

Electromagnetic Structure of the $Z_c(3900)$

E. Wilbring^a, H.-W. Hammer^a, U.-G. Meißner^{a,b}

^aHelmholtz–Institut für Strahlen- und Kernphysik (Theorie) and Bethe Center for Theoretical Physics, Universität Bonn, D-53115 Bonn, Germany

^bInstitut für Kernphysik (IKP-3), Institute for Advanced Simulation (IAS-4) and Jülich Center for Hadron Physics, Forschungszentrum Jülich, D-52425 Jülich, Germany

Abstract

The observation of the exotic quarkonium state $Z_c(3900)$ by the BESIII and Belle collaborations supports the concept of hadronic molecules. Charmonium states interpreted as such molecules would be bound states of heavy particles with small binding energies. This motivates their description using an effective theory with contact interactions. In particular, we focus on the electromagnetic structure of the charged state $Z_c(3900)$. Using first experimental results concerning spin and parity, we interpret it as an S -wave molecule and calculate the form factors as well as charge and magnetic radii up to next-to-leading order. We also present first numerical estimations of some of these observables at leading order.

1. Introduction

In 2013 the BESIII and Belle collaborations reported the observation of a charged state in the charmonium sector called $Z_c(3900)$ [1, 2] (this state was also seen at CLEO [3]). The measured mass and width are

$$M_Z = (3899.0 \pm 8.5) \text{ MeV} \quad \Gamma_Z = (46 \pm 30) \text{ MeV} . \quad (1)$$

Soon after its discovery it was proposed that these states are S -wave hadronic molecules [4, 5]. Later a further analysis in this framework and different approaches for the substructure like tetraquarks were presented in Ref. [6]. In this work, we will assume that the Z_c is indeed a hadronic molecule. A comparison with combinations of charm mesons shows that this states lies close to a specific threshold and thus favors this interpretation. Denoting the wave function with the respective particle name, one can identify their main component to be [4, 6]

$$Z_c = \frac{1}{\sqrt{2}} (\bar{D}D^* + D\bar{D}^*) . \quad (2)$$

First, the constituents are charm mesons whose masses are far above 1 GeV and second, their binding energy is rather small, i.e. below the pion mass m_π or at least of that order. Hence, for the description of these states one can use in a first approximation a pionless effective field theory where the constituents are treated as non-relativistic, point-like particles which only interact via contact interactions. Such an short-range effective theory is called EFT($\not{\pi}$) and is a commonly used tool in nuclear physics [7–10]. Its expansion parameter is Q/m_π , where Q is the scale of the modulus p of the internal momenta \mathbf{p} of the involved particles which are - according to the small binding energy of the molecule, $|B| \lesssim 30 \text{ MeV}$ - below the pion mass m_π . We already remark here that the characteristics of the Z_c are such that EFT($\not{\pi}$) is barely applicable, but it still allows for some first insights into the electromagnetic structure of this exotic particle.

In particular, using the charm meson masses from Ref. [11] the (averaged) binding energy B can be obtained as

$$\begin{aligned} B &= \frac{1}{2} [(M_+ + M_{*0} - M_Z) + (M_0 + M_{*+} - M_Z)] \\ &= (-23.13 \pm 6.01) \text{ MeV} . \end{aligned} \quad (3)$$

The negative value implies a resonance instead of a charm meson bound state solution. Neglecting the imaginary part of the binding momentum γ (cf. Sec. 2), we treat the $Z_c(3900)$ as a virtual state with negative scattering length. The binding momentum obtained from the extracted binding energy, Eq. (3), is

$$\gamma = (-211.6 \pm 27.5) \text{ MeV} , \quad (4)$$

where the uncertainty is entirely from the one in the binding energy. In leading order, the electromagnetic properties are independent of the sign of γ . Our EFT expansion parameter is Q/m_π and since the binding momentum γ should be a low-energy scale proportional to Q to provide a proper counting scheme, one concludes that $|\gamma|$ must be smaller than $m_\pi \approx 140 \text{ MeV}$. This is not the case for our estimate of γ in Eq. (4). However, one can use the predictions for the Z_c as first, *rough* model estimations and we will do so in the following. Our predictions can be made more quantitative by explicitly including pions in the theory.

