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Few-Baryon Systems

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Systems of few nucleons can be solved in a numerically precise manner on supercomputers, which allows to probe nuclear forces in an unprecedented manner. We present achievements and challenges for three- and four- nucleon bound states, three-nucleon scattering reactions and electron induced processes in the three-nucleon system. The need for three-nucleon forces is documented. We also demonstrate that the presently available most modern hyperon-nucleon forces fail to describe the lightest few-body hypernuclei.

1 Introduction

The question, what is the nature and what are the properties of nuclear forces, is as old as nuclear physics. Despite an intensive research of more than half a century this question is still not satisfactorily answered. As we know now, the reason is the composite nature of the nucleons, where the subnucleonic degrees of freedom are governed by a mathematically very complex nonlinear field theory, the Quantum Chromodynamics. A rigorous treatment is not yet possible. Instead one still relies on a more effective picture, going back to Yukawa, that nuclear forces are generated by the exchange of various types of mesons between the nucleons. Qualitatively this mechanism describes quite well the rich set of measured nucleon-nucleon (NN) scattering data at not too high energies. When fine tuned and augmented by additional phenomenological structures it leads to several so-called high precision NN interaction models, which perfectly describe the NN data set.

Now, the NN system alone is apparently not all of nuclear physics. There are hundreds of nuclei with many nucleons bound together by nuclear forces. One would like to understand quantitatively the amount of binding energy, which is defined as the mass of the nucleus minus the nucleon number times the mass of a nucleon. That difference, the binding energy, is smaller than zero and it requires an input of energy to separate the nucleus into free nucleons. The overall attractive nuclear forces work against that separation . One can also hit a nucleus by a nucleon leading to a so called nuclear reaction process with many facets,which one would like to understand theoretically. There are of course many more questions to be posed but already these most elementary ones serve to illustrate the basic challenge: One has to solve the dynamical equation for the motion of the nucleons under the action of nuclear forces in order to answer these questions. For low energy nuclear physics this is the Schrödinger equation of quantum mechanics.

At this point enters the spirit of few-nucleon physics. That Schrödinger equation can be solved on supercomputers in a numerically rigorous manner for a small number of nucleons, not yet for a large number. Therefore systems of say 3,4 nucleons are an ideal test laboratory to probe nuclear forces and to find out whether the present day nuclear force models are good enough to lead to correct binding energies and to a correct description of nuclear reaction processes. When we talked up to now of nuclear forces we meant forces acting between two nucleons. The composite nature of the nucleons, however, and thus the possibility of internal excitations, lead in the very natural manner, as we shall sketch below, to additional new types of forces, three-nucleon forces. Trivially they can act the first time, when three nucleons are together. Will that go on and one has to expect proper four-nucleon forces if four nucleons are together ? Then the task of understanding of nuclear physics would be hopeless. Thus we face a very first question: will NN forces as determined from NN scattering data be sufficient to a large extent to also describe systems of 3,4.. nucleons correctly or will many-nucleon forces play a never ending role ? We shall describe the present inside in our first result section.

Normal nuclei are composed of neutrons and protons, to which we referred generically as nucleons. There exist more particles in nature with similar masses as the nucleons. Though they are not stable, they live long enough to be combined with nucleons to form new types of nuclei, hypernuclei. It is a great theoretical challenge to study the strong forces acting between these new types of particles, called hyperons (Λ's and Σ's), and the nucleons, as well as the forces among the hyperons. Again light systems are an ideal laboratory since they are accessible by supercomputers. Our results will be displayed in the second result section.

Finally we would like to select one further approach to study nuclear forces, namely scattering electrons on nuclei. The electron emits a photon, which is absorbed by the nuclear system and triggers nuclear reactions. This is an important tool to probe nuclear forces and is intensively pursued experimentally at electron facilities all over the world. We present our results in the third result section and end with an outlook.

