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The Barnes-Hut Tree Algorithm

and its highly scalable parallel implementation PEPC

10.09.2013 | Mathias Winkel | Jülich Supercomputing Centre

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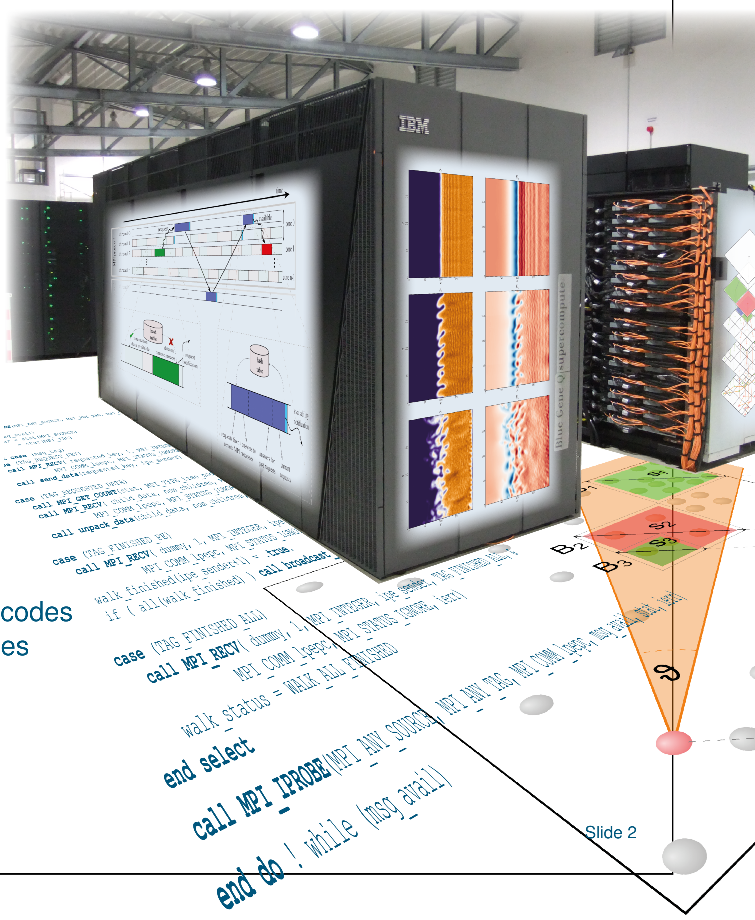


Electrostatics recap – Particle representation

Potential due to general charge distribution $\rho(r)$ is given by Poisson's equation:

$$\nabla^2\phi(\vec{r}) = -4\pi\rho(\vec{r}),$$

which has a general solution

$$\phi(\vec{r}) = \int_{V'} \frac{\rho(\vec{r}')}{|\vec{r}-\vec{r}'|} d^3r'.$$

Discretization of charge density with point particles

$$\rho(\vec{r},t) = \sum_j q_j \delta(\vec{r}_j - \vec{r})$$

yields pairwise potential sum

$$\phi_i(\vec{r}_i,t) = \sum_{j\neq i} \frac{q_j}{|\vec{r}_i - \vec{r}_j|}.$$

Dynamics governed by Newton's law:

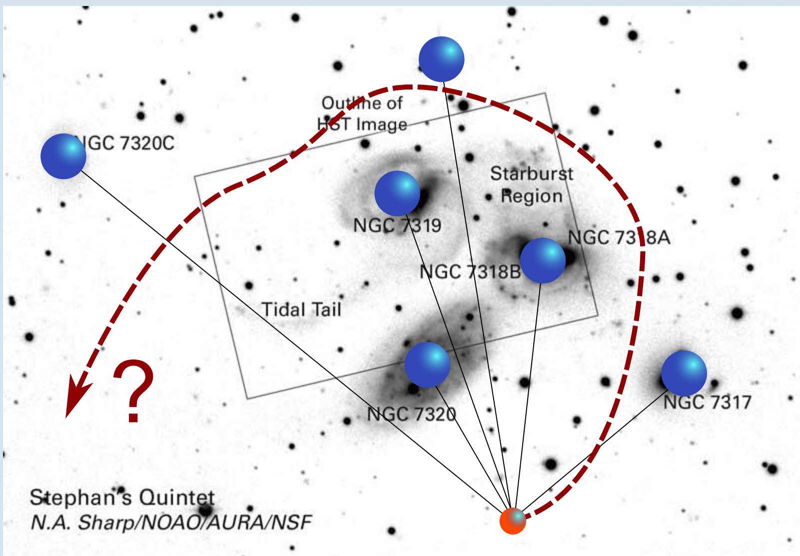
$$\frac{\mathrm{d}}{\mathrm{d}t}\vec{v}_i = -\frac{q_i}{m_i}\nabla\phi(\vec{r}_i) \quad ; \qquad \frac{\mathrm{d}}{\mathrm{d}t}\vec{r}_i = \vec{v}_i.$$

$\longrightarrow$ Basis for MD with charged particles, gravitational dynamics, . . .

Multipole algorithms

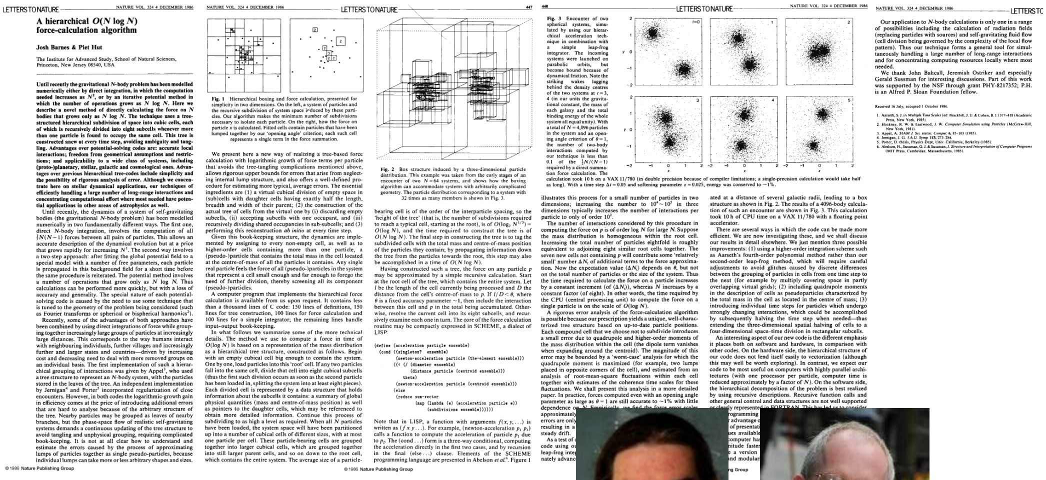
The idea behind. . .

Approximate galaxies by point masses



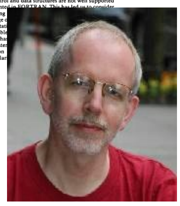
Multipole algorithms

The pioneers


LETTERS TO NATURE

A hierarchical $O(N \log N)$ force-calculation algorithm

Josh Barnes & Piet Hut



Barnes-Hut Tree Code:

particle-cluster interactions

[J. Barnes & P. Hut, Nature **324**, 446 (1986)]
[A. Appel, SIAM J. on Sci. and Stat. Comp. **6**, 85 (1985)]

Josh Barnes Piet Hut

Multipole expansion

Better precision with more expansion terms

- applied for potential of a particle cloud with M particles around origin and observer at $\vec{a}$:

$$\phi\left(\vec{a}, \{\vec{r}_1 \dots \vec{r}_M\}\right) = \sum_{i=1}^{M} q_i \frac{1}{|\vec{a} - \vec{r}_i|}$$

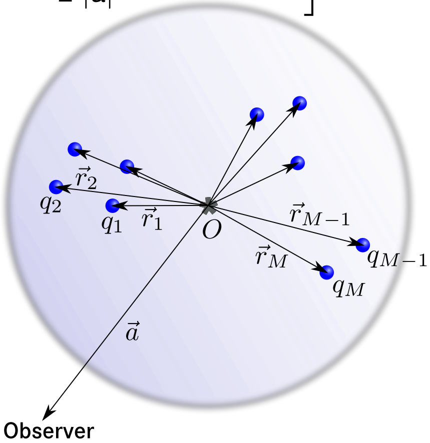
$$= \left[\frac{1}{|\vec{a}|} \cdot Q + \frac{1}{|\vec{a}|^3} \cdot \vec{a}^{\mathsf{T}} \vec{Q} + \frac{1}{2} \frac{1}{|\vec{a}|^5} \cdot \vec{a}^{\mathsf{T}} \bar{Q} \vec{a} + \dots \right]$$

- with multipole moments

monopole $\quad Q = \sum_{i=1}^{M} q_i$

dipole $\quad \vec{Q} = \sum_{i=1}^{M} q_i \cdot \vec{r}_i$

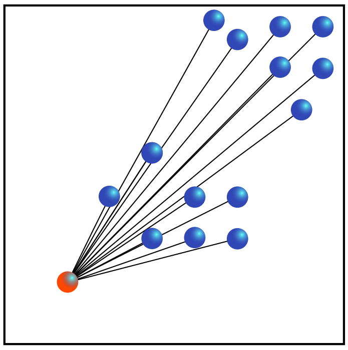
quadrupole $\quad \bar{Q} = \sum_{i=1}^{M} q_i \cdot \left[\vec{r}_i \otimes \vec{r}_i - |\vec{r}|^2 \right]$