For the charm molecule Z_c the analysis of its decay channel shows that the quantum numbers $I(J^P) = 1(1^+)$ are favored [1, 4], which agrees with the S -wave hypothesis. Thus the Z_c can be described by an effective field theory with S -wave contact interactions similar to the loosely-bound charm molecule $X(3872)$ [12–14]. The structure of the Z_c state can then be calculated analog to the deuteron treated in EFT($\not{\pi}$) as a baryonic molecule of nucleons in a relative S -wave with total spin $J = 1$ [15–18]. Besides this analogy, there is also a conceptual connection to the so-called Halo EFT developed in Refs. [19, 20] and in particular to its application in Ref. [21].

2. Formalism

Following Refs. [14, 15, 21], one can write the effective Lagrangian up to next-to-leading order (NLO) for the positively charged Z_c as

$$\begin{aligned}
\mathcal{L} = & (D^+)^\dagger \left(i\partial_t + eA_0 + \frac{(\nabla + ie\mathbf{A})^2}{2M_+} \right) D^+ \\
& + (\bar{D}^0)^\dagger \left(i\partial_t + \frac{\nabla^2}{2M_0} \right) \bar{D}^0 \\
& + (D^{*+})^\dagger \left(i\partial_t + eA_0 + \frac{(\nabla + ie\mathbf{A})^2}{2M_{*+}} \right) D^{*+} \\
& + (\bar{D}^{*0})^\dagger \left(i\partial_t + \frac{\nabla^2}{2M_{*0}} \right) \bar{D}^{*0} \\
& + Z^\dagger \left[\eta \left(i\partial_t + eA_0 + \frac{(\nabla + ie\mathbf{A})^2}{2M_Z} \right) + \Delta \right] Z \\
& - g \left[Z^\dagger (D^+ \bar{D}^{*0} + D^{*+} \bar{D}^0) + (D^+ \bar{D}^{*0} + D^{*+} \bar{D}^0)^\dagger Z \right] \\
& - \mu_+(D^+)^\dagger \mathbf{U} \cdot \mathbf{B} D^+ - \mu_{*+}(D^{*+})^\dagger \mathbf{U} \cdot \mathbf{B} D^{*+} \\
& - \mu_Z Z^\dagger \mathbf{U} \cdot \mathbf{B} Z. \tag{5}
\end{aligned}$$

This Lagrangian and our formalism are generic for non-relativistic particles. The kinetic terms for the D and D^* mesons simply reproduce the free Schrödinger equation which is linear in the time derivative. Moreover, Z is an auxiliary field describing the molecular state $Z_c(3900)$, g is the coupling constant for ZDD^* interactions, Δ is a constant and $\eta = \pm 1$ is a phase whose sign depends on the effective range.

We stress that the Z_c is not treated as a dynamical degree of freedom. We use the "dimeron" formalism of Kaplan [22] which is convenient to treat non-perturbative contact interactions. If the auxiliary field is integrated out, the Lagrangian (5) reduces to one with contact interactions between the D and D^* mesons. If the phase η is negative, the Z field is a ghost but the scattering amplitude for the D and D^* mesons is unitary and sensible. This method is well established in effective field theory treatments of resonant interactions in nuclear and particle physics [23].

Note also that the spin structure of the D^* and Z_c is hidden in Eq. (5) since the interactions are spin-independent. Since there are only S -wave interactions, the spin state of the Z_c is always equal to the spin state of the constituent D^* and remains unchanged in the ZDD^* interaction vertex.

Minimal substitution with electric charge e induces an A_0 interaction with the constituents and the molecule itself, leading to the Feynman diagrams which contribute to the charge form factor. The magnetic form factor instead comes from the terms in the last line of Eq. (5), where $\mathbf{B} = \nabla \times \mathbf{A}$ is the magnetic field and \mathbf{U} is the spin-1 generalization of the Pauli vector for spin-1/2 particles, with $(U_k)_{jl} = -i\epsilon_{kjl}$ being the generators of the rotation group. Those terms are proportional to the magnetic moment μ of the respective charged particle and describe purely magnetic interactions of the spatial part of the vector potential A_i with both the charged D mesons and Z_c . Other contributions to the Lagrangian, for instance those relevant for the quadrupole



Figure 1: Full Z propagator (thick solid line) as sum over charm meson loops. The double solid line represents the bare Z propagator, the single solid line stands for D^* mesons, the dashed one for D mesons and the dots represent higher order terms.

form factor or effects due to the anomalous magnetic moment of neutral D mesons, are at least one order higher and thus not shown. Finally, we note that the Z_c binding energy scale of order 20 MeV is small compared to the D - D^* mass splitting of about 140 MeV. Our Lagrangian is only valid in a small energy region close to the Z_c mass and thus heavy quark symmetry is not relevant.