2 Results

The Schrödinger equation for n nucleons reads

$$\left(\sum_{i=1}^{n} T_i + \sum_{i<j}^{n} V_{ij} + \sum_{i<j<k}^{n} V_{ijk} + \dots \right) \Psi = E\Psi \tag{1}$$

The Hamiltonian acting on the wavefunction Ψ is composed of the kinetic energy (T_i), the NN interactions (V_{ij}), three-nucleon interactions (V_{ijk}) and possible higher-order nuclear forces. In configuration space this is a 3n-fold partial differential equation. In case of bound states E is directly related to the binding energy of a nucleus and in case of nuclear reactions E is related to the energy by which a nucleon hits a nucleus. In the form of Eq. (1) the numerical task is not yet tractable even on the most powerful present day supercomputers.The wave function Ψ describes the very complex correlations in the motion of the n nucleons: two nucleons cannot approach each other too closely due to the strong short range repulsion of NN forces and 3N forces impose even more complex restrictions. A break-through was established in the form of the Faddeev-Yakubovsky equations (FY) which break Ψ into pieces according to the various sequences of subclusters of the whole n- body problem. These pieces of Ψ are responsible only for certain subcluster correlations and not for all at the same time as Ψ. These pieces obey a set of mathematically well defined coupled equations, the FY equations.

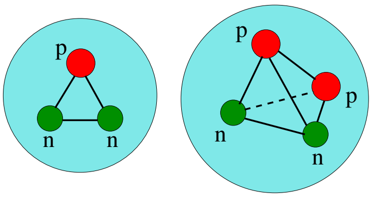

Figure 1. Schematic picture of the nucleon locations in ^{3}H and ^{4}He.

The motion of the interacting nucleons can be classified according to the many ways orbital angular momenta and spins are coupled to a total conserved angular momentum. This introduces a great amount of analytical insight and reduces the number of variables dramatically. Nevertheless the resulting number of unknowns for the discretized form of the FY equations is quite large. It amounts to typically 10^5 in a three-body system and up to 10^8 in a four-body system. In other words one has to handle (essentially) full matrices up to the order $10^8 \times 10^8$. Apparently this requires supercomputers and our results are based on both, highly vectorized as well as massively parallelized codes.

Let us now regard the problems and our results.

2.1 The Lightest Nuclei and Nuclear Reactions

The starting point are the most modern high precision NN forces as mentioned in the introduction. What will be the outcome if these pair forces act between two neutrons and one proton, like in case of ^{3}H, or between two protons and two neutrons in case of ^{4}He ? Will the Schrödinger equation or the mathematically equivalent FY equations yield the correct masses of these nuclei and thus the correct binding energies ? How are the nucleons spatially arranged in these light nuclei ?

We show in Fig. 1 their most probable symmetric geometries as they are predicted by Ψ from Eq. (1). The most probable pair distances are close to 1 fm=10^{-13} cm (1 fm is a typical unit for the range of nuclear forces). The blue background indicates that all orientations in space of these geometrical patterns have to be superimposed. The amount of extension of the nucleons shown as little balls is just an artistic view. The binding energies (in units of MeV) are presented in Table 1 for various high precision NN forces and compared to the experimental values. We see, theory and experimental values disagree. The theoretical numbers are smaller in magnitude, which tells that the NN forces alone do not bind the nuclei as strongly together as in nature.

This result should not come as a surprise in view of the remarks made in the introduction. 3N forces might act in nature in addition. To understand their properties is a hot topic right now. One very natural mechanism for a 3N force is depicted in Fig. 2, where processes following one another in time are shown from left to right. First three nucleons move upwards along vertical lines. Then nucleon number 1 emits a pion, which is absorbed by nucleon 2 and converts thereby nucleon 2 into an excited state, the famous Δ-excitation of the nucleon. In that intermediate state there are no longer three nucleons in their ground state present but one of them is excited and appears as a new particle, the Δ with different quantum numbers. Further on in time the Δ gets deexcited by emitting a pion which is

