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One Approach of Providing Quality and Reliability Programs in Conditions of Uncertainty

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An algorithm for determining the membership function and the degree of criticality of elements (fuzzy FMECA algorithm) is developed and applied in this paper in terms of ranking elements based on their criticality by applying the theory of fuzzy sets.

Keywords: Quality, Reliability, Uncertainty, FMECA

Introduction

Decision-making in quality management is conducted under conditions of uncertainty. The fuzzy set theory provides a conceptual framework for solving problems with vague and inaccurate (unclear) information, statements, or uncertainties, which are predominantly non-statistical in nature. With a quality management problem, the source of the inaccuracy and uncertainty lies not only in the presence of random events, but also in the inability to operate with exact data due to the complexity of the system, that is, the inaccuracy of defining constraints and goals. The application of fuzzy set theory to decision-making tasks in quality management modeling helps too: structuring the problem, operating it accurately and clearly, developing possible solutions, select the appropriate choice (so that decision-makers consider all significant conditions and consequences), finding possible and acceptable ways to execute a decision.

Research Methodology

Failure Modes, Effects and Criticality Analysis (FMECA) are a significant technique for ensuring quality and reliability programs. FMECA can be applied to many problems. It is reduced to methods with varying levels of analysis and modifications. The FMECA method can be defined as a systematic set of procedures designed to identify and evaluate potential product failures, determine measures and activities to eliminate or reduce the possibility of failure occurrence and documenting the previous two procedures. The method was developed for US military use as a technique for assessing reliability in terms of determining the consequences of different types of technical systems failures. As an official document, in the form of a US military standard, this method dates from 1949 under the code MIL-P-1629 and entitled "Procedures for Performing a Failure Mode, Effects and Criticality Analysis."[1]

The FMECA method is an inductive method of performing a qualitative analysis of the quality, reliability and safety of a system from the lowest to the highest structural level. It is based on consideration of all potential failures of system components and their consequences on the system in order to identify failure modes that have the most severe consequences on the usable performance of the system under consideration. The conception of the procedure is the most general because it allows the analysis to be started at any level of system breakdown and directed to higher or lower structural levels. Testing product reliability in design involves assessing the degree of failure criticality using the FMECA procedure.[1] In addition, the calculation of the criticality level of cancellation is performed according to the expression:

$$C_{ij}^{(r)}=\alpha_{ij}\cdot\beta_{ij}^{(r)}\cdot\lambda_i\cdot t_i \quad \ldots (1)$$

$$C_i=\Sigma_j\, C_{ij} \quad \ldots (2)$$

Where are:

$C_{ij}^{(r)}$- the degree of criticality of the j-th type of failure that causes the r-th category $(r = I, II, \ldots)$ consequence of the element i,

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C_i - criticality failure degree of the element *i*,

λ_i - failure intensity for element *i*,

t - work duration of the element *i*,

α_{ij} - the "weight" of the *j*-th type of failure of the element *i*, $0 \leq \alpha_{ij} \leq 1$,

$\beta_{ij}^{(r)}$ - conditional probability that the *j*-th type of failure of the element *i* will lead to the *r*-th category due to the failure (as recommended),

k_i - number of failure types of the element *i.*

FMECA procedure using terms for criticality of failure modes and criticality of elements ends with ranking of elements according to size of degree of criticality $C_{ij}^{(r)}$ or C_i. It is important to note that while values λ_i and t_i in the expression for $C_{ij}^{(r)}$ are not dependent on the opinion of the expert (designer) and are conditioned by the type of elements being analyzed, their functional purpose, methodology for calculating the failure rate during design, choice α_{ij} and $\beta_{ij}^{(r)}$ is completely dependent on the competence of the expert and information (designer), which performs the analysis. In this situation, to increase the analysis objectivity, value is appropriate α_{ij} and β_{ij} set not precisely but at intervals $\alpha_{ij} \in [\underline{\alpha}_{ij};\ \alpha_{ij}]$, $\beta_{ij} \in [\underline{\beta}_{ij};\ \overline{\beta}_{ij}]$, ie imprecise sets.[2] In this way, the task of elaborating the criticality determination process arises $C_{ij}^{(r)}$ and C_i, considering that some baseline data are given in the form of imprecise sets.