Multipole algorithms

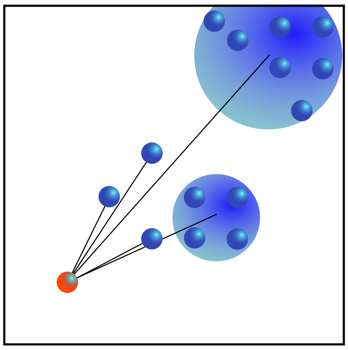
. . . different flavors

Direct Summation



total interactions: $\mathcal{O}(N^2)$

Treecode



$\mathcal{O}(N \log N)$

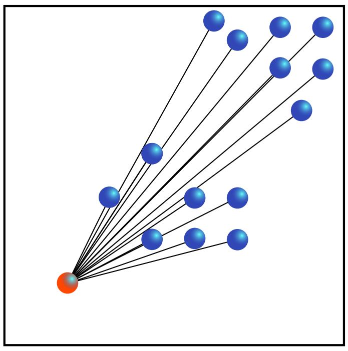
- Clustering of remote particle groups
- Interaction via their multipole expansion
- Hierarchical algorithm for finding interaction partners

[J. Barnes & P. Hut, Nature **324**, 446 (1986)]

Multipole algorithms

. . . different flavors

Direct Summation

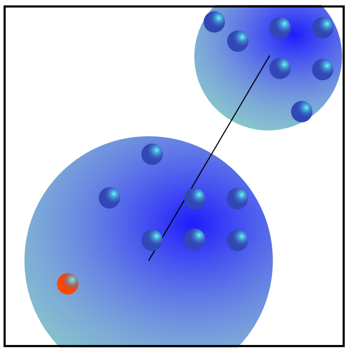


total interactions: $\mathcal{O}(N^2)$

- Clustering of remote and local particle groups
- Interaction via their multipole expansions and elaborate shifting/conversion operators
- Hierarchical algorithm for finding interaction partners

Fast Multipole Method

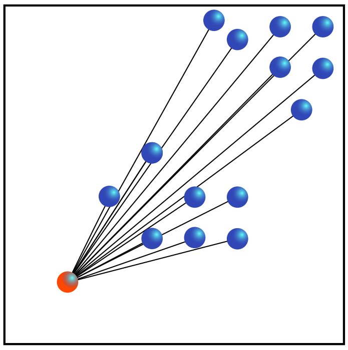
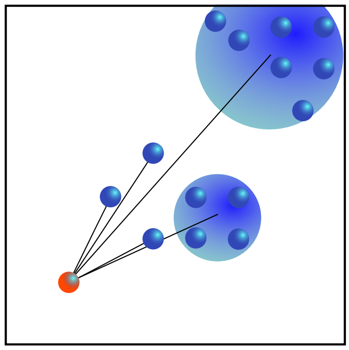
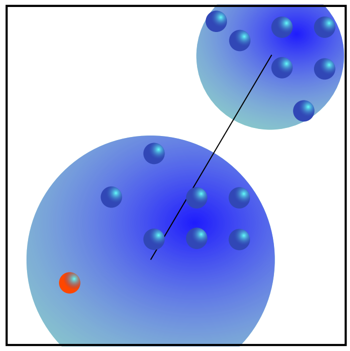


$\mathcal{O}(N)$

[L. Greengrad & V. Rokhlin, J. Copmp. Phys. **73**, 325 (1987)]

Multipole algorithms

. . . different flavors

Direct Summation	Treecode	Fast Multipole Method
		
total interactions: $\mathcal{O}(N^2)$	$\mathcal{O}(N \log N)$	$\mathcal{O}(N)$

- good load balancing
- kernel flexibility
- simple parallelization

$\Longleftrightarrow$

- less floating point ops
- more logistics
- elaborate parallelization

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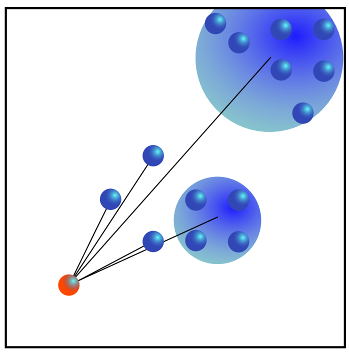
Multipole algorithms

. . . different flavors

- straightforward load-balancing for dynamic simulations
- extensibility
- exchange of interaction kernel

Treecode



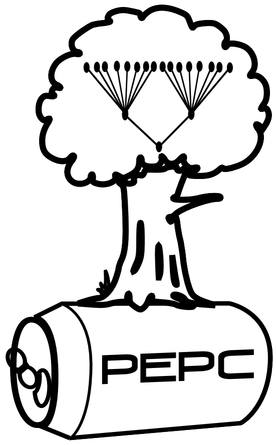
$\mathcal{O}(N \log N)$

$$\phi(\vec{r},t) = \sum_{i=1}^{N}{}' q_i\, \varphi_i(\vec{r} - \vec{r}_i(t))$$

- Coulomb/Gravitation: $\varphi(\vec{r}) \propto \frac{1}{|\vec{r}|}$
- reg. Coulomb: $\varphi(\vec{r}) \propto \frac{1}{\sqrt{\vec{r}^2+\alpha^2}}$
- vector-valued potentials (vortex method)
- . . .

PEPC – The Pretty Efficient Parallel Coulomb Solver

- developed at JSC since 2003
- several modifications to original algorithm and parallelization
- world records in scalability and particle number for Barnes-Hut tree codes
- part of the ScaFaCoS library
  (with some restrictions due to technical reasons)



[M. Winkel *et al.*, Comp. Phys. Comm. 187, 880–889 (2012)]
PEPC is open-source, freely available via `www.fz-juelich.de/ias/jsc/pepc`
mailing list for reaching all developers: `pepc@fz-juelich.de`

The Barnes-Hut Tree Algorithm

Part II: The Barnes-Hut tree code

10.09.2013 | Mathias Winkel

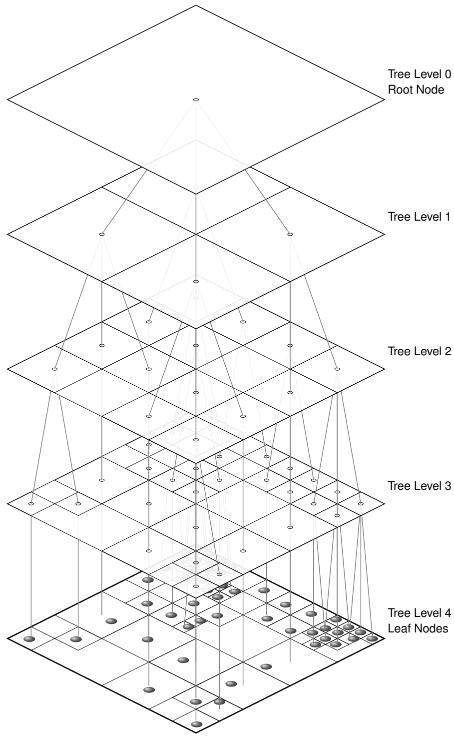
The Barnes-Hut tree code

Tree construction

- Recursive spatial subdivision...
- ...until every particle lies in its own box
  $\longrightarrow$ tree leaves
- Lower-level boxes
  $\longrightarrow$ tree nodes

$\rightarrow$ Hierarchy of spatially coarsened particle groups

$\rightarrow$ Potential interaction partners



Multipole shifting rules

- center-of-charge

$$\hat{\vec{r}}_{\mathrm{coc}} = \frac{\sum_{i=1}^{N_c} |Q_i| \cdot \vec{r}_{\mathrm{coc},i}}{\sum_{i=1}^{N_c} |Q_i|}$$

- shift vector

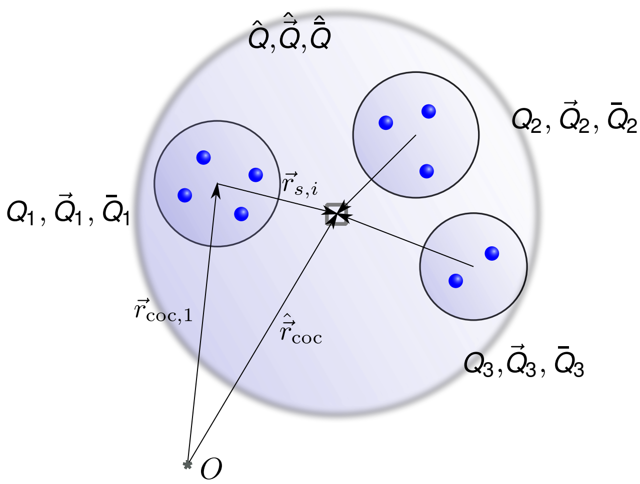
$$\vec{r}_{\mathrm{s},i} = \hat{\vec{r}}_{\mathrm{coc}} - \vec{r}_{\mathrm{coc},i}$$

