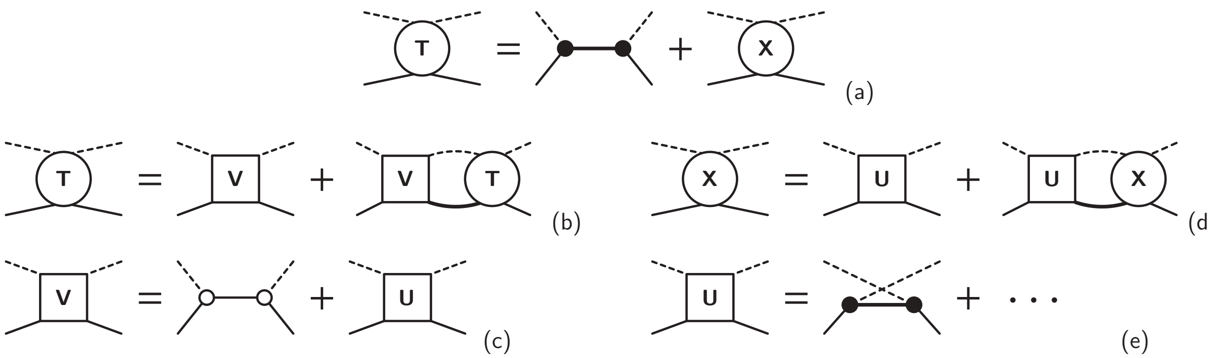

FIG. 1. Generic structure of the pion-nucleon T matrix employing pions and nucleons as the only hadronic degrees of freedom [15]. (a) Splitting of T into s-channel pole part and nonpole X. (b) Bethe-Salpeter integral equation for T, with (c) the driving term V according to Eq. (6). (d) Bethe-Salpeter integral equation for nonpole X, with (e) nonpole driving term U. Dressed vertices are solid circles; undressed ones open circles. Dressed (internal) nucleons are shown as thick lines; undressed ones as thin lines; pions as dashed lines. Note that the s-channel pole term in the driving term V is bare [because it gets dressed by the equation (b) itself], whereas in the full theory, all mechanisms in the nonpole U are fully dressed via Dyson-Schwinger-type mechanisms, as depicted in Fig. 2.

full amplitude T into its s-channel pole part and its nonpole part X, i.e., [15]

$$T = |F\rangle\, S\, \langle F| + X, \tag{2}$$

relevant for some of the present considerations.[3] The first term here contains the nucleon propagator S that provides the s-channel pole. The F are the fully dressed πNN vertices related to the bare vertex f by

$$|F\rangle = |f\rangle + XG_0\, |f\rangle\,, \tag{3}$$

which is part of the nonlinear Dyson-Schwinger-type equations shown in Fig. 2. Both T and X are obtained as solutions of Bethe-Salpeter-type integral equations according to

$$T = V + VG_0T, \tag{4}$$

and

$$X = U + UG_0X, \tag{5}$$

as depicted in Figs. 1(b) and 1(d), respectively. The respective driving terms V and U differ by the bare s-channel diagram, as shown in Figs. 1(c) and 1(e), i.e.,

$$V = |f\rangle\, S_0\, \langle f| + U, \tag{6}$$

where S_0 stands for the bare nucleon propagator. G_0 in Eqs. (3)–(5) describes the intermediate propagation of free pion and nucleon states that share the same total four-momentum of the process.

[3] We follow here the notation of Ref. [15], i.e., we do not use the usual notation of T^{P} and T^{NP} for the pole and nonpole contributions of T, respectively, because the corresponding indices tend to clutter up the equations. For the same reason, we use U instead of V^{NP} for the driving term of the nonpole Bethe-Salpeter equation (5). Furthermore, as in Ref. [15], the bra and ket notation is used here simply as a quick way to see whether a vertex F describes $N \to \pi N$, which would be $|F\rangle$, or $\pi N \to N$, which is written as $\langle F|$. This avoids the excessive use of adjoint daggers (†) and makes the equations easier to read. The bras and kets are not to be misconstrued as Hilbert-space vectors.

The fully dressed electromagnetic nucleon current J^μ derived in Ref. [15] is shown in Fig. 3. Formally, it results from applying the gauge-derivative procedure [15] to the dressed nucleon propagator S; however, it can be understood very simply as attaching a photon line to the propagator diagrams in Fig. 2(a) in all possible ways. To further understand the details of this structure, we mention that one of the simplest physical manifestations of the nucleon current occurs in the pion photoproduction process off the nucleon (shown in Fig. 4), because here the nucleon current provides one of the factors of the s-channel pole term (the other being the hadronic πNN production vertex). It should not be surprising, therefore, that much of the detailed internal structure of the current can be understood by the same mechanisms that contribute to the pion photoproduction amplitude M^μ. Substituting Fig. 4(b) for parts of Fig. 3(a), Fig. 5 shows that all internal dynamics of the nucleon current J^μ depicted in Fig. 3(a) may be represented equivalently in terms of loops over one-nucleon irreducible contributions to the pion photoproduction, with the exception of one loop involving the Kroll-Ruderman current [15]. This close relationship of the dressing mechanisms of the nucleon current forms the basis of the results presented below.

A. Gauge invariance of the nucleon current

The dressed current J^μ must satisfy gauge invariance; hence, it must obey the Ward-Takahashi identity

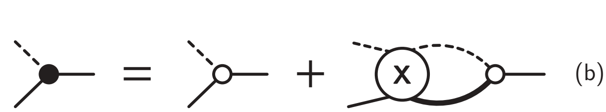

FIG. 2. Dressing mechanisms for (a) the nucleon propagator S and (b) the πNN vertex F according to Eq. (3) that appears in the nucleon's self-energy contribution Σ shown in (a) as a loop. The notation is the same as in Fig. 1.