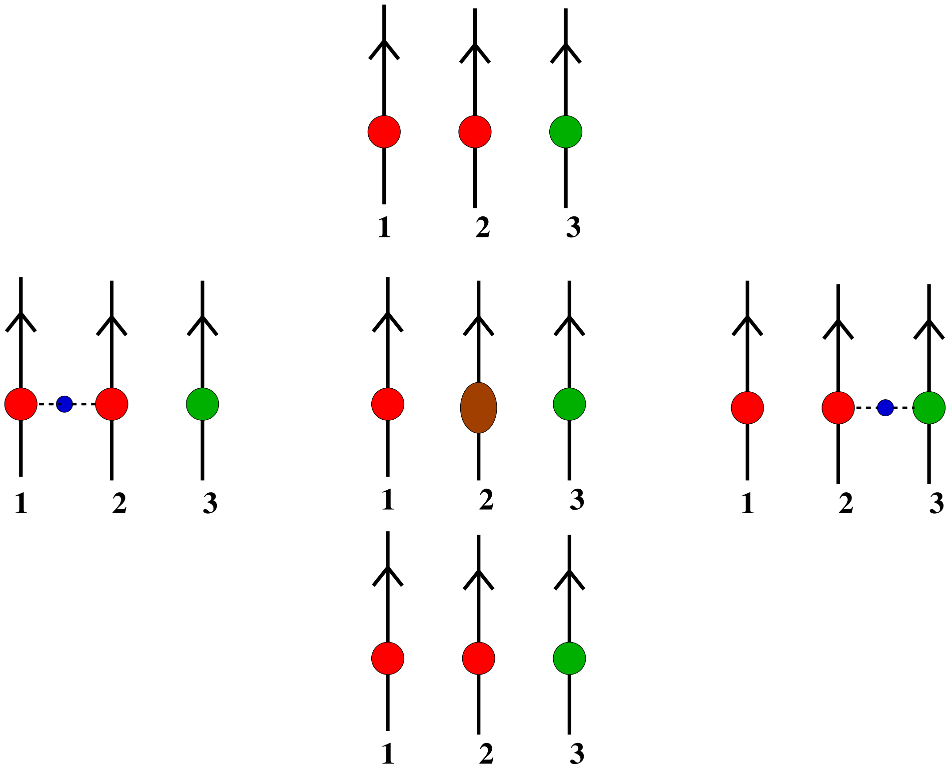

Figure 2. Two-pion-exchange 3NF.

then absorbed by nucleon 3. We end up again with 3 nucleons. This process induces a force between the three nucleons. In mathematical terms one leaves the Hilbert space of 3 nucleons at the intermediate state and consequently that process can not be reduced to a sequence of pair forces between nucleons. It is a proper 3N force mechanism.

NN force	^{3}H	^{4}He
CD Bonn	-8.013	-26.26
AV18	-7.628	-24.28
Nijm I	-7.741	-24.98
Nijm II	-7.659	-24.56
Nijm'93	-7.668	-24.53
Experimental value	-8.482	-28.30

Table 1. ^{3}H and ^{4}He binding energies obtained with different NN force models compared to the experimental values.

nuclear forces	^{3}H	^{4}He
CD Bonn + TM	-8.478	-29.15
AV18 + TM	-8.478	-28.84
AV18 + Urbana IX	-8.484	-28.50
Experimental value	-8.482	-28.30

Table 2. ^{3}H and ^{4}He binding energies obtained with different nuclear force models compared to the experimental values.

Related to the mechanism of Fig. 2 is the Tucson - Melbourne (TM) 3NF. It contains one free parameter, which can be used to fix the 3N binding energy. Thus like NN data are used to adjust model parameters in the NN force the 3N binding energy is used to adjust a model parameter in the 3NF. This is not as bad as it might look in the very first moment. Though one has lost the predictive power for the 3N binding energy one can now predict the 4N binding energy. Furthermore there is a huge amount of other 3N observables in the realm of nuclear reactions, which can now be predicted.

Table 2 shows the adjusted ^{3}H binding energies (the partially not perfect adjustment is of no relevance) and the predictions for ^{4}He[1]. It also includes predictions based on a second very popular 3NF, the Urbana IX. The additional 3NF's improve significantly the ^{4}He results, but there is now a small overbinding. At present time that small overestimation can be considered as an indication for the possible action of a very small 4N force of repulsive character.

2.2 Nuclear Reactions for Three Nucleons

The famous classical 3- body problem is still unsolved analytically ; the quantum mechanical one is not easier and only thanks to supercomputers it can now be solved[2]. One can distinguish elastic scattering of a nucleon on a deuteron or inelastic processes, where the deuteron breaks apart and in the final state three nucleons emerge from the interaction region. While the three nucleons are close together within the range of nuclear forces classically spoken they follow very complex intricate trajectories. Quantum mechanically one has to solve the Schrödinger equation in the form of FY equations nonperturbatively, because the nuclear forces are rather strong. The nucleon as a spin 1/2 particle can be experimentally prepared in two spin states, say up and down; the deuteron with overall spin 1 in 8 spin states (vector and tensor polarized). This refers for instance to the states of the two reaction partners before the reaction takes place, but also the final states. can be separated according to their different spin orientations. Thus there is quite a rich number of possible scattering processes starting from a certain spin orientation and ending up in the same or another one in the final state. The probabilities for all these detailed processes are governed by the spin- dependent nuclear forces. In case of the deuteron break up process the three outgoing nucleons share the total available energy in a continuously distributed manner and they can leave the reaction zone into arbitrary directions. This together with spin orientations provides an even richer amount of information when measured to challenge the Schrödinger equation together with its dynamical input, the nuclear forces. Lack of space forces us to select just four examples which demonstrate successes and failures of our present day understanding.