Development of algorithm for determination of elements functionality and criticality (fuzzy FMECA)

An expert (researcher) may assume that there are some values $\lambda_{ij} \in [\underline{\lambda}_{ij};\ \overline{\lambda}_{ij}]$ more possible than others. In the range $[\underline{\lambda}_{ij};\ \overline{\lambda}_{ij}]$ specifies the membership function: $0 \leq \mu(\lambda_{ij}) \leq 1$. If a value $\lambda_{ij1} \in [\underline{\lambda}_{ij};\ \overline{\lambda}_{ij}]$ more possible than value $\lambda_{ij2} \in [\underline{\lambda}_{ij};\ \overline{\lambda}_{ij}]$, it means that $\mu(\lambda_{ij1}) > \mu(\lambda_{ij2})$.

In that case, criticality $C_{ij}^{(r)}$ will belong to the section $[\underline{C}_{ij}^{(r)},\ \overline{C}_{ij}^{(r)}]$, where are:

$$\underline{C}_{ij}^{(r)} = \alpha_{ij} \cdot \beta_{ij}^{(r)} \cdot \underline{\lambda}_i \cdot t_i\ ,$$
$$\overline{C}_{ij}^{(r)} = \alpha_{ij} \cdot \beta_{ij}^{(r)} \cdot \overline{\lambda}_i \cdot t_i$$

Membership function for arbitrary value $C_{ij}^{(r)*}$ it is equal:

$$\mu\left[C_{ij}^{(r)*}\right] = \mu\left(k \cdot \lambda_{ij}^{*}\right),\ \mu[C_{ij}^{(r)*}] = \mu(k \cdot \lambda_{ij}^{*}) \qquad \dots (3)$$

where is: $C_{ij}^{(r)*} = \alpha_{ij} \cdot \beta_{ij}^{(r)} \cdot t_i \cdot \lambda_{ij}^{*} = k \cdot \lambda_{ij}^{*}$. $\qquad \dots (4)$

Criticality C_i for element *i* is also represented in the form of a fuzzy set, since the components C_{ij} given in the form of fuzzy set.

From the expression for C_i follows:

$\underline{C}_i = \Sigma_j \underline{C}_{ij}$, $\overline{C}_i = \Sigma_j \overline{C}_{ij}$.

In accordance with the principle of expanding the representation of fuzzy sets, a formula for calculating the membership function is obtained:

$\mu(C_i)$ *for* $\forall C_i^{*}$.

$$\mu(C_i^{*}) = \max \min\{\mu_1(C_{i1});\ \mu_2(C_{i2}); \dots;\ \mu_i(C_{ii})\}\ , \dots (5)$$

where is: $C_{i1} + C_{i2} + C_{ii} = C_i^{*}$.

Thus, for calculating $\mu(C_i^{*})$ all possible sets must be identified $C_{i1} + C_{i2} + \dots + C_{ii}$, whose sum is C_i^{*}, then determine the minimum of $\mu(C_{ij})$ and finally a maximum of $\mu(C_{ij})$. To select the most critical elements in the process FMECA using fuzzy values λ the following criterion can be applied[3]: criticality of element *i*, which is given by a fuzzy set $[\underline{C}_i, \overline{C}_i]$, is greater than criticality of element *k* which is given by a fuzzy set $[\underline{C}_k, \overline{C}_k]$, if: $C_{imax} > C_{kmax}$, where are[4]:

C_{i1max} - the maximum point of the segment $[\underline{C}_{i1},\ \overline{C}_{i1}]$, for which it is $\mu(C_{1max})$ maximum on the segment $[\underline{C}_{i1},\ \overline{C}_{i1}]$,

C_{i2max} - the maximum point of the segment $[\underline{C}_{i2},\ \overline{C}_{i2}]$, for which it is $\mu(C_{2max})$ maximum on the segment $[\underline{C}_{i2},\ \overline{C}_{i2}]$.