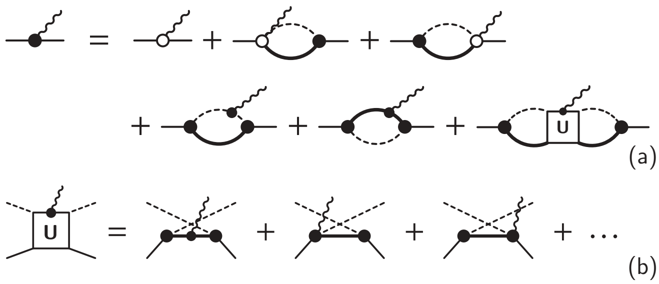

FIG. 3. (a) Structure of the full electromagnetic nucleon current J^μ employing nucleons and pions as the only hadronic degrees of freedom [15]. The open circle of the first term on the right-hand side is the bare current J_0^μ and open-circle four-point functions in the next two diagrams depict the Kroll-Ruderman contact current. The last diagram subsumes the intermediate contributions of the interaction current U^μ arising from the photon interacting with the internal mechanisms of the one-nucleon irreducible (i.e., nonpolar) πN interaction U. (b) The interaction current U^μ; explicitly shown are only the lowest-order contributions that follow from the photon interacting with the u-channel Born term of πN scattering [cf. Fig. 1(e)]. The $\gamma\pi NN$ four-point vertices of the last two diagrams subsume the interaction of the photon with the interior of the fully dressed πNN vertex (cf. last diagram in Fig. 6).

(WTI) [18,19],

$$k_\mu J^\mu(p', p) = S^{-1}(p')Q_N - Q_N S^{-1}(p), \tag{7}$$

where p and p' are the incoming and outgoing nucleon four-momenta, respectively, and $k = p' - p$ is the (incoming) photon momentum; Q_N is the nucleon's charge operator. This *off-shell constraint* ensures a conserved current for nucleons that are on-shell, i.e., when $p'^2 = p^2 = m^2$.

Without lack of generality, we may write the nucleon current as

$$J^\mu(p', p) = J_s^\mu(p', p) + T^\mu(p', p), \tag{8}$$

where J_s^μ is the *minimal* current that satisfies the WTI (7). Minimal is used here in the sense that one cannot find a current with simpler analytic structures that still satisfies the nucleon WTI [17]. (However, there is a freedom regarding the symmetry properties of this minimal current; see footnote 4.)

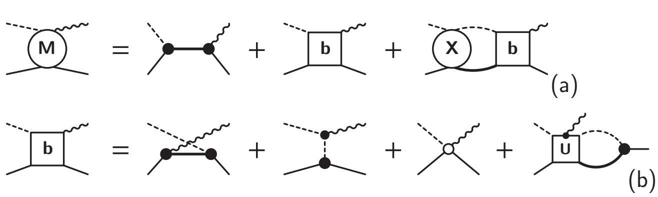

FIG. 4. (a) Pion photoproduction current M^μ [15]. The diagrams show the splitting of the production current M^μ into the s-channel pole term and the remaining one-nucleon irreducible contributions, with the final-state interaction mediated by the nonpole part X of the pion-nucleon T matrix. (b) Structure of the Born-type contribution b^μ, as given in Eq. (23). The four diagrams on the right-hand side depict, in the order given, the u- and t-channel contributions, the Kroll-Ruderman contact term, and the loop involving the πN interaction current U^μ of Fig. 3(b).

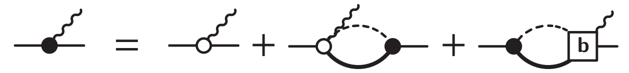

FIG. 5. Alternative depiction of the nucleon current J^μ of Fig. 3(a) employing the Born-type pion-production contribution b^μ from Fig. 4(b).

In other words, the four-divergence of J_s^μ is given by

$$k_\mu J_s^\mu(p', p) = S^{-1}(p')Q_N - Q_N S^{-1}(p), \tag{9}$$

and T^μ thus is the transverse remainder, with

$$k_\mu T^\mu(p', p) = 0. \tag{10}$$

By construction, this transversality must be manifest globally and it is not subject to any particular kinematic or dynamic restrictions.

B. Minimal nucleon current

Let us write the dressed propagator for a nucleon with four-momentum p in a generic manner as

$$S(p) = \frac{1}{\not{p}\, A(p^2) - m\, B(p^2)}, \tag{11}$$

where $A(p^2)$ and $B(p^2)$ are the two independent scalar dressing functions constrained by the residue conditions

$$A(m^2) = B(m^2) \tag{12a}$$

and

$$A(m^2) + 2m^2 \frac{d[A(p^2) - B(p^2)]}{dp^2}\bigg|_{p^2=m^2} = 1. \tag{12b}$$

From the residue condition alone, one cannot in general conclude that $A(m^2) = B(m^2) = 1$; in the structureless case, however, we have $A \equiv B \equiv 1$. (Note, however, that even though there are no explicit p^2-dependent dressing functions in the latter case, implicit dressing effects are present nevertheless owing to the fact that the mass m is the physical mass.)

Following Ball and Chiu [17], the minimal nucleon current that satisfies the WTI (9) is given by

$$J_s^\mu(p', p) = (p' + p)^\mu \frac{S^{-1}(p')Q_N - Q_N S^{-1}(p)}{p'^2 - p^2} + \left[\gamma^\mu - \frac{(p'+p)^\mu}{p'^2 - p^2}\not{k}\right] Q_N \frac{A(p'^2) + A(p^2)}{2}. \tag{13}$$

The first term here on the right-hand side is sufficient to produce the WTI, but the second part (which is transverse) is necessary to fully cancel the $1/(p'^2 - p^2)$ singularity, as can be seen explicitly by recasting J_s^μ in the equivalent form

$$J_s^\mu(p', p) = \gamma^\mu Q_N \frac{A(p'^2) + A(p^2)}{2} + (p' + p)^\mu Q_N \times \left[\frac{\not{p}' + \not{p}}{2} \frac{A(p'^2) - A(p^2)}{p'^2 - p^2} - m \frac{B(p'^2) - B(p^2)}{p'^2 - p^2}\right]. \tag{14}$$

In fact, J_s^μ is the *unique* current that satisfies the WTI and also is nonsingular and symmetric[4] in p' and p. Moreover, as can be seen from (14), for structureless nucleons, this reduces to the usual γ^μ Dirac current. And, invoking the generalized Gordon identity

$$(p'+p)^\mu = -i\sigma^{\mu\nu}k_\nu + \not{p}'\gamma^\mu + \gamma^\mu \not{p}, \tag{15}$$

the on-shell matrix element of J_s^μ is easily found as

$$\bar{u}J_s^\mu u = \bar{u}(p')e\delta_N\left\{\gamma^\mu + i\frac{\sigma^{\mu\nu}k_\nu}{2m}[A(m^2)-1]\right\}u(p). \tag{16}$$

Note that there is no k^2 dependence here, i.e., this result does not depend on whether the photon is real or virtual. This is consistent with the fact that minimal currents that satisfy the WTI as a rule cannot depend on the photon four-momentum, since such a dependence *always* sits in transverse contributions [14]. We point out in this context that the $\sigma^{\mu\nu}k_\nu$ contribution here must not be confused with the usual Pauli current, i.e., its coefficient is *not* directly related to the anomalous magnetic moment of the nucleon (which should be obvious because the entire current J_s^μ vanishes for the neutron).

