Quantitative Verification of a Force-based Model for Pedestrian Dynamics

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Summary. This paper introduces a spatially continuous force-based model for simulating pedestrian dynamics. The main intention of this work is the quantitative description of pedestrian movement through bottlenecks and in corridors. Measurements of flow and density at bottlenecks will be presented and compared with empirical data. Furthermore the fundamental diagram for the movement in a corridor is reproduced. The results of the proposed model show a good agreement with empirical data.

1 Introduction

One application of pedestrian dynamics is the enhancement of the safety of people in complex buildings and in big mass events e.g., sporting events, religious pilgrimages, etc. where there is a risk of disaster. Thanks to computer simulations, it is possible to forecast the emergency egress and optimise the evacuation of large crowds. Another aspect of pedestrian dynamics is the comfort of passengers in pedestrian facilities e.g., airports, railway stations, shopping malls, etc. Those facilities have to be designed in a way to ensure minimal travel times and maximal capacities. For these applications, robust and quantitatively validated models are necessary.

A wide spectrum of models have been designed to simulate pedestrian dynamics. Generally those models can be classified into macroscopic and microscopic models. In macroscopic models the system is described by mean values of characteristics of pedestrian streams e.g., density and flow, whereas microscopic models consider the movement of individual persons separately. Microscopic models can be subdivided into several classes e.g., rule-based and force-based models. For a detailed discussion we refer to [1]. In this work we focus on spatially continuous force-based models.

Force-based models take Newton's second law of dynamics as a guiding principle. Thus, the movement of each pedestrian is defined by:

$$\overrightarrow{F_i} = \sum_{j \neq i}^{\tilde{N}} \overrightarrow{F_{ij}^{\text{rep}}} + \sum_{B} \overrightarrow{F_{iB}^{\text{rep}}} + \overrightarrow{F_i^{\text{drv}}} = m_i \overrightarrow{a_i}, \tag{1}$$

where $\overrightarrow{F_{ij}^{\text{rep}}}$ denotes the repulsive force from pedestrian j acting on pedestrian i, $\overrightarrow{F_{iB}^{\text{rep}}}$ is the repulsive force emerging from borders and $\overrightarrow{F_i^{\text{drv}}}$ is a driving force. m_i is a constant with dimensions of mass and $\tilde{N}$ the number of neighbouring pedestrians. Repulsive forces model the collision-avoidance performed by pedestrians. Whereas the driving force models the intention of a pedestrian to move to some destination. The set of equations (1) for all pedestrians results in a high-dimensional system of second order ordinary differential equations. The time evolution of the positions and velocities of all pedestrians is obtained by numerical integration.

Most force-based models describe the movement of pedestrians qualitatively well. Collective phenomena like lane formations [2, 3, 4], oscillations at bottlenecks [2, 3], the "faster-is-slower" effect [5, 6], clogging at exit doors [3, 4] etc. are reproduced. These achievements indicate that these models are promising candidates. However, a qualitative description is not sufficient if reliable statements about critical processes, e.g., emergency egress, are requested. Moreover, implementations of models do not rely on one sole approach. Especially in high density situations simple numerical treatment has to be supplemented by additional techniques to obtain reasonable results. Examples are restrictions on state variables and sometimes even totally different procedures replacing the above equations of motion (1) to avoid partial and total overlapping among pedestrians [5, 4] or negative and high velocities [2].

We address the possibility of describing reasonably and in a quantitative manner the movement of pedestrians, with a modelling approach as simple as possible. For a systematic verification of our model we measure the fundamental diagram, the flow through bottlenecks and the density inside and in front of the entrance of a bottleneck. In the next section, we propose such a model which is solely based on the equation of motion (1). Furthermore the model incorporates free parameters which allow calibration to fit quantitative data.

2 Definition of the model

Our model is based on the Centrifugal Force Model (CFM) [4]. The CFM takes into account the distance between pedestrians as well as their relative velocities. Pedestrians are modelled as circles with constant diameter. Their movement is a direct result of superposition of repulsive and driving forces acting on the centre of each pedestrian. Repulsive forces acting on pedestrian i from other pedestrians in their neighbourhood and eventually from walls, stairs, etc. to prevent collisions and overlapping (Fig. 1). The driving force, however, adds a positive term to the resulting force, to enable movement of pedestrian i in a certain direction with a given desired speed $\parallel \overrightarrow{V_i^0} \parallel$. The mathematical expression of the driving force as introduced initially in [2] is used:

$$\overrightarrow{F_i^{\text{drv}}} = m_i \frac{\overrightarrow{V_i^0} - \overrightarrow{V_i}}{\tau}, \tag{2}$$

with a time constant τ.