This criterion is presented graphically in Fig. 1. The described procedure for determining the function of belonging and ranking of elements according to the degree of criticality is shown by the general algorithm in Fig. 2.

Algorithm fuzzy FMECA application

We considered three elements *(i, m, p)*, with two potential types of failure *(j = 1, 2; n = 1, 2, q= 1, 2)*. Membership functions are determined on the basis of the developed algorithm Fuzzy FMECA, shown in

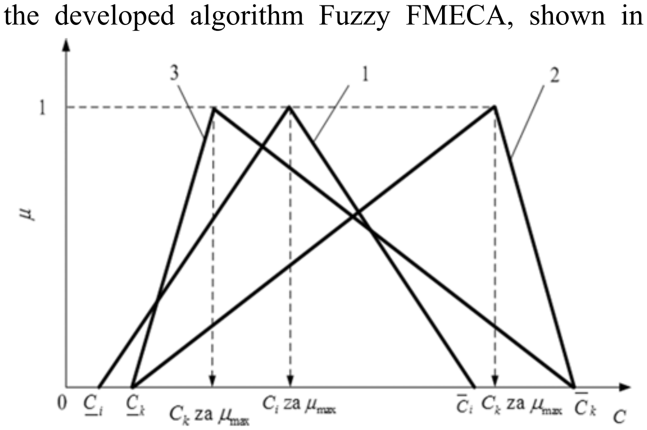

Fig. 1 — The selection of the maximum degree of criticality of a given fuzzy set: $C_{imax} < C_{kmax}$ (functions 1 and 2), $C_{imax} > C_{kmax}$ (functions 1 and 3)

Fig. 2. Specified the membership functions $\mu(C_i)$. If : $j=1,2;$ $\underline{C}_{i1}=2;$ $,\overline{C}_{i1}=7;$ $\underline{C}_{i2}=1;$ $,\overline{C}_{i2}=8$ and if membership functions $\mu(C_{i1})$ and $\mu(C_{i2})$ linear (Fig. 3)[5,6]:

$$\mu_1(C_{i1}) = 0,2 \cdot (C_{i1} - 2),$$
$$\mu_2(C_{i2}) = 0,143 \cdot (C_{i2} - 8).$$

It is obvious that:

$$C_i = \underline{C}_{i1} + \underline{C}_{i2} = 2 + 1 = 3,$$
$$C_i = \overline{C}_{i1} + \overline{C}_{i2} = 7 + 8 = 15.$$

It's being chosen $\forall C_i \in [3;15]$, for example, for $C_i=6$. Then the following pairs are possible C_{i1} and C_{i2} who build the sum $C_i=6$:

$C_{i1}=2;$ $C_{i2}=4$ *or* $C_{i1}=3$ *or* $C_{i2}=3$ *or* $C_{i1}=4,5;$ $C_{i2}=1,5;$ $C_{i1}=5;$ $C_{i2}=1$

In accordance with the principle of extension, it is obtained:

$\mu(C_i = 6) = max\{min[\mu_1(2);\ \mu_2(4)];\ min[\mu_1(3);\ \mu_2(3)];\ min[\mu_1(4,5);\ \mu_2(1,5)];\ min[\mu_1(5);\ \mu_2(1)]\}$