1. *Minimal current taken half on-shell*

Of particular interest for many applications is to consider the half-on-shell reduction of the current. We shall do so here for an incoming on-shell nucleon interacting with a photon followed by the subsequent propagation of an off-shell nucleon, but the following considerations can be readily translated into describing the reversed situation where the outgoing nucleon is on-shell. Thus, half-on-shell, with an incoming nucleon spinor $u(p)$ on the right and an outgoing propagator $S(p+k)$ on the left, using Eq. (15), this results in

$$\begin{aligned} SJ_s^\mu u &\equiv S(p+k)J_s^\mu(p+k,p)u(p) \\ &= \left(\frac{1}{\not{p}+\not{k}-m}j_1^\mu + \frac{2m}{s-m^2}j_2^\mu\right)Q_N\,u(p), \end{aligned} \tag{17}$$

where $s=(p+k)^2$ and $p^2=m^2$. The (dimensionless) auxiliary currents are given by

$$j_1^\mu = \gamma^\mu(1-\kappa_1) + \frac{i\sigma^{\mu\nu}k_\nu}{2m}\kappa_1 \tag{18a}$$

and

$$j_2^\mu = \frac{(2p+k)^\mu}{2m}\kappa_1 + \frac{i\sigma^{\mu\nu}k_\nu}{2m}\kappa_2, \tag{18b}$$

with (dimensionless) *independent* coefficient functions

$$\kappa_1 = m^2\frac{[B(s)-A(s)][A(s)+A(m^2)]}{sA^2(s)-m^2B^2(s)} \tag{19a}$$

and

$$\kappa_2 = \frac{A(m^2)-A(s)}{2A(s)} + \frac{A(s)+B(s)}{2A(s)}\kappa_1. \tag{19b}$$

The on-shell values at $s=m^2$ of both coefficients are identical, i.e.,

$$\kappa_1(m^2) = \kappa_2(m^2) = A(m^2)-1. \tag{20}$$

This means they both vanish in the structureless limit, thus leaving in Eq. (17) only the usual γ^μ Dirac current together with a structureless propagator. All effects of the dressing thus reside in the terms that depend on the κ_i $(i=1,2)$ whose overall contributions are easily seen to be transverse.

We emphasize that Eq. (17) is exact and that it includes *all* possible dressing mechanisms. Its four-divergence, in particular, is given by

$$k_\mu\, S(p+k)J_s^\mu(p+k,p)\,u(p) = Q_N\,u(p), \tag{21}$$

and the resulting expression does not involve any dressing effects whatsoever. Any approximation, therefore, that only involves the coefficient functions κ_i will have no bearing on the gauge-invariance contribution of any term containing Eq. (17). This is of direct and immediate relevance to the treatment of pion photoproduction presented in the following.

III. PION PHOTOPRODUCTION REVISITED

To see how the gauge-invariant minimal current contribution J_s^μ of Eq. (13) can be utilized for a practically useful description of the full nucleon current J^μ, we need to revisit the photoproduction of the pion because, as alluded to above, the internal dynamics of the current is closely related to mechanisms found in this production process, as seen in Fig. 5.

Following Ref. [15], we start by writing the production current as

$$M^\mu = F_sSJ^\mu + b^\mu + XG_0b^\mu, \tag{22}$$

which is a self-evident transcription of Fig. 4(a). The first term on the right-hand side provides the s-channel current $M_s^\mu = F_sSJ^\mu$ which contains the nucleon pole; F_s describes the πNN vertex with s-channel kinematics.[5] In the last term, X provides the πN final-state interaction (FSI) of the production current. The current b^μ subsuming the Born-type mechanisms is given by

$$b^\mu = M_u^\mu + M_t^\mu + m_{\rm KR}^\mu + U^\mu G_0\,|F\rangle\,. \tag{23}$$

[4]It is not necessary to make J_s^μ symmetric; to render it nonsingular, it suffices to write the last factor in Eq. (13) as

$$\frac{A(p'^2)+A(p^2)}{2} \to \eta A(p'^2) + (1-\eta)A(p^2)$$

instead, where η is an arbitrary parameter. However, since there is nothing that distinguishes the dynamics of the incoming nucleon from that of the outgoing nucleon, it seems natural to choose $\eta = 1/2$ which then indeed does make the nonsingular form (13) unique.

[5]Within the context of Eq. (22), using the notation discussed in footnote 3, we could write the s-channel vertex F_s equivalently as $|F\rangle$, i.e., the same way as in Eq. (2). We choose not to do that here to maintain consistency with the notation of the generalized WTI in Eq. (25) where the explicit Mandelstam indices $x=s,u,t$ provide a better description of the kinematic context of the respective πNN vertices F_x.

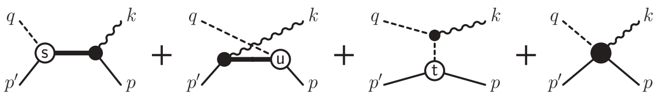

FIG. 6. Generic pion photoproduction diagrams $\gamma + N \to \pi + N$, with the s-, u-, and t-channel pole diagrams M_s^μ, M_u^μ, and M_t^μ, respectively, and the contact-type interaction current $M_{\rm int}^\mu$ that subsumes all final-state interaction [cf. Eq. (24)]. The labels (s, u, t) at the πNN vertices allude to the usual Mandelstam variables s, u, and t that describe the respective kinematic situations. The four-momenta here are the ones used in Eq. (51).

This four-point current (which is also an essential ingredient of the nucleon current, as shown in the last diagram of Fig. 5) describes the four terms appearing on the right-hand side of Fig. 4(b), which are, respectively, the u- and t-channel currents, the Kroll-Ruderman current [21], and the loop integration involving the πN interaction current U^μ of Fig. 3(b). Generically, the overall structure of M^μ is presented in Fig. 6 [22], with the first three diagrams containing the respective s-, u-, and t-channel pole contributions, and everything else—including the FSI contributions—being subsumed in

$$M_{\rm int}^\mu = m_{\rm KR}^\mu + U^\mu G_0 |F\rangle + XG_0 b^\mu, \tag{24}$$

which is the contact-type nonpolar four-point interaction current depicted in the last diagram of Fig. 6.