The auxiliary field Z is not dynamical but due to its coupling to the constituents it is dressed by D meson loops, see Fig. 1. This leads to a full Z propagator

$$iS_Z = \frac{i}{(S_Z^0)^{-1} - \Sigma}, \tag{6}$$

in terms of the bare propagator iS_Z^0 and the self-energy Σ . To avoid an unnatural scaling of the scattering length (similar to that discussed in Ref. [24]), one calculates the self-energy in the PDS scheme introduced in Refs. [25, 26]; see Ref. [27] for an alternative solution to this problem. In terms of the PDS scale Λ_{PDS} and the mean reduced mass $\omega_Z = \frac{1}{2}(\frac{M_+M_{*0}}{M_+M_{*0}} + \frac{M_0M_{*+}}{M_0M_{*+}})$ of the molecule, one obtains

$$\Sigma = -\frac{\omega_Z g^2}{\pi} \left[i \sqrt{2\omega_Z \left(p_0 - \frac{p^2}{M_Z} \right) + i\epsilon + \Lambda_{\text{PDS}}} \right]. \tag{7}$$

From the full propagator of the Z field at NLO,

$$\begin{aligned}
iS_Z(p_0, \mathbf{p}) = & \frac{\pi i}{\omega_Z g^2} \left[\frac{\pi \Delta}{\omega_Z g^2} + \frac{\pi \eta}{\omega_Z g^2} \left(p_0 - \frac{p^2}{2M_Z} \right) \right. \\
& \left. + \Lambda_{\text{PDS}} + i \sqrt{2\omega_Z \left(p_0 - \frac{p^2}{2M_Z} \right) + i\epsilon} \right]^{-1}, \tag{8}
\end{aligned}$$

one can deduce the scattering length a and the effective range r_0 of the Z_c by matching the scattering amplitude $-iT = (-ig)^2 iS_Z(E = k^2/(2\omega_Z), 0)$ to the effective range expansion (ERE)

$$T_{\text{ERE}}^{(2)} = \frac{\pi}{\omega_Z} \frac{1}{\frac{1}{a} - \frac{1}{2}r_0 k^2 + ik}. \tag{9}$$

At NLO the following relations are obtained:

$$\frac{1}{a} = \frac{\pi \Delta}{\omega_Z g^2} + \Lambda_{\text{PDS}}, \tag{10}$$

$$r_0 = -\frac{\pi \eta}{\omega_Z^2 g^2}. \tag{11}$$

In addition this result fixes the phase introduced in the Lagrangian Eq. (5). To get a positive value for the effective range, the phase η must be equal to minus one.

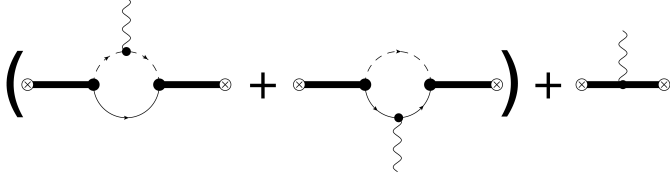


Figure 2: Diagrams contributing to the charge and magnetic form factor of the Z_c at NLO. While the left diagrams in parenthesis are LO, the right diagram is the NLO contribution. The thick solid line represents the full Z propagator, the thin solid line stands for D^* and the dashed line for D mesons. The incoming photons are indicated by wavy lines and the application of wave function renormalization factors is indicated by the crosses.

Furthermore, we calculate the wave function renormalization constant Z defined as the residue of the pole of the propagator in Eq. (8). At NLO one finds

$$Z_{NLO} = \frac{\pi\gamma}{\omega_Z^2 g^2} \frac{1}{1 - r_0\gamma}, \quad (12)$$

where the Z_c binding momentum $\gamma = \text{sgn}(\mathbf{B})\sqrt{2\omega_Z|\mathbf{B}|}$ was introduced whose definition is chosen such that one takes care of both, bound *and* virtual states.