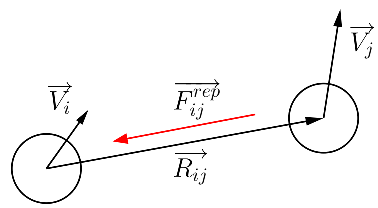


Fig. 1. The direction of the repulsive force pedestrian j acting on pedestrian i.

The definition of the repulsive force in the CFM expresses several principles. First, the force between two pedestrians decreases with increasing distance. In the CFM it is inversely proportional to their distance. Given the position of two pedestrians i and j, the direction vector between their centers is defined as:

$$\overrightarrow{R_{ij}} = \overrightarrow{R_j} - \overrightarrow{R_i}, \quad \overrightarrow{e_{ij}} = \frac{\overrightarrow{R_{ij}}}{\parallel \overrightarrow{R_{ij}} \parallel}. \tag{3}$$

Furthermore, the repulsive force takes into account the relative velocity between pedestrian i and pedestrian j. The following special definition provides that slower pedestrians are not affected by the presence of faster pedestrians in front of them:

$$V_{ij} = \frac{1}{2}[(\overrightarrow{V_i} - \overrightarrow{V_j}) \cdot \overrightarrow{e_{ij}} + |(\overrightarrow{V_i} - \overrightarrow{V_j}) \cdot \overrightarrow{e_{ij}}|]. \tag{4}$$

As in general pedestrians react only to obstacles and pedestrians that are within their perception, the reaction field of the repulsive force is reduced to the angle of vision of each pedestrian (180°), by introducing the coefficient:

$$K_{ij} = \frac{1}{2}\frac{\overrightarrow{V_i} \cdot \overrightarrow{e_{ij}} + \mid \overrightarrow{V_i} \cdot \overrightarrow{e_{ij}} \mid}{\parallel \overrightarrow{V_i} \parallel}. \tag{5}$$

With the definitions in Eqs. (3), (4) and (5), the repulsive force between two pedestrians is formulated as:

$$\overrightarrow{F_{ij}^{\text{rep}}} = -m_i K_{ij} \frac{V_{ij}^2}{\parallel \overrightarrow{R_{ij}} \parallel} \overrightarrow{e_{ij}}\,. \tag{6}$$

In [7] it was shown that the introduction of a "collision detection technique" (CDT), see [4] for the definition, is necessary to mitigate overlapping among pedestrians.

In the following, we will discuss why volume exclusion is not guaranteed by Eq. (6) and meanwhile introduce our modifications of the repulsive force. Due to the quotient in Eq. (6) when the distance is small, low relative velocities lead to an unacceptably small force. Consequently, partial or total overlapping are not prevented. Introducing the intended speed in the numerator of the repulsive force eliminates this side-effect. Furthermore, the modified repulsive force and driving force (2) compensate at low velocities, which damps oscillations.

Since faster pedestrians require more space than slower pedestrians, due to increasing step sizes [8], the diameter of pedestrian i depends linearly on its velocity:

$$D_i = d_a + d_b \parallel \overrightarrow{V_i} \parallel, \tag{7}$$

with free parameters d_a and d_b. We define the distance between pedestrian i and pedestrian j as:

$$\text{dist}_{ij} = \parallel \overrightarrow{R_{ij}} \parallel - \frac{1}{2}(D_i(\parallel \overrightarrow{V_i} \parallel) + D_j(\parallel \overrightarrow{V_j} \parallel)). \tag{8}$$

By taking these aspects into account, the definition of the modified repulsive force reads

$$\overrightarrow{F_{ij}^{\text{rep}}} = -m_i K_{ij} \frac{(\nu \parallel \overrightarrow{V_i^0} \parallel + V_{ij})^2}{\text{dist}_{ij}} \overrightarrow{e_{ij}}, \tag{9}$$

where ν is a parameter which adjusts the strength of the force. Due to these changes we can do without the extra CDT which dominates the dynamics in [4] in case of formation of dense crowds.