A. Gauge invariance of the photoproduction current

The full production current M^μ must obey gauge invariance formulated in terms of the generalized Ward-Takahashi identity [15,23],

$$\begin{aligned} k_\mu M^\mu = &-F_s S(p+k) Q_i S^{-1}(p) \\ &+ S^{-1}(p') Q_f S(p'-k) F_u \\ &+ \Delta_\pi^{-1}(q) Q_\pi \Delta_\pi(q-k) F_t, \end{aligned} \tag{25}$$

where the four-momenta are those shown in Fig. 6. The vertices F_x here correspond to the (fully dressed) πNN vertices F in the respective kinematic situations corresponding to the Mandelstam variables $x = s, u, t$, as shown in Fig. 6. The propagators for the nucleon and pion are denoted by S and Δ_π, respectively, and the charge operators for the initial and final nucleon and for the outgoing pion are Q_i, Q_f, and Q_π, respectively. Obviously, this expression vanishes for on-shell hadrons and thus provides a conserved current. This *off-shell* formulation of gauge invariance, however, goes beyond that by providing a *local* constraint on the gauge invariance of the photoproduction current that is similar to requiring the usual Ward-Takahashi identity for the single-particle currents [18], which for the nucleon is given in Eq. (7). Both requirements (25) and (7) (and its analog for the pion) are essential for the internal consistency of microscopic formulations of photoprocesses.

For the practical implementation of gauge invariance to be discussed below, however, it is easier to use the equivalent constraint for the four-divergence of the interaction current $M_{\rm int}^\mu$, *viz.* [15,20],

$$k_\mu M_{\rm int}^\mu = -\tilde{F}_s e_i + \tilde{F}_u e_f + \tilde{F}_t e_\pi, \tag{26}$$

which follows immediately from the generic structure shown in Fig. 6 assuming the validity of the generalized Ward-Takahashi identity (25) for the current M^μ. Here, the $\tilde{F}_x$ are the vertices F_x of Eq. (25) stripped of their isospin operators τ that now appear in $e_i = \tau Q_i$, $e_f = Q_f \tau$, and $e_\pi = Q_\pi \tau$, which are the charges for all external hadron legs in an appropriate isospin basis (with all corresponding indices and summations suppressed). The relation $e_i = e_f + e_\pi$, therefore, describes charge conservation for the pion photoproduction process. We emphasize that the gauge-invariance condition (26) is an *off-shell* constraint that must always be true, i.e., it is not restricted to special kinematic or dynamic situations. With the single-particle WTIs given, Eqs. (25) and (26) provide completely equivalent formulations of gauge invariance for the pion production process.

B. Reformulating the FSI contribution

To utilize Eq. (26) in the following, we first need to rewrite the current M^μ of Eq. (22) to isolate the interaction current $M_{\rm int}^\mu$ in a practically useful manner. To this end, we split the Born current b^μ into its longitudinal and transverse parts,

$$b^\mu = b_{\rm L}^\mu + b_{\rm T}^\mu, \tag{27}$$

as indicated by the respective indices L and T, and write the FSI term of Eq. (22) equivalently as

$$\begin{aligned} XG_0 b^\mu = &\, XG_0 b_{\rm L}^\mu + XG_0\big[\big(M_u^\mu + M_t^\mu\big)_{\rm T} + T_X^\mu\big] \\ &+ XG_0\big[\big(m_{\rm KR}^\mu + U^\mu G_0|F\rangle\big)_{\rm T} - T_X^\mu\big]. \end{aligned} \tag{28}$$

We have introduced here an as yet undetermined *transverse* current T_X^μ that cancels out in the last two terms; its choice, therefore, is of no consequence for the full formalism. Inserting this into Eq. (22), we may write

$$\begin{aligned} M^\mu = &\, F_s S J^\mu + M_u^\mu + M_t^\mu + M_c^\mu \\ &+ XG_0\big[\big(M_u^\mu + M_t^\mu\big)_{\rm T} + T_X^\mu\big], \end{aligned} \tag{29}$$

where

$$\begin{aligned} M_c^\mu = &\, (1 + XG_0)\big(m_{\rm KR}^\mu + U^\mu G_0|F\rangle\big) \\ &+ XG_0\big(M_u^\mu + M_t^\mu\big)_{\rm L} - XG_0 T_X^\mu. \end{aligned} \tag{30}$$

We shall now exploit the freedom of choosing the undetermined transverse current and put

$$T_X^\mu = \big(M_c^\mu\big)_{\rm T}, \tag{31}$$

which results in

$$M^\mu = F_s S J^\mu + B^\mu + XG_0 B_{\rm T}^\mu \tag{32}$$

for the photoproduction amplitude, where

$$B^\mu = M_u^\mu + M_t^\mu + M_c^\mu. \tag{33}$$

These equations are depicted in Fig. 7. Owing to the choice (31), Eq. (32) is very close in structure to Eq. (22) with, however, an explicit FSI loop contribution that contains only the transverse restriction $B_{\rm T}^\mu$ of B^μ. This particular loop

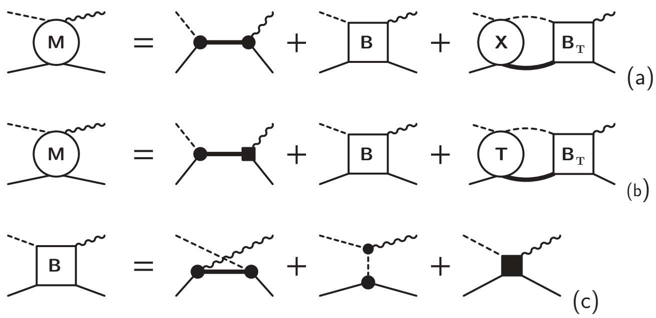

FIG. 7. Representations of the photoproduction current M^μ (equivalent to those of Fig. 4), with (a) loop integration over the nonpole amplitude X [see Eq. (32)] and (b) loop integration over the full πN T matrix [see Eq. (47)]. The details of the four-point box B are shown in (c), with the last diagram subsuming the contact-type current mechanisms M_c^μ shown in Fig. 8. $\mathrm{B_T}$ denotes the restriction of B to transverse contributions. The different nucleon-current contributions appearing in the s-channel terms of (a) and (b) are given in Fig. 9.

integration, therefore, does not contribute when evaluating the four-divergence of M^μ using Eq. (32).