- shifting rules for multipole moments

$$\hat{Q} = \sum_{i=1}^{N_c} Q_i$$

$$\hat{\vec{Q}} = \sum_{i=1}^{N_c} \left(\vec{Q}_i - Q_i \cdot \vec{r}_{\mathrm{s},i} \right)$$

$$\hat{\bar{Q}} = \sum_{i=1}^{N_c} \left(\bar{Q}_i - \vec{r}_{\mathrm{s},i} \otimes \vec{Q}_i - (\vec{r}_{\mathrm{s},i} \otimes \vec{Q}_i)^{\mathrm{T}} + \vec{r}_{\mathrm{s},i} \otimes \vec{r}_{\mathrm{s},i} \cdot Q_i \right)$$



The Barnes-Hut tree code

Tree construction

- Recursive spatial subdivision…
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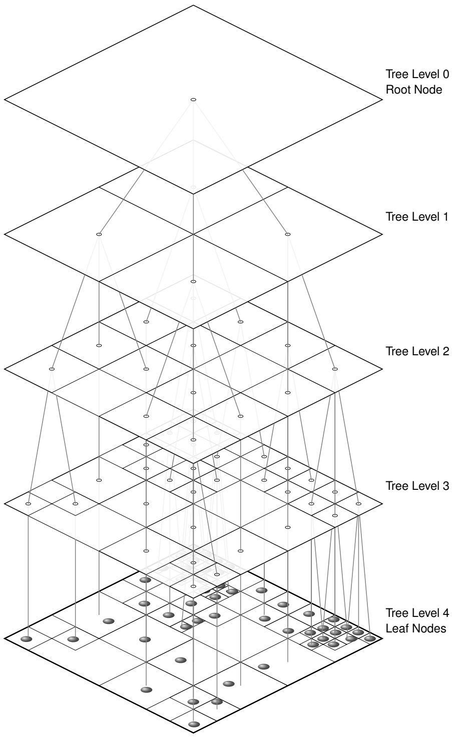
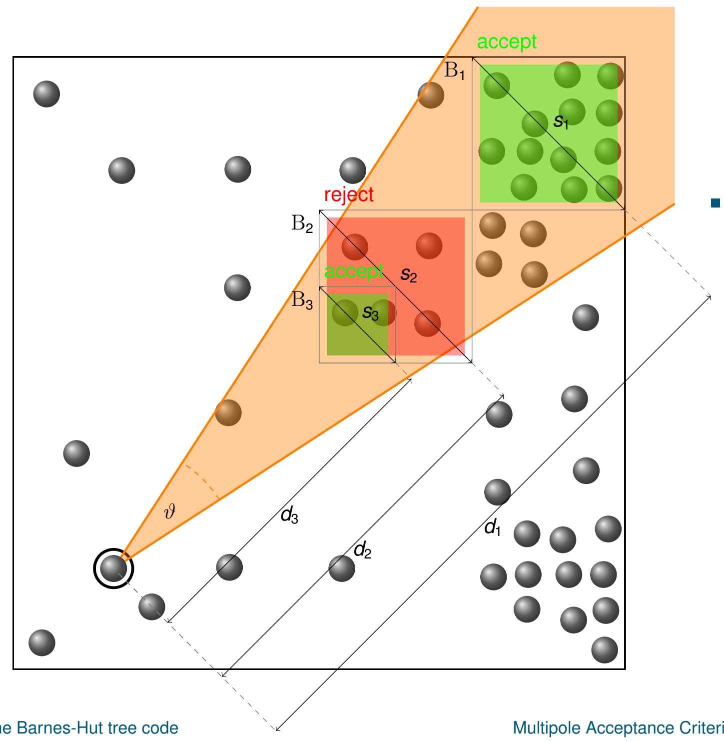
How to systematically identify interaction partners ??



The Barnes-Hut tree code

Multipole Acceptance Criterion (MAC)

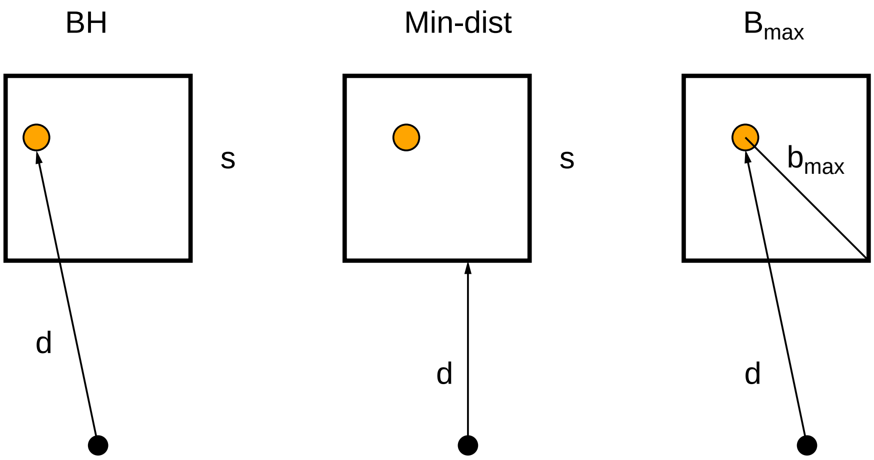


- cluster substructure ignored if

$$\frac{\text{clustersize}}{\text{clusterdistance}} = \frac{s}{d} \leq \vartheta$$

$$\mathcal{O}(N \log N): \ 0.0 < \vartheta$$
$$\mathcal{O}(N^2): \ 0.0 = \vartheta$$

MAC variations



Tree traversal - identification of interaction partners

```
for all particle in particle_list do
    todo_stack.clear()
    node ← root                          traversal for the particle starts at root node
    repeat
        if (MAC_OK(particle, node)) and not (particle ∈ node) then      MAC evaluation
                                                                   interaction allowed
            call INTERACT(particle, node)
              due to interacting with this node, its children do not have to be considered
        else
                                          the MAC requires the node to be further resolved
            todo_stack.push(node.children)                    proceed vertically in tree
        end if
    until IS_INVALID(node ← todo_stack.pop())
                          todo_stack is empty for this particle – its traversal is complete
end for
```

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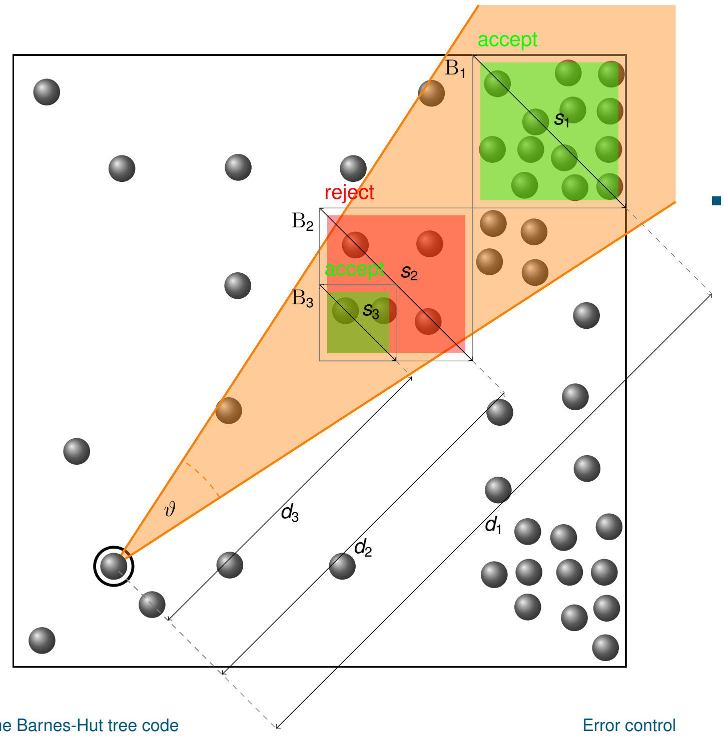
The Barnes-Hut tree code

Multipole Acceptance Criterion (MAC)



- cluster substructure ignored if

$$\frac{\text{clustersize}}{\text{clusterdistance}} = \frac{s}{d} \leq \vartheta$$

$$\mathcal{O}(N \log N): \; 0.0 < \vartheta$$
$$\mathcal{O}(N^2): \; 0.0 = \vartheta$$

What about precision / error ??

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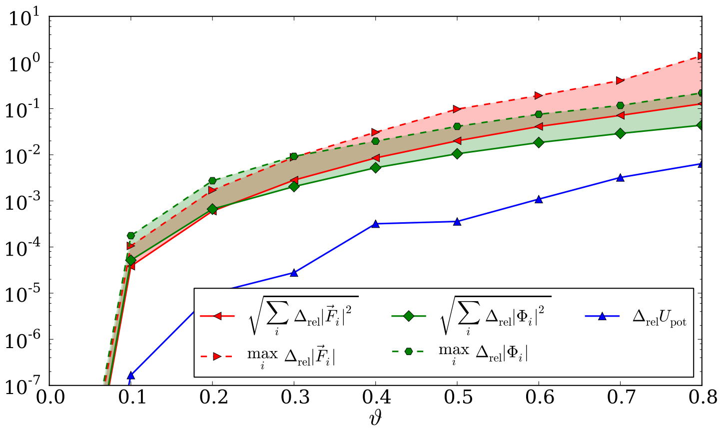
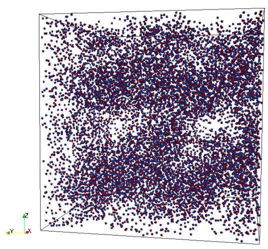
Precision of the tree code depending on ϑ





Example system: 12,960 particles open-boundary extract of a melting NaCl crystal that includes some density variations

↗ Hands-On Session (Wednesday)