The non-relativistic matrix elements of the electromagnetic current J^μ can be expanded in terms of form factors. For a vector particle state $|\mathbf{p}, \varepsilon_i\rangle$ with three momentum \mathbf{p} and linear polarization vector ε_i , one can write in the Breit-frame[15, 21]

$$\langle \mathbf{p}', \varepsilon_j | J^0 | \mathbf{p}, \varepsilon_i \rangle = -ie \left[G_c(q) \delta_{ij} + \frac{1}{2M^2} G_q(q) \left(q_i q_j - \frac{q^2 \delta_{ij}}{n-1} \right) \right], \quad (13)$$

$$\langle \mathbf{p}', \varepsilon_j | J^k | \mathbf{p}, \varepsilon_i \rangle = -i \frac{e}{2M} \left[G_m(q) (\delta_j^k q_i - \delta_i^k q_j) \right]. \quad (14)$$

Here, $\mathbf{q} = \mathbf{p}' - \mathbf{p}$ is the three-momentum transfer with $q_\mu q^\mu = q_0^2 - q^2$ and n is the number of space-time dimensions (here $n = 4$). The charge (G_c), magnetic (G_m) and quadrupole (G_q) form factors are normalized in the following way [28]:

$$\begin{aligned} G_c(q=0) &= 1, \\ \frac{e}{2M} G_m(q=0) &= \mu, \\ \frac{1}{M^2} G_q(q=0) &= \kappa, \end{aligned} \quad (15)$$

in terms of the magnetic moment μ of the particle and its quadrupole moment κ .

According to Eq. (13), the charge form factor is determined by the diagrams shown in Fig. 2 in which the incoming photons are A_0 photons and where the initial and final spin of the Z_c are equal. The charge form factor at NLO of the Z_c is then given by

$$\begin{aligned} G_c^{(NLO)}(q) &= \frac{1}{1 - \gamma r_0} \left[\frac{M_+ \gamma}{q\omega_Z} \arctan\left(\frac{q\omega_Z}{2M_+ \gamma}\right) \right. \\ &\quad \left. + \frac{M_{**} \gamma}{q\omega_Z} \arctan\left(\frac{q\omega_Z}{2M_{**} \gamma}\right) - \gamma r_0 \right]. \end{aligned} \quad (16)$$

This is indeed correctly normalized to one. A second observable besides the form factor itself is the expectation value of the squared charge radius $\langle r_c^2 \rangle$. It can be found from the low-energy expansion of the form factor, $G_c(q) = 1 - \langle r_c^2 \rangle q^2 / 6 + \mathcal{O}(q^4)$. At NLO one finds

$$\langle r_c^2 \rangle = \frac{1}{1 - \gamma r_0} \frac{\omega_Z^2}{4\gamma^2} \left(\frac{1}{M_+^2} + \frac{1}{M_{**}^2} \right). \quad (17)$$

If the incoming photons in Fig. 2 are A_i photons, one can choose the polarizations in Eq. (14) and calculate the amplitude of the corresponding diagrams to get the magnetic form factor G_m of the Z_c . At NLO it is given by

$$\begin{aligned} G_m^{(NLO)}(q) &= \frac{2M_Z}{e(1 - \gamma r_0)} \left[\frac{\mu_+ M_+ \gamma}{2 q\omega_Z} \arctan\left(\frac{q\omega_Z}{2M_+ \gamma}\right) \right. \\ &\quad \left. + \frac{\mu_{**} M_{**} \gamma}{2 q\omega_Z} \arctan\left(\frac{q\omega_Z}{2M_{**} \gamma}\right) - \gamma r_0 \mu_Z \right]. \end{aligned} \quad (18)$$

So the difference to the charge form factor are the magnetic moments in front of each term. With the normalization condition in Eq. (15), it is possible to identify a relation between the magnetic moment of the molecule Z_c and that of its charged constituents D^+ and D^{*+}

$$\mu_Z = \frac{1}{4} (\mu_+ + \mu_{**}). \quad (19)$$

In addition, one can determine the expectation value of the squared magnetic radius $\langle r_m^2 \rangle$ by expanding the magnetic form factor up to second order in q . At NLO one finds

$$\langle r_m^2 \rangle = \frac{1}{1 - \gamma r_0} \frac{\omega_Z^2}{4\gamma^2} \left[\frac{1}{\mu_+ + \mu_{**}} \left(\frac{\mu_+}{M_+} + \frac{\mu_{**}}{M_{**}} \right) \right]. \quad (20)$$

According to Eq. (13), the quadrupole form factor is determined by A_0 photon interactions in Fig. 2 with a spin change between the initial and the final state. The A_0 interaction itself cannot induce such a spin change because A_0 is just the time-like component of the 4-vector potential A_μ . Thus one needs additional operators in the Lagrangian which project on specific polarization states ε_i . According to Ref. [15] such operators first appear at NLO and thus the quadrupole form factor $G_q(q)$ vanishes at leading order.