We show in Fig. 3 a schematic set up for elastic nucleon-deuteron scattering. We see the initial nucleon at the left approaching the deuteron (in the middle) . Then after the elastic scattering has taken place in a certain reaction volume in space nucleon and deuteron separate from each other and are registered under certain angles θ and θ' in detectors. The arrows on the initial nucleon (one of them in bracket) indicate two possible spin orientations and the three arrows on the deuteron indicate a situation, where one averages over the various spin orientations of the deuteron. In other words this set up corresponds to the case that only the initial nucleon is polarized (spin oriented). Fig. 4 shows the angular distribution of the scattered nucleon for a totally unpolarized situation, where also an average over the two spin orientations of the initial nucleon has been performed. That distribution

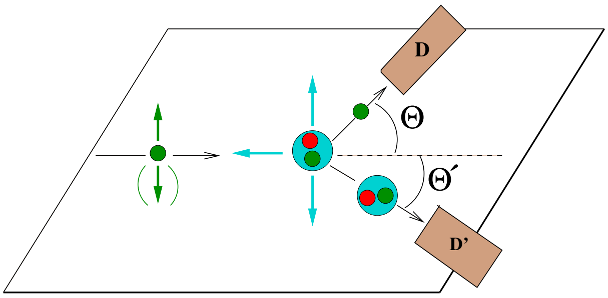

Figure 3. Elastic nucleon-deuteron scattering.

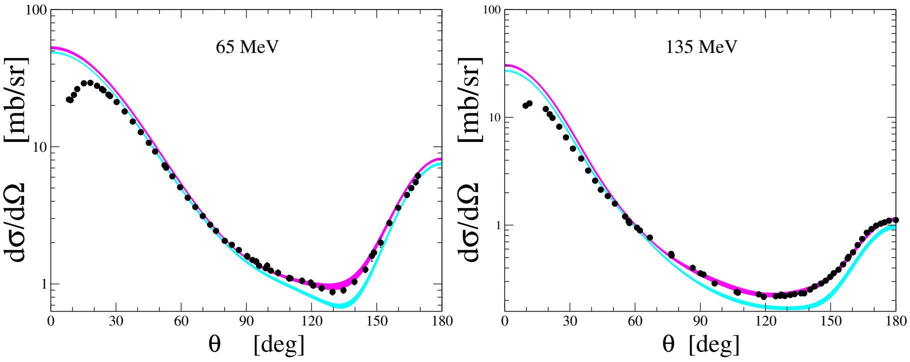

Figure 4. Angular distribution of nucleons in elastic nucleon-deuteron scattering.

is a measure for the probability that a nucleon is elastically scattered into a certain angle θ, whereas the deuteron is scattered into a corresponding angle θ'. The left and right parts refer to two different energies, 65 and 135 MeV of the incoming nucleon, respectively. The blue bands comprise the predictions based on the present day most modern high precision NN forces and fail to reproduce the experimental results, given as dots. If we add the 3NF adjusted in ^{3}H as described above theory, as given by the red band, moves without any further modification right away into the data. We consider this to be a beautiful signature of a 3NF effect[3].

Lets move on to observables which depend on spin orientations in Fig. 5[4]. The left figure shows the situation that a difference of angular distributions is formed for nucleon spin up and down related to their sum. One talks of an analyzing power A_y. The right figure displays a corresponding analyzing power now with a spin flip of the deuteron leading to $A_y(d)$. Again we see the two bands. For $A_y(d)$ the 3NF moves theory beautifully into the data, but for $A_y(N)$ the 3NF effect which we have taken into account up to now is negligible and one is left with a serious problem, called the low-energy A_y puzzle.