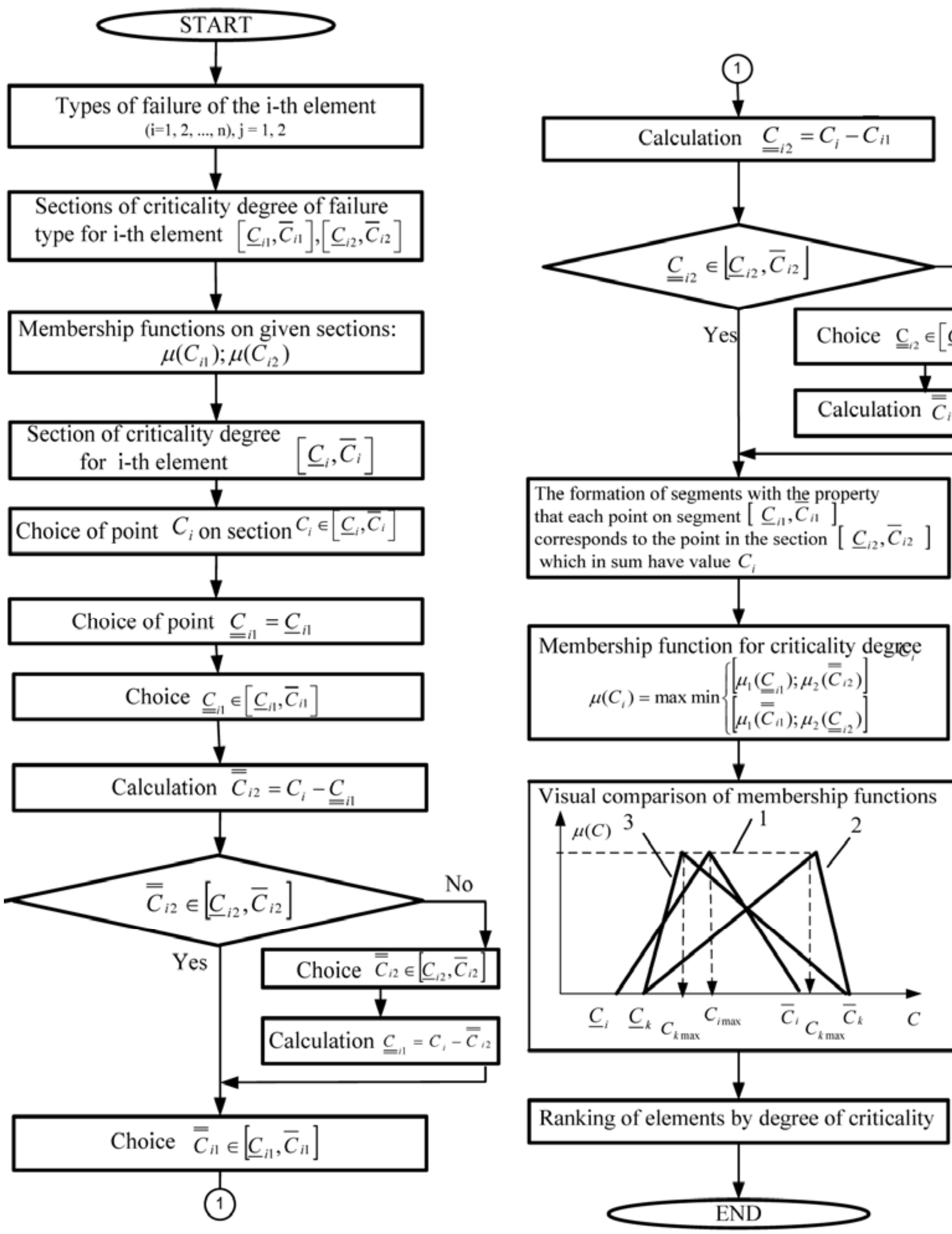

Fig. 2 — The algorithm for determining memebership functions and ranking elements by degree of criticality

$\mu(C_i = 6) = max\{min[0; 0,572], min[0,2; 0,715]; min[0,5; 0,929]; min[0,6; 0]\}$

$\mu(C_i = 6) = max\{0; 0,2; 0,5; 0,6\} = 0,6.$

This is how the affiliation function is obtained for $C_i = 6$, equal $\mu(C_i = 6) = 0,6$. Checking the value of the degree of criticality $C_i = 6$ for element i:

Step 1: $\overline{\overline{C}}_{i1} = \underline{C}_{i1} = 2,\ x = 6 - 2 = 4,\ \overline{\overline{C}}_{i2} = x = 4,$

Step 2: -

Step 3: $\overline{\overline{C}}_{i1} = 7,\ \underline{C}_{i2} = 6 - 7 = -1 \notin [1,8],$

Step 4: $\underline{\underline{C}}_{i2} = 1$

Step 5: $\overline{\overline{C}}_{i1} = 6 - 1 = 5,$

Sections: $[\underline{\underline{C}}_{i1}, \overline{\overline{C}}_{i1}] = [2,5]$ and $[\underline{\underline{C}}_{i2}, \overline{\overline{C}}_{i2}] = [1,4]$

The check: $2 + 4 = 5 + 1 = 6 = C_i$.

In the same way, the values of the membership function were obtained for the other values of the element criticality at interval [3,15]. On Fig. 4 shown the obtained dependence of the memebership function $\mu(C_i)$ and criticality degree C_i of element i of the system at interval [3,15]. Then, determined the memebership function $\mu(C_m)$. If $n=1,2;\ \underline{C}_{m1}=2;\ \overline{C}_{m1}=4;\ \underline{C}_{m2}=3;\ \overline{C}_{m2}=9$. If membership functions $\mu(C_{m1})$ and $\mu(C_{m2})$ are linear:

$\mu_1(C_{m1}) = 0,5 \cdot (C_{m1} - 2),$

$\mu_2(C_{m2}) = -0,167 \cdot (C_{m2} - 9).$

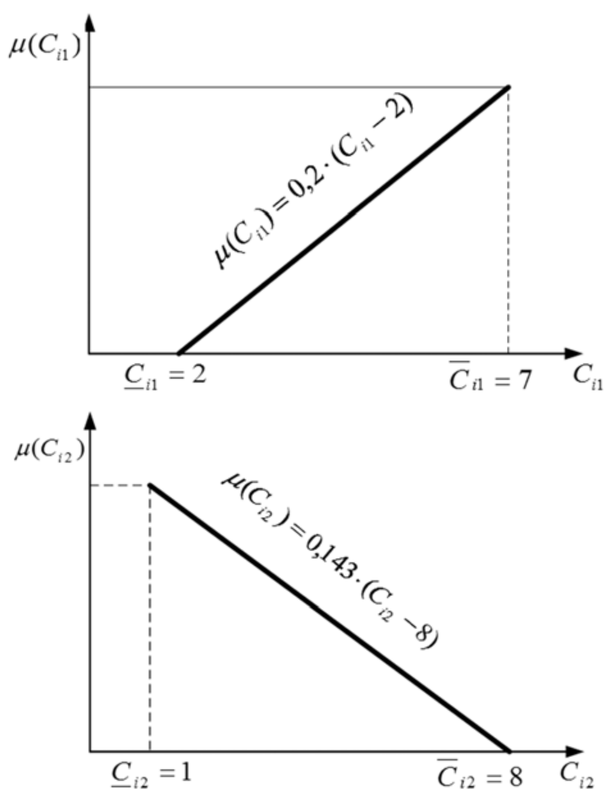

Fig. 3 — Dependencies of membership functions $\mu(C_{i1})$ and $\mu(C_{i2})$

Using the algorithm in Fig. 2, the membership function for the element m is determined. On Fig. 5 shown the obtained dependence of the memebership function $\mu(C_m)$ and criticality degree C_m of element m of the system at interval [5,13].