3. Results

Except for the masses, widths and $I(J^P)$ there are no experimental data available for the $Z_c(3900)$, hence one can predict observables only at LO. Furthermore, the leading order magnetic form factor and the magnetic radius are proportional to the magnetic moments of the constituent mesons. As these are experimentally unknown both are not accessible even at LO. Thus only the charge form factor and the charge radius at LO can be predicted.

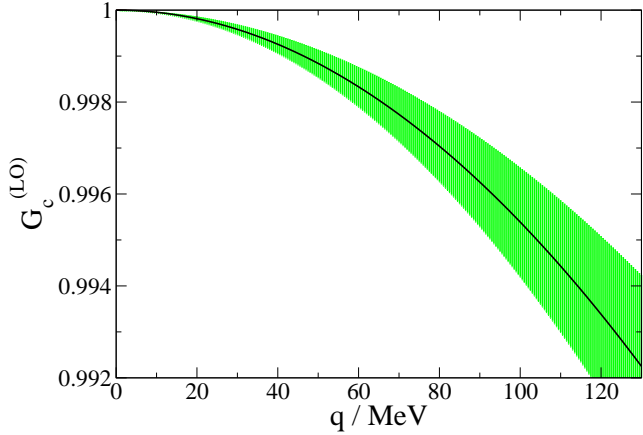


Figure 3: LO charge form factor $G_c^{(LO)}(q)$ of the $Z_c(3900)$. The error band includes only the uncertainty in the binding energy and is depicted in green.

The results for the scattering length and charge radius of the Z_c obtained from the extracted binding momentum, Eq. (4) are

$$\begin{aligned} a &= (-0.93 \pm 0.12) \text{ fm} , \\ \langle r_c^2 \rangle &= (0.11 \pm 0.03) \text{ fm}^2 , \end{aligned} \quad (21)$$

where the uncertainties are entirely from the one in the binding energy, Eq. (3). Due to the large expansion parameter, we refrain from estimating the errors due to higher orders. We reiterate that our predictions for the Z_c should thus be considered model estimates. Also, note that the charge radius in Eq. (21) should be added in quadrature the charge radii of the constituent $D^{(*)}$ mesons which are treated as pointlike in our theory.

The charge form factor of the Z_c at LO is shown in Fig. 3. Furthermore, there is a correlation between the leading order charge radius and the binding energy as shown in Fig. 4.

4. Summary

In this work the electromagnetic structure of the exotic meson $Z_c(3900)$ interpreted as an S -wave charm meson molecule with spin $J = 1$ was investigated in the framework of EFT($\not\Lambda$). Similar to the description of the deuteron in this effective theory, the charge, magnetic and quadrupole form factor were formally analyzed up to NLO. In particular, the charge and magnetic radius were obtained. Due to the poor experimental data on the Z_c state only the LO charge radius and charge form factor could be estimated. Furthermore, we derived a relation between the magnetic moment of the molecule and of its constituents, see Eq. (19). Given the experimental mass of the $Z_c(3900)$, the binding momentum of the $Z_c(3900)$ is a bit too large to allow for a quantitative description of its properties in EFT($\not\Lambda$). Thus our results should be viewed as as first, *rough* model estimations of the electromagnetic properties of the $Z_c(3900)$. To sharpen

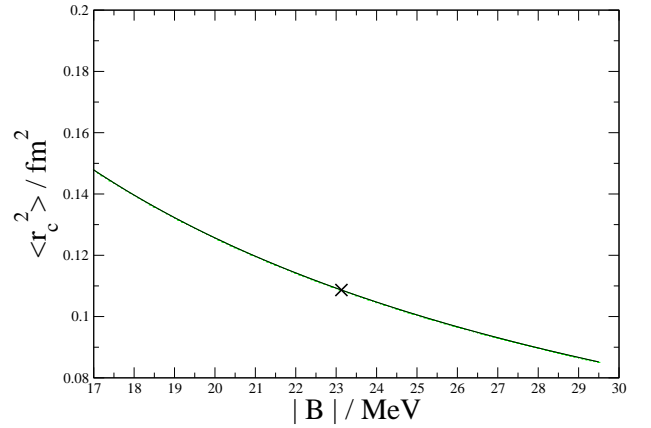


Figure 4: Correlation between the expectation value of the squared charge radius $\langle r_c^2 \rangle$ and the modulus of the binding energy B of the $Z_c(3900)$ at LO. The correlation is shown for binding energies within the errors of Eq. (3). The central value Eq. (21) is represented by the cross and the error band is depicted in green.

these predictions, one has to go beyond the pionless EFT and include explicit pions.