In case of the deuteron break up process three nucleons emerge from the reaction region. Due to kinematical constraints (energy and momentum conservation) it is sufficient to detect two nucleons, which fixes then also the momentum of the third nucleon. We select

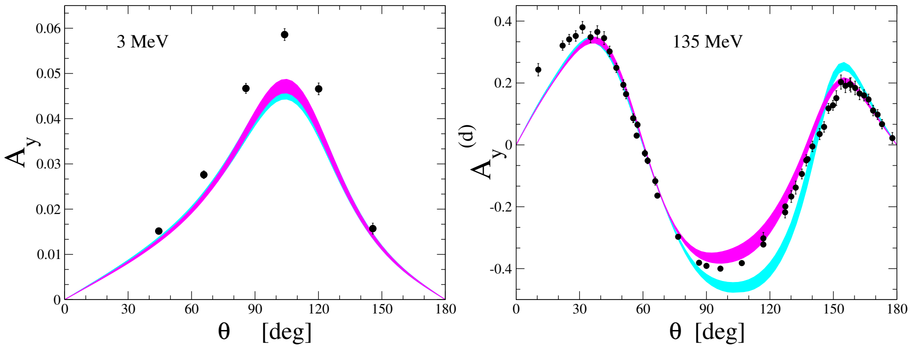

Figure 5. Nucleon and deuteron vector analyzing powers in elastic nucleon-deuteron scattering.

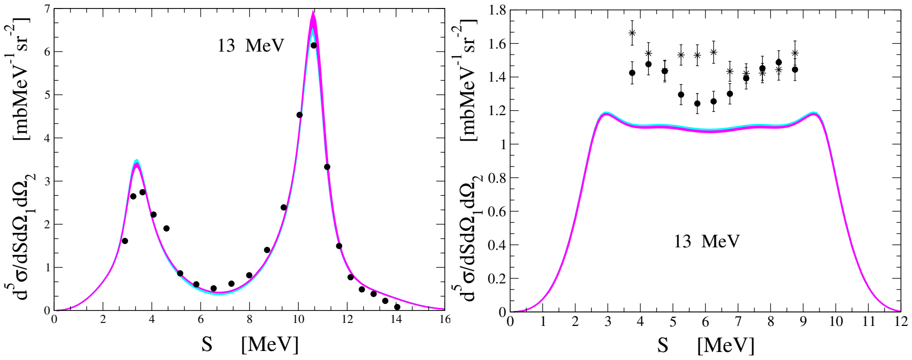

Figure 6. Examples of the breakup cross sections.

in Fig. 6 two examples out of an unlimited number of different breakup configurations^2. In both cases the frequency of the break up events (cross section) is displayed as a function of the way the energy is distributed over the three final particles (parameter S). The left figure is for a situation that two of the final nucleons leave the interaction region in the same direction; in the right figure the final three nucleons leave under 120 ° interparticle angles in a specially oriented plane. We see a rich structure in the number of events in the first case, which is beautifully described in theory when only NN forces act (3NF effects are negligible) and we see a clear discrepancy in the second case. Though the data fluctuate a bit there are clearly more events in nature than in theory and 3NF effects considered up to now turned out to be negligible.

Apparently there are successes and failures. These numerically rigorous calculations stimulated world wide experimental activities at several accelerator facilties to enrich and improve the experimental data and to challenge theory even stronger. One can say that these few-nucleon studies are a perfect tool to probe and finally fix the Hamiltonian underlying low- energy nuclear physics.

2.3 The Lightest Hypernuclei

Hyperon-nucleon (YN) forces are not strong enough to form two-body bound states. Thus the lightest hypernucleus ${}^{3}_{\Lambda}$H, the so called hypertriton, plays the role of the deuteron in hypernuclear physics. Next come the two hypernuclei ${}^{4}_{\Lambda}$H and ${}^{4}_{\Lambda}$He. We shall investigate these three nuclei using the FY equations and the most modern YN forces from the Nijmegen group together with the high precision nuclear forces used before. The numerical challenge is much higher than for nuclei since there are more coupled equations due to the distinguishability of the hyperon from the nucleon and due to an important dynamical effect. There are strong transitions which change the Λ-hyperon into a Σ-hyperon during the interaction of the hyperon with a nucleon. This $\Lambda - \Sigma$ conversion brings in an interesting facet: the hypertriton is at one moment a system of Λ, neutron and proton and in another moment a system of Σ, neutron and proton. Our results[6] for the separation energies (there is a fixed link to binding energies) based on two versions of the Nijmegen YN forces are displayed in Table 3. The lightest hypernucleus ${}^{3}_{\Lambda}$H is well described by only one of the two YN forces. Because of the very low separation energy its dominant structure is a deuteron surrounded far out by a Λ particle. The other appearance of the same hypernucleus is a deformed deuteron with a Σ-hyperon sitting very close by. This fascinating fluctuating picture is depicted schematically in Fig. 7. Quantitatively the most probable distance of the Λ-hyperon from the center of mass of the two nucleons (deuteron) is 11 fm, which means that the Λ particle in ${}^{3}_{\Lambda}$H remains most of the time outside the range of the strong interaction. Nuclei with this astonishing property are called "halo" nuclei and ${}^{3}_{\Lambda}$H is a very spectacular one.