Proper error control

Generalized algebraic kernel (Coulomb, Plummer, r^{-n}, etc.):

$$g_\lambda(r) = \sum_{l=0}^{\lambda} \frac{a_l \cdot r^{2l}}{(r^2+\sigma^2)^{\lambda+\frac{1}{2}}}$$

Plummer:

$$g_0(r) = \frac{a_0}{(r^2+\sigma^2)^{\frac{1}{2}}}$$

High order (vortex):

$$g_3(r) = a_3 \cdot \frac{r^2+\frac{3}{2}\sigma^2}{(r^2+\sigma^2)^{\frac{3}{2}}}$$

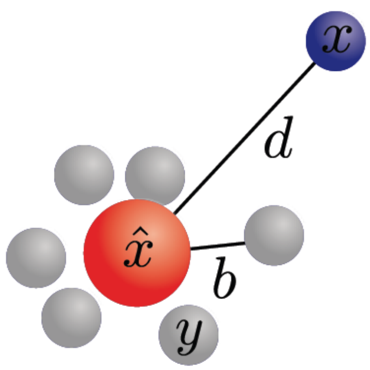
[R. Speck, PhD Thesis, U. Wuppertal (2011)]

Error bound

Can show that error per interaction has upper bound

$$\varepsilon^{\tau}_{(p+1)} \leq \mathcal{D}_{2\tau-1}(p+1) \cdot \frac{M_0}{(d-b)^{2\tau}} \cdot \left(\frac{b}{d-b}\right)^{p+1}$$

with monopole term M_0, $b = \sup|y-\hat{x}|$, polynomial $\mathcal{D}$ of rank $2\tau - 1$.



[R. Speck, PhD Thesis, U. Wuppertal (2011)]

The Barnes-Hut Tree Algorithm

Part III: Periodic boundary conditions for tree codes

10.09.2013 | Mathias Winkel

The Barnes-Hut tree code

Tree construction

- Recursive spatial subdivision...
- ...until every particle lies in its own box
  $\longrightarrow$ tree leaves
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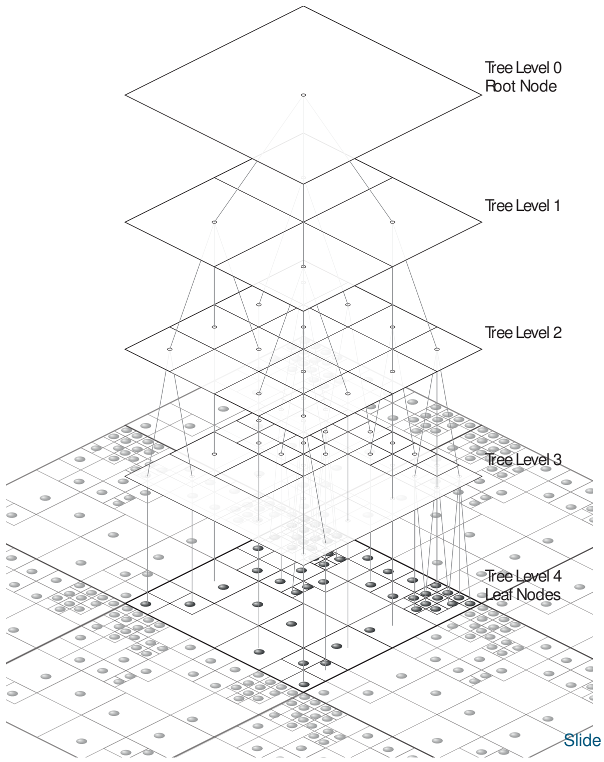
$\rightarrow$ Hierarchy of spatially coarsened particle groups

$\rightarrow$ Potential interaction partners

but

- $\mathcal{O}(10^{23})$ particles still infeasible
- Artificial surfaces

$\longrightarrow$ Periodic boundaries



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Periodic boundary conditions for tree codes

An adaptation from the Fast Multipole Method

Bipolar expansion of the inverse distance

$$\frac{1}{|\vec r_1 - (\vec r_2 + \vec n)|} = \sum_{l=0}^{\infty}\sum_{m=-l}^{l}\sum_{j=0}^{\infty}\sum_{k=-j}^{j} (-1)^j\, \mathcal{O}_l^m(\vec r_1)\, \mathcal{M}_{l+j}^{m+k}(\vec n)\, \mathcal{O}_j^k(\vec r_2)$$

- Multipole coefficients

$$\mathcal{O}_l^m(\vec r = [r,\theta,\varphi]) = \frac{r^l}{(l+m)!} \mathrm{P}_{lm}(\cos\theta) e^{-im\varphi}$$

- Taylor coefficients

$$\mathcal{M}_l^m(\vec r = [r,\theta,\varphi]) = \frac{(l-m)!}{r^{l+1}} \mathrm{P}_{lm}(\cos\theta) e^{im\varphi}$$

- associated Legendre polynomials $\mathrm{P}_{lm}(z)$

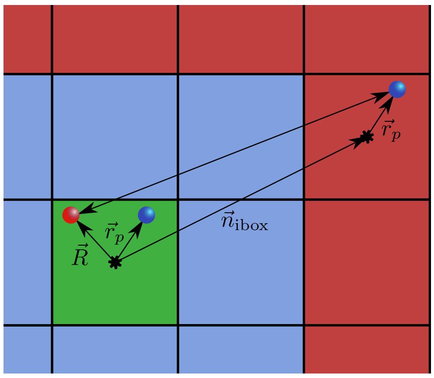
↗ FMM-Lecture by Ivo Kabadshow (Thursday)

Periodic boundary conditions for tree codes

An adaptation from the Fast Multipole Method

- Periodic continuation of the interaction potential

$$\Phi^{\mathrm{lat}}(\vec R) = \sum_{\vec n \in \mathbb{Z}^3} \sum_{p=1}^{N} \frac{q_p}{\left|\vec R - (\vec r_p + \vec n)\right|}$$



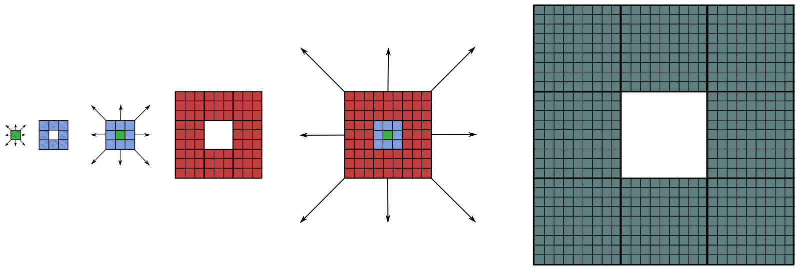
$$\Phi^{\mathrm{lat}}(\vec R) = \sum_{l=0}^{\infty}\sum_{m=-l}^{l} \mathcal{O}_l^m(\vec R) \underbrace{\sum_{j=0}^{\infty}\sum_{k=-j}^{j} (-1)^j \underbrace{\sum_{\vec n \in \mathbb{Z}^3} \mathcal{M}_{l+j}^{m+k}(\vec n)}_{\mathcal{L}_{l+j}^{m+k}} \underbrace{\sum_{p=1}^{N} q_p\, \mathcal{O}_j^k(\vec r_p)}_{\omega_j^k}}_{\mu_l^{m,\mathrm{cent}}}$$

$$|\vec n| > |\vec R| + |\vec r_p|$$

[M. Challacombe *et al.*, J. Chem. Phys. **107**, 10131 (1997)]

Periodic boundary conditions for tree codes

A renormalization approach for the real space sum



Number of included boxes after n iterations

$$N_n^{\mathrm{FF},dim} \propto 3^{n \cdot dim}$$

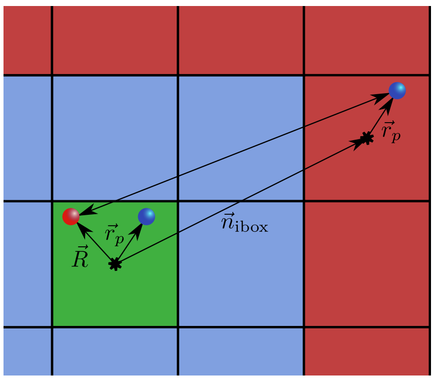
[G. Kudin & G. Scuseria, J. Chem. Phys. **121**, 2886 (2004)]

Periodic boundary conditions for tree codes

An adaptation from the Fast Multipole Method

- Periodic continuation of the interaction potential

$$\Phi^{\mathrm{lat}}(\vec{R}) = \sum_{\vec{n} \in \mathbb{Z}^3} \sum_{p=1}^{N} \frac{q_p}{\left|\vec{R} - (\vec{r}_p + \vec{n})\right|}$$



$$\Phi^{\mathrm{lat}}(\vec{R}) = \underbrace{\sum_{l=0}^{\infty} \sum_{m=-l}^{l} \mathcal{O}_l^m(\vec{R}) \sum_{j=0}^{\infty} \sum_{k=-j}^{j} (-1)^j \underbrace{\sum_{\vec{n} \in \mathbb{Z}^3} \mathcal{M}_{l+j}^{m+k}(\vec{n})}_{\mathcal{L}_{l+j}^{m+k}} \underbrace{\sum_{p=1}^{N} q_p\, \mathcal{O}_j^k(\vec{r}_p)}_{\omega_j^k}}_{\mu_l^{m,\mathrm{cent}}}$$

$$|\vec{n}| > |\vec{R}| + |\vec{r}_p|$$

[M. Challacombe *et al.*, J. Chem. Phys. **107**, 10131 (1997)]