Finally, the membership function $\mu(C_p)$ is determined. If: $q=1,2;\ \underline{C}_{p1}=4;\ \overline{C}_{p1}=9;\ \underline{C}_{p2}=3;\ \overline{C}_{p2}=5$; then membership functions $\mu(C_{p1})$ and $\mu(C_{p2})$ are linear:

$$\mu_1(C_{p1}) = 0,2 \cdot (C_{p1} - 2),\ \ \mu_2(C_{p2}) = -0,5 \cdot (C_{p2} - 5).$$

Membership function for element p determined as in the case of the elements i and m. On Fig. 6 is shown the obtained dependence of the memebership function $\mu(C_p)$ and criticality degree

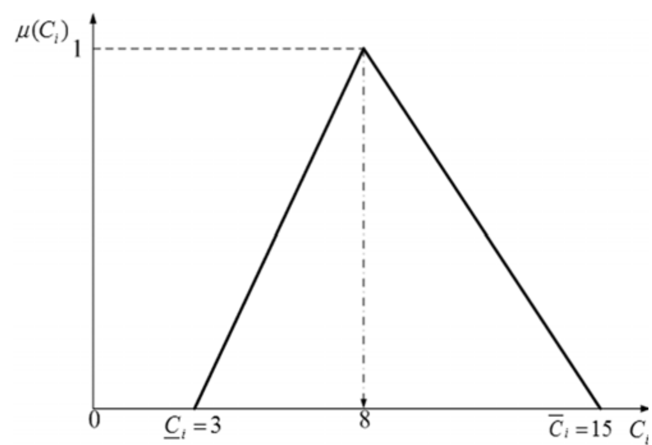

Fig. 4 — Dependence of membership function $\mu(C_i)$ and criticality degree C_i for element i of system at interval [3,15], $C_i = 8$ - the highest possible element criticality value

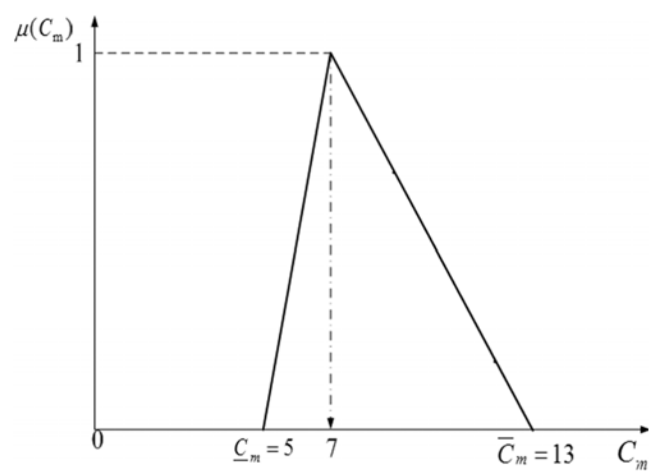

Fig. 5 — Dependence of membership function $\mu(C_m)$ and criticality degree C_m for element m of system at interval [5,13], $C_m = 7$ - the highest possible element criticality value

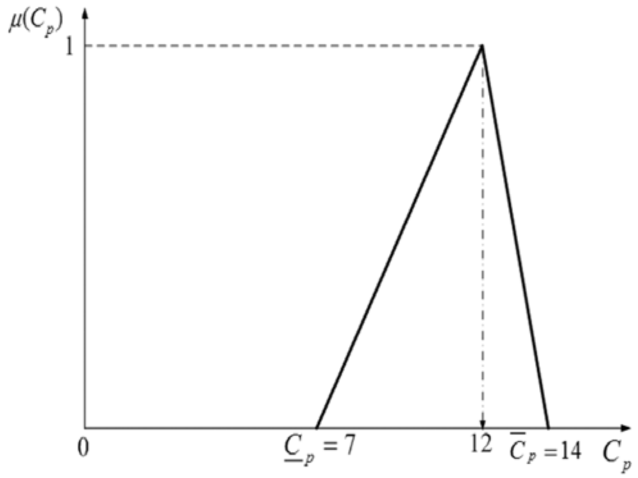

Fig. 6 — Dependence of membership function μ(C_p) and criticality degree C_p for element pof system at interval [7,14], C_p = 12 - the highest possible element criticality value