With the choice (31), we may then recast Eq. (30) in the implicit form

$$M_c^\mu = m_{\rm KR}^\mu + U^\mu G_0\,|F\rangle + UG_0\big(M_u^\mu + M_t^\mu + M_c^\mu\big)_{\rm L}, \quad (34)$$

which is shown in Fig. 8. The longitudinal part of this equation constitutes an integral equation for $(M_c^\mu)_{\rm L}$, and for the transverse part we have

$$\big(M_c^\mu\big)_{\rm T} = \big(m_{\rm KR}^\mu + U^\mu G_0\,|F\rangle\big)_{\rm T}, \quad (35)$$

which is given by the transverse projections of the first two diagrams on the right-hand side of Fig. 8.

The interaction current of Eq. (24) can now be written as

$$M_{\rm int}^\mu = M_c^\mu + XG_0\big(M_u^\mu + M_t^\mu + M_c^\mu\big)_{\rm T}, \quad (36)$$

where the explicit loop-integration over the nonpolar FSI amplitude X is transverse. Its four-divergence vanishes and thus we have

$$k_\mu M_{\rm int}^\mu = k_\mu M_c^\mu. \quad (37)$$

It is this equality, in particular, that is of central importance for practical purposes since it will allow us to exploit the gauge-invariance condition (26) fully in terms of the properties of the contact-type current M_c^μ. Any approximation of this

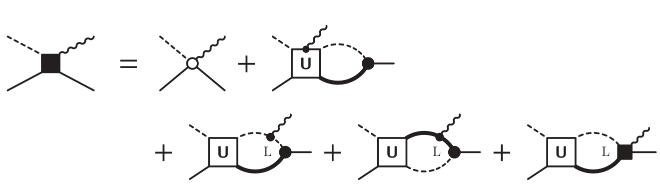

FIG. 8. Diagrammatic representation of Eq. (34), with the solid square four-point vertex depicting the contact-type current M_c^μ. The indices L in the loops of the last three diagrams signify that only the longitudinal parts of the respective photon couplings are to be taken into account. These three diagrams, therefore, do not contribute for real photons.

current, therefore, can be understood as an approximation of the mechanisms subsumed in Fig. 8.

Of particular importance in this respect is the fact that for real photons, only the transverse parts of the currents contribute to physical observables. Therefore, as an immediate consequence of the choice (31), effectively any approximation of M_c^μ is a direct approximation of $U^\mu G_0\,|F\rangle$ because of Eq. (35). Note also that the structural closeness between Eqs. (22) and (32) becomes even closer for real photons since the differences between b^μ, B^μ, and $B_{\rm T}^\mu$ are irrelevant in this case. This is easily seen explicitly by equating the right-hand sides of Eqs. (22) and (32), which produces

$$\begin{aligned} b^\mu &= B^\mu - UG_0B_{\rm L}^\mu \\ &= B_{\rm T}^\mu + (1-UG_0)B_{\rm L}^\mu \end{aligned} \quad (38)$$

when using Eq. (5) and the splitting $B^\mu = B_{\rm T}^\mu + B_{\rm L}^\mu$, i.e., for transverse real photons effectively both b^μ in Eq. (22) and B^μ in Eq. (32) are represented by $B_{\rm T}^\mu$.

IV. DRESSING THE NUCLEON CURRENT

Let us now turn back to the question of how to describe the dressing of the nucleon current. According to Fig. 5, the dressed current may be written as [15]

$$J^\mu = J_{\rm b}^\mu + \langle F|\,G_0 b^\mu, \quad (39)$$

where b^μ subsumes the mechanism given in Eq. (23), and the modified bare current $J_{\rm b}^\mu$ corresponds to the first two diagrams on the right-hand side of Fig. 3,

$$J_{\rm b}^\mu = J_0^\mu + \big\langle m_{\rm KR}^\mu\big|G_0|F\big\rangle\,. \quad (40)$$

Here, J_0^μ is the (true) bare current and the second term is the loop containing the Kroll-Ruderman current $m_{\rm KR}^\mu$ with, however, the pion coming *into* the contact vertex instead of going out.

To rewrite Eq. (39) employing the results of Sec. III B, we replace b^μ using Eq. (38) to find

$$J^\mu = J_{\rm b}^\mu + \langle f|\,G_0B_{\rm L}^\mu + \langle F|\,G_0B_{\rm T}^\mu, \quad (41)$$

where the relationship (3) between dressed and undressed vertices, F and f, respectively, was used. We emphasize in this respect that the clear separation found here of bare vertex f and longitudinal current $B_{\rm L}^\mu$ on the one hand, and dressed vertex F and transverse current $B_{\rm T}^\mu$ on the other hand is a direct consequence of the choice (31) for T_X^μ, i.e., the choice (31) is unique in this regard. Let us write the nucleon current as

$$J^\mu = \tilde{J}_s^\mu + \langle F|\,G_0\big(M_u^\mu + M_t^\mu + M_c^\mu\big)_{\rm T}, \quad (42)$$

where

$$\tilde{J}_s^\mu = J_{\rm b}^\mu + \langle f|\,G_0\big(M_u^\mu + M_t^\mu + M_c^\mu\big)_{\rm L}. \quad (43)$$

For the sake of clarity, we have expanded here $B_{\rm T}^\mu$ and $B_{\rm L}^\mu$ using

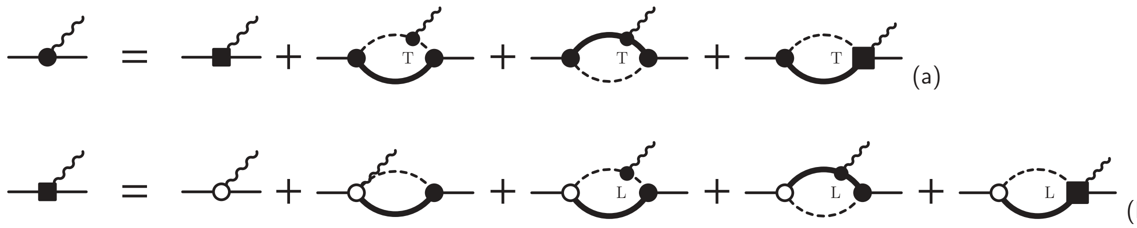

FIG. 9. (a) Dressed nucleon current J^μ according to Eq. (42). The first term on the right-hand side (with a solid square vertex) depicts $\tilde{J}_s^\mu$ which subsumes the mechanisms shown in part (b). Diagrams labeled T or L correspond to the transverse and longitudinal contributions given in Eqs. (42) and (43), respectively. The solid-square four-point vertices in the last diagrams of each line depict M_c^μ given in Fig. 8. The phenomenological approximations discussed in Sec. V affect the (square-shaped) three- and four-point vertices of $\tilde{J}_s^\mu$ and M_c^μ, respectively.

the explicit form (33). Equations (42) and (43) are depicted in Fig. 9.