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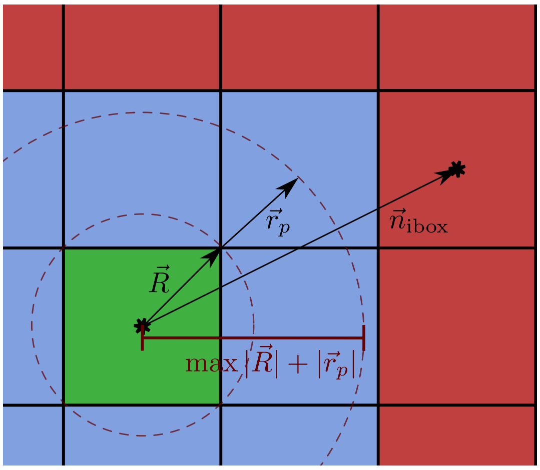
Near field contribution

- near-field treated separately to satisfy convergence criterion

$$|\vec{R}| + |\vec{r}_p| < |\vec{n}|$$

- simple loop through all NF cells
  - worst case: $N^{\text{eff}} = 27 \cdot N$
  - usually significantly less additional interactions
- too non-cubic cell violates convergence criterion

$\longrightarrow$ increase size of near field



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Dipole (Extrinsic-to-Intrinsic) Correction

- electrostatic potential in a lattice (C, L: shape, volume, ... of unit cell/full crystal)

$$\underbrace{\Phi(\vec{r}, C, L)}_{\text{Real space}} = \underbrace{\Phi^{\text{int}}(\vec{r})}_{\text{Ewald}} + \Phi^{\text{ext}}(\vec{r}, C, L)$$

- for cubic unit cell:

$$\Phi^{\text{ext}}(\vec{r}, C, L) = \frac{4\pi}{3V}(\vec{r} - \vec{r}_0) \sum_{k=1}^{N} q_k \vec{r}_k - \frac{2\pi}{3V} \sum_{k=1}^{N} q_k |\vec{r}_k|^2$$

(result of infinite summation of conditionally convergent dipole contribution)

$$\Phi^{\text{Ewald}} = \Phi^{\text{far-field}} - \Phi^{\text{ext}}$$

- alternatively: correction of unit cell dipole moment $\vec{d}$ with ficticious charges

$$\vec{p}_1 = (1,0,0)^{\text{T}} \quad \vec{p}_2 = (0,1,0)^{\text{T}} \quad \vec{p}_3 = (0,0,1)^{\text{T}} \quad \vec{p}_4 = (0,0,0)^{\text{T}}$$

$$q_1 = \vec{d}^{(1)}/L^{(1)} \quad q_2 = \vec{d}^{(2)}/L^{(2)} \quad q_3 = \vec{d}^{(3)}/L^{(2)} \quad q_4 = -(q_1 + q_2 + q_3)$$

[Redlack & Grindlay, J. Chem. Phys. **101**, 5024 (1994)]
[Kudin, Chem. Phys. Lett. **283**, 61 (1998)]

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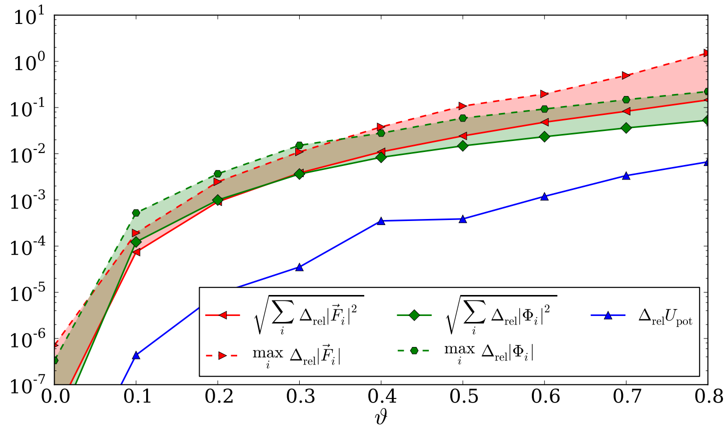
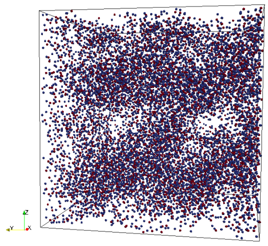
Precision of the periodic tree code depending on ϑ

Comparison with standard Ewald method

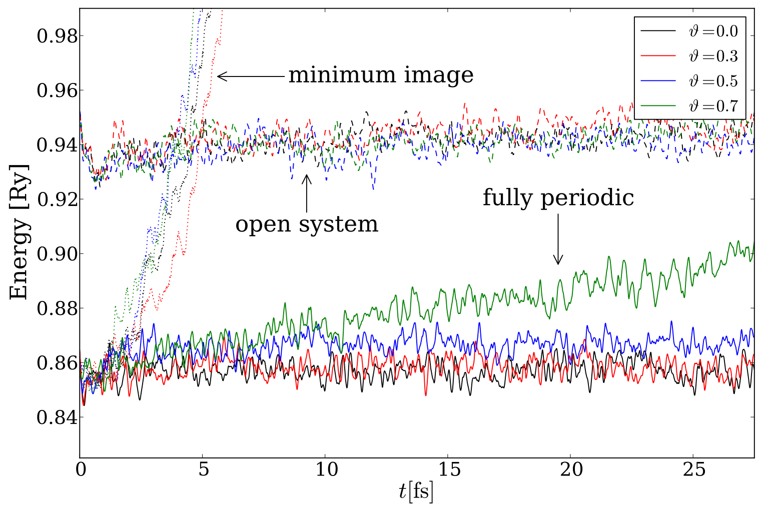


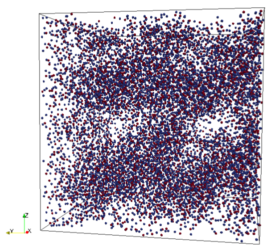


Example system: 12,960 particles melting NaCl crystal that includes some density variations, periodically continued

Precision of the periodic tree code depending on ϑ

Energy conservation with leap-frog integration





Example system: 12,960 particles melting NaCl crystal that includes some density variations, periodically continued

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Periodic boundary conditions for tree codes Precision with periodic boundary conditions Slide 32

The Barnes-Hut Tree Algorithm

Part IV: Parallelization of Barnes-Hut tree codes

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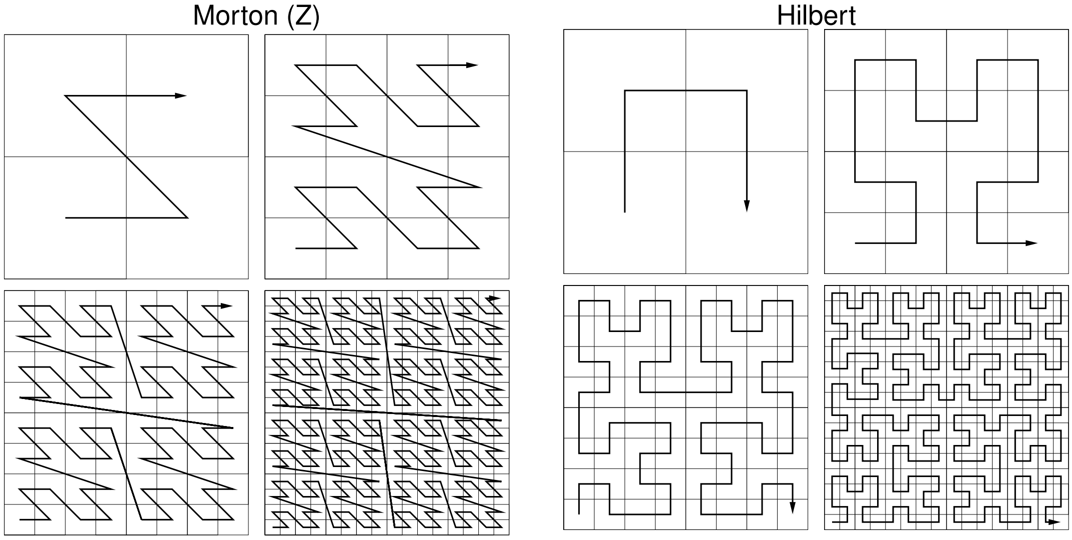
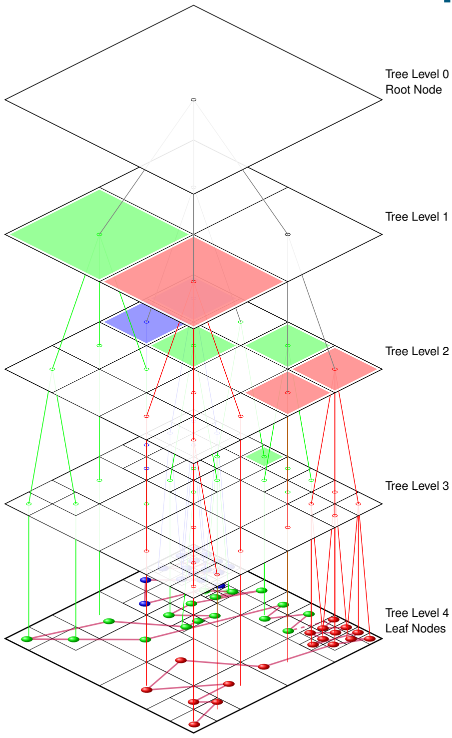
Space Filling Curve

A continuous function $f : \mathcal{I} \to \mathbb{R}^n$ of the compact set $\mathcal{I} \subset \mathbb{R}$ into $\mathbb{R}^n$ ($n \geq 2$) is called space filling curve if its image $f_*(\mathcal{I})$ has a Jordan content (area, volume, ...) greater 0.