The repulsive force between two pedestrians i and j is infinite at contact and decreasing with increasing distance between i and j. Since the repulsive force as defined in Eq. (9) does not vanish, the summation over all other pedestrians leads to a complexity of $O(N^2)$. To deal with this problem and to consider a limited range of pedestrian interaction only the influence of neighbouring pedestrians is taken into account. Two pedestrians are said to be neighbours if their distance is within a certain cut-off radius $R_c = 2.5$ m. To guarantee robust numerical integration a two-sided Hermite-interpolation of the repulsive force is implemented (see Fig. 2). The interpolation guarantees that for each pair i, j with a distance in the interval $[R'_c, R_c]$ the norm of the repulsive force between them decreases smoothly to zero. R'_c is set to $R_c - 0.1$ m. For distances in the interval $[S_{\max}, R_{\text{eps}}]$ the interpolation avoids an increase of the force to infinity, to reach a maximum value of $F_{\max} = 1000$ N. R_{eps} is set to 0.1 m and $S_{\max}$ to -5 m.

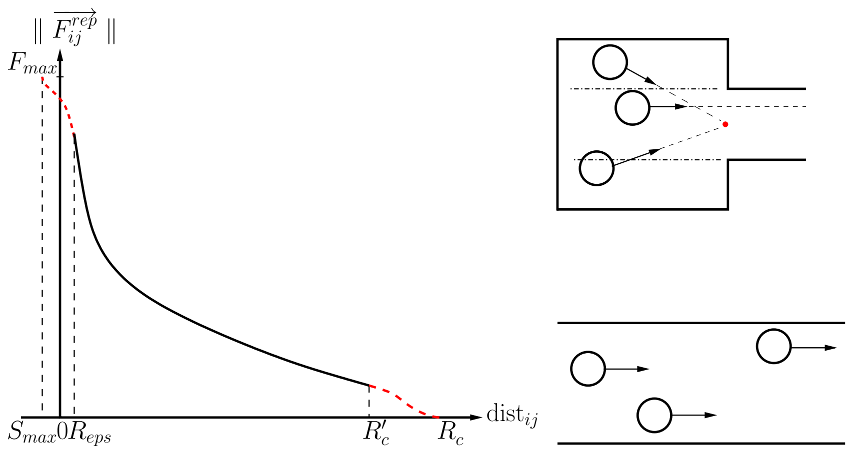


Fig. 2. Left: The interpolation of the repulsive force between pedestrians i and j. Right: Direction of pedestrians in corridors and bottlenecks.

The desired direction of a pedestrian is set to be parallel to the walls of the corridor. In the bottleneck case it is set towards the centre of the entrance to the bottleneck if the pedestrian is outside the range of the bottleneck. That is if he can not "see" the exit of the bottleneck. Otherwise, the desired direction is chosen parallel to the length of the bottleneck (Fig. 2).

3 Simulation results

The initial value problem (1) was solved using an Euler scheme with fixed-step size $\Delta t = 0.01$ s. The desired speeds of pedestrians are Gaussian distributed with mean $\mu = 1.34$ m/s and standard deviation $\sigma = 0.26$ m/s. The constant τ in Eq. (2) is set to 0.5 s. For simplicity, the mass, m_i is set to unity. Several parameter values were tested. The free parameters in Eqs. (9) and (7) are set to $\nu = 0.2$, $d_a = 0.3$ m and $d_b = 0.2$ s. With this parameter set the results of the simulations are in good agreement with empirical data.

To verify the ability of the model to reproduce the fundamental diagram, measurements in corridors of different widths were performed. The length of the corridor is 20 m and its width is 2 m. The shape of the reproduced velocity-density relation is in good agreement with the empirical data [9, 10, 11, 12], see Fig. 3).

Furthermore, the flow of 60 pedestrians through the bottleneck as described in [13] was simulated. The width of the bottleneck was changed from 0.8 m to 1.2 m in steps of 0.1 m (Fig. 3).