Acknowledgments This work is supported in part by the EU Integrated Infrastructure Initiative HadronPhysics3 and the DFG through funds provided to the Sino-German CRC 110 ‘‘Symmetries and the Emergence of Structure in QCD’’.

References

- [1] M. Ablikim *et al.* [BESIII Collaboration], arXiv:1303.5949 [hep-ex].
- [2] Z. Q. Liu *et al.* [Belle Collaboration], arXiv:1304.0121 [hep-ex].
- [3] T. Xiao, S. Dobbs, A. Tomaradze and K. K. Seth, arXiv:1304.3036 [hep-ex].
- [4] Q. Wang, C. Hanhart and Q. Zhao, arXiv:1303.6355 [hep-ph].
- [5] F. -K. Guo, C. Hidalgo-Duque, J. Nieves and M. P. Valderrama, arXiv:1303.6608 [hep-ph].
- [6] M. B. Voloshin, arXiv:1304.0380 [hep-ph].
- [7] U. van Kolck, Prog. Part. Nucl. Phys. **43**, 337 (1999) [nucl-th/9902015].
- [8] S. R. Beane, P. F. Bedaque, W. C. Haxton, D. R. Phillips and M. J. Savage, In *Shifman, M. (ed.): At the frontier of particle physics, vol. 1* 133-269 [nucl-th/0008064].
- [9] P. F. Bedaque and U. van Kolck, Ann. Rev. Nucl. Part. Sci. **52**, 339 (2002) [nucl-th/0203055].
- [10] E. Epelbaum, H.-W. Hammer and U.-G. Meißner, Rev. Mod. Phys. **81**, 1773 (2009) [arXiv:0811.1338 [nucl-th]].
- [11] J. Beringer *et al.* [Particle Data Group Collaboration], Phys. Rev. D **86**, 010001 (2012).
- [12] E. Braaten and M. Kusunoki, Phys. Rev. D **69**, 074005 (2004) [hep-ph/0311147].
- [13] M. T. AlFiky, F. Gabbiani and A. A. Petrov, Phys. Lett. B **640**, 238 (2006) [hep-ph/0506141].
- [14] D. L. Canham, H.-W. Hammer and R. P. Springer, Phys. Rev. D **80**, 014009 (2009) [arXiv:0906.1263 [hep-ph]].
- [15] J. -W. Chen, G. Rupak and M. J. Savage, Nucl. Phys. A **653**, 386 (1999) [nucl-th/9902056].
- [16] D. R. Phillips, Czech. J. Phys. **52**, B49 (2002) [nucl-th/0203040].
- [17] S. R. Beane and M. J. Savage, Nucl. Phys. A **694**, 511 (2001) [nucl-th/0011067].

- [18] D. B. Kaplan, M. J. Savage and M. B. Wise, Phys. Rev. C **59**, 617 (1999) [nucl-th/9804032].
- [19] C. A. Bertulani, H.-W. Hammer and U. Van Kolck, Nucl. Phys. A **712**, 37 (2002) [nucl-th/0205063].
- [20] P. F. Bedaque, H.-W. Hammer and U. van Kolck, Phys. Lett. B **569**, 159 (2003) [nucl-th/0304007].
- [21] H.-W. Hammer and D. R. Phillips, Nucl. Phys. A **865**, 17 (2011) [arXiv:1103.1087 [nucl-th]].
- [22] D. B. Kaplan, Nucl. Phys. B **494**, 471 (1997) [nucl-th/9610052].
- [23] H.-W. Hammer and L. Platter, Ann. Rev. Nucl. Part. Sci. **60**, 207 (2010) [arXiv:1001.1981 [nucl-th]].
- [24] D. B. Kaplan, M. J. Savage and M. B. Wise, Nucl. Phys. B **478**, 629 (1996) [nucl-th/9605002].
- [25] D. B. Kaplan, M. J. Savage and M. B. Wise, Phys. Lett. B **424**, 390 (1998) [nucl-th/9801034].
- [26] D. B. Kaplan, M. J. Savage and M. B. Wise, Nucl. Phys. B **534**, 329 (1998) [nucl-th/9802075].
- [27] U. van Kolck, Nucl. Phys. A **645**, 273 (1999) [nucl-th/9808007].
- [28] M. J. Zuilhof and J. A. Tjon, Phys. Rev. C **22**, 2369 (1980).