Morton (Z) Hilbert



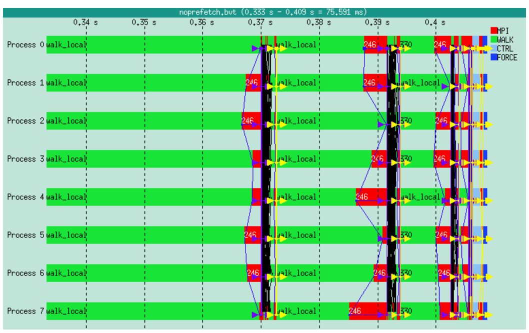
Parallel tree buildup



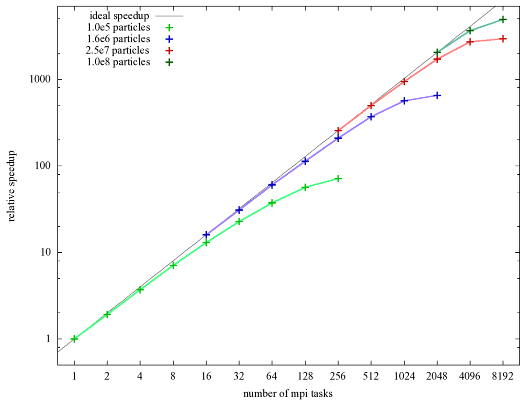
- thread particles onto space-filling curve
- partition space-filling curve (i.e. particles and tree branches above them) across MPI ranks
- distribute local roots ('branch nodes') across all MPI ranks
- build global tree above branch nodes

[J. Barnes & P. Hut, Nature **324**, 446-449 (1986)]
[M. S. Warren & J. K. Salmon, Proc. of the 1993 ACM/IEEE Conf. on Supercomputing, 12–21 (1993)]

Scaling of original pure MPI scheme



Vampir trace with 8 cores

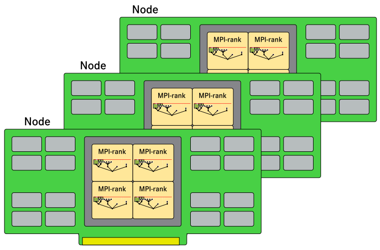


- reason for scalability problems beyond 8k cores: load-balancing issues during multi-pass tree-walks

[R. Speck, L. Arnold, P. Gibbon, J. Comp. Sci. 2, 138 (2011)]

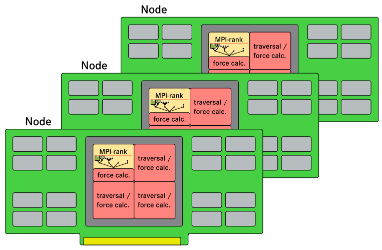
Hybrid parallelization: What do we gain?

previously:



one MPI-rank per processor core

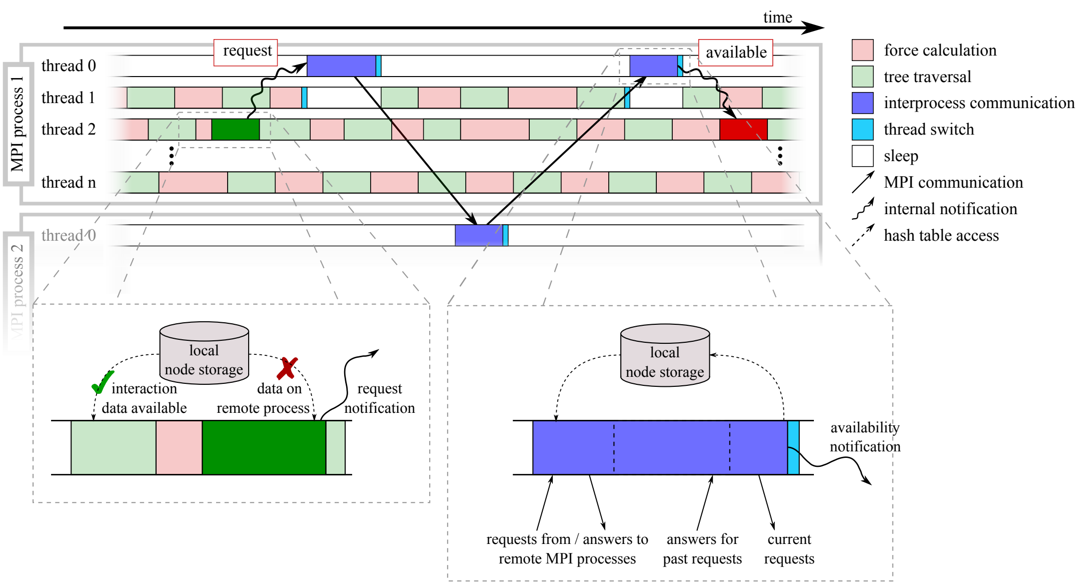
now:



one MPI-rank and several threads per compute node

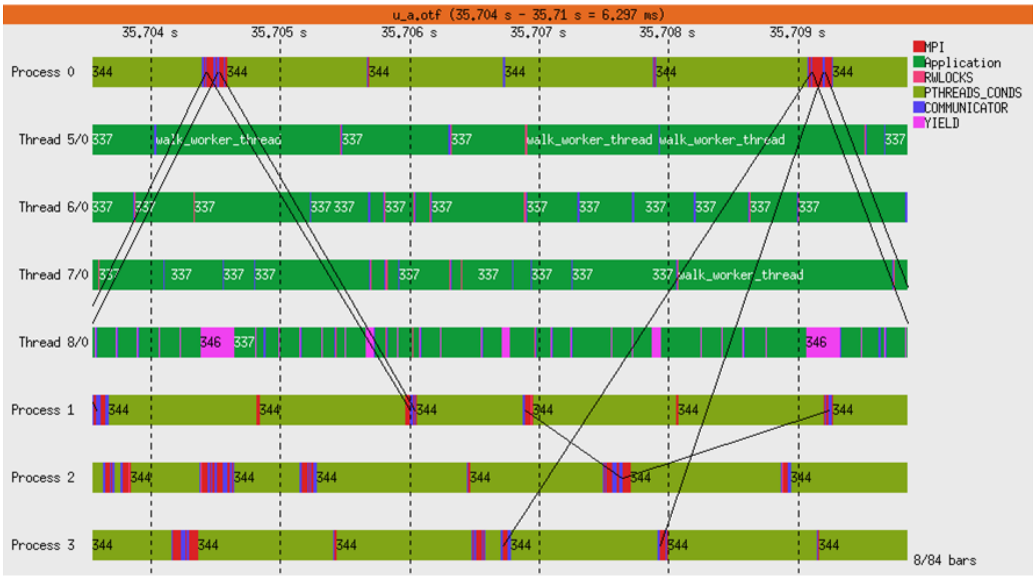
- only one tree per compute node instead of per core
- reduction of number of MPI ranks per node leads to:
  - fewer multipole/tree-node copies = less MPI communication and lower storage needed
  - reduced effective answer latency for individual tree node requests
- potential for overlap of computation and communication via dedicated communicator thread

Hybrid parallelization: MPI & PThreads in detail



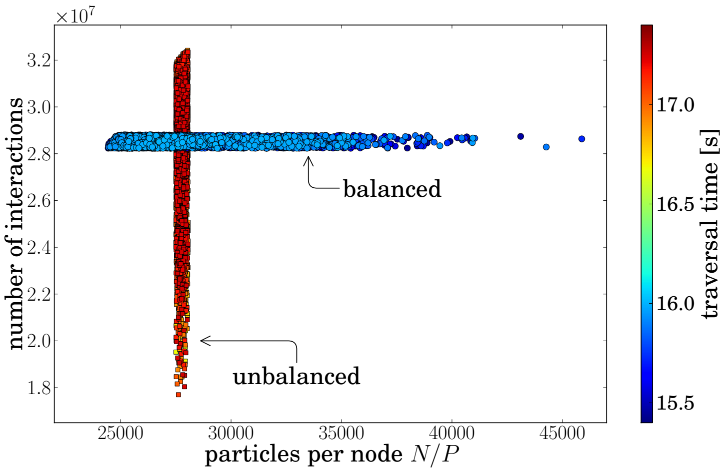
[M. Winkel *et al.*, Comp. Phys. Comm. 187, 880–889 (2012)]

Hybrid parallelization: MPI & PThreads in detail



[M. Winkel *et al.*, Comp. Phys. Comm. 187, 880–889 (2012)]

Load balancing



- number of interactions per MPI rank significant, not number of particles
- total runtime reduces significantly
- based on workload of previous simulation timestep