A third validation comes from measurements of density inside the bottleneck as well as in front of the entrance to the bottleneck. The density in

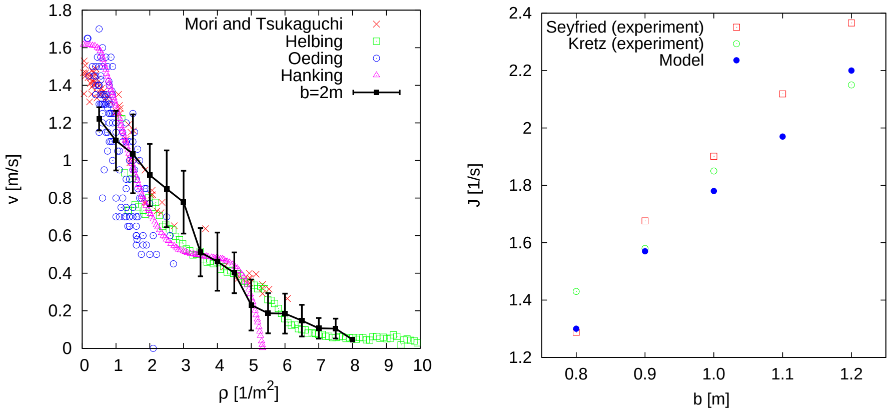
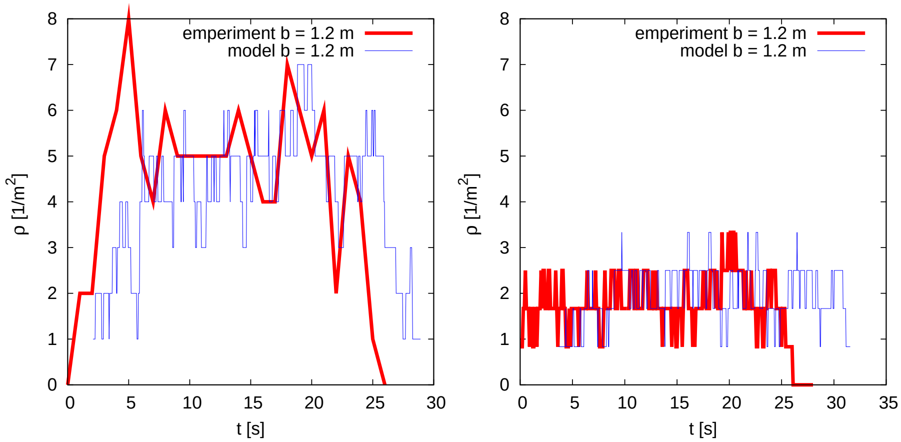


Fig. 3. Left: The fundamental diagram in comparison with empirical data. For other values of the corridor's width (1 m and 4 m), the simulation results are also in good agreement with the empirical data. Right: Flow measurement with the modified CFM in comparison with empirical data.

front of the entrance to the bottleneck is presented in Fig. 4(a). The results are in good agreement with the experimental data in [13]. Additionally, the measured density values inside the bottleneck are in accordance with the published empirical results in [14], see Fig. 4(b). One remarks that the density in front of the bottleneck is much higher than the density in the bottleneck. This difference reflects typical dynamics at bottlenecks, which is reproduced by our model.

4 Conclusions

We have proposed modifications of a spatially continuous force-based model [4] to describe quantitatively the movement of pedestrians in 2D-space. Besides being a remedy for numerical instabilities in CFM the modifications simplify the approach of Yu et al. [4] since we can dispense with their extra "collision detection technique" without deteriorating performance. The implementation of the model is straightforward and does not use any restrictions on the velocity. Simulation results show good agreement with empirical data. Nevertheless, the model contains free parameters that have to be tuned adequately to adapt the model to a given scenario. Further improvement of the model could be made by including, for example, a density dependent repulsive force.



(a) Density in front of the entrance to the bottleneck

(b) Density inside the bottleneck

Fig. 4. Density measurements: The simulation results (blue lines) are in good agreement with the empirical data presented in [15] and [14]. The difference between the density in front and inside the bottleneck as well as the amplitude of the fluctuations are given correctly. The width of the bottleneck is 1.2 m. Also for other values of the width a good agreement between simulation results and empirical data is found.

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