NN and YN forces	${}^{3}_{\Lambda}$H	${}^{4}_{\Lambda}$H	${}^{4}_{\Lambda}$H*	${}^{4}_{\Lambda}$He	${}^{4}_{\Lambda}$H*
Nijm'93 + SC89	0.143	1.80	–	2.14	0.02
Nijm'93 + SC97e	0.023	1.47	0.73	1.54	0.72
Experimental value	0.130 ± 0.050	2.04	1.00	2.39	1.24

Table 3. Theoretical and experimental separation energies of the lightest hypernuclei.

Regarding now the separation energies for the 4-body hypernuclei we face serious discrepancies as displayed in Table 3. Neither the separation energies in relation to the ground state nor to the excited state (marked with a star) are acceptably close to the data and this for both YN force models. It is only due to our rigorous calculations that this has been discovered. Previous approximate evaluations have hidden these facts. Because of the lack of hyperon beams the YN data set is extremely scarce and therefore the separation energies of the light hypernuclei have to serve as the important test cases for probing theoretical models of YN forces. Without supercomputers this could not have been carried out.

2.4 Probing Nuclear Dynamics by Photons

The interaction of a photon with a charged particle is weak and can be treated perturbatively. This opens the way to probe the strong dynamics, which can not be treated perturbatively, in a well controlled manner. Worldwide electron and photon facilities, like MAMI

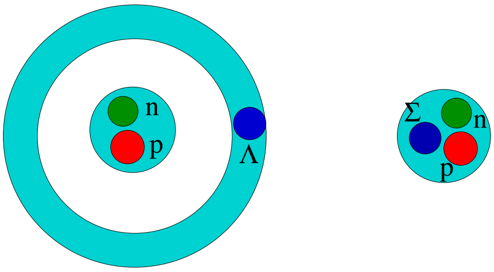

Figure 7. Schematic picture of the Λ and Σ appearances in the ${}^3_\Lambda$H wave function.

and ELSA in Germany, take advantage of that to investigate nuclei and nucleons themselves. With respect to nuclei the very lightest ones are more and more in the focus, since few- nucleon systems can be treated rigorously. Regarding for instance electron scattering on ^{3}He various processes can occur: elastic scattering where in the final state ^{3}He emerges intact and inelastic scattering, where in the final state fragments of ^{3}He occur, either a proton and a deuteron or two protons and a neutron. Moreover one can fix the spin orientations of the electron and of the nucleus ^{3}He in the initial state before the reaction takes place, which increases the amount of information one can extract from the measurement. Here we have to restrict ourselves to just one process, inclusive electron scattering of polarized electrons on polarized ^{3}He: $\overrightarrow{{}^3\mathrm{He}}(\vec{e}, e')$. This is illustrated in Fig. 8. On the left we see an electron emerging from the accelerator and its spin (red arrow) is oriented parallel to its velocity. Then the electron emits a photon, which travels to the right and the electron changes its direction of flight by an angle θ. This final electron (denoted by e') is then registered in a detector. The photon hits the nucleus ^{3}He and breaks it apart. The outcoming nucleons are not measured in that so called inclusive reaction.But on the way out of the reaction zone they interact strongly with each other by the nuclear forces, which is indicated by the wiggly lines. It is a property of the nuclear forces that the total spin of ^{3}He is carried to more than 90 % by the neutron (in green), while the spins of the two protons are oriented opposite to each other, which cancels their effect. This opens the fascinating possibility to extract neutron properties if one selects judiciously those pieces of the photon interaction with the nucleon which are sensitive to its spin. This can be achieved by performing the measurement as depicted in Fig. 8 twice, one with the spin of the initial electron pointing into the direction of flight and one pointing into the opposite direction. Then one forms the ratio of the difference of the number of measured events to their sum. That ratio called an asymmetry, A, is plotted in Fig. 9 as a function of the energy ω which the photon carries and injects into the nucleus. First of all we see the experimental results presented by open circles. Then we see the green curve, which describes the theoretical results by neglecting all interactions among the final nucleons. This is not at all the case in nature and it has to be taken into account properly and then leads to the blue curve, which now already comes closer to the data. The last very important dynamical ingredient comes from a closer look into the mechanism of the photon absorption by ^{3}He. There are basically two processes illustrated in Fig. 10. Either the photon is absorbed just by one nucleon and the others are spectators or it is absorbed by the charged mesons during their flight from one nucleon