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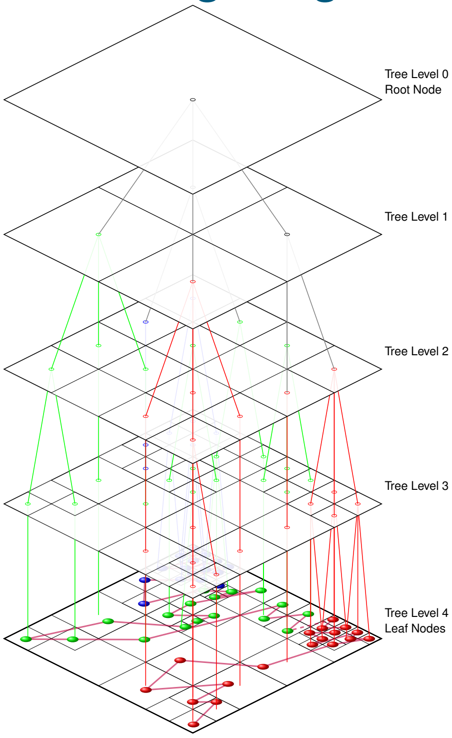
Prefetching / Eager sending



- requests for tree nodes processed by remote communicator thread
- additional payload information in request: particle position
- provisional traversal by remote communicator
  - identification of all necessary tree nodes (not just one level)
  - answer with child, grandchild, . . . nodes

$\longrightarrow$ less requests, avoids latency for sequential requests

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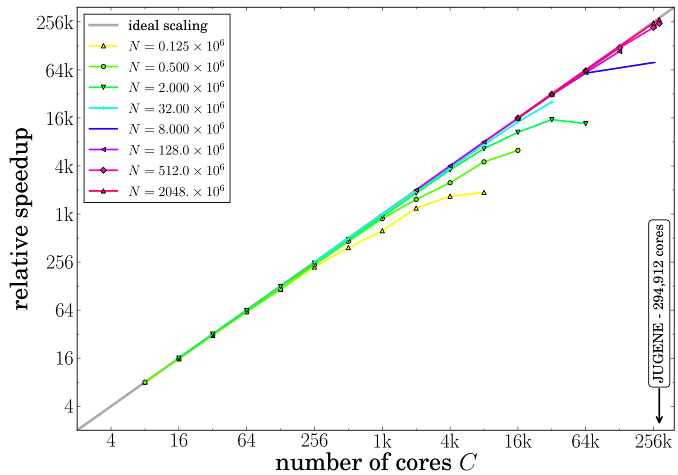
Hybrid parallelization: MPI & PThreads in action

Scaling things up – Homogeneous, quasi-neutral cube



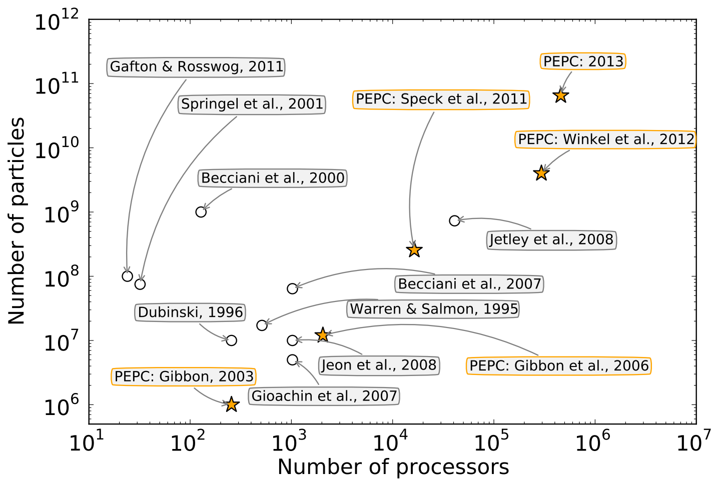
strong scaling of tree traversal

- up to 2×10^9 particles on 294 912 processors
- load-balanced

PEPC – The Pretty Efficient Parallel Coulomb Solver

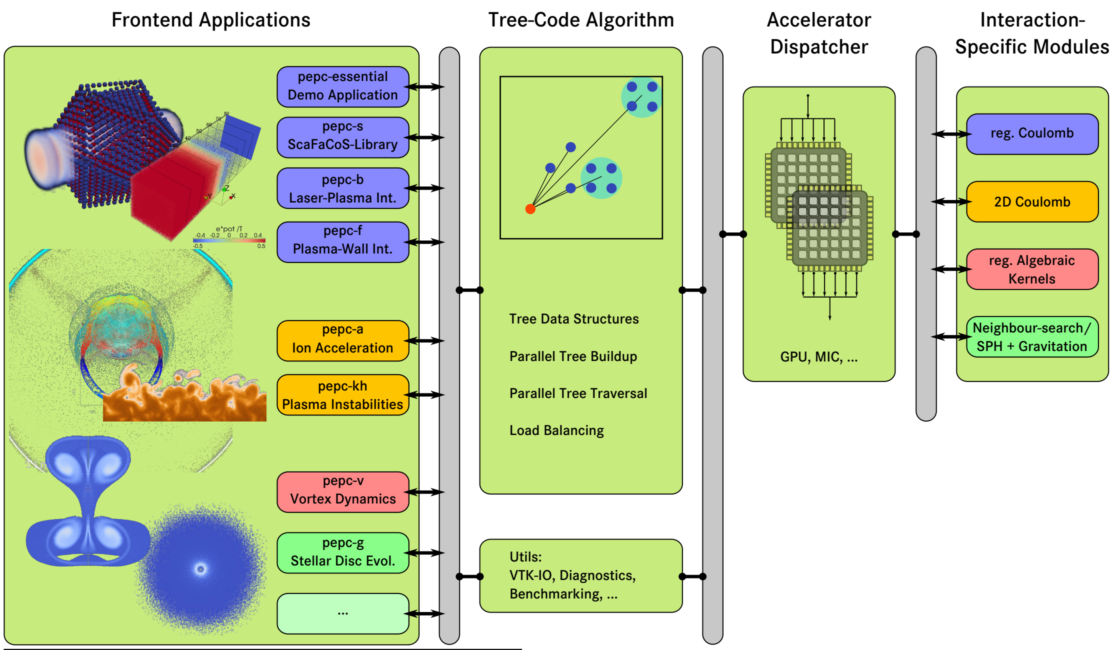
World records in scalability and particle number for Barnes-Hut tree codes

- Parallelization:
  MPI + pthreads
- Target Architectures:
  - GNU, OSX, ARM, . . .
  - JuGene (BG/P)
    2 billion particles on
    294,912 processors
  - JUQUEEN (BG/Q)
    65 billion particles on
    458,752 processors
  - JuRoPa (Nehalem cluster)
- sophisticated inter- & intra-node load balancing
- unique modular concept



PEPC is open-source, freely available via `www.fz-juelich.de/ias/jsc/pepc`
mailing list for reaching all developers: `pepc@fz-juelich.de`

PEPC – The Pretty Efficient Parallel Coulomb Solver



[M. Winkel *et al.*, Comp. Phys. Comm. **187**, 880 (2012)]

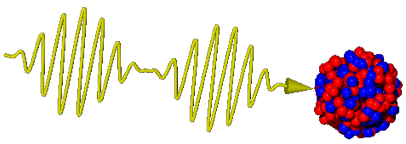
The Barnes-Hut Tree Algorithm

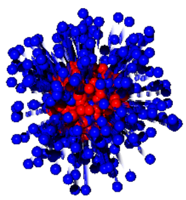
Part V: Applications

10.09.2013 | Mathias Winkel

Strongly coupled plasmas

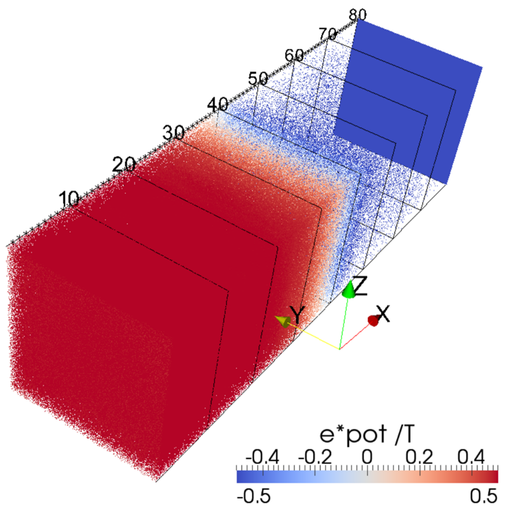
- Transport coefficients of warm, dense matter
- collective effects in nano clusters
- Strong coupling + non-equilibrium
- Laser-solid interactions, stellar interior, inertial fusion

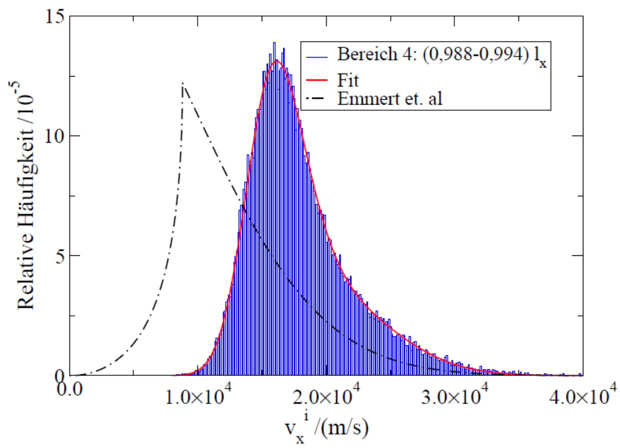




[M. Winkel, PhD thesis, RWTH Aachen (2013)]