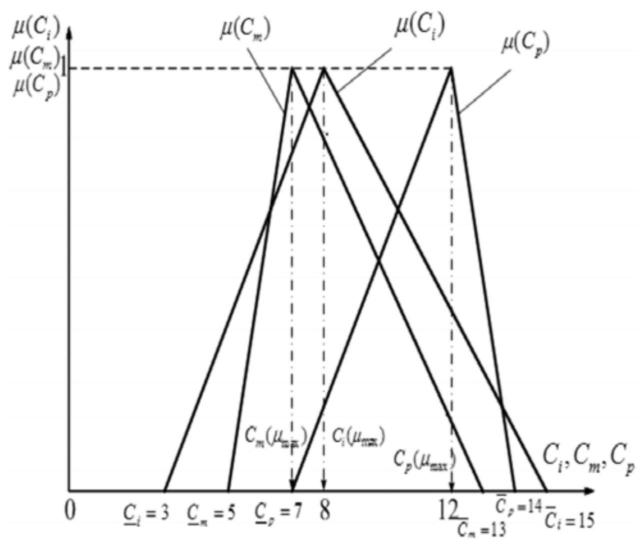

Fig. 7 — Obtained dependencies of membership functions μ(C_i), μ(C_m) and μ(C_p) and criticality degrees C_i, C_m, C_p for elements i, m, p of the system

C_p of element *p* of the system at interval [7,14]. In accordance with the algorithm in Fig. 2, the comparison of membership functions and the ranking of elements according to the degree of criticality are performed. In Fig. 7 is given a comparative representation of the obtained dependencies of the membership functions $\mu(C_i)$, $\mu(C_m)$ and $\mu(C_p)$ for degrees of criticality C_i, C_m, C_p of elements *i, m, p* of the system, respectively. The degree of criticality of the elements *i, m* and *p* can be ranked as follows: $C_p > C_i > C_m$. The highest degree of criticality has element *p*, and the smallest – element *m*.

Conclusions

In this way was given a generalization of the FMECA methodology when some starting data were given not specified but by fuzzy sets, i.e. change intervals with appropriate membership functions. An algorithm for calculating the fuzzy criticality of a failure type with corresponding membership functions is proposed. A procedure for comparing the fuzzy criticality of the analyzed elements was developed and applied. An algorithm for determining the membership function and the degree of criticality of elements (fuzzy FMECA algorithm) is developed in this paper in terms of ranking elements based on their criticality by applying the theory of fuzzy sets. Analysis of the development and application of the concept of indeterminacy can be developed and applied to other methods that find application in quality management modeling.

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References

1 Buyukozkan G & Gocer F, An extension of ARAS methodology under interval valued intuitionistic fuzzy environment for digital supply chain, *Appl Soft Comput*, **69** (2018) 634–654.

2 Milunovic S**,** Development and Application of the Concept of Uncertainty in Modeling in Quality Management, *MSc Thesis*, Technical faculty, Cacak, Serbia, 2005.

3 Lee Y C, Chung P H & Shyu J Z, Performance Evaluation of Medical Device Manufacturers Using a Hybrid Fuzzy MCDM, *J Sci Ind Res*, **76(1)** (2017) 28–31.

4 Fernández-Gámez M A, Del Castillo A, Alaminos D, Santos J A C & Alcoforado E, Corporate Reputation, Financial Performance and Earnings Quality, *J Sci Ind Res*, **78(1)**, (2019) 15–18.

5 Renjith V R, Jose Kalathil M, Kumar P H & Madhavan D, Fuzzy FMECA (failure mode effect and criticality analysis) of LNG storage facility, *J Loss Prev Process Ind,* **56**, (2018), 537–547.

6 Wang L, Gao Y, Xu W, Hong K, Wang B & Chen X, An Extended FMECA Method and Its Fuzzy Assessment Model for Equipment Maintenance Management Optimization, *J Fail Anal and Preven*, Published online: 08 February 2019.