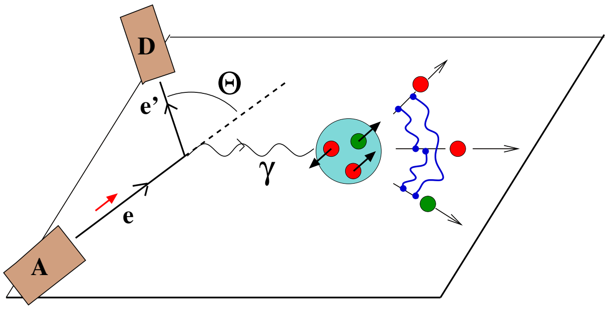

Figure 8. Schematic view of the ${}^3\vec{\mathrm{He}}(\vec{e}, e')$ experiment.

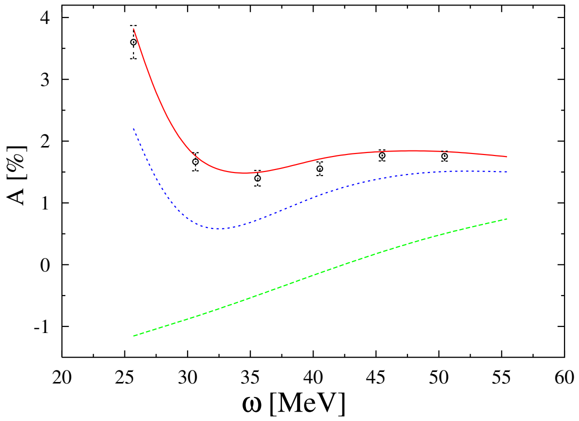

Figure 9. The cross section asymmetry A with respect to the initial electron helicity in the ${}^3\vec{\mathrm{He}}(\vec{e}, e')$ experiment.

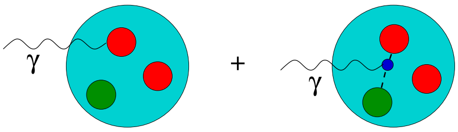

Figure 10. Two mechanisms of the photon absorption in nuclei.

to another. It is the inclusion of the second type of the photon absorption process which finally brings theory right into the data (red curve)[7].

This is of course an important result and insight by itself but one should add that this experiment and theoretical analysis has been performed also with the aim to extract electromagnetic properties of the neutron, namely its electromagnetic form factors. This has been achieved beautifully[8] and can, because of lack of space, not described here further. The knowledge of the electromagnetic form factors provides important information how charges and magnetic properties are distributed inside a neutron. Since there are no free neutron targets experiments on the deuteron or ^{3}He, each with one neutron inside, are mandatory to extract that information on the neutron. The theoretical analysis of that requires as we have seen a profound knowledge of the nuclear forces and a very reliable understanding of these light nuclei and of the interaction processes going on among the nucleons after the photon has been absorbed.We were able to perform the investigations of these electromagnetically induced processes based on the basic research described before.

Another important remark should be added. The preparation of spin oriented ^{3}He nuclei necessary for this sort of experiments required world wide efforts and was carried through in view of gaining access to the properties of the neutron. Thus it was pure basic research. It could not been foreseen that polarized ^{3}He can be used to "illuminate" the human lungs and to localize defects of various types in the lungs in an unprecedented clear manner. Polarized ^{3}He, developed on the basis of pure nuclear research is a beautiful example for a spin off having important application in medicine.

3 Outlook

Our results clearly demonstrate that the aim to establish the basic Hamiltonian for low-energy nuclear physics expressed in nucleon degrees of freedom is reachable. Three-nucleon forces will thereby play an important role. Chiral perturbation theory, used as a new approach to nuclear forces, will provide an input linked to the underlying QCD. Intensive research along this new line is being performed[5]. Without supercomputers nuclear dynamics could not be probed and the growing amount of experimental data could not be analyzed.

Acknowledgments

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