It is obvious, of course, that since J^μ and $\tilde{J}_s^\mu$ differ only by transverse pieces, their four-divergences coincide. In other words, they both satisfy the full nucleon WTI (7). We may, therefore, without lack of generality employ the minimal Ball-Chiu current J_s^μ of Eq. (13) and write

$$\tilde{J}_s^\mu = J_s^\mu + \tilde{T}_s^\mu, \tag{44}$$

where $\tilde{T}_s^\mu$ is the *transverse* remainder defined by this relation, i.e.,

$$\tilde{T}_s^\mu = \left(\tilde{J}_s^\mu - J_s^\mu\right)_{\mathrm{T}} = \left(J_{\mathrm{b}}^\mu - J_s^\mu\right)_{\mathrm{T}}. \tag{45}$$

The (exact) nucleon current then reads

$$J^\mu = J_s^\mu + \tilde{T}_s^\mu + \langle F|\, G_0\left(M_u^\mu + M_t^\mu + M_c^\mu\right)_{\mathrm{T}}, \tag{46}$$

where the details of any given dressing scheme implied by the underlying hadronic Lagrangians determine the elements on the right-hand side. Dynamically the most complex contributions here, and therefore the most challenging ones in numerical applications, are those that arise from the contact-type current M_c^μ contained implicitly in $\tilde{T}_s^\mu$ and explicitly in the last term of Eq. (46). In the following section, we discuss approximations of $\tilde{T}_s^\mu$ and M_c^μ that will help render this result useful in practical applications.

V. APPLICATION TO PION PHOTOPRODUCTION

Inserting the (exact) nucleon current (42) into the s-channel term of the photoproduction current (32) and using the splitting (2) of T into its pole and nonpole contributions, we immediately find

$$\begin{aligned} M^\mu = {} & F_s S \tilde{J}_s^\mu + M_u^\mu + M_t^\mu + M_c^\mu \\ & + T G_0\left(M_u^\mu + M_t^\mu + M_c^\mu\right)_{\mathrm{T}}, \end{aligned} \tag{47}$$

which expresses the final-state interaction in terms of the full πN T matrix instead of just its nonpolar part X. This equation is depicted in Fig. 7(b).

By construction, this reformulation of the photoproduction current M^μ is completely equivalent formally to both Eqs. (22) and (32). From a practical point of view, however, formulating the final-state interaction in terms of the full T matrix, as in Eq. (47), possesses some advantages over doing so in terms of the nonpolar X. We note, in particular, that in numerical applications, one must truncate the tower of (nonlinear) Dyson-Schwinger-type equations summarized in Figs. 1–5 because their exact self-consistent solution would require enormous computational resources that, in general, are not available. As a consequence, the splitting of T into its pole part and the nonpolar X will depend on the adopted approximation scheme, thus making it non-unique. Its individual pieces, therefore, may exhibit undesirable numerical artifacts [24] that are absent from the full T matrix in Eq. (47) since T is much closer to the actual observables.

A. Approximating $\tilde{J}_s^\mu$

Another advantage of the formulation (47) results from the fact that its physical matrix elements only require the half-on-shell expression of $S\tilde{J}_s^\mu$. The very fact that $\tilde{J}_s^\mu$ already provides the full WTI (7), yet its only transverse contribution stems from the modified bare term J_{b}^μ, suggests that $\tilde{J}_s^\mu$ is very close to being minimal in the sense of the Ball-Chiu current J_s^μ, Eq. (13), whose properties were discussed in Sec. II B. Therefore, if one neglects the transverse remainder $\tilde{T}_s^\mu$ from Eq. (44), i.e.,

$$\tilde{T}_s^\mu \to \tilde{T}_s^\mu = 0, \tag{48}$$

we have $S\tilde{J}_s^\mu \to SJ_s^\mu$ for which one may then employ the result (17) for the corresponding half-on-shell elements. In actual calculations, one can then use the two coefficient functions κ_1 and κ_2 appearing in the auxiliary currents of Eqs. (18) as fit parameters, which is an excellent approximation of the dressing effects inherent in the product SJ_s^μ when taken half on-shell. This assertion is corroborated by the preliminary numerical results for pion photoproduction of Ref. [24].

B. Approximating M_c^μ

The approximation (48) does not change the gauge-invariance properties of the corresponding expressions for both the nucleon current itself and for the production current M^μ because changes of transverse contributions do not alter the WTI (7) for the nucleon current J^μ or the generalized WTI (25) for the production current M^μ. More general truncations of the full Dyson-Schwinger structure, however, will very likely result in the violation of gauge invariance of M^μ, and one then needs to introduce gauge-invariance preserving (GIP) procedures to restore it.

The basic mechanism for this restoration was first given by Drell and Lee [25] in a tree-level approximation of the entire interaction current $M^\mu_{\rm int}$ that satisfies the constraint (26). The Drell-Lee current was later rediscovered by Ohta [26] using analytic expansions of the vertex functions subjected to minimal substitution. The dynamically more sophisticated prescription put forward by Haberzettl, Nakayama, and Krewald [20] generalizes the basic procedure of Ref. [27] (which in turn generalizes the Drell-Lee mechanism [25]) to allow the inclusion of the full FSI contribution appearing in Eq. (22) in terms of X. This procedure based on the generalized Ward-Takahashi identity (25) for the production current [15,23] is not unique, of course, since the generalized WTI does not constrain transverse current contributions.

We will exploit this ambiguity here and provide an alternative gauge-invariant approximation of the amplitude M^μ that is based on Eq. (47). It is one of the big advantages of the present formulation that as long as the electromagnetic nucleon and pion currents satisfy their individual WTIs, any violation of the gauge invariance of M^μ resulting from truncations necessitated by practicality can always be expressed in terms of approximations of the contact-type current M^μ_c. The present procedure is distinguished from the formulation given in Ref. [20] by the choice (31) for T^μ_X which results in M^μ_c appearing in both the photoproduction current M^μ and the mechanisms of the nucleon current J^μ in Eq. (42). In other words, any approximation of M^μ_c will consistently affect the photoproduction and the nucleon currents in much the same way as they are consistently linked in the exact Dyson-Schwinger formalism.