Plasma-wall interaction in tokamaks

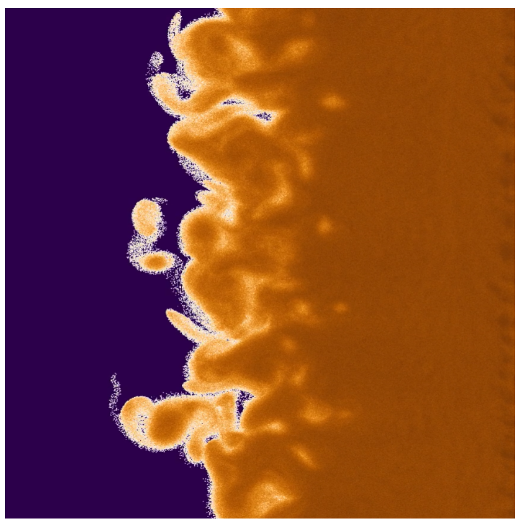




[B. Berberich, PhD thesis (2011); C. Salmagne (IEK-4), in progress]

Magnetized Plasmas

- Kelvin-Helmholtz instabilities at plasma-vacuum interfaces
- Driving mechanism:
  1. magnetised plasma slab, $n =$ const., in contact with vacuum
  2. different Larmor radii lead to different density gradient scales
  3. charge separation creates a sheared electric field
  4. sheared $\vec{E} \times \vec{B}$ flow feeds KH instability

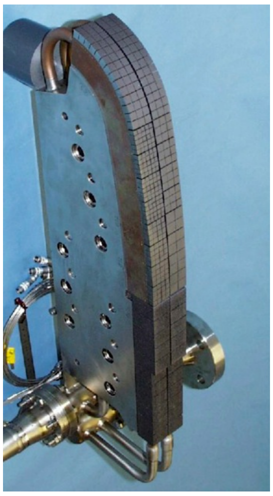


[B. Steinbusch (JSC), in progress]

Member of the Helmholtz-Association

Boundary element method

- plasma wall interaction in fusion vessels
- complex geometry: castellated tiles
- metal walls, free charges
- real world geometries can be modeled in a CAD program
- Supported boundary conditions:
  - (mixed) Dirichlet and Neumann
  - periodic
  - metal wall with floating potential



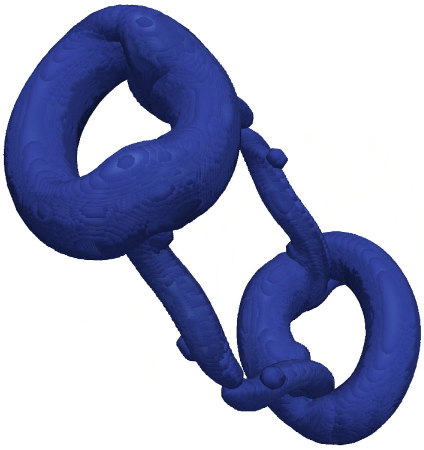
M. Hellwig (IEK-4)

[B. Steinbusch (JSC), in progress]

Member of the Helmholtz-Association

Vortex fluids

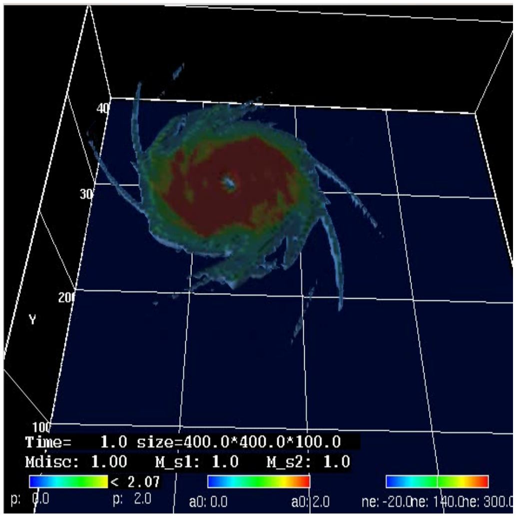
- Mathematical equivalence between Navier-Stokes vortex equations and magnetostatics
- Vortex particles must overlap $\rightarrow$ remeshing
- Multipole expansion of *smoothed* vorticity kernel



[Robert Speck, PhD thesis, U. Wuppertal]

Self-gravitating accretion discs

- Planet formation & disc dynamics with first principles particle simulation (SPH + gravity)
- nearest neighbour search kernel
- Here: $M_{\text{disc}} = 0.5 M_{\odot}$
- Observed: $M_{\text{disc}} = 0.01 - 0.1 M_{\odot}$; $M_E = 10^{-4} M_{\text{disc}}$



[Andreas Breslau, Susanne Pfalzner, MPI Radioastronomie Bonn]

Contributors

- Paul Gibbon (Head of group; JSC): first parallel implementation at JSC (2003)
- Michael Hofmann, (U. Chemnitz): parallel key sort
- M. W. (JSC): hybrid MPI-Pthread algorithm; periodic boundaries; strongly-coupled plasmas
- Benedikt Steinbusch (JSC): hybrid scaling, performance, boundary element method, wall-plasma interactions
- Dirk Brömmel (SLPP; JSC): novel architectures, accelerators, GPU
- Christian Salmagne (IEK-4): wall-plasma interactions
- Andreas Breslau (MPIfR Bonn): near-neighbour search; SPH; protoplanetary discs

- Robert Speck (JSC, U. Lugano): vortex model
- Lukas Arnold (JSC, now U. Wuppertal): novel architectures, performance
- Helge Hübner (JSC, now U. Hamburg): Hilbert curve, memory analysis, branch node optimizations
- Benjamin Berberich (IEK-4, now Carl-Zeiss): gyrokinetics; 3rd-party collisions; wall-plasma interactions

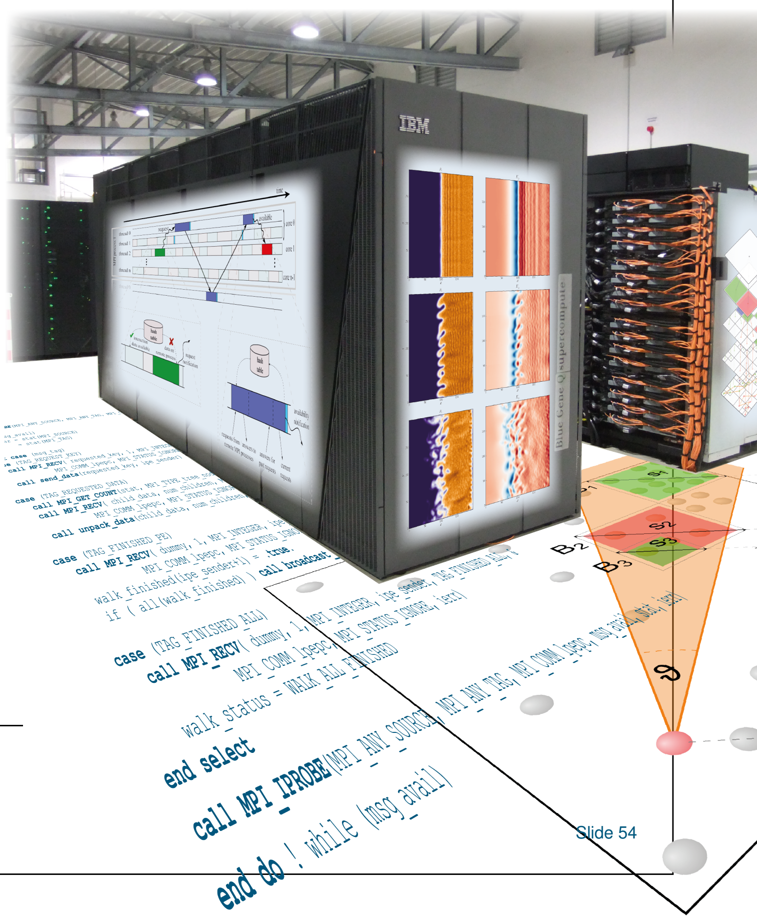
Further reading

- M. Winkel, R. Speck, H. Hübner, L. Arnold, R. Krause, P. Gibbon:
  A massively parallel, multi-disciplinary Barnes-Hut tree code for extreme-scale N-body simulations
  Comp. Phys. Commun. (2012).
- R. Speck, L. Arnold, P. Gibbon:
  Scaling and Efficiency of the PEPC Library
  J. Comp. Sci., **2**, 138 (2011).
- P. Gibbon, R. Speck, L. Arnold, M. Winkel, H. Hübner:
  Parallel Tree Codes,
  in Fast Methods for Long-Range Interactions in Complex Systems, Summer School, 6-10 September 2010, Jülich.
- P. Gibbon, R. Speck, A. Karmakar, L. Arnold, W. Frings, B. Berberich, D. Reiter, M. Mašek:
  Progress in Mesh-Free Plasma Simulation with Parallel Tree Codes
  IEEE Trans. Plasma Sci. **38**, 2367-2376, (2010).
- S. Pfalzner, P. Gibbon:
  Many Body Tree Methods in Physics,
  Cambridge University Press, New York (September 2005), ISBN 0-521-01916-8.

Thank you...

...for your attention.

Questions?



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