As shown in the description leading up to Eq. (37), any approximation of M^μ_c chosen to satisfy the interaction-current condition (26) will preserve gauge invariance as a matter of course. Restricting the present discussion to real photons for simplicity, it follows from Eq. (35) that approximations of M^μ_c can be understood basically as approximations of the loop integral over the five-point contact-type mechanisms subsumed in U^μ, given by the second diagram on the right-hand side of Fig. 8. The various mechanisms entering the πN interaction current U^μ are discussed in some detail in Ref. [15]. Following the procedures described in Ref. [20], there are various levels of sophistication at which the constraint (26) can be implemented, depending on how much of the detailed dynamics of U^μ can be incorporated in a particular application. It is shown in Ref. [20] how any one of these interaction-current mechanisms can be approximated in a gauge-invariant manner by utilizing the structure of their underlying *independent* hadronic four-point interactions. For the case of the lowest-order contributions to U^μ depicted explicitly in Fig. 3(b), for example, this entails constructing a gauge-invariant approximation that uses the underlying u-channel exchange interaction shown in Fig. 1(e). In general, the procedures described in Ref. [20] permit one to find a GIP approximation for any $(n+1)$-point current arising from attaching the photon to an n-point hadronic mechanism. This is made possible by the consistent microscopic implementation of local gauge invariance in terms of generalized Ward-Takahashi identities at all levels of the underlying reaction dynamics.

For the present purpose, it is not necessary to duplicate the discussion of Ref. [20]. We, therefore, restrict the present application of the procedure to the simplest possible case in which the *entire* contact-type current M^μ_c is approximated without any regard for the details of its internal mechanisms. While more sophisticated approximations could easily be constructed following Ref. [20] (and might even be warranted for some applications), they would not add anything structurally new to the present discussion.

At the simplest possible level, the dressed πNN vertices are described by phenomenological form factors which we write as

$$\tilde{F}_x = G_\lambda \tilde{f}_x, \tag{49}$$

where the scalar function $\tilde{f}_x$ provides the phenomenological functional form of the vertex (normalized to unity when all hadron legs are on-shell) and G_λ its coupling structure. As in Eq. (26), the tilde indicates that the vertex has been stripped of its isospin dependence (i.e., we have, for example, $F_s Q_i = \tilde{F}_s e_i = G_\lambda \tilde{f}_s e_i$), and $x = s, u, t$ indicates the kinematic context in which the vertex appears. The coupling operator is written as

$$G_\lambda = g\,\gamma_5 \left(\lambda + \frac{1-\lambda}{2m}\,\not{q}\right), \tag{50}$$

where g is the coupling strength, q is the outgoing pion four-momentum, and λ dials between pseudovector ($\lambda = 0$) and pseudoscalar ($\lambda = 1$) coupling.[6] Following Refs. [20,27], we may then approximate all of M^μ_c by the phenomenological GIP current

$$\begin{aligned} M^\mu_c &\to M^\mu_c \\ &= -(1-\lambda) g \frac{\gamma_5\gamma^\mu}{2m} \tilde{f}_t e_\pi - G_\lambda \left[e_i \frac{(2p+k)^\mu}{s-p^2}(\tilde{f}_s - \hat{F}) \right. \\ &\quad + e_f \frac{(2p'-k)^\mu}{u-p'^2}(\tilde{f}_u - \hat{F}) \\ &\quad \left. + e_\pi \frac{(2q-k)^\mu}{t-q^2}(\tilde{f}_t - \hat{F}) \right] \\ &\quad + g e \gamma_5 \frac{i\sigma^{\mu\nu}k_\nu}{4m^2}\tilde{\kappa}_N, \end{aligned} \tag{51}$$

where the momenta shown in Fig. 6 are being used. The first term here provides a dressed version of the Kroll-Ruderman

[6]Note in this context that phenomenological form factors are intended to mock up the fully dressed vertex. Hence, even if one starts out with a fully chiral-symmetric pseudovector bare vertex, the dressed vertex, in general, would no longer be pure pseudovector. The ansatz (50) accounts for this fact in a phenomenological manner. Phenomenologically, of course, λ could also be chosen as an s-, u-, or t-dependent function, depending on the dynamical context of the vertex.

current[7] and the other three terms supply the gauge-invariance-preserving corrections for the s-, u-, and t-channel contributions. The subtraction function $\hat{F}$ must be chosen such that any one of the three terms in the square brackets remains finite if the corresponding denominators go to zero. For specific choices of how to achieve this, see Ref. [20]. The simplest possible such choice is $\hat{F} = 1$ which corresponds to the original Drell-Lee current [25,26]. In any case, this phenomenological expression is then nonsingular and, moreover, it clearly satisfies the gauge-invariance condition (26) because the $\hat{F}$-dependent terms do not contribute to the four-divergence because of charge conservation in the form $e_i - e_f - e_\pi = 0$.

The additional (transverse) last term in Eq. (51) is not needed to preserve the gauge invariance of the photoproduction current M^μ (and correspondingly it is absent from the considerations of Ref. [20]). It is needed here, however, to provide consistency with the nucleon current J^μ of Eq. (46) which also features M_c^μ as one of its dynamical ingredients. The coefficient $\tilde{\kappa}_N$ of this additional transverse $\sigma^{\mu\nu}k_\nu$ current needs to be fixed such that the on-shell matrix elements of the current (46) reproduce Eq. (1), in particular, when we use the approximation (48). The factors in this term ensure that $\tilde{\kappa}_N$ is dimensionless. In practice, as is the case for the present application to pion photoproduction, the actual on-shell matrix elements of the nucleon current usually never enter the calculations, and one may then use $\tilde{\kappa}_N$ as an additional fit parameter that accounts for the current being partially off-shell. Altogether then, for given phenomenological vertices $\tilde{F}_x$ and subtraction function $\hat{F}$, the dressing structure of the nucleon current is parametrized by three parameters, $\tilde{\kappa}_N$ in Eq. (51), and κ_1 and κ_2 contained in the half-on-shell result (17) via the auxiliary currents (18).

VI. DISCUSSION AND SUMMARY

Based on the field-theory approach of Haberzettl [15], we have presented here a formulation of the dressed electromagnetic current of the nucleon that is microscopically consistent with the reaction mechanisms inherent in meson photoproduction. The goal was to equivalently rewrite the original expressions of the full formalism in a manner that retains as much as possible of its original dynamical structure while at the same time presenting options for meaningful approximations which in practice are necessary to render the equations manageable. The consistency requirement, in particular, led to a novel approximation scheme for pion photoproduction, different from what was proposed in Ref. [20]. The resulting expressions are summarized diagrammatically in Fig. 7 for pion photoproduction and in Fig. 9 for the dressed nucleon current.

The full theory presented up to and including Eq. (47) in Sec. V is exact. The guiding principle for the construction of the corresponding equations was the consistent and complete implementation of local gauge invariance at all levels of the reaction mechanisms in a manner that lends itself to transparent approximation schemes. In doing so we followed the basic strategy of Ref. [20] with, however, one important and essential difference. Instead of choosing the optional transverse current T_X^μ as zero, as was done in Ref. [20], we now choose it so that the resulting expression (42) for the dressed nucleon current exhibits a clean separation of transverse and longitudinal contributions that makes it straightforward to implement a phenomenological description of the dressing effects which preserves gauge invariance through the use of the minimal Ball-Chiu current J_s^μ of Eq. (13).

The phenomenological use of J_s^μ for the nucleon current makes the description of pion photoproduction particularly simple when the FSI loop of the production current M^μ is written in terms of the full πN T matrix (instead of with its nonpole part X) because the s-channel term of the form (17) resulting from the approximation (48) then admits a very simple approximation by utilizing the effective dressing functions κ_1 and κ_2 as two fit parameters.

Another obvious advantage of the present scheme is that for real photons, in particular, the effective structure of the resulting photoproduction current remains very close to the full formalism even if the loops over the five-point-current contributions U^μ are approximated with the phenomenological contact current of Eq. (51), since for real photons the longitudinal contributions that make up the structural difference between the currents M^μ of Fig. 4 and of Fig. 7 are irrelevant, as discussed in the context of Eq. (38).

The approximations discussed here in detail concern replacing the current $\tilde{J}_s^\mu$ by the minimal Ball-Chiu current J_s^μ and the contact current M_c^μ of Eq. (34) by the phenomenological GIP expression (51). It should be clear, however, that this still leaves a formidable self-consistency problem because, as can be read off Fig. 9(a), the nucleon current J^μ also appears in one of the loops on the right-hand side. In practice, therefore, instead of solving this self-consistency problem iteratively, one might truncate it at the lowest level by employing the usual simplified on-shell expression (1) for the current in the loop.

The obvious first application of the present dressing formalism for the nucleon current is pion photoproduction, of course, since it was the consistency requirement with this process that inspired the formalism in the first place. As mentioned, this application is underway already [24], and the preliminary results obtained so far are very encouraging. In other words, the present approach is not just formally correct but the approximations suggested by its formal structure indeed lead to an excellent description of the data. Other possible applications include any process that may benefit from a detailed microscopic description of the nucleon current. Obvious candidates are other meson production processes with both real and virtual photons off the nucleon, Compton scattering off the nucleon, and NN bremsstrahlung. For virtual photons, in particular, the present formalism may also be helpful in extracting the functional behavior of electromagnetic form factors from the data.

ACKNOWLEDGMENT

This work is partly supported by the FFE Grant No. 41788390 (COSY-58).

[7]Note that $(1-\lambda)\tilde{f}_t = 1 + [(1-\lambda)\tilde{f}_t - 1]$, i.e., the original bare Kroll-Ruderman term $m_{\rm KR}^\mu$ survives in this GIP current and the phenomenological dressing comes in via the additional $(1-\lambda)\tilde{f}_t - 1$ contribution.

[1] E. Klempt and J. M. Richard, Rev. Mod. Phys. **82**, 1095 (2010).
[2] A. Pich, Rep. Prog. Phys. **58**, 565 (1995); G. Ecker, Prog. Part. Nucl. Phys. **35**, 1 (1995); S. Scherer, Adv. Nucl. Phys. **27**, 277 (2003); V. Bernard and U.-G. Meißner, Annu. Rev. Nucl. Part. Sci. **53**, 33 (2007).
[3] F. Iachello, A. D. Jackson, and A. Lande, Phys. Lett. B **43**, 191 (1973).
[4] M. Gari and W. Krümpelmann, Phys. Lett. B **274**, 159 (1992); **282**, 483(E) (1992).
[5] J. Friedrich and Th. Walcher, Eur. Phys. J. A **17**, 607 (2003).
[6] M. A. Belushkin, H. W. Hammer, and U.-G. Meißner, Phys. Rev. C **75**, 035202 (2007).
[7] C. Crawford *et al.*, Phys. Rev. C **82**, 045211 (2010).
[8] G. Eichmann, arXiv:1104.4505.
[9] A. Bincer, Phys. Rev. **118**, 855 (1960).
[10] E. M. Nyman, Nucl. Phys. A **154**, 97 (1970).
[11] H. W. L. Naus and J. H. Koch, Phys. Rev. C **36**, 2459 (1987).
[12] P. C. Tiemeijer and J. A. Tjon, Phys. Rev. C **42**, 599 (1990).
[13] Y. Surya and F. Gross, Phys. Rev. C **53**, 2422 (1996).
[14] J. H. Koch, V. Pascalutsa, and S. Scherer, Phys. Rev. C **65**, 045202 (2002).
[15] H. Haberzettl, Phys. Rev. C **56**, 2041 (1997).
[16] A. N. Kvinikhidze and B. Blankleider, Phys. Rev. C **60**, 044003 (1999); **60**, 044004 (1999).
[17] J. S. Ball and T.-W. Chiu, Phys. Rev. D **22**, 2542 (1980).
[18] J. C. Ward, Phys. Rev. **78**, 182 (1950); Y. Takahashi, Nuovo Cimento **6**, 371 (1957).
[19] S. Weinberg, *The Quantum Theory of Fields, Vol. I. Foundations* (Cambridge University Press, New York, 1995).
[20] H. Haberzettl, K. Nakayama, and S. Krewald, Phys. Rev. C **74**, 045202 (2006).
[21] N. M. Kroll and M. A. Ruderman, Phys. Rev. **93**, 233 (1954).
[22] M. Gell-Mann and M. L. Goldberger, Phys. Rev. **96**, 1433 (1954).
[23] E. Kazes, Nuovo Cimento **13**, 1226 (1959).
[24] F. Huang, M. Döring, H. Haberzettl, J. Haidenbauer, C. Hanhart, S. Krewald, U.-G. Meißner, and K. Nakayama (unpublished).
[25] S. D. Drell and T. D. Lee, Phys. Rev. D **5**, 1738 (1972).
[26] K. Ohta, Phys. Rev. C **40**, 1335 (1989).
[27] H. Haberzettl, C. Bennhold, T. Mart, and T. Feuster, Phys. Rev. C **58**, R40